SUPPLEMENTARY MATERIALS FOR

Enhancing the search ability of hybrid LSHADE for global optimization of interplanetary trajectory design

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ARTICLE HISTORY
Compiled December 12, 2021

1. Difference from LSHADE_SPA_CMA

CMA-ES strategy is also used in LSHADE_SPA_CMA. But there are still several differences between LSHADE_SPA_CMA and HLSHADE.

- LSHADE_SPA_CMA uses LSHADE and CMA-ES in parallel. It divides the population into two sub-populations using LSHADE and CMA-ES for evolution. CMA-ES is employed as a global optimizer in the whole search space. However, HLSHADE employs LSHADE_SPS_EIG as a global optimizer to find an optimal solution, then uses CMA-ES as a local optimizer to enhance the local exploitation ability around the optimal solution.
- HLSHADE uses interior-point method to strengthen the local search ability while LSHADE_SPA_CMA does not. The interior-point method is an algorithm for solving linear programming or nonlinear convex optimization problems. It is found through experiments that after the interior-point method is added, the convergence performance of HLSHADE in the later stage is significantly improved. This is also one of the major differences between HLSHADE and LSHADE_SPA_CMA.
- HLSHADE proposes the adaptive strategy to control the two important triggering parameters which is different from LSHADE_SPA_CMA. HLSHADE proposes two triggering parameters to adaptively control the evaluation times of LSHADE_SPS_EIG and CMA-ES. If LSHADE_SPS_EIG or CMA-ES in HLSHADE can successfully update the optimal solution, it is awarded more function evaluations. Otherwise, it is punished next time to reduce the number of evaluations.

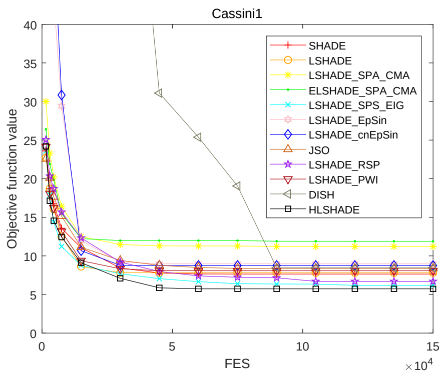
2. Comparison with LSHADE and its variants

Table S1. Experimental results of HLSHADE, LSHADE and its variants over 25 independent runs on GTOP benchmarks with 150,000 FES.

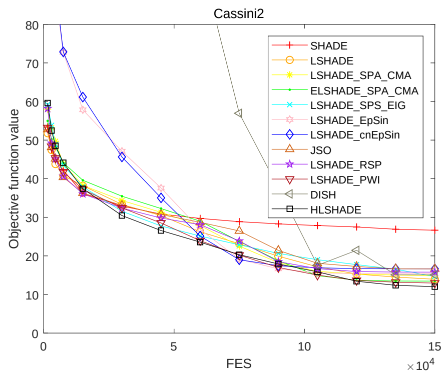
Benchmarks	Paradigm	SHADE	LSHADE	LSHADE_SPA_CMA	ELSHADE_SPA_CMA	LSHADE_SPS_EIG	LSHADE_EpSin	LSHADE_cnEpSin	JSO	LSHADE_RSP	LSHADE_PWI	DISH	HLSHADE
Cassini1	Best	5.3034	5.3034	**4.9307**	5.3034	**4.9307**	**4.9307**	5.3034	**4.9307**	**4.9307**	5.3034	**4.9307**	**4.9307**
	Worst	12.5420	12.5420	12.9230	12.5420	11.0320	12.5420	12.5420	16.7113	12.5285	12.5420	12.5420	11.0320
	Mean	7.6453	7.8108	11.2180	11.8921	6.1740	8.9733	8.7473	8.3794	6.7026	8.0739	8.4553	**5.7308**
	Std	2.9425	3.0846	1.8838	1.5755	2.1634	3.0713	3.0959	3.4872	2.6299	3.2490	3.4218	1.5934
Cassini2	Best	23.7073	8.6134	9.6542	8.7305	11.0216	12.7552	12.7553	12.6338	8.6116	**8.6089**	12.7553	8.8101
	Worst	28.9330	18.0148	20.6896	15.9227	19.0964	20.6825	20.6827	21.2732	20.7999	16.2092	20.6861	15.9932
	Mean	26.6519	13.9403	14.8141	13.4666	14.6810	15.3601	16.6430	16.6163	15.7879	12.9543	15.0864	**12.0239**
	Std	1.0849	2.6007	2.6454	1.4793	1.9279	2.0820	2.7025	2.4525	3.0367	2.1976	2.1311	2.2353
Gtoc1	Best	-1236202.0	-1386716.5	-1336019.6	-1499476.8	-1487003.7	-1160542.5	-1459683.0	-1445330.0	-1320180.0	-1357808.5	-1128011.4	**-1576143.0**
	Worst	-923930.3	-904675.9	-793963.5	-812370.9	-878412.5	-825934.5	-1062965.6	-788504.0	-844024.0	-899078.4	-804075.3	-901531.2
	Mean	-1039539.6	-1064373.2	-1047944.0	-1030969.3	**-1205745.4**	-1004980.8	-1165152.5	-1098266.3	-1030000.0	-1060396.2	-1008750.0	-1193033.1
	Std	77426.8	101487.2	133498.1	130043.6	172273.4	92000.2	94832.6	170221.6	123700.0	97799.7	82303.9	151251.4
Rosetta	Best	7.3481	1.8891	1.3832	1.9024	2.6013	1.3834	1.8849	**1.3584**	1.3672	1.3837	1.8848	1.5528
	Worst	12.6275	3.1865	3.4450	3.3885	5.6791	1.9969	1.9967	3.1775	3.2033	3.6937	3.1917	4.0974
	Mean	9.7916	2.0487	2.4219	2.7776	4.2042	**1.9145**	1.9310	1.9860	2.0199	2.3484	2.2319	2.4967
	Std	1.3117	0.3448	0.6113	0.4334	0.8137	0.1157	0.0398	0.0373	0.3748	0.6140	0.5492	0.6857
Sagas	Best	704.1721	284.0758	932.5784	932.5765	22.9860	970.1247	376.9455	334.3450	282.4860	282.4860	376.9455	**18.5826**
	Worst	976.3123	970.4253	970.2622	970.1247	970.6765	980.3716	980.3716	980.2460	980.2460	970.3738	980.2471	970.1247
	Mean	938.6692	938.5558	955.5534	945.3415	497.3500	976.2039	952.6074	946.8352	942.0447	923.9599	946.6035	**281.7730**
	Std	74.7324	136.8377	16.6573	16.6537	358.3584	4.9621	117.9069	125.3773	137.9263	143.2508	118.9981	294.7937
Messenger	Best	15.6614	10.6427	10.1820	**8.7016**	10.2302	**8.7016**	**8.7016**	9.2554	8.7441	11.4076	10.2029	9.0582
	Worst	18.2451	13.7719	13.8447	14.0160	13.8272	13.5152	12.9049	12.9491	13.0307	13.6386	14.1514	13.7744
	Mean	16.7842	12.5250	12.1774	12.4942	12.6263	12.0068	11.7913	**11.6464**	11.8474	12.5295	12.0271	11.7519
	Std	0.6317	0.7101	0.7752	1.1876	0.9223	1.4818	1.0926	0.8766	1.1066	0.5967	1.1783	1.2589
Messenger-full	Best	23.0321	11.9458	12.527	13.1885	10.7932	13.8731	9.9925	14.0444	13.8441	14.2139	13.4733	**5.2185**
	Worst	28.4419	16.8247	15.2846	17.1817	17.6064	16.6794	15.7829	16.7746	17.1912	16.6848	17.0253	14.2408
	Mean	25.3261	15.3164	14.2044	15.3260	14.0647	15.4267	14.5908	15.7763	15.6718	15.3207	15.3225	**10.5344**
	Std	1.4316	0.8872	0.8871	0.9843	2.0393	0.7192	1.0696	0.6908	0.8892	0.7077	0.8678	2.5158

Table S2. Results of the multiple-problem Wilcoxon's test for HLSHADE, LSHADE and its variants at $\alpha = 0.05$ and at $\alpha = 0.1$ significance level based on the mean values.

