AIAA Journal
Vol. 48, No. 6, June 2010

Enhancement of a Class of Adjoint Design Methods via Optimization of Parameters

Sangho Kim*
Konkuk University, Seoul 143-701, Republic of Korea
Kaveh Hosseini†
Exponent, Inc., Menlo Park, California 94025
Kasidit Leoviriyakit‡
Airbus UK, Bristol, England BS99 7AR, United Kingdom
and
Antony Jameson§
Stanford University, Stanford, California 94305

DOI: 10.2514/1.42985

Nomenclature

C_D	=	drag coefficient
C_L	=	lift coefficient
C_p	=	pressure coefficient
C_W	=	structural weight coefficient
$\mathcal{F}$	=	design variable
$\mathcal{G}$	=	gradient
$\bar{\mathcal{G}}$	=	modified gradient using Sobolev inner product
i, j, k	=	cell indices in the three computational coordinate directions
I	=	cost function
M_∞	=	freestream Mach number
R	=	residual
$\bar{R}$	=	implicitly smoothed residual
r	=	scaled spectral radii of the flux Jacobian matrices
α	=	angle of attack
α_l	=	weighing coefficients in cost function
β	=	smoothing coefficient factor
δ	=	variation
ϵ	=	gradient smoothing coefficient
ε	=	residual smoothing coefficient
λ	=	step size
ξ	=	computational coordinate

Introduction

SINCE the adjoint method was introduced to aerodynamic shape optimization in 1980s [1,2], this method has become a popular choice for design problems involving fluid flow and has been successfully used for the aerodynamic design of complete aircraft configurations [3–6].

In the authors' previous works on the use of control theory, every surface mesh point was used as a design variable. Using this approach, the complete design space of all airfoil shapes that can be represented by a given number of surface points can be spanned. A problem of this choice is that the smoothness of the aerodynamic shape may not be preserved. Similar to the implicit residual smoothing [7], the gradient was implicitly smoothed to preserve the first derivative continuity of the solution in the development of the adjoint formulation.

The implicit gradient smoothing may also be regarded as a preconditioner that allows the use of much larger steps for the search procedure and leads to a large reduction in the number of design iterations needed for convergence [8]. The efficiency of the steepest descent method results from the elimination of the need for exact gradients and the effectiveness of implicit smoothing as a preconditioner.

Determining the value of the smoothing coefficient and the size of the step for the shape change in the steepest descent method are critical for a successful design in practice. Moreover, as computational fluid dynamics (CFD) analysis and design codes become more complex, there is an increasing number of input parameters, and it is also very time consuming to find a proper combination of parameters.

In this work, the existing adjoint-based design method was tuned by determining the control parameters of the CFD analysis and design process with a nonlinear gradient-based optimization package, SNOPT [9]. Three approaches of enhancing the Euler adjoint design methods have been investigated. First, the convergence of the Euler and adjoint solutions was accelerated by optimizing the coefficients of the residual smoothing scheme and the Courant number. Second, the input parameters, the gradient smoothing coefficient, and the step size of the Euler adjoint design methods were optimized such that the best aerodynamic shape can be achieved in a given number of design iterations. Finally, the SNOPT software has also been used to provide line searches of the shape optimization parameters at each step. The numerical results will be presented to show the utility of the integration of the SNOPT package and the adjoint software.

Residual Smoothing Parameter Optimization

The SNOPT package was successfully used by Hosseini and Alonso [10] to optimize various input parameters for explicit Euler and Navier–Stokes flow solvers, including the multistage coefficients of the modified Runge–Kutta schemes. The use of residual smoothing can accelerate the convergence of flow solution and, consequently, the design optimization.

Description of Implicit Residual Smoothing

The general idea behind this technique is to increase the time step limit by replacing the residual at one cell in the flowfield by a

Presented as Paper 0448 at the 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV, 10–13 January 2005; received 16 January 2009; revision received 30 January 2010; accepted for publication 28 February 2010.

*Assistant Professor, Department of Aerospace Information Engineering. Member AIAA.
†Engineer, 149 Commonwealth Drive. Student Member AIAA.
‡Wing Shape Design Engineer, Filton. Student Member AIAA.
§Thomas V. Jones Professor of Engineering, Department of Aeronautics and Astronautics. Fellow AIAA.

