Enhancement of Dielectric Constant in Metal-Dielectric Meta-materials

Oleg Rybin*
Department of Information Technology,
Kharkov State University of Food Technology and Trade, Kharkov, Ukraine

A two component meta-material as copper cylindrical wires embedded periodically in the host dielectric material (wax) is studied with a viewpoint of measuring and analysing its electric and magnetic properties in the frequency range from 0.5 to 5 GHz. A large enhancement of the effective dielectric constant relative to the dielectric constant of the matrix material is observed in the considered periodic metal-dielectric structures. The increase of enhancement of the effective dielectric constant with the increase of wires diameter is also observed. A method for measuring the effective dielectric and magnetic constants of meta-materials is also presented briefly to compare it with the results obtained by numerical simulations in the considered frequency range. An analysis of both measurement and simulation results is carried out. A comparison of such analysis with the appropriative analysis of similar meta-material structures with magnetic metal (iron) wires conducted in the earlier work is also presented.

1. Introduction

Wire-dielectric composite periodic structures [1-6] have attracted a significant attention of researchers for a long time because of their numerous applications in wireless communications and microwave engineering. Initially, these structures were of high interest with a viewpoint of studying the electromagnetic wave propagation and scattering problems [7-9]. Recently, a new class of artificial periodical media known as meta-materials [10-14] was separated out. Meta-materials in the microwave frequency range are of special interest because their desired properties as conventional materials are seriously degraded at frequencies above 1 GHz [15].

In this paper, we study the effective behavior of meta-materials, which can be used as substrates in strip line technology. The meta-material structures have been made from copper cylindrical wires embedded periodically in the wax matrix within two orientations of bottom wire layer, vertical and parallel with respect to the waveguide vector (Fig.1). Also, two cases of mutual orientation, parallel and perpendicular, for the wire layers were considered. The modified strip line method [16] has been used to measure the S-parameters using a network analyzer when a meta-material is located like a substrate under a wire (it plays the role of a strip) between 0.5 and 5 GHz (Fig.2). These S-parameters were also simulated using the Finite-Difference Time-Domain (FDTD) method for both with and without the meta-materials under the strip wire. Two kind of simulations were made. First, supposing that metal wires were inside the dielectric (non-homogenous model). Second, supposing that one homogenous material with its dielectric and magnetic constants equal to the effective constants of the meta-materials (homogeneous model). In order to complete the electromagnetic analysis of the presented metamaterial structures, FDTD calculations were performed in correspondence to each measurement (orientation) of a meta-material for comparison and with the assumption that it has infinite periodic structure. The effective constants were calculated from the numerically simulated S-parameters and were compared with the values obtained from "the strip simulations". The results obtained are consistent between them and in good agreement with the experimental results in the frequency range of measurements. Essential enhancement of the dielectric constant of meta-materials relative to dielectric constant of matrix material samples of the same geometry was observed. At the same time, no significant enhancement of the effective magnetic constant was found.

2. Materials and methods

2.1 Experimental procedure

The presented meta-material samples have been fabricated by a wax rapid prototyping machine produced by Solidscape Co., USA. The produced

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*rybin.oleg@gmail.com

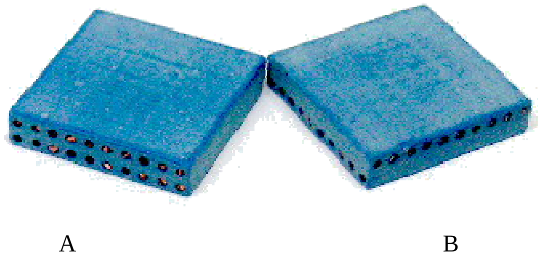

Fig.1: Meta-materials made from two parallel layers of Cu wires embedded inside wax in two different combinations.