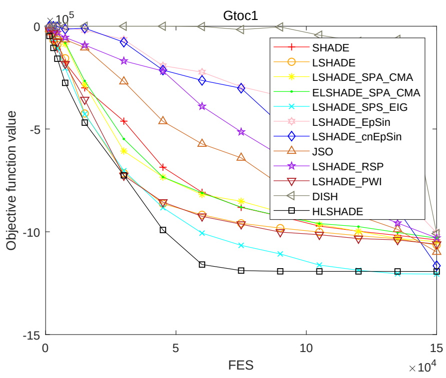
HLSHADE vs.	R^+	R^-	p value	at $\alpha = 0.05$	at $\alpha = 0.1$
SHADE	28.0	0.0	0.015626	+	+
LSHADE	27.0	1.0	0.031260	+	+
LSHADE_SPA_CMA	27.0	1.0	0.031260	+	+
ELSHADE_SPA_CMA	28.0	0.0	0.015626	+	+
LSHADE_SPS_EIG	21.0	7.0	0.204894	=	=
LSHADE_EpSin	26.0	2.0	0.046880	+	+
LSHADE_cnEpSin	26.0	2.0	0.046880	+	+
LSHADE_RSP	26.0	2.0	0.046880	+	+
JSO	25.0	3.0	0.078120	=	+
LSHADE_PWI	27.0	1.0	0.031260	+	+
DISH	27.0	1.0	0.031260	+	+



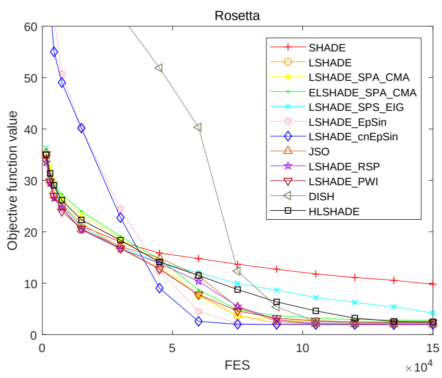
(a) Convergence graph on Cassini1



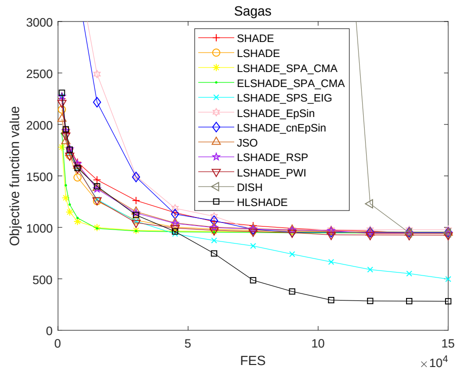
(b) Convergence graph on Cassini2



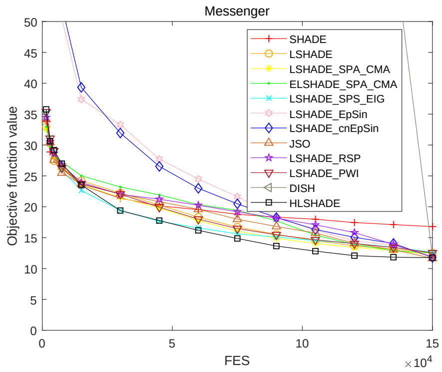
(c) Convergence graph on Gtoc1



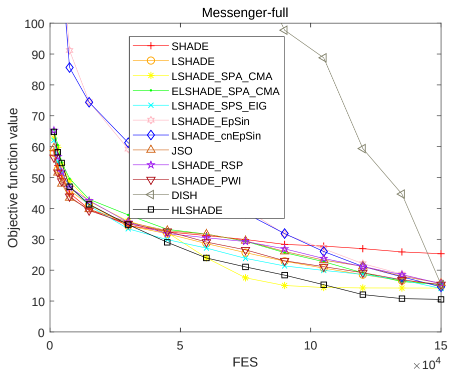
(d) Convergence graph on Rosetta



(e) Convergence graph on Sagas



(f) Convergence graph on Messenger



(g) Convergence graph on Messenger-full

Figure S1. Evolution of the mean objective function values derived from the twelve algorithms versus FES on GTOP benchmarks

3. Comparison with CEC competition algorithms

Table S3. Experimental results of HLSHADE and six CEC competition algorithms over 25 independent runs on GTOP benchmarks with 150,000 FES.

Benchmarks	Paradigm	UMOEAsII	EBOwithCMAR	MVMO	HSES	GA-MPC	PaDE	HLSHADE
Cassini1	Best	5.3034	**4.9307**	5.0497	5.3034	5.3034	5.3034	**4.9307**
	Worst	12.5420	12.5420	11.2133	17.3027	12.5420	12.5420	11.0320
	Mean	6.7948	6.9778	7.0373	13.5999	9.9812	10.2045	**5.7308**
	Std	2.7328	2.8692	2.3395	2.3090	3.3202	2.9190	1.5934
Cassini2	Best	8.6222	12.7553	9.5380	20.1271	**8.6095**	8.6484	8.8101
	Worst	16.5458	21.1941	20.5413	27.5058	18.3662	15.5439	15.9932
	Mean	12.4008	17.5697	15.9132	23.4964	**11.3303**	12.0893	12.0239
	Std	2.4953	2.1266	3.1744	1.6215	3.5255	1.3252	2.2353
Gtoc1	Best	-1331056.4	-1511530.0	-1451400.0	-1333336.3	-1351272.8	-1351757.5	**-1576143.0**
	Worst	-904337.5	-844296.0	-1051900.0	-68659.1	-500355.2	-892394.8	-901531.2
	Mean	-1079644.8	-1165356.4	**-1253000.0**	-529560.9	-987759.3	-1058492.7	-1193033.1
	Std	113064.4	156336.5	106500.0	398670.3	278818.6	110222.4	151251.4
Rosetta	Best	**1.3796**	1.8851	1.7982	4.3636	1.4098	5.5610	1.5528
	Worst	3.6740	2.0525	5.4293	11.8721	7.5471	11.6838	4.0974
	Mean	2.3645	**1.9528**	2.6176	6.8087	3.6265	8.3286	2.4967
	Std	0.6993	0.0547	0.9641	2.3119	1.5789	1.4409	0.6857
Sagas	Best	376.9455	334.1228	932.4109	657.8691	970.1247	665.6507	**18.5826**
	Worst	990.1673	980.2471	980.2476	1499.305	990.3011	921.1000	970.1247
	Mean	924.7390	927.3344	954.2597	1087.8092	974.9240	906.9732	**281.7730**
	Std	160.2547	159.2257	19.5434	160.4327	7.5261	50.2983	294.7937
Messenger	Best	9.5585	**8.6529**	8.7750	13.9617	11.7824	11.0899	9.0582
	Worst	13.2353	13.4830	13.6415	21.9148	17.9876	13.8582	13.7744
	Mean	11.4240	**11.1798**	11.4310	18.7122	14.4290	12.5566	11.7519
	Std	0.8987	1.2415	1.4054	2.1817	1.8877	0.7203	1.2589
Messenger-full	Best	7.3378	12.0402	10.9329	13.8362	7.8310	9.8742	**5.2185**
	Worst	15.7117	15.6099	15.9520	19.3070	18.5866	20.5243	14.2408
	Mean	12.0935	13.8476	15.0949	16.1591	12.8084	15.9325	**10.5344**
	Std	2.4793	0.8767	1.0015	1.8211	3.2797	3.0098	2.5158

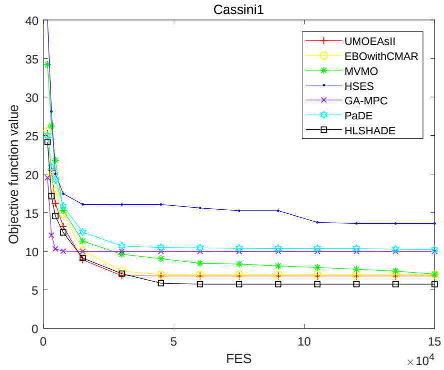
Table S4. Results of the multiple-problem Wilcoxon's test for HLSHADE and six CEC competition algorithms at $\alpha = 0.05$ and at $\alpha = 0.1$ significance level based on the mean values.

HLSHADE vs.	R^+	R^-	p value	at $\alpha = 0.05$	at $\alpha = 0.1$
UMOEAsII	25.0	3.0	0.051913	=	+
EBOwithCMAR	25.0	3.0	0.051913	=	+
MVMO	19.0	9.0	0.352542	=	=
HSES	28.0	0.0	0.014248	+	+
GA-MPC	27.0	1.0	0.022494	+	+
PaDE	28.0	0.0	0.014248	+	+

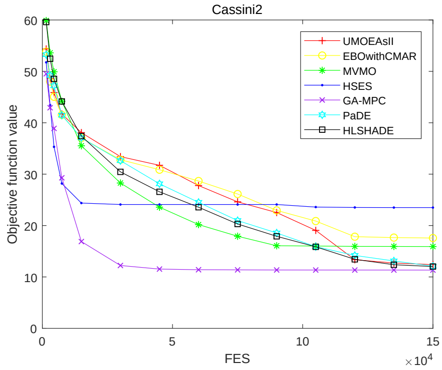
4. Comparison with algorithms in PyGMO

To further verify the efficiency of HLSHADE, eleven algorithms from the software platform of PyGMO which is provided by ESA-ACT are selected. PyGMO includes GTOP benchmarks and a number of well-known algorithms, which is a recognized interplanetary trajectory test platform. The eleven algorithms include Differential Evolution (DE), Self-adaptive DE (jDE), Particle Swarm Optimization (PSO), Simple Genetic Algorithm (SGA), Covariance Matrix Adaptation Evolution Strategy (CMA-ES), Artificial Bee Colony (ABC), DE-1220, Monotonic Basin Hopping (MBH), DE with p-best crossover (MDE_pBX), Monte Carlo Search (MC), and Improved Harmony Search (IHS). Because Messenger benchmark is not included in PyGMO, six GTOP benchmarks are used. The basic parameter settings for each compared algorithm in this paper are as follows, and more details can be found in PyGMO.

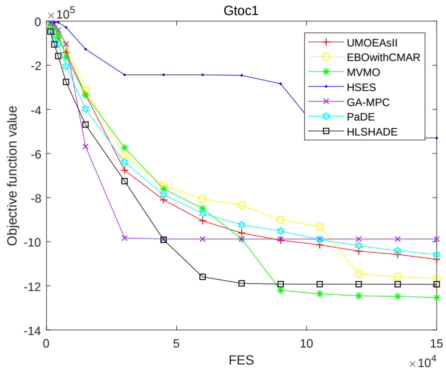
- DE: PopSize = 100, gen = 1,500, F = 0.5 ,CR = 0.5, variant = 2, ftol = 1e-030, xtol = 1e-030;
- jDE: PopSize = 100, gen = 1,500, variant = 2, self_adaptation = 1, memory = 0, ftol = 1e-030, xtol = 1e-030;
- PSO: PopSize = 100, gen = 1,500, omega = 0.7298, eta1 = 2.05, eta2 = 2.05, variant = 5, topology = 2, topology param = 4;
- SGA: PopSize = 100, gen = 1,500, CR = 0.95, M = 0.02, elitism = 1, mutation = GAUSSIAN(0.1), selection = ROULETTE, crossover = EXPONENTIAL;
- CMA-ES: cc = -1, cs = -1, c1 = -1, cmu = -1, sigma0 = 0.5, ftol = 1e-030, xtol = 1e-030, memory = 0;
- ABC: PopSize = 100, gen = 1,500, limit = 20;
- DE-1220: PopSize = 100, gen = 1,500, self_adaptation = 2, variants = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], memory = 0, ftol = 1e-030, xtol = 1e-030;
- MBH: gen = 1500, Compass Search stop = 1,500, perturb = 0.05;
- MDE_pBX: gen = 1,500, percentage = 0.15, meanexponent = 1.5, ftol = 1e-030, xtol = 1e-030;
- MC: iter = 150,000;
- IHS: iter = 150,000, phmcr = 0.85, ppar_min = 0.35, ppar_max = 0.99, bw_min = 1e-005, bw_max = 1;
- HLSHADE: max_eval = 150,000, PopSize = 18 × D.



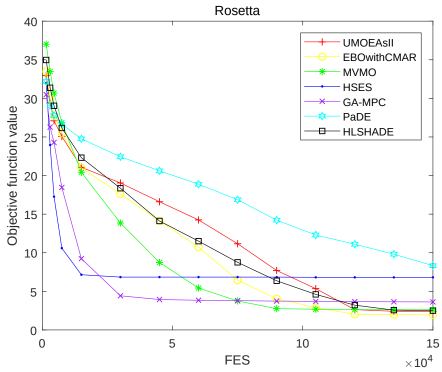
(a) Convergence graph on Cassini1



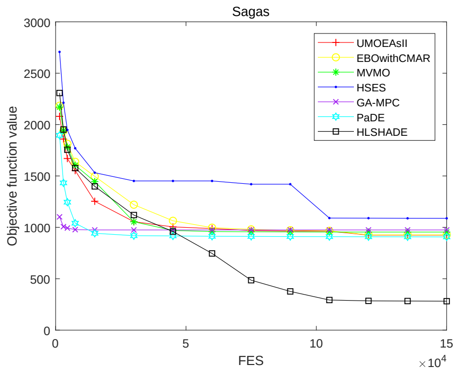
(b) Convergence graph on Cassini2



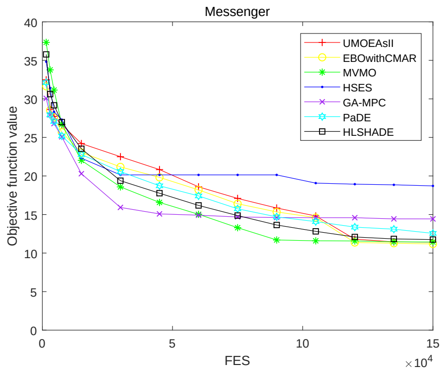
(c) Convergence graph on Gtoc1



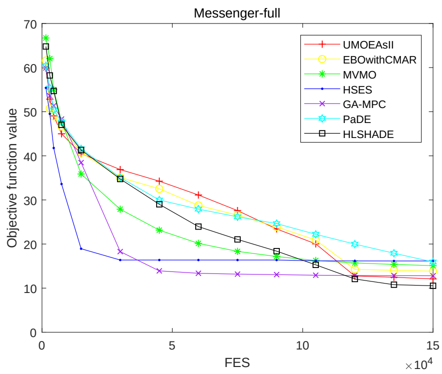
(d) Convergence graph on Rosetta



(e) Convergence graph on Sagas



(f) Convergence graph on Messenger



(g) Convergence graph on Messenger-full

Figure S2. Evolution of the mean objective function values derived from the seven algorithms versus FES on GTOP benchmarks

Table S5. Experimental results of HLSHADE and eleven algorithms in PyGMO over 25 independent runs on GTOP benchmarks.

