Progress of Theoretical Physics, Vol. 72, No. 4, October 1984

Entropy Production in the Inflationary Universe

Masahiro MORIKAWA and Misao SASAKI

Department of Physics, Kyoto University, Kyoto 606

(Received May 21, 1984)

Entropy production mechanism during the vacuum energy dominated stage of the inflationary universe is considered. We propose a thermalization mechanism of vacuum energy due to Higgs particles produced by the temporal change of the background classical Higgs field which subsequently decay into other particles. Then the dissipation coefficient associated with the classical Higgs field is evaluated for an specific decay process and implications of the result are discussed.

§ 1. Introduction

As a model of the early universe, the new inflationary scenario has recently attracted the interests of many physicists because it is thought to solve some of the fundamental problems in cosmology such as the horizon problem and the flatness problem.[1] The scenario is based on a grand unified theory (GUT) and it introduces the exponentially expanding era due to the vacuum energy of the Higgs field which drives the GUT phase transition. By this extreme expansion, the first of the above-mentioned problems can be solved.

It is intuitively expected that immediately after the inflation, the vacuum energy is converted into the radiation energy efficiently, and that the de Sitter expansion is supplanted by the Friedmann expansion. And at this transient era, the huge amount of entropy more than 10^{86}, which is observed in our present universe, is expected to be produced. This extreme entropy solves the flatness problem and the generated heat makes the ordinary baryon generation mechanism[2] work, provided that the energy transformation takes place sufficiently fast compared with the expansion rate of the universe. Therefore a crucial point of the scenario is whether the efficient thermalization is possible or not, with which we shall deal in this paper.

Several mechanisms of the thermalization have been proposed so far. Albrecht, Steinhardt, Turner and Wilczek[3] performed a numerical analysis of the thermalization process with a dissipation coefficient introduced as a parameter by hand. Abbott, Farhi and Wise[4] considered a particle production process by applying the effective action method to a damped harmonic oscillator. However, unfortunately, their methods cannot predict the instantaneous entropy production rate at an intermediate time, but gives only the total number of particles, which they interpret as entropy, produced through the whole process. Especially, it is not clear from their result that whether a non-zero value of a vacuum expectation value of the Higgs field $\langle\phi\rangle$ promotes the particle production or that of a time derivative of the expectation value $d\langle\phi\rangle/dt$ does. Only in the latter case, the entropy production can be associated since a dissipative process must involve $d\langle\phi\rangle/dt$. Hosoya, Sakagami and Takao[5] evaluated a dissipation coefficient for the motion of a thermal expectation value of the Higgs field in $\lambda\phi^4$ model. They considered a small linear deviation from the thermal equilibrium and used the method of Zubarev's nonequilibrium statistical operator.[6] They obtained a dissipation coefficient proportional to the Boltzmann factor, $\exp(-\beta H)$. Owing to this factor, their method cannot be applied to an early stage of the thermalization process because at that time, a temperature of the universe is considered to be vanishingly small due to the de Sitter expansion. Even if we take into account the effect of the Hawking temperature intrinsic to de Sitter space,[7] we cannot get sufficient thermalization[8] for the standard model of GUTS in which the Hawking temperature is of order 10^9 GeV. To summarize, the entropy production must arise from a cold, almost zero-temperature situation, and must originate from the non-zero value of $d\langle\phi\rangle/dt$ rather than $\langle\phi\rangle$ itself.

In §2, we propose a thermalization process due to particle production which is caused by the temporal variation of $\langle\phi\rangle$. Then the entropy production is estimated in the form of a dissipation coefficient by coarse-graining off-diagonal elements of the density matrix. In §3, a perturbation method is developed which enables us to evaluate the dissipation coefficient systematically. Applications of the mechanism to the inflationary universe scenario and physical interpretations of our results are presented in §4. Further problems and implications are discussed in §5.

§ 2. Dissipation mechanism due to particle production

Keeping in mind the conditions of effective thermalization mentioned in §1, here we shall proceed to propose and investigate a thermalization machanism due to particle production. In general, a phase transition is driven by a temporal change of an order parameter from zero to some finite value. For a $\lambda\phi^4$ model, the vacuum expectation value $\langle\phi\rangle$ corresponds to the order parameter. At the same time, $\langle\phi\rangle$ determines the mass and the symmetry of the elementary excitations (or quasi-particles) which inhabit the background $\langle\phi\rangle$. Therefore, according to the temporal development of $\langle\phi\rangle$, the definition of quasi-particles varies from time to time. Consequently, the vacuum state at some time develops into a many particle state at a later time. This mechanism of particle production from vacuum is essentially the same as that of black hole evaporation[9] or that of cosmological particle production.[7],[10]

For simplicity, we model the Higgs field which drives the GUT phase transition by the $\lambda\phi^4$ model:

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4!}\phi^4\,. \tag{2·1}$$

An appropriate expectation value of ϕ field is denoted by $\phi_c\equiv\langle\phi\rangle$, and the fluctuation from ϕ_c by $\phi_q\equiv\phi-\langle\phi\rangle$. At this stage, we do not specify the state with respect to which the expectation value is taken. However, let us adopt the Heisenberg representation and assume the spatial uniformity of the state. Thanks to them, the averaging operation $\langle\cdots\rangle$ and ∂_μ commute each other. Rewriting Eq. (2·1) by ϕ_c and ϕ_q, we obtain

$$\mathcal{L}=\mathcal{L}_c+\mathcal{L}_q+\mathcal{L}_I\,, \tag{2·2}$$

where

$$\mathcal{L}_c=\frac{1}{2}(\partial_\mu\phi_c)^2-\frac{m^2}{2}\phi_c{}^2-\frac{\lambda}{4!}\phi_c{}^4\,, \tag{2·3}$$

$$\mathcal{L}_q=\frac{1}{2}(\partial_\mu\phi_q)^2-\frac{m^2}{2}\phi_q{}^2-\frac{\lambda}{4!}\phi_q{}^4, \tag{2·4}$$

$$\mathcal{L}_I=\partial_\mu\phi_c\cdot\partial_\mu\phi_q-m^2\phi_c\phi_q-\frac{\lambda}{3!}\phi_c{}^3\phi_q-\frac{\lambda}{4}\phi_c{}^2\phi_q{}^2-\frac{\lambda}{3!}\phi_c\phi_q{}^3. \tag{2·5}$$

From

$$\left\langle\frac{\delta\mathcal{L}}{\delta\phi_c}\right\rangle=0,$$

the equation of motion for ϕ_c is given by

$$\left(\Box+m^2+\frac{\lambda}{2}\langle\phi_q{}^2\rangle\right)\phi_c+\frac{\lambda}{3!}(\phi_c{}^3+\langle\phi_q{}^3\rangle)=0. \tag{2·6}$$

From

$$\frac{\delta\mathcal{L}}{\delta\phi_q}-\left\langle\frac{\delta\mathcal{L}}{\delta\phi_q}\right\rangle=0,$$

the equation of motion for ϕ_q is given by

$$\left(\Box+m^2+\frac{\lambda}{2}\phi_c{}^2\right)\phi_q+\frac{\lambda}{2}\phi_c(\phi_q{}^2-\langle\phi_q{}^2\rangle)+\frac{\lambda}{3!}(\phi_q{}^3-\langle\phi_q{}^3\rangle)=0. \tag{2·7}$$

We note that the field ϕ_q effectively possesses a time dependent mass through $\phi_c{}^2(t)$. Accordingly, the definition of the positive-frequency modes of the Klein-Gordon operator changes from time to time. That is, even if we set up the vacuum state at some time, it develops into a many particle state in due course of time.[10] The term proportinal to $\phi_c\phi_q{}^2$ in Eq. (2·7) also includes the effect of particle production; the triple particle production by the decay of an external field ϕ_c. The terms proportional to $\langle\phi_q{}^2\rangle$ and $\langle\phi_q{}^3\rangle$ in Eq. (2·6) represent the recoil of the particle production, in which all the dissipative processes such as entropy production are to show their effects.

