Enhanced propagation of aviation noise in complex environments: A hybrid approach

Joyce Rosenbaum
Environmental Scientist

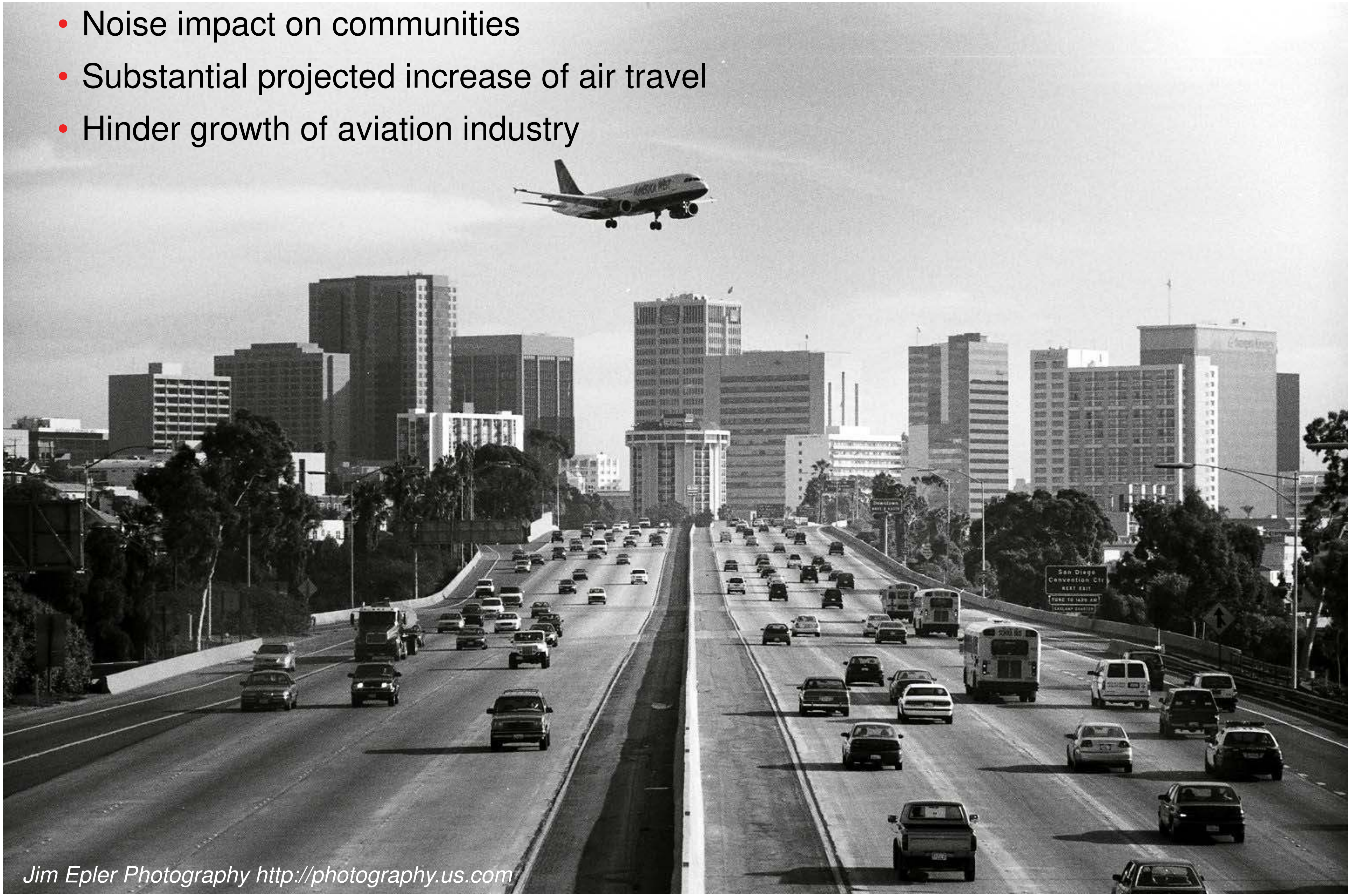
September 25, 2013

17th Workshop of the Aeroacoustics Specialists' Committee of CEAS

Outline

Statement of the Problem

- Noise impact on communities
- Substantial projected increase of air travel
- Hinder growth of aviation industry



Jim Epler Photography http://photography.us.com

Background

Current Capabilities: Integrated Noise Model (INM)

- Includes spherical spreading and atmospheric absorption
- Ground impedance, terrain, and meteorology included through simplified approximations

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Goal: Investigate enhancements to modeling capabilities of Federal Aviation Administration's Aviation Environmental Design Tool (FAA AEDT) and Integrated Noise Model (INM) in complex environments, such as National Parks.

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Result: Hybrid Propagation Model (HPM)

- Numerical model designed to predict aviation noise levels under complicated propagation conditions.
- Collaborative research between FAA, Volpe and the Pennsylvania State University (PSU)
- HPM is a composite of 3 propagation methods:
  - Parabolic equation (PE)
  - Fast field program (FFP)
  - Straight ray-trace (Ray)
- Methods chosen for complimentary strengths

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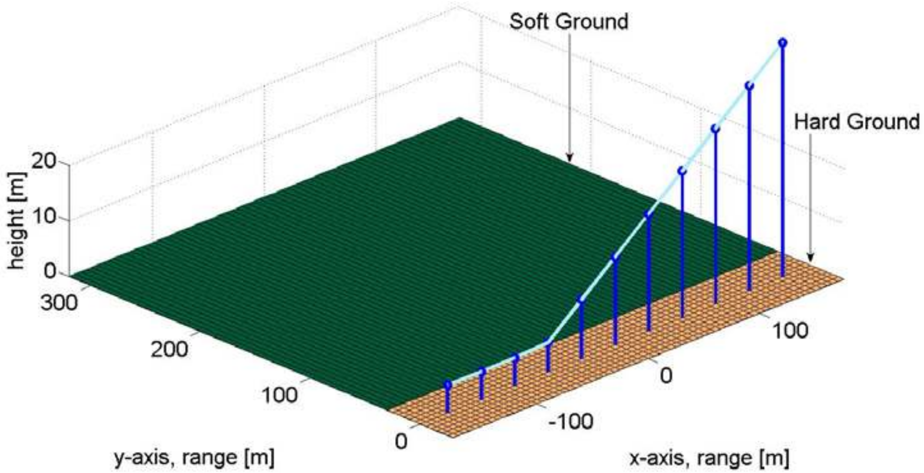
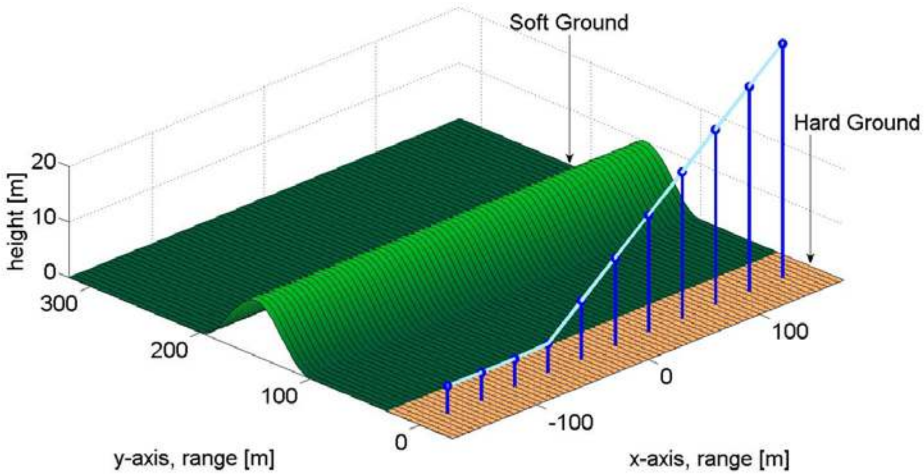
Supporting Rationale:
Simple take-off flight path – Flat and uneven terrain (PE)

- Downward refracting atmosphere
  $c = c_0 + b*\ln(z/z_0+1)$, $b = 1$ m/s

- Ground Impedance Discontinuity
  - Hard (tan): 20,000 cgs Rayls
  - Soft (green):150 cgs Rayls





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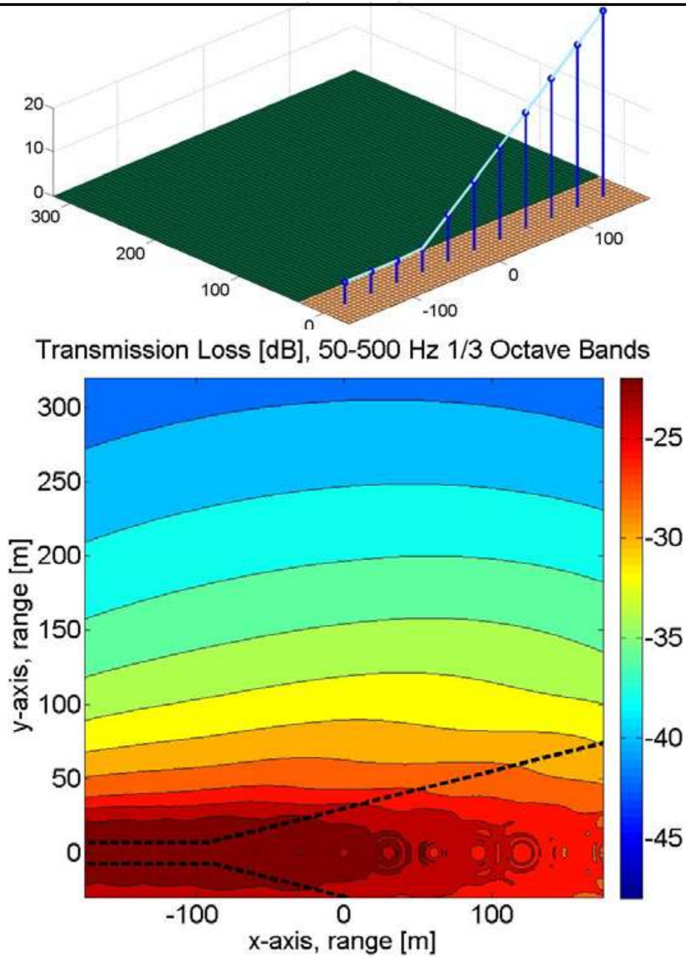

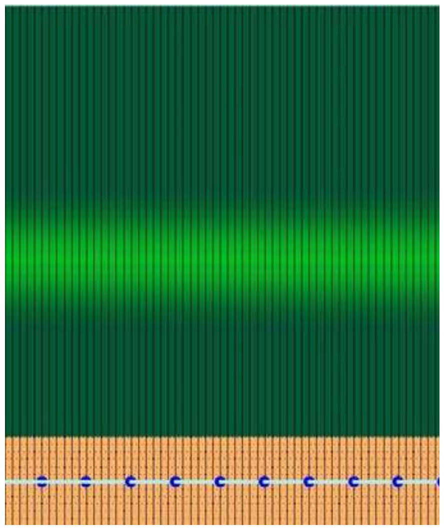

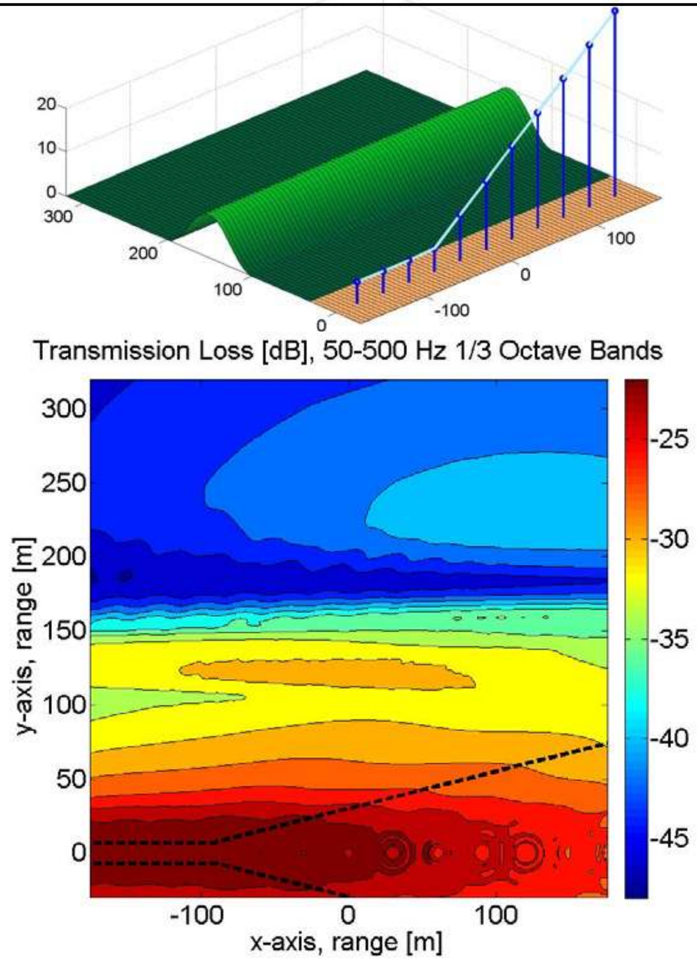

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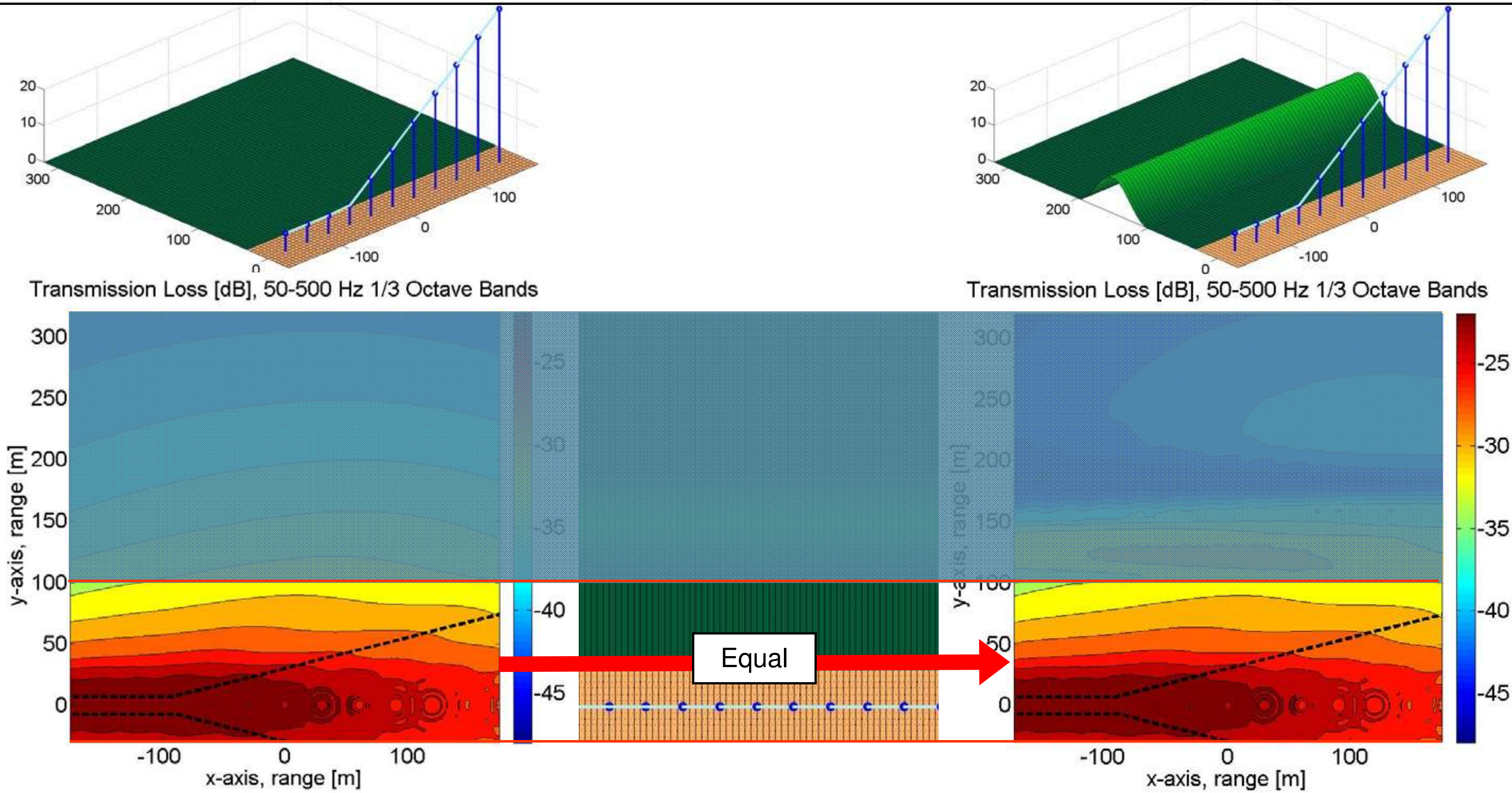