Table 1 Results for residual smoothing optimization

Case: Euler	β_i	β_j	β_k	CFL^*	R_{200}
Reference or initial	0.6	0.6	0.6	9.0	1.631E − 06
Optimal	0.459	0.471	0.450	7.18	1.914E − 08

weighted average of the residuals at the neighboring cells. The average is calculated implicitly:

$$(1-\varepsilon_i\delta_{xx})(1-\varepsilon_j\delta_{yy})(1-\varepsilon_k\delta_{zz})\bar{R}_{i,j,k} = R_{i,j,k} \tag{1}$$

where $\varepsilon_{i,j,k}$ control the level of smoothing.

The smoothing parameters in each direction should satisfy the following inequalities.

$$\varepsilon_{i,j,k} = \max\left\{\frac{1}{4}\left[\left(\frac{CFL^*}{CFL} r_{i,j,k}\right)^2 - 1\right], 0\right\} \tag{2}$$

where CFL^* is the new Courant number, and CFL is the maximum allowable Courant number for the scheme without implicit residual averaging. A complete discussion of the stability character and overall benefit of this acceleration method is provided by Martinelli [11].

Optimization of Residual Smoothing

In the usual practice of residual averaging, the coefficients are modified by

$$\varepsilon_i^* = \beta_i\varepsilon_i, \qquad \varepsilon_j^* = \beta_j\varepsilon_j, \qquad \varepsilon_k^* = \beta_k\varepsilon_k \tag{3}$$

Here, the proper values of β_i, β_j, β_k, and CFL^* depend on the flight conditions, flow conditions, and aspect ratio of the meshes, etc. Moreover, they must be empirically tuned by trial and error. In the present work, the values of β_i, β_j, β_k, and CFL^* for the three-dimensional Euler calculations are optimized using the SNOPT package such that a maximum flow convergence level can be reached within 200 multigrid cycles.

Results of Residual Smoothing Optimization

Euler calculations on the Boeing 747 wing–body configuration at a fixed angle of attack, $\alpha = 2.3$ deg, and $M_\infty = 0.87$ using a C-H grid of size $192 \times 32 \times 32$ has been performed. SNOPT calls the Euler solver for the parameter optimization. The optimization formulation is summarized as follows:

Table 2 Results for gradient smoothing optimization

Case	ϵ	λ	Design iterations	$C_{D_{\mathrm{final}}}(\Delta C_D)$
Parameter by trial and error	8.0	0.2	17	0.01007(0.00085)
Parameter by SNOPT	4.0	0.253	10	0.00986(0.00106)

$$\min\ R_{200}(\beta_{i,j,k}, CFL^*) \quad \text{subject to } \beta_{i,j,k}\text{: } 0 \le \beta_{i,j,k} \le 1 \quad \text{and} \quad CFL^*\text{: } 1 \le CFL^* \le 10$$

The resulting optimized residual smoothing coefficients and CFL^* number are listed in Table 1. Using these optimized parameters, the average density residual after 200 multigrid iterations, R_{200}, was improved by two orders of magnitude. A similar improvement in adjoint solution convergence has also been obtained using the same optimized parameters. As a result, the necessary flow and adjoint convergence levels of 10^{-4} for the accurate adjoint gradients were achieved with a fewer number of multigrid cycles, as shown in Fig. 1, and the overall adjoint design method could ideally be improved by 30%.

Enhancement of Shape Optimization Procedure

Description of Implicit Gradient Smoothing

Let $\mathcal{F}$ represent the design variable and $\mathcal{G}$ the gradient. An improvement could then be made with a shape change using the steepest descent method:

