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# Entropy generation in hydrodynamic slip flow over a vertical plate with convective boundary †

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**Abstract**

The present article aims to report the effects of hydrodynamic slip on entropy generation in the boundary layer flow over a vertical surface with convective boundary condition. Suitable similarity transformations are used to transform the fundamental equations of hydrodynamic and thermal boundary layer flow into ordinary differential equations. The governing equations are then solved numerically using the shooting method and the velocity and the temperature profiles are obtained for various values of parameters involved in the governing equations. The expressions for the entropy generation number and the Bejan number are presented and the results are discussed graphically and quantitatively for the slip parameter, the local Grashof number, the Prandtl number, the local convective heat transfer parameter, the group parameter and the local Reynolds number. It is observed that due to the presence of slip, entropy production in a thermal system can be controlled and reduced.

*Keywords*: Boundary layer flow; Vertical plate; Hydrodynamic slip; Entropy generation

## 1. Introduction

The boundary layer flow over a flat surface has been studied extensively due to its wide applications in the fields of aerodynamics, mechanical engineering, chemical engineering etc. Blasius [1] was the first who investigated a steady two-dimensional boundary layer flow past a stationary flat plate. Pohlhausen [2] analyzed the heat transfer of the problem. Howarth [3] made hand computations for the problem using the Runge-Kutta scheme. Abbussita [4] established the existence of the solution for the flow past a flat plate. Since then, different scientists and engineers obtained the solution of the problem using different analytical and numerical techniques [5-7]. In all the above mentioned investigations, the no-slip condition at the boundary had been assumed. However, a situation may arise when the no-slip boundary condition is not appropriate. Slip velocity is a phenomenon that occurs due to non-adherence of the fluid to a solid boundary.

The fluids having slip velocity have many applications in technology such as polishing of surfaces and in micro devices. In the literature, there is a scarcity of studies of slip flow over a flat surface. Martin and Boyd [8] studied the effects of slip flow on the boundary layer flow over a flat plate. Vedantam [9] studied the same problem with three different models for the slip flow. Fang and Lee [10] extended the problem to moving plate. Aziz [11] investigated the influence of hydrodynamic slip on boundary layer flow over a flat plate with constant heat flux at the boundary. Bhattacharyya and Layek [12] studied the slip flow and heat transfer over a flat plate under the influence of magnetic field. Mehmood and Ali [13] studied the injection flow past a porous plate. Munawar et al. [14] commented on flow and heat transfer of viscoelastic fluid over a semi-infinite horizontal moving flat plate.

The study of convective heat transfer has much importance in high-temperature processes like gas turbines, nuclear plants, thermal energy storage, etc. Bataller [15] discussed the effects of thermal radiation and convective surface heat transfer on boundary layer flow in Blasius and Sakiadis flow. A similarity solution for laminar thermal boundary layer over a flat plate with a convective boundary condition was reported by Aziz [16]. Makinde [17, 18] studied the effects of buoyancy force over a stationary plate and the internal heat generation effects on moving vertical plate under convective boundary condition.

In thermodynamic analysis of flow and heat transfer processes, one thing of core interest is to improve the thermal systems to avoid energy losses and fully utilize energy resources. Second law analysis in terms of entropy generation rate is a

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useful tool to predict the performance of engineering processes by investigating the irreversibility arising during the processes. Different sources such as heat transfer and viscous dissipation are responsible for the production of entropy. Since the pioneering work done by Bejan [21], many investigations have been made on entropy generation analysis. Sahin [22] investigated the second law analysis for a viscous fluid in a circular duct with isothermal boundary conditions. Mahmud and Fraser [23] applied the second law analysis to heat and fluid flow due to forced convection inside a channel. The entropy generation in boundary layer flow was investigated by Arpaci and Selamet [24]. Odat et al. [25] studied the effect of magnetic field on entropy generation due to laminar forced flow past a horizontal plate. Saouli and Mahmud [26] discussed entropy generation in a falling liquid film along an inclined heated plate. Tshehla et al. [27] studied the irreversibility effects in a pipe flow with temperature dependent viscosity and convective cooling. Makinde [28] investigated the entropy generation in hydromagnetic variable viscosity boundary layer flow over a flat plate in the presence of thermal radiation and Newtonian heating.

The purpose of the present study is to observe the effects of hydrodynamic slip on entropy generation in a viscous flow over a vertical plate with convective boundary condition. The velocity and the temperature distribution are determined by solving the momentum and energy equation with a numerical technique, the shooting method. The expression for entropy generation rate is evaluated and the effects various parameters arising in the problem are analyzed.