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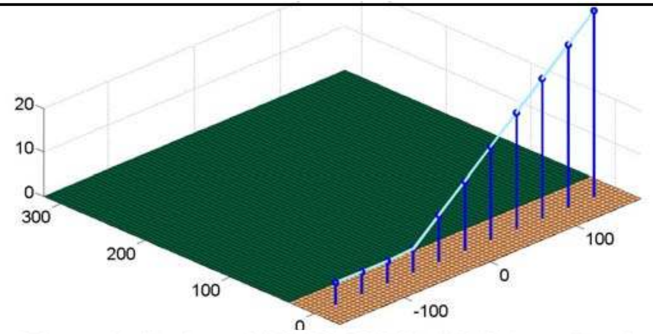

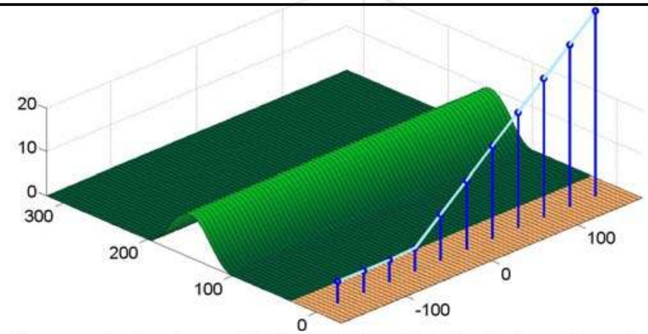

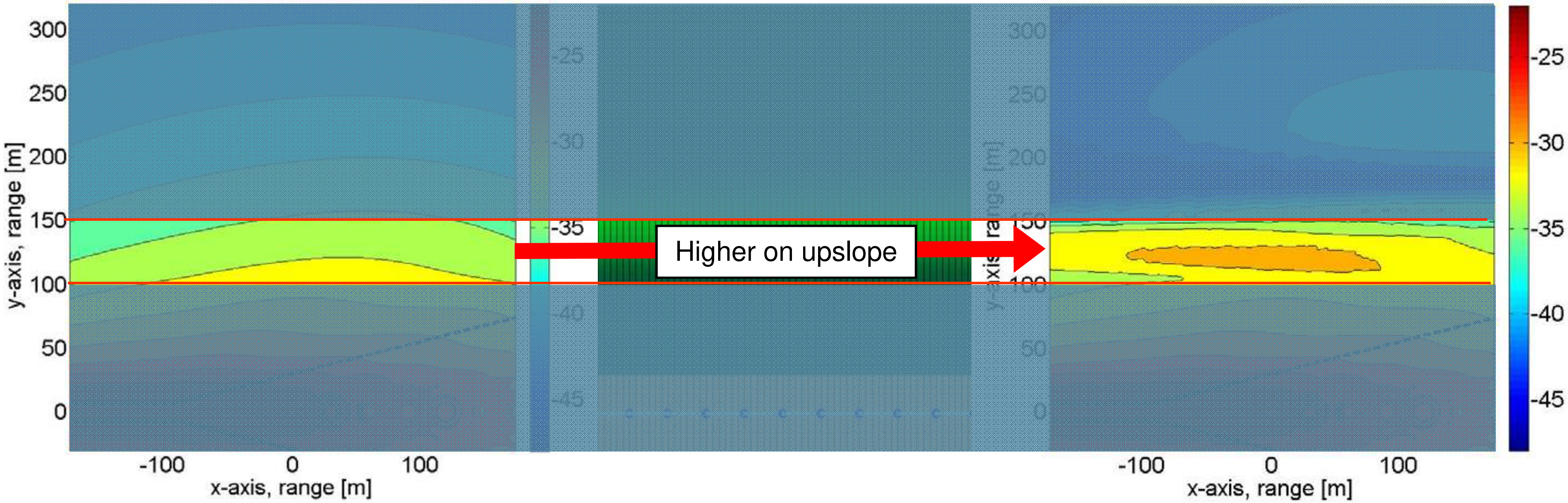

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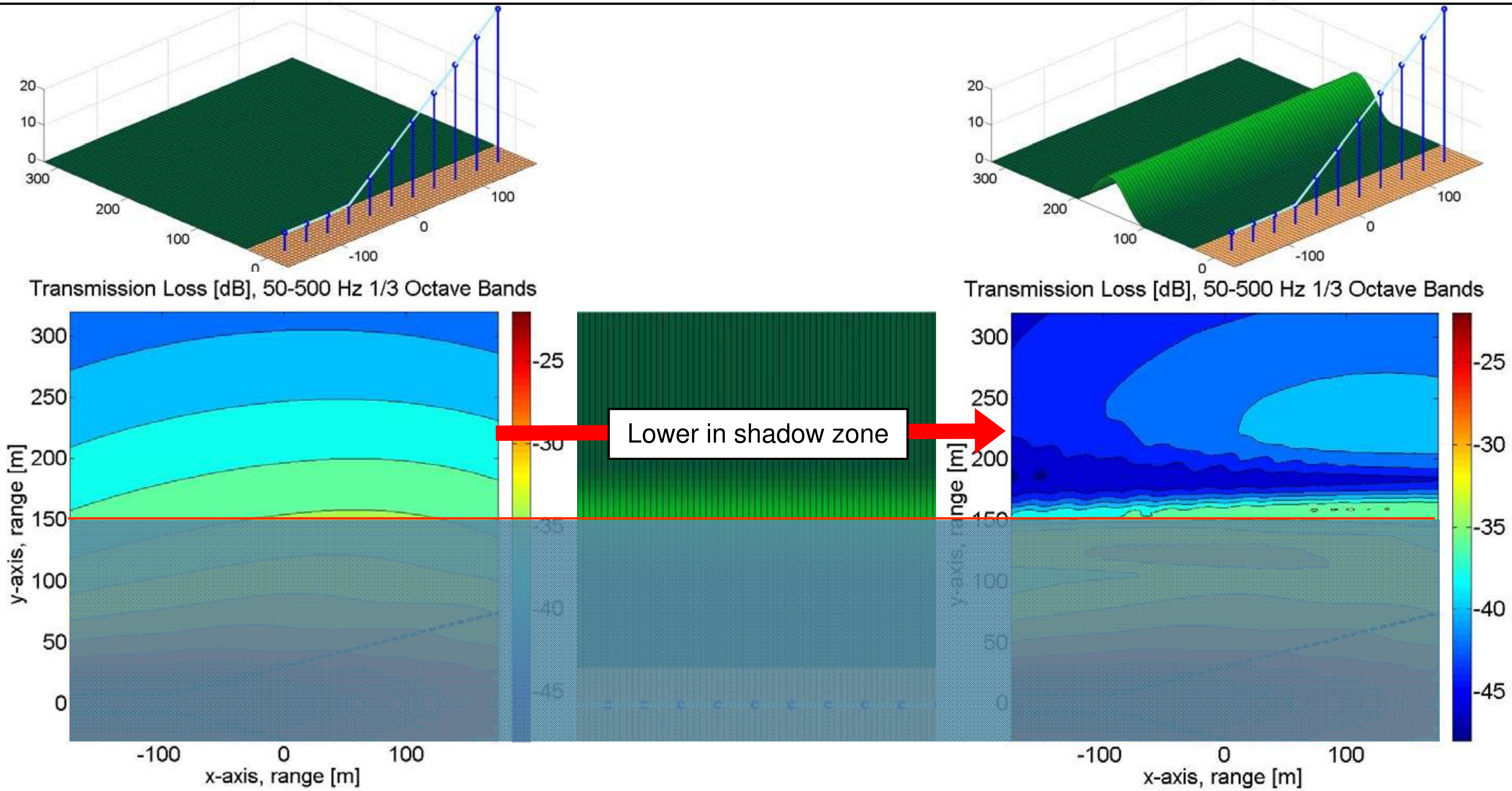

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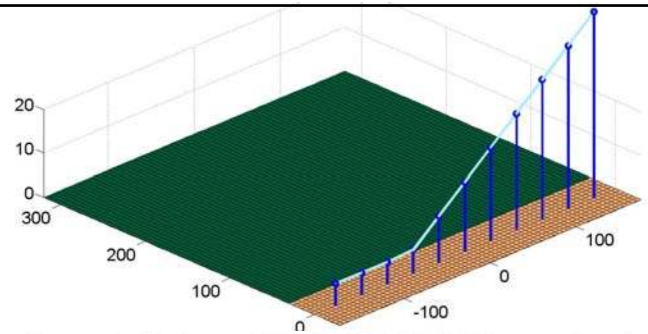

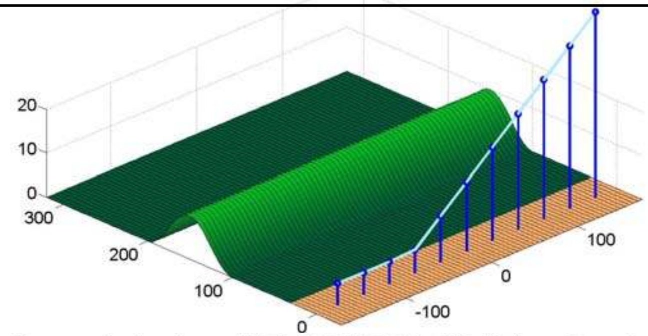

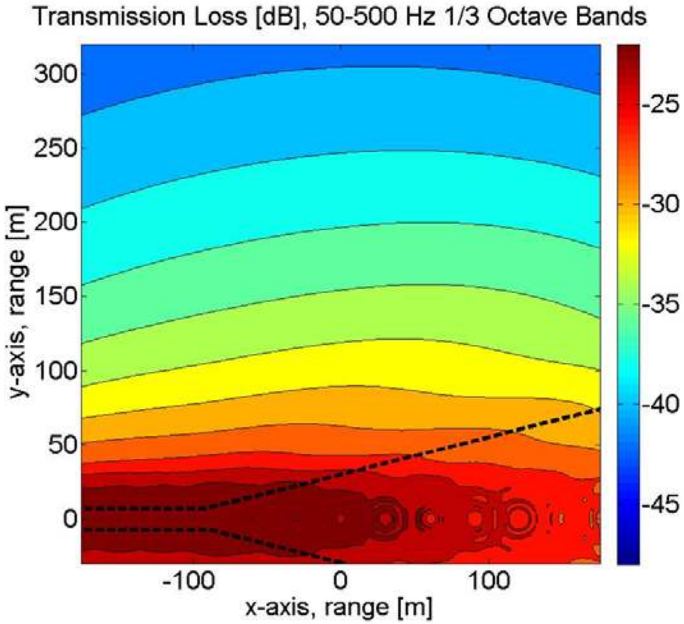

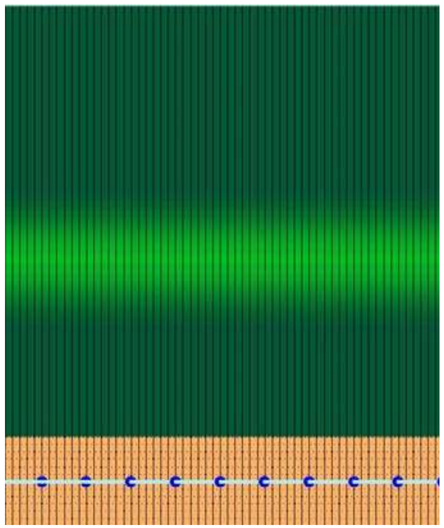

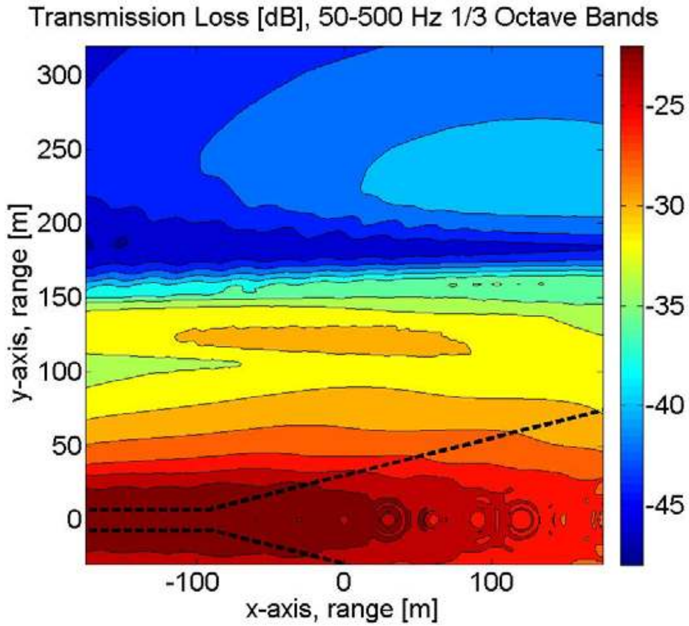

Modeling Approach: Parabolic Equation (PE)

- Numerical method that models a monopole source above a ground surface
- Addresses one source frequency at a time
- Increased accuracy of low frequency noise propagation

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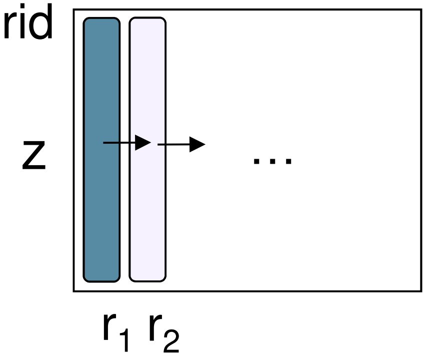
Generalized Terrain Parabolic Equation (PE) method

- Derived from the one-way Helmholtz equation
- Start field extrapolated in range on the grid



Modeling Approach: Parabolic Equation (PE) Fast Field Program (FFP)

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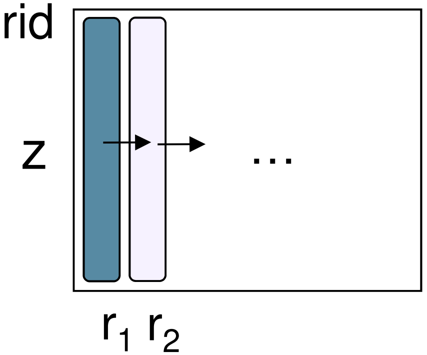

Fast Field Program (FFP) method

- Derived from the Helmholtz equation assuming homogeneous horizontal layers of the atmosphere with constant wave number
- Employs a transform from the horizontal spatial domain to the horizontal wave number domain and extrapolates between horizontal layers

