Enhanced Narrowband Signal Detection and Estimation with a Synthetic Antenna Array for Location Applications

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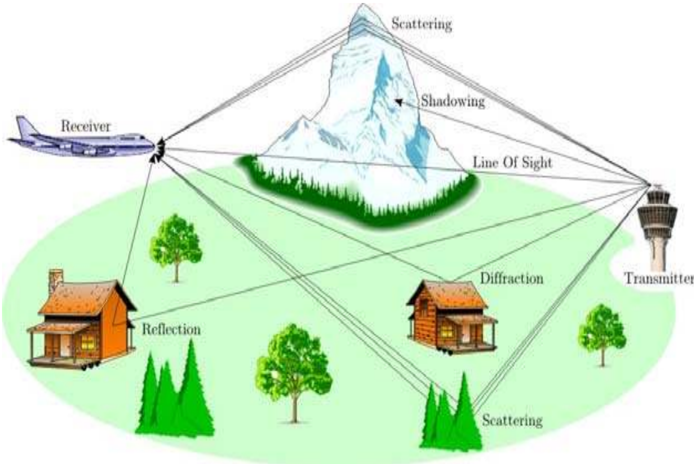
Outline

- Motivation and applications of antenna array systems
- Signal detection challenges in fading environments
- Signal detection performance by using a static antenna
- Signal detection performance by utilizing a moving antenna
    - Analysis of processing gain in uncorrelated Channel
    - Analysis of processing gain in correlated Channel
- Data collection and performance verification
- Conclusions

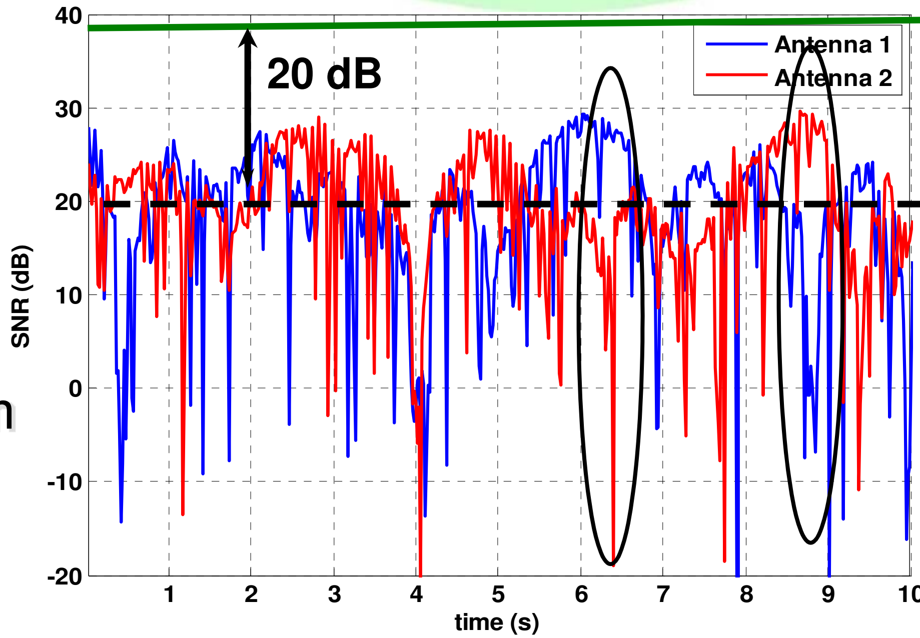
Applications of Antenna Array in Multipath Environment



- In a dense multipath scattering environment, fading appears to be a random function of antenna location
- To reduce the fading margin required, the receiver can use multiple spatially separated antennas
- Spatial diversity
  - Assume 4 branch diversity system

$$P_1(10\ dB) = 9.5 \times 10^{-2}$$

$$P_4(10\ dB) = 8.2 \times 10^{-5}$$



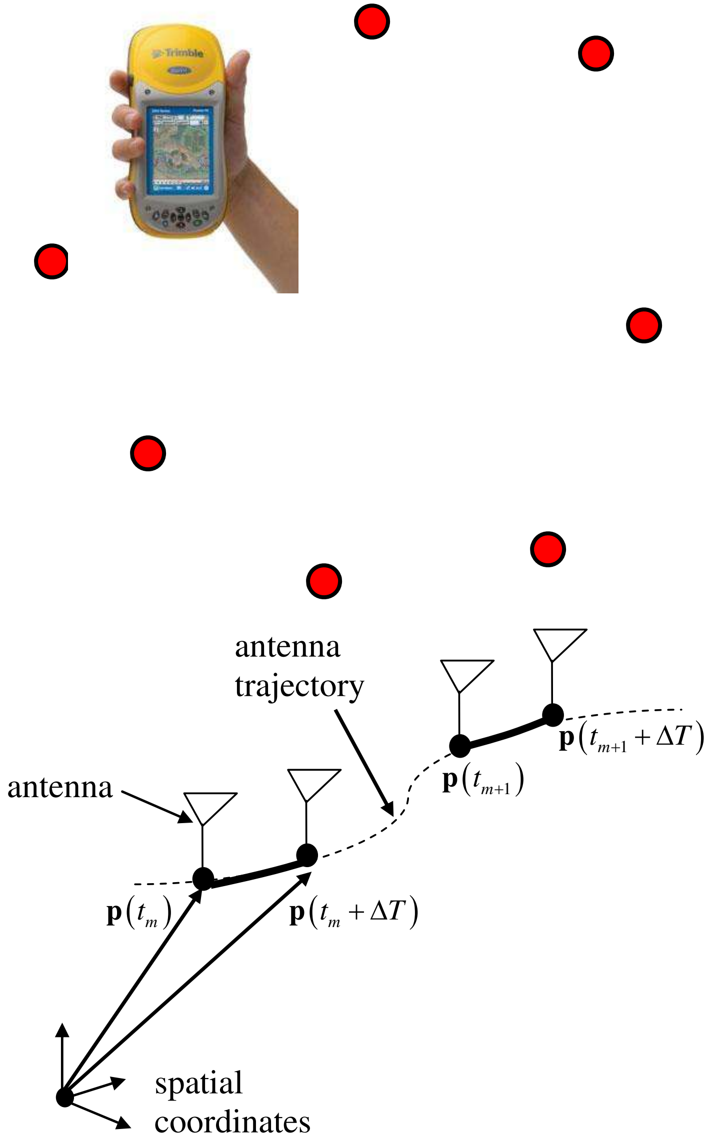
Synthetic Array Concept

- The size of the antenna array is incompatible with the small size of the handheld receiver

- The only means of realizing the potential spatial processing is to move antenna

- For a direct comparison with the stationary antenna, the constraint

$$T = M \ \Delta T$$



Problem Definition in Dense Fading

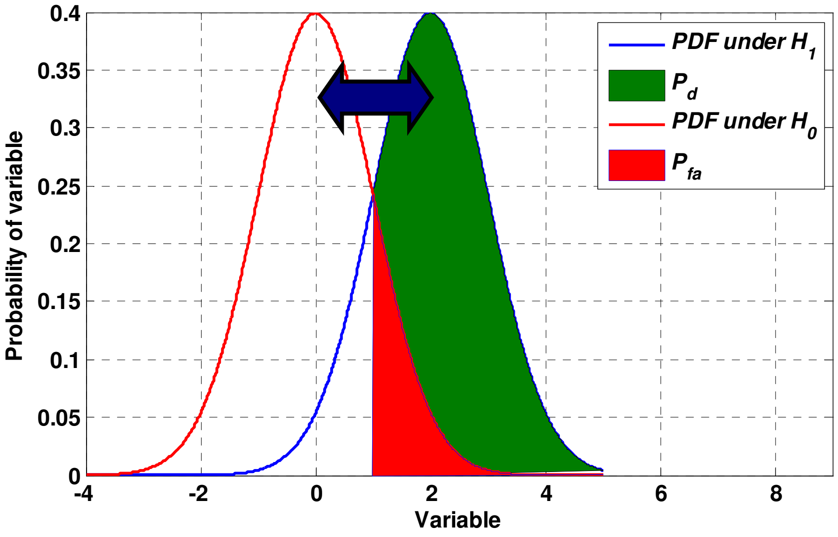
- Consider a single channel handheld receiver detecting narrowband signals in Rayleigh fading

- Static Antenna
  - Signal coherency is maintained
  - Signal will be subject to statistically large fading losses
- Moving Antenna
  - The coherency of the signal is decreasing leading to processing losses
  - The spatial diversity contained in the snapshot data more than compensates for this loss
- Evaluating processing gain by moving an antenna instead of keeping it stationary in Rayleigh fading
- Comparing the detection performance of Estimator-Correlator (EC) with that of Equal Gain (EG) combiner

Detection Problem

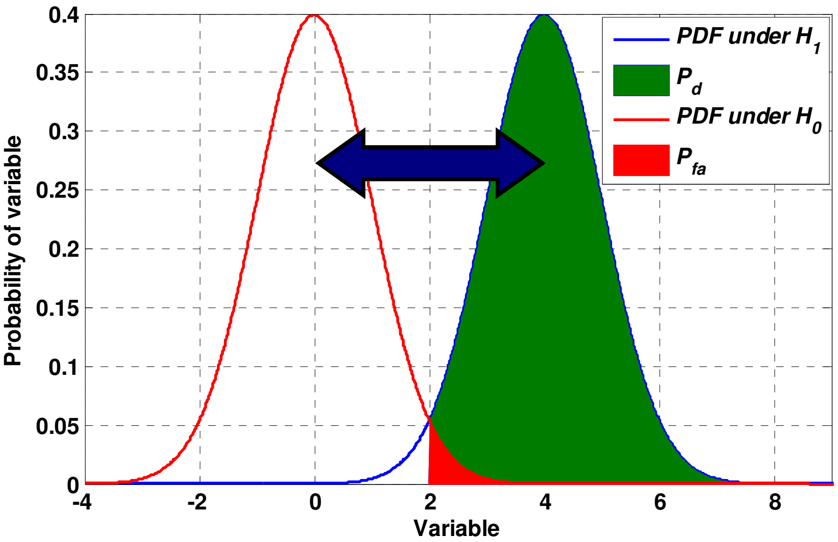
- The optimal detection processing based on the log likelihood Ratio Test (LRT) chooses H_1 if

$$L(\mathbf{x}) = \frac{p(\mathbf{x} \mid H_1)}{p(\mathbf{x} \mid H_0)} > \gamma$$

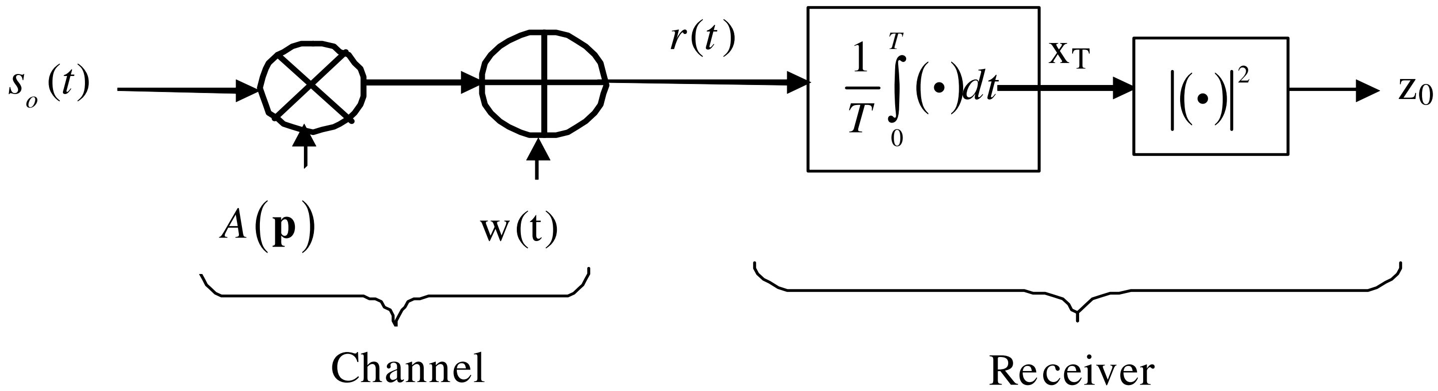


- The LRT reduces to the Estimator-Correlator (EC) formulation resulting in a sufficient statistic given as

$$z_{EC} = \mathbf{x}^H \mathbf{C}_s \left(\mathbf{C}_s + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{x}$$