## 2. Mathematical description of the problem

Consider the two-dimensional steady state incompressible boundary layer flow with heat transfer by convection over a vertical plate. The fluid is assumed to flow over the right surface of the plate with a uniform velocity $U_\infty$. The stream of the cold fluid has temperature $T_\infty$ and the right surface of the plate is heated by convection from a hot fluid at temperature $T_f$, which provides a heat transfer coefficient $h_f$ as shown in Fig. 1. Thus, the governing equations for the flow and heat transfer can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where $u$ and $v$ are the $x$ and $y$ components of the velocities respectively, $\nu$ is the kinematic viscosity of the fluid, $\rho$ is the density of the fluid, $c_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity of the fluid, $g$ is the gravitational constant and $\beta$ is the thermal expansion coeffi-

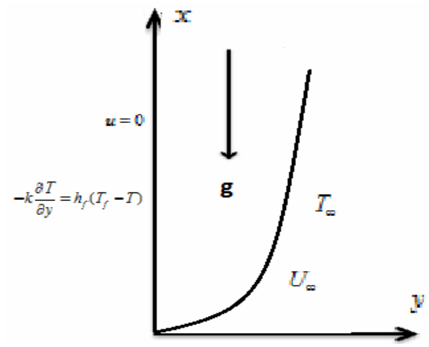


Fig. 1. Schematic diagram of the problem.

cient.

The boundary conditions for the velocity field are:

$$u(x,0)=\frac{(2-\sigma)}{\sigma}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=0},\ v(x,0)=0, \tag{4}$$

$$u(x,\infty)=U_\infty \tag{5}$$

where $\lambda$ is the mean free path and $\sigma$ is the tangential momentum accommodation coefficient. The boundary conditions for temperature at the surface and far into the cold fluid are:

$$-k\frac{\partial T}{\partial y}=h_f[T_f-T(x,0)], \tag{6}$$

$$T(x,\infty)=T_\infty. \tag{7}$$

Introducing the following dimensionless quantities:

$$\left.\begin{aligned}&\eta=y\sqrt{\frac{U_\infty}{\nu x}},\ u=U_\infty f'(\eta), v=\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(\eta f'-f),\\&\theta=\frac{T-T_\infty}{T_f-T_\infty}\end{aligned}\right\} \tag{8}$$

where prime denotes the differentiation with respect to $\eta$. Substituting Eq. (8) into Eqs. (1)-(7), we have:

$$f'''+\frac{1}{2}ff''+Gr_x\theta=0, \tag{9}$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr Ec\,(f'')^2=0. \tag{10}$$

The corresponding boundary conditions are

$$\left.\begin{aligned}&f'(0)=\frac{2-\sigma}{\sigma}Kn_x \mathrm{Re}_x^{1/2}f''(0)=Kf''(0),\\&f(0)=0,\ \ f'(\infty)=1\end{aligned}\right\} \tag{11}$$

$$\theta'(0)=-Bi_x[1-\theta(0)],\quad \theta(\infty)=0. \tag{12}$$

where $Kn_x$ and $\mathrm{Re}_x$ are the local Knudsen number and local Reynolds number defined by

$$Kn_x = \frac{\lambda}{x}, \quad \mathrm{Re}_x = \frac{U_\infty x}{\nu} \tag{13}$$

and $K$ is the slip (rarefaction) parameter defined as

$$K = \frac{2-\sigma}{\sigma} Kn_x \mathrm{Re}^{1/2}{}_x . \tag{14}$$

Also, the local Biot number, Prandtl number and local Grashof number are given as

$$Bi_x = \frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}}, \; \mathrm{Pr} = \frac{\mu c_p}{k}, \; Gr_x = \frac{g\beta x(T_f - T_\infty)}{U_\infty{}^2}. \tag{15}$$

Here, we assume

$$h_f = cx^{-1/2}, \quad \beta = mx^{-1} \tag{16}$$

where $c$ and $\beta$ are the constants. Substituting Eq. (16) into the parameters $Bi_x$ and $Gr_x$ we have

$$Bi = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}}, \quad Gr = \frac{mg(T_f - T_\infty)}{U^2{}_\infty}. \tag{17}$$

## 3. Numerical solution

The coupled non-linear differential Eqs. (9) and (10) with the boundary conditions Eqs. (11) and (12) are solved numerically using the shooting technique, and the calculations are made by utilizing the symbolic software MATHEMATICA.

