# Enhanced Lumped Parameter Model for Photolithography

Chris A. Mack
FINLE Technologies
Austin, TX 78716

*Abstract*

Enhancements to the lumped parameter model for semiconductor optical lithography are introduced. These enhancements allow the lumped parameter to calculate resist sidewall angle as well as resist linewidth in an approximate but extremely fast manner. The model shows the two main contributors to resist slope: development effects due to the time required for the developer to reach the bottom of the photoresist, and absorption effects resulting in a reduced exposure at the bottom of the resist.

## I. Introduction

Modeling is an increasingly important tool in the study of optical lithography. Most efforts in lithographic model development involve primary parameter models, in which the effects of the primary physical parameters of the process are described based on first principles. The goal of primary parameter simulation is to accurately and rigorously describe and predict a wide variety of lithographic phenomena. Another class of models, called lumped parameter models, uses a relatively small set of simple, lumped parameters to describe one or two important effects in lithography. The simplicity of these models is, of course, their advantage and their disadvantage. However, in cases where approximate results are adequate and quick (possibly real time) calculations are required, lumped parameter models are the best choice.

In this paper, significant enhancements to Hershel and Mack's Lumped Parameter Model (LPM) [1,2] will be introduced. The original LPM took as its input an aerial image of a one-dimensional mask feature through focus and two lumped parameters, the resist contrast and the effective resist thickness, and predicted the resulting focus-exposure process window and Bossung curves (at speeds 2 to 4 orders of magnitude faster than primary parameter models). The enhanced LPM will add the calculation of resist sidewall angle by the addition of a third lumped parameter, the effective resist absorption. This sidewall angle output, which will include both absorption and development time effects on sidewall angle, will then be used to create an overlapping sidewall angle process window along with the resist linewidth process window.

0-8194-1492-1/94/$6.00

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

## II A Simple Development Rate Model

The mathematical description of the resist process incorporated in the lumped parameter model uses a simple photographic model relating development time to exposure, while the image simulation is derived from the standard optical parameters of the lithographic tool. A very simple development rate model is used based on the assumption of a constant contrast. Before proceeding, however, let us define a few terms needed for the derivations that follow. Let $E$ be the nominal exposure energy (i.e., the intensity in a large clear area times the exposure time), $I(x)$ the normalized image intensity, and $I(z)$ the relative intensity variation with depth into the resist. It is clear that the exposure energy as a function of position within the resist ($E_{xz}$) is just $E\, I(x)I(z)$ where $x$=0 is the center of the mask feature and $z$=0 is the top of the resist. Defining logarithmic versions of these quantities,

$$\varepsilon = \ln[E]\,, \quad i(x) = \ln[I(x)]\,, \quad i(z) = \ln[I(z)] \qquad (1)$$

and the logarithm of the energy deposited in the resist is

$$\ln[E_{xz}] = \varepsilon + i(x) + i(z) \qquad (2)$$

The photoresist contrast ($\gamma$) is defined theoretically as [3]

$$\gamma \equiv \frac{d \ln r}{d \ln E_{xz}} \qquad (3)$$

where $r$ is the resulting development rate from an exposure of $E_{xz}$. Note that the base e definition of contrast is used here. If the contrast is assumed constant over the range of energies of interest, equation (3) can be integrated to give a very simple expression for development rate. In order to evaluate the constant of integration, let us pick a convenient point of evaluation. Let $\varepsilon_o$ be the energy required to just clear the photoresist in the allotted development time, $t_{dev}$, and let $r_o$ be the development rate which results from an exposure of the this amount. Carrying out the integration gives

$$r(x,z) = r_o\, e^{\gamma(\varepsilon + i(x) + i(z) - \varepsilon_o)} = r_o \left[\frac{E_{xz}}{E_o}\right]^{\gamma} \qquad (4)$$