Modeling Approach: Parabolic Equation (PE) Fast Field Program (FFP) Straight Ray

- Numerical method that models a monopole source above a ground surface
- Addresses one source frequency at a time
- *Increased accuracy of low frequency noise propagation*

Generalized Terrain Parabolic Equation (PE) method

Fast Field Program (FFP) method

Straight Ray method

- Superposition of direct and reflected rays, following straight ray paths

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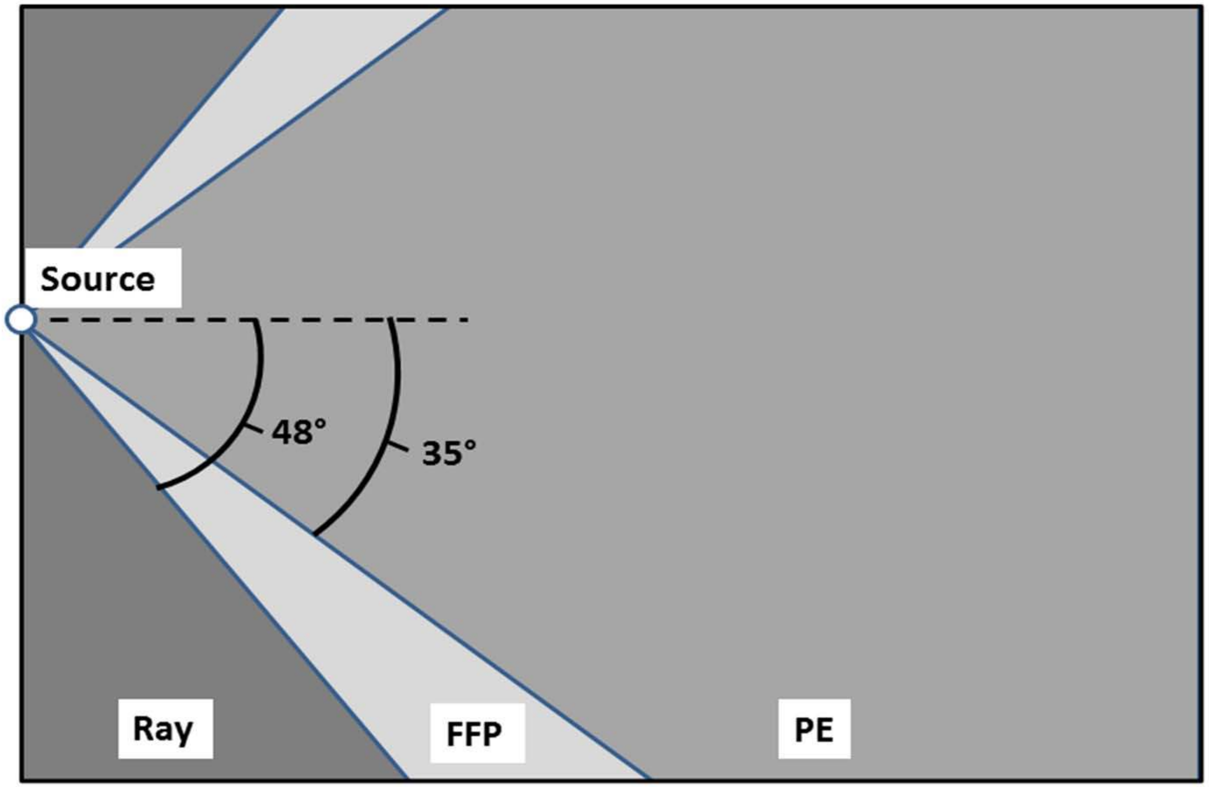
Hybrid Propagation Model

- 3 models are joined in two-dimensional vertical plane
- FFP and Ray fill in regions where PE model is not valid, and ensure full coverage for aircraft noise model



Combination of component models in HPM in the vertical plane

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Comparing Properties of the Component Models

- 3 types of propagation methods in toolbox, all with benefits and limitations

	Source Representation *spectrum*	**Source-Receiver Geometry** *elevation angle from source*	**Propagation Effects** *terrain, ground, meteorology*	**Runtime**
PE	• Full frequency range	• Inaccurate at elevation angles >35 degrees	• Includes range-dependent effects (terrain, ground, meteorology)	• Very slow for high frequencies, long propagation ranges, high altitude sources (days, full spectrum)
FFP	• Full frequency range	• Inaccurate at very high elevation angles >72.5 degrees (window dependent)	• Limited to layered atmosphere, no range-dependent effects	• Fairly slow for high frequencies, long propagation ranges, high altitude sources (hours, full spectrum, short ranges)
Ray	• High frequency assumption	• Accurate at all elevation angles	• Limited to homogeneous atmosphere, no range-dependent effects	• Fast (seconds)

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- Evaluate tradeoffs between increasing accuracy and decreasing runtimes

  For which conditions can faster methods be substituted with minimal effect on accuracy?

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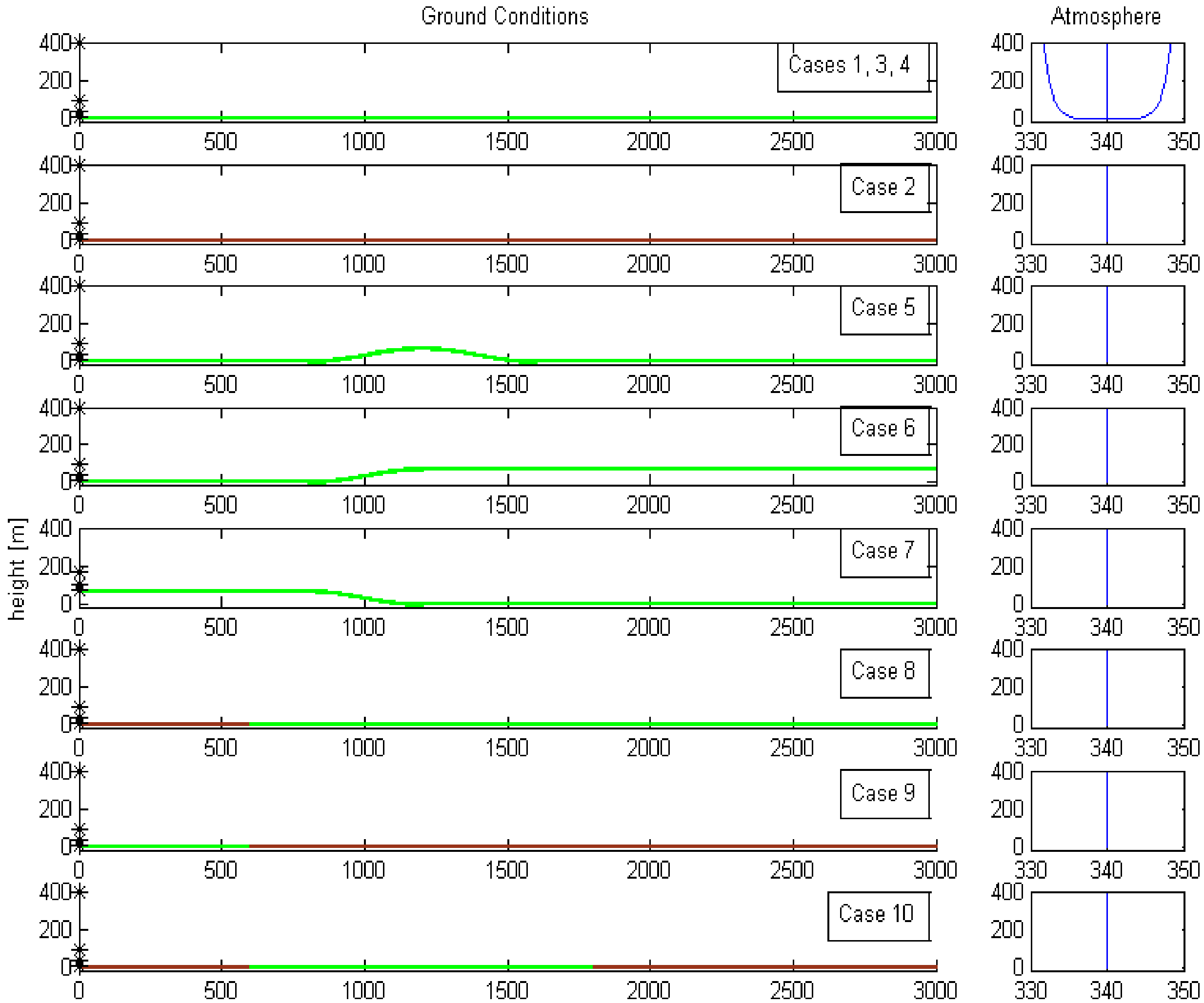
Model Comparison

HPM, FFP (only) and Ray (only) compared for 10 test cases (where appropriate)

Varied terrain, ground type and atmospheric conditions



Diagrams of the ground and atmospheric conditions of the ten test cases (green lines indicate soft ground, brown lines hard ground).

Takeaways

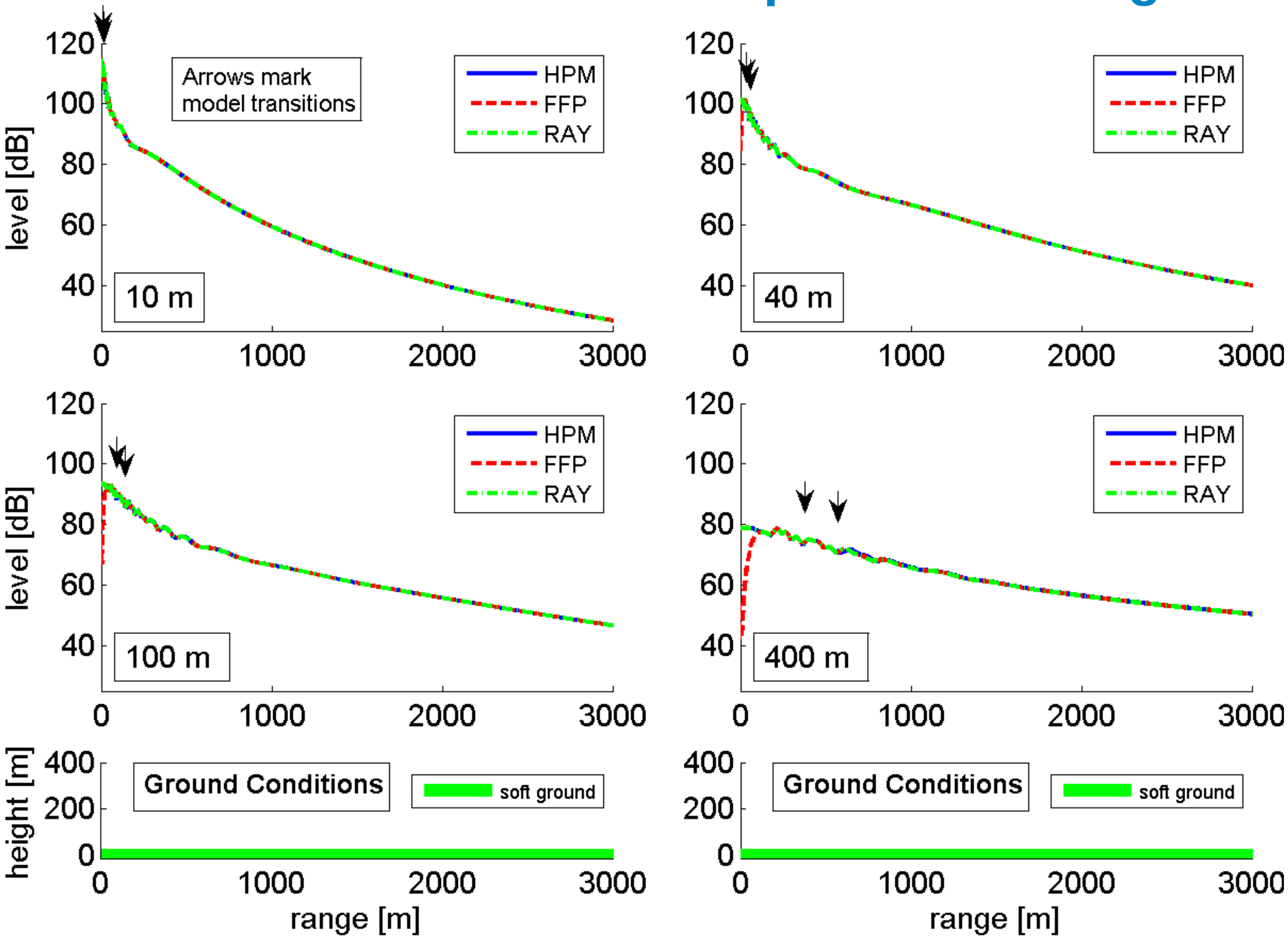
	Use Full Model	Use Simple Model
More Intuitive	• Line of sight is blocked by a terrain feature	• Ground is flat and atmosphere is homogeneous
↑	• Receiver is in/near a shadow zone, or the atmosphere supports multiple ground reflections. (Can use FFP)	• Source is high, and receiver is far from a ground type transition
↓	• Source is low, and receiver is near a ground type transition	• A terrain feature may exist, but does not break the line of sight or significantly change the angle of ground reflection
Less Obvious	• Terrain shape significantly changes the angle of reflection off a soft ground surface	• Source has a very high altitude

Takeaways

	Use Full Model	Use Simple Model
More Intuitive ↑		• Ground is flat and atmosphere is homogeneous
↓ Less Obvious		

Model Comparison: Flat, soft ground, homogeneous atm

Ground is flat and atmosphere is homogeneous



4 source heights (10 m, 40 m, 100 m, 400 m)

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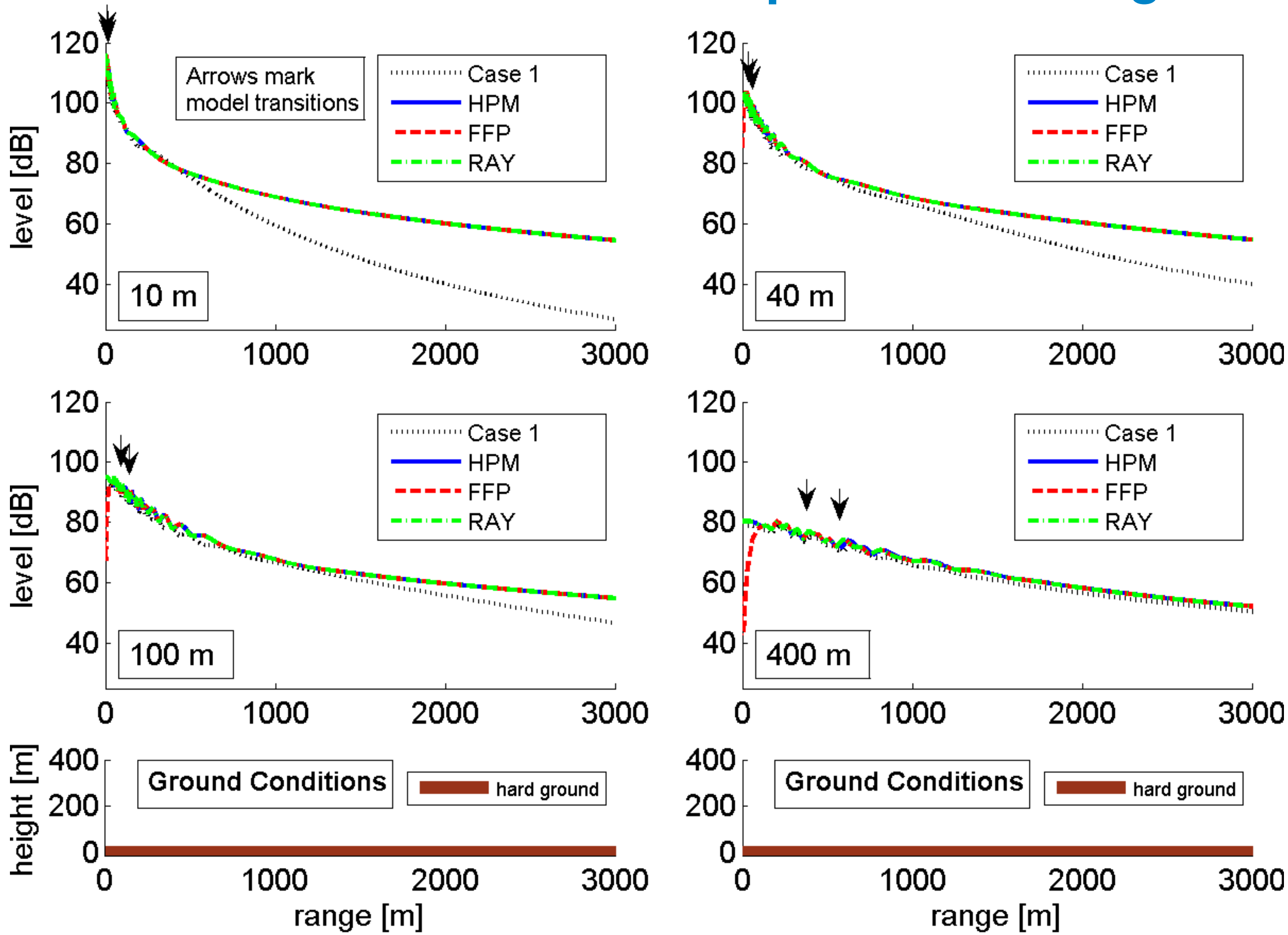
Model Comparison: Flat, hard ground, homogeneous atm

