Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Ensemble Kalman filter based state estimation in 2D shallow water equations using Lagrangian sensing and state augmentation

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***Abstract*— We present a state estimation method for two-dimensional shallow water equations in rivers using Lagrangian drifter positions as measurements. The aim of this method is to compensate for the lack of knowledge of upstream and downstream boundary conditions in rivers that causes inaccuracy in the velocity field estimation by releasing drifters equipped with GPS receivers. The drifters report their positions and thus provide additional information of the state of the river. This information is incorporated into shallow water equations by using *Ensemble Kalman Filtering* (EnKF). The proposed method is based on the discretization of the governing non-linear equations using the finite element method in unstructured meshes. We incorporate the drifter positions into the unknown state, which directly exploits the Langrangian nature of the measurements. The performance of the method is assessed with twin experiments.**

I. INTRODUCTION

In river hydraulics, atmospheric sciences, and oceanography, forecasts are based on solutions of underlying partial differential equations and assimilation of measurement data into these models [1], [2], [3], [4]. The knowledge of the boundary conditions (and initial condition) of the partial differential equations are crucial for the reliability of the computed solutions. In river hydraulics, for example, the boundary conditions are collected from gauge stations from different parts of the river.

When modeling large river systems the boundary conditions might not always be known at the boundaries of the computational domain. In theory, we could extend the domain to cover a larger area, but the limitations of computational resources, such as memory and time, may prevent this. Measurements are sometimes too sparsely sampled in time to provide good results when used as boundary conditions for models. Furthermore, the installation of more accurate portable pressure sensors and velocity profilers is time consuming and the sensors are very expensive.

To compensate for the lack of knowledge in a case in which the boundary information is not accurate, Lagrangian sensors can be deployed into an area of interest. We can use measurements collected from these sensors to improve the output of underlying model.

Lagrangian measurements are obtained when the sensors are moving with the flow itself, reporting their location and possibly local conditions (such as speed, temperature, salinity, etc). Examples of these sensors in use can be found in oceanography [3], [5], [6] (often referred to as drifters or floats), in river hydraulics [4], and in traffic modeling (mobile sensors) [7]. The measurements are usually incorporated into an underlying partial differential equation model to improve realtime estimates or to provide more reliable forecasts. Langrangian measurements have also been used to predict particle motion in oceans [8].

The fundamental idea in the use of Lagrangian measurements is that while specific positions of the drifters do not provide information about the underlying flow phenomena, the subsequent measurements provide information about the velocity field when drifters advect to a new position over some time interval. The velocity field of the flow is often referred to as the Eulerian velocity field.

The present work focuses on river hydraulics. More specifically, we seek to incorporate Lagrangian measurements into a two-dimensional shallow water model with poorly known upstream and downstream boundary conditions using an *Ensemble Kalman Filter* (EnKF). The area of interest is a section of the Sacramento River in California.

The Ensemble Kalman filter is an alternative to the traditional *Extended Kalman filter* (EKF). With nonlinear dynamics, the linearization required in EKF (for example, in case of implicit time stepping) can be computationally too demanding. EnKF is a sequential data assimilation method which uses Monte Carlo or ensemble integrations. By integrating an ensemble of model states forward in time, it is possible to compute the mean and error covariances needed at analysis times (the time instants when measurements are used to update model output) [9], [10]. The analysis scheme in EnKF uses traditional update equations of the *Kalman Filter* (KF), except that the Kalman gain is computed using the error covariances provided by the ensemble of model states.

Whereas many large scale oceanographical problems are modeled using rectangular domains and Cartesian grids with large cell sizes and using simple boundary conditions for the flow, these type of methods are not directly applicable for smaller scale river flows. In this article we propose an inverse modeling method which can exploit all the necessary

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nonlinearities and complex geometries that are encountered in river modeling. The prediction step needed in the EnKF is made using a fully nonlinear *Shallow Water Equation* (SWE) solver package Telemac 2D [11], which operates on an unstructured triangular grid.

The contributions of this article are as follows. We employ state estimation for nonlinear 2D shallow water equations using Lagrangian measurements in rivers without deriving Eulerian flow velocities from the measurements. The method is tested by using experimental measurement data for bathymetry and boundary conditions. The geometry of the area is complex and thus we use an unstructured finite element mesh. The unknown state variables define a distributed parameter system and the Lagrangian sensors are incorporated into the state. This state augmentation approach, compared to traditional approaches, makes it possible to use a time-independent observation model which means that the observation model does not change, even though the locations from which the observations are collected change over time.

The rest of the article is organized as follows. In Section II, we present the Shallow Water Equations and Langragian sensor dynamics. In Section III, we define the state estimation problem, that is, the assimilation of Lagrangian data into the Shallow Water Equations. In Section IV, the proposed method is tested with simulations. Finally, in Section V we present conclusions and future milestones related to sensor deployments in the Sacramento delta.

II. FORWARD PROBLEM

A. Two-dimensional Shallow Water Equations

The governing hydrodynamic equations are [11], [12]:

$$\frac{\partial h}{\partial t} + \vec{u} \cdot \nabla h + h \nabla \cdot \vec{u} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u = -g \frac{\partial \eta}{\partial x} + F_x + \frac{1}{h} \nabla \cdot (h \nu_t \nabla u) \tag{2}$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \nabla v = -g \frac{\partial \eta}{\partial y} + F_y + \frac{1}{h} \nabla \cdot (h \nu_t \nabla v) \tag{3}$$

The friction forces are given by the following Manning law:

$$F_x = -\frac{1}{\cos\alpha} \frac{g m^2}{h^{4/3}} u \sqrt{u^2 + v^2} \tag{4}$$

$$F_y = -\frac{1}{\cos\alpha} \frac{g m^2}{h^{4/3}} v \sqrt{u^2 + v^2} \tag{5}$$

where h is the total depth of water, $\vec{u} = (u, v)$ is the velocity field, g is the acceleration of gravity, η is the free surface elevation, ν_t is the coefficient of turbulence diffusion obeying the so called k-epsilon model [11], $\alpha = \alpha(x, y)$ is the slope of the bottom, and m is the Manning coefficient. Finally, t is time and x, y are horizontal space coordinates. The boundary and initial conditions are given by

$$u(x,y,t)\big|_{\partial\Omega_{\text{land}}} = 0, \quad v(x,y,t)\big|_{\partial\Omega_{\text{land}}} = 0, \tag{6}$$

$$(u(x,y,t), v(x,y,t))\big|_{\partial\Omega_{\text{upstream}}} = f(x,y,t), \tag{7}$$

$$\eta(x,y,t)\big|_{\partial\Omega_{\text{downstream}}} = g(x,y,t), \tag{8}$$

$$u(x,y,0) = u_0, \quad v(x,y,0) = v_0, \quad h(x,y,0) = h_0, \tag{9}$$

where $\partial\Omega$ represents the boundaries of our computational domain and f, g are known functions (in the present case obtained from experimental measurements and a *Delta Simulation Model II* (DSM2) numerical tool [13]).

To clarify the notation, from now on we denote the concatenated vector of flow variables (u, v, h) by θ_F.

B. Lagrangian drifters

In addition to the above equations describing the flow dynamics, we model the deployed drifters as passive Lagrangian tracers. Hence, the drifters move with the local fluid velocity, obeying the following equations:

$$\frac{\mathrm{d}x_D(t)}{dt} = u[x_D(t), y_D(t), t], \tag{10}$$

$$\frac{\mathrm{d}y_D(t)}{dt} = v[x_D(t), y_D(t), t], \tag{11}$$

with the initial conditions

$$x_D(0) = x_{D,0}, \quad y_D(0) = y_{D,0}. \tag{12}$$

In (10) and (11) the dimension of x_D and y_D is the same as the total number of deployed drifters. In the sequel we denote the concatenated vector of drifter positions (x_D, y_D) by θ_D.

C. Numerical solution

The numerical solution of the 2D shallow-water equations and drifter positions, i.e., the *forward problem*, is computed using a commercial hydrodynamic software Telemac 2D [11]. Telemac 2D uses a streamline upwind Petrov-Galerkin based finite element solver for hydrodynamic equations.

III. STATE ESTIMATION PROBLEM

A. State-space equations

In the state estimation the aim is to extract information about the unknown quantity and use the measurements to provide additional information.

In this state estimation problem we employ a state-augmentation approach that has been earlier introduced in oceanography (see [6], [14], [15] for more details). In this approach, Lagrangian drifter positions are embedded in the state vector and the new state vector is written as

$$\theta = \begin{pmatrix} \theta_F \\ \theta_D \end{pmatrix}.$$

This approach makes it possible to take the Lagrangian nature of observations into account in a way which does not require deriving Eulerian quantities from Lagrangian measurements.

By inspecting (1)–(12) we note that there is a one-way coupled system from flow variables θ_F to drifter positions θ_D, that is, the drifter positions can be solved after the flow variables have been solved. Let us express this system by F_k, where subindex k refers to time step. Furthermore, due to the uncertainties in the modeling we add a stochastic term w_k to represent these modeling errors. The noise process w_k, with covariance Q_k, is a Gaussian noise process that is used to model inaccuracies in the evolution model (discretization error, poorly known inputs etc., see for example [16]).

Thus, we can write a nonlinear discrete stochastic state space model of the form:

$$\theta_{k+1} = F_k(\theta_k) + w_k. \tag{13}$$

Here θ_k is the predicted system state at time step k and $F_k(\theta_k)$ is one time step in the (non-linear) discretized shallow water and drifter model. Formula (13) is usually called the state evolution equation.

For the observations, we use an additive noise model

$$y_k = H_k\theta_k + \varepsilon_k. \tag{14}$$

The observation vector is y_k and the observation model that relates the state variables to the measurements is H_k. The noise process ε_k is the measurement noise with covariance R_k. Without loss of generality, noise processes are assumed to have zero mean. Furthermore, with these choices, the observation model H_k becomes simply $H_k = (\mathbf{0}\ I)$ where $\mathbf{0}$ is a zero matrix and I is the identity operator.

B. Ensemble Kalman Filter (EnKF)

In the filtering problem, the aim is to compute conditional expectations

$$\theta_{k|k} = \mathbb{E}(\theta_k | y_k, \ldots, y_1).$$

In the case of linear observation and evolution equations and Gaussian noise processes, the recursive Kalman filter algorithm can be used for determining the estimates of conditional expectation $\theta_{k|k}$ and covariance $\Gamma_{k|k}$. If evolution and/or observation equations are nonlinear, the Extended Kalman Filter can be applied [17]. The use of the EKF also requires that the operator $F_k(\cdot)$ is differentiable, which is not always the case in practice. However, to avoid the difficult linearization of the model F_k and to preserve the higher order statistics, which may be lost in the linearization, we employ the Ensemble Kalman filter.

For the state space model (13)–(14), the EnKF algorithm can be summarized as in [9], [1]:

1) Initialization: An ensemble of N states $\xi_{0|0}^{(i)}$ are generated to represent the uncertainty in θ_0.
2) Time update:

$$\xi_{k|k-1}^{(i)} = F(\xi_{k-1|k-1}^{(i)}) + w_{k-1}^{(i)} \tag{15}$$

$$\theta_{k|k-1} = \frac{1}{N}\sum_{i=1}^{N} \xi_{k|k-1}^{(i)} \tag{16}$$

$$E_{k|k-1} = [\xi_{k|k-1}^{(1)} - \theta_{k|k-1}, \ldots, \xi_{k|k-1}^{(N)} - \theta_{k|k-1}] \tag{17}$$

3) Measurement update:

$$\Gamma_{k|k-1} = \frac{1}{N-1} E_{k|k-1} E_{k|k-1}^{\mathrm{T}} \tag{18}$$

$$K_k = \Gamma_{k|k-1} H_k^{\mathrm{T}} [H_k \Gamma_{k|k-1} H_k^{\mathrm{T}} + R_k]^{-1} \tag{19}$$

$$\xi_{k|k}^{(i)} = \xi_{k|k-1}^{(i)} + K_k[y_k - H_k \xi_{k|k-1}^{(i)} + \varepsilon_k^{(i)}] \tag{20}$$

where the ensemble of state vectors are generated with the realizations $w_k^{(i)}$ and $\varepsilon_k^{(i)}$ of the noise processes w_k and ε_k, respectively. In the previous equations, an important step is that at measurement times, each measurement is represented by an ensemble. This ensemble has the actual measurement y_k as mean and the variance of the ensemble is used to represent measurement errors. This is done by adding perturbations $\varepsilon_k^{(i)}$ to measurements drawn from a distribution with zero mean and covariance equal to measurement error covariance matrix R_k. This ensures that the updated ensemble has a variance that is not too low [10]. The graphical illustration of the EnKF algorithm is presented in Fig. 1.

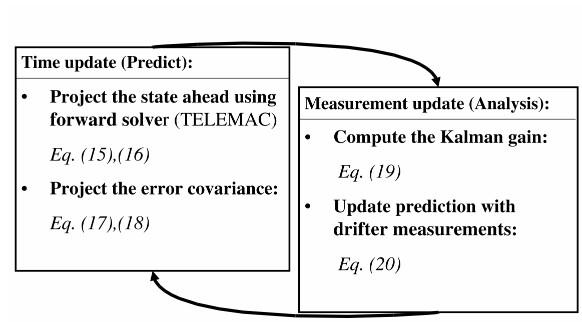


Fig. 1. Graphical illustration of the Ensemble Kalman Filter algorithm.

IV. NUMERICAL RESULTS

A. Simulating the drifter trajectories

The domain of interest is the Sacramento River, upstream of the intersection with the Georgiana Slough, in the Sacramento delta. The bathymetry is provided by United States Geological Survey (USGS) as shown in Fig. 2. The 930m long section of the Sacramento River is 85m wide in its narrowest part and 190m wide in its largest part.

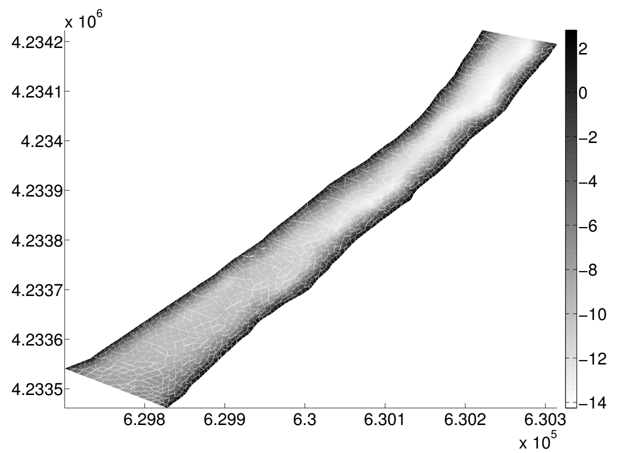


Fig. 2. Bathymetry in the Sacramento River (m). The bathymetry on this 930m section of the Sacramento River goes from -14m in the deepest part to +2m on the river banks.

The software package Telemac 2D is used to perform a non linear forward simulation from which the drifter position measurements and so called *true state* of the river is obtained. The true state and the drifter measurements are computed in a denser mesh (2972 nodes, 5484 triangular elements) compared to the mesh used in the solution of the inverse problem (1428 nodes, 2567 triangular elements). The use of different meshes is necessary when one wants to avoid the so-called *inverse crime*. This term refers to an overly optimistic setup of a twin experiment in which the simulated measurements are computed using the same underlying forward model and/or discretization of the forward model that is used in the solution of the inverse problem (see [18]).

The boundary conditions for models (Fig. 3) are computed using the Delta Simulation Model II (DSM2). It is a model of the San Francisco Bay and Sacramento Delta that provides discharge and surface elevation at various locations every hour. The bottom friction is modeled using Manning's law. The Manning coefficient is chosen to be constant in time and space and equal to 0.02, corresponding to a straight gravel bottom (see [19]). The simulation runs for two and a half hours before the release of the drifters so that a stable state is reached. The drifters are released starting at 1:00AM on June 29th 2006. This time period was chosen to provide a highly variable flow in Sacramento river.

We release a total of twenty four drifters during the experiment. The first eight drifters are released at 1:00AM on one line at the upstream boundary. Then at 1:20AM, we release eight more drifters at the upstream boundary, followed by eight more at 1:40AM. Drifter positions are then recorded every 30 seconds until the end of the experiment at 2:00AM. Fig. 4 shows the drifter trajectories and the snapshots of the drifter positions corresponding to the three releases of the drifters.

