

# Entropy Generation in Viscoelastic Fluid Over a Stretching Surface

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## 1. Introduction

Due to the increasing importance in processing industries and elsewhere when materials whose flow behavior cannot be characterized by Newtonian relationships, a new stage in the evolution of fluid dynamics theory is in progress. An intensive effort, both theoretical and experimental, has been devoted to problems of non-Newtonian fluids. The study of MHD flow of viscoelastic fluids over a continuously moving surface has wide range of applications in technological and manufacturing processes in industries. This concerns the production of synthetic sheets, aerodynamic extrusion of plastic sheets, cooling of metallic plates, etc.

(Crane, 1970) considered the laminar boundary layer flow of a Newtonian fluid caused by a flat elastic sheet whose velocity varies linearly with the distance from the fixed point of the sheet. (Chang, 1989; Rajagopal et al., 1984) presented an analysis on flow of viscoelastic fluid over stretching sheet. Heat transfer cases of these studies have been considered by (Dandapat & Gupta, 1989, Vajravelu & Rollins, 1991), while flow of viscoelastic fluid over a stretching surface under the influence of uniform magnetic field has been investigated by (Andersson, 1992).

Thereafter, a series of studies on heat transfer effects on viscoelastic fluid have been made by many authors under different physical situations including (Abel et al., 2002, Bhattacharya et al., 1998, Datti et al., 2004, Idrees & Abel, 1996, Lawrence & Rao, 1992, Prasad et al., 2000, 2002). (Khan & Sanjayanand, 2005) have derived similarity solution of viscoelastic boundary layer flow and heat transfer over an exponential stretching surface.

(Cortell, 2006) have studied flow and heat transfer of a viscoelastic fluid over stretching surface considering both constant sheet temperature and prescribed sheet temperature. (Abel et al., 2007) carried out a study of viscoelastic boundary layer flow and heat transfer over a stretching surface in the presence of non-uniform heat source and viscous dissipation considering prescribed surface temperature and prescribed surface heat flux.

(Khan, 2006) studied the case of the boundary layer problem on heat transfer in a viscoelastic boundary layer fluid flow over a non-isothermal porous sheet, taking into account the effect a continuous suction/blowing of the fluid, through the porous boundary. The effects of a transverse magnetic field and electric field on momentum and heat transfer characteristics in viscoelastic fluid over a stretching sheet taking into account viscous dissipation and ohmic dissipation is presented by (Abel et al., 2008). (Hsiao, 2007) studied

the conjugate heat transfer of mixed convection in the presence of radiative and viscous dissipation in viscoelastic fluid past a stretching sheet. The case of unsteady magnetohydrodynamic was carried out by (Abbas et al., 2008). Using Kummer's functions, (Singh, 2008) carried out the study of heat source and radiation effects on magnetohydrodynamic flow of a viscoelastic fluid past a stretching sheet with prescribed power law surface heat flux. The effects of non-uniform heat source, viscous dissipation and thermal radiation on the flow and heat transfer in a viscoelastic fluid over a stretching surface was considered in (Prasad et al., 2010). The case of the heat transfer in magnetohydrodynamic flow of viscoelastic fluids over stretching sheet in the case of variable thermal conductivity and in the presence of non-uniform heat source and radiation is reported in (Abel & Mahesha, 2008). Using the homotopy analysis, (Hayat et al., 2008) looked at the hydrodynamic of three dimensional flow of viscoelastic fluid over a stretching surface. The investigation of biomagnetic flow of a non-Newtonian viscoelastic fluid over a stretching sheet under the influence of an applied magnetic field is done by (Misra & Shit, 2009). (Subhas et al., 2009) analysed the momentum and heat transfer characteristics in a hydromagnetic flow of viscoelastic liquid over a stretching sheet with non-uniform heat source. (Nandeppanavar et al., 2010) analysed the flow and heat transfer characteristics in a viscoelastic fluid flow in porous medium over a stretching surface with surface prescribed temperature and surface prescribed heat flux and including the effects of viscous dissipation. (Chen, 2010) studied the magneto-hydrodynamic flow and heat transfer characteristics viscoelastic fluid past a stretching surface, taking into account the effects of Joule and viscous dissipation, internal heat generation/absorption, work done due to deformation and thermal radiation. (Nandeppanavar et al., 2011) considered the heat transfer in viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink in the presence of a magnetic field