Ground is flat and atmosphere is homogeneous



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Takeaways

	Use Full Model	Use Simple Model
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↕		
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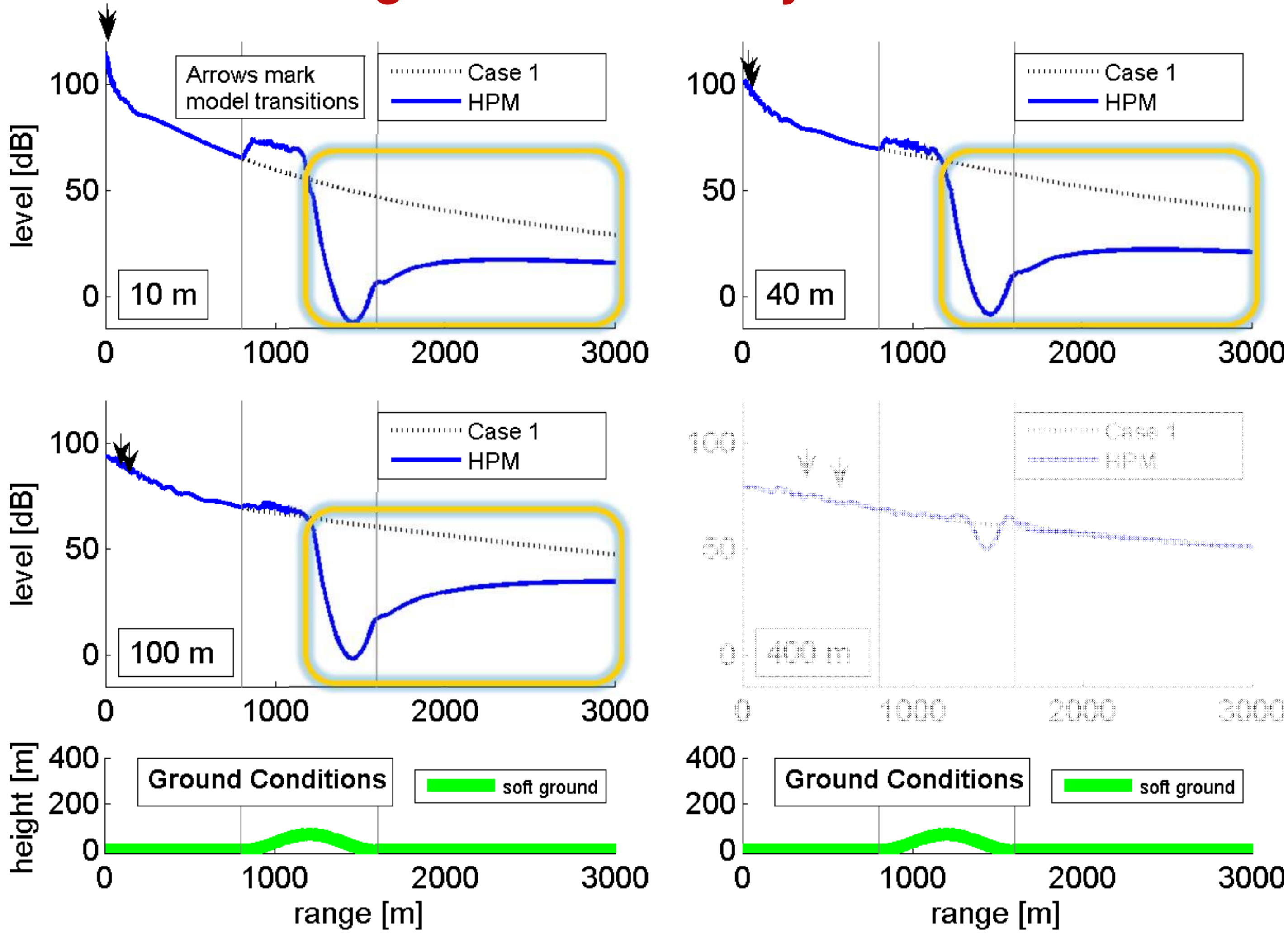
Model Comparison: Soft ground, hill, homogeneous atm

Line of sight is blocked by a terrain feature



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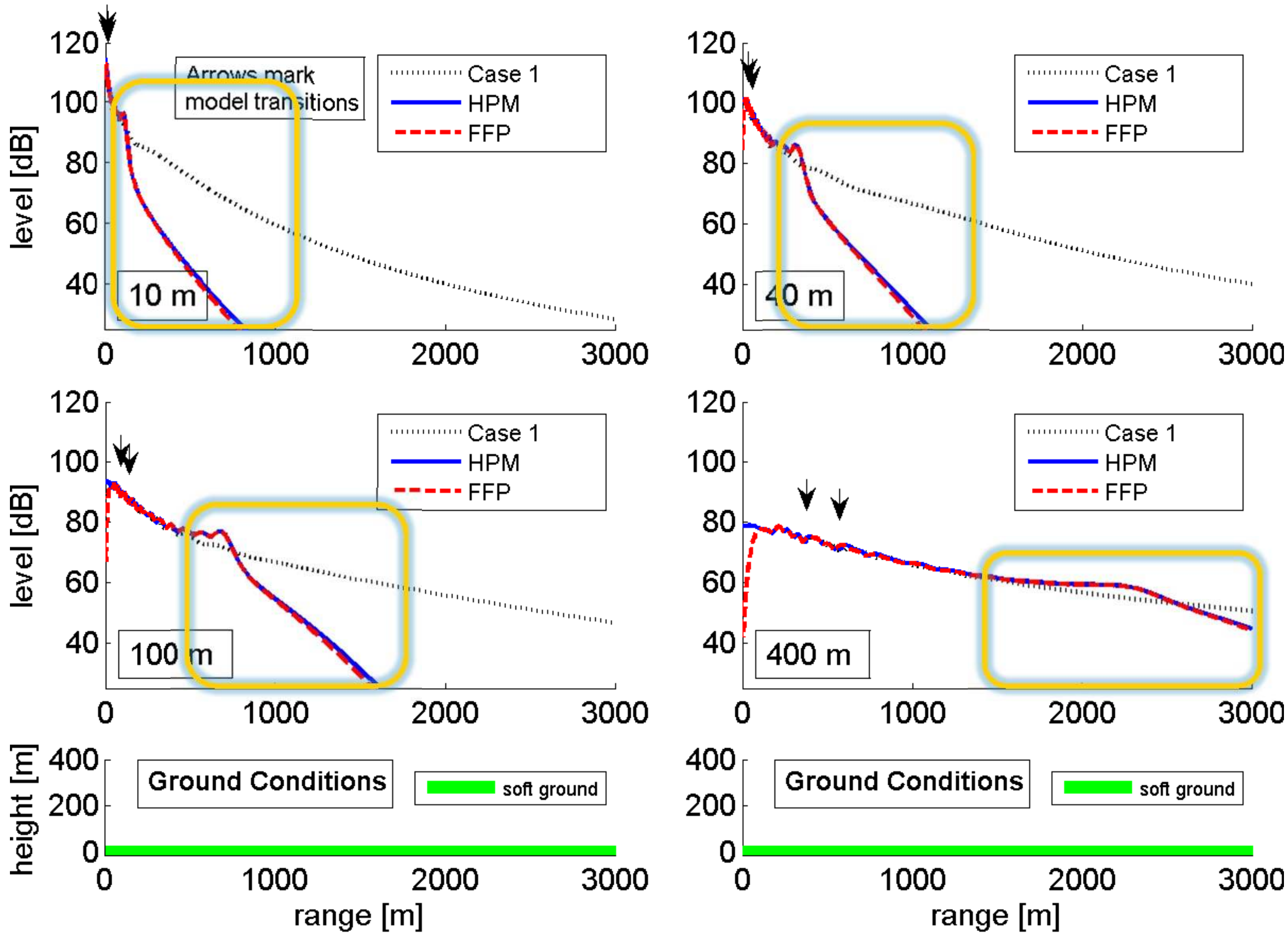
Model Comparison: Soft, flat ground, upward refracting atm

Receiver is in/near shadow zone



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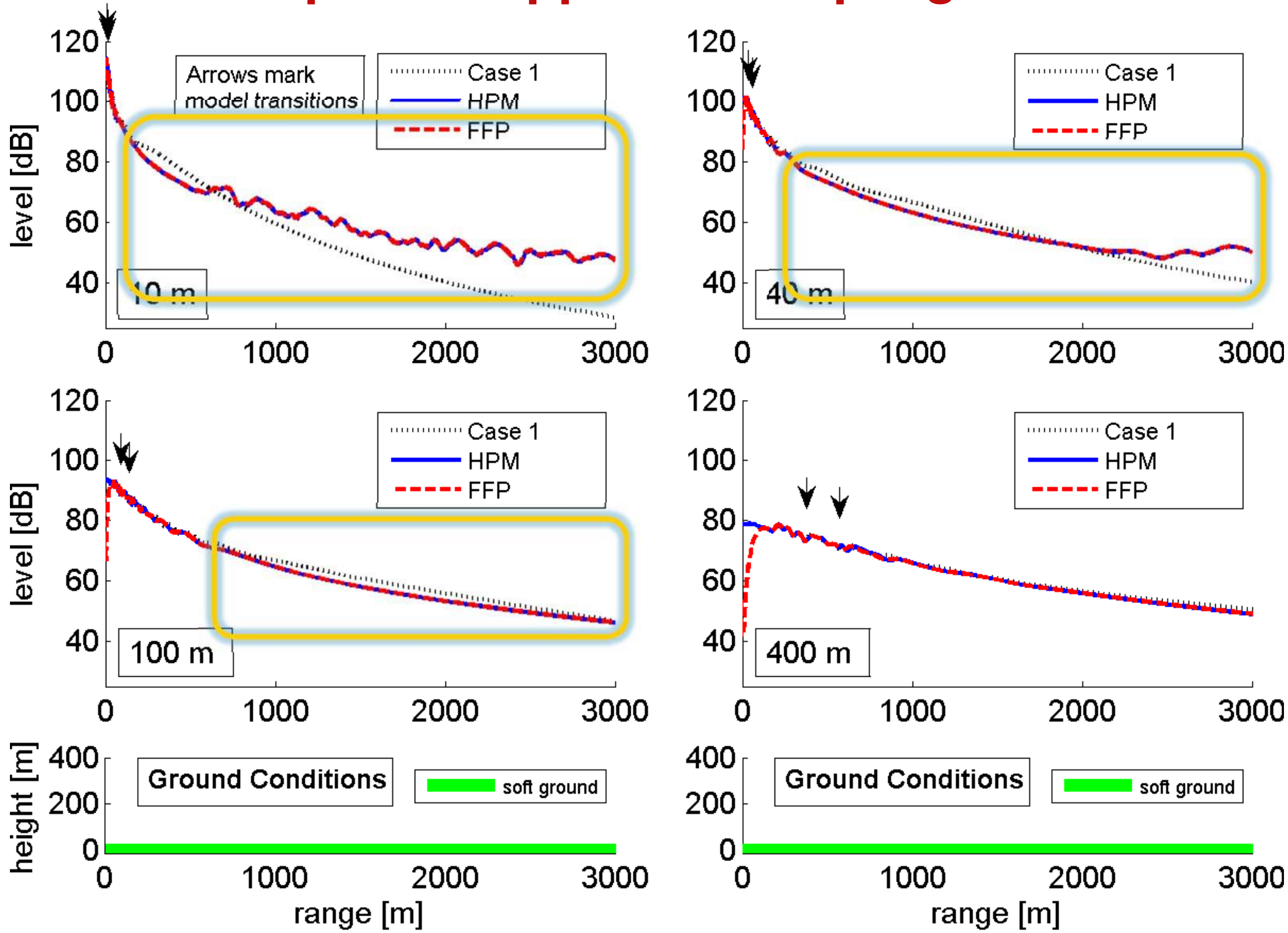
Model Comparison: Soft, flat ground, downward refracting atm