B. Data assimilation experiment

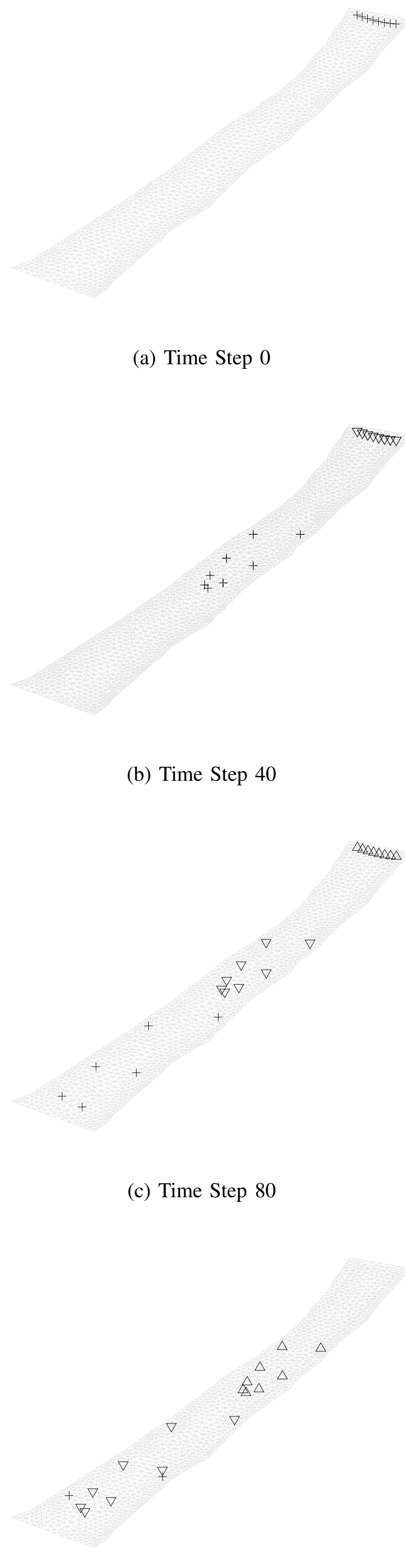
At first, an initial ensemble for the EnKF is created. We generate an ensemble of size $N = 100$ as follows. The idea is to produce N different initial velocity fields representating the different states of the river. In order to do this, we use data from the DSM2 model, which are available for a period of eight years. From these data, we extract statistical characteristics of the boundary conditions: a mean value of discharge upstream, a mean value of surface elevation downstream and a covariance matrix of those two quantities. From this, we generate a set of discharges and free surface elevations $(q^{(i)},\eta^{(i)})$, $i = 1,\dots,N$ that correspond to the statistical characteristics deduced from the historical data. For each discharge $q^{(i)}$, we deduce a velocity profile

$$(u^{(i)},v^{(i)})|_{\partial\Omega_{\text{upstream}}},$$

that is normal to the upstream boundary and proportional to the square root of the water depth at the upstream boundary. The downstream boundary condition is the surface elevation obtained directly from the historical data statistics:

$$\eta^{(i)}|_{\partial\Omega_{\text{downstream}}} = \eta^{(i)}.$$

In order to obtain realistic initial states for the EnKF algorithm, an 2.5 hour stabilization run is performed as



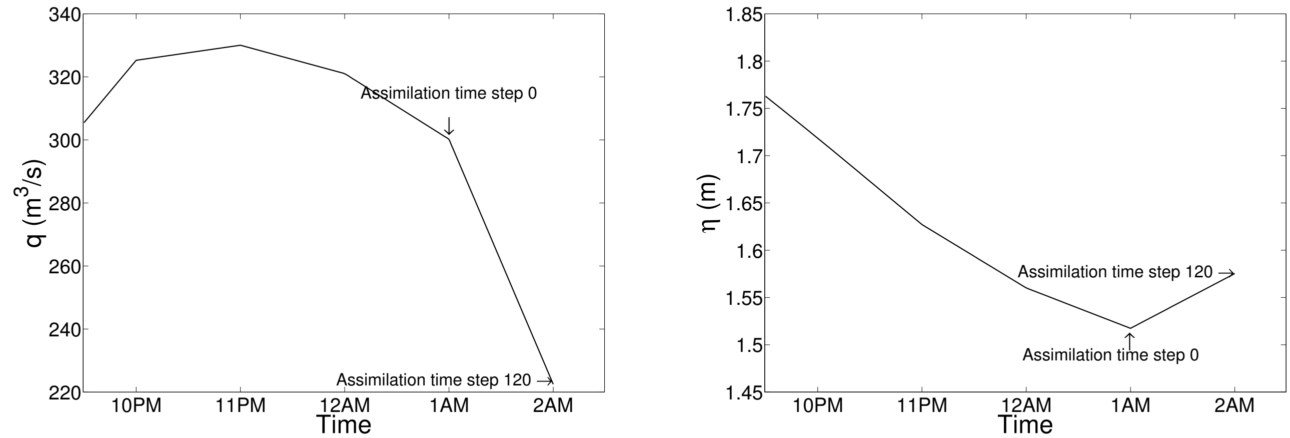
(a) Time Step 0

(b) Time Step 40

(c) Time Step 80

(d) Time Step 120

Fig. 4. Drifter trajectories and their release positions. 8 drifters are released at time step 0 (×), 8 drifters at time step 40 (▽) and 8 drifters at time step 80 (△). For measurement purposes, the drifter positions are sampled with a 30-second-time-interval until the time step 120.



(a) Upstream discharge

(b) Downstream free surface elevation

Fig. 3. The solid line represents the linearly interpolated DSM2 boundary conditions used in Telemac as boundary conditions.

follows. Forward solver Telemac 2D is started from zero velocity and it runs for 2.5 hours for each ensemble member using the set $(u^{(i)}, v^{(i)})|_{\partial\Omega_{\text{upstream}}}, \eta^{(i)}$ as boundary conditions. By doing this, numerically stable and physically meaningful initial states for flow variables $(u_0^{(i)}, v_0^{(i)}, h_0^{(i)})$ are obtained. Furthermore, the zero velocity initial condition that we use to instantiate Telemac 2D is not affecting the flow variables when the state estimation starts.

When stable states are reached, state estimation starts at 1:00 AM on June 29th 2006, with a total running time of 120 times a 30-second time step. In the state estimation procedure we do not update the velocity at the nodes that correspond to the land boundary. This is to make sure that we do not violate the no-slip condition. The initial boundary conditions for each ensemble member at time step zero are $(u^{(i)}, v^{(i)})|_{\partial\Omega_{\text{upstream}}}, \eta^{(i)}$; these are obtained from the last time step of the stabilizing run.

We set the standard deviation of the measurement noise ε_k to be 0.5 m in both x and y directions. This indicates our confidence on the measurement data, and can be seen as a realistic choice for the accuracy of GPS receivers.

The state noise models the discrepancy between the model and reality. The state noise for the flow variables is deduced from the properties of the ensemble members. The covariance matrix Q_k for the flow variables is computed as the covariance of the ensemble. We further assume that the mesh nodes corresponding to the land nodes are not affected by the state noise in order not to violate the no-slip condition on the land boundaries. In addition, we assume that there is a small uncertainty in the model which predicts the drifter positions. This uncertainty is modeled by a white noise, that is the covariance matrix for the drifters is a diagonal matrix. This uncertainty is indicated by setting the standard deviation of the state noise in the covariance matrix Q_k corresponding to all drifter coordinates to be 0.5 m in both x and y directions.

We compare the results of the EnKF algorithm with the results obtained from a *reference case*. The estimates for the reference case are obtained by propagating the initial ensemble forward in time without updating the state with the drifter measurements. In this case, the only knowledge about the boundary conditions in the river comes from historical data. This knowledge is thus contained in the ensemble and we use it to obtain our best estimate.

Fig. 5 shows the evolution in time of the relative error on the computation of the velocity field $\vec{u} = (u, v)$ with and without the drifters, computed using the following equation:

$$\varepsilon(t) = \sqrt{\frac{\sum_{i=1}^{\text{NNode}} \|\vec{u}_{\text{True}}(\mathbf{x}_i, t) - \vec{u}(\mathbf{x}_i, t)\|^2}{\sum_{i=1}^{\text{NNode}} \|\vec{u}_{\text{True}}(\mathbf{x}_i, t)\|^2}}, \qquad (21)$$

where NNode refers to the number of nodes in the mesh that is used in the inversion, and subindex "True" refers to the true state. Results from the reality simulation are interpolated to the same mesh where the state estimation is performed in order to compute this error.

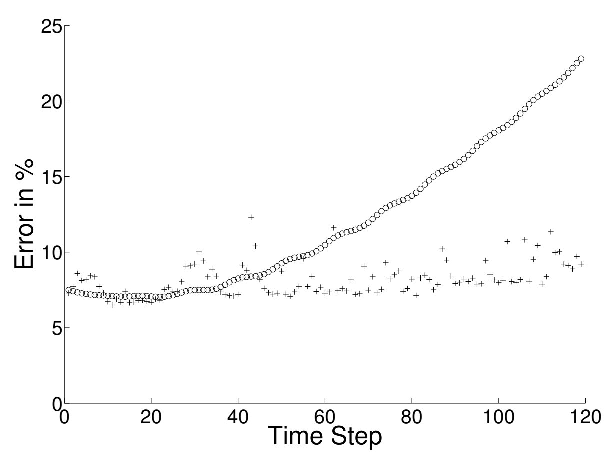


Fig. 5. Time evolution of the relative error of the velocity for EnKF estimate (+) and the reference case (○). The EnKF estimate captures the variations in the velocity field and thus, has a smaller error.

Fig. 5 shows the results of the state estimation problem and the reference case (forward simulation with no update of the state using the drifter measurements). At first, the reference case shows a small error, comparable to the error of the EnKF estimate. Indeed, the mean value of the discharge in the ensemble corresponds to the initial value of the discharge in reality. Thus, the velocity field obtained from the reference case in the first time steps is very close to the real one and the relative RMS error is low. Then the discharge in the river decreases and the reference case estimate diverges from the true state.

At later time steps in Fig. 5, the results of the state estimation problem show a remarkable improvement compared to the reference case. The assimilation of the drifter positions allows to catch the variation of the flow and thus correct the velocity field.

However, there is no improvement in the water depth relative error. The error is the same and approximately equal to 3% for all the simulations. This is justified by the fact that the information carried by the drifter measurements are exclusively related to the dynamics of the flow, allowing to reconstruct a correct velocity field, but giving no information about the water depth in the river.

Also, to give a view on the error in the velocity fields in the river, Fig. 6 shows the absolute error in the magnitudes of velocity fields at time step 0, 60 and 120 for the EnKF estimate (6a, 6c, 6e) and the reference case (6b, 6d, 6f). The velocity field resulting from the inverse problem is clearly closer to the true state than the velocity field deduced from the forward simulation without using the Lagrangian measurements.

By looking at Fig. 6 we can see that for the first time step, the absolute error for both simulations is quite small and around 2-5 cm/s. Then, while the reference case diverges from the reality and its error goes up until around 10 cm/s, the EnKF estimate error stays stable around 2-5cm/s. This is because the drifters catch the variation in the flow and thus correct the velocity field. One can also see that the error is bigger on the land boundary. This might be a consequence of the absence of drifters in those areas. Indeed, the drifter trajectories are mainly in the middle of the river and the velocity field cannot be corrected in the area with few or no drifter passage.

V. CONCLUSIONS AND FUTURE WORK

In this article we have presented a state estimation method for 2D shallow water equations in river hydraulics using Lagrangian measurement data. The solution method is based on the state augmentation and the use of ensemble Kalman filter. By using state augmentation it is possible to use the drifter positions as measurements directly without any mappings to Eulerian flow variables. The state augmentation approach has been earlier successfully introduced to oceanography in [14], [15], [6].

The performance of the method has been demonstrated using a twin experiment setting in which the true state and the measurement data have not been obtained using the same discretization of the underlying flow model. From the results a promising performance of the method can be seen. The method is capable of correcting the velocity field, predicted using poorly known boundary conditions, toward the true state of the river compared to the results obtained when no drifter measurements are used to correct the velocity field.

Because of the nature of river hydraulics, the correction of the surface elevation remains as a challenging task for the future. Although we used varying surface elevation to generate the ensemble, the fact that the drifters do not give a reliable value for their height do not allow a good correction for the surface elevation. One key point in the method proposed in this article was, however, to demonstrate that we can improve the estimates of the velocity field regardless of the fact that the boundary conditions used in the inversion are erroneous compared to those that were used to generate the true state.

Although the ensemble Kalman filter avoids the difficulties in the linearization of the forward model, the computational burden remains quite heavy. With the ensemble size $N = 100$ one assimilation step takes approximately 3.5 minutes to compute with a 2.2 GHz Pentium dual core processor. This can be seen as a small drawback with the current implementation of the method when considering real time applications. However, the forward solver can be parallelized. Also, the ensemble members are independent, so the computational burden can be distributed among multiple computers in the future.

Future works using the method include the use of real data collected from the GPS equipped drifters deployed into Sacramento Delta. Fig. 7 shows drifter trajectories extracted from GPS measurements during preliminary field test of the data collection protocols and drifter deployment on November 16th 2007.

With real data, the noise model for GPS measurements can also be embedded into the measurement update part of the EnKF algorithm. Also, when using real data, the correct state noise modeling plays an important role. State noise term should include information about the important stochastic properties of the flow that might be lost when using sparsely discretized shallow water equations.

Furthermore, the sensitivity of the method with respect to different deployment strategies and the optimal number of drifters is a future research topic.

VI. ACKNOWLEDGMENTS

The authors gratefully acknowledge Prof. Mark Stacey from UC Berkeley for giving valuable information about the river hydraulics during this work.