Although the forgoing research works have covered a wide range of problems involving the flow and heat transfer of viscoelastic fluid over stretching surface they have been restricted, from thermodynamic point of view, to only the first law analysis. The contemporary trend in the field of heat transfer and thermal design is the second law of thermodynamics analysis and its related concept of entropy generation minimization.

Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes. Different sources are responsible for generation of entropy such as heat transfer and viscous dissipation (Bejan, 1979, 1982). The analysis of entropy generation rate in a circular duct with imposed heat flux at the wall and its extension to determine the optimum Reynolds number as function of the Prandtl number and the duty parameter were presented by (Bejan, 1979, 1996). (Sahin, 1998) introduced the second law analysis to a viscous fluid in circular duct with isothermal boundary conditions. In another paper, (Sahin, 1999) presented the effect of variable viscosity on entropy generation rate for heated circular duct. A comparative study of entropy generation rate inside duct of different shapes and the determination of the optimum duct shape subjected to isothermal boundary condition were done by (Sahin, 1998). (Narusawa, 1998) gave an analytical and numerical analysis of the second law for flow and heat transfer inside a rectangular duct. In a more recent paper, (Mahmud & Fraser, 2002a, 2002b, 2003) applied the second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by (Saouli & Aïboud-Saouli, 2004). As far as the effect of a magnetic field on the entropy generation is

concerned, (Mahmud et al., 2003) studied the case of mixed convection in a channel. The effects of magnetic field and viscous dissipation on entropy generation in a falling film and channel were studied by (Aïboud-saouli et al., 2006, 2007). The application of the second law analysis of thermodynamics to viscoelastic magnetohydrodynamic flow over a stretching surface was carried out by (Aïboud & Saouli 2010a, 2010b).

The objective of this paper is to study the entropy generation in viscoelastic fluid over a stretching sheet with prescribed surface temperature in the presence of uniform transverse magnetic field.

## 2. Formulation of the problem

In two-dimensional Cartesian coordinate system  $(x, y)$  we consider magneto-convection, steady, laminar, electrically conduction, boundary layer flow of a viscoelastic fluid caused by a stretching surface in the presence of a uniform transverse magnetic field and a heat source. The  $x$ -axis is taken in the direction of the main flow along the plate and the  $y$ -axis is normal to the plate with velocity components  $u, v$  in these directions.

Under the usual boundary layer approximations, the flow is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left( u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

The constant  $k_0 = -\frac{\alpha_1}{\rho}$  is the viscoelastic parameter.

The boundary conditions are given by

$$y = 0, u = u_p = \lambda x, v = 0 \quad (3a)$$

$$y = \infty, u = 0, \frac{\partial u}{\partial y} = 0 \quad (3b)$$

The heat transfer governing boundary layer equation with temperature-dependent heat generation (absorption) is

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \quad (4)$$

The relevant boundary conditions are

$$y = 0, T = T_p = A \left( \frac{x}{l} \right)^2 + T_\infty \quad (5a)$$

$$y = \infty, T = T_\infty \quad (5b)$$

### 3. Analytical solution

The equation of continuity is satisfied if we choose a dimensionless stream function  $\Psi(x, y)$  such that

$$u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x} \quad (6)$$

Introducing the similarity transformations

$$\eta = y\sqrt{\frac{\lambda}{\nu}}, \Psi(x, y) = x\sqrt{\nu\lambda}f(\eta) \quad (7)$$

