Equal-Gain Diversity Receiver Performance in Wireless Channels

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***Abstract*—Performance analysis of equal-gain combining (EGC) diversity systems is notoriously difficult only more so given that the closed-form probability density function (pdf) of the EGC output is only available for dual-diversity combining in Rayleigh fading. In this paper, a powerful frequency-domain approach is therefore developed in which the average error-rate integral is transformed into the frequency domain, using Parseval's theorem. Such a transformation eliminates the need for computing (or approximating) the EGC output pdf (which is unknown), but instead requires the knowledge of the corresponding characteristic function (which is readily available). The frequency-domain method also circumvents the need to perform multiple-fold convolution integral operations, usually encountered in the calculation of the pdf of the sum of the received signal amplitudes. We then derive integral expressions for the average symbol-error rate of an arbitrary two-dimensional signaling scheme, with EGC reception in Rayleigh, Rician, Nakagami-m, and Nakagami-q fading channels. For practically important cases of second- and third-order diversity systems in Nakagami fading, both coherent and noncoherent detection methods for binary signaling are analyzed using the Appell hypergeometric function. A number of closed-form solutions are derived in which the results put forward by Zhang are shown to be special cases.**

***Index Terms*—Diversity systems, equal-gain combining, frequency-domain analysis, generalized fading channels, mobile radio systems.**

I. Introduction

In recent years, diversity reception and multi-level modulation schemes have received considerable attention for facilitating high-rate data transmission over wireless links. Equal-gain diversity (EGC) is of a practical interest because it provides performance comparable to optimal maximal-ratio combining (MRC) technique but with greater simplicity. Surprisingly, published results concerning EGC receiver performance in fading have been scarce when compared to those for other diversity combining methods such as MRC. This lack may have stemmed from the difficulty of finding the probability density function (pdf) of the EGC output signal-to-noise ratio (SNR) (traditionally, the performance is evaluated by averaging the conditional error probability (CEP) over the EGC output pdf), which depends on the square of a sum of L fading amplitudes. A closed-form solution to the pdf of this sum has been elusive for more than 80 years (dating back to Lord Rayleigh himself [1]) and indeed, even for the case of Rayleigh fading (mathematically simplest distribution) no solution exists for $L > 2$ [2]. No closed-form expressions for the pdf of a sum of Nakagami-m, Nakagami-q, or Rician random variables (RVs) exist either.

Clearly, if the average error rate can be evaluated without knowing the pdf of the combiner output then the EGC case can be analyzed readily [3], [4]. One way to achieve this is to eliminate the pdf using Parseval's theorem to transform the error integral into the frequency domain. Since the Fourier transform (FT)[1] of the pdf is the characteristic function (CHF) and since the FT of the conditional-error probability can be expressed analytically, the resulting integral solution is both general and exact. This approach is employed to evaluate the exact performance of EGC receivers for MQAM over Nakagami-m fading [4]. Zhang [3] also developed a CHF approach to analyze the EGC performance of binary signaling schemes over Rayleigh fading. In this paper, we extend [3] and [4] to obtain the average symbol-error rate (ASER) of a broad class of coherent, differentially coherent, and noncoherent modulation formats with predetection EGC in different fading environments. Subsequently, we show that the generic expression can be further simplified given one of four particular forms of the CEP. The contributions of this paper include the following: 1) derivation of simple yet exact analytical expressions for the ASER of binary and two-dimensional signal constellations in Rayleigh, Rician, Nakagami-m, and Nakagami-q fading channels with EGC; 2) derivation of the CHF of the fading amplitude in Nakagami-q channel and derivation of an alternative expression for the CHF of the fading amplitude in Rician channel; 3) highlight the use of a "desirable exponential form" of the Gaussian probability integral $Q(x)$ for performing averages over fading amplitudes; and 4) derivation of some closed-form solutions for coherent, differentially coherent, and noncoherent binary modulation formats in Nakagami-m fading. To the best of the authors' knowledge, closed-form formulas for the EGC in Nakagami-m fading are not available in the literature.

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[1] In this paper, we define the FT of a function $g(t)$ as $G(\omega) = E(e^{j\omega t}) = \int_{-\infty}^{\infty} g(t)e^{j\omega t}\,dt$ to be consistent with the definition for the CHF and its inverse FT used in [5, p. 35]. This differs from the usual definition of FT by a negative sign in the exponential.

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Previous related studies on EGC diversity include the following. In [6], the pdf of a dual diversity EGC output in Rayleigh fading was derived. For higher order of diversity, [1] made use of a small argument approximation [7]. In [2], the author devised an approximate infinite series technique to compute the pdf for the sum of independent Rayleigh RVs. Applying this technique, [8] and [9] analyzed the performance of EGC for coherent and differential binary signaling schemes in Nakagami-m and Rician fading channels. Subsequently, [3] presented some closed-form solutions for binary signaling schemes in a Rayleigh fading channel. In [10], the authors presented yet another approximate solution for the binary phae-shift keying (BPSK) case on Nakagami-m fading channel using Hermite integration. This has been extended in [16] by considering two-dimensional signal constellations in a variety of fading environments. More recently, [11] provided an accurate analysis for six 16-ary signal constellations with EGC by using the approximate infinite series technique initially developed in [2].