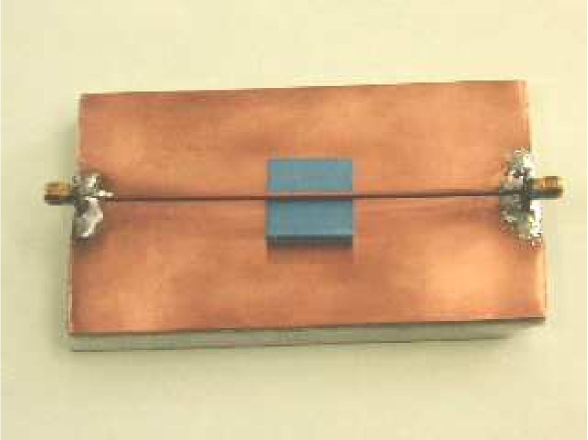

Fig.2: Resonator used for measurement of meta-materials and one meta-material located under the wire and at its center.

samples have only hollows inside them (Fig.1) of size 22.5 mm × 4.5 mm × 22.5 mm. The size of the unit cell is 2.25mm × 2.25 mm × 4.5mm. The hollows inside each unit cell and embedded in the center have diameter 1 mm and length 22.5mm. Copper wires with the same size were inserted inside these. Finally, the two layers of ten wires have been created in two possible orientations inside all samples and consistent with those presented in the previous paragraph. Also, one more sample with the same size, but without hollows, has been made for reference.

The basic device for measuring the effective constants of meta-materials is a modified strip line resonator by using the wire against the strip (Fig.2). The impedance of this resonator was different than 50 Ohm, and for this reason it gave resonances relative to its size and the material sample located as substrate under the center of its wire. This device consists of a copper layer as ground, with size 120 mm × 70 mm, copper wire as the strip with diameter 2 mm and length 110 mm and the coaxial connectors. The experimental S-parameters have been measured with a HP8722D network analyzer, calibrated at 50 Ohm and connected to the connectors of the above mentioned resonator. The meta-material samples were located in the center of the strip-wire and in contact with it during all measurements.

2.2 Model of calculation of the effective constants

In order to get the information about effective constants, the basic approach is the Effective Medium Theory (EMT) [4,17-19] that is used in this study. According to this theory, the effective constants of non-homogeneous samples in an appropriate frequency range are identical with the homogenous ones of the same geometry. We suppose that constants found from FDTD simulations were studied only in parallel direction relative to the applied electric field vector of the incident electromagnetic wave (vertical to the ground of the resonator). Due to the heterogeneity of the cross section, a transverse electromagnetic (TEM) wave cannot be propagated. However, for low frequencies, longitudinal components of the microwave fields can be neglected as compared to the transversal for the sample laid on the ground of a strip resonator used for measurements. So, the hypothesis of a quasi-TEM mode is valid. It is important due to the fact that the field pattern of the fundamental quasi-TEM mode propagating in a strip line is suitable for the characterization of rectangular magnetic sample [20,21] and because of the necessity to make electromagnetic analysis for the purpose of characterization of the sample under study.

Of course, EMT cannot be applied for whole frequency range, but the theory is applied for a frequency range in which the wavelength of the electromagnetic wave propagating through a sample under study is greater as compared to the dimensions of the unit cell. So, the hypothesis, for a possibility to use EMT for observing the effective internal constants, is quite valid for quasi-transverse electromagnetic wave of low frequencies. That is why all calculations of the complex dielectric and magnetic constants were carried out for frequency about 0.1 GHz in this study. Finally, in order to apply EMT to the presented meta-material structures, it was supposed that the wavelength of incident wave is much greater than the diameter of wires and the distance between them in the frequency area of measurements.

In this paper, the effective dielectric and magnetic constants are evaluated via measured (or calculated) S-parameters through a simple mathematical model presented in [22] and developed for a flat bulk sample.

As our samples have electric and magnetic anisotropy, it is therefore expected to find different behavior of the effective constants in different orientations of the samples relative to the direction of the resonator strip wire.

3. Results and discussion

In order to simplify the description of our analysis, a relative classification of all measurement cases were carried out. The case, where the sample B of Fig.2 has its top wires vertical to the resonator strip wire, has been called as "v-p sample" measurement. The case, where the sample B of Fig.2 has its top wires parallel to the resonator strip wire, has been called as "p-v sample" measurement. "p-p sample" measurement case is the case of measurement where the sample A of Fig.2 has its wires parallel to the resonator strip wire. Finally, "v-v sample" measurement case is the case of measurement where the sample A of Fig.2 has its wires vertical to the resonator strip wire.