In a realistic situation, the produced particles are expected to decay subsequently into lighter particles and finally into radiation. The time-irreversible entropy production, with which we are concerned, is associated with these processes essentially. As an example, we shall consider the decay of Higgs particles into fermion pairs and evaluate the dissipative effect associated with it in the next section. However, in this section, we only assume the existence of such processes and keep our arguments as general as possible.

In what follows, we first consider the particle production process due to the time varying effective mass of ϕ_q. Then we discuss how the produced particles induce the dissipative effect on ϕ_c. For this purpose, we neglect the terms cubic and quartic in ϕ_q in the Lagrangian (2·2) for the moment. Writing ϕ_q as ϕ, the Lagrangian for ϕ under the present approximation is

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}M^2\phi^2, \tag{2·8}$$

where $M^2(t)\equiv m^2+(\lambda/2)\phi_c{}^2(t)$ is the effective mass of the ϕ field. Defining the canonical momentum by $\pi\equiv\delta\mathcal{L}/\delta\dot\phi=\dot\phi$, the corresponding Hamiltonian density becomes

$$\mathcal{H}=\pi\dot{\phi}-\mathcal{L}=\frac{1}{2}\pi^2+\frac{1}{2}(\nabla\phi)^2+\frac{1}{2}M^2\phi^2\,. \tag{2·9}$$

Although the definition of a "particle" in the case of a time-dependent Klein-Gordon operator is ambiguous generally, we adopt the following definition which seems to be most natural. Namely, the creation and annihilation operators of a particle at time t are defined in the manner that they diagonalize the Hamiltonian at time t.[11)]

Fourier decomposing $\phi(x)$ and $\pi(x)$ as

$$\phi(x)=\int\frac{d^3k}{(2\pi)^{3/2}}\tilde{\phi}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \tag{2·10}$$

$$\pi(x)=\int\frac{d^3k}{(2\pi)^{3/2}}\tilde{\pi}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \tag{2·11}$$

and imposing the canonical commutation relation

$$[\tilde{\phi}_{\boldsymbol{k}}(t),\ \tilde{\pi}_{\boldsymbol{k}'}(t)]=i\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \tag{2·12}$$

as usual, the Heisenberg equations are given by

$$\dot{\tilde{\phi}}_{\boldsymbol{k}}(t)=\tilde{\pi}_{-\boldsymbol{k}}(t), \tag{2·13}$$

$$\dot{\tilde{\pi}}_{\boldsymbol{k}}(t)=-\omega_{\boldsymbol{k}}(t)^2\tilde{\phi}_{-\boldsymbol{k}}(t), \tag{2·14}$$

where $\omega_{\boldsymbol{k}}(t)^2\equiv\boldsymbol{k}^2+M^2(t)$.

Then defining the creation and annihilation operators at time t by

$$a_{\boldsymbol{k}}(t)=(2\omega_{\boldsymbol{k}}(t))^{-1/2}(\omega_{\boldsymbol{k}}(t)\tilde{\phi}_{\boldsymbol{k}}(t)+i\tilde{\pi}_{-\boldsymbol{k}}(t)), \tag{2·15}$$

$$a_{\boldsymbol{k}}^{\dagger}(t)=(2\omega_{\boldsymbol{k}}(t))^{-1/2}(\omega_{\boldsymbol{k}}(t)\tilde{\phi}_{-\boldsymbol{k}}(t)-i\tilde{\pi}_{\boldsymbol{k}}(t)), \tag{2·16}$$

the Hamiltonian is instantaneously diagonalized:

$$H(t)=\frac{1}{2}\int\frac{d^3k}{(2\pi)^{3/2}}\omega_{\boldsymbol{k}}(t)[a_{\boldsymbol{k}}(t)a_{\boldsymbol{k}}^{\dagger}(t)+a_{\boldsymbol{k}}^{\dagger}(t)a_{\boldsymbol{k}}(t)]. \tag{2·17}$$

The operators $a_{\boldsymbol{k}}(t)$ and $a_{\boldsymbol{k}}^{\dagger}(t)$ are related to $a_{\boldsymbol{k}}(0)$ and $a_{\boldsymbol{k}}^{\dagger}(0)$ by a Bogoliubov transformation

$$\begin{bmatrix}a_{\boldsymbol{k}}(t)\\ a_{-\boldsymbol{k}}^{\dagger}(t)\end{bmatrix}=\begin{bmatrix}\alpha_{\boldsymbol{k}}(t) & \beta_{\boldsymbol{k}}(t)\\ \beta_{\boldsymbol{k}}^{*}(t) & \alpha_{\boldsymbol{k}}^{*}(t)\end{bmatrix}\begin{bmatrix}a_{\boldsymbol{k}}(0)\\ a_{-\boldsymbol{k}}^{\dagger}(0)\end{bmatrix}. \tag{2·18}$$

The coefficients $\alpha_{\boldsymbol{k}}(t)$ and $\beta_{\boldsymbol{k}}(t)$ are expressed respectively as[9)]

$$\alpha_{\boldsymbol{k}}(t)=\frac{1}{2}(\omega_{\boldsymbol{k}}(t)\omega_{\boldsymbol{k}}(0))^{-1/2}(U_{\boldsymbol{k}}(t)\omega_{\boldsymbol{k}}(t)+i\dot{U}_{\boldsymbol{k}}(t)), \tag{2·19}$$

$$\beta_{\boldsymbol{k}}(t)=\frac{1}{2}(\omega_{\boldsymbol{k}}(t)\omega_{\boldsymbol{k}}(0))^{-1/2}(U_{\boldsymbol{k}}^{*}(t)\omega_{\boldsymbol{k}}(t)+i\dot{U}_{\boldsymbol{k}}^{*}(t)), \tag{2·20}$$

where the function $U_{\boldsymbol{k}}(t)$ satisfies the equation:

$$\ddot{U}_{\boldsymbol{k}}(t)=-\omega_{\boldsymbol{k}}^2(t)U_{\boldsymbol{k}}(t) \tag{2·21}$$

with the initial conditions

$$U_k(0)=1 \quad \text{and} \quad \dot{U}_k(0)=-i\omega_k(0). \tag{2·22}$$

It is easy to check that Eqs. (2·19) and (2·20) guarantee the canonical transformation property of Eq. (2·18), that is the condition

$$|\alpha_k(t)|^2-|\beta_k(t)|^2=1\,. \tag{2·23}$$

Now let us introduce the variables $x_k(t)$ and $y_k(t)$ defined by

$$x_k(t)=\langle a_k^\dagger(t)a_k(t)\rangle, \tag{2·24}$$

$$y_k(t)=\langle a_k(t)a_{-k}(t)\rangle, \tag{2·25}$$

which are respectively diagonal and off-diagonal elements of the density matrix. If we choose the state to be the vacuum one at $t=0$(i.e., the state annihilated by $a_k(0)$), they are related to $\alpha_k(t)$ and $\beta_k(t)$ as

$$x_k(t)=|\beta_k(t)|^2, \qquad y_k(t)=\alpha_k(t)\beta_k(t), \tag{2·26}$$

and in particular, $x_k(t)$ represents the number of particles created in $\boldsymbol{k}$-mode by the time t. In general, the variables $x_k(t)$ and $y_k(t)$ satisfy the following equations:

$$\dot{x}_k(t)=(\dot{\omega}_k(t)/\omega_k(t))\mathrm{Re}\,y_k(t), \tag{2·27}$$

$$\dot{y}_k(t)=(\dot{\omega}_k(t)/\omega_k(t))\left(x_k(t)+\frac{1}{2}\right)-2i\omega_k(t)y_k(t), \tag{2·28}$$

which will be of a great importance in the following discussions.

Let us now derive the form of the dissipation term, which should appear in the effective equation of motion for ϕ_c, and the entropy production rate. Under the present approximation that we keep terms up to quadratic in ϕ_q in the original Lagrangian, the term proportional to $\langle\phi_q^3\rangle$ in the equation of motion for ϕ_c, Eq. (2·6) is neglected. Therefore, the effect of dissipation must appear through the term proportional to $\langle\phi_q^2\rangle$ at least if it is ever present. Hence the task we have to do is to evaluate $\langle\phi_q^2\rangle$.

By using Eqs. (2·10), (2·15), (2·16), (2·24) and (2·25), we can express $\langle\phi_q^2(t)\rangle$ in terms of $x_k(t)$ and $y_k(t)$ as follows:

$$\begin{aligned}\langle\phi_q^2(t)\rangle&=\int\frac{d^3k d^3k'}{(2\pi)^3}(2\omega_k(t))^{-1/2}(2\omega_{k'}(t))^{-1/2}\langle(a_k(t)e^{ik\cdot x}\\&\quad+a_k^\dagger(t)e^{-ik\cdot x})(a_{k'}(t)e^{ik'\cdot x}+a_{k'}^\dagger(t)e^{-ik'\cdot x})\rangle\\&=\int\frac{d^3k}{(2\pi)^3 2\omega_k(t)}[2\mathrm{Re}\langle a_k(t)a_{-k}(t)\rangle+2\langle a_k^\dagger(t)a_k(t)\rangle+1]\\&=\int\frac{d^3k}{(2\pi)^3 2\omega_k(t)}[2\mathrm{Re}\,y_k(t)+2x_k(t)+1].\end{aligned} \tag{2·29}$$

The system of Eqs. (2·6), (2·27), (2·28) and (2·29) forms a closed set of equations for the variables $\phi_c(t)$, $x_k(t)$, and $y_k(t)$. Therefore, one might think that the sole problem is to solve this set of equations, numerically for example. But the solution to it would never accompany the effect of genuine dissipation because the system possesses the time reversal symmetry. In order to associate the entropy production with the system, further manipulation which introduces some kind of the time irreversibility is needed.

Among the variables $x_k(t)$, $y_k(t)$ and $\phi_c(t)$, $\phi_c(t)$ is the order parameter which describes the symmetry of the universe and $x_k(t)$ is a diagonal element of the density matrix which describes the particle number density. Therefore, they can be regarded as the variables which determine the gross properties of the system. While the variable $y_k(t)$, being an off-diagonal element and representing the correlation among produced particles, varies more rapidly than $x_k(t)$ as seen from Eqs. (2·27), (2·28) provided $\dot{\omega}_k(t) \ll \omega_k^2(t)$, namely, $\phi_c(t)$ varies slowly. Therefore, it is expected that the information carried by the variable $y_k(t)$ is easily destroyed by a small perturbation to the system and that the entropy production is associated with this loss of information.

From Eq. (2·28), $y_k(t)$ is solved formally as

$$y_k(t)=\int_0^t dt'\exp\left[-2i\int_{t'}^t dt''\omega_k(t'')\right]\frac{\dot{\omega}_k(t')}{2\omega_k(t')}(2x_k(t')+1)+y_k(0)\exp\left[-2i\int_0^t dt''\omega_k(t'')\right]. \tag{2·30}$$

If this is inserted into Eqs. (2·27) and (2·29), $\dot{x}_k(t)$ and $\langle\phi_q^2(t)\rangle$ become complicated non-local functions of time which depend on the precise behavior of the system's history.*) However, if a dissipative process should ever be present, the correlation among produced particles would be destroyed sufficiently rapidly and $\dot{x}_k(t)$ and $\langle\phi_q^2(t)\rangle$ should become local functions of time which involves $\phi_c(t)$ and $\dot{\phi}_c(t)$ only. At a glance of Eq. (2·30), we find the above requirement implies the existence of a negative imaginary part in $\omega_k(t)$ such that

$$\mathrm{Im}\,\omega_k(t)\equiv-\frac{1}{2}\Gamma_k(t);\qquad \Gamma_k(t)\gg\Delta t_k(t), \tag{2·31}$$

where

$$(\Delta t_k(t))^{-1}\equiv\left(\ln\left|\frac{\dot{\omega}_k(t)}{\omega_k(t)}\left(x_k(t)+\frac{1}{2}\right)\right|\right)^{\cdot}. \tag{2·32}$$

Provided that this is the case, Re $y_k(t)$ is given by

$$\mathrm{Re}\,y_k(t)=\frac{\dot{\omega}_k(t)}{\omega_k(t)}\left(x_k(t)+\frac{1}{2}\right)\tau_k(t), \tag{2·33}$$

where

$$\tau_k(t)=\begin{cases}8\omega_k^2(t)/\Gamma_k^3(t) & \text{for } \omega_k\ll\Gamma_k, \quad (2·34\text{a})\\ \Gamma_k(t)/(2\omega_k^2(t)) & \text{for } \omega_k\gg\Gamma_k. \quad (2·34\text{b})\end{cases}$$

The quantity $\tau_k(t)$ represents a reduction time within every time-interval of which the realization of created particles and the back-reaction associated with it occur.[12),**)] Then

*) Even if we performed the momentum integral in Eq. (2·29), this non-locality would not disappear.

**) Kodama[12)] introduced the concept of the reduction time in the case of particle production in Friedmann universe.

Eq. (2·29) becomes

$$\langle\phi_q^2(t)\rangle=\int\frac{d^3k}{(2\pi)^3 2\omega_k(t)}\left[1+2x_k(t)+\frac{\dot{\omega}_k(t)}{\omega_k(t)}(2x_k(t)+1)\tau_k(t)\right]. \tag{2·35}$$

Note that the first and the second terms in the square brackets are related to the mass renormalization of the field ϕ_c. In fact, the first term, which is divergent, is present even when $\dot{\omega}_k(t)=x_k(t)=0$ and is simply canceled by the usual mass counter term. While the second term contributes to the particle-distribution-dependent mass renormalization, m_R^2. That is,

$$\begin{aligned} m_R^2(t)&=m^2+\frac{\lambda}{2}\int\frac{d^3k}{(2\pi)^3 2\omega_k(t)}(1+2x_k(t)) \\ &=m_r^2(t)+\frac{\lambda}{4\pi^2}\int_0^\infty dk\frac{k^2 x_k(t)}{\omega_k(t)}, \end{aligned} \tag{2·36}$$

where m_r^2 is the renormalized mass with no particle distribution. For a thermal equilibrium, $x_k(t)=(\exp(\omega_k(t)/T)-1)^{-1}$ and at the high temperature limit ($m\ll T$), we have

$$m_R^2(t)=m_r^2(t)+\frac{\lambda}{24}T^2, \tag{2·37}$$

which is a well-known result. In the following arguments, we shall not touch on these kinds of renormalization problems. This is because what we are interested in is dissipative effects which arise from imaginary parts of Feynman diagrams. Therefore, for $\lambda\phi^4$ model, being renormalizable, we do not have to worry about their real parts.

The dissipation term stems from the third term of Eq. (2·35) and the equation of motion for ϕ_c becomes

$$(\Box+m_R^2)\phi_c+\frac{\lambda}{3!}\phi_c^3+F(\phi_c)\dot{\phi}_c=0, \tag{2·38}$$

where the dissipation coefficient $F(\phi_c)$ is given by

$$F(\phi_c)=\frac{\lambda^2\phi_c^2}{16\pi^2}\int_0^\infty dk\frac{k^2\tau_k(t)}{\omega_k^3(t)}(2x_k(t)+1). \tag{2·39}$$

We emphasize that this dissipation coefficient was derived without assuming any particular properties of the state under consideration. Therefore, the formula holds regardless of whether it is a thermal equilibrium one or not. In this respect, we call the present mechanism quantum dissipation. Now from Eqs. (2·27) and (2·33), the incoherent (i.e., irreversible) particle production rate is given by

$$\dot{x}_k(t)=\left(\frac{\dot{\omega}_k(t)}{\omega_k(t)}\right)^2\left(x_k(t)+\frac{1}{2}\right)\tau_k(t). \tag{2·40}$$

Positivity of the quantities $F(\phi_c)$ and $\dot{x}_k$ is observed, which is the consequence of coarse-graining the quantum correlation $y_k(t)$. The part which is proportional to $x_k(t)$ are interpreted as the "induced" dissipation effect. It should be noted that the vacuum energy loss rate infered from Eq. (2·38), $F(\phi_c)\dot{\phi}_c^2$, is consistently equal to the energy generation rate associated with the incoherent particle production, $\int(d^3k/(2\pi)^3)\dot{x}_k(t)\omega_k(t)$, where $\dot{x}_k(t)$ is given by Eq. (2·40).

§ 3. Evaluation of the dissipation coefficient

Up to now, our arguments have been quite formal. In order to apply the formula (2·39) to a specific situation, it is necessary to develop a method for evaluating $\tau_k(t)$. In addition, it is more preferable if the method allows us to evaluate higher order effects which have been neglected so far. In this section, we present such a method and evaluate $\tau_k(t)$ for a couple of specific cases.

The method we develop is essentially the same as the usual perturbation theory for interacting fields. The perturbation is composed of two parts. One is that due to an explicit time dependence through $\phi_c(t)$, which gives rise to the production of ϕ_q quanta. The other is that due to interactions of ϕ_q with other fields, which gives rise to the relaxation of the produced ϕ_q quanta. Here we assume that the interaction time characteristic of the former is greater than that of the latter. This is equivalent to the assumption of a slowly varying $\phi_c(t)$, adopted in the previous section. The corresponding asymptotic field is defined to satisfy the linearized equation of motion instantaneously at a certain reference time t_0 which we arbitrarily choose.

Writing ϕ_q by ϕ as before, the total Lagrangian for the field ϕ is from Eq. (2·2),

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}m^2\phi^2-\frac{\lambda}{4!}\phi^4+\partial_\mu\phi_c\cdot\partial_\mu\phi-m^2\phi_c\phi$$

$$-\frac{\lambda}{3!}\phi_c{}^3\phi-\frac{\lambda}{4}\phi_c{}^2\phi^2-\frac{\lambda}{3!}\phi_c\phi^3+\mathcal{L}_{\text{int}}\,, \tag{3·1}$$

where $\mathcal{L}_{\text{int}}$ represents possible interactions of ϕ with other fields such as fermions and gauge bosons. Using the equation of motion for ϕ_c, Eq. (2·6), the terms linear in ϕ can be expressed in terms of $\langle\phi^2\rangle$ and $\langle\phi^3\rangle$ which are essentially non-local in time. Thus $\mathcal{L}$ is rewritten as

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}m^2\phi^2-\frac{\lambda}{4!}\phi^4-\frac{\lambda}{4}\phi_c{}^2\phi^2-\frac{\lambda}{3!}[\phi_c\phi^3-3\phi_c\phi\langle\phi^2\rangle-\phi\langle\phi^3\rangle]+\mathcal{L}_{\text{int}}\,. \tag{3·2}$$

Let us split it into two parts:

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1\,, \tag{3·3}$$

where

$$\mathcal{L}_0=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}m^2\phi^2-\frac{\lambda}{4}\phi_c{}^2(t_0)\phi^2-\frac{\lambda}{3!}\phi_c(t_0)\phi^3-\frac{\lambda}{4!}\phi^4+\mathcal{L}_{\text{int}} \tag{3·4}$$

and

$$\mathcal{L}_1(t)=-\frac{\lambda}{4}(\phi_c{}^2(t)-\phi_c{}^2(t_0))\phi^2-\frac{\lambda}{3!}[(\phi_c(t)-\phi_c(t_0))(\phi^3-3\phi\langle\phi^2(t)\rangle)$$

$$-3\phi_c(t_0)\phi\langle\phi^2(t)\rangle]+\frac{\lambda}{3!}\phi\langle\phi^3(t)\rangle. \tag{3·5}$$

The former part $\mathcal{L}_0$ is the Lagrangian we would obtain if the time development of ϕ_c were frozen at $t=t_0$, and the latter part $\mathcal{L}_1$ is the rest of $\mathcal{L}$ which is explicitly time-dependent.

The evaluation of the dissipation coefficient is done as follows. The quantities to be evaluated are $\langle\phi^2\rangle$ and $\langle\phi^3\rangle$. In order to do so, we first perform the perturbation expansion with respect to $\mathcal{L}_1$. The full field ϕ, whose dynamics is determined by the full Lagrangian $\mathcal{L}$, is related to the field $\hat{\phi}$, whose dynamics is determined by $\mathcal{L}_0$, as

$$\phi(t)=U^{\dagger}(t,\ t_0)\hat{\phi}(t)U(t,\ t_0), \tag{3·6}$$

where

$$\begin{aligned}U(t,\ t_0)&=\exp[iH_0(t-t_0)]\exp[-iH(t-t_0)]\\&=T\exp[i\int_{t_0}^{t}dt'\int d^3x'\mathcal{L}_1[\hat{\phi}(x')]]\end{aligned} \tag{3·7}$$

and

$$\hat{\phi}(t)=\exp[iH_0(t-t_0)]\hat{\phi}(t_0)\exp[-iH_0(t-t_0)] \tag{3·8}$$

with H_0 being the Hamiltonian corresponding to $\mathcal{L}_0$. As the state with respect to which the expectation value is taken, we choose the vacuum at $t=t_0$, namely, the state$|0\rangle$ such that $H(t_0)|0\rangle=0$ where H is the total Hamiltonian corresponding to $\mathcal{L}$. Since what we are interested in is the quantum dissipation, this choice seems to be reasonable. Perturbatively expanding $U(t,\ t_0)$ in Eq. (3·6), we can obtain the expressions for $\langle\phi^2\rangle$ and $\langle\phi^3\rangle$ in terms of the field $\hat{\phi}$. Then, by taking into account the interaction terms in $\mathcal{L}_0$, the dissipation coefficient can be explicitly evaluated. The above procedure is graphically rephrased as follows: First, perform skeleton expansion with respect to $\mathcal{L}_1$ and then radiatively correct the lines in each skeleton with respect to $\mathcal{L}_0$.

Let us evaluate $\langle\phi^2\rangle$ first. Up to the first order in λ, it is given by

$$\begin{aligned}\langle\phi^2(t)\rangle&=\langle\hat{\phi}^2(t)\rangle\\&\quad-\frac{i\lambda}{4}\int_{t_0}^{t}dt'\int d^3x'\langle\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle(\phi_c^2(t')-\phi_c^2(t_0))\\&\quad+\frac{i\lambda}{4}\int_{t_0}^{t}dt'\int d^3x'\langle\hat{\phi}^2(x')\hat{\phi}^2(x)\rangle(\phi_c^2(t')-\phi_c^2(t_0))\\&=\langle\hat{\phi}^2(t)\rangle\\&\quad+\frac{\lambda}{2}(\phi_c^2(t)-\phi_c^2(t_0))\int_{t_0}^{t}dt'\int d^3x'\mathrm{Im}\langle T\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle\\&\quad+\frac{\lambda}{2}\int_{t_0}^{t}dt'\int d^3x'(\phi_c^2(t')-\phi_c^2(t))\mathrm{Im}\langle T\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle.\end{aligned} \tag{3·9}$$

Note that the odd power terms of ϕ in $\mathcal{L}_1$ do not contribute at this order. Here the term immediately after the second equality is related to the usual divergent mass renormalization. However, the second term is a new one which was absent in the usual perturbation theory and it is also divergent. Although we do not know what to do with this term in the strict sense, we presume it is also related to the mass renormalization. This is supported by the fact that the space-time integral of it turns out to have nothing to do with the imaginary part of the inverse of the full propagator (at least in the lowest order), and by the fact that this term does not contain any factor which shall give rise to a time derivative of $\phi_c(t)$. Furthermore, if one accepts the idea of time-dependent mass renormalization and assumes the mass of $\hat{\phi}$ quanta at time t is $M^2(t)=m^2+(\lambda/2)\phi_c^2(t)$, this term cancels completely. This is similar to the idea of temperature-dependent mass renormalization for a system which evolves adiabatically; one assumes the system to be static when performing the renormalization but the resulting temperature dependent mass becomes eventually function of time. This procedure is usually justified by the fact that the real part of a propagator represents the abiabatic part of the evolution. Therefore let us assume the same is true for our case and the mass of $\hat{\phi}$ quanta is time-dependently renormalized. Then the only possible sourse of dissipation is the last term and this turns out to be the case.

Thanks to the assumption of the slow temporal development of $\phi_c(t)$ and the rapid decay of the quantum correlation, $\mathrm{Im}\langle T\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle$, we can approximate

$$\phi_c^2(t')-\phi_c^2(t)\simeq 2\phi(t)_c\dot{\phi}_c(t)\cdot(t'-t). \tag{3·10}$$

On the other hand, the quantum correlation $\langle T\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle$ can be decomposed as

$$\langle T\hat{\phi}^2(x)\hat{\phi}^2(x')\rangle = G_4(x,x,x',x')+2G_2(x,x')^2+G_2(x,x)G_2(x',x'), \tag{3·11}$$

where $G_n(x_1,\cdots,x_n)$ denotes the n-point connected Green function. Note that contribution of the terms that contain tadpole graphs is apparently higher order and therefore discarded in Eq. (3·11) from the beginning. On the right-hand side of Eq. (3·11), G_4 involves the 4-point vertex and is at least of order λ, hence can be neglected at the lowest order. The last term G_2G_2 is always real and local thus does not contribute to the expression of $\langle\phi^2(t)\rangle$. Therefore the relevant term of Eq. (3·9) takes the form

$$\langle\phi^2(t)\rangle_d=4\lambda\phi_c(t)\dot{\phi}_c(t)\mathrm{Im}\int_{t_0}^{t}dt'\cdot(t'-t)\int d^3x' G_2(x,x')^2\,, \tag{3·12}$$

where the subscript d to $\langle\phi^2(t)\rangle$ denotes the part of $\langle\phi^2(t)\rangle$ which causes the dissipation of ϕ_c. Since the quantum correlation dies out sufficiently fast, we may replace the lower limit of the time integral t_0 by $-\infty$ without affecting the value of Eq. (3·9). In addition, the integrand should be independent of the spatial coordinate x because of the assumption of spatial uniformity. Then, defining

$$D(t)\equiv-\int_0^{\infty}dt'\cdot t'\int d^3x' G_2(x,x')^2 \tag{3·13}$$

and denoting the part of the dissipation coefficient which is due to $\langle\phi^2(t)\rangle_d$ by $F_2(\phi_c)$, we obtain

$$F_2(\phi_c)=\lambda^2\phi_c^2\mathrm{Im}\,D\,. \tag{3·14}$$

Due to the interaction terms in $\mathcal{L}_0$, the mass in the denominator of the propagator acquires an imaginary part: $M^2\to M^2-i\Sigma(p^2)(\Sigma(p^2)\text{:real})$ as well as the renormalization. Thus we have

$$G_2(x,x')=\langle T\hat{\phi}(x)\hat{\phi}(x')\rangle$$

$$=-i\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x-x')}}{M^2-i\Sigma(p^2)-p^2}$$

$$=-\int\frac{d^3p}{(2\pi)^3}\frac{e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}e^{-i\omega|t-t'|}e^{-(\Gamma_p/2)|t-t'|}}{2(\omega_p-i\Gamma_p/2)}, \tag{3·15}$$

where

$$(\omega_p-i\Gamma_p/2)^2\equiv\mathbf{p}^2+M^2-i\Sigma(M^2), \quad \text{or} \quad \Gamma_p=\frac{\Sigma(M^2)}{\omega_p} \tag{3·16}$$

and $\omega_p\gg\Gamma_p$ has been assumed. Then, D is expressed as

$$D(t)=\int\frac{d^3p}{(2\pi)^3}\frac{(-1/4)}{(\omega_p-i\Gamma_p/2)^2}\int_0^\infty dt\cdot te^{-2i(\omega_p-i\Gamma_p/2)t}$$

$$=\int\frac{d^3p}{(2\pi)^3}\frac{1}{16(\omega_p-i\Gamma_p/2)^4}. \tag{3·17}$$

Therefore, in the case $\omega_p\gg\Gamma_p$, Eq. (3·14) is expressed as

$$F_2(\phi_c)=\frac{\lambda^2\phi_c{}^2}{16\pi^2}\int_0^\infty dp\cdot\mathbf{p}^2\cdot\Gamma_p/\omega_p{}^5=\frac{\lambda^2\phi_c{}^2}{16\pi^2}\frac{\Sigma}{M^3}\int_0^\infty dx\cdot x^2(1+x^2)^{-3}. \tag{3·18}$$