Signal Detection by a Static Antenna



- The optimal NP detection processing is

$$z_0 = |x_T|^2, \quad x_T = \int_0^T r(t) s_o(t)^* dt$$

- The *average signal to noise ratio*, ρ given as

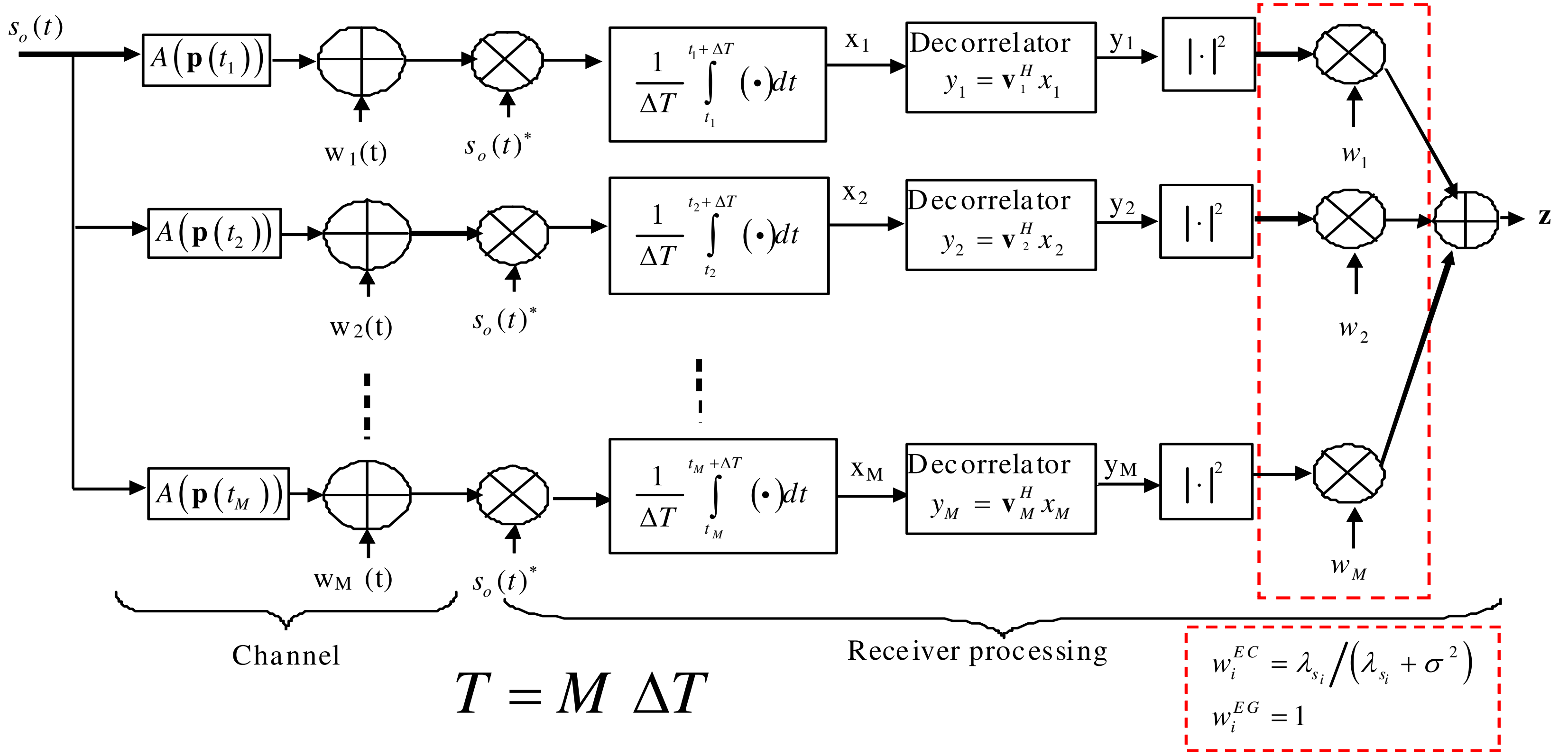
$$\rho \equiv \frac{T\sigma_A^2}{N_o}$$

- The Detection performance can be represented by

$$P_{fa} = \exp(-\gamma), \quad P_{det} = \exp\left(\frac{-\gamma}{(1+\rho)}\right)$$

$$\rho = \frac{\ln(P_{fa})}{\ln(P_{det})} - 1$$

Signal Detection by a Moving Antenna



- In correlated Rayleigh fading the optimum detector in Gaussian signal and noise model is Estimator-Correlator (EC)
- The test statistics is

$$z_{EC} = \sum_{m=1}^{M} \frac{\lambda_{s_m}}{\lambda_{s_m} + \sigma^2} |y_m|^2 \qquad z_{EG} = \sum_{m=1}^{M} |y_m|^2$$