Benchmarks	Paradigm	DE	jDE	PSO	SGA	CMA-ES	ABC	DE-1220	MBH	MDE	MC	IHS	HLSHADE
Cassini1	Best	5.3200	5.3034	5.3062	5.7691	10.9965	5.7888	**4.9307**	5.5174	5.3034	15.7190	11.5960	**4.9307**
	Worst	11.0426	10.9965	12.5781	21.5356	17.3027	10.9020	17.2673	17.1296	16.7116	15.7190	28.1179	11.0320
	Mean	5.7524	**5.4943**	8.0715	13.4506	15.8657	7.3362	8.5811	9.4762	9.8229	15.7190	19.6871	5.7308
	Std	1.0856	1.0217	2.8008	4.0483	2.0156	1.2543	3.9057	3.7252	3.4481	0.0000	4.3315	1.5934
Cassini2	Best	15.2789	16.0018	11.2899	14.2186	18.6835	13.7028	9.3085	21.0515	16.7185	54.5696	36.8589	**8.8101**
	Worst	24.2901	25.5085	23.3031	30.2832	25.9048	23.9788	25.3773	39.9882	23.6304	54.5696	53.7474	15.9932
	Mean	20.7682	20.5340	17.5791	23.8495	22.7294	18.1520	18.7384	31.8087	19.2994	54.5696	45.9349	**12.0239**
	Std	2.1518	2.0851	2.6761	3.6736	1.9482	2.2400	4.3105	5.2069	1.5147	0.0000	4.1098	2.2353
Gtoc1	Best	-1037449.8	-996622.0	-1274940.2	-736418.9	-1189160.1	-888316.8	-1388382.1	-561047.2	-1279836.1	-13459.3	-146658.8	**-1576143.0**
	Worst	-525986.0	-426332.7	-472590.0	-13632.7	-68283.3	-387094.4	-525444.0	-173578.6	-535416.7	-5463.5	-5779.6	-901531.2
	Mean	-703421.6	-650891.3	-886615.3	-194014.3	-744227.4	-636520.1	-874156.3	-365048.0	-933255.7	-6005.7	-24373.7	**-1193033.1**
	Std	105299.2	130067.8	206251.4	222298.7	279024.6	135691.2	215842.0	118928.2	186642.1	1870.6	29798.7	151251.4
Rosetta	Best	5.9884	4.8021	2.1247	4.9463	2.3974	5.2828	2.3599	8.0650	**1.3468**	31.4984	22.0351	1.5528
	Worst	11.4576	11.1571	9.5985	17.4165	8.3325	9.7775	13.4383	21.0927	8.0188	40.5910	34.1857	4.0974
	Mean	8.8906	8.3265	5.9176	11.0304	3.8762	7.9511	6.2808	15.5271	2.5305	39.5056	27.4361	**2.4967**
	Std	1.5359	1.6538	1.7602	3.6239	1.1050	1.1440	2.7374	3.1841	1.4460	2.4950	2.8382	0.6857
Sagas	Best	339.7481	218.3113	78.5858	331.5902	92.9502	32.2400	99.6094	822.4027	334.2383	1975.6650	1401.9609	**18.5826**
	Worst	998.4001	987.7864	980.5872	1685.2883	1431.8453	934.8273	990.1673	1609.1574	998.2244	2051.1175	2452.0392	970.1247
	Mean	721.5120	663.8832	793.5563	1076.3671	730.9669	375.5439	748.3194	1204.4717	954.2162	2048.0994	1916.9474	**281.7730**
	Std	150.0681	227.3871	277.5440	281.9395	382.8495	212.8317	313.5975	225.9511	126.8973	14.7856	243.7374	294.7937
Messenger-full	Best	13.3504	15.9521	12.9234	11.9525	12.6515	12.3928	10.7861	18.3942	13.0608	47.8903	37.9870	**5.2185**
	Worst	25.5574	24.6548	20.2483	32.5251	20.4092	21.8737	18.6565	45.2742	16.7893	47.8903	61.1433	14.2408
	Mean	20.1813	20.1603	16.4134	19.3908	14.3437	17.5674	15.4157	30.6209	15.2339	47.8903	51.6815	**10.5344**
	Std	3.1119	2.1456	1.8579	4.6269	2.1323	2.4039	2.4969	6.9045	1.0470	0.0000	5.0622	2.5158

Table S5 shows that DE-1220 and HLSHADE obtain the best solution (4.9307) on Cassini1 benchmark, at the same time, the average performance of jDE is better than the other algorithms. On Rosetta benchmark, MDE_pBX obtains the best solution (1.3468). Except that, it is evident that HLSHADE is superior to the other algorithms in both best value and mean value on Cassini2, Gtoc1, Sagas, and Messenger-full benchmarks.

Table S6 and Table S7 also present the statistical analysis results according to the multiple-problem Wilcoxon's test and the Friedman's test, respectively. It can be seen from Table S6 that HLSHADE obtains higher R+ values than R- values in all the cases. Furthermore, the p values of all the cases except jDE are less than $\alpha = 0.05$. The Friedman's test results in Table S7 indicate that HLSHADE obtains the best ranking (1.1667) among the twelve algorithms. In summary, the above comparison clearly demonstrates that HLSHADE is significantly better than the competitors.

Table S6. Results of the multiple-problem Wilcoxon's test for HLSHADE and eleven algorithms in PyGMO at $\alpha = 0.05$ and at $\alpha = 0.1$ significance level based on the mean values.

HLSHADE vs.	R^+	R^-	p value	at $\alpha = 0.05$	at $\alpha = 0.1$
DE	21.0	0.0	0.03126	+	+
jDE	20.0	1.0	0.06250	=	+
PSO	21.0	0.0	0.03126	+	+
SGA	21.0	0.0	0.03126	+	+
CMA-ES	21.0	0.0	0.03126	+	+
ABC	21.0	0.0	0.03126	+	+
DE-1220	21.0	0.0	0.03126	+	+
MBH	21.0	0.0	0.03126	+	+
MDE-pBX	21.0	0.0	0.03126	+	+
MC	21.0	0.0	0.03126	+	+
IHS	21.0	0.0	0.03126	+	+

Table S7. Average rankings of the twelve algorithms according to the Friedman's test based on the mean values.

Algorithm	Ranking
DE	6.1667
jDE	5.3333
PSO	4.3333
SGA	8.8333
CMA-ES	5.6667
ABC	4.8333
DEA1220	4.8333
MBH	9.3333
MDE_pBX	4.6667
MC	11.5000
IHS	11.3333
HLSHADE	**1.1667**

5. Experimental analysis of control parameter

In order to determine the initial values of parameters ρ_1 and ρ_2, the parameter sensitivity experiment is done. The experiment tests a set of values of ρ_1 and ρ_2. The values of ρ_1 are set as $\{5, 10, 15, 20, 25, 30\}$ and the values of ρ_2 are set as $\{3, 5, 10, 15\}$. The experimental results are shown in Table S8-S9. The smaller the Friedman's test ranking value is, the better the corresponding setting is. It can be seen from Table S8-S9 that $\rho_1 = 20$ and $\rho_2 = 10$ are the better choice parameters to HLSHADE.

Table S8. Average Rankings of the HLSHADE settings with different ρ_1. These ranking values are obtained by Friedman test in terms of best result among 25 runs on GTOP problems.

Parameter setting	Ranking
$\rho_1 = 5$	5.8571
$\rho_1 = 10$	4.5714
$\rho_1 = 15$	3.5714
$\rho_1 = 20$	**2.1429**
$\rho_1 = 25$	2.4286
$\rho_1 = 30$	2.4286

Table S9. Average Rankings of the HLSHADE settings with different ρ_2. These ranking values are obtained by Friedman test in terms of best result among 25 runs on GTOP problems.

Parameter setting	Ranking
$\rho_2 = 3$	2.5714
$\rho_2 = 5$	2.4286
$\rho_2 = 10$	**2.2857**
$\rho_2 = 15$	2.7143

6. Experiment analysis of each part of HLSHADE

To further confirm the effectiveness of each part of HLSHADE, HLSHADE is compared with LSHADE_SPS_EIG, LSHADE_SPS_EIG+CMA-ES, LSHADE_SPS_EIG+Interior-point on GTOP. In Table S10 and Table S11, it can be seen that HLSHADE performs significantly better than LSHADE_SPS_EIG, LSHADE_SPS_EIG+CMA-ES, LSHADE_SPS_EIG+Interior-point on 5, 6, 5 test problems. HLSHADE obtains the 2 similar results with LSHADE_SPS_EIG. HLSHADE is worse than LSHADE_SPS_EIG+CMA-ES, LSHADE_SPS_EIG+Interior-point on 1 and 2 problems. Moreover, the results shown in Table S12 indicate that HLSHADE has the best ranking (1.7143) among the four algorithms, LSHADE_SPS_EIG+Interior-point and LSHADE_SPS_EIG+CMA-ES are ranked second and third, respectively. In general, the above comparison clearly demonstrates that the different parts can effectively improve the search performance of HLSHADE.