Let $f = x_1, f' = x_2, f'' = x_3, \theta = x_4, \theta' = x_4$. Eqs. (9) and (10) are then transformed into a system of first order differential equations as follows:

$$\left.\begin{aligned} x_1' &= x_2, \\ x_2' &= x_3, \\ x_3' &= -\frac{1}{2}x_1 x_3 - Gr\, x_4, \\ x_4' &= x_5, \\ x_5' &= -\frac{1}{2}\mathrm{Pr}\; x_1 x_5 - \mathrm{Pr}\, Ec\; x_3{}^2 \end{aligned}\right\} . \tag{18}$$

Subject to the following initial conditions:

$$\left.\begin{aligned} &x_1(0) = 0,\; x_2(0) = Kx_3(0),\; x_3(0) = s_1, \\ &x_4(0) = s_2,\; x_4(0) = -Bi(1 - s_2). \end{aligned}\right\} \tag{19}$$

In order to solve Eqs. (18) and (19) as an initial value problem, we need values of $s_1$ and $s_2$. By making initial guess values for $s_1$ and $s_2$, the solutions of Eqs. (18) and (19) are obtained by applying fourth order Runge-Kutta method. A step size of 0.001 is used to obtain the numerical solution and the accuracy goal is kept equal to $10^{-7}$ as the convergence criteria. The semi-infinite domain is truncated at suitable distance where the effects of boundary layers are negligible. The values of plate surface temperature, local skin-friction coefficient and local Nusselt number, which are respectively proportional to $\theta(0), f''(0), -\theta'(0)$, can be worked out and their numerical values are presented in a tabular form.

## 4. Entropy generation

According to Woods [28], the local entropy generation rate is defined as

$$S_G = \frac{k}{T_f{}^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T_f}\left(\frac{\partial u}{\partial y}\right)^2 . \tag{20}$$

Clearly, it can be seen from Eq. (20) that there are two sources of entropy generation. The first term on the right hand side is the entropy generation due to heat transfer and the second term is the entropy generation due to fluid friction. In terms of dimensionless variables, the entropy generation has the form

$$Ns = \frac{S_G}{S_o} = \left(\frac{\eta^2}{4\mathrm{Re}^2{}_x} + \frac{1}{\mathrm{Re}_x}\right)\theta'^2 + \frac{Br}{\Omega}\frac{1}{\mathrm{Re}_x} f''^2 \tag{21}$$

where $S_o = \frac{k(T_f - T_\infty)^2 U^2{}_\infty}{T_f{}^2 \upsilon^2}$, $\Omega = \frac{T_f}{T_f - T_\infty}$, $Br = \mathrm{Pr}\; Ec$ , are characteristic entropy generation rate, dimensionless temperature difference and the Brinkman number, respectively. Thus, the dimensionless form of entropy generation in Eq. (21) can be expressed as follows:

$$Ns = N_H + N_F, \tag{22}$$

where $N_H = \left(\frac{\eta^2}{4\mathrm{Re}^2{}_x} + \frac{1}{\mathrm{Re}_x}\right)\theta'^2,$ (23)

and $N_F = \frac{Br}{\Omega}\frac{1}{\mathrm{Re}_x} f''^2$ (24)

is the local entropy generation due to heat transfer and fluid friction, respectively. In order to understand the entropy generation mechanism, the irreversibilites of heat transfer and fluid flow processes are analyzed. For this reason, we define the irreversibility distribution ratio $\phi$ . This is the ratio between entropy generation due to fluid friction $N_F$ to the entropy generation due to heat transfer $N_H$ .

$$\phi = \frac{N_F}{N_H} \tag{25}$$

The heat transfer irreversibility is dominant for the range $0 \le \phi < 1$ and when $\phi > 1$, the irreversibility due to fluid friction dominates. When $\phi = 1$, the contribution of heat transfer entropy generation $N_H$ is equal to fluid friction $N_F$. Another alternative irreversibility distribution parameter is the Bejan number $Be$, which is the ratio of entropy generation due to heat transfer to the total entropy generation.

$$Be = \frac{N_H}{Ns} = \frac{1}{1+\phi} \tag{26}$$

Clearly, the Bejan number ranges from 0 to 1. When the value of $Be$ is greater than 0.5, the irreversibility due to heat transfer dominates, whereas $Be$ < 0.5 refers to irreversibility due to viscous dissipation. When $Be = 0.5$, the contribution of the heat transfer and fluid friction entropy generation are equal.