$$\delta\mathcal{F} = -\lambda\mathcal{G} \tag{4}$$

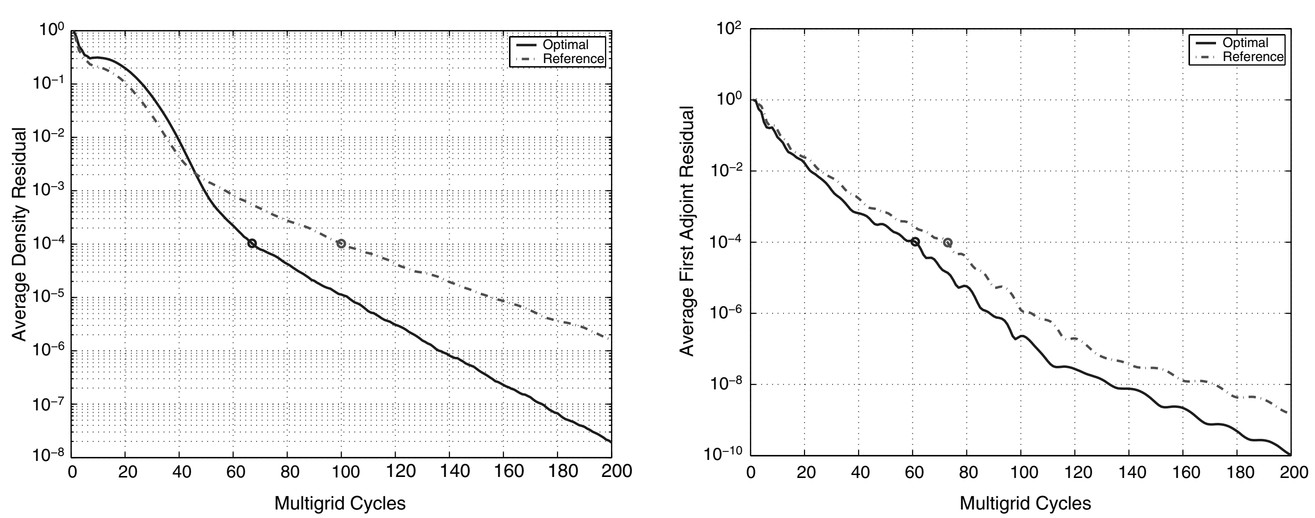
In fact, however, the gradient $\mathcal{G}$ is generally of a lower smoothness class than the shape $\mathcal{F}$. To preserve the smoothness, we redefine the gradient corresponding to a weighted Sobolev inner product of the form

$$\langle u, v\rangle = \int\left(uv + \epsilon\frac{\partial u}{\partial\xi}\frac{\partial v}{\partial\xi}\right)\mathrm{d}\xi \tag{5}$$

Thus, we define a modified gradient $\bar{\mathcal{G}}$ such that

$$\delta I = \langle\bar{\mathcal{G}}, \delta\mathcal{F}\rangle \tag{6}$$

In the one-dimensional case, taking $\bar{\mathcal{G}} = 0$ at the end points, integration by parts yields



a) Flow convergence history

b) Adjoint convergence history

Fig. 1 Residual smoothing optimization.

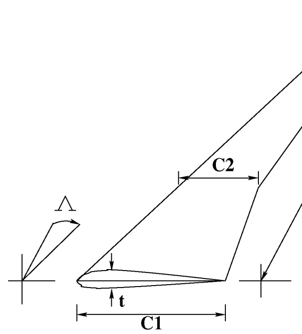


Fig. 2 Simplified wing planform of a transport aircraft.

$$\delta I = \int \left(\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial \bar{\mathcal{G}}}{\partial \xi_1} \right) \delta \mathcal{F} \, \mathrm{d}\xi \tag{7}$$

Then $\bar{\mathcal{G}}$ is obtained by solving the smoothing equation:

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial}{\partial \xi_1} \bar{\mathcal{G}} = \mathcal{G} \tag{8}$$

In the multidimensional case, the smoothing is applied in product form. Finally, we set

$$\delta \mathcal{F} = -\lambda \bar{\mathcal{G}} \tag{9}$$

with the result that

$$\delta I = -\lambda \langle \bar{\mathcal{G}}, \bar{\mathcal{G}} \rangle \qquad < 0 \tag{10}$$

unless $\bar{\mathcal{G}} = 0$, and correspondingly, $\mathcal{G} = 0$.