Table S10. Effectiveness analysis results of different parts of HLSHADE over 25 independent runs on GTOP benchmark.

Benchmarks	Paradigm	LSHADE_SPS_EIG	LSHADE_SPS_EIG+CMA-ES	LSHADE_SPS_EIG+Interior-point	HLSHADE
Cassini1	Best	**4.9307**	4.9603	**4.9307**	**4.9307**
	Worst	11.0320	11.0322	11.2909	11.0320
	Mean	6.1740	6.2034	**5.7275**	5.7308
	Std	2.1634	2.1440	1.6404	1.5934
Cassini2	Best	11.0216	8.8951	9.0246	**8.8101**
	Worst	19.0964	17.1455	16.4910	15.9932
	Mean	14.6810	12.6766	11.5895	**12.0239**
	Std	1.9279	2.2586	2.2753	2.2353
Gtoc1	Best	-1487003.7	-1415497.4	-1575080.5	**-1576143.0**
	Worst	-878412.5	-890431.3	-810325.4	-901531.2
	Mean	**-1205745.4**	-1202111.5	-1189554.2	-1193033.1
	Std	172273.4	134355.5	178853.6	151251.4
Rosetta	Best	2.6013	1.9702	1.8849	**1.5528**
	Worst	5.6791	6.0532	4.7176	4.0974
	Mean	4.2042	2.9706	**2.4755**	2.4967
	Std	0.8137	1.0560	0.6829	0.6857
Sagas	Best	22.9860	18.6192	21.4422	**18.5826**
	Worst	970.6765	970.1408	970.1247	970.1247
	Mean	497.3500	303.2228	337.6468	**281.7730**
	Std	358.3584	332.0371	336.2037	294.7937
Messenger	Best	10.2302	9.9364	9.9893	**9.0582**
	Worst	13.8272	14.6693	14.6408	13.7744
	Mean	12.6263	11.8264	11.8041	**11.7519**
	Std	0.9223	1.2642	1.0835	1.2589
Messenger-full	Best	10.7932	7.1101	7.2363	**5.2185**
	Worst	17.6064	14.8663	16.1554	14.2408
	Mean	14.0647	11.8617	10.8502	**10.5344**
	Std	2.0393	2.4667	1.9955	2.5158

Table S11. Results of the Wilcoxon's rank sum test between HLSHADE and different parts of HLSHADE over 25 independent runs on GTOP.

Benchmarks	LSHADE_SPS_EIG	LSHADE_SPS_EIG+CMA-ES	LSHADE_SPS_EIG+Interior-point
Cassini1	=	+	−
Cassini2	+	+	+
Gtoc1	=	−	+
Rosetta	+	+	−
Sagas	+	+	+
Messenger	+	+	+
Messenger-full	+	+	+
	5+/ 2=/ 0−	6+/ 0=/ 1−	5+/ 0=/ 2−

"+", "−", "=" denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon's rank sum test at $\alpha = 0.05$.

Table S12. Average rankings of between HLSHADE and different parts of HLSHADE according to the Friedman's test based on the mean values.

Algorithm	Ranking
LSHADE_SPS_EIG	3.4286
LSHADE_SPS_EIG+CMA-ES	2.8571
LSHADE_SPS_EIG+Interior-point	2.0000
HLSHADE	**1.7143**

7. Experiments on CEC2017 benchmark suite

To further test the performance of HLSHADE on the CEC benchmark suite, more experiments have been done to examine it on CEC2017 benchmark suite. HLSHADE is compared with the five algorithms including LSHADE, LSHADE_SPS_EIG, PaDE, DISH, and LSHADE_PWI on D=10, 30, 50. As for the five algorithms used in the comparison, the corresponding parameter settings are the same in the recommended settings on CEC2017 competition when conducting algorithm validation under this test suite. Each of the six algorithms under tests is executed 51 runs with respect to the test functions. The parameters of the experiment are set as follows:

(1) LSHADE: max_eval=$10,000 \times D$, PopSize=$18 \times D$, MinPopSize=4
(2) LSHADE_SPS_EIG: max_eval=$10,000 \times D$, PopSize=$18 \times D$, MinPopSize=4
(3) PaDE: max_eval=$10,000 \times D$, PopSize=$round(25 \times logD \times sqrtD)$, F = 0.8, CR = 0.6, MinPopSize=4, pmax=pmin=0.11
(4) DISH: max_eval=$10,000 \times D$, PopSize=$floor(25 \times logD \times sqrtD)$, pmax=0.25, pmin=0.125
(5) LSHADE_PWI: max_eval=$10,000 \times D$, PopSize=$18 \times D$, MinPopSize=4
(6) HLSHADE: max_eval=$10,000 \times D$, PopSize=$18 \times D$, MinPopSize=4

HLSHADE is compared with the performance of these five algorithms in Table S13, Table S15 and Table S17. Table S13 shows the results of Wilcoxon's rank sum test between HLSHADE and the compared algorithms on D=10. It can be seen that HLSHADE performs significantly better than LSHADE, LSHADE_SPS_EIG, PaDE, DISH, and LSHADE_PWI on 12, 12, 10, 14, and 12 test problems respectively. HLSHADE is worse on 9, 8, 12, 8, and 10 test problems respectively. Furthermore, HLSHADE obtains the 8 similar results with PaDE, DISH, and LSHADE_PWI, and the 10 similar results with LSHADE_SPS_EIG, and the 9 similar results with LSHADE. From Table S15 and Table S17, it can be seen that the performance of HLSHADE is worse than other five algorithms on benchmark functions at D=30 and D=50.

To further analyze the performances of the algorithms, the experimental results on CEC2017 are discussed according to the following 3 categories: simple functions (F1-F10), hybrid functions (F11-F20) and composition functions (F21-F30).

On ten simple functions, it can be seen that the performance of HLSHADE is worse than other five algorithms at D=10, D=30 and D=50 from Table S14, Table S16 and Table S18.

On ten hybrid functions, in Table S14, HLSHADE is better than LSHADE_SPS_EIG and DISH in 5 and 6 functions respectively, while it is worse than LSHADE, PaDE, and LSHADE_PWI in 4, 6, 5 functions at D=10. Moreover, from Table S16, the performance of HLSHADE is similar to LSHADE_SPS_EIG and LSHADE_PWI, but is worse than LSHADE, PaDE, and DISH on ten hybrid functions at D=30. From Table S18, it can be seen that the performance of HLSHADE is better or similar to the compared algorithms at D=50, except for DISH.

At last, on ten composition functions, it can find that HLSHADE is much better than other five algorithms on ten composition functions at D=10 from Table S14. HLSHADE is better than LSHADE, LSHADE_SPS_EIG, PaDE, DISH, and LSHADE_PWI on 9, 6, 8, 8, and 9 functions respectively, while it is worse than LSHADE, LSHADE_SPS_EIG, PaDE, DISH, and LSHADE_PWI on 1, 1, 2, 2, and 1 at D=10. In Table S16, HLSHADE is better or similar than LSHADE, LSHADE_SPS_EIG, PaDE, and LSHADE_PWI, respectively. In Table S18, the performance of HLSHADE is worse than the compared algorithms.

At the same time, according to HLSHADE's analysis results on GTOP benchmarks, it can be seen that HLSHADE has better than the compared algorithms including LSHADE, LSHADE_SPS_EIG, PaDE, DISH, and LSHADE_PWI.

Based on the above experimental analysis, it is interesting to be found that HLSHADE has better performance on GTOP benchmarks and composition functions at D=10 and D=30 on CEC2017 test suite, but it has poor performance when solving simple functions and partially hybrid functions on CEC2017 test suite, especially for the 50-dimensional functions.