As an example of the use of the above development rate expression and to further illustrate the relationship between $r_o$ and the dose to clear, consider the standard dose to clear experiment where a large clear area is exposed and the thickness of photoresist remaining is measured. The definition of development rate,

$$r = \frac{dz}{dt} \qquad (5)$$

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

can be integrated over the development time. If $\varepsilon = \varepsilon_o$, the thickness remaining is by definition zero, so that

$$t_{dev} = \int_0^D \frac{dz}{r} = \frac{1}{r_o}\int_0^D e^{-\gamma\, i(z)}\, dz \qquad (6)$$

where *i(x)* is zero for an open frame exposure. Based on this equation, one can now define an effective resist thickness, $D_{eff}$, which will be very useful in the derivation of the lumped parameter model which follows.

$$D_{eff} = r_o t_{dev}\, e^{\gamma\, i(D)} = e^{\gamma\, i(D)} \int_0^D e^{-\gamma\, i(z)}\, dz = \int_0^D \left[\frac{I(z)}{I(D)}\right]^{-\gamma} dz \qquad (7)$$

## III. Segmented Development

Equation (4) is an extremely simple-minded model relating development rate to exposure energy based on the assumption of a constant resist contrast. In order to use this expression, we will develop a phenomenological explanation for the development process. This explanation will be based on the assumption that development occurs in two steps: a vertical development to a depth $z$, followed by a lateral development to position $x$ (measured from the center of the mask feature) as shown in Figure 1.

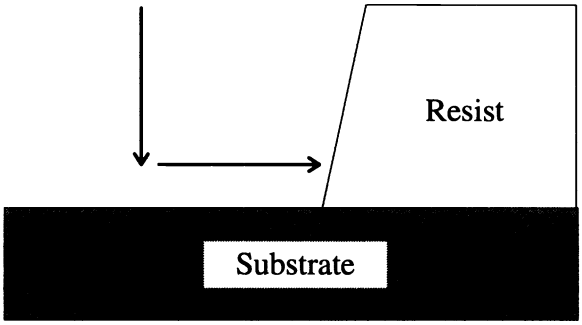

Figure 1. Illustration of segmented development: development proceeds first vertically, then horizontally, to the final resist sidewall.

A development ray, which traces out the path of development, starts at the point $(x_o,0)$ and proceeds vertically until a depth $z$ is reached such that the resist to the side of the ray has been exposed more than the resist below the ray. At this point the development will begin horizontally. The time needed to develop in both vertical and horizontal directions, $t_z$ and $t_x$

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

respectively, can be computed from equation (4). The development time per unit thickness of resist is just the reciprocal of the development rate.

$$\frac{1}{r(x,z)} = \tau(x,z) = \tau_o\, e^{-\gamma(\varepsilon + i(x) + i(z))} \tag{8}$$

where

$$\tau_o = \frac{1}{r_o} e^{\gamma \varepsilon_o} \tag{9}$$

The time needed to develop to a depth $z$ is given by

$$t_z = \tau_o\, e^{-\gamma\varepsilon}\, e^{-\gamma i(x_o)} \int_0^z e^{-\gamma\, i(z')} dz' \tag{10}$$

Similarly, the horizontal development time is

$$t_x = \tau_o\, e^{-\gamma\varepsilon}\, e^{-\gamma i(z)} \int_{x_o}^{x} e^{-\gamma\, i(x')} dx' \tag{11}$$

The sum of these two times must equal the total development time.

$$t_{dev} = \tau_o\, e^{-\gamma\varepsilon} \left[ e^{-\gamma i(x_o)} \int_0^z e^{-\gamma\, i(z')} dz' \;+\; e^{-\gamma i(z)} \int_{x_o}^{x} e^{-\gamma\, i(x')} dx' \right] \tag{12}$$