We find this has the same form as Eq. (2·39) with $x_k(t)$ set equal to zero and $\tau_k(t)$ given by Eq. (2·34b), except that the numerical factor is twice the case of Eq. (2·39). This difference may be interpreted as follows. In Eq. (2·39), only the off-diagonal element $y_k(t)$ is smeared out to yield the effective equation of motion for $\phi_c(t)$ and $x_k(t)$. While in Eq. (3·18), both the diagonal element $x_k(t)$ and the off-diagonal one $y_k(t)$ are smeared out to yield the effective equation of motion for $\phi_c(t)$. Then the latter case should be associated with a greater loss of information and is expected to yield a larger dissipation coefficient. The answer to the question which is more realistic procedure may depend on a situation one deals with.

Next, we consider $\langle\phi^3(t)\rangle$. Following the same procedure as before, we obtain

$$\begin{aligned}\langle\phi^3(t)\rangle=&\langle\hat{\phi}^3(t)\rangle\\
&+\frac{\lambda}{3}(\phi_c(t)-\phi_c(t_0))\int_{-\infty}^t dt'\int d^3x'\,\mathrm{Im}\langle T\hat{\phi}^3(x)(\hat{\phi}^3(x')-3\langle\phi^2(x')\rangle\hat{\phi}(x'))\rangle\\
&-\frac{\lambda}{3}\int_{-\infty}^t dt'\int d^3x'\langle\phi^3(t')\rangle\mathrm{Im}\langle T\hat{\phi}^3(x)\hat{\phi}(x')\rangle\\
&-\lambda\phi_c(t_0)\int_{-\infty}^t dt'\int d^3x'\,\mathrm{Im}\langle T\hat{\phi}^3(x)\hat{\phi}(x')\rangle\langle\phi^2(x')\rangle\\
&+\frac{\lambda}{2}\int_{-\infty}^t dt'\int d^3x'(\phi_c{}^2(t')-\phi_c{}^2(t_0))\mathrm{Im}\langle T\hat{\phi}^3(x)\hat{\phi}^2(x')\rangle\\
&+\frac{\lambda}{3}\int_{-\infty}^t dt'\int d^3x'(\phi_c(t')-\phi_c(t))\mathrm{Im}\langle T\hat{\phi}^3(x)(\hat{\phi}^3(x')-3\langle\phi^2(x')\rangle\hat{\phi}(x'))\rangle.\end{aligned} \tag{3·19}$$

Note that the first four terms of the right-hand side do not contain any factor which shall give rise to the time derivative of $\phi_c(t)$. Thus they have nothing to do with dissipation. For the divergences appearing in these terms, we adopt the idea of time-dependent renormalization by appealing to the same kind of arguments we have given before and assume it to be properly canceled. The fifth term is a higher order contribution because it is proportional to the vacuum expectation value of odd power of $\hat{\phi}$. Thus the last term is relevant,

$$\langle\phi^3(t)\rangle_d=\frac{\lambda}{3}\int_{-\infty}^{t}dt'\int d^3x'(\phi_c(t')-\phi_c(t))\mathrm{Im}\langle T\hat{\phi}^3(x)(\hat{\phi}^3(x')-3\langle\phi^3(x')\rangle\hat{\phi}(x'))\rangle. \quad (3\cdot20)$$

The quantum correlation factor is decomposed as

$$\langle T\hat{\phi}^3(x)(\hat{\phi}^3(x')-3\langle\phi^3(x')\rangle\hat{\phi}(x'))\rangle=6G_2(x,x')^3+(\text{higher order terms}). \quad (3\cdot21)$$

Thanks to the assumption of slow variation of $\phi_c(t)$ as before, we get

$$\langle\phi^3(t)\rangle=2\lambda\dot{\phi}_c(t)\mathrm{Im}\int_{-\infty}^{t}dt'\int d^3x'\cdot(t'-t)G_2(x,x')^3. \quad (3\cdot22)$$

Defining

$$E(t)\equiv-\int_0^{\infty}dt'\cdot t'\int d^3x'G_2(x,x')^3 \quad (3\cdot23)$$

and denoting the part of the dissipation coefficient which is due to $\langle\phi^3(t)\rangle_d$ by $F_3(\phi_c)$, we obtain

$$F_3(\phi_c)=\frac{\lambda^2}{3}\mathrm{Im}E\,. \quad (3\cdot24)$$

Inserting the expression (3·15) for G_2 into Eq. (3·23), E is expressed as

$$E(t)=\frac{1}{8}\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(\tilde{\omega}_1\tilde{\omega}_2\tilde{\omega}_3)^{-1}(\tilde{\omega}_1+\tilde{\omega}_2+\tilde{\omega}_3)^{-2}, \quad (3\cdot25)$$

where

$$\tilde{\omega}_1=+(\boldsymbol{k}_1^2+M^2-i\Sigma(M^2))^{1/2},$$
$$\tilde{\omega}_2=+(\boldsymbol{k}_2^2+M^2-i\Sigma(M^2))^{1/2},$$
$$\tilde{\omega}^3=+(\boldsymbol{k}_1^2+\boldsymbol{k}_2^2+M^2-i\Sigma(M^2))^{1/2}. \quad (3\cdot26)$$

Thus $F_3(\phi_c)$ is given by

$$F_3(\phi_c)=\frac{\lambda^2\Sigma}{96\pi^4M}\left[\int_0^{\infty}dx_1\int_0^{\infty}dx_2x_1^2x_2^2\frac{1}{A_1A_3^2}\left(\frac{A_2^2}{2A_1A_3}+\frac{A_2}{A_1^2}-1\right)\right], \quad (3\cdot27)$$

where

$$A_1=\Omega_1+\Omega_2+\Omega_3,\quad A_2=\Omega_1\Omega_2+\Omega_2\Omega_3+\Omega_3\Omega_1,\quad A_3=\Omega_1\Omega_2\Omega_3;$$
$$\Omega_1=(1+x_1^2)^{1/2},\quad \Omega_2=(1+x_2^2)^{1/2},\quad \Omega_3=(1+x_1^2+x_2^2)^{1/2}. \quad (3\cdot28)$$