An alternative irreversibility distribution parameter called the Bejan number is defined as follows:

$$Be = \frac{Entropy\ generation\ due\ to\ heat\ transfer}{Total\ entropy\ generation}. \tag{27}$$

Clearly, the Bejan number ranges from 0 to 1. When the value of $Be$ is greater than 0.5, the irreversibility due to heat transfer dominates, whereas $Be$ < 0.5 refers to irreversibility due to viscous dissipation. When $Be = 0.5$, the contribution of the heat transfer and fluid friction entropy generation are equal.

## 5. Results and discussion

Numerical calculations have been carried out for different values of the physical parameters involved in the considered problem. In Tables 1 and 2, a comparison of our work with the work reported by Aziz [16] and Makinde [17] is made in the absence of slip parameter. From here, good agreement is observed among the studies, which shows the accuracy and validity of our numerical scheme. Table 3 presents the values of the local skin friction coefficient and the local Nusselt number for different values of the parameters involved in the considered problem. It is observed that the skin friction coefficient decreases whereas the heat transfer rate at the surface of the plate increases with an increase in the slip (rarefaction) parameter and the Prandtl number. On the other hand, an increase in the local Grashof number and the convective heat transfer parameter causes an increase in the skin friction and the rate of transfer at the surface. Fig. 2 depicts the effects of slip (rarefaction) parameter on the velocity profile. It is noticed that as the fluid becomes more rarefied, the velocity of the fluid increases. Fig. 3 shows that an increase in the intensity of convective surface heat transfer $Bi_x$ causes an increase in the velocity of the fluid. However, it is observed that the effects of the local Grashof number on the velocity profile are more pronounced than the convection parameter $Bi_x$ as shown in Fig. 4.

Table 1. Computation showing comparison of values of $f''(0)$ with Aziz [15] and Makinde [16] for $K = 0$.

| Gr | Pr | Bi | $f''(0)$ Makinde | $f''(0)$ Present |
|---|---|---|---|---|
| 0 | 0.72 | 0.05 | | 0.3321 |
| 0 | 0.72 | 0.10 | | 0.3321 |
| 0 | 0.72 | 0.40 | | 0.3321 |
| 0.5 | 0.72 | 0.10 | 0.4970 | 0.4970 |
| 1.0 | 0.72 | 0.10 | 0.6320 | 0.6320 |
| 0.1 | 3.00 | 0.10 | 0.3493 | 0.3493 |
| 0.1 | 7.00 | 0.10 | 0.3427 | 0.3427 |

Table 2. Computation showing comparison of values of $-\theta'(0)$, $\theta(0)$ with Aziz [15] and Makinde [16] for $K = 0$.

| Gr | Pr | Bi | $-\theta'(0)$ Aziz | $-\theta'(0)$ Makinde | $-\theta'(0)$ Present | $\theta(0)$ Aziz | $\theta(0)$ Aziz | $\theta(0)$ Present |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.72 | 0.05 | 0.0428 | | 0.0428 | 0.1447 | | 0.1447 |
| 0 | 0.72 | 0.10 | 0.0747 | | 0.0747 | 0.2528 | | 0.2528 |
| 0 | 0.72 | 0.40 | 0.1700 | | 0.1700 | 0.5750 | | 0.5750 |
| 0.5 | 0.72 | 0.10 | | 0.0761 | 0.0761 | | 0.2386 | 0.2386 |
| 1.0 | 0.72 | 0.10 | | 0.0770 | 0.0770 | | 0.2295 | 0.2295 |
| 0.1 | 3.00 | 0.10 | | 0.0830 | 0.0830 | | 0.1695 | 0.1695 |
| 0.1 | 7.00 | 0.10 | | 0.0867 | 0.0867 | | 0.1327 | 0.1327 |

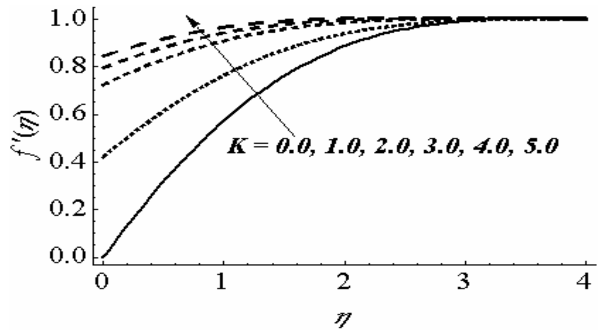


Fig. 2. Effects of slip (rarefaction) parameter $K$ on $f'(\eta)$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Ec = 0.5$, $Bi_x = 0.1$.

