

Epipolar Geometry in Stereo, Motion and Object Recognition

A Unified Approach

by

Gang Xu

*Department of Computer Science,
Ritsumeikan University,
Kusatsu, Japan*

and

Zhengyou Zhang

*INRIA Sophia-Antipolis,
Sophia-Antipolis, France*



KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

CONTENTS

Forward by Olivier Faugeras	xiii
Forward by Saburo Tsuji	xv
PREFACE	xvii
1 INTRODUCTION	1
1.1 Vision Research	1
1.2 Multiple View Problems in Vision	2
1.3 Organization of this Book	4
1.4 Notation	5
2 CAMERA MODELS AND EPIPOLAR GEOMETRY	7
2.1 Modeling Cameras	7
2.1.1 Pinhole Camera and Perspective Projection	7
2.1.2 Perspective Projection Matrix and Extrinsic Parameters	10
2.1.3 Intrinsic Parameters and Normalized Camera	12
2.1.4 The General Form of Perspective Projection Matrix	14
2.2 Perspective Approximations	16
2.2.1 Orthographic and Weak Perspective Projections	17
2.2.2 Paraperspective Projection	20
2.2.3 Affine Cameras	24
2.3 Epipolar Geometry Under Full Perspective Projection	26
2.3.1 Concepts in Epipolar Geometry	26
2.3.2 Working with Normalized Image Coordinates	29

2.3.3	Working with Pixel Image Coordinates	33
2.3.4	Working with Camera Perspective Projection Matrices	37
2.3.5	Fundamental Matrix and Epipolar Transformation	39
2.4	A General Form of Epipolar Equation for Any Projection Model	42
2.4.1	Intersecting Two Optical Rays	42
2.4.2	The Full Perspective Projection Case	45
2.5	Epipolar Geometry Under Orthographic, Weak Perspective, Paraperspective and General Affine Projections	46
2.5.1	Orthographic and Weak Perspective Projections	47
2.5.2	Paraperspective Projection	56
2.5.3	The General Affine Camera	61
2.6	Epipolar Geometry Between Two Images with Lens Distortion	65
2.6.1	Camera Distortion Modelling	66
2.6.2	Computating Distorted Coordinates from Ideal Ones	68
2.6.3	Epipolar Constraint Between Two Images with Distortion	69
2.7	Summary	70
2.A	Appendix	71
2.A.1	Thin and Thick Lens Camera Models	71
2.A.2	Inverse and Pseudoinverse Matrices	75
3	RECOVERY OF EPIPOLAR GEOMETRY FROM POINTS	79
3.1	Determining Fundamental Matrix Under Full Perspective Projection	79
3.1.1	Exact Solution with 7 Point Matches	81
3.1.2	Analytic Method with 8 or More Point Matches	81
3.1.3	Analytic Method with Rank-2 Constraint	87
3.1.4	Nonlinear Method Minimizing Distances of Points to Epipolar Lines	88
3.1.5	Nonlinear Method Minimizing Distances Between Observation and Reprojection	91
3.1.6	Robust Methods	93

3.1.7	Characterizing the Uncertainty of Fundamental Matrix	106
3.1.8	An Example of Fundamental Matrix Estimation	111
3.1.9	Defining Epipolar Bands by Using the Estimated Uncertainty	117
3.2	Determining Fundamental Matrix for Affine Cameras	120
3.2.1	Exact Solution with 4 Point Matches	121
3.2.2	Analytic Method with More than 4 Point Matches	121
3.2.3	Minimizing Distances of Points to Epipolar Lines	126
3.2.4	Minimizing Distances Between Observation and Reprojection	127
3.2.5	An Example of Affine Fundamental Matrix Estimation	129
3.2.6	Charactering the Uncertainty of Affine Fundamental Matrix	131
3.2.7	Determining Motion Equation in the 2D Affine Motion Case	133
3.3	Recovery of Multiple Epipolar Equations by Clustering	136
3.3.1	The Problem in Stereo, Motion and Object Recognition	137
3.3.2	Definitions and Assumptions	138
3.3.3	Error Analysis of Motion Parameters	139
3.3.4	Estimating Covariance Matrix	141
3.3.5	The Maximal Likelihood Approach	142
3.3.6	Robust Estimation Using Exponential of Gaussian Distribution	145
3.3.7	A Clustering Algorithm	147
3.3.8	An Example of Clustering	148
3.4	Projective Reconstruction	149
3.4.1	Projective Structure from Two Uncalibrated Images	149
3.4.2	Computing Camera Projection Matrices	151
3.4.3	Reconstruction Techniques	153
3.4.4	Use of Projective Structure	155
3.5	Affine Reconstruction	156
3.5.1	Affine Structure from Two Uncalibrated Affine Views	156
3.5.2	Relation to Previous Work	159
3.5.3	Experimental Results	159