## IV. The Lumped Parameter Model

The above equation can be used to derive some interesting properties of the resist profile. For example, how would a small change in exposure energy $\Delta\varepsilon$ affect the position of the resist profile $x$? A change in overall exposure energy will not change the point at which the development ray changes direction. Thus, the depth $z$ is constant. Differentiating equation (12) with respect to log-exposure energy, the following equation can be derived:

$$\left.\frac{dx}{d\varepsilon}\right|_z = \frac{\gamma\, t_{dev}}{\tau(x,z)} = \gamma\, t_{dev}\, r(x,z) \tag{13}$$

Since the $x$ position of the development ray endpoint is just one half of the linewidth, equation (13) defines a change in critical dimension (CD) with exposure energy. To put this expression in a more useful form, take the log of both sides and use the development rate expression (4) to give

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

$$\ln\left(\frac{dx}{d\varepsilon}\right) = \ln\left(\gamma\, t_{dev}\, r_o\right) + \gamma\left(\varepsilon + i(x) + i(z) - \varepsilon_o\right) \tag{14}$$

Rearranging ,

$$\varepsilon = \varepsilon_o - i(x) - i(z) + \frac{1}{\gamma}\ln\left(\frac{dx}{d\varepsilon}\right) - \frac{1}{\gamma}\ln\left(\gamma\, t_{dev}\, r_o\right) \tag{15}$$

where $\varepsilon$ is the (log) energy needed to expose a feature of width $2x$. Equation (15) is the differential form of the lumped parameter model and relates the CD versus log-exposure curve and its slope to the image intensity. A more useful form of this equation is given below; however, some valuable insight can be gained by examining equation (15). In the limit of very large $\gamma$, one can see that the CD versus exposure curve becomes equal to the aerial image. Thus, exposure latitude becomes *image limited.* For small $\gamma$, the other terms become significant and the exposure latitude is *process limited.* Obviously, an image limited exposure latitude represents the best possible case.

A second form of the lumped parameter model can also be obtained in the following manner. Applying the definition of development rate to equation (13) or, alternatively, solving for the slope in equation (15), yields

$$\frac{d\varepsilon}{dx} = \frac{1}{\gamma\, t_{dev} r_o} e^{-\gamma\,(\varepsilon + i(x) + i(z) - \varepsilon_o)} \tag{16}$$

Before proceeding, let us introduce a slight change in notation that will make the role of the variable $\varepsilon$ more clear. As originally defined, $\varepsilon$ is just the nominal exposure energy. In equations (14) through (16), it takes the added meaning as the nominal energy which gives a linewidth of $2x$. To emphasize this meaning, we will replace $\varepsilon$ by $\varepsilon(x)$ where the interpretation is not a variation of energy with $x$, but rather a variation of $x$ (linewidth) with energy. Using this notation, the energy to just clear the resist can be related to the energy which gives zero linewidth.

$$\varepsilon_o = \varepsilon(0) + i(x = 0) \tag{17}$$

Using this relation in equation (16),

$$\frac{d\varepsilon}{dx} = \frac{1}{\gamma\, t_{dev} r_o} e^{-\gamma\, i(z)}\, e^{-\gamma\,(\varepsilon(0) - \varepsilon(x))}\, e^{-\gamma\,(i(0) - i(x))} \tag{18}$$

Invoking the definitions of the logarithmic quantities,

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

$$\frac{dE}{dx} = \frac{E(x)}{\gamma D_{eff}} \left[ \frac{E(0)I(0)}{E(x)I(x)} \right]^{\gamma} \tag{19}$$

where equation (7) has been used and the linewidth is assumed to be measured at the resist bottom (i.e., $z = D$). Equation (19) can now be integrated.

$$\int_{E(0)}^{E(x)} E^{\gamma-1} dE = \frac{1}{\gamma D_{eff}} [E(0)I(0)]^{\gamma} \int_{0}^{x} I(x')^{-\gamma} dx' \tag{20}$$

giving

$$\frac{E(x)}{E(0)} = \left[ 1 + \frac{1}{\gamma D_{eff}} \int_{0}^{x} \left( \frac{I(x')}{I(0)} \right)^{-\gamma} dx' \right]^{\frac{1}{\gamma}} \tag{21}$$

Equation (21) is the integral form of the lumped parameter model. Using this equation, one can generate a normalized CD vs. exposure curve by knowing the image intensity, $I(x)$, the effective resist thickness, $D_{eff}$, and the contrast, $\gamma$.