Atmosphere supports multiple ground reflections



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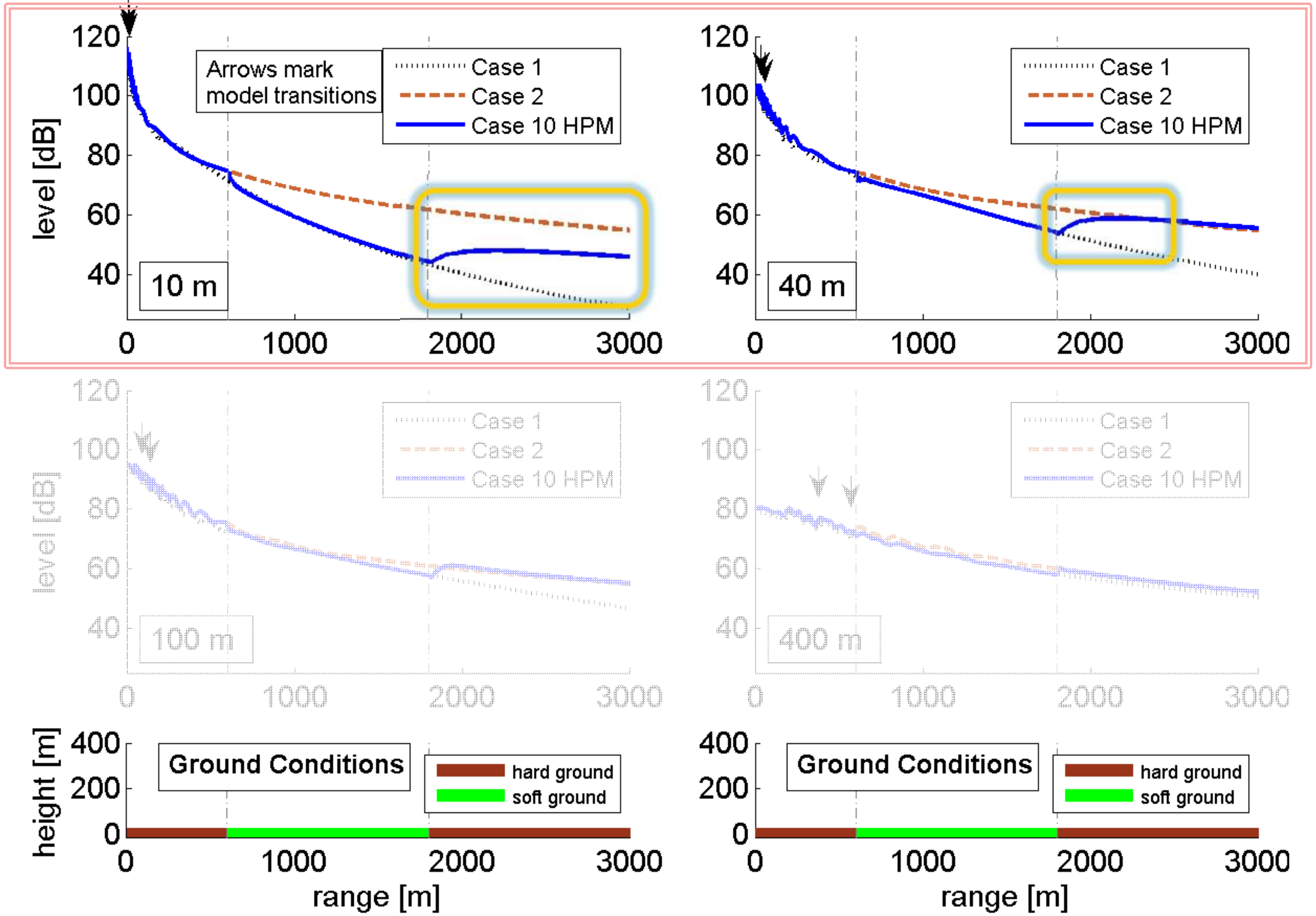
Model Comparison: Flat, hard to soft to hard ground

Source is low and receiver is near a ground type transition



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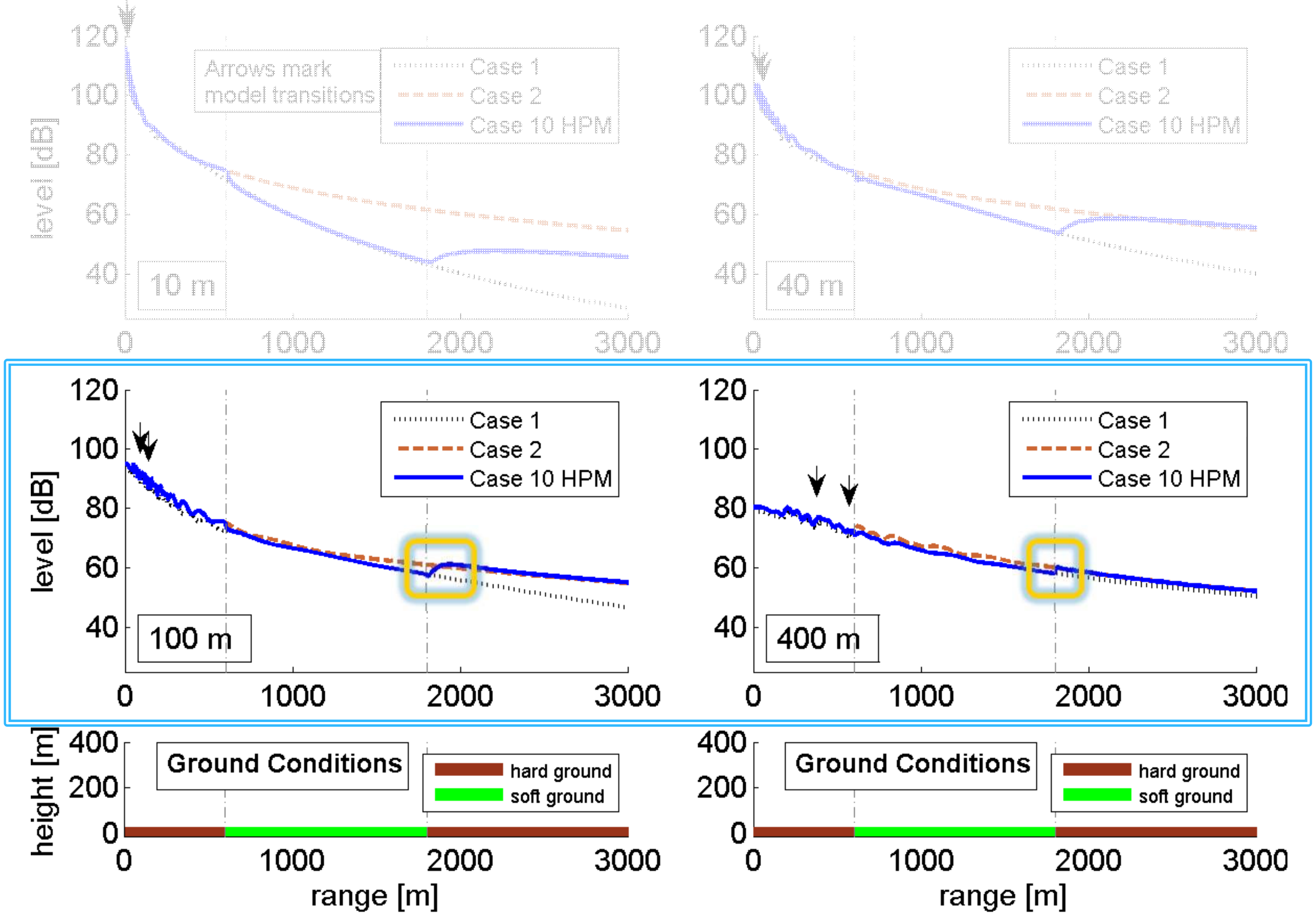
Model Comparison: Flat, hard to soft to hard ground

Source is high and receiver far from a ground type transition



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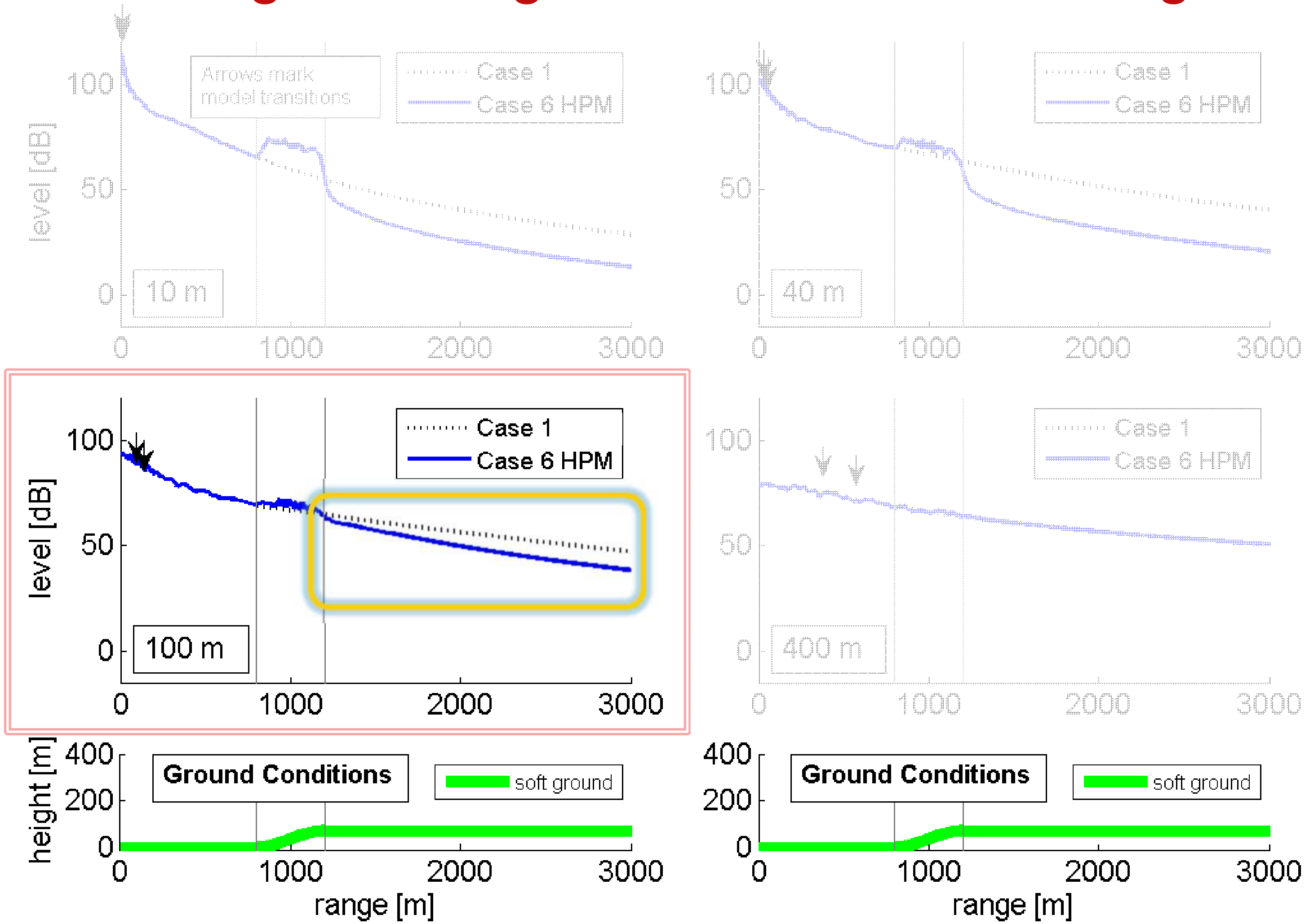
Model Comparison: Soft ground, upward sloping terrain

Terrain changes the angle of reflection off a soft ground



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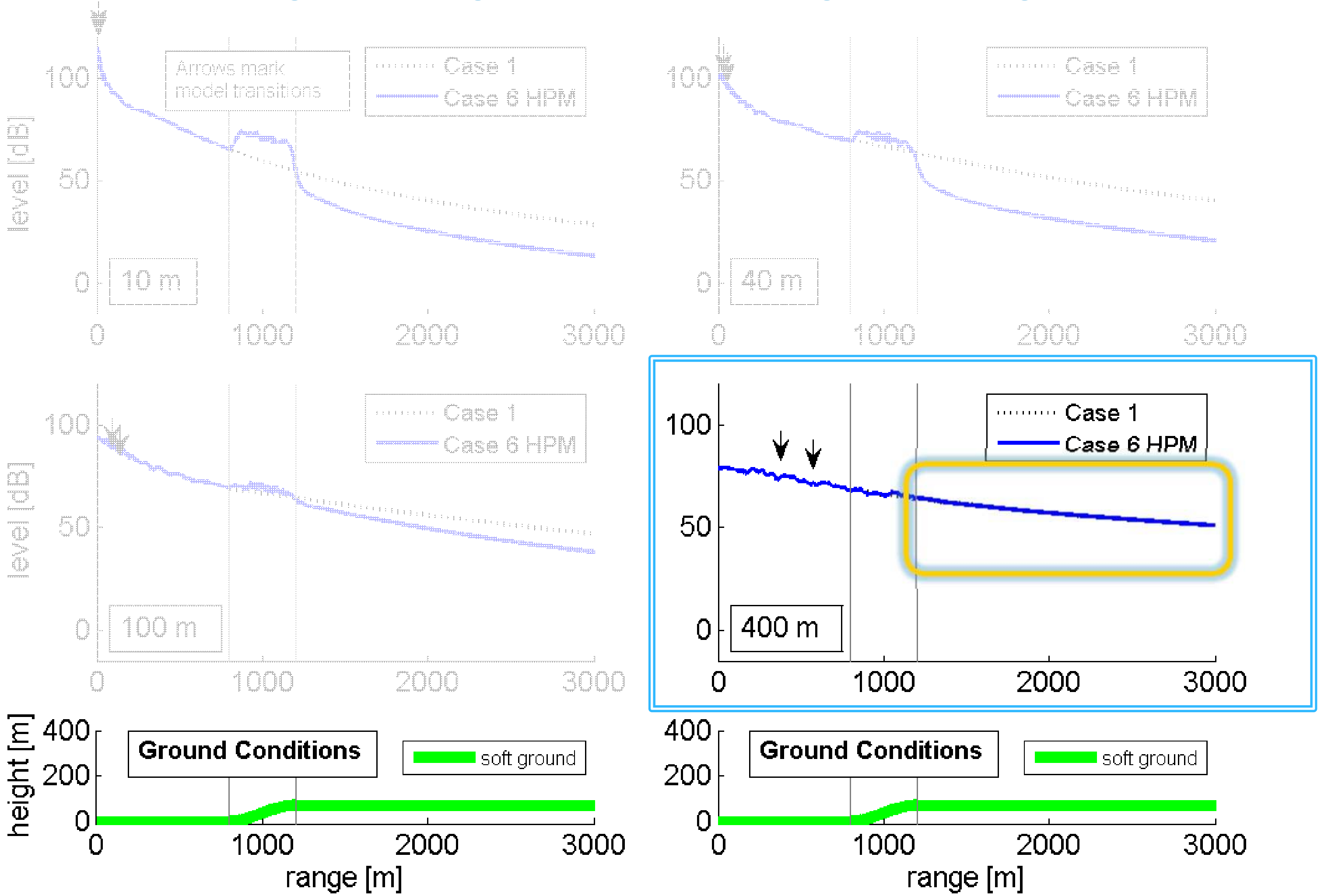
Model Comparison: Soft ground, upward sloping terrain

No LOS blockage or significant change to angle of reflection



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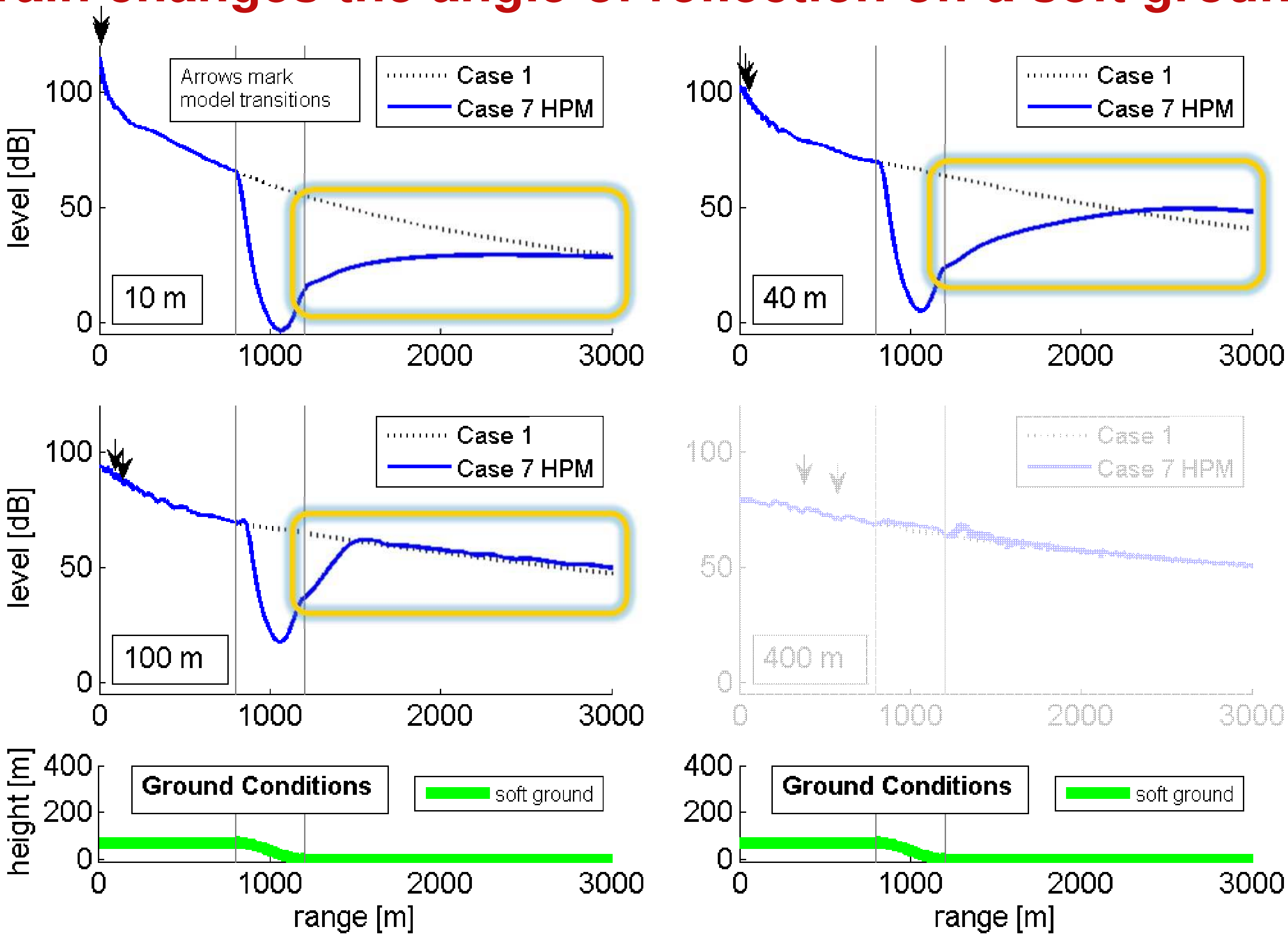
Model Comparison: Soft ground, downward sloping terrain