There are several reasons for this phenomenon and the future of work has been found:

(1) HLSHADE is designed to solve the interplanetary trajectory optimization problem. Based on the search space characteristics analysis of interplanetary trajectory optimization on GTOP, this type of problems have the characteristics of the extreme non-linearity search space, a large number of locally optimal solutions, and sensitivity to initial conditions. So, the global search algorithm without local search is difficult to find the optimal solution of interplanetary trajectory optimization problems. It can be seen that the hybrid global and local search algorithms (such as HLSHADE and UMOEAsII) are more conducive to solving the interplanetary trajectory optimization problems.

(2) Since the local search strategy of HLSHADE consists of two steps, they consume a lot of function evaluation times in local search, which is a waste for simple functions on CEC2017 test suite. Therefore, HLSHADE has a poor performance on simple functions and partially hybrid functions on CEC2017 test suite.

(3) DISH has good global search on CEC2017 test suite, especially for high-

dimensional functions. This inspires us to make hybrid improvements based on DISH and its variants in future work to solve the interplanetary trajectory optimization problems.

Table S13. Experimental results of HLSHADE and five algorithms on CEC2017 benchmark suite at D=10.

	LSHADE			LSHADE_SPS_EIG			PaDE			DISH			LSHADE_PWI			HLSHADE	
	Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std
F1	0.00E+00	0.00E+00	=	1.18E−06	5.45E−06	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F2	0.00E+00	0.00E+00	=	6.34E−07	2.14E−06	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	7.61E−06	3.75E−05
F3	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F4	0.00E+00	0.00E+00	=	4.58E−02	2.21E−02	+	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F5	2.40E+00	9.78E−01	−	3.22E+00	1.09E+00	−	2.42E+00	8.72E−01	−	2.17E+00	7.63E−01	−	2.64E+00	7.40E−01	−	4.23E+00	9.51E−01
F6	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F7	1.22E+01	7.85E−01	−	1.33E+01	1.04E+00	−	1.23E+01	8.13E−01	−	1.20E+01	6.03E−01	−	1.22E+01	6.68E−01	−	1.44E+01	1.50E+00
F8	2.56E+00	1.09E+00	−	3.05E+00	1.01E+00	−	2.39E+00	7.73E−01	−	2.05E+00	1.06E+00	−	2.48E+00	8.76E−01	−	4.29E+00	1.13E+00
F9	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F10	2.30E+01	3.83E+01	−	4.75E+01	7.04E+01	−	3.80E+01	5.23E+01	−	4.65E+01	5.66E+01	−	3.28E+01	5.14E+01	−	8.75E+01	7.32E+01
F11	2.93E−01	6.54E−01	−	1.24E+00	7.10E−01	+	6.33E−01	8.17E−01	−	2.53E−10	1.80E−09	−	3.29E−01	6.63E−01	−	9.36E−01	7.01E−01
F12	1.96E+01	5.02E+01	+	3.40E+01	5.32E+01	+	2.15E+01	5.16E+01	+	1.44E+01	3.88E+01	+	1.21E+02	7.03E+01	+	7.35E−02	1.09E−01
F13	3.20E+00	2.48E+00	+	4.66E+00	2.75E+00	+	1.45E+00	2.28E+00	=	3.15E+00	2.28E+00	+	3.54E+00	2.51E+00	+	1.86E+00	1.21E+00
F14	1.30E−01	1.46E−01	−	1.20E−01	1.51E−01	−	2.91E−01	4.92E−01	−	8.16E−01	2.77E+00	=	1.33E−01	1.59E−01	−	5.65E−01	4.65E−01
F15	2.08E−01	2.09E−01	=	3.88E−01	4.15E−01	+	1.29E−01	1.87E−01	−	2.77E−01	2.20E−01	+	1.82E−01	2.09E−01	=	1.75E−01	2.58E−01
F16	3.84E−01	1.91E−01	=	4.35E−01	1.95E−01	=	3.75E−01	1.70E−01	−	8.24E−01	4.24E−01	+	3.66E−01	1.60E−01	−	3.98E−01	2.72E−01
F17	1.27E−01	1.37E−01	−	8.33E−02	1.25E−01	−	1.36E−01	1.75E−01	−	1.05E+00	2.84E+00	=	1.30E−01	1.40E−01	−	7.08E−01	6.45E−01
F18	1.90E−01	2.09E−01	+	4.07E−01	4.04E−01	+	2.39E−01	2.04E−01	+	2.88E−01	2.15E−01	+	2.20E−01	1.96E−01	+	7.12E−02	1.60E−01
F19	1.23E−02	1.62E−02	−	1.67E−02	1.02E−02	−	1.22E−02	1.06E−02	−	1.30E−02	1.28E−02	−	1.30E−02	1.06E−02	−	2.80E−02	1.94E−02
F20	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	1.13E+00	3.92E+00	+	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F21	1.83E+02	4.01E+01	+	9.80E+01	1.40E+01	=	1.45E+02	5.11E+01	+	1.39E+02	5.06E+01	+	1.00E+02	2.93E+00	+	1.00E+02	1.01E−08
F22	1.00E+02	4.04E−08	+	9.17E+01	2.57E+01	+	9.67E+01	1.69E+01	+	1.00E+02	6.24E−02	+	1.00E+02	4.01E−02	+	4.27E+01	4.59E+01
F23	3.03E+02	1.48E+00	+	3.05E+02	1.57E+00	+	2.96E+02	4.23E+01	+	3.02E+02	1.57E+00	+	3.04E+02	1.35E+00	+	2.88E+02	7.27E+01
F24	3.08E+02	6.93E+01	+	2.12E+02	1.17E+02	+	2.65E+02	1.02E+02	+	3.12E+02	6.25E+01	+	3.11E+02	6.32E+01	+	1.00E+02	2.83E+01
F25	4.14E+02	2.21E+01	+	3.98E+02	1.49E−01	+	4.13E+02	2.16E+01	+	4.06E+02	1.75E+01	+	4.14E+02	2.20E+01	+	3.98E+02	7.11E−02
F26	3.00E+02	1.11E−13	+	2.94E+02	4.20E+01	+	3.00E+02	0.00E+00	+	3.00E+02	0.00E+00	+	3.00E+02	9.09E−14	+	2.00E+02	1.43E+02
F27	3.89E+02	1.54E−01	+	3.89E+02	6.99E−01	=	3.93E+02	1.86E+00	+	3.89E+02	1.97E−01	+	3.89E+02	1.89E−01	+	3.89E+02	7.48E−01
F28	3.64E+02	1.23E+02	+	2.76E+02	8.15E+01	=	3.17E+02	6.74E+01	+	3.31E+02	8.77E+01	+	3.76E+02	1.31E+02	+	2.82E+02	7.13E+01
F29	2.34E+02	2.95E+00	−	2.38E+02	4.27E+00	−	2.34E+02	2.87E+00	−	2.34E+02	3.00E+00	−	2.34E+02	3.10E+00	−	2.41E+02	6.06E+00
F30	4.01E+02	1.67E+01	+	4.76E+02	5.29E+01	+	3.98E+02	1.31E+01	−	3.95E+02	3.47E−02	−	4.23E+02	2.26E+01	+	4.01E+02	2.66E+01
			12+/9−/9=			12+/8−/10=			10+/12−/8=			14+/8−/8=			12+/10−/8=		

"+", "−", "=" denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon's rank sum test at $\alpha = 0.05$.

Table S14. Comparison between HLSHADE and five algorithms on three types of functions on CEC2017 benchmark suite at D=10.

Algorithm	LSHADE	LSHADE_SPS_EIG	PaDE	DISH	LSHADE_PWI
F1-F10	0+/4−/6=	1+/4−/5=	0+/4−/6=	0+/4−/6=	0+/4−/6=
F11-F20	3+/4−/3=	5+/3−/2=	2+/6−/2=	6+/2−/2=	3+/5−/2=
F21-F30	9+/1−/0=	6+/1−/3=	8+/2−/0=	8+/2−/0=	9+/1−/0=

"+", "−", "=" denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon's rank sum test at $\alpha = 0.05$.