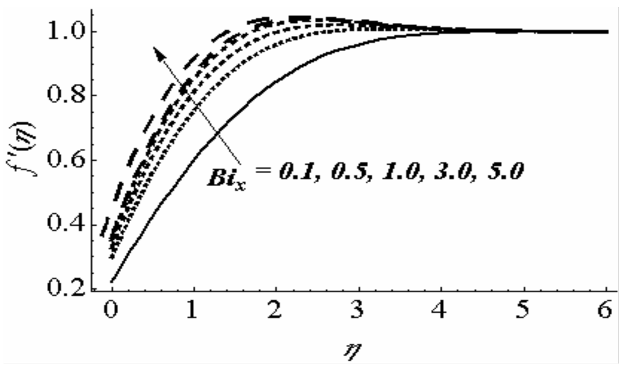


Fig. 3. Effects of the local Biot number $Bi_x$ on $f'(\eta)$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Ec = 0.1$, $K = 0.5$.

Table 3. Computation showing values of $f''(0), -\theta'(0), \theta(0)$ for various values of physical parameters.

| K | Gr | Pr | Bi | $f''(0)$ | $-\theta'(0)$ | $\theta(0)$ |
|---|---|---|---|---|---|---|
| 0.0 | 0.1 | 0.72 | 0.10 | 0.36881 | 0.07507 | 0.24922 |
| 1.0 | | | | 0.31381 | 0.07867 | 0.21326 |
| 2.0 | | | | 0.25197 | 0.08013 | 0.19868 |
| 3.0 | | | | 0.20622 | 0.08085 | 0.19140 |
| 1.0 | 0.5 | 0.72 | 0.10 | 0.38176 | 0.07961 | 0.20380 |
| | 1.0 | | | 0.45089 | 0.08043 | 0.19561 |
| | 1.5 | | | 0.50977 | 0.08105 | 0.18949 |
| | 2.0 | | | 0.56193 | 0.08153 | 0.18462 |
| 1.0 | 0.1 | 1.00 | 0.10 | 0.31016 | 0.08087 | 0.19125 |
| | | 3.00 | | 0.30172 | 0.08702 | 0.12976 |
| | | 5.00 | | 0.29930 | 0.08930 | 0.10698 |
| | | 7.10 | | 0.29805 | 0.09067 | 0.09327 |
| 1.0 | 0.1 | 0.72 | 0.10 | 0.31381 | 0.07867 | 0.21326 |
| | | | 0.50 | 0.34502 | 0.21562 | 0.56876 |
| | | | 1.00 | 0.35792 | 0.27704 | 0.72295 |
| | | | 3.00 | 0.37116 | 0.34307 | 0.88564 |

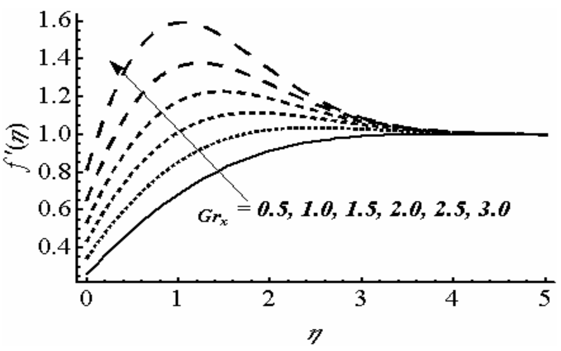


Fig. 4. Effects of the local Grashof numberon $f'(\eta)$ when $\Pr = 0.72$, $K = 0.5$, $Ec = 0.5$, $Bi_x = 0.1$ .

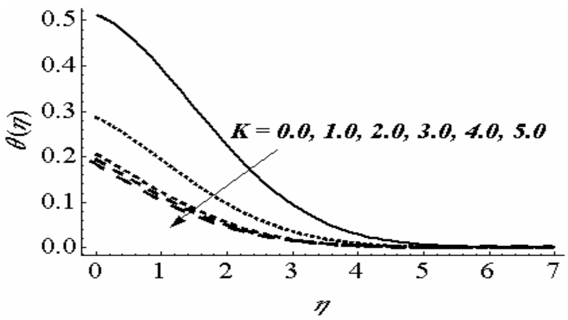


Fig. 5. Effects of slip (rarefaction) parameter $K$ on $\theta(\eta)$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Ec = 0.5$, $Bi_x = 0.1$ .