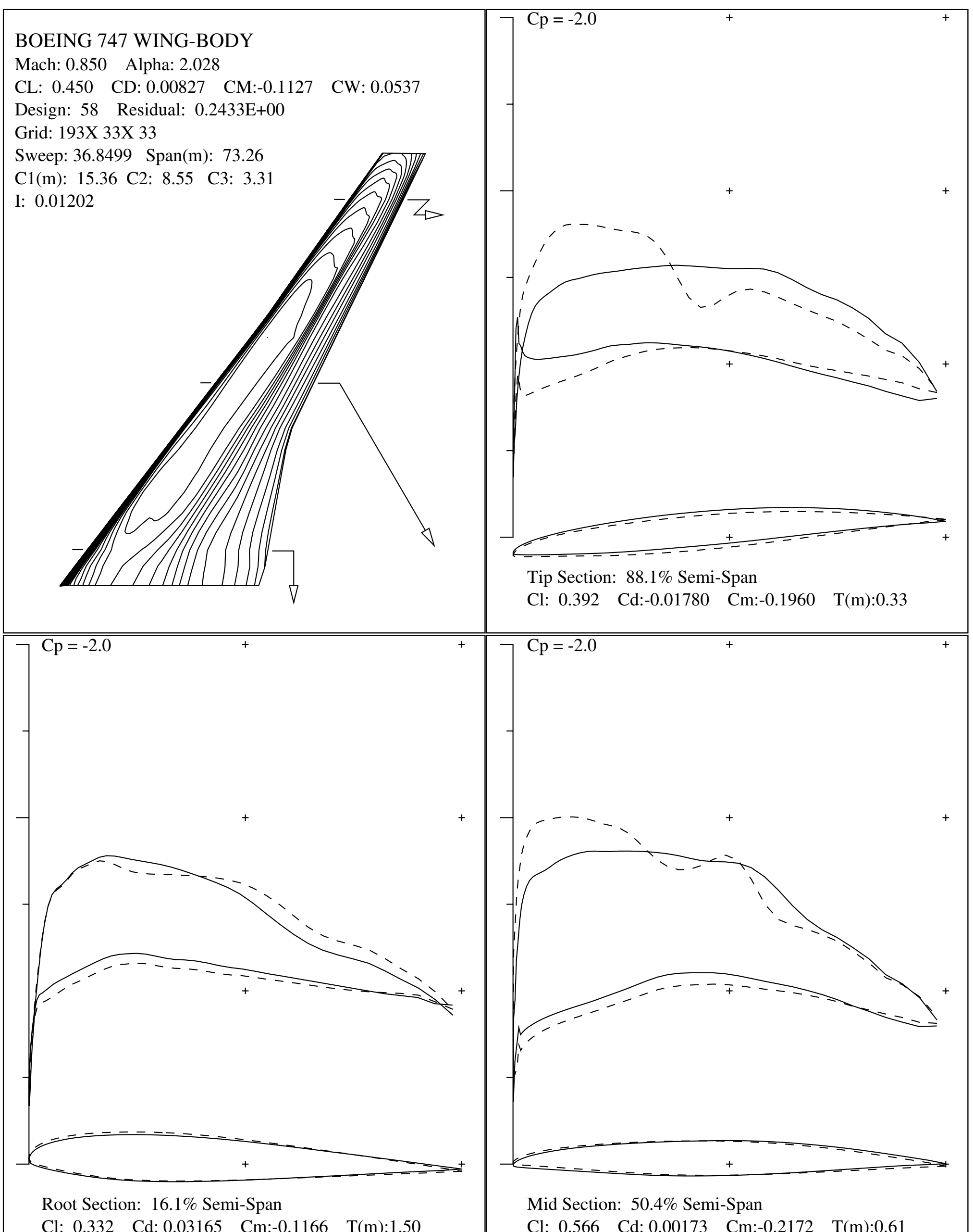


Fig. 3 Boeing 747 wing and planform redesign (initial C_p: dotted line; redesigned C_p: solid line). At fixed $C_L = 0.45$, drag and wing weight were minimized after 58 adjoint design iterations using SNOPT line searches.

When second-order central differencing is applied to Eq. (8), the equation at a given node, i, can be expressed as

$$\bar{\mathcal{G}}_i - \epsilon(\bar{\mathcal{G}}_{i+1} - 2\bar{\mathcal{G}}_i + \bar{\mathcal{G}}_{i-1}) = \mathcal{G}_i, \qquad 1 \leq i \leq n \tag{11}$$

where n is the number of design variables equal to the number of mesh points in this case. Then,

$$\bar{\mathcal{G}} = A\mathcal{G} \tag{12}$$

where A is the $n \times n$ tridiagonal matrix such that

$$A^{-1} = \begin{bmatrix} 1+2\epsilon & -\epsilon & 0 & . & 0 \\ \epsilon & . & . & & \\ 0 & . & . & . & \\ . & & . & . & -\epsilon \\ 0 & & & \epsilon & 1+2\epsilon \end{bmatrix} \tag{13}$$

Now using the steepest descent method in each design iteration, a step, $\delta\mathcal{F}$, is taken such that

$$\delta\mathcal{F} = -\lambda A\mathcal{G} \tag{14}$$

where the implicit smoothing may be regarded as a preconditioner.