## V. Sidewall Angle

The lumped parameter model allows the prediction of linewidth by developing down to a depth $z$ and laterally to a position $x$, which is one half of the final linewidth. Typically, the bottom linewidth is desired so that the depth chosen is the full resist thickness. By picking different values for $z$, different $x$ positions will result, giving a complete resist profile. One important result that can be calculated is the resist sidewall slope and the resulting sidewall angle. To derive an expression for the sidewall slope, let us first rewrite equation (12) in terms of the development rate.

$$t_{dev} = \int_{0}^{z} \frac{dz'}{r(0,z')} + \int_{0}^{x} \frac{dx'}{r(x',z)} \tag{22}$$

Taking the derivative of this expression with respect to $z$,

$$0 = \int_{0}^{z} \frac{d\tau}{dz} dz' + \frac{1}{r(0,z)} + \int_{x_o}^{x} \frac{d\tau}{dz} dx' + \frac{1}{r(x,z)} \frac{dx}{dz} \tag{23}$$

The derivative of the reciprocal development rate can be calculated from equation (4) or (8),

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

$$\frac{d\tau}{dz} = -\gamma\,\tau(x,z)\frac{d\ln[E_{xz}]}{dz} \tag{24}$$

As one would expect, the variation of development rate with depth into the resist depends on the variation of the exposure dose with depth. Consider a simple example where bulk absorption is the only variation of exposure with $z$. For an absorption coefficient of $\alpha$, the result is

$$\frac{d\ln[E_{xz}]}{dz} = -\alpha \tag{25}$$

Using equations (24) and (25) in (23),

$$-\alpha\gamma\left(\int_0^z \tau\, dz' + \int_{x_o}^x \tau\, dx'\right) = \frac{1}{r(0,z)} + \frac{1}{r(x,z)}\frac{dx}{dz} \tag{26}$$

Recognizing the term in parentheses as simply the development time, the reciprocal of the resist slope can be given as

$$-\frac{dx}{dz} = \frac{r(x,z)}{r(0,z)} + \alpha\gamma\, t_{dev}\, r(x,z) = \frac{r(x,z)}{r(0,z)} + \alpha\frac{dx}{d\varepsilon} \tag{27}$$

Equation (27) shows two distinct contributors to sidewall angle. The first is the development effect. Because the top of the photoresist is exposed to developer longer than the bottom, the top linewidth is smaller resulting in a sloped sidewall. This effect is captured in equation (27) as the ratio of the development rate at the edge of the photoresist feature to the development rate at the center. Good sidewall slope is obtained making this ratio small. The second term in equation (27) describes the effect of optical absorption on the resist slope. High absorption or poor exposure latitude will result in a reduction of the resist sidewall angle.

The results of the lumped parameter model are shown in Figures 2-5. Figure 2 shows that by applying equation (21) for aerial images through focus, an entire focus-exposure matrix can be computed (taking only 1-2 seconds on a 486 PC computer). Figure 3 shows the resulting sidewall angle for this matrix predicted using equation (27). The results of Figures 2 and 3 can be plotted in contours resulting in the focus-exposure process window plot of Figure 4. Both a linewidth specification of ±10% and a sidewall angle specification of 80° can be plotted to give a window of acceptable focus and exposure values. Finally, by fitting rectangles of various heights and widths inside this process window, a plot of exposure latitude versus depth-of-focus is obtained as shown in Figure 5. This type of graph is extremely revealing, indicating the important trade-off between the requirements of exposure latitude and depth-of-focus.