Synthetic Array Detection Performance in Uncorrelated Rayleigh fading

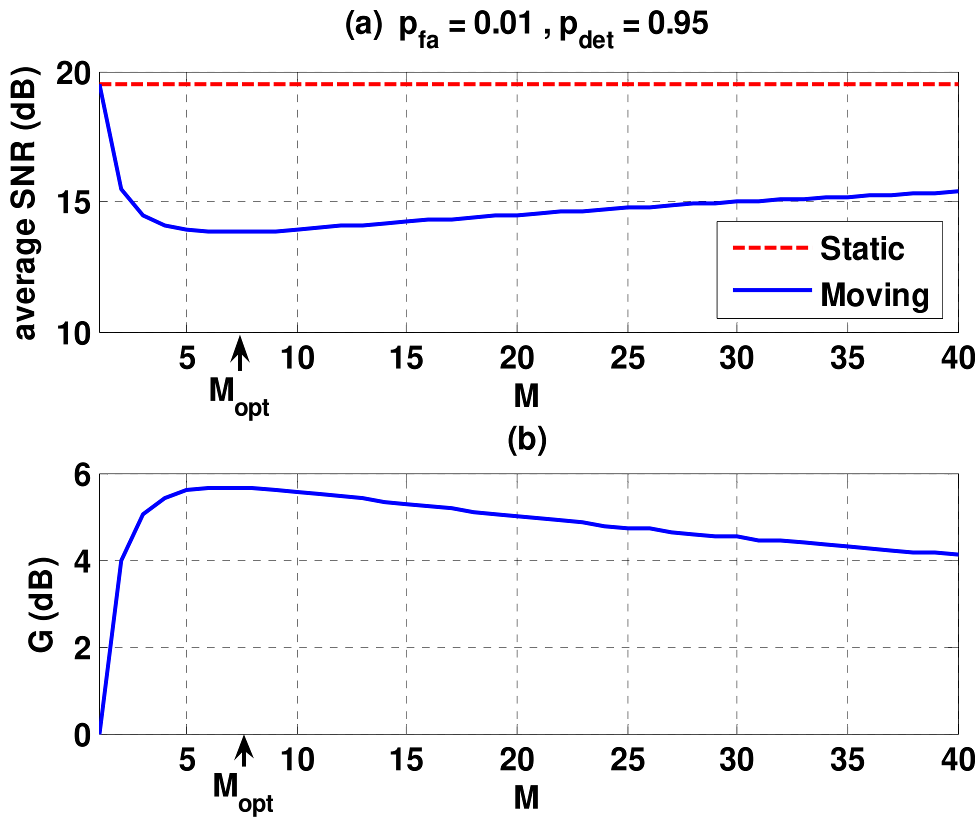
- In uncorrelated Rayleigh the optimum detector in Gaussian model is EG combiner
- The test statistics is $z_1 = \sum_{m=1}^{M} \left| x_m \right|^2$
- Detection performance

$$P_{fa} = Q_{x_{2M}^2}(\gamma) \qquad P_{\det} = Q_{x_{2M}^2}\left(\frac{\gamma}{\frac{\rho}{M} + 1} \right)$$

where

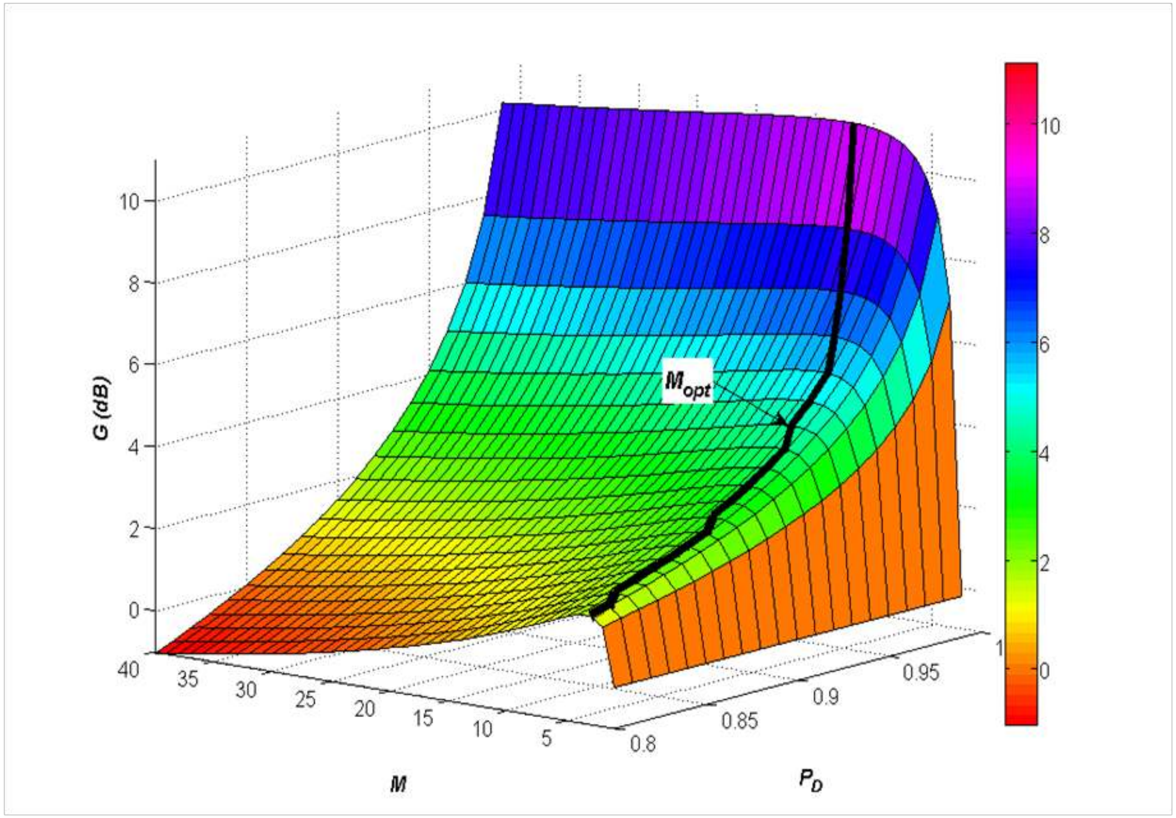
$$Q_{x_{2M}^2}(x) = \exp\left(-\frac{1}{2} x \right) \sum_{k=0}^{M-1} \frac{(0.5x)^k}{k!}$$

Required SNR and Processing Gain for Stationary and Moving Antenna



- Average required SNR for static and moving antenna is compared as a function of the target parameters with several values of M
- G is the processing gain which is defined by $G = 10\log(\rho_s / \rho_m)$

Processing Gain of the Synthetic Antenna Array



- Synthetic array gain with respect to static antenna versus P_D and M for P_{FA}=0.01
- The black line represents the optimum M as a function of P_D