Momentum equation (2) becomes

$$f''^2(\eta) - f(\eta)f''(\eta) = f'''(\eta) - \frac{k_0\lambda}{\nu} [2f'(\eta)f'''(\eta) - f(\eta)f^{(4)}(\eta) - f''^2(\eta)] - Mn f'(\eta) \quad (8)$$

where  $Mn = \frac{\sigma B_0^2}{\rho\lambda}$

Now let us seek a solution of Eq. (6) in the form

$$f'(\eta) = e^{-\alpha\eta} \quad (\alpha > 0) \quad (9)$$

which is satisfied by the following boundary conditions:

$$\eta = 0, f(0) = 0, f'(0) = 1 \quad (10a)$$

$$\eta = \infty, f'(\infty) = 0, f''(\infty) = 0 \quad (10b)$$

On substituting (7) into (6) and using boundary conditions (10a) and (10b) the velocity components take the form

$$u = \lambda x f'(\eta) \quad (11)$$

$$v = -\sqrt{\nu\lambda} f(\eta) \quad (12)$$

Where  $k_1 = \frac{k_0\lambda}{\nu}$  is the viscoelastic parameter, and

$$\alpha = \sqrt{\frac{1+Mn}{1-k_1}} \quad (13)$$

Defining the dimensionless temperature

$$\Theta(\eta) = \frac{T - T_\infty}{T_p - T_\infty} \quad (14)$$

and using (9), (11), (12), Eq. (14) and the boundary conditions (5a) and (5b) can be written as

$$\Theta''(\eta) + \frac{\text{Pr}}{\alpha}(1 - e^{-a\eta})\Theta'(\eta) - (2\text{Pr}e^{-a\eta} - \beta)\Theta(\eta) = 0 \quad (15)$$

$$\eta = 0, \Theta(0) = 1 \quad (16a)$$

$$\eta = \infty, \Theta(\infty) = 0 \quad (16b)$$

Where  $\text{Pr} = \frac{\mu C_P}{k}$  and  $\beta = \frac{QV}{\lambda k}$  are respectively the Prandtl number and the heat/sink parameter.

Introducing the variable

$$\xi = \frac{\text{Pr}}{\alpha^2} e^{-a\eta} \quad (17)$$

And inserting (17) in (15) we obtain

$$\xi\Theta''(\xi) + \frac{\text{Pr}}{\alpha}\left(1 - \frac{\text{Pr}}{\alpha^2} + \xi\right)\Theta'(\xi) - \left(2 - \frac{\beta}{\alpha^2\xi}\right)\Theta(\xi) = 0 \quad (18)$$

And (16a) and (16b) transform to

$$\xi = \frac{\text{Pr}}{\alpha^2}, \Theta\left(\frac{\text{Pr}}{\alpha^2}\right) = 1 \quad (19a)$$

$$\xi = 0, \Theta(0) = 0 \quad (19b)$$

The solution of Eq. (18) satisfying (19a) and (19b) is given by

$$\Theta(\xi) = \left(\frac{\alpha^2}{\text{Pr}}\xi\right)^{a+b} \frac{M\left(a+b-2, 2b+1, -\xi\right)}{M\left(a+b-2, 2b+1, -\frac{\text{Pr}}{\alpha^2}\right)} \quad (20)$$

The solution of (20) in terms of  $\eta$  is written as

$$\Theta(\xi) = e^{-\alpha(a+b)\eta} \frac{M\left(a+b-2, 2b+1, -\frac{\text{Pr}}{\alpha^2}e^{-a\eta}\right)}{M\left(a+b-2, 2b+1, -\frac{\text{Pr}}{\alpha^2}\right)} \quad (21)$$

where  $a = \frac{\text{Pr}}{2\alpha^2}$ ,  $b = \frac{\sqrt{\text{Pr}^2 - 4\alpha^2\beta}}{2\alpha^2}$  and  $M\left(a+b-2, 2b+1, -\frac{\text{Pr}}{\alpha^2}e^{-a\eta}\right)$  is the Kummer's function.

#### 4. Second law analysis

According to (Woods, 1975), the local volumetric rate of entropy generation in the presence of a magnetic field is given by

$$S_G = \frac{k}{T_\infty^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_\infty} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_\infty} u^2 \quad (22)$$

Eq. (22) clearly shows contributions of three sources of entropy generation. The first term on the right-hand side of Eq. (22) is the entropy generation due to heat transfer across a finite temperature difference; the second term is the local entropy generation due to viscous dissipation, whereas the third term is the local entropy generation due to the effect of the magnetic field. It is appropriate to define dimensionless number for entropy generation rate  $N_s$ . This number is defined by dividing the local volumetric entropy generation rate  $S_G$  to a characteristic entropy generation rate  $S_{G0}$ . For prescribed boundary condition, the characteristic entropy generation rate is

$$S_{G0} = \frac{k(\Delta T)^2}{l^2 T_\infty^2} \quad (23)$$

therefore, the entropy generation number is

$$N_s = \frac{S_G}{S_{G0}} \quad (24)$$

using Eq. (9), (21) and (22), the entropy generation number is given by

$$N_s = \frac{4}{X^2} \Theta^2(\eta) + \text{Re}_i \Theta'^2(\eta) + \text{Re}_i \frac{Br}{\Omega} f'^2(\eta) + \frac{BrHa^2}{\Omega} f'^2(\eta) \quad (25)$$

where  $\text{Re}_i$  and  $Br$  are respectively the Reynolds number and the Brinkman number.  $\Omega$  and  $Ha$ , are respectively the dimensionless temperature difference and the Hartman number. These number are given by the following relationships

$$\text{Re}_i = \frac{u_i l}{\nu}, \quad Br = \frac{\mu u_p^2}{k\Delta T}, \quad \Omega = \frac{\Delta T}{T_\infty}, \quad Ha = B_0 l \sqrt{\frac{\sigma}{\mu}} \quad (26)$$

## 5. Results and discussion

The flow and heat transfer in a viscoelastic fluid under the influence of a transverse uniform magnetic field has been solved analytically using Kummer's functions and analytic expressions of the velocity and temperature have been used to compute the entropy generation. Figs. 1 and 2 show the variations of the longitudinal velocity  $f'(\eta)$  and the transverse velocity  $f(\eta)$  as function of  $\eta$  for several values of magnetic parameter  $Mn$ . It can be observed that  $f'(\eta)$  decreases with  $\eta$  and  $f(\eta)$  increases with  $\eta$  asymptotically for  $Mn$  keeping constant. For a fixed position  $\eta$ , both  $f'(\eta)$  and  $f(\eta)$  decreases with  $Mn$ , thus the presence of the magnetic field decreases the momentum boundary layer thickness and increase the power needed to stretch the sheet.

The effects of the viscoelastic parameter  $k_1$  on the longitudinal velocity  $f'(\eta)$  and the transverse velocity  $f(\eta)$  are illustrated on figs. 3 and 4. As it can be seen, for a fixed value of  $\eta$ , both  $f'(\eta)$  and  $f(\eta)$  decrease as viscoelastic parameter rises. This can be explained by

the fact that, as the viscoelastic parameter increases, the hydrodynamic boundary layer adheres strongly to the surface, which in turn retards the flow in the longitudinal and the transverse directions.

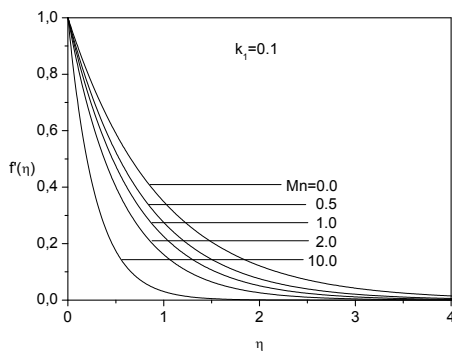


Fig. 1. Effect of the magnetic parameter on the longitudinal velocity.

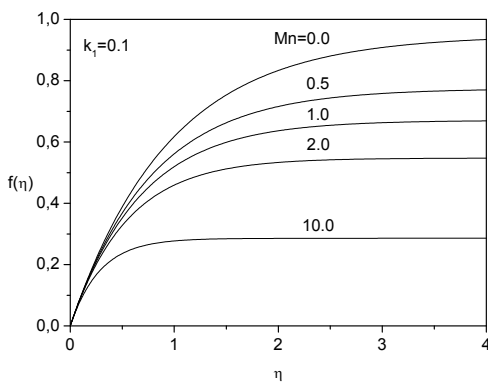


Fig. 2. Effect of the magnetic parameter on the transverse velocity.

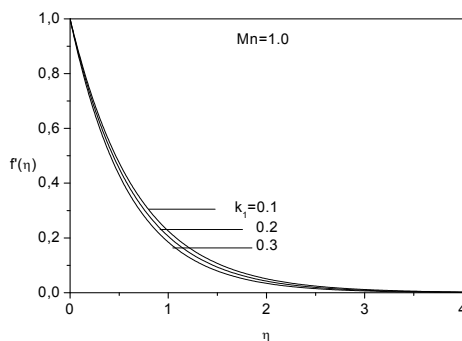


Fig. 3. Effect of the viscoelastic parameter on the longitudinal velocity.

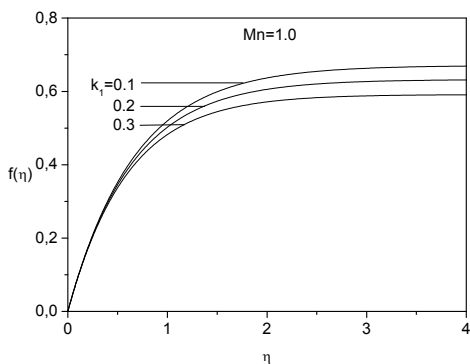


Fig. 4. Effect of the viscoelastic parameter on the transverse velocity.

Fig. 5 depicts the temperature profiles  $\Theta(\eta)$  as function of  $\eta$  for different values of the Prandtl number  $Pr$ . As it can be noticed,  $\Theta(\eta)$  decreases with  $\eta$  whatever is the value of the Prandtl number, for a fixed value of  $\eta$ , the temperature  $\Theta(\eta)$  decreases with an increase in Prandtl number which means that the thermal boundary layer is thinner for large Prandtl number.

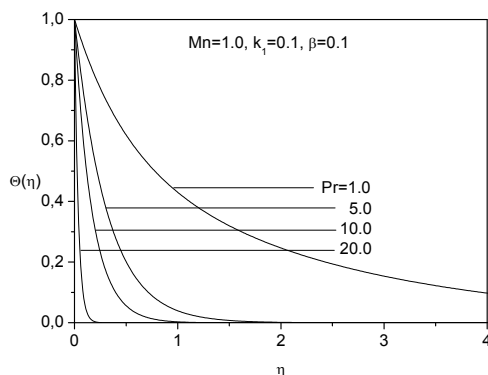


Fig. 5. Effect of the Prandtl number on the temperature.

The temperature profiles  $\Theta(\eta)$  as function of  $\eta$  for different values of the magnetic  $Mn$  are plotted in fig. 6. An increase in the magnetic parameter  $Mn$  results in an increase of the temperature; this is due to the fact that the thermal boundary layer increases with the magnetic parameter. Fig. 7 represents graphs of temperature profiles  $\Theta(\eta)$  as function of  $\eta$  for various values of the heat source/sink parameter  $\beta$ . For fixed value of  $\eta$ , the temperature  $\Theta(\eta)$  augments with the heat source/ sink parameter  $\beta$ . This is due to the fact that the increase of the heat source/sink parameter means an increase of the heat generated inside the boundary layer leading to higher temperature profile.

The influence of the magnetic parameter  $Mn$  on the entropy generation number  $N_s$  is shown on fig. 8. The entropy generation number  $N_s$  decreases with  $\eta$  for  $Mn$  keeping constant. For



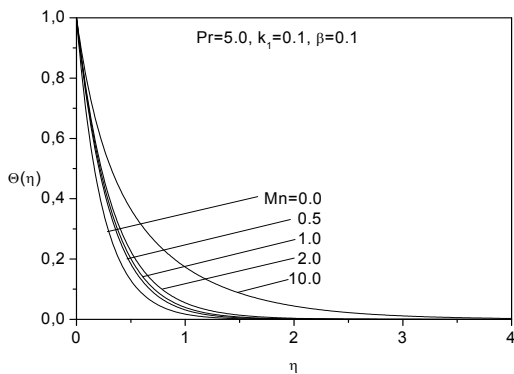


Fig. 6. Effect of the magnetic parameter on the temperature.

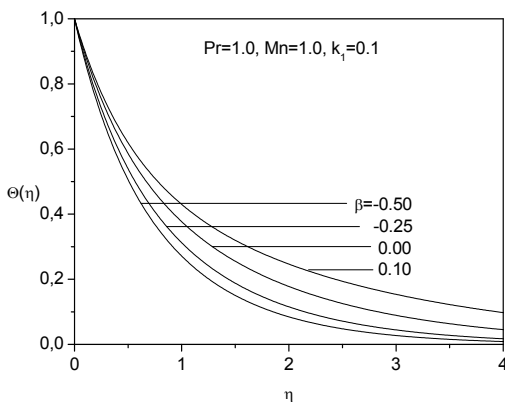


Fig. 7. Effect of the heat source/sink parameter on the temperature.

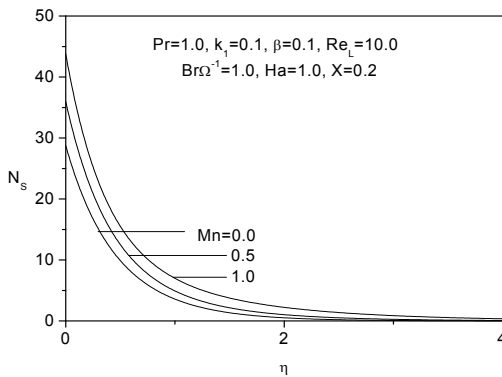


Fig. 8. Effect of the magnetic parameter on the entropy generation number.

fixed value of  $\eta$ , the entropy generation number increases with the magnetic parameter, because the presence of the magnetic field creates more entropy in the fluid. Moreover, the stretching surface acts as a strong source of irreversibility.

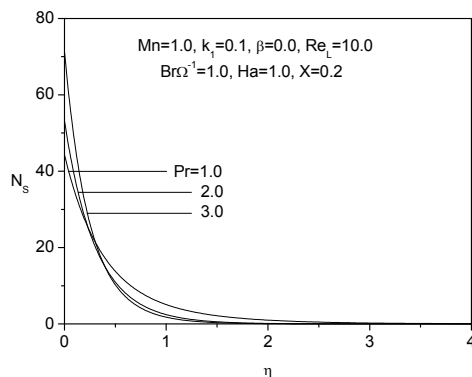


Fig. 9. Effect of the Prandtl number on the entropy generation number.

Fig. 9 illustrates the effect of the Prandtl number  $Pr$  on the entropy generation number  $N_s$ . The entropy generation number is higher for higher Prandtl number near the surface, but as  $\eta$  increases, the entropy generation number shows different variation. This is due to the fact that according to fig. 6, the temperature profiles decrease sharply with the increase of the Prandtl number.

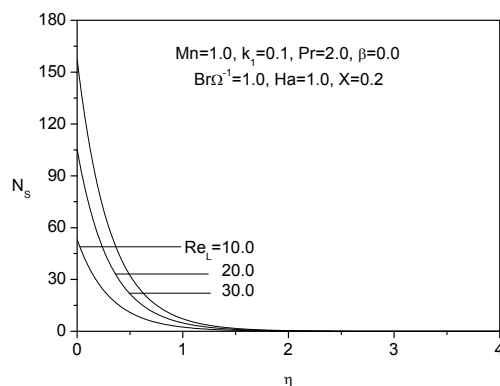


Fig. 10. Effect of the Reynolds number on the entropy generation number.

The influence of the Reynolds number  $Re_i$  on the entropy generation number is plotted on fig. 10. For a given value of  $\eta$ , the entropy generation number increases as the Reynolds number increases. The augmentation of the Reynolds number increases the contribution of the entropy generation number due to fluid friction and heat transfer in the boundary layer.

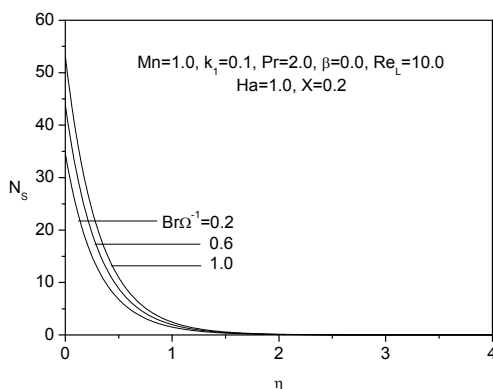


Fig. 11. Effect of the dimensionless group on the entropy generation number.

The effect of the dimensionless group parameter  $Br\Omega^{-1}$  on the entropy generation number  $N_s$  is depicted in fig. 11. The dimensionless group determines the relative importance of viscous effect. For a given  $\eta$ , the entropy generation number is higher for higher dimensionless group. This is due to the fact that for higher dimensionless group, the entropy generation numbers due to the fluid friction increase.

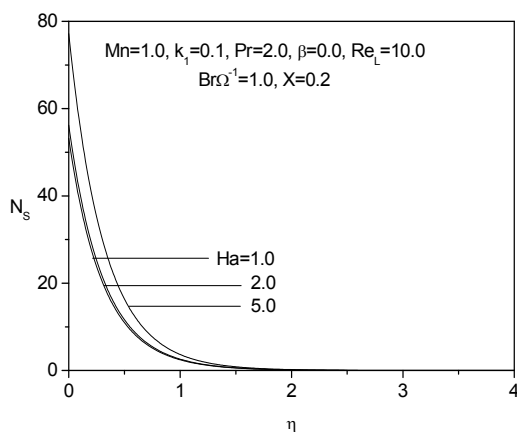


Fig. 12. Effect of the Hartman number on the entropy generation number.

The effect of the Hartman number  $Ha$  on the entropy generation number  $N_s$  is plotted in fig. 12. For a given  $\eta$ , as the Hartman number increases, the entropy generation number increases. The entropy generation number is proportional to the square of Hartman number which is proportional to the magnetic field. The presence of the magnetic field creates additional entropy.

## 6. Conclusion

The velocity and temperature profiles are obtained analytically and used to compute the entropy generation number in viscoelastic magnetohydrodynamic flow over a stretching surface

The effects of the magnetic parameter and the viscoelastic parameter on the longitudinal and transverse velocities are discussed. The influences of the Prandtl number, the magnetic parameter and the heat source/sink parameter on the temperature profiles are presented. As far as the entropy generation number is concerned, its dependence on the magnetic parameter, the Prandtl, the Reynolds, the Hartmann numbers and the dimensionless group are illustrated and analyzed.