In Fig.3, both experimental and simulated S-parameters are shown, while the resonator is unloaded. Fitting of these data gives the effective length of resonator wire, which was found equal to 104 mm. This value is kept constant every time during the rest of simulations. Also, experimental and simulated S-parameters, when the resonator is loaded with a wax sample with no hollows but same size with presented samples, were shown. Fitting procedure gave a dielectric constant of the wax equal to 2.2. This value was also kept constant in the cases of non-homogenous and homogenous models during simulations.

In Fig.4, both experimental and calculated S-parameters are shown, while the resonator was loaded with different kind of meta-material samples. In all cases, simulated S-parameters for homogenous and non-homogenous models are presented. The reason of using non-homogenous model is to verify the strength of our FDTD model for all devices with real characteristics of materials and wires that compose our meta-materials relative to measured experimental results. The same model, with no wires inside, the same size of materials, position, and effective constants, was used (homogeneous model).

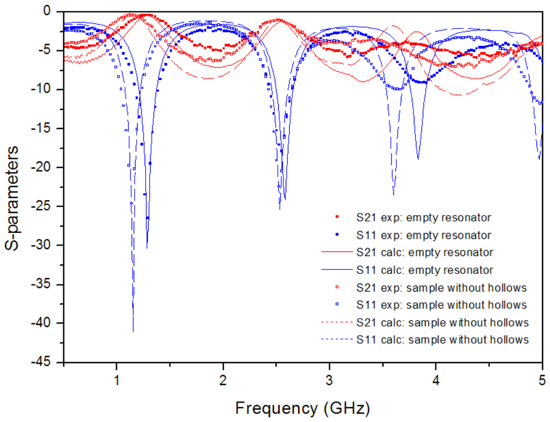

Fig.3: Measured and calculated S-parameters of the empty resonator and for the resonator loaded with a material without hollows and with the same size like the meta-materials under study.

In the case of p-p and v-p samples, the effective dielectric constants equal to 2.4 and 2.5, respectively, were found. The value of effective magnetic constants was found equal to 1. We can observe a very small shrinking of spectra of S-parameters relative to S-parameters found for the case of material without hollows (Fig.3). For this reason, an obvious enhancement of dielectric constants was not observed relative to the sample with no hollows that has dielectric constant equals 2.2. In the case of p-v and v-v samples, the shrinking of spectra of S-parameters became more obvious and an enhancement of dielectric constants was found equal to 3.6 and 4.5, respectively, in the case of homogeneous model applied in our simulations. The observed enhancement is caused by a large local enhancement of the electro-magnetic field's electric component due to reflections on metallic wires. Except for that, a small enhancement of the magnetic constant for above mentioned cases was found equal to 1.1 and 1.15, respectively. The last enhancements can logically be referred to an error of the measurement related to the presented measurement device so far as it is difficult to expect any magnetic enhance-ment without having no magnetic ingredients which compose the presented meta-material samples. This error of measurement is mostly due to the fact that it is quite difficult to keep TEM in an open resonator comparable to a closed one, but the presented measurement resonator is quite simple and cheap in fabrication. Moreover, it uses quite simple mathematical model which will be mentioned later.

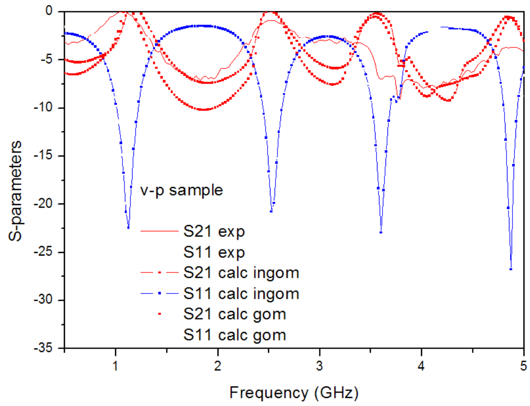

Fig.4a: S-parameters of resonator measured and calculated for meta-materials with v-p orientation of Cu-layers using the homogenous and non-homogenous model of simulation.