Channel Correlation Coefficient

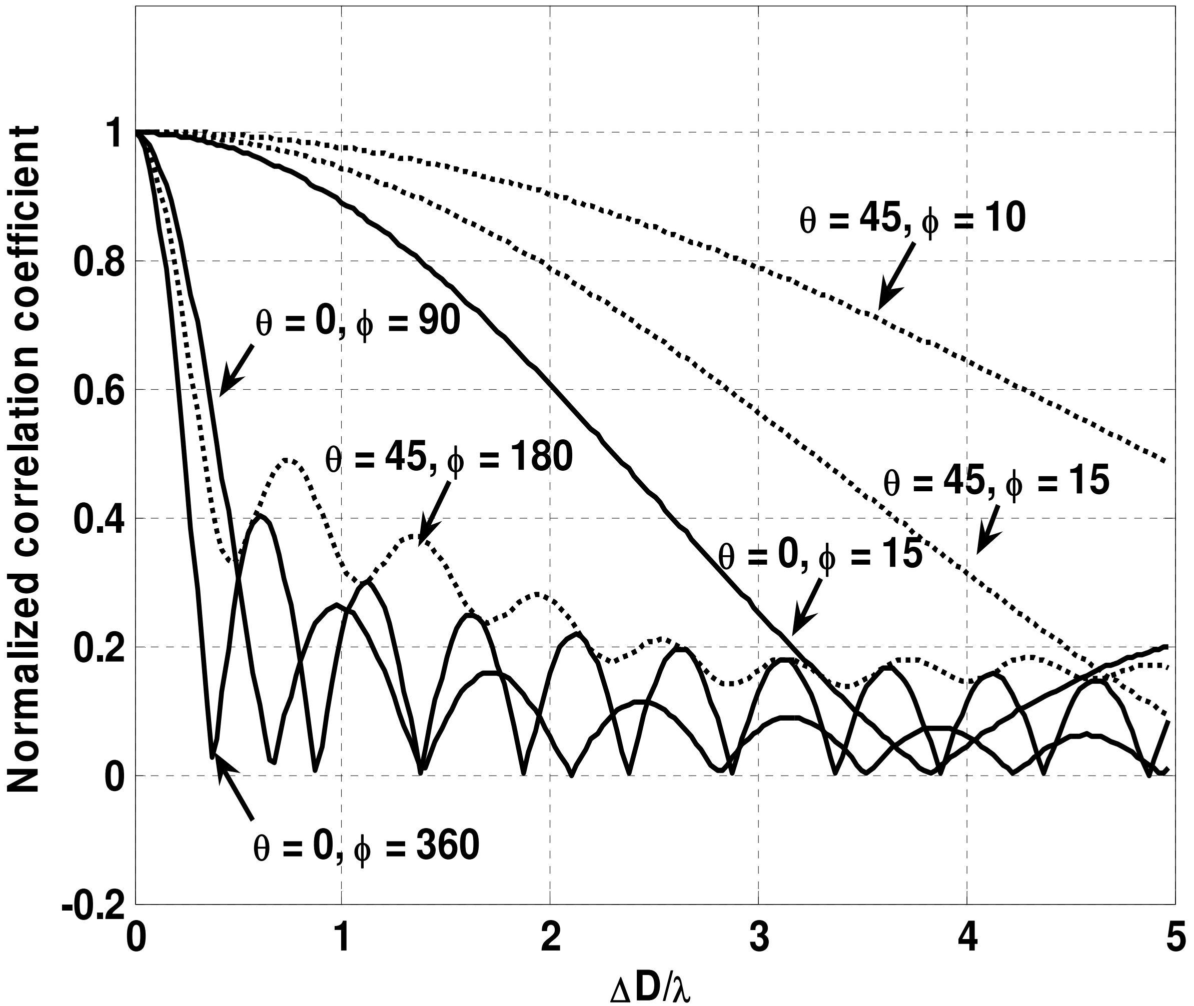
- The received signal

$$s(t) = A(\theta)\, s_o(t)$$

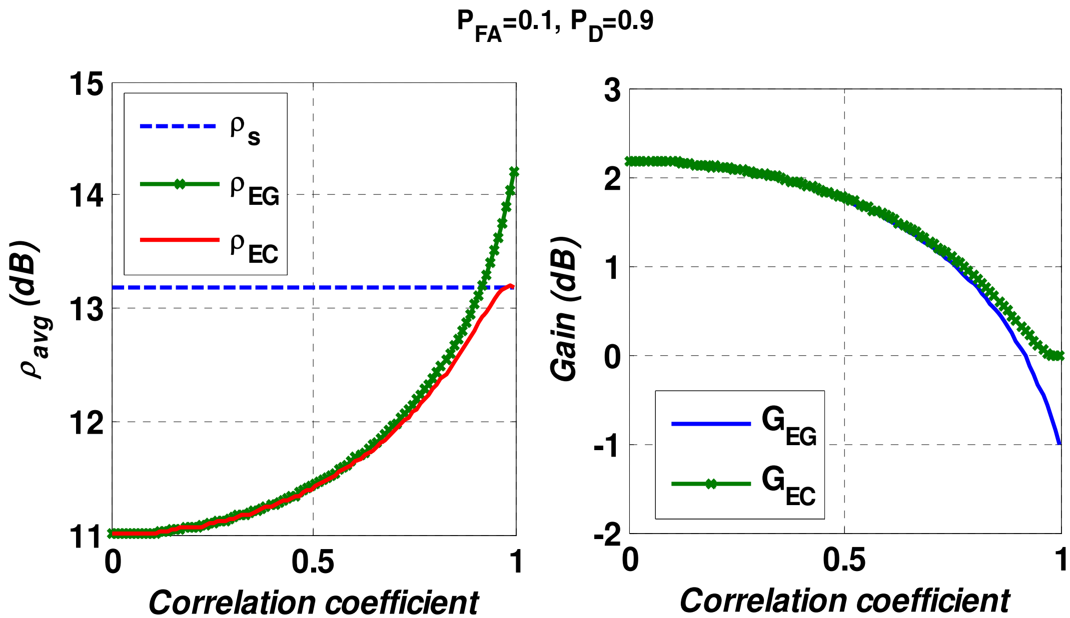
- The correlation coefficient can be defined by