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

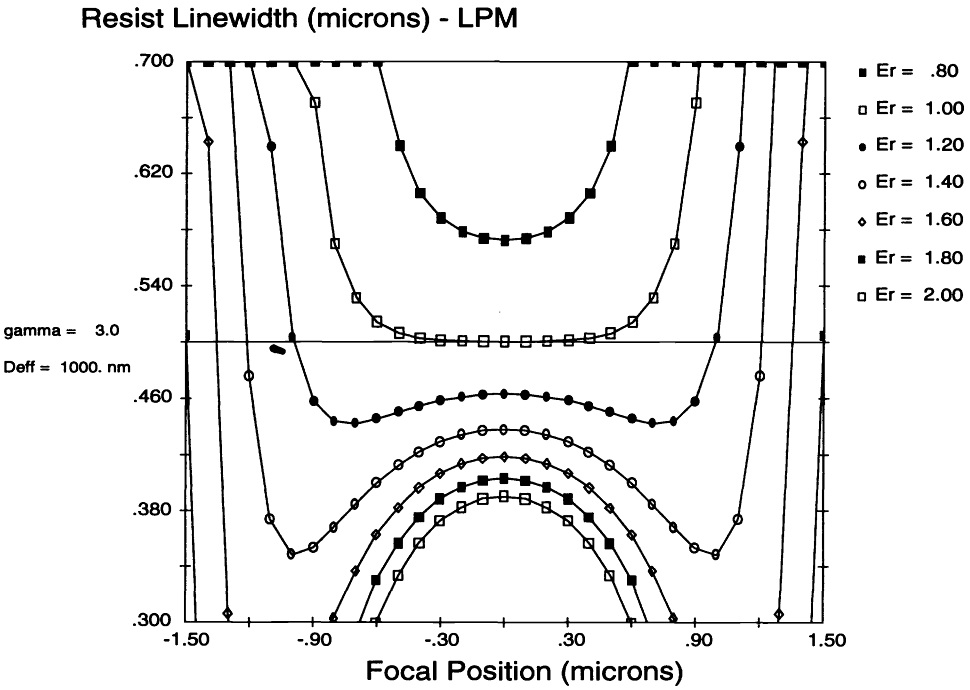

Figure 2. Bossung curves as predicted by the Lumped Parameter Model (0.5 μm lines/spaces for an *i*-line stepper, NA = 0.55, $\sigma = 0.45$, using the lumped parameters $\gamma = 3.0$ and $D_{eff} = 1.0$ μm).