Fig. 5 illustrates that the thermal boundary layer thickness decreases with an increase in the slip parameter. The effects of the local Grashof number on the thermal boundary layer thickness are presented in Fig. 6. The temperature profile increases as $Gr_x$ increases. However, the thermal boundary layer thickness decreases.

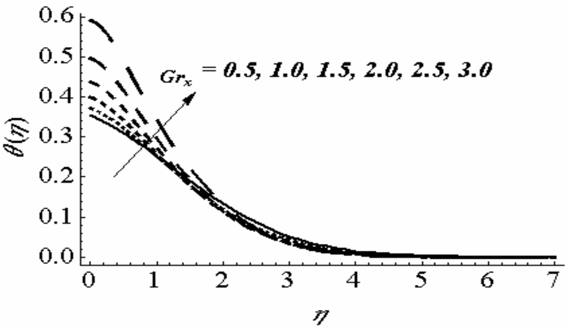


Fig. 6. Effects of the local Grashof numberon $\theta(\eta)$ when $\Pr = 0.72$, $K = 0.5$, $Ec = 0.5$, $Bi_x = 0.1$ .

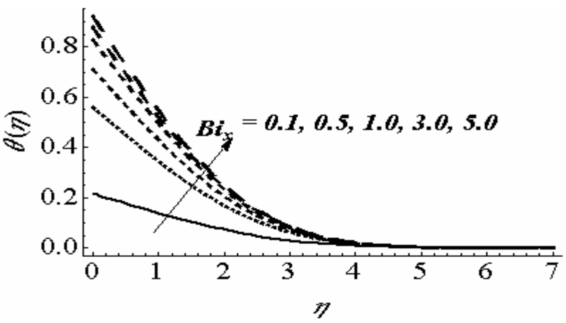


Fig. 7. Effects of the local Biot number $Bi_x$ on $\theta(\eta)$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Ec = 0.1$, $K = 0.5$ .

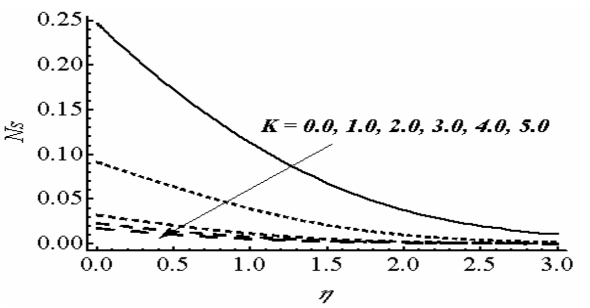


Fig. 8. Effects of slip (rarefaction) parameter $K$ on $Ns$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Bi_x = 0.1$, $Br/\Omega = 1.0$, $\mathrm{Re}_x = 2.0$.

Fig. 7 depicts that with an increase in the intensity of local Biot number due to convective surface heat transfer, the thermal boundary layer becomes thicker.

The influence of the slip on the entropy generation number $Ns$ is presented in Fig. 8. With an increase in $K$ , the skin friction decreases, ultimately resulting in less entropy production. The entropy generation rate increases with an increase in local Grashof number as illustrated in Fig. 9. Fig. 10 shows that the entropy generation number increases as $Bi_x$ increases. Fig. 11 illustrates that the entropy generation number increases with the group parameter $Br/\Omega$ .

The Bejan number is investigated in Figs. 12-15 for different physical parameters. The effects of slip parameter $K$ on the Bejan number are shown in Fig. 12. The effects of fluid

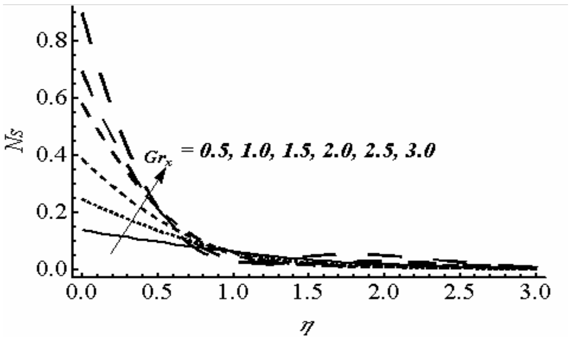


Fig. 9. Effects of local Grashof numberon $Ns$ when $\Pr = 0.72$, $K = 0.5$, $Bi_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