Next, we shall evaluate Γ_p and $F(\phi_c)$ explicitly, by specifying the form of $\mathcal{L}_{\rm int}$ in Eq. (3·4). In the realistic cosmological situation at the GUT phase transition, the Higgs field interacts with light fermions or with gauge bosons. We choose the Yukawa coupling with a light fermions as a demonstration;

$$\mathcal{L}_{\rm int} = -f\phi\bar{\psi}\psi\,. \tag{3·29}$$

The imaginary part of the self energy of $\hat{\phi}$ field, Σ which is defined in Eq. (3·15) due to this interaction is given by

$$\Sigma(p^2) = \frac{f^2}{12\pi}\left(1+\frac{4\mu^2}{p^2}\right)^{1/2}(p^2+2\mu^2)\theta(p^2-4\mu^2), \tag{3·30}$$

where μ denotes the fermion mass. From Eq. (3·16), the corresponding decay width is given by

$$\Gamma_p = \frac{f^2M^2}{12\pi\omega_p}, \tag{3·31}$$

where we have assumed $\mu \ll M$. Provided $f^2 \ll 12\pi$, we have $\omega_p \gg \Gamma_p$ and Eq. (3·18) can be used, yielding the dissipation coefficient which originates from $\langle\phi^2(t)\rangle$ as

$$F_2(\phi_c) = \frac{\lambda^2\phi_c{}^2 f^2}{3072\pi^2 M}\,. \tag{3·32}$$

In the case $m^2 \ll \lambda\phi_c{}^2$, this is reduced to

$$F_2(\phi_c) = \frac{\sqrt{2}\lambda^{3/2}f^2}{3072\pi^2}\cdot\phi_c\,. \tag{3·33}$$

If we use Eq. (3·30) to Eq. (3·27), we get the dissipation coefficient which originates from $\langle\phi^3(t)\rangle$:

$$F_3(\phi_c) = \frac{\lambda^2 f^2 M}{1152\pi^5}\left[\int_0^\infty dx_1\int_0^\infty dx_2 x_1{}^2 x_2{}^2\frac{1}{A_1A_3{}^2}\left(\frac{A_2{}^2}{2A_1A_3}+\frac{A_2}{A_1{}^2}-1\right)\right]. \tag{3·34}$$

In the case $m^2 \ll \lambda\phi_c{}^2$, this is reduced to

$$F_3(\phi_c) = \frac{\lambda^{5/2}f^2\phi_c}{1152\sqrt{2}\pi^5}\left[\int_0^\infty dx_1\int_0^\infty dx_2 x_1{}^2 x_2{}^2\frac{1}{A_1A_3{}^2}\left(\frac{A_2{}^2}{2A_1A_3}+\frac{A_2}{A_1{}^2}-1\right)\right]. \tag{3·35}$$

We can show that the ϕ self-interactions do not contribute to the relaxation of the produced ϕ quanta. Let us use the spectral representation for $G_2(x)$,[13]

$$G_2(x) = \int_0^\infty d\kappa^2\rho(\kappa^2)\Delta_F(x;\kappa^2), \tag{3·36}$$

where

$$\Delta_F(x;\kappa^2) = -i\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ipx}}{\kappa^2-p^2-i\varepsilon}, \tag{3·37}$$

and ρ, the spectral function, includes the effect of two-body and three-body decays by the self-interactions in $\mathcal{L}_0$, Eq. (3·4). The dissipation coefficient $F_2(\phi_c)$ then takes the form

$$F_2(\phi_c) = -\lambda^2\phi_c{}^2{\rm Im}\int_0^\infty d\kappa_1{}^2\int_0^\infty d\kappa_2{}^2\rho(\kappa_1{}^2)\rho(\kappa_2{}^2)\left[\int_0^\infty dt\cdot t\int d^3x\Delta_F(x;\kappa_1{}^2)\Delta_F(x;\kappa_2{}^2)\right]. \tag{3·38}$$

The term inside the square brackets of this expression becomes

$$\int \frac{d^3p}{(2\pi)^3} \frac{i}{4\omega_1\omega_2}\left[\pi\delta'(\omega)+i\frac{\mathcal{P}}{\omega^2}\right], \tag{3·39}$$

where the prime to the δ -function denotes the derivative with respect to ω and $\mathcal{P}$ denotes the principal value, and

$$\omega=\omega_1+\omega_2\,,$$
$$\omega_1=+(\varkappa_1{}^2+\boldsymbol{p}^2)^{1/2}, \qquad \omega_2=+(\varkappa_2{}^2+\boldsymbol{p}^2)^{1/2}. \tag{3·40}$$

It is apparent that the imaginary part of Eq. (3·39) is zero since ω is always positive. Thus Eq. (3·38) vanishes. Similarly the dissipation coefficient due to $\langle\phi^3\rangle$, $F_3(\phi_c)$, also vanishes for the self-coupling interactions. This is essentially because the self-coupling cannot induce any decay process.

§ 4. Physical interpretation

In the present section, let us clarify the origin of the entropy production and discuss the meaning of the dissipation coefficient obtained in the previous section. The Higgs particles are produced from the mixing of positive and negative frequency modes due to the temporal development of $\phi_c(t)$ field. If these particles are left intact, their quantum correlation never fades out.

In general particle production problems, if we literally interpret $x_k(t)$ in Eqs. (2·27) and (2·28) as the total number of produced particles in $\boldsymbol{k}$-mode during a time interval from 0 to t, peculiar things may happen. For example, consider the scalar particle production due to the cosmic expansion in a radiation dominated closed Friedmann universe. The quantity $x_k(t)$ can be evaluated with the initial condition (Fulling condition)[14] which is considered to be reasonable even in the neighborhood of the initial singularity. Using the Fulling condition, Ishihara and Nariai have shown that although $x_k(t)$ increases monotonically immediately after the big bang, it decreases monotonically immediately before the big crunch.[15] This does not reconcile with our ordinary thermodynamical sense.

In order that the produced particles should give rise to entropy production, there must exist interactions among the produced particles and the other fields. The interactions promote diffusion of energy in the huge phase-space of all particle modes and species. A particle, produced in a heavy mass mode, subsequently decays into lighter modes, and this successive cascade continues to the end where the whole phase-space is occupied with some weight and the diffusion flow ceases. This diffusion in the phase-space is the very origin of the irreversible entropy production.

From this point of view, the formula of dissipation coefficients, Eqs. (2·39), (3·18) and (3·27) can be interpreted as follows. At the beginning of the cosmic thermalization, the phase-space is almost empty. The particle production due to the temporal development of ϕ_c is then a process of injecting the background energy into heavy mass modes in the phase-space. The strength of the injection is therefore proportional to $\dot{\phi}_c$. Subsequently the energy injected is expected to diffuse into lighter mass modes in the phase-space. The diffusion velocity is expected to be determined by the decay rate of the particles, Γ_p. Hence, the entropy generation rate should be proportional to the product of the energy injection strength $\dot{\phi}_c$ and the diffusion velocity Γ_p. The dissipation coefficients, Eqs. (2·39), (3·18) and (3·27), really possess this very form, provided $\omega_p \gg \Gamma_p$. On the contrary, the dissipation coefficient derived by Hosoya, Sakagami and Takao[5] is inversely proportional to the interaction strength Γ_p in the same situation $\omega_p \gg \Gamma_p$. This is interpreted as follows: Their perturbation expansion is performed around a thermal equilibrium. Therefore the greater is the interaction Γ_p, the shorter is the relaxation time to the thermal equilibrium. Accordingly the system is closer to the thermal equilibrium and the entropy production is smaller. In the present case, however, the perturbation expansion is performed around the vacuum. Therefore the greater is the interaction Γ_p, the faster is the diffusion in the phase space, accordingly the greater is the entropy production. But for modes $\omega_p \ll \Gamma_p$, the dissipation coefficient is inversely proportional to Γ_p. A possible interpretation would be that a thermal equilibrium is rapidly attained for these modes due to the large interaction strength. Therefore the situation is expected to be similar to that discussed by Hosoya, Sakagami and Takao.