Terrain changes the angle of reflection off a soft ground



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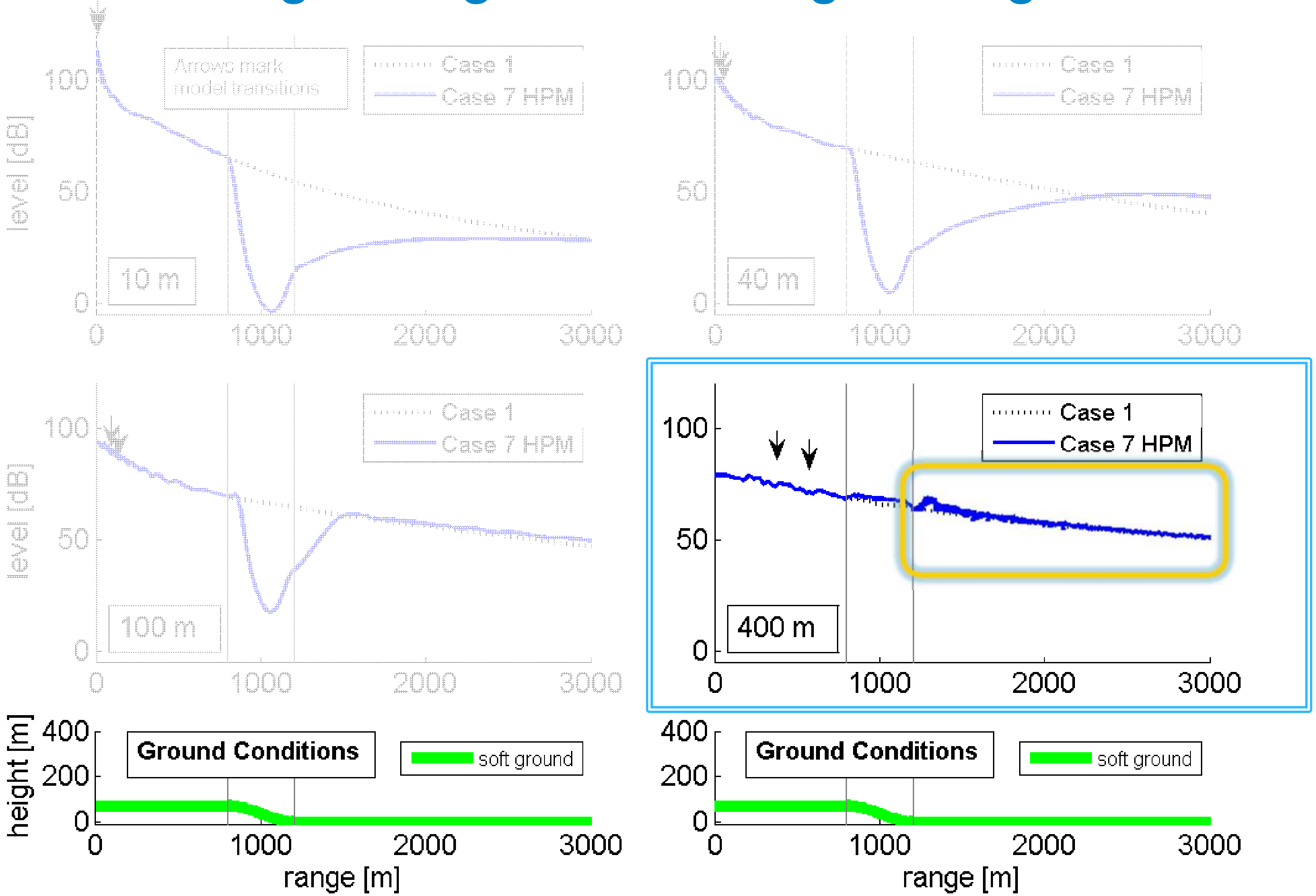

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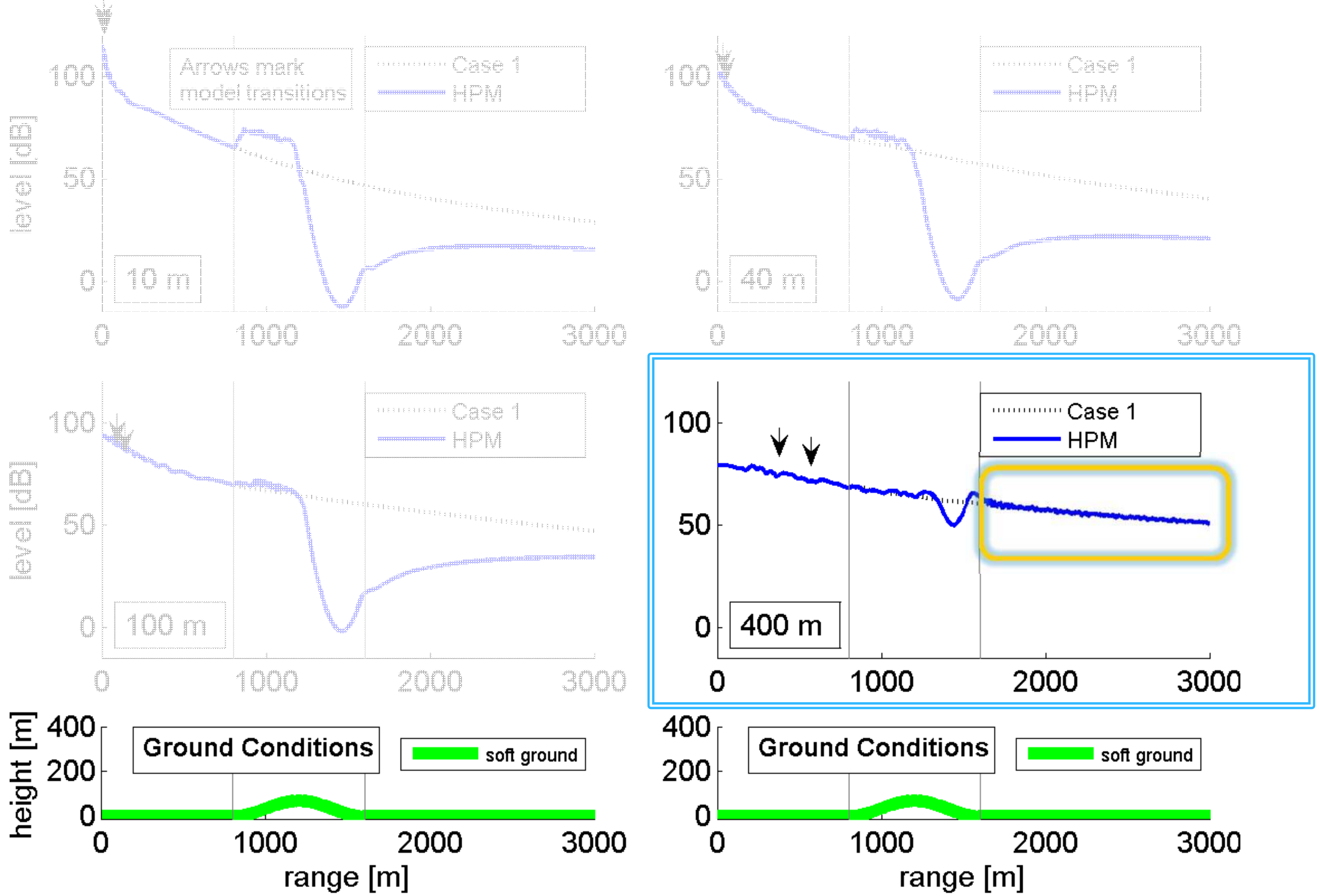

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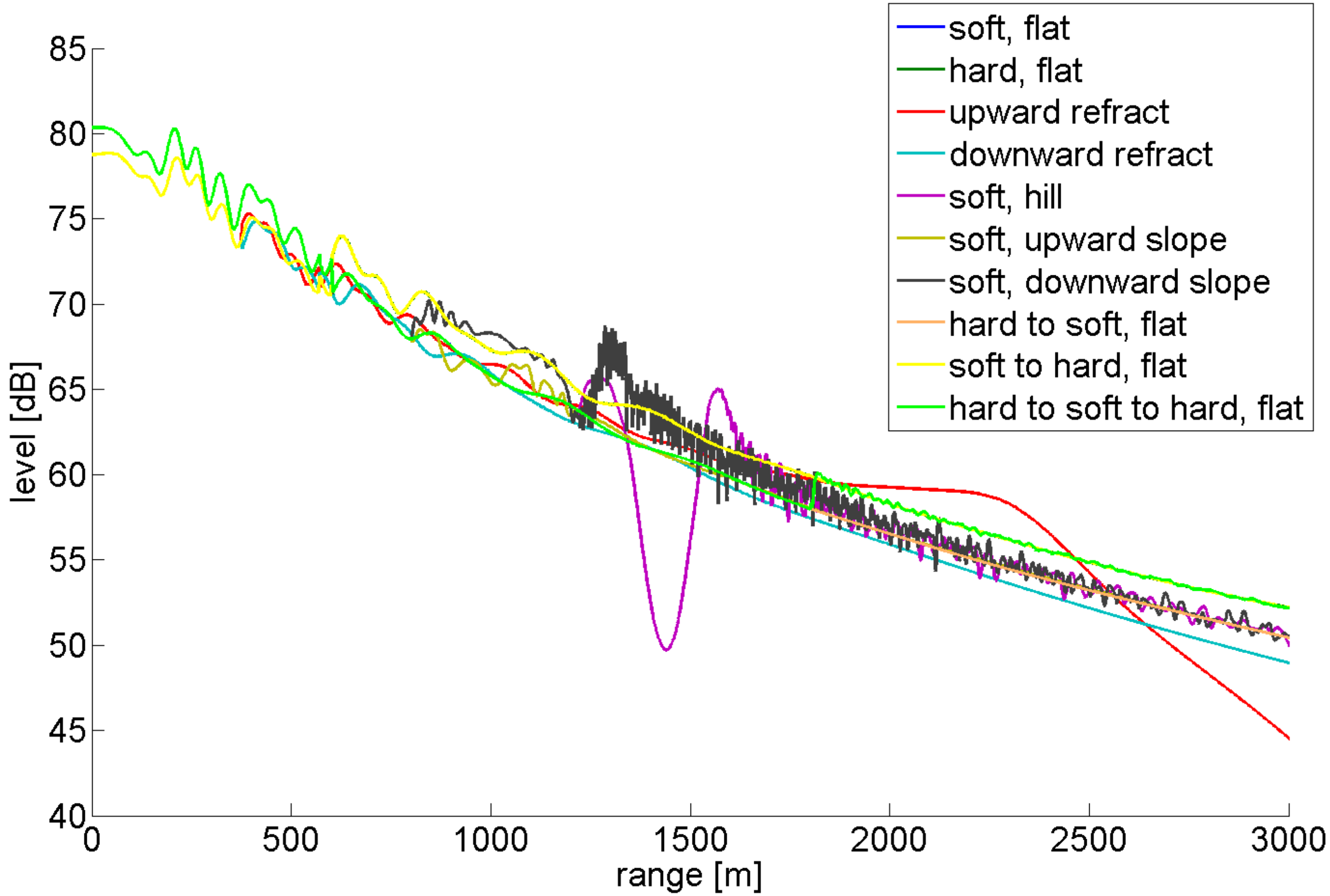
Model Comparison: All Cases, 400 m Source Height

Source has a very high altitude



10 cases, source at 400 m height

Model Comparison: All Cases, 400 m Source Height

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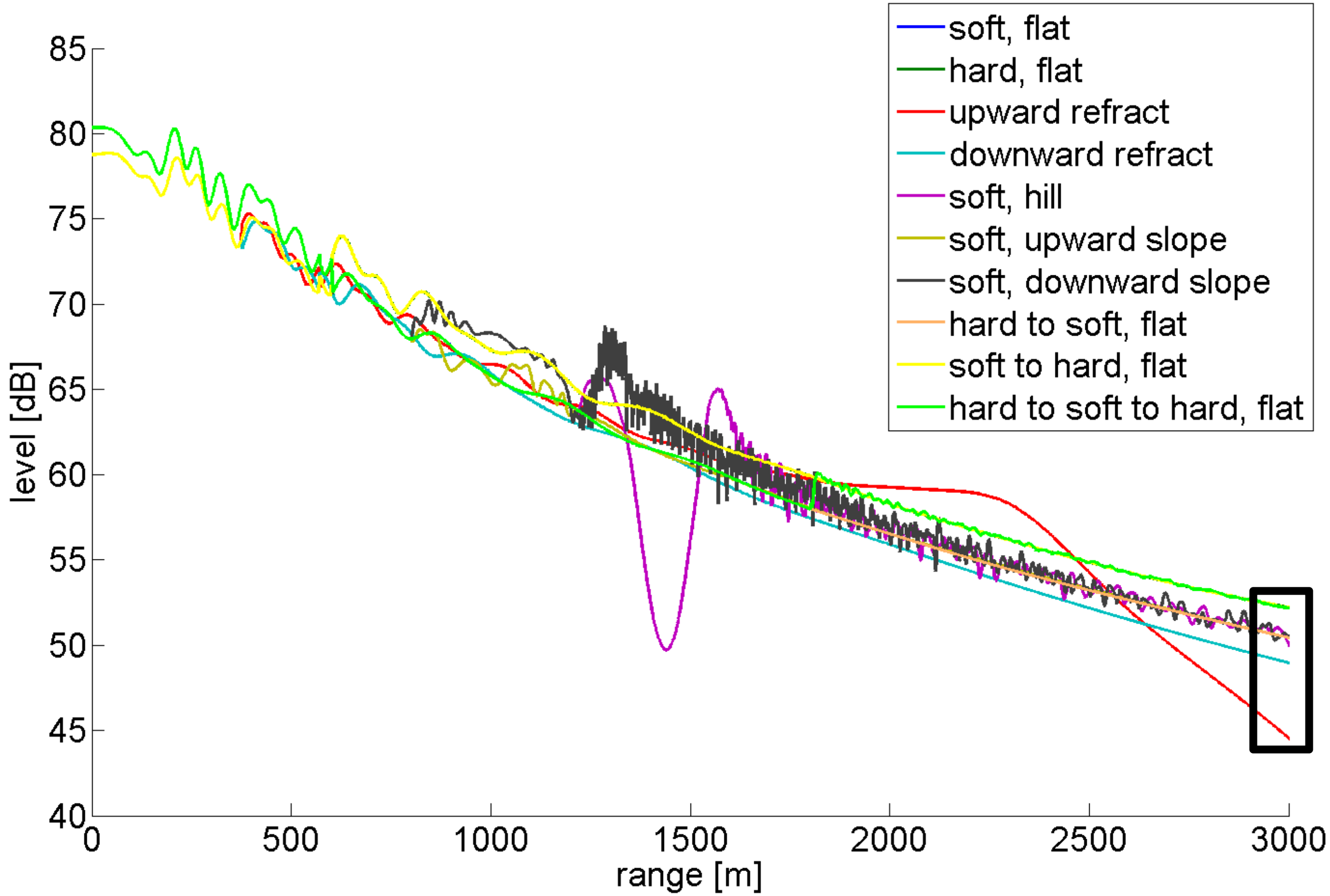