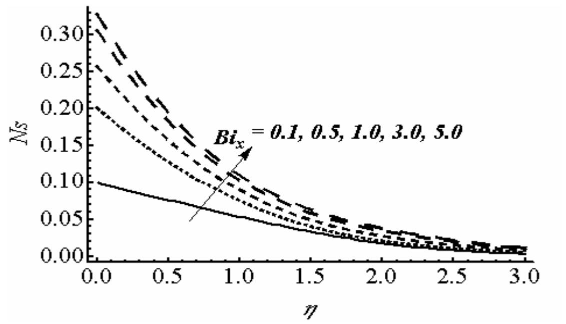


Fig. 10. Effects of local Biot number on $Ns$ when $\Pr = 0.72$, $K = 0.5$, $Gr_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

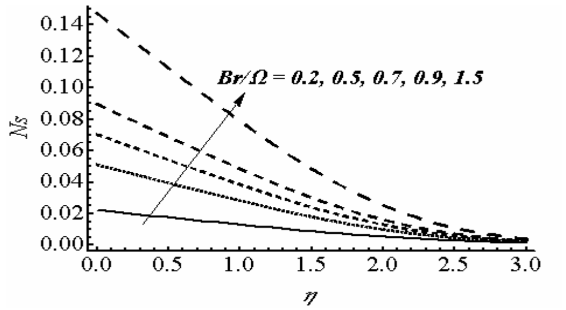


Fig. 11. Effects of group parameter $Br/\Omega$ on $Ns$ when $\Pr = 0.72$, $K = 0.5$, $Gr_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

friction irreversibility weaken near the surface as the slip parameter increases while far away in the flow regime, the heat transfer irreversibility is dominant. Fig. 13 shows the effects of $Gr_x$ on the Bejan number. The Bejan number increases as $Gr_x$ augments and decreases as one goes downstream. For large $Gr_x$, three variational trends are noticed. Near the plate, fluid friction irreversibility dominates, and then afterwards, heat transfer irreversibility dominates over fluid friction irreversibility and attains a peak value within the boundary layer region. In the main flow regime, fluid friction irreversibility again develops, and far away from the boundary layer region, heat transfer irreversibility controls the entropy production. Moreover, it is noteworthy that within the boundary layer region, heat transfer irreversibility becomes dominant earlier for large values of local Grashof number as compared to

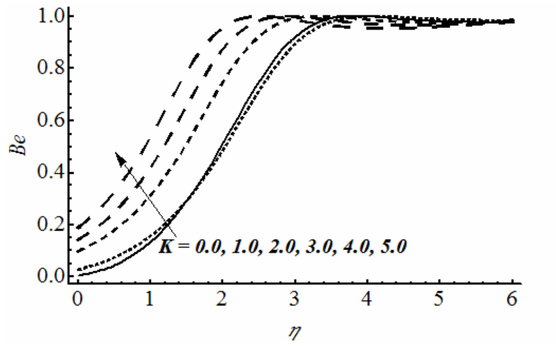


Fig. 12. Effects of slip (rarefaction) parameter $K$ on $Be$ when $\Pr = 0.72$, $Gr_x = 0.5$, $Bi_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

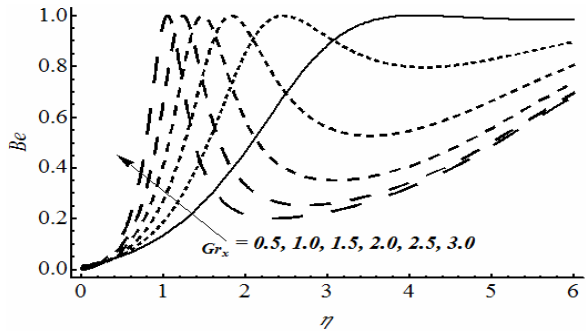


Fig. 13. Effects of local Grashof number on $Be$ when $\Pr = 0.72$, $K = 0.5$, $Bi_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

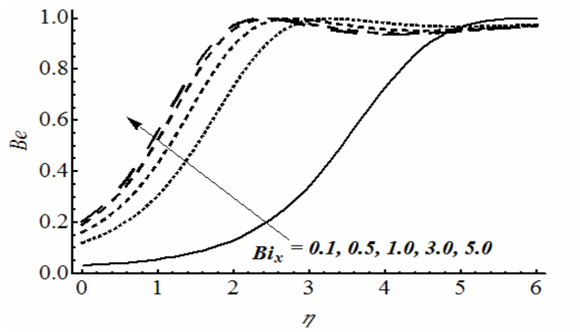


Fig. 14. Effects of local Biot number on $Be$ when $\Pr = 0.72$, $K = 0.5$, $Gr_x = 0.5$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