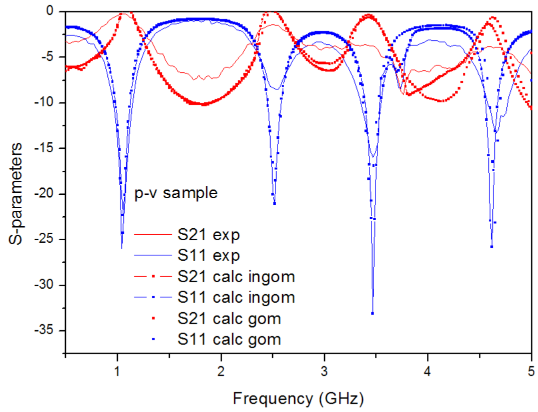

Fig.4b: S-parameters of resonator measured and calculated for meta-materials with p-v orientation of Cu-layers using the homogenous and non-homogenous model of simulation.

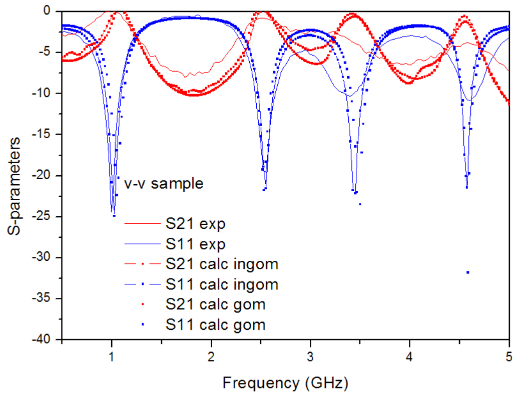

Fig.4c: S-parameters of resonator measured and calculated for meta-materials with v-v orientation of Cu-layers, using homogenous and non-homogenous model of simulation.

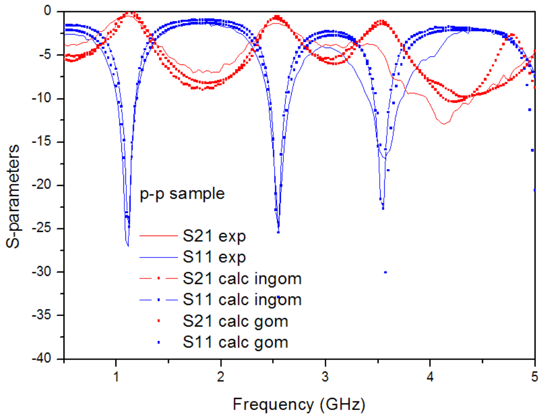

Fig.4d: S-parameters of resonator measured and calculated for meta-materials with p-p orientation of Cu-layers using the homogenous and non-homogenous model of simulation.

The above mentioned results enable us to conclude that shrinking of spectra of S-parameters becomes larger when going from p-p to v-p to p-v to v-v sample. It is because a larger shrinking should correspond to a larger dielectric constant for the same geometry and size of a homogenous material.

An analysis of the curves in Fig.4 has also shown the same effective behavior of meta-materials as we found above using the homogenous model and the modified strip method.

S-parameters spectrum of the considered samples has “anomalous” peaks between 3.7 GHz and 4.3 GHz except for the v-v case. Experiments have shown that these “anomalous” peaks are caused by the presence of metallic wires inside the material (these peaks do not exist at spectra of homogenous model). Peaks presence is caused by the size of wires, which gives resonances at corresponding frequencies. The distribution of S-parameters shows that the existence of above mentioned “anomalous” resonances corresponds to the case when any wire plane or both of them are parallel to the resonator wire.