Enhancement of the Design Optimization Procedure

SNOPT is used to set ϵ and λ, which removes the burden of selecting these values manually. We have proposed two enhancement approaches.

First, SNOPT optimization of Euler adjoint design parameters, ϵ and λ, has been made such that an optimum aerodynamic shape can be reached in a given number of design iterations. In this approach, because a full adjoint shape optimization will be carried out in every SNOPT iteration for parameter optimization, there will be a huge disadvantage in terms of computational savings. However, once SNOPT finds an optimal set of design parameters, an even better optimum may be achieved by the adjoint methods using these optimized parameters.

Second, the SNOPT package can provide line searches to decide the values of design parameters at every adjoint shape design iteration. An improvement in the robustness of the shape design methods without any noticeable addition of computational cost will be a target in this approach. This approach may be challenging because the function evaluation for SNOPT should now include the effect of the shape change as well.

Results of the First Enhancement Approach: SNOPT Parameter Optimization for Boeing 747 Wing–Body Configuration Wing Redesign

A drag minimization of the Boeing 747 wing–body configuration was performed by modifying the wing with a fixed fuselage shape. An Euler calculation was carried out for the wing–body configuration at a fixed coefficient of lift, $C_L = 0.45$, and $M_\infty = 0.85$ using a C-H grid of size $192 \times 32 \times 32$. SNOPT calls 10 adjoint design iterations in its ϵ and λ optimization iteration such that the adjoint design code using these parameters can produce the optimal shape within 10 shape design iterations. The optimal values of ϵ and λ were found to be 4.0 and 0.253, and using these values, the total wing drag coefficient was reduced from 0.01092 to 0.00986 at the fixed $C_L = 0.45$. The design results with and without SNOPT parameter optimization are listed in Table 2.

Results of the Second Enhancement Approach: SNOPT Line Search for Boeing 747 Wing Planform Optimization

Finally, SNOPT was used to carry out line searches for the best step parameters at each adjoint design cycle. The design methodology has been extended to a wing planform optimization [12]. In addition to the shape changes in the wing section, larger scale changes such as changes in the wing planform were considered to obtain a realistic optimum design. Because these larger scale changes directly affect the structural weight, we redesigned both the wing section and planform to minimize a cost function including both drag and structural weight in terms of the form

$$I = \alpha_1 C_D + \alpha_2 C_W \tag{15}$$

The wing section was modeled by surface mesh points, and the wing planform was modeled by the design variables shown in Fig. 2 as the root chord (c_1), midspan chord (c_2), tip chord (c_3), span (b), sweepback (Λ), and wing thickness ratio (t).

This design case is quite complicated due to the increased number of design parameters. In the previous sections, a total of eight design parameters, λ, ϵ, and the step sizes of six wing planform parameters have been updated at every adjoint design iteration.

Figure 3 shows the result after 58 design iterations using SNOPT with line searches for the design parameters. The total cost function, $I = C_D + 0.07C_W$, has reduced from 0.01431 to 0.01202. The design convergence history is shown in Fig. 4.

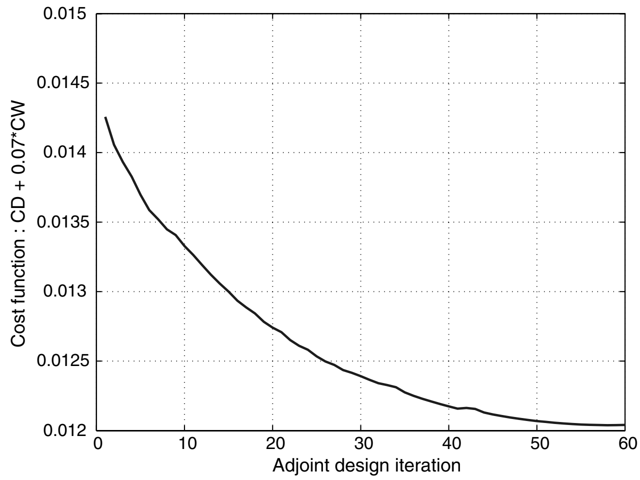


Fig. 4 Adjoint shape design convergence history using SNOPT line searches.