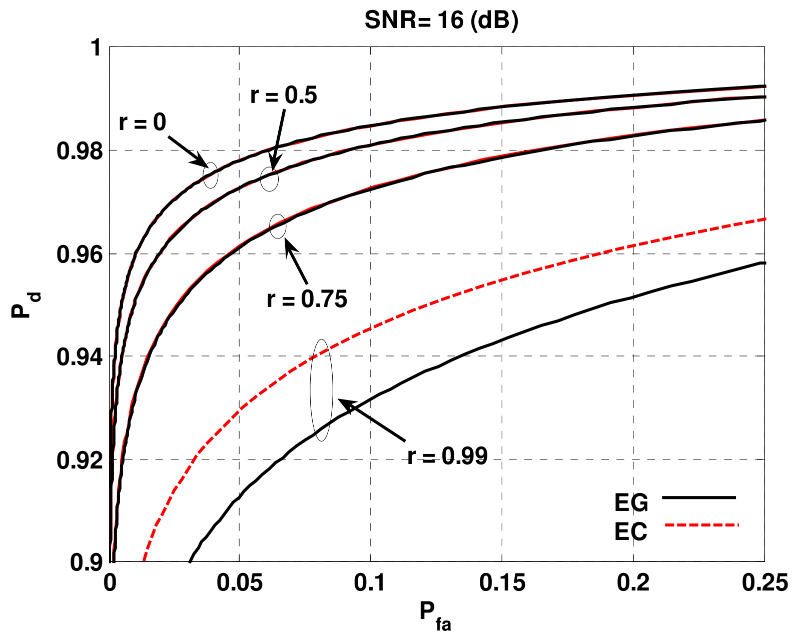
$$\mathbf{C_s} = \alpha\Psi$$

- where θ is mean of Angle of Arrival and Φ is angle spread



Performance of EC and EG in Correlated Rayleigh

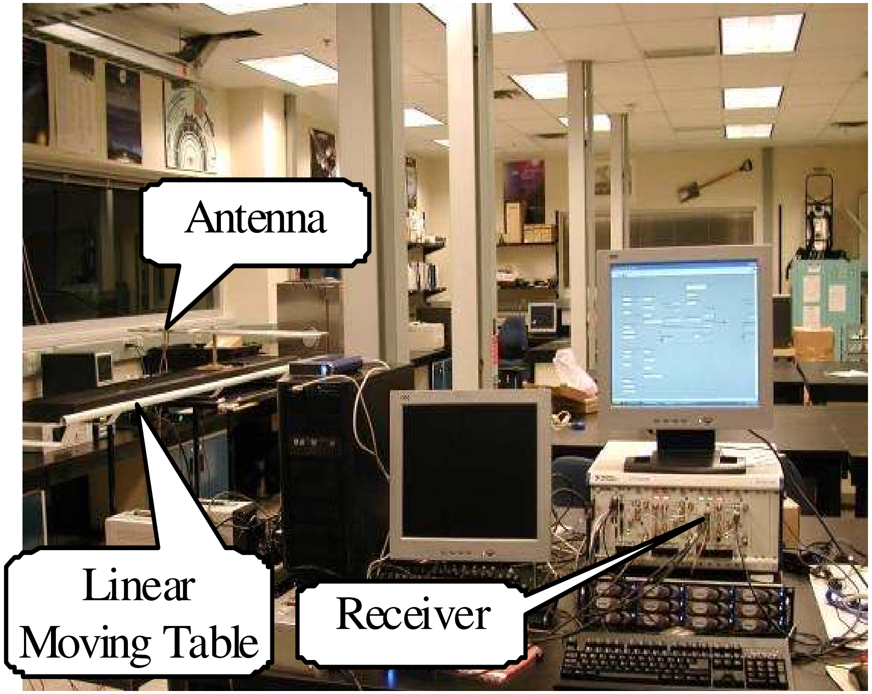




- Required SNR and equivalent gain of the dual antenna relative to the signal antenna processing schemes
- EC has better or identical performance as the static antenna

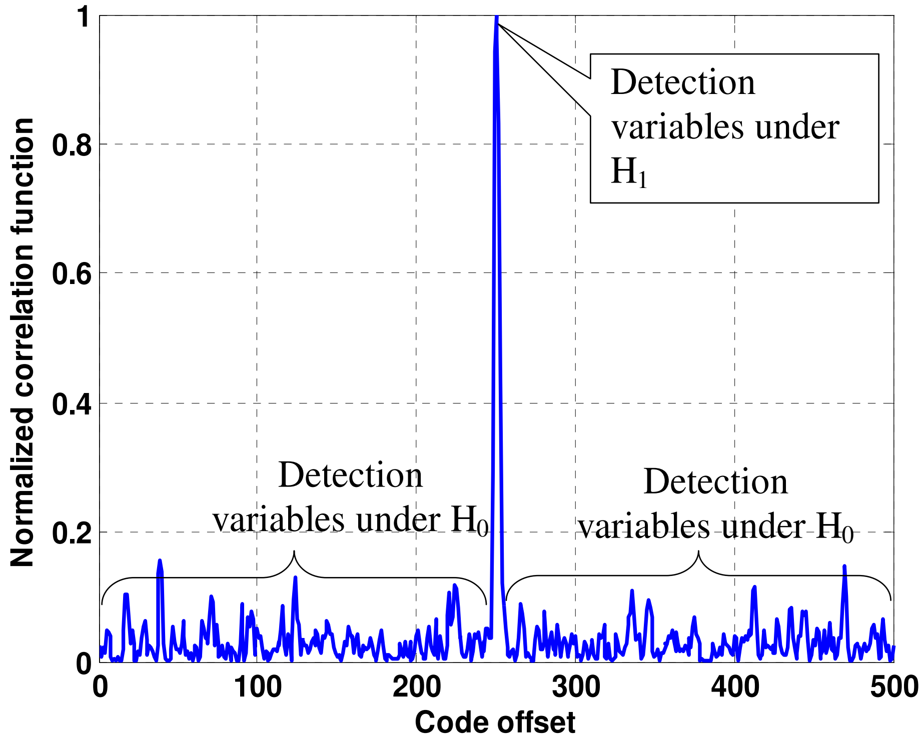
- ROC curves versus *r* for given SNR=16 dB and *M=2*
- For moderate correlation coefficient performance of EG and EC are almost identical