Table S15. Experimental results of HLSHADE and five algorithms on CEC2017 benchmark suite at D=30.

	LSHADE			LSHADE_SPS_EIG			PaDE			DISH			LSHADE_PWI			HLSHADE	
	Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std
F1	0.00E+00	0.00E+00	=	4.37E+02	5.82E+02	+	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	5.43E−06	3.84E−05
F2	0.00E+00	0.00E+00	−	1.26E−06	2.17E−06	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	9.34E+10	1.18E+10
F3	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	1.45E−07	8.13E−08
F4	5.86E+01	4.57E−14	=	8.50E+01	4.83E−01	+	5.57E+01	1.42E+01	=	5.86E+01	4.18E−14	=	5.86E+01	4.62E−14	=	5.63E+01	1.11E+01
F5	6.40E+00	1.51E+00	−	2.34E+01	4.69E+00	−	8.55E+00	1.71E+00	−	6.51E+00	2.07E+00	−	6.20E+00	1.56E+00	−	3.51E+01	5.60E+00
F6	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	1.88E−08	8.22E−08	−	1.07E−08	3.72E−08	−	1.41E−03	9.86E−03
F7	3.73E+01	1.23E+00	−	5.57E+01	4.66E+00	−	3.86E+01	1.86E+00	−	3.73E+01	1.52E+00	−	3.75E+01	1.37E+00	−	6.93E+01	7.42E+00
F8	6.99E+00	1.62E+00	−	2.52E+01	3.82E+00	−	8.80E+00	2.03E+00	−	7.06E+00	2.14E+00	−	7.10E+00	1.57E+00	−	3.70E+01	9.31E+00
F9	0.00E+00	0.00E+00	−	5.65E+00	4.41E+00	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	1.98E+01	3.76E+01
F10	1.41E+03	1.92E+02	−	1.29E+03	2.49E+02	−	1.53E+03	2.03E+02	−	1.52E+03	2.27E+02	−	1.46E+03	1.83E+02	−	1.71E+03	2.67E+02
F11	1.67E+01	2.17E+01	+	1.08E+01	3.21E+00	−	1.68E+01	2.34E+01	+	6.21E+00	1.40E+01	−	2.21E+01	2.62E+01	+	1.48E+01	5.06E+00
F12	1.04E+03	3.74E+02	+	5.03E+02	2.52E+02	+	1.03E+03	3.84E+02	+	1.18E+02	8.61E+01	=	1.09E+03	3.51E+02	+	1.59E+02	1.24E+02
F13	1.59E+01	6.51E+00	−	9.92E+01	5.38E+01	+	1.36E+01	6.47E+00	−	1.86E+01	4.53E+00	−	2.60E+01	8.48E+00	=	2.38E+01	5.30E+00
F14	3.26E+01	6.24E+00	−	3.46E+01	1.22E+01	−	2.19E+01	3.44E+00	−	3.48E+01	5.66E+00	−	3.33E+01	6.39E+00	−	4.66E+01	1.24E+01
F15	3.66E+00	1.71E+00	−	1.63E+01	6.45E+00	+	3.06E+00	1.68E+00	−	1.29E+00	9.52E−01	−	2.45E+01	2.60E+01	+	5.61E+00	1.76E+00
F16	5.20E+01	5.57E+01	−	2.11E+02	1.17E+02	−	8.94E+01	8.59E+01	−	3.55E+01	4.83E+01	−	4.17E+01	4.29E+01	−	2.91E+02	1.08E+02
F17	3.22E+01	6.82E+00	−	3.66E+01	1.30E+01	−	3.01E+01	7.24E+00	−	3.77E+01	6.66E+00	−	3.22E+01	6.37E+00	−	4.64E+01	1.38E+01
F18	2.18E+01	1.11E+00	=	2.72E+01	6.43E+00	+	2.17E+01	1.07E+00	=	2.05E+01	2.88E+00	−	6.04E+01	3.01E+01	+	2.19E+01	2.80E+00
F19	5.72E+00	1.88E+00	−	1.29E+01	4.60E+00	+	5.18E+00	1.42E+00	−	6.06E+00	1.92E+00	−	7.95E+00	2.09E+00	=	8.67E+00	1.74E+00
F20	3.37E+01	4.76E+00	−	3.55E+01	3.71E+01	−	3.71E+01	7.76E+00	−	3.27E+01	7.21E+00	−	3.17E+01	6.43E+00	−	6.35E+01	4.23E+01
F21	2.07E+02	1.32E+00	−	2.28E+02	4.40E+00	+	2.08E+02	1.57E+00	−	2.07E+02	1.81E+00	−	2.08E+02	1.38E+00	−	2.21E+02	4.51E+01
F22	1.00E+02	9.20E−14	=	1.00E+02	1.11E−13	=	1.00E+02	1.00E−13	=	1.00E+02	2.11E−13	=	1.00E+02	1.17E−13	=	1.00E+02	2.88E−07
F23	3.52E+02	3.42E+00	−	3.71E+02	5.68E+00	−	3.45E+02	3.29E+00	−	3.52E+02	3.39E+00	−	3.52E+02	2.39E+00	−	3.84E+02	6.15E+00
F24	4.27E+02	1.64E+00	−	4.45E+02	5.14E+00	−	4.21E+02	2.28E+00	−	4.28E+02	2.09E+00	−	4.27E+02	1.71E+00	−	4.67E+02	7.01E+00
F25	3.87E+02	2.61E−02	+	3.87E+02	2.00E−02	+	3.87E+02	3.00E−02	+	3.87E+02	7.84E−03	−	3.87E+02	2.50E−02	+	3.87E+02	4.20E−02
F26	9.45E+02	4.68E+01	=	1.19E+03	5.22E+01	=	8.73E+02	4.24E+01	=	9.62E+02	4.45E+01	=	9.51E+02	3.66E+01	=	7.15E+02	5.85E+02
F27	5.05E+02	4.58E+00	+	5.00E+02	3.49E+00	=	5.06E+02	7.10E+00	+	4.98E+02	7.04E+00	=	5.04E+02	4.34E+00	+	4.99E+02	4.63E+00
F28	3.11E+02	3.36E+01	+	3.00E+02	1.98E+00	=	3.17E+02	3.95E+01	+	3.02E+02	1.45E+01	=	3.34E+02	5.48E+01	+	3.00E+02	8.87E−12
F29	4.34E+02	7.13E+00	=	4.06E+02	3.70E+01	−	4.32E+02	7.47E+00	=	4.39E+02	8.98E+00	=	4.34E+02	6.91E+00	=	4.27E+02	4.46E+01
F30	1.97E+03	3.65E+01	+	2.26E+03	2.40E+02	+	2.05E+03	6.17E+01	+	1.97E+03	1.50E+01	+	2.10E+03	1.20E+02	+	1.95E+03	6.44E+00
		6+/17−/7=			10+/15−/5=			6+/17−/7=			1+/20−/9=			8+/14−/8=			

“+”, “−”, “=” denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon’s rank sum test at $\alpha = 0.05$.

Table S16. Comparison between HLSHADE and five algorithms on three types of functions on CEC2017 benchmark suite at D=30.

Algorithm	LSHADE	LSHADE_SPS_EIG	PaDE	DISH	LSHADE_PWI
F1-F10	0+/7−/3=	2+/7−/1=	0+/7−/3=	0+/7−/3=	0+/7−/3=
F11-F20	2+/7−/1=	5+/5−/0=	2+/7−/1=	0+/9−/1=	4+/4−/2=
F21-F30	4+/3−/3=	3+/3−/4=	4+/3−/3=	1+/4−/5=	4+/3−/3=

“+”, “−”, “=” denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon’s rank sum test at $\alpha = 0.05$.

Table S17. Experimental results of HLSHADE and five algorithms on CEC2017 benchmark suite at D=50.