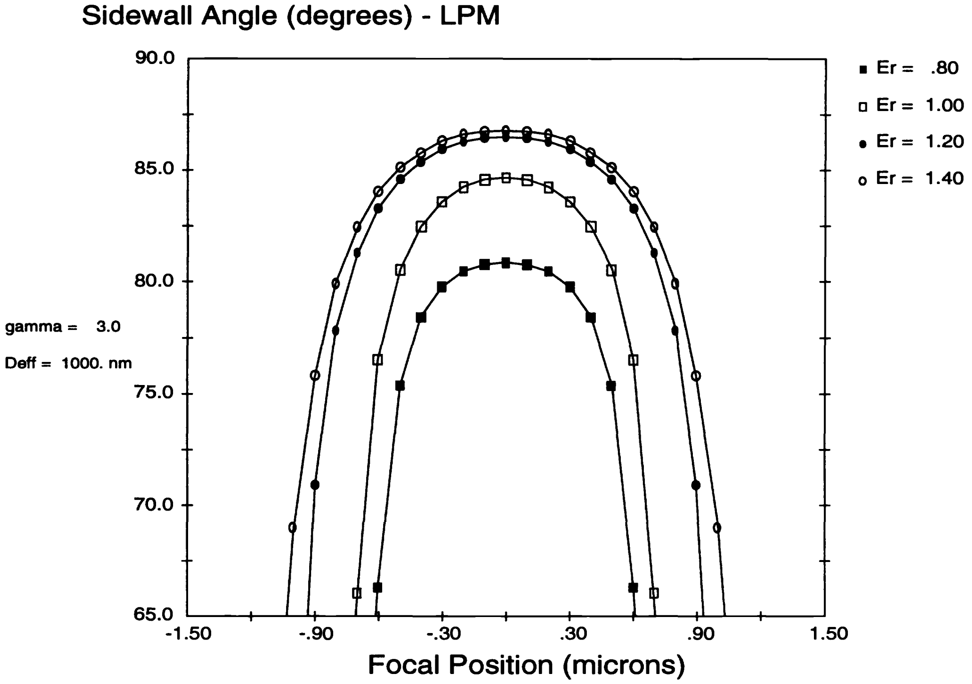

Figure 3. Sidewall angle versus focus and exposure as predicted by the Lumped Parameter Model (0.5 μm lines/spaces for an *i*-line stepper, NA = 0.55, $\sigma = 0.45$, using the lumped parameters $\gamma = 3.0$, $D_{eff} = 1.0$ μm, and an effective resist absorption coefficient $\alpha = 0.3\ \mu m^{-1}$).

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

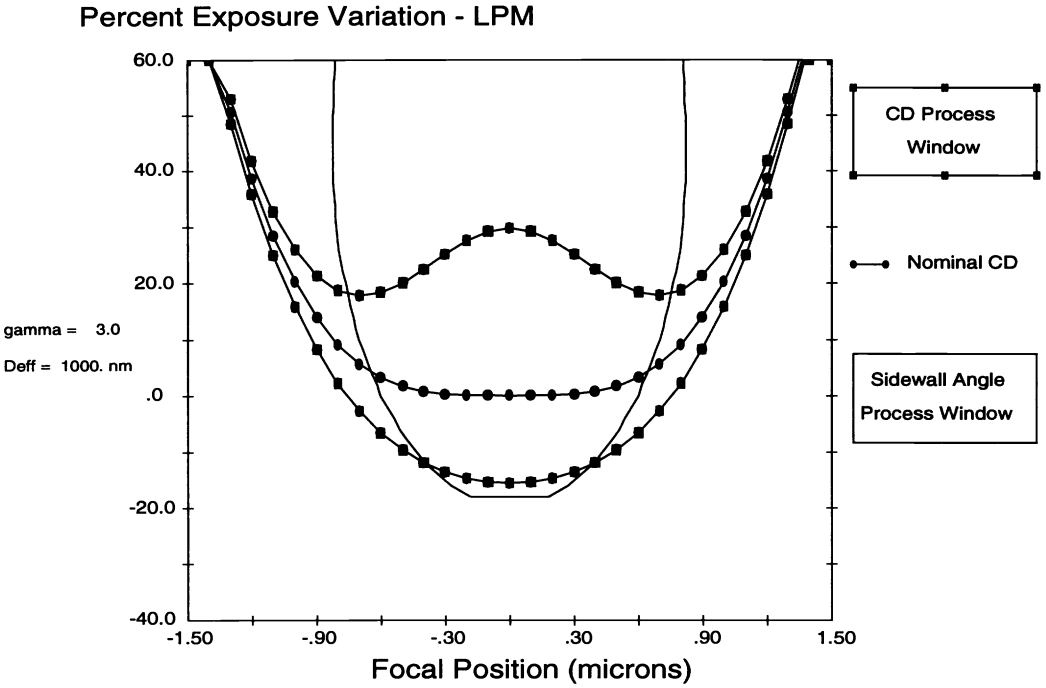

Figure 4. Process window resulting from the LPM predictions of Figures 1 and 2 for a linewidth specification of ±10% and a sidewall angle specification of 80°.