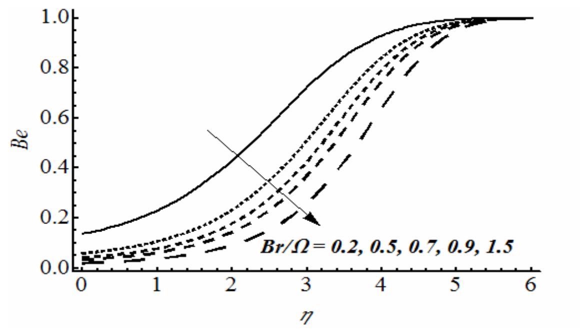


Fig. 15. Effects of group parameter $Br/\Omega$ on $Be$ when $\Pr = 0.72$, $K = 0.5$, $Gr_x = 0.1$, $Br/\Omega = 1.0$, $\text{Re}_x = 2.0$.

smaller values due to strong buoyancy effects. Fig. 14 depicts the effects of local Biot number $Bi_x$ on $Be$. With an increase in value of $Bi_x$, fluid friction irreversibility at the surface decreases and effects of heat transfer irreversibility start to appear. In the free stream region, the heat transfer irreversibility effects are fully dominant. The influence of the group parameter $Br/\Omega$ on the Bejan number is presented in Fig. 15. The fluid friction irreversibility dominates at the surface of the plate with increase in $Br/\Omega$ and in the flow regime, the heat transfer irreversibility dominates.

## 6. Conclusions

In the present study, an analysis is carried out for the entropy generation rate in hydrodynamic boundary layer flow over a vertical plate in the presence of slip velocity and convective boundary condition. The velocity and the temperature profiles are obtained numerically and the entropy generation number and the Bejan number are computed. The effects of different physical parameters on the velocity and the temperature profiles are shown and their influence on the entropy generation is also discussed. It is observed that the velocity increases by increasing the slip parameter $K$, the local Grashof number $Gr_x$ and the local Biot number $Bi_x$. The slip parameter $K$ and local Grashof number $Gr_x$ have decreasing effects on thermal boundary layer, and the local Biot number $Bi_x$ causes the thermal boundary layer thickness to increase. It is concluded that the entropy production in the fluid can be reduced and controlled by increasing slip at the boundary wall. Also, increasing $Bi_x$ enhances entropy production. Therefore, the entropy can be minimized by reducing the convection through boundaries. It is observed from the study that fluids flowing with high Reynolds number depreciate the entropy production. The effects of the local Grashof number $Gr_x$ and the group parameter $Br/\Omega$ on $Ns$ are increasing. Fluid friction irreversibility dominates at the surface of the plate and the heat transfer irreversibility effects are dominant in the main flow regime.

The results obtained through this article depict that the optimal design and the efficient performance of a flow system or a thermally designed system can be improved by choosing the appropriate values of the physical parameters. This will enable us to reduce the effects of entropy generated within the system.

## Nomenclature

Br : Brinkman number
Be : Bejan number
$Bi_x$ : Local Biot number
$c_p$ : Specific heat at constant pressure
Ec : Eckert number
$Gr_x$ : Local Grashof number
k : Thermal conductivity
K : Slip (rarefaction) parameter
$Kn_x$ : Local Knudsen number
Ns : Entropy generation number
$N_H$ : Entropy generation due to heat transfer
$N_F$ : Entropy generation due to fluid friction
Pr : Prandtl number
$\mathrm{Re}_x$ : Local Reynolds number
$S_G$ : Volumetric rate of entropy generation
$S_{G_o}$ : Characteristic entropy generation rate
$T$ : Temperature of the fluid
$T_f$ : Temperature of the hot fluid
$T_\infty$ : Temperature of the ambient fluid
$u, v$ : Velocity components in x and y directions
$x, y$ : Spatial coordinates

### *Greek symbols*

$\eta$ : Similarity variable
$\theta$ : Dimensionless temperature
$\mu$ : Coefficient of viscosity
$\nu$ : Kinematic viscosity
$\rho$ : Density of fluid
$\Omega$ : Dimensionless temperature difference

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