	LSHADE			LSHADE_SPS_EIG			PaDE			DISH			LSHADE_PWI			HLSHADE	
	Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std		Mean	Std
F1	0.00E+00	0.00E+00	=	2.52E+03	1.43E+03	+	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00
F2	0.00E+00	0.00E+00	−	1.29E+03	3.90E+03	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	1.96E+11	1.94E+10
F3	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	0.00E+00	0.00E+00	=	5.45E−07	5.74E−07
F4	6.36E+01	4.90E+01	+	7.65E+01	2.86E+01	+	7.51E+01	4.73E+01	+	5.94E+01	5.07E+01	=	5.26E+01	4.88E+01	=	2.87E+01	9.47E-01
F5	1.20E+01	1.81E+00	−	5.82E+01	9.33E+00	−	1.71E+01	2.50E+00	−	8.86E+00	2.98E+00	−	1.13E+01	2.32E+00	−	8.48E+01	1.38E+01
F6	4.14E−05	2.12E−04	−	9.50E−06	6.89E−06	−	1.29E−03	3.27E−03	−	1.28E−07	1.91E−07	−	1.18E−05	8.09E−05	−	1.93E−02	4.03E−02
F7	6.32E+01	2.14E+00	−	1.04E+02	8.43E+00	−	6.48E+01	2.10E+00	−	6.25E+01	2.30E+00	−	6.28E+01	1.64E+00	−	1.46E+02	1.25E+01
F8	1.09E+01	1.94E+00	−	5.58E+01	7.04E+00	−	1.75E+01	2.48E+00	−	8.73E+00	2.59E+00	−	1.15E+01	1.77E+00	−	8.13E+01	1.49E+01
F9	0.00E+00	0.00E+00	−	7.30E+01	2.89E+01	−	5.27E−03	2.13E−02	−	0.00E+00	0.00E+00	−	0.00E+00	0.00E+00	−	3.86E+02	4.32E+02
F10	3.17E+03	2.67E+02	=	2.74E+03	5.28E+02	−	3.13E+03	3.27E+02	−	3.39E+03	3.42E+02	=	3.19E+03	2.99E+02	=	3.28E+03	3.88E+02
F11	4.85E+01	9.69E+00	=	4.74E+01	5.01E+00	=	6.45E+01	1.25E+01	+	2.42E+01	3.71E+00	−	5.05E+01	1.06E+01	=	4.91E+01	7.13E+00
F12	2.19E+03	5.07E+02	+	8.65E+03	4.03E+03	+	2.18E+03	4.96E+02	+	1.41E+03	3.68E+02	+	4.56E+03	1.83E+03	+	1.18E+03	3.01E+02
F13	6.54E+01	3.40E+01	+	4.60E+02	4.31E+02	−	5.37E+01	2.49E+01	=	3.24E+01	2.11E+01	−	1.13E+02	6.20E+01	+	4.93E+01	1.91E+01
F14	2.41E+02	7.67E+01	−	3.65E+02	1.45E+02	−	3.03E+01	3.64E+00	−	2.75E+02	1.18E+02	−	2.51E+02	8.03E+01	−	4.27E+02	1.22E+02
F15	4.14E+01	1.13E+01	+	1.29E+02	4.77E+01	+	4.36E+01	1.07E+01	+	2.15E+01	2.15E+00	−	2.87E+02	5.95E+01	+	3.25E+01	4.91E+00
F16	4.26E+02	1.24E+02	−	5.96E+02	1.17E+02	=	3.70E+02	9.51E+01	−	4.46E+02	1.38E+02	−	3.89E+02	1.19E+02	−	6.45E+02	1.34E+02
F17	2.37E+02	7.35E+01	−	3.71E+02	1.33E+02	−	2.79E+02	5.15E+01	−	2.36E+02	8.40E+01	−	2.59E+02	6.68E+01	−	4.72E+02	1.05E+02
F18	4.29E+01	1.60E+01	+	6.08E+01	1.84E+01	+	4.12E+01	1.32E+01	+	2.31E+01	1.34E+00	−	1.95E+02	5.87E+01	+	2.74E+01	3.28E+00
F19	2.37E+01	4.50E+00	+	3.60E+01	6.71E+00	+	2.88E+01	8.57E+00	+	1.30E+01	3.07E+00	−	7.71E+01	1.72E+01	+	2.03E+01	3.24E+00
F20	1.64E+02	6.61E+01	−	2.47E+02	1.34E+02	−	1.64E+02	6.45E+01	−	1.42E+02	7.02E+01	−	1.82E+02	7.58E+01	−	2.93E+02	9.37E+01
F21	2.13E+02	2.64E+00	−	2.62E+02	7.89E+00	+	2.18E+02	2.28E+00	−	2.11E+02	2.78E+00	−	2.13E+02	2.64E+00	−	2.90E+02	1.13E+01
F22	1.89E+03	1.81E+03	+	6.47E+02	1.21E+03	=	6.04E+02	6.04E+02	=	2.72E+03	1.65E+03	+	1.92E+03	1.82E+03	+	6.05E+02	1.29E+03
F23	4.32E+02	4.10E+00	−	4.80E+02	9.13E+00	−	4.27E+02	5.86E+00	−	4.31E+02	5.69E+00	−	4.32E+02	3.88E+00	−	5.16E+02	1.16E+01
F24	5.09E+02	2.44E+00	−	5.41E+02	7.81E+00	−	5.04E+02	5.03E+00	−	5.11E+02	2.68E+00	−	5.09E+02	2.53E+00	−	6.07E+02	1.80E+01
F25	4.83E+02	1.18E+01	+	4.81E+02	5.65E+00	+	5.02E+02	3.01E+01	+	4.80E+02	1.62E+00	+	4.82E+02	1.17E+01	+	4.80E+02	5.03E−03
F26	1.18E+03	4.79E+01	−	1.70E+03	1.00E+02	−	1.11E+03	7.46E+01	−	1.18E+03	4.85E+01	−	1.17E+03	4.35E+01	−	2.10E+03	1.03E+02
F27	5.44E+02	2.15E+01	=	5.21E+02	1.55E+01	−	5.40E+02	1.19E+01	=	5.18E+02	1.11E+01	−	5.33E+02	1.23E+01	−	5.41E+02	1.19E+01
F28	4.78E+02	2.39E+01	+	4.59E+02	2.80E−05	=	4.98E+02	1.96E+01	+	4.59E+02	4.30E−13	=	4.74E+02	2.27E+01	+	4.59E+02	3.49E−09
F29	3.51E+02	1.05E+01	−	3.78E+02	1.92E+01	−	3.51E+02	1.10E+01	−	3.68E+02	1.49E+01	−	3.53E+02	1.07E+01	−	4.42E+02	6.70E+01
F30	6.85E+05	1.04E+05	+	6.89E+05	7.55E+04	+	6.20E+05	4.39E+04	+	6.17E+05	4.39E+04	+	6.59E+05	7.53E+04	+	5.93E+05	2.38E+04
			10+/15−/5=			9+/16−/5=			9+/16−/5=			4+/21−/5=			9+/16−/5=		

"+", "−", "=" denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon's rank sum test at $\alpha = 0.05$.

Table S18. Comparison between HLSHADE and five algorithms on three types of functions on CEC2017 benchmark suite at D=50.

Algorithm	LSHADE	LSHADE_SPS_EIG	PaDE	DISH	LSHADE_PWI
F1-F10	1+/6−/3=	2+/7−/1=	1+/7−/2=	0+/6−/4=	0+/6−/4=
F11-F20	5+/4−/1=	4+/4−/2=	5+/4−/1=	1+/9−/0=	5+/4−/1=
F21-F30	4+/5−/1=	3+/5−/2=	3+/5−/2=	3+/6−/1=	4+/6−/0=

"+", "−", "=" denote that the performance of HLSHADE is respectively better than, worse than, and similar to the compared algorithms according to the Wilcoxon's rank sum test at $\alpha = 0.05$.