This paper has the following organization. Section II illustrates the applications of two distinct frequency-domain methods in the analysis of EGC diversity systems over generalized fading channels. The generality of the CHF method based on Parseval's theorem is highlighted in Section II-A by explicitly deriving the FT of the CEP for a broad range of modulation schemes. A thorough discussion on another variation of CHF method (that exploits a desirable exponential integral representation for Gaussian probability integral) is presented in Section II-B. While the latter approach tends to yield considerably simpler and computationally more efficient formulas, its application, however, is restricted to coherent binary signaling schemes. The derivation of closed-form average bit-error probability expressions for CPSK and CFSK in conjunction with EGC on Rayleigh and Nakagami-m fading environments are outlined in Section III. Selected numerical results are also presented in Section IV. Finally, the main points are summarized in Section V.

II. Error Probability Analysis

In an EGC combiner (see Fig. 1), the output of different diversity branches are first co-phased and weighted equally before being summed to give the resultant output. The instantaneous SNR at the output of the EGC combiner is $\gamma = x^2$, where x is defined as

$$x = \sqrt{\frac{E_s}{LN_0}} \sum_{l=1}^{L} \alpha_l \qquad (1)$$

for which α_l is the fading amplitude that may be modeled as a Rayleigh, Rician, Nakagami-m, or a Nakagami-q RV, E_s/N_0 is the SNR per symbol and L denotes the diversity order. Table I summarizes the CEPs for binary and M-ary signaling constellations. By recognizing the alternative exponential form for the complementary error functions, i.e., $\mathrm{erfc}(\sqrt{\gamma}) = (2/\pi)\int_0^{\pi/2} \exp(-\gamma \csc^2\theta)\,d\theta$ [12][2] and

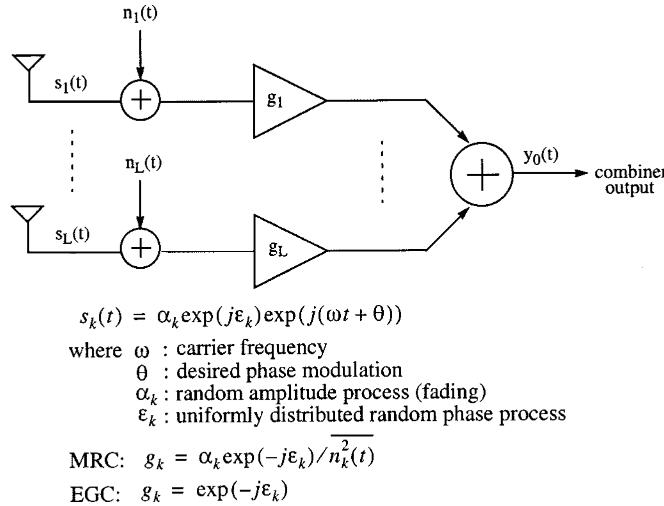

Fig. 1. Gain combining predetection diversity systems.

$\mathrm{erfc}^2(\sqrt{\gamma}) = (4/\pi)\int_0^{\pi/4} \exp(-\gamma \csc^2\theta)\,d\theta$ [14],[3] we can see that each entry in the table is a special case of the following generic form:

$$P_s(\varepsilon|x) = \sum_u \int_0^{\eta_u} a_u(\theta)\exp[-x^2 b_u(\theta)]\,d\theta \qquad (2)$$

where $x = \sqrt{\gamma}$, index u corresponds to the number of distinct exponential integrals, and both $a_u(\theta)$ and $b_u(\theta)$ are coefficients independent of γ though they may be dependent on θ. In some cases (e.g., BDPSK), $a_u(\theta) = \delta(\theta)$, where $\delta(\cdot)$ denotes the impulse function. The ASER in fading channels can be obtained by averaging the CEP over the pdf of the combined signal amplitude at the output of the EGC combiner, namely

$$P_s = \int_0^{\infty} P_s(\varepsilon|x) p_x(x)\,dx \qquad (3)$$

where $p_x(\cdot)$ denotes the pdf of RV x.

If the fading amplitudes are assumed to be independent, then the evaluation of the ASER [for the CEP given by (2)], using the classical solution in the form of (3), will require an L-fold convolution integral. This is because we can replace the L-fold average in (4)

$$\begin{aligned} P_s = \sum_u \int_o^{\eta_u} a_u(\theta) \int_0^{\infty} \cdots \int_0^{\infty} \\ \times \exp\left[-b_u(\theta)\frac{E_s}{LN_0}\left(\sum_{l=1}^{L}\alpha_l\right)^2\right] \\ \times p_{\alpha_1}(\alpha_1)\cdots p_{\alpha_L}(\alpha_L)\,d\alpha_1\cdots d\alpha_L\,d\theta \qquad (4) \end{aligned}$$

with a single average over x. Finding this $p_x(\cdot)$ (i.e., determining the convolution of the pdfs of α_l) can be very tedious and complicated, particularly for large L. Also, notice that we cannot partition the L-fold integral illustrated in (4) into a product of one-dimensional integrals. This is possible for MRC (e.g., [10]) but not for EGC because of the presence of the $\alpha_l\alpha_k$ cross-product terms. As such, it is more insightful if we transform the

[2]This alternative form may also be obtained from [13, eq. (7.4.11)].

[3]This form can be obtained using [13, eqs. (7.4.12), (7.4.11)].

TABLE I
INSTANTANEOUS SER OF SEVERAL COMMON MODULATION SCHEMES

Modulation Scