Open access • Journal Article • DOI:10.1007/S10483-008-0210-1

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Published on: 18 Feb 2008 - Applied Mathematics and Mechanics-english Edition (Springer Netherlands)

Topics: Forced convection, Brinkman number, Darcy's law, Péclet number and Heat transfer

Entropy Generation Analysis of Thermally Developing Forced Convection in a Fluid-Saturated Porous Medium

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Abstract

Entropy generation for thermally developing forced convection in a porous medium bounded by two isothermal parallel plates is investigated analytically on the basis of the Darcy flow model where the viscous dissipation effects had also been taken into account. A parametric study showed that decreasing the group parameter and the Péclet number increases the entropy generation while for the Brinkman number the converse is true. Heatline visualization technique is applied with an emphasis on Br<0 case where there is somewhere that heat transfer changes direction at some streamwise location *to the wall* instead of its original direction, i.e. *from the wall*.

1. Introduction

Minimization of entropy generation in any thermodynamic system leads to efficient use of exergy which is always destroyed partially or totally as a consequence of the second law of thermodynamics. The entropy generation is related to fluid flow and heat transfer irreversibility. The contributions of various mechanism and design features on the different irreversibility terms often compete with one another so that one may seek an optimized design to minimize the amount of entropy generation which leads to minimal lost work [1].

Bejan [2] has investigated the problem in detail for heat and fluid flow problems in clear fluid (of solid material) case and developed the work to the porous counterparts. Being relevant to a lot of industrial application, entropy generation resulting from heat and fluid flow through porous medium became a popular field of investigation, see for instance [3-14]. However, some moot points of the previous papers have been highlighted for a better understanding of the issue [15-17].

To the best knowledge of the authors, there exists no published article dealing analytically with the case of second law analysis of thermally developing forced convection neither in a porous medium nor in a clear fluid case. The aim of this paper is to fill this gap in the literature. Since the problem is a forced convection one with the Darcy flow model employed, the velocity field is already prescribed and is known to be a slug one so that one may obtain the temperature distribution by solving the conduction-like energy equation. However, one has to cope with the entropy generation in terms of heat transfer irreversibility (HTI) and fluid flow irreversibility (FFI) to find the sources of entropy generation locally and also to investigate the total entropy generation along the duct.

2. Analysis

Assuming the hydrodynamically fully developed flow, there exists a unidirectional flow in the streamwise direction while, at the same time, the flow is thermally developing; as shown in figure 1.

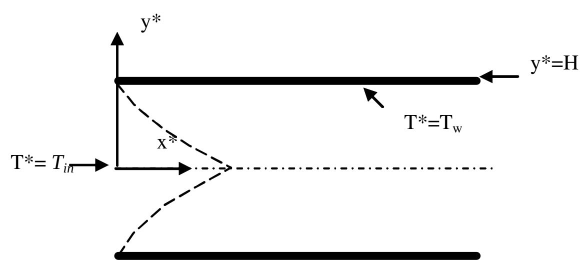


Fig. 1 Definition Sketch

Heat and fluid flow analysis

The Darcy flow model assumes that

$$-\frac{\mu}{K}u^* + P = 0 \tag{1}$$

where u* is the filtration velocity, μ is the fluid viscosity, K is the permeability, and P is the applied pressure gradient.

The thermal energy equation in the absence of heterogeneity, anisotropy, axial conduction, and, property variation becomes

$$\rho c_p u^* \frac{\partial T^*}{\partial x^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu u^{*2}}{K}. \tag{2}$$

where *k* is the porous medium thermal conductivity, ρ is the fluid density, and C_p is the specific heat at constant pressure.

The energy equation in its dimensionless form becomes

$$\hat{u}\frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial y^2} + Br\hat{u}^2. \tag{3}$$

with the following definitions

$$x = x^* \frac{\mu\alpha}{PKH^2}, y = \frac{y^*}{H}, \theta = \frac{T - T_w}{T_{in} - T_w}, \hat{u} = \frac{u}{\int_0^1 u dy} \quad (4)$$

The Brinkman number is defined as

$$Br = \frac{\mu U^2 H^2}{\mathrm{K}k(T_{in} - T_w)} \quad (5)$$

The appropriate boundary conditions to solve the energy equation are

$$\theta(0, \mathrm{y}) = 1, \quad \frac{\partial\theta(\mathrm{x},0)}{\partial \mathrm{y}} = 0, \quad \theta(\mathrm{x},1) = 0. \quad (6\text{-a,b,c})$$

Using the method of separation of variables, it is a straight forward task to find the solution to equation (3) subject to the aforementioned boundary conditions as

$$\theta(\mathrm{x}, \mathrm{y}) = \sum_{\mathrm{n}=0} \mathrm{b_n} \cos(\lambda_\mathrm{n} \mathrm{y}) \exp(-\lambda_\mathrm{n}^2 \mathrm{x}) + 0.5\mathrm{Br}(1 - y^2). \quad (7)$$

where the constants b_n may be found as

$$\mathrm{b_n} = \frac{2(-1)^\mathrm{n}\left(1 - \frac{\mathrm{Br}}{\lambda_\mathrm{n}^2}\right)}{\lambda_\mathrm{n}}. \quad (8)$$

with $\lambda_n=(2n+1)\pi/2$.

Having found the velocity distribution (based on the Darcy momentum equation) and the temperature distribution by equation (7), one readily starts the second law analysis.

Entropy generation analysis

It is known that entropy is generated through heat and fluid flow and the amount of volumetric entropy generation may be found in terms of HTI and FFI as follows

$$\dot{S}_{gen} = HTI + FFI \quad (9)$$

where HTI may be found as

$$HTI = k\left(\frac{\nabla T}{T}\right)^2 = \frac{k}{T^2}\left(\left(\frac{\partial T}{\partial x^*}\right)^2 + \left(\frac{\partial T}{\partial y^*}\right)^2\right) \quad (10)$$

and also FFI can be obtained by

$$FFI = \frac{\mu u^{*2}}{T\mathrm{K}} \quad (11)$$

In dimensionless form HTI becomes

$$HTI = \frac{k\Omega^2}{H^2} \frac{(\frac{\partial\theta}{\partial x})^2 \frac{1}{Pe^2} + (\frac{\partial\theta}{\partial y})^2}{\left(1+\Omega\theta\right)^2} \tag{12}$$

For dimensionless FFI one obtains

$$FFI = \frac{k\Omega^2}{H^2} \frac{\mu U^2 H^2}{\mathrm{K}k\Omega(T_i - T_w)\left(1+\Omega\theta\right)} = \frac{k\Omega^2}{H^2} \frac{Br}{\Omega\left(1+\Omega\theta\right)} \tag{13}$$

Consequently the dimensionless entropy generation becomes

$$\dot{S}_{gen} = \frac{k\Omega^2}{H^2\left(1+\Omega\theta\right)^2} \left(\left(\frac{\partial\theta}{\partial x}\right)^2 \frac{1}{Pe^2} + \left(\frac{\partial\theta}{\partial y}\right)^2 + \frac{Br}{\Omega}\left(1+\Omega\theta\right) \right) \tag{14}$$

Wherein the Peclet number is defined as $Pe= \rho c_p HU/k$ with U being the inlet velocity.

In particular from equations (12-14) the entropy generation becomes

$$\frac{\dot{S}_{gen}}{\frac{k\Omega^2}{H^2}} = \frac{\left(\begin{aligned} &\left(\sum_{\mathrm{n}=0} \lambda_{\mathrm{n}}^2 \mathrm{b}_{\mathrm{n}} Pe^{-1} \cos(\lambda_n \mathrm{y}) \exp(-\lambda_{\mathrm{n}}^2 \mathrm{x})\right)^2 + \\ &\left(\sum_{\mathrm{n}=0} \lambda_n \mathrm{b}_{\mathrm{n}} \cos(\lambda_n \mathrm{y}) \exp(-\lambda_{\mathrm{n}}^2 \mathrm{x}) + \mathrm{Br}y\right)^2 \\ &+ \left(\frac{Br}{\Omega} + Br \sum_{n=0} \mathrm{b}_{\mathrm{n}} \cos(\lambda_n \mathrm{y}) \exp(-\lambda_{\mathrm{n}}^2 \mathrm{x}) + 0.5 Br^2 (1-y^2)\right) \end{aligned} \right)}{\left(1 + \Omega \sum_{\mathrm{n}=0} \mathrm{b}_{\mathrm{n}} \cos(\lambda_n \mathrm{y}) \exp(-\lambda_{\mathrm{n}}^2 \mathrm{x}) + 0.5 \mathrm{Br}\Omega(1-y^2)\right)^2} \tag{15}$$

The dimensionless temperature difference is called group parameter (GP for short) in our work and is defined as

$$\Omega = \frac{T_{in} - T_w}{T_w} \tag{16}$$

The reader's attention is drawn to the point that our definition is different from those previously addressed group parameter as Br/Ω.

For large values of x, the temperature shows no change with x and the temperature distribution reduces to a parabolic one (for its series part vanishes). For the fully developed region the entropy generation becomes

$$\dot{S}_{gen} = \frac{2k}{H^2} \frac{G + y^2}{\left(\mathrm{G} - y^2\right)^2} \tag{17}$$

with G being defined as

$$G = 1 + \frac{2}{\Omega Br} \tag{18}$$

A modified Brinkman number, Br*, may be defined as

$$Br^* = \Omega Br = \frac{\mu U^2 H^2}{\mathrm{K} k T_w} \quad (19)$$

This modified Brinkman number shows the heat generated as a result of viscous dissipation divided by the maximum wall heat transfer. One refers to T_w in the denominator of Br* as a temperature difference, (T_w -0), measured in Kelvin where 0^oK is supposed as the minimum temperature in engineering applications so that this temperature difference is the maximum one that may happen for a system at T_w, here the duct wall.

Heatline visualization

The concept of heatfunction introduced by Bejan [2] shows the heat transfer details in a convection problem, i.e. to enable a convenient, visual representation of heat flow in a two-dimension convection (forced or natural) problem as a result of both diffusion and advection. As Bejan [2] argues, isotherms are as informative in a convection heat transfer problem as constant-pressure lines in a momentum transfer problem so that there is a need for a new definition like heatfunction to satisfy the energy equation intrinsically. At the same time, heatlines are lines along which heat flows. The following set of equations are applied in this problem

$$\begin{aligned} \frac{\partial H}{\partial \mathrm{x}} &= \frac{\partial \theta}{\partial \mathrm{y}} \\ \frac{\partial H}{\partial y} &= \theta - Brx \end{aligned} \quad (20)$$

Solving the above set of equations one finds that

$$H(\mathrm{x},\mathrm{y}) = \sum_{\mathrm{n}=0} \frac{\mathrm{b_n}}{\lambda_n} \sin(\lambda_n \mathrm{y}) \exp(-\lambda_\mathrm{n}^2 \mathrm{x}) + \mathrm{Br}\left(\frac{1}{2} - \frac{y^2}{6} - x\right)\mathrm{y}. \quad (21)$$

Note that according to Bejan [2], one should choose the lowest temperature of the system as the reference temperature for more informative heatline distribution, see [18] for example. This means that θ in equation (20) should be replaced by $\theta-\theta_{in}$ when $T_{in}<T_w$.

3. Results and Discussion

A channel length to width ratio of 5 is found to be large enough for the flow to become fully developed. Half of the domain is presented due to the channel symmetry in figures 2-5. Figure 2 shows the heatlines and energy flux vectors (see Hooman et al. [19]) for Br=1, and Pe=1. Observe that energy flux vectors are tangent to heatlines, as expected.

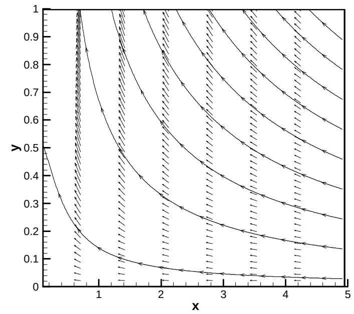


Figure 2 The heatlines and energy flux vectors for Br=1, Pe=1.

Moving from the heatlines to entropy generation, figure 3-a is presented to show the local dimensionless entropy generation contours. It is understood that the highest rate of entropy generation takes place just after the flow enters the channel and in this zone the maximum entropy generation happens in a region adjacent to the wall. This conclusion is inline with that of [9]. Fig 3-b is presented to show this zoomed region. These two figures are found for the case Br=1, and Pe=1. It may be concluded the top left corner of the channel is the place where the most exergy is destroyed for the incoming fluid, at a uniform temperature T_{in}, contacts the wall at a different temperature T_w and this results in an increased HTI compared to downstream regions.

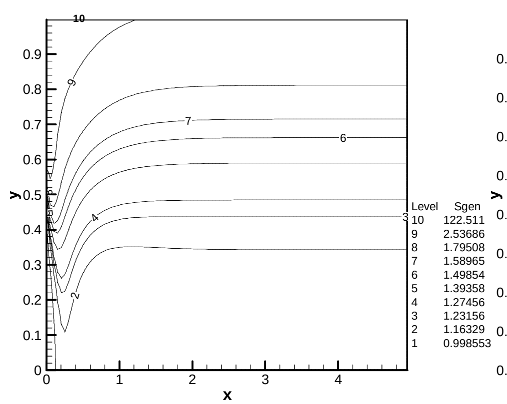


Figure 3-a The local entropy generation in the whole domain

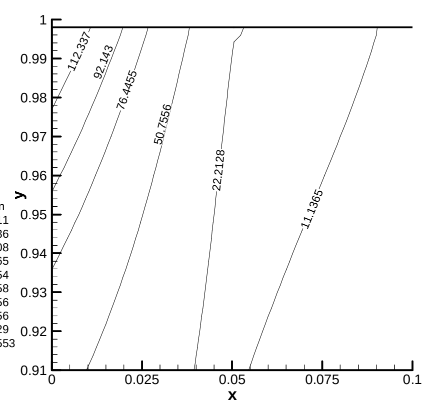


Figure 3-b The local entropy generation in the zoomed region

Figure 4 shows the area-weighted average of the entropy generation in the duct. This average is calculated by integrating the local entropy generation over the cross-section. It seems that effects of Pe and group parameter are somehow similar and, at the same time, opposed to those of Br. Increasing Pe implies a shrink in the thermal conductivity for a fixed mass flow rate. This reduced thermal conductivity plunges the entropy generation rate for being linearly related to it. Besides, an increase in the group parameter results in a fall in the dimensionless entropy generation rate as a result of lower FFI. On the contrary, increasing Br enhances HTI and, as a result, boosts the total entropy generation rate.

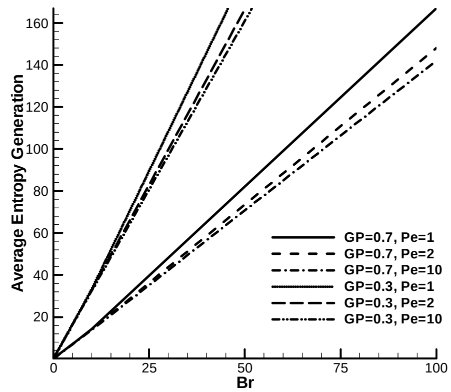


Figure 4 Average entropy generation versus Br.

Figure 5 shows the developing Nusselt number for some values of Br. It is observed that the fully developed Nu is independent of Br while for the developing Nu the situation is somehow different in such a way that higher Br results in higher developing Nu similar to the previous reports on similar cases, see [20-22] for example.

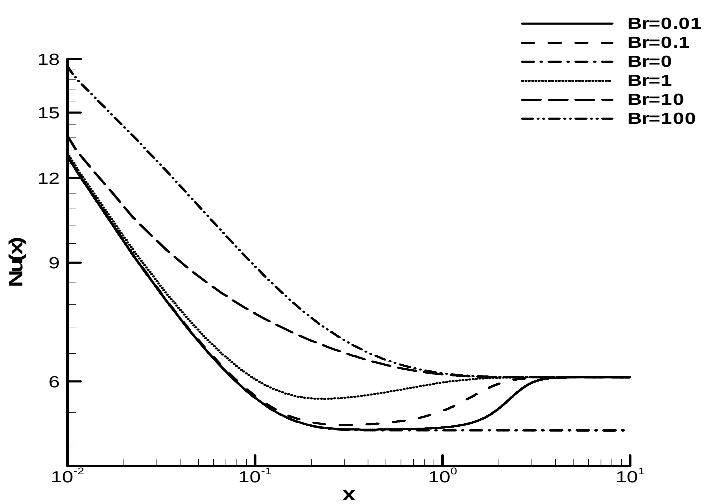
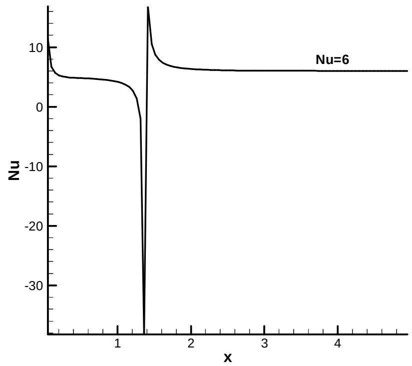
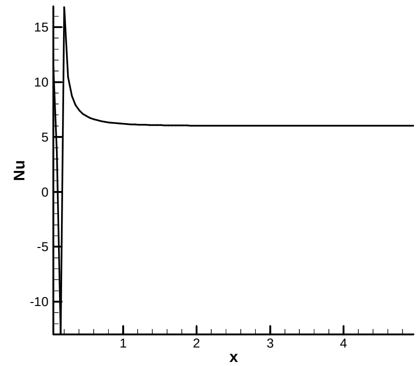


Figure 5 Nusselt number versus x for some values of Br.

A challenging problem is the negative Br case where the wall is to be cooled while the internal heat generation is an opposing effect. Figure 6 shows the Nusselt number versus x for two different Br values.



(a)



(b)

Figure 6 The Nusselt number versus x for a) Br=-0.1 b) Br=-5

A jump is observed in both of the above figures but clearly with lower Br the jump occurs closer to the duct entrance similar to what reported by Hooman et al. [23]. Note that the smaller absolute value of Br is, the longer the entry length is. Larger Br will help viscous dissipation to defeat over wall flux (heating or cooling) sooner and the fully developed region will be reached in a much smaller value of x. Even a small amount of viscous dissipation (nonzero Br) leads to a jump in the fully developed Nu to a value which is then independent of Br. A difference between the effect of positive Br and the effect of negative Br is ostentatious. The case Br>0 corresponds to incoming fluid being heated at the walls. The viscous dissipation produces a distribution of positive heat sources, and this reinforces the heating effect as the fluid moves downstream. As x increases the value of the Nusselt number passes through a minimum. For very large values of Br the value of Nu changes only slowly with x and when Br is large enough the curve will not experience a minimum. The case Br<0 corresponds to incoming fluid being cooled at the walls, and this cooling at the walls is opposed by the heating due to viscous dissipation in the bulk of the fluid. This was previously reported in some articles but in none of the above articles a visual representation of the problem was possible.

Presentation of heatlines has the advantage that one observes something that could not be shown by means of just isotherms. Mainly for this reason, heatlines are presented in figure 7. A region, at the entrance, is remarkable where heatlines are stretched from the wall to the bulk. Moving downstream, one observes a change in the direction of the heatlines from bulk to wall.