Zoom in over 2900 – 3000 m range)

10 cases, source at 400 m height

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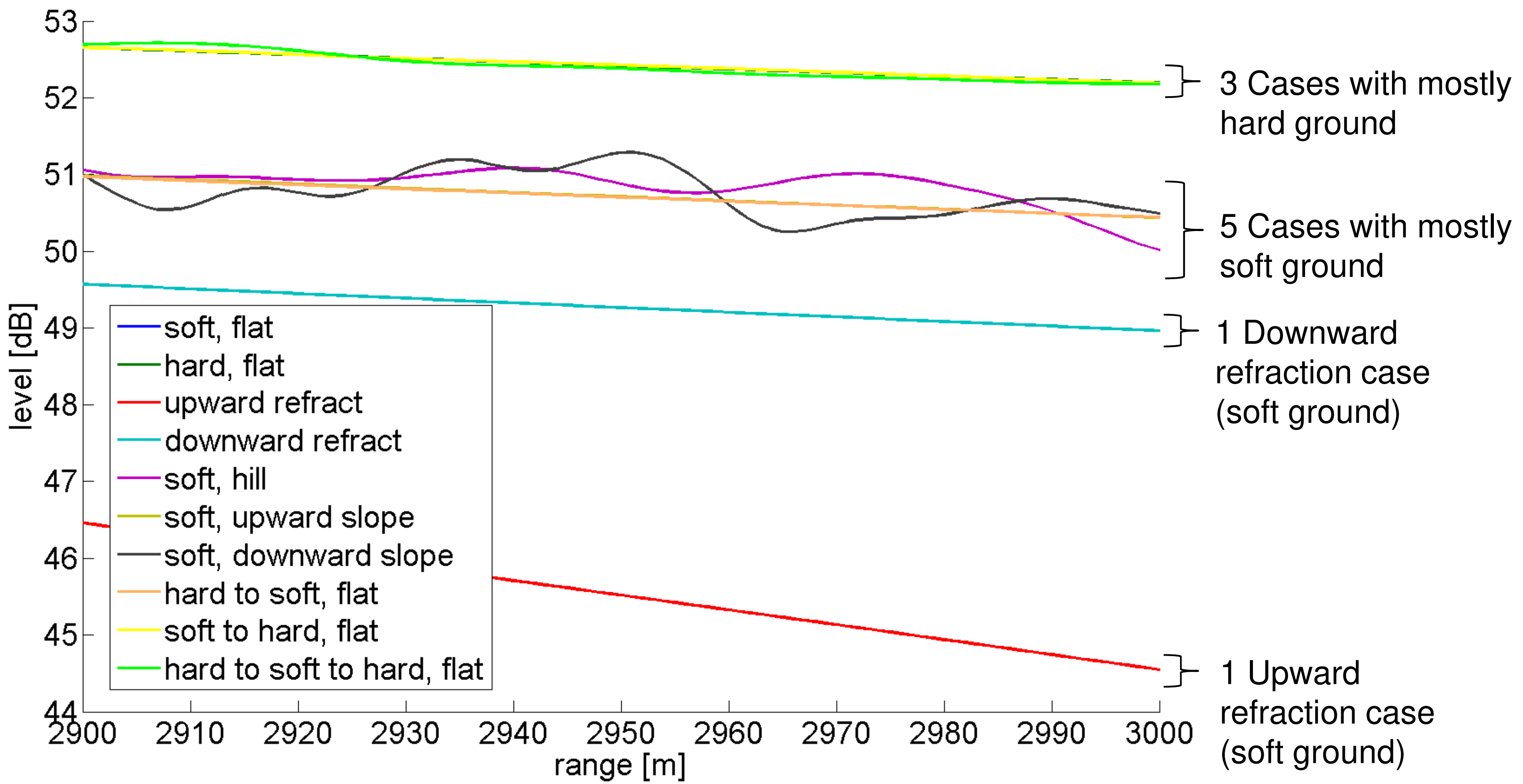
Model Comparison: All Cases, 400 m Source Height

Source has a very high altitude



10 cases, source at 400 m height

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Summary

- Ten different sets of propagations conditions were run with the HPM and its component FFP and ray trace models for 4 source heights, using a 747-400 aircraft spectrum
- The results of the cases were analyzed and compared to provide insight into the effects of the different propagation mechanisms on noise level predictions
- Conditions were identified that did not require the use of the full HPM
- The investigation is part of the long-term goal to integrate more accurate propagation methods into AEDT, allowing for accurate modeling of complex noise propagation conditions, while keeping runtimes manageable

Conclusions

Instead of using a broad brush approach for propagation modeling, sets of conditions can be parsed by assessing the needs of an individual case, and assigning an appropriate model to decrease runtime without significantly sacrificing accuracy.

Use Full Model	Use Simple Model
• Line of sight is blocked by a terrain feature	• Ground is flat and atmosphere is homogeneous
• Receiver is in/near a shadow zone, or the atmosphere supports multiple ground reflections. (Can use FFP)	• Source is high, and receiver is far from a ground type transition
• Source is low, and receiver is near a ground type transition	• A terrain feature may exist, but does not break the line of sight or significantly change the angle of ground reflection
• Terrain shape significantly changes the angle of reflection off a soft ground surface	• Source has a very high altitude

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Acknowledgements

The authors gratefully acknowledge the support of the FAA Office of Environment and Energy, FAA Western-Pacific Region, and the National Park Service (NPS) Natural Sounds and Night Skies Division.

Contact Information

Joyce Rosenbaum
Computer Sciences Corporation
E-mail: Joyce.Rosenbaum.CTR@dot.gov

Eric Boeker
United States Department of Transportation
Volpe National Transportation Systems Center
Environmental Measurement & Modeling Division, RVT-41
E-mail: Eric.Boeker@dot.gov

Thank You

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Extra Slides

Parabolic Equation Method: Derivation

3D Helmholtz Equation: $$\nabla^2 p + k^2 p = 0$$

Parabolic Equation Method: Derivation

3D Helmholtz Equation: $$\nabla^2 p + k^2 p = 0$$

↓

- –Cylindrical coordinates
- –Neglect azimuthal variation
- –Substitute $q = p\sqrt{r}$
- –Assume far-field

Parabolic Equation Method: Derivation

3D Helmholtz Equation:

$$\nabla^2 p + k^2 p = 0$$

$$\downarrow$$

2D Helmholtz in *q*:

$$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

Parabolic Equation Method: Derivation

3D Helmholtz Equation: $$\nabla^2 p + k^2 p = 0$$

↓

2D Helmholtz in *q*: $$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

↓

- Group *z*-dependent terms
- Divide into forward and backward propagating terms
- Neglect backward propagating sound

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Parabolic Equation Method: Derivation

3D Helmholtz Equation: $$\nabla^2 p + k^2 p = 0$$

↓

2D Helmholtz in *q*: $$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

↓

$$\left[\frac{\partial}{\partial r} - i\sqrt{H}\right]\left[\frac{\partial}{\partial r} + i\sqrt{H}\right] q = 0$$

where $H(z) = \frac{\partial^2}{\partial z^2} + k^2(z)$

Parabolic Equation Method: Derivation

3D Helmholtz Equation:

$$\nabla^2 p + k^2 p = 0$$

↓

2D Helmholtz in *q*:

$$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

↓

One-way Helmholtz:

$$\frac{\partial q}{\partial r} - i\sqrt{H}q = 0$$

where $H(z) = \frac{\partial^2}{\partial z^2} + k^2(z)$

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Parabolic Equation Method: Derivation

3D Helmholtz Equation:

$$\nabla^2 p + k^2 p = 0$$

↓

2D Helmholtz in *q*:

$$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

↓

One-way Helmholtz:

$$\frac{\partial q}{\partial r} - i\sqrt{H}q = 0$$

where $H(z) = \frac{\partial^2}{\partial z^2} + k^2(z)$

↓

–Substitute solution with slowly varying envelope function $q = \psi(r,z)e^{jk_a r}$

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Parabolic Equation Method: Derivation

3D Helmholtz Equation:

$$\nabla^2 p + k^2 p = 0$$

↓

2D Helmholtz in *q*:

$$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k^2 q = 0$$

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One-way Helmholtz:

$$\frac{\partial q}{\partial r} - i\sqrt{H} q = 0$$

where $H(z) = \frac{\partial^2}{\partial z^2} + k^2(z)$

↓

One-way Helmholtz in ψ:

$$\frac{\partial \psi}{\partial r} + i\left(k_a - \sqrt{H}\right)\psi = 0$$

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Parabolic Equation Method: Derivation

One-way Helmholtz in ψ: $$\frac{\partial \psi}{\partial r} + i\left(k_a - \sqrt{H}\right)\psi = 0$$

↓

–Substitute approximation

$$\sqrt{H} \approx k_a \frac{1 + \frac{3}{4}s}{1 + \frac{1}{4}s}$$

where $s = \dfrac{k^2(z) - k_a^2}{k_a^2} + \dfrac{1}{k_a^2}\dfrac{\partial^2}{\partial z^2}$

Parabolic Equation Method: Derivation

One-way Helmholtz in ψ:
$$\frac{\partial \psi}{\partial r} + i\left(k_a - \sqrt{H}\right)\psi = 0$$

Wide-Angle Parabolic Equation:
$$\left(1 + \frac{1}{4}s\right)\frac{\partial \psi}{\partial r} = \frac{1}{2} i k_a s \psi$$

Parabolic Equation Method: Derivation

Wide-Angle Parabolic Equation

$$\left(1+\frac{1}{4}s\right)\frac{\partial\psi}{\partial r}=\frac{1}{2}ik_a s\psi$$

The central difference formula is applied to represent $\partial^2\psi/\partial z^2$ within the variable *s* as

$$\left(\frac{\partial^2\psi}{\partial z^2}\right)_{z_j} \rightarrow \frac{\psi_{j+1}-2\psi_j+\psi_{j-1}}{(\Delta z)^2}$$

Parabolic Equation Method: Derivation

Wide-Angle Parabolic Equation

$$\left(1+\frac{1}{4}s\right)\frac{\partial\psi}{\partial r}=\frac{1}{2}ik_a s\psi$$

The equation is integrated over *r*

The Crank-Nicholson approximation is applied to represent the integral of ψ over *r*

$$\int_{r}^{r+\Delta r}\vec{\psi}\,dr \rightarrow \frac{1}{2}\left[\vec{\psi}(r+\Delta r)+\vec{\psi}(r)\right]\Delta r$$

Parabolic Equation Method: Derivation

- An equation of the form

$$M_2 \vec{\psi}(r+\Delta r) = M_1 \vec{\psi}(r)$$

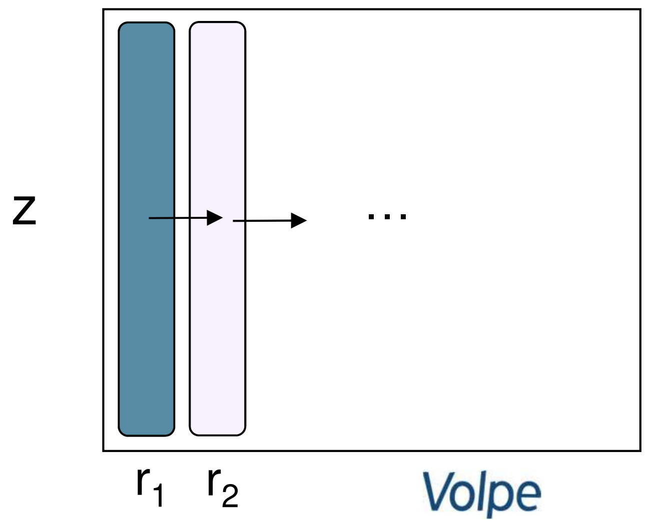
results where M_1 and M_2 are tridiagonal matrices

- To calculate the sound field, the starting field

$$\vec{\psi}(0) \equiv \psi(0,z) = q(0,z)$$

is required as input

- A Gaussian function starting field is the standard representation for an omnidirectional monopole source



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Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r dk_r$$

$$P(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r dr$$

Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r dk_r$$

$$P(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r dr$$

↓

- Substitute $q = p\sqrt{r}$
- Impose a far-field approximation
- Transform the cylindrical coordinate Helmholtz equation, neglecting azimuthal variation
- Assume wave number is constant in each layer of the atmosphere

Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r dk_r$$

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↓

1D Height-dependent transformed Helmholtz:

$$\frac{\partial^2 Q}{\partial z^2} + k_z^2 Q = -S_\delta \delta(z - z_s)$$

Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r \, dk_r$$

$$P(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r \, dr$$

↓

1D Height-dependent transformed Helmholtz:

$$\frac{\partial^2 Q}{\partial z^2} + k_z^2 Q = -S_\delta \delta(z - z_s)$$

↓

–Write down solution for 1D Helmholtz equation of this form

–Discretize equation for layered atmosphere

Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r dk_r$$

$$P(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r dr$$

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1D Height-dependent transformed Helmholtz:

$$\frac{\partial^2 Q}{\partial z^2} + k_z^2 Q = -S_\delta \delta(z - z_s)$$

↓

1D discretized Helmholtz solution:

$$Q_j = A_j \exp(ik_{zj} z) + B_j \exp(-ik_{zj} z)$$

for $z_j \le z \le z_{j+1}$

Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r dk_r$$

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$$Q_j = A_j \exp(ik_{zj} z) + B_j \exp(-ik_{zj} z)$$

for $z_j \le z \le z_{j+1}$

↓

–Use the discretized solution of Q at z and $z + \Delta z$, and their derivatives to extrapolate the sound field between layers of the atmosphere

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Fast Field Program Method: Derivation

Hankel Transform Pair:

$$p(r,z) = \int_0^\infty P(k_r,z) J_0(k_r r) k_r \, dk_r$$

$$P(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r \, dr$$

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$$\frac{\partial^2 Q}{\partial z^2} + k_z^2 Q = -S_\delta \delta(z - z_s)$$

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1D discretized Helmholtz solution:

$$Q_j = A_j \exp(ik_{zj} z) + B_j \exp(-ik_{zj} z)$$

for $z_j \le z \le z_{j+1}$

↓

Relations between z and $z + \Delta z$ in layer j

$$Q_j(z + \Delta z) = \cos(k_{zj}\Delta z) Q_j(z) + k_{zj}^{-1} \sin(k_{zj}\Delta z) Q'_j(z)$$

$$Q'_j(z + \Delta z) = -k_{zj} \sin(k_{zj}\Delta z) Q_j(z) + \cos(k_{zj}\Delta z) Q'_j(z)$$