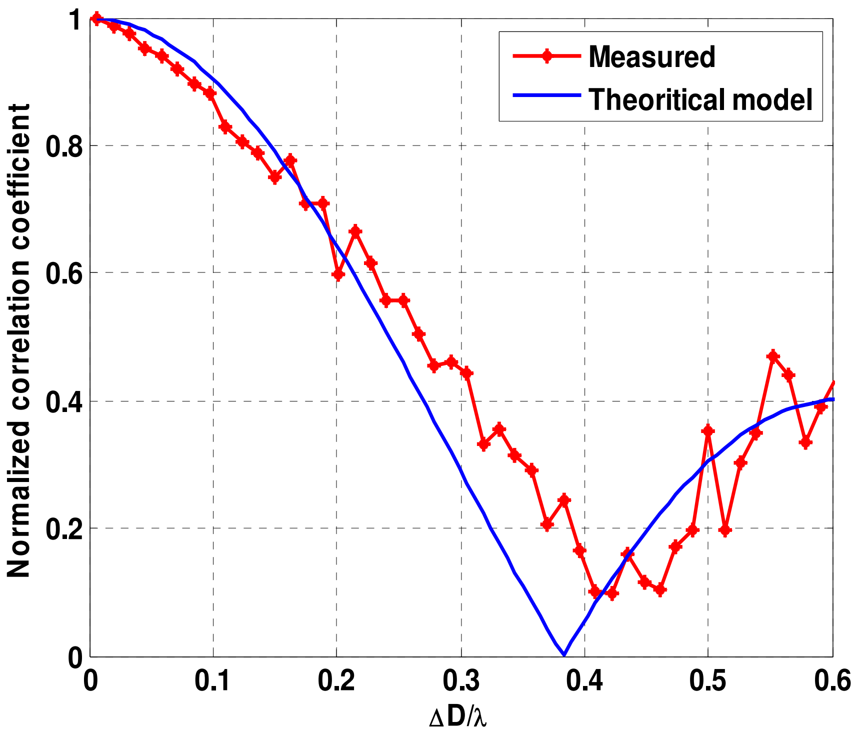
Indoor Data Collections



- GPS L1 C/A code
- Data collection in different indoor environments
- Linear moving table is used to realize the synthetic array
- The antenna was moved by 2 cm/s

Correlation Functions and Indoor Channel Correlation Coefficient





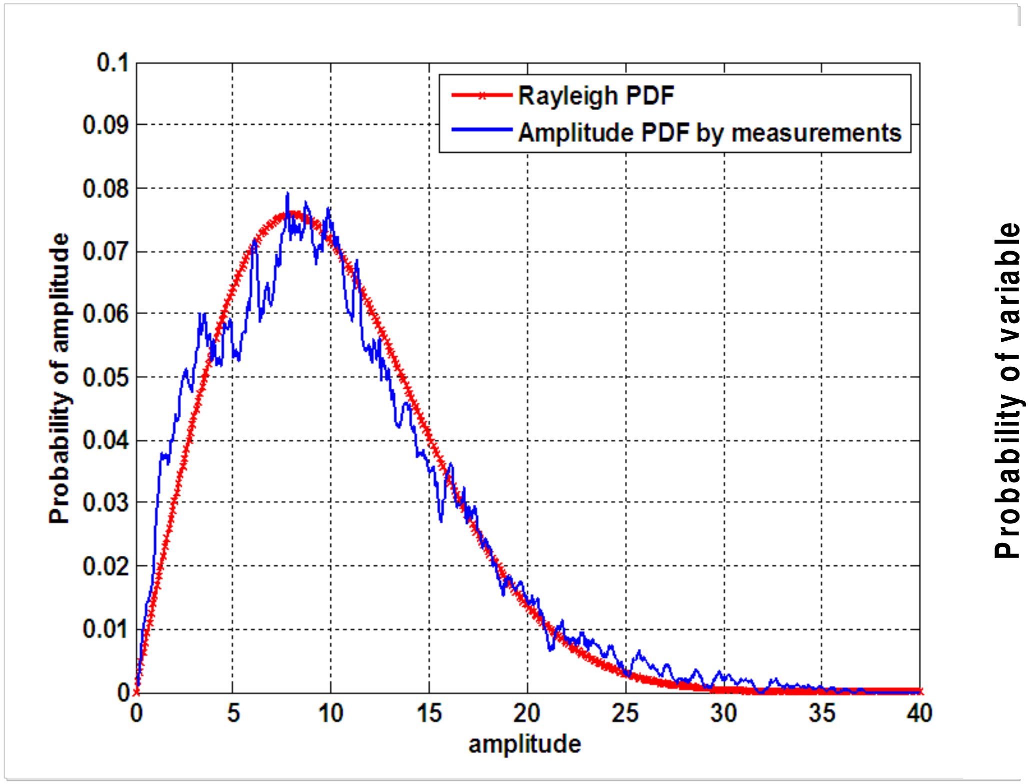
- Detection variables used for estimating PDF under H_0 and H_1