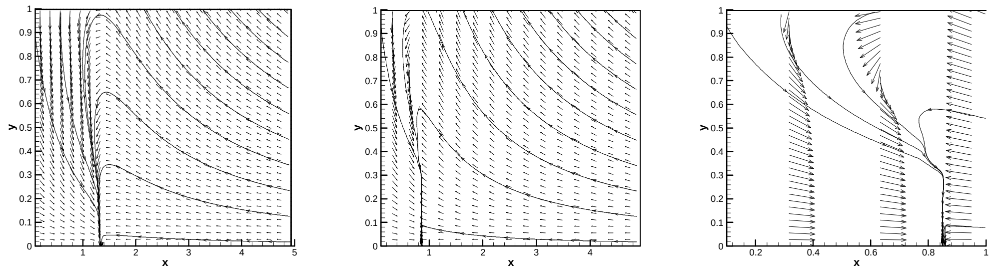


Figure 7 Heatlines and energy flux vectors for Pe=5 and a) Br=-0.1 b,c) Br=-5.

Figures 7-a,b shows the heatlines in the whole domain for two different Br values while figure 7-c shows the heatlines in a zoomed region (by the jump location for Br=-5, Pe=5). It is clearly observed that passing through the jump region, the heat transfer direction changes from wall to bulk of the flow to a completely inverse direction.

For the fully developed region the solution to the temperature distribution is a parabolic one corresponds to the viscous dissipation effects only since the wall heat flux decays downstream and the local entropy generation is indicated in figure 8.

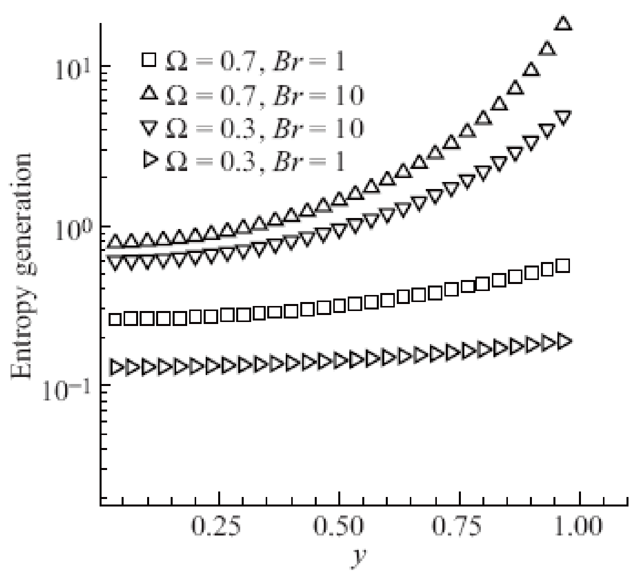


Fig. 8 Entropy generation in the fully developed region

One observes that the plots corresponding to higher Br stand over those of smaller Br and for a fixed Br the curves for low group parameter stand over those of high counterparts, as expected from the linear dependence of the fully developed temperature distribution on Br. Close to the walls, the entropy generation rate increases for higher HTI in this region (as opposed to zero heat flux in the duct centerline due to symmetry). At the duct centerline, unlike the clear fluid case, entropy generation will not vanish as a result of non-zero FFI. A uniform velocity distribution, due to slug flow assumption, in the duct cross-section allows FFI to vary only with the local temperature (in an inverse-linear fashion).

4. Conclusion

Thermally developing forced convection in the entrance region of a parallel plate channel, with the effects of viscous dissipation included, is investigated analytically based on the Darcy flow model. Heatlines and energy flux vector are also presented for a more comprehensive study of the problem. The second law of thermodynamics showed that Pe, Br, and Ω are parameters affecting the entropy generation so that one should use a proper combination of the aforementioned parameters to design a system with the least exergy destruction.

Acknowledgments

The first author, the scholarship holder, acknowledges the support provided by The University of Queensland in terms of UQILAS, Endeavor IPRS, and School Scholarship.

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