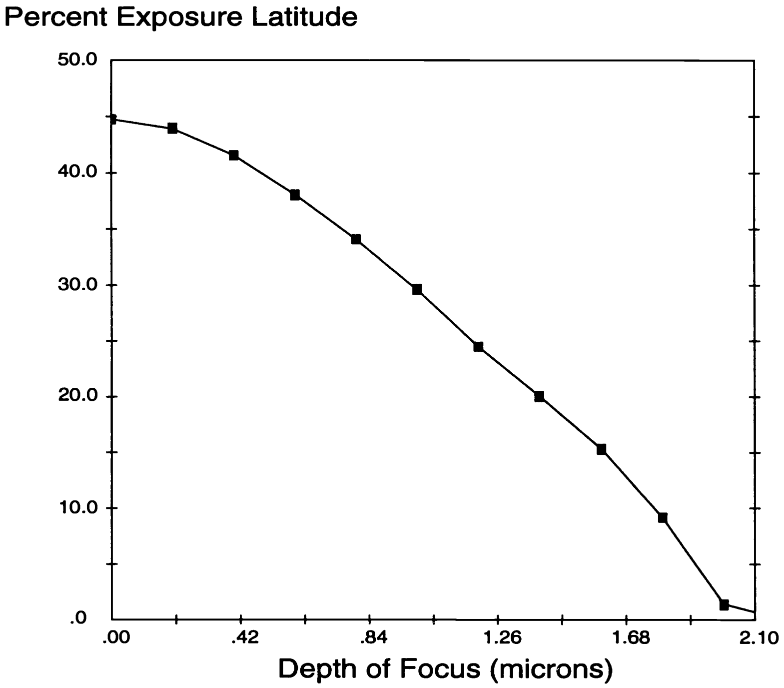

Figure 5. By fitting rectangles inside the linewidth process window of Figure 3, a plot of the maximum rectangle height (exposure latitude) for any given rectangle width (depth-of-focus) can be obtained.

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms

## VI Conclusions

The lumped parameter model is based on a simple model for development rate and a phenomenological description of the development process. The result is an equation which predicts the change in linewidth with exposure for a given aerial image. The major advantage of the lumped parameter model is its extreme ease of application to a lithography process. The two parameters of the model, resist contrast and effective thickness, can be determined by the collection of linewidth data from a standard focus-exposure matrix. This data is routinely available in most production and development lithography processes; no extra or unusual data collection is required. The result is a simple and fast model which can be used as an initial predictor of results or as the engine of a lithographic control scheme.

Additionally, this paper introduces the use of the lumped parameter model to predict the sidewall angle of the resulting photoresist profile. The model shows the two main contributors to resist slope: development effects due to the time required for the developer to reach the bottom of the photoresist, and absorption effects resulting in a reduced exposure at the bottom of the resist.

Finally, the lumped parameter model presents a simple understanding of the optical lithography process. The potential of the model as a learning tool should not be underestimated. In particular, the model emphasizes the competing roles of the aerial image and the photoresist process in determining linewidth control. This fundamental knowledge lays the foundation for further investigations into the behavior of optical lithography systems.

## References

1. R. Hershel and C. A. Mack, "Lumped Parameter Model for Optical Lithography," Chapter 2, Lithography for VLSI, VLSI Electronics - Microstructure Science, R. K. Watts and N. G. Einspruch, eds., Academic Press (New York:1987) pp. 19-55.

2. C. A. Mack, A. Stephanakis, R. Hershel, "Lumped Parameter Model of the Photolithographic Process," *Kodak Microelectronics Seminar, Proc.*, (1986) pp. 228-238.

3. C. A. Mack, "Lithographic Optimization Using Photoresist Contrast," *KTI Microlithography Seminar, Proc.*, (1990) pp. 1-12, and *Microelectronics Manufacturing Technology*, Vol. 14, No. 1 (Jan. 1991) pp. 36-42.

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 06/02/2014 Terms of Use: http://spiedl.org/terms