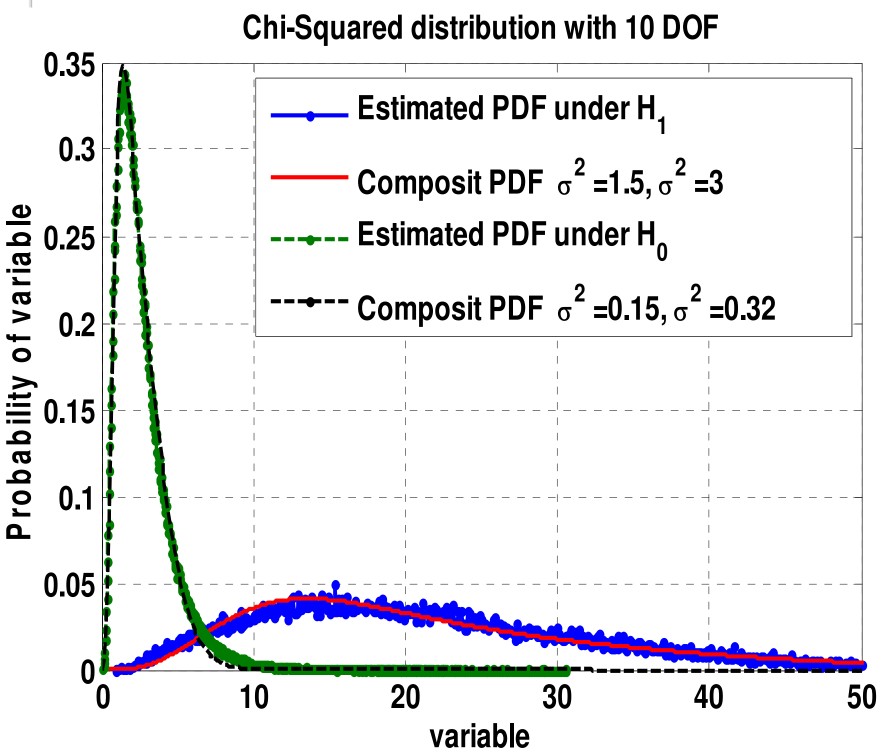
- Theoretical model is

$$J_0\left(2\pi\Delta D / \lambda\right)$$

- ΔD is antenna spacing and λ is the wavelength

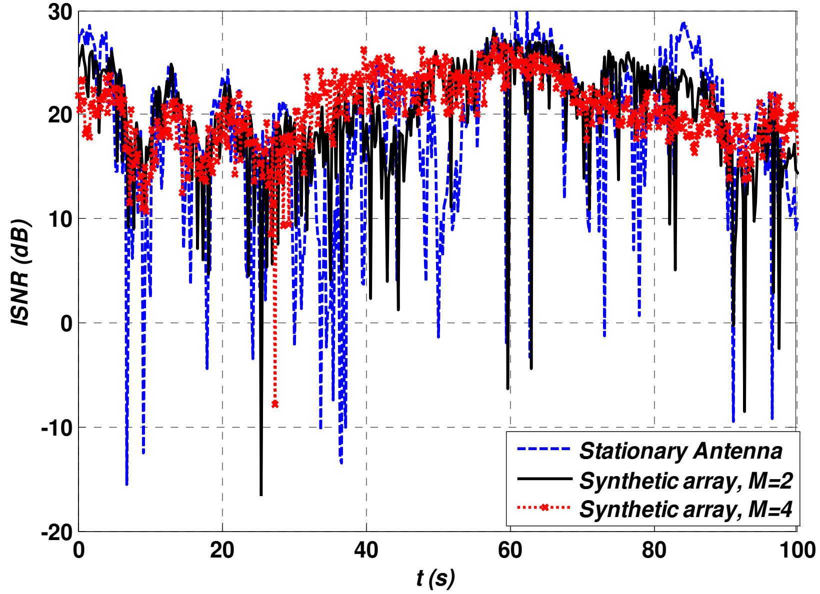
Indoor GNSS Channel Measurements and Verification



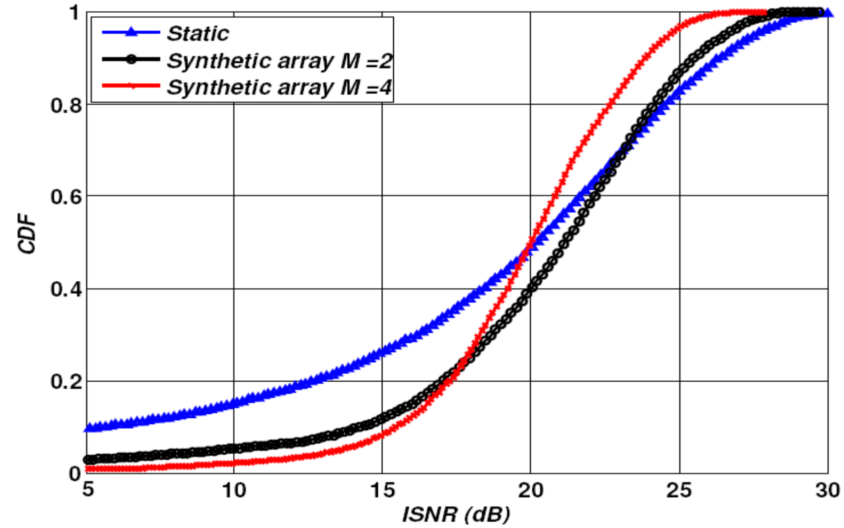


- PDF of the receiver signal amplitude and theoretical Rayleigh PDF fit
- Theoretical and measured PDF under H_0 and H_1 states

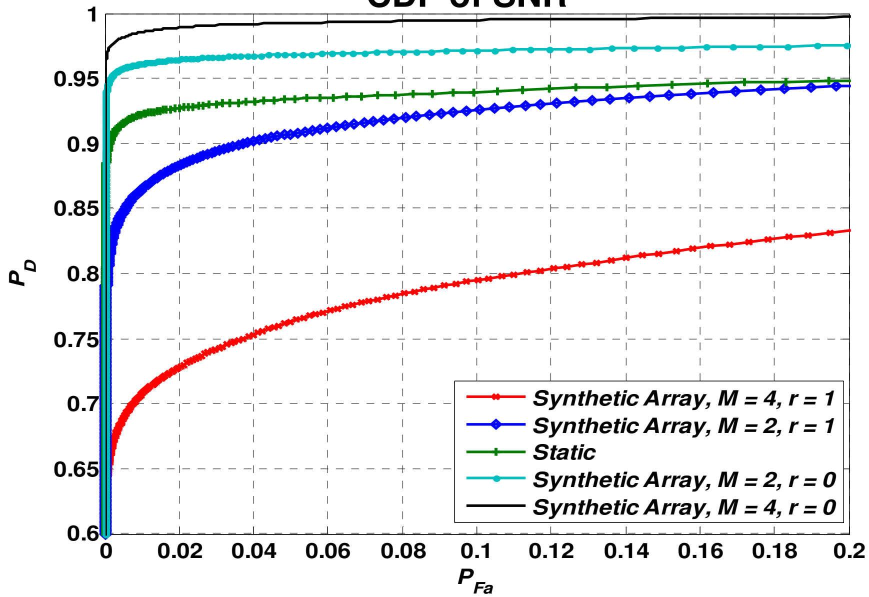
Detection Performance



SNR measurements



CDF of SNR



ROC Curves

- Synthetic array reduces the fading effect
- ROC performance of the synthetic array is higher than the static antenna

Conclusions

- The detection performance of a narrow bandwidth wireless signal subjected to Rayleigh fading has been considered for a single antenna handheld receiver
- Of specific interest was to determine the merits of moving the antenna while capturing the signal that provides diversity gain
- It was shown that substantial processing gains are possible by moving the antenna
- Substantial processing gains are possible by moving the antenna
- Experimental measurements were done to verify the assumption of the Rayleigh fading and also to verify the calculated processing gain
- Good agreement between the experimental and theoretical results was obtained