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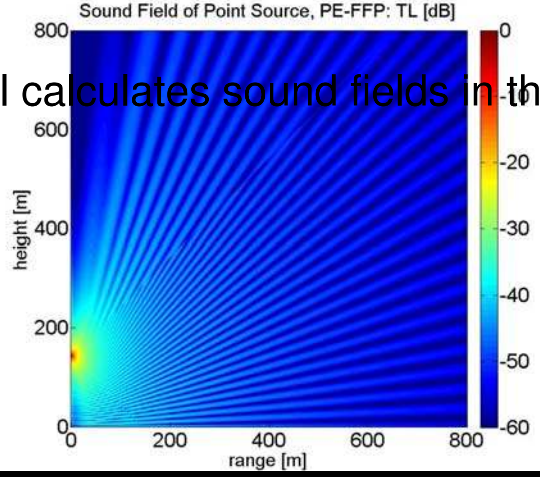
Fast Field Program Method: Derivation

- Implement boundary conditions between the ground and atmosphere and between atmosphere layers
- Manipulate the inverse transform into the form of a Fourier transform
- Use the inverse transform to numerically calculate the sound field

Model expanded from 2D to pseudo-3D

1

Model calculates sound fields in the vertical plane



Model expanded from 2D to pseudo-3D

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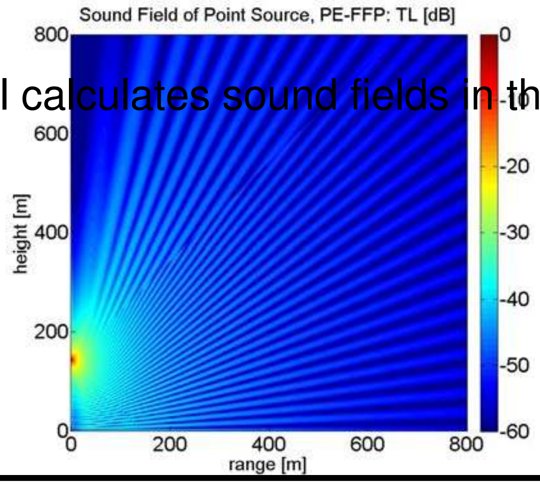
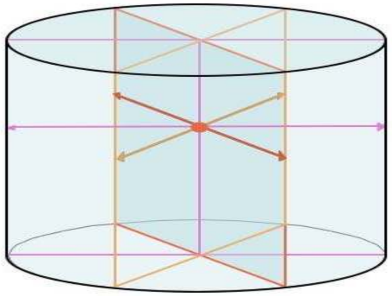
Model calculates sound fields in the vertical plane



2

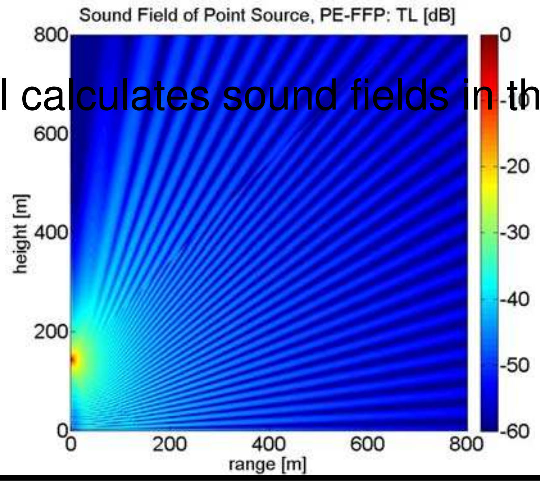
Several runs are performed radially outward from the source at specified angle increments.



Model expanded from 2D to pseudo-3D

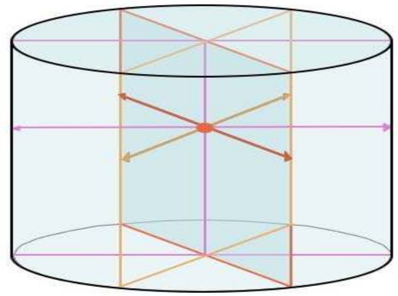
1

Model calculates sound fields in the vertical plane



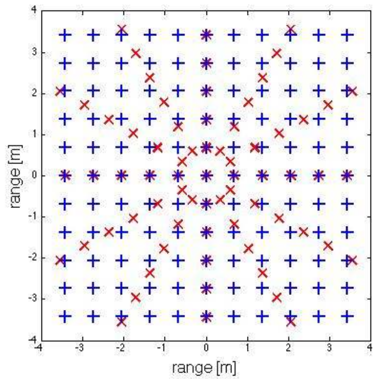
2

Several runs are performed radially outward from the source at specified angle increments.



3

Sound field on a Cartesian grid is interpolated (4 point linear) from polar coordinate grid points.



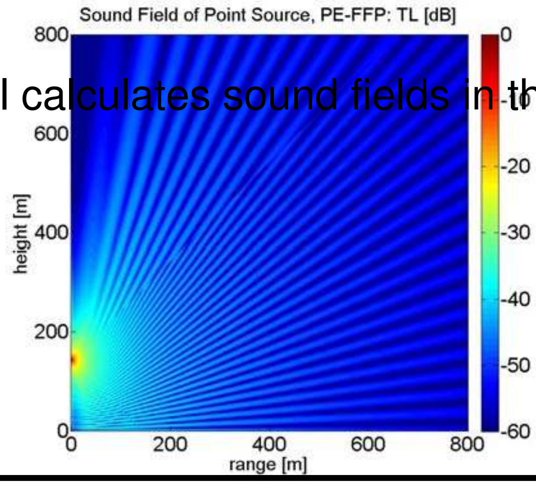
x points calculated

\+ points interpolated

Model expanded from 2D to pseudo-3D

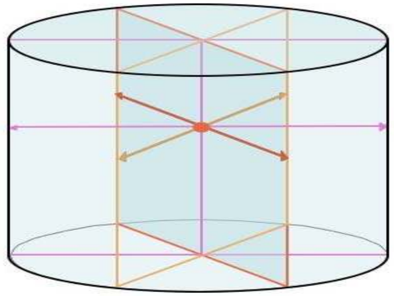
1

Model calculates sound fields in the vertical plane



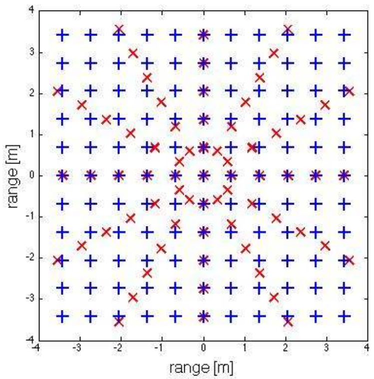
2

Several runs are performed radially outward from the source at specified angle increments.



3

Sound field on a Cartesian grid is interpolated (4 point linear) from polar coordinate grid points.

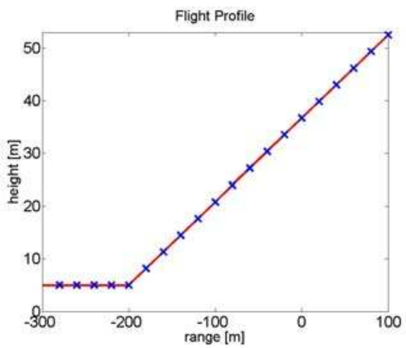


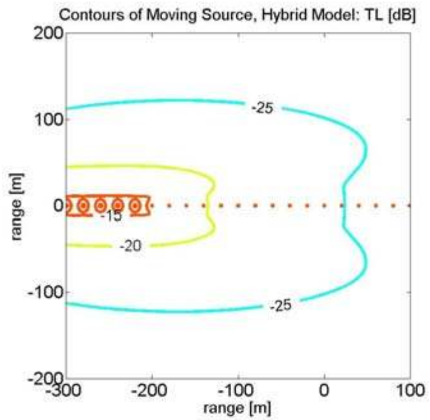
x points calculated

+ points interpolated

4

For a given flight profile, sound fields from point sources at all represented locations are added incoherently together.





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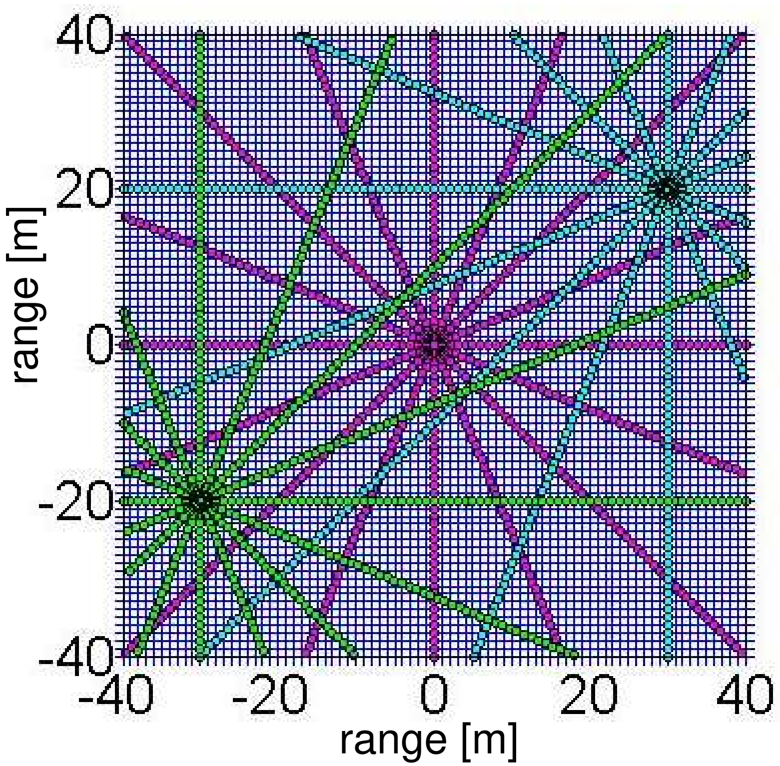
3D Range-Dependent Effect Capabilities

1

Rectangular grid data for terrain and ground impedance is interpolated to individual polar coordinate systems for each source



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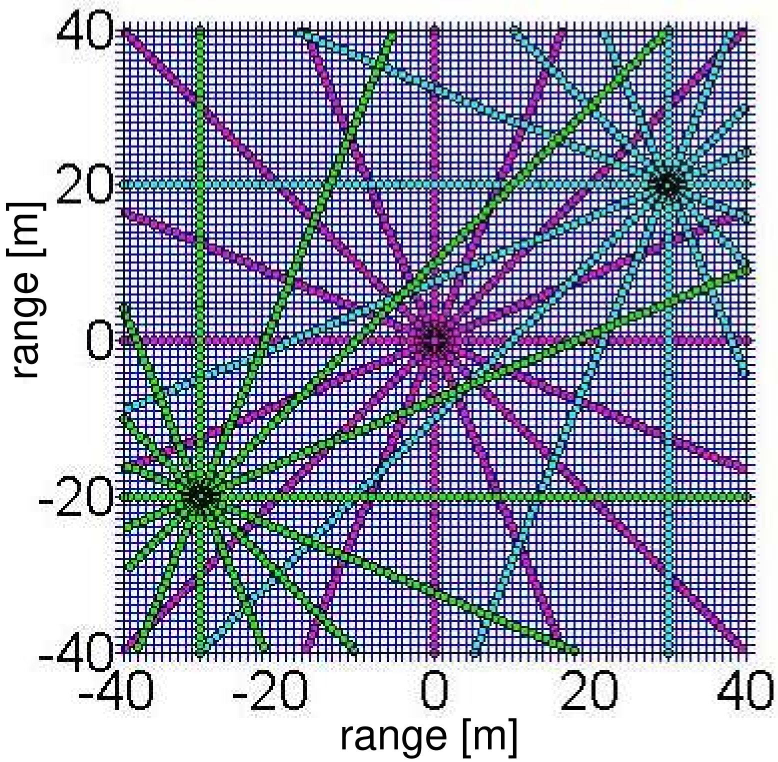
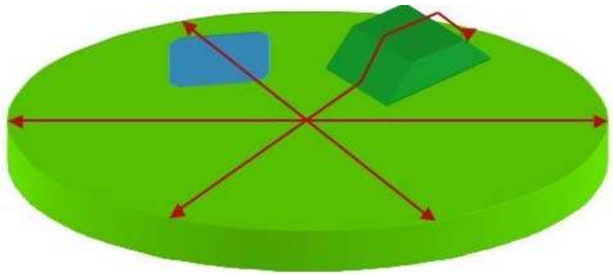
3D Range-Dependent Effect Capabilities

1 Rectangular grid data for terrain and ground impedance is interpolated to individual polar coordinate systems for each source

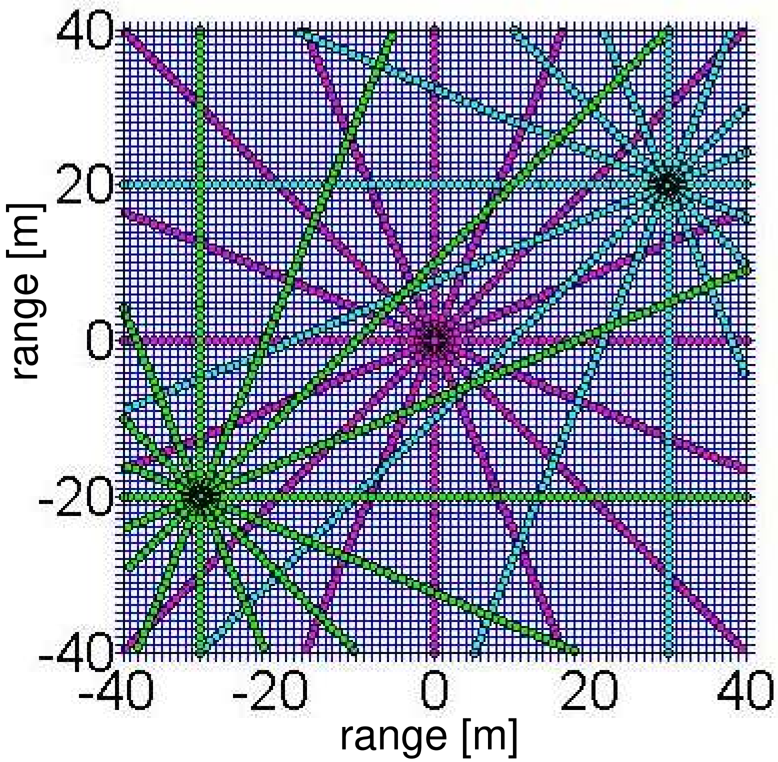


2 Interpolated terrain and impedance data is used for each radial run from each representative source along the flight path

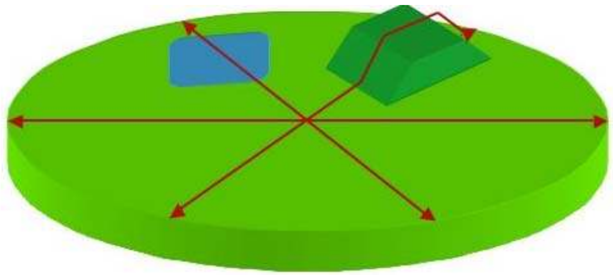


3D Range-Dependent Effect Capabilities

1 Rectangular grid data for terrain and ground impedance is interpolated to individual polar coordinate systems for each source



2 Interpolated terrain and impedance data is used for each radial run from each representative source along the flight path



3 Data from all source runs is combined to obtain sound fields that include range-dependent effects