Now let us turn our attention to the original question whether the sufficient thermalization is possible or not. In the previous section, we have obtained dissipation terms, Eqs. (3·33) and (3·35), which are proportional to $\phi_c \dot{\phi}_c$. This form is the same as that assumed by Albrecht, Steinhardt, Turner and Wilczek in their numerical analysis.[3] Thus we can borrow their results. According to them, the numerical factor in front of ϕ_c in the dissipation coefficient must be greater than 10^{-5} in order to thermalize the vacuum energy more than 60%. (Albrecht et al. used the Coleman-Weinberg type potential with initial conditions:$\langle\phi\rangle = T = 3\times10^8$GeV, $\langle\phi\rangle^{\cdot} = 0$.) Unfortunately our numerical factor seems to be somewhat smaller than 10^{-5}. However there are several factors which may increase the dissipation coefficient and they must be taken into account. Possible factors are as follows. The greater is the number of the species of fields which couple to the Higgs field, the greater is the dissipation. An induced dissipation due to the factor $(2x_k+1)$ in Eq. (2·39) and the decay strength Γ_p due to the gauge boson coupling to Higgs field would increase the dissipation. Also there may be the effect of the de Sitter expansion. Further, if the temperature increases to some extent by the present quantum dissipation mechanism, a thermal dissipation mechanism due to Hosoya, Sakagami and Takao[5] may set in. In order to answer the question whether the thermalization is sufficiently promoted so that the inflation is followed by the standard hot universe or not, we have to investigate the whole scenario elaborately taking into account all the factors mentioned above.

§ 5. Concluding remarks

To conclude this paper, let us briefly comment on some unsolved problems and possible generalizations of the present analysis.

First, we comment on the problem concerning the meaning and the legitimacy of coarse-graining off-diagonal elements of the density matrix. We did not give the precise method of this coarse-graining procedure in the present analysis, but as indicated by the fact that Eq. (2·39) differs from Eq. (3·18) by the factor of two, the resultant dissipation coefficient seems to depend on how we perform the coarse-graining. Whether one could remove this ambiguity is an open problem.

Second, a possible connection of the present work with generation of density fluctuations in the early universe should be mentioned. Basically dissipation and fluctuation are closely related. In fact, within the scope of linear response theories, they are firmly connected with each other through the dissipation-fluctuation theorem. However it is not clear whether a similar theorem holds or not in our manifestly non-equilibrium situation. If this is the case, we expect that physical fluctuations are related to the imaginary part of the convolution of several propagators rather than the bare propagator itself, which has been proposed to be the origin of density fluctuations by Hawking.[16] In order to clarify the origin of density fluctuations, we have to generalize the present analysis to the case of space-dependent, as well as time-dependent, background ϕ_c. There, two competing effects should be kept in mind. First, the particle production process itself may enhance the spatial non-uniformity. As is expected from the induced dissipation factor $(2x_k(t)+1)$ in Eq. (2·39), a denser distribution of particles results in more effective particle production. This effect, if followed by the gravitational instability, may cause appreciable spatial non-uniformity in the distribution of particles. Contrary to that, there exists the effect which makes the system uniform. The dissipation term proportional to $\dot{\phi}_c$, in Eq. (2·38), is a consequence of temporal non-locality in the equation of motion for ϕ_c. If we allowed spatial dependence in ϕ_c, the spatial non-locality would give rise to an extra term proportional to $\nabla\phi_c$. For example, the extra term to the right-hand side of Eq. (3·12) would be

$$4\lambda\phi_c(x)\nabla\phi_c(x)\cdot\int_{-\infty}^{t}dt'\int d^3x(\boldsymbol{x}-\boldsymbol{x}')\mathrm{Im}\,G_2(x,x')^2\,. \qquad (5\cdot1)$$

The latter term is expected to reduce spatial inhomogeneities of $\phi_c(x)$.

Finally, some other possibilities of generalizing the present analysis are as follows. We have tentatively adopted the particle definition which instantaneously diagonalize the Hamiltonian. But instead, we could have chosen another definition such as Parker and Fulling's adiabatic particle picture[17] because there is no unique particle picture in the case of time-dependent background. As for applications to a realistic inflationary universe model, generalization to the case of a multi-dimensional and/or composite Higgs field is also interesting and necessary. In addition, an analysis which includes gravitational effects is probably important.

Acknowledgements

We would like to thank Professor C. Hayashi and Professor H. Sato for their continuous encouragement. We also would like to thank Professor S. Tanaka, Professor T. Kugo, Dr. M. Bando, and Dr. H. Hata for valuable comments.

Recently the present topic has been discussed by the authors of Ref. 5) on the basis of a formalism quite different from ours. We are grateful to Professor A. Hosoya for informing us of their results and discussing the topic with us.

References

1) K. Sato, Month. Not. Roy. Astron. Soc. **195** (1981), 467.
A. H. Guth, Phys. Rev. **D23** (1981), 347.
A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48** (1982), 1220.
A. D. Linde, Phys. Lett. **108B** (1982), 389; **114B** (1982), 431.
S. W. Hawking and I. G. Moss. Phys. Lett. **110B** (1982), 35.
2) M. Yoshimura, Phys. Rev. Lett. **41** (1978), 281.

S. Weinberg, Phys. Rev. Lett. **42** (1979), 850.
3) A. Albrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. **48** (1982), 1437.
4) L. F. Abbott, E. Farhi and M. B. Wise, Phys. Lett. **117B** (1982), 29.
See also, A. D. Dolgov and A. D. Linde, Phys. Lett. **116B** (1982), 329.
5) A. Hosoya, M. Sakagami and M. Takao, Ann. of Phys. **154** (1984), 229.
A. Hosoya and M. Sakagami, Phys. Rev. **D29** (1984), 2228.
6) D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Plenum, New York, 1974).
7) G. W. Gibbons and S. W. Hawking, Phys. Rev. **D15** (1977), 2738.
A. S. Lapedes, J. Math. Phys. **19** (1978), 2289.
8) A. Hosoya, private communication.
9) S. W. Hawking, Commun. Math. Phys. **43** (1975), 199.
10) L. Parker, Phys. Rev. **183** (1969), 1057.
For review, see, e.g., B. S. DeWitt, Phys. Rep. **91C** (1975), 295.
11) B. L. Hu, S. A. Fulling and L. Parker, Phys. Rev. **D8** (1973), 2377.
12) H. Kodama, Prog. Theor. Phys. **67** (1982), 1415.
13) See, e.g., N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (John Wiley & Sons, New York, 1980).
14) S. A. Fulling, General Rel. and Gravi. **10** (1979), 807.
15) H. Ishihara and H. Nariai, Prog. Theor. Phys. **68** (1982), 1147.
16) S. W. Hawking, Phys. Lett. **115B** (1982), 295.
17) L. Parker and S. A. Fulling, Phys. Rev. **D9** (1974), 341.