3.6	Summary	161
3.A	Appendix	164
3.A.1	Approximate Estimation of Fundamental Matrix from General Matrix	164
3.A.2	Estimation of Affine Transformation	165
4	RECOVERY OF EPIPOLAR GEOMETRY FROM LINE SEGMENTS OR LINES	167
4.1	Line Segments or Straight Lines	168
4.2	Solving Motion Using Line Segments Between Two Views	173
4.2.1	Overlap of Two Corresponding Line Segments	173
4.2.2	Estimating Motion by Maximizing Overlap	175
4.2.3	Implementation Details	176
4.2.4	Reconstructing 3D Line Segments	179
4.2.5	Experimental Results	180
4.2.6	Discussions	192
4.3	Determining Epipolar Geometry of Three Views	194
4.3.1	Trifocal Constraints for Point Matches	194
4.3.2	Trifocal Constraints for Line Correspondences	199
4.3.3	Linear Estimation of \mathbf{K} , \mathbf{L} , and \mathbf{M} Using Points and Lines	200
4.3.4	Determining Camera Projection Matrices	201
4.3.5	Image Transfer	203
4.4	Summary	204
5	REDEFINING STEREO, MOTION AND OBJECT RECOGNITION VIA EPIPOLAR GEOMETRY	205
5.1	Conventional Approaches to Stereo, Motion and Object Recognition	205
5.1.1	Stereo	205
5.1.2	Motion	206
5.1.3	Object Recognition	207
5.2	Correspondence in Stereo, Motion and Object Recognition as 1D Search	209
5.2.1	Stereo Matching	209

5.2.2	Motion Correspondence and Segmentation	209
5.2.3	3D Object Recognition and Localization	210
5.3	Disparity and Spatial Disparity Space	210
5.3.1	Disparity under Full Perspective Projection in the Parallel Camera Case	211
5.3.2	Disparity under Full Perspective Projection in the General Case	211
5.3.3	Disparity under Weak Perspective and Paraperspective Projections	214
5.3.4	Spatial Disparity Space and Smoothness	215
5.3.5	Correspondence as Search for Surfaces and Contours in SDS	218
5.4	Summary	219
6	IMAGE MATCHING AND UNCALIBRATED STEREO	221
6.1	Finding Match Candidates by Correlation	223
6.1.1	Extracting Points of Interest	223
6.1.2	Matching Through Correlation	224
6.1.3	Rotating Correlation Windows	225
6.2	Unique Correspondence by Relaxation and Robust Estimation of Epipolar Geometry for Perspective Images	227
6.2.1	Measure of the Support for a Match Candidate	228
6.2.2	Relaxation Process	230
6.2.3	Detection of False Matches	232
6.3	Unique Correspondence by Robust Estimation of Epipolar Geometry for Affine Images	233
6.3.1	Discarding Unlikely Match Candidates	233
6.3.2	Generating Local Groups of Point Matches for Clustering	234
6.4	Image Matching with the Recovered Epipolar Geometry	235
6.4.1	A Simple Implementation for Matching Corners	236
6.4.2	A Simple Implementation for Matching Edges	236
6.5	An Example of Matching Uncalibrated Perspective Images	237
6.6	An Example of Matching Uncalibrated Affine Images	241
6.7	Summary	243

7	MULTIPLE RIGID MOTIONS: CORRESPONDENCE AND SEGMENTATION	247
7.1	Problems of Multiple Rigid Motions	247
7.2	Determining Epipolar Equations for Multiple Rigid Motions	249
7.2.1	The Algorithm	249
7.2.2	Experimental Results	250
7.3	Matching and Segmenting Edge Images with Known Epipolar Equations	253
7.3.1	Representing the Problem in ESDS	253
7.3.2	A Support Measure for Selection from Multiple Candidates	255
7.3.3	Experimental Results	256
7.4	Transparent Multiple Rigid Motions	258
7.5	Summary and Discussions	260
7.6	Appendix: SVD Algorithm for Structure and Motion Recovery with Known Epipolar Equations	263
8	3D OBJECT RECOGNITION AND LOCALIZATION	269
8.1	Introduction: 3D Model vs 2D Model, and Single Model View vs. Multiple Model Views	269
8.2	Recognition and Localization with Single Model View with Model Views	271
8.2.1	Is a Single View Sufficient for 3D Object Recognition?	271
8.2.2	Matching Model View with Input View as Uncalibrated Stereo Images	272
8.2.3	An Example	273
8.3	Recognition and Localization with Multiple Model Views	278
8.3.1	Intersection of Epipolar Lines	279
8.3.2	Basis Vectors	279
8.3.3	Determining Coefficients	282
8.3.4	An Example	286
8.4	Summary	287
9	CONCLUDING REMARKS	291
	REFERENCES	293
	INDEX	309