FDTD simulations for free space case were carried out in this study. These simulations used non-homogenous model in which supposed meta-material samples with thickness equal to 4.5 mm were to be expanded periodically and vertically to the incident electromagnetic wave. The wave vector of this wave is parallel to the direction of strip wire. The electric field is vertical to the wave vector and unit cells of meta-materials in the same way as we took measurements in the samples where the electric field was vertical to the ground. The unit cells of the above mentioned periodical structures is shown in Fig.5. In Fig.6, S-parameter graphs for free space simulations were presented. The change of first resonant peak position of meta-materials relative to its thickness towards the smaller frequency value corresponds to an enhancement of effective dielectric constant in this frequency region. This enhancement is as much larger as the peak shift to the left is larger. In order to calculate the effective constants, the approach of [22] was used. We can observe a huge enhancement of the effective dielectric constant with an increase of wires diameter (Fig.7). For example, it has been calculated that effective dielectric constant equals 172 if the volume fraction equals 0.772. As opposed to the above mentioned dielectric constant enhancement, enhancement of the effective magnetic constant was not found for the simulations in free space. The last result also stands for the idea that the enhancement of magnetic constant is not observed for the presented samples.

As we can see from Fig.7, the curves of effective dielectric constant are nonlinear. It is caused by the non-uniform character of the Bergman spectrum of meta-materials considered. The Bergman spectrum is discrete, away from percolation threshold and tends to be continuous as the wires come closer, and finally touch each other. The effective permittivity close to percolation threshold is scaled with an exponent [23].

The presented results are quite compatible with the results presented in [16], where iron wires were studied. It enables us to conclude about a weak capability for iron to get an excessive magnetic moment due to a magnetic field component of external microwave. Furthermore, the proposed strip wire resonator with the above described measurement procedure presents a basis of for an alternative cheap broadband measurement system for microwave characterizing bulk materials and meta-materials relative to other measurement systems exist at present.

4. Conclusion

The effective electric and magnetic properties of two different metal-dielectric meta-material structures have been studied in the frequency range of 0.5 GHz until 5 GHz. Enhancement of the effective dielectric constant was found in all of the considered samples by using a modified strip method. The existence of dimensional resonances was found in the considered frequency range during S-parameter measurements.

The giant enhancement of the effective dielectric constant with the increase of wires diameter was found through free space simulations. A behavior of effective dielectric constant curves has been explained via the concept of Bergman spectrum. The study was also oriented towards the use of these meta-material structures as substrates devices.

The measurement resonator and the above mentioned measurement procedure were presented in detail as a basis for an alternative cheap broadband measurement system for microwave characterizing bulk materials and meta-materials relative to other measurement systems existing at present.