From the results the following conclusions could be drawn:

- The velocities depend strongly on the magnetic and the viscoelastic parameters.
- The temperature varies significantly with the Prandtl number, the magnetic parameter and the heat source/sink parameter.
- The entropy generation increases with the increase of the Prandtl, the Reynolds, the Hartmann numbers and also with the magnetic parameter and the dimensionless group.
- The surface acts as a strong source of irreversibility.

## 7. Nomenclature

$A$	constant, K
$\vec{B}_0$	uniform magnetic field strength, Wb.m <sup>-2</sup>
$Br$	Brinkman number, $Br = \frac{\mu u_0^2}{k\Delta T}$
$C_p$	specific heat of the fluid, J.kg <sup>-1</sup> .K <sup>-1</sup>
$f$	dimensionless function
$Ha$	Hartman number $Ha = B_0 l \sqrt{\frac{\sigma}{\mu}}$
$k$	thermal conductivity of the fluid, W.m <sup>-1</sup> .K <sup>-1</sup>
$k_1$	viscoelastic parameter, $k_1 = \frac{k_0 \lambda}{\nu}$
$k_0$	viscoelastic parameter, m <sup>2</sup>
$l$	characteristic length, m
$M$	Kummer's function
$Mn$	magnetic parameter, $Mn = \frac{\sigma B_0^2}{\rho \lambda}$
$N_s$	entropy generation number, $N_s = \frac{S_G}{S_{G0}}$
$Pr$	Prandtl number, $Pr = \frac{\mu C_p}{k}$
$Q$	rate of internal heat generation or absorption, W.m <sup>-3</sup> .K <sup>-1</sup>

$Re_i$	Reynolds number based on the characteristic length, $Re_i = \frac{u_i l}{\nu}$
$S_G$	local volumetric rate of entropy generation, $W.m^{-3}.K^{-1}$
$S_{G0}$	characteristic volumetric rate of entropy generation, $W.m^{-3}.K^{-1}$
$T$	temperature, K
$u$	axial velocity, $m.s^{-1}$
$u_i$	plate velocity based on the characteristic length, $m.s^{-1}$
$u_p$	plate velocity, $m.s^{-1}$
$v$	transverse velocity, $m.s^{-1}$
$x$	axial distance, m
$X$	dimensionless axial distance, $X = \frac{x}{l}$
$y$	transverse distance, m
$\alpha$	positive constant
$\beta$	heat source/sink parameter, $\beta = \frac{Qv}{\lambda k}$
$\lambda$	proportional constant, $s^{-1}$
$\eta$	dimensionless variable, $\eta = y\sqrt{\frac{\lambda}{\nu}}$
$\xi$	dimensionless variable, $\xi = \frac{Pr}{\alpha^2} e^{-\alpha\eta}$
$\mu$	dynamic viscosity of the fluid, $kg.m^{-1}.s^{-1}$
$\nu$	kinematic viscosity of the fluid, $m^2.s^{-1}$
$\Delta T$	temperature difference, $\Delta T = T_p - T_\infty$
$\Omega$	dimensionless temperature difference, $\Omega = \frac{\Delta T}{T_\infty}$
$\Theta$	dimensionless temperature, $\Theta = \frac{T - T_\infty}{T_p - T_\infty}$
$\rho$	density of the fluid, $kg.m^{-3}$
$\sigma$	electric conductivity, $\Omega^{-1}.m^{-1}$

#### subscripts

$P$	plate
$\infty$	far from the sheet

## 8. References

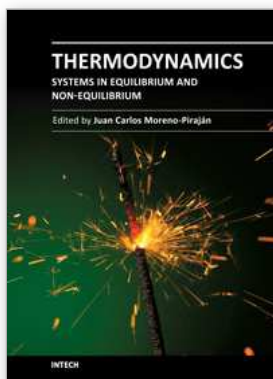
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