Conclusions

The feasibility of improving the existing adjoint method by finding an optimal combination of flow analysis and design input parameters has been investigated in this work. A nonlinear gradient-based optimization package, SNOPT, was chosen as a direct numerical tool for parameter optimization. Various numerical tests, in conjunction with the three-dimensional Euler adjoint design software, SYN88, were carried to investigate the benefits out of using this choice of parameter optimization.

First, SNOPT has found the improved residual smoothing parameters and Courant number for the Euler and the adjoint solutions. Second, SNOPT has found the gradient smoothing parameters and the step size of the Euler adjoint design methods. There were no computational advantages in these approaches due to the necessary additional time cost for SNOPT to find the parameters. However, once the parameters were found, the convergence of both the Euler and the adjoint solutions has been accelerated and the adjoin design method has been more effective. Finally, using SNOPT with line searches for the best design parameters at each design cycle for the Boeing 747 wing planform optimization problem, the robustness of the design process has been improved and the huge amount of time and effort required to find a proper set of eight input design parameters has been removed.

The numerical results show that the adjoint design method can be improved in shape design speed, performance, and stability by integrating the method with a parameter optimization tool such as, but not limited to, SNOPT. The benefits may be greater for parameter optimization for a complex system in which, in particular, neither an analytical nor a trial-and-error approach is practical.

Acknowledgments

This work has benefited greatly from the support of the U.S. Air Force Office of Scientific Research under grant AF F49620-98-1-2005 and partially from grant T007F3510001-07F015400310 from the Aviation Safety R&D Program funded by the Ministry of Land, Transport and Maritime Affairs of the Korean government.

References

[1] Pironneau, O., *Optimal Shape Design for Elliptic Systems*, Springer–Verlag, New York, 1984.

[2] Jameson, A., "Aerodynamic Design via Control Theory," *Journal of Scientific Computing*, Vol. 3, Sept. 1988, pp. 233–260. doi:10.1007/BF01061285

[3] Mohammadi, B., and Pironneau, O., "Shape Optimization in Fluid Mechanics," *Annual Review of Fluid Mechanics*, Vol. 36, 2004, pp. 255–279. doi:10.1146/annurev.fluid.36.050802.121926

[4] Baysal, O., and Eleshaky, M. E., "Aerodynamic Design Optimization Using Sensitivity Analysis and Computational Fluid Dynamics," AIAA Paper 91-0471, Jan. 1991.

[5] Nielson, E. J., and Anderson, W. K., "Aerodynamic Design Optimization on Unstructured Meshes Using the Navier–Stokes Equations," *AIAA Journal*, Vol. 37, 1999, pp. 1411–1419. doi:10.2514/2.640

[6] Nemec, M., and Zingg, D. W., "Newton-–Krylov Algorithm for Aerodynamic Design Using the Navier–Stokes Equations," *AIAA Journal*, Vol. 40, No. 6, 2002, pp. 1146–1154. doi:10.2514/2.1764

[7] Jameson, A., "Transonic Flow Calculations," Princeton Univ. Rept. MAE 1651, Princeton, NJ, April 1984.

[8] Jameson, A., and Vassberg, J. C., "Studies of Alternative Numerical Optimization Methods Applied to the Brachistochrone Problem," *Computational Fluid Dynamics Journal*, Vol. 9, 2000, pp. 281–296.

[9] Gill, P. E., Murray, W., and Saunders, M. A., "User's Guide For SNOPT (Ver. 6.0): A Fortran Package for Large-Scale Nonlinear Programming," Dept. of Mathematics, Univ. of California, San Diego, CA, Dec. 2002.

[10] Hosseini, K., and Alonso, J. J., "Practical Implementation and Improvement of Preconditioning Methods for Explicit Multistage Flow Solvers," AIAA Paper 2004-0763, Jan. 2004.

[11] Martinelli, L., "Calculation of Viscous Flows with a Multigrid Method," Ph.D. Dissertation, Princeton University, Princeton, NJ, 1987.

[12] Leoviriyakit, K., "Wing Optimization via an Adjoint Method," Ph.D. Dissertation, Stanford University, Stanford, CA, Dec. 2004.

T. Zang
Associate Editor