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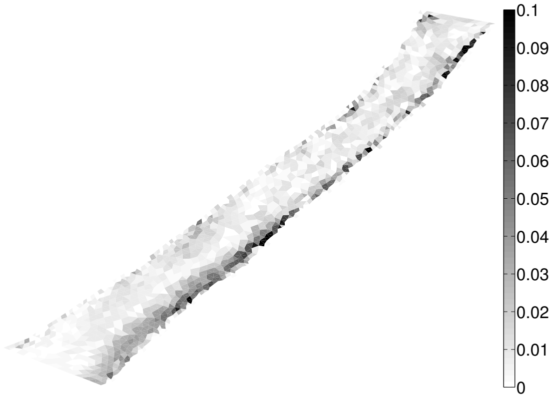
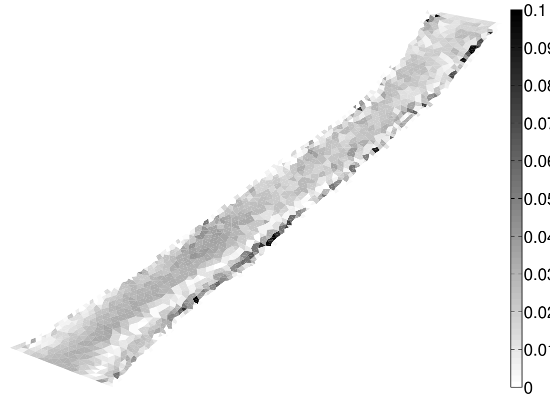
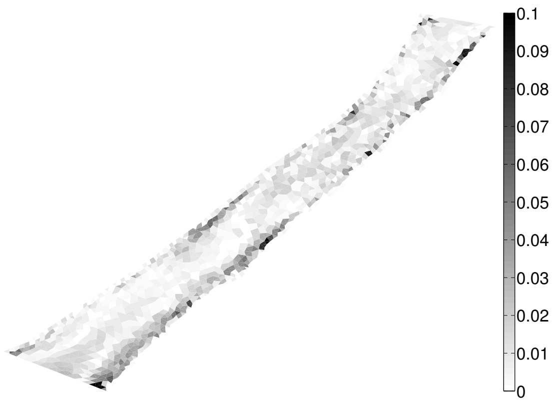


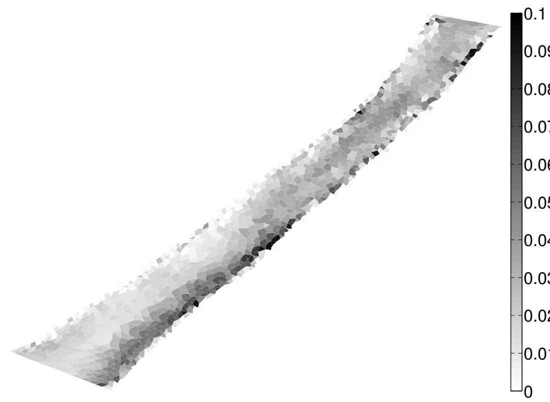
(a) Error in the EnKF estimate at time step 0



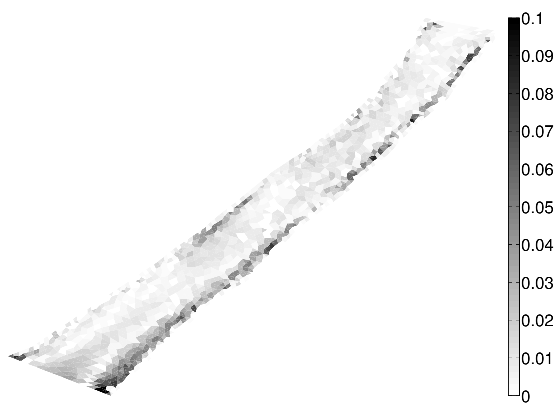
(b) Error in the reference case estimate at time step 0



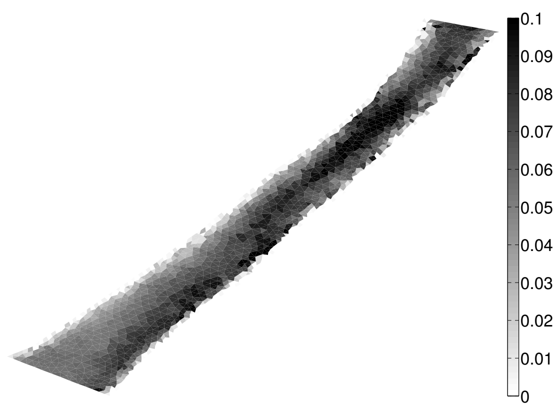
(c) Error in the EnKF estimate at time step 60



(d) Error in the reference case estimate at time step 60



(e) Error in the EnKF estimate at time step 120



(f) Error in the reference case estimate at time step 120

Fig. 6. Comparison of the absolute error in the velocity fields (m/s) for the EnKF estimate and the forward simulation of the ensemble members at time steps 0, 60 and 120. The absolute error is small, around 2-5 cm/s for the EnKF estimate. The reference simulation which is not using Lagrangian measurements sees its error increasing progressively until reaching 10 cm/s.

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Fig. 7. An example of drifter trajectories extracted from GPS measurements in the Sacramento River on November 16th 2007.

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