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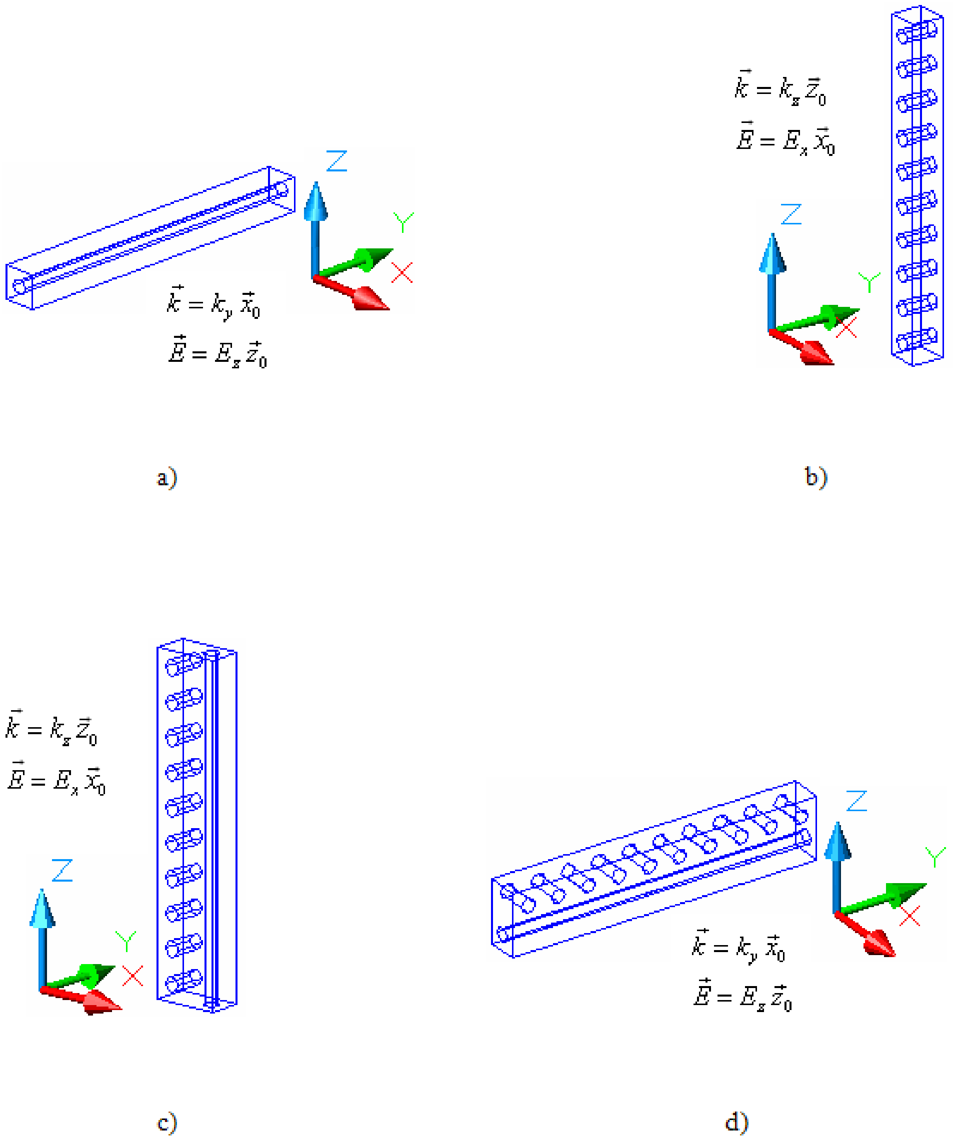

Fig.5: Unit cells for free space simulations for cases: a) parallel-parallel, 2.25×22.5×2.25 b) v-v, 2.25×2.25×22.5 c) p-v and v-p with wires is vertical to the wave vector, 4.5×2.25×22.5 d) p-v and v-p with wires is parallel to the wave vector, 2.25×22.5×4.5.

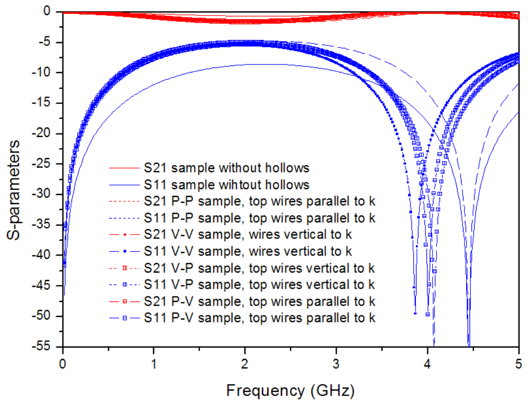

Fig.6: S-parameters of metamaterials calculated in free space method for different orientations of wave vector relative to the orientation of wires inside the samples.

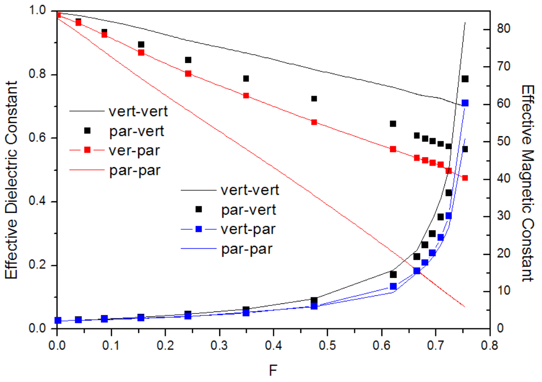

Fig.7: Change of effective dielectric and magnetic constant of metamaterial versus volume fraction F at frequency of 0.1 GHz with volume fraction of unit cell, in a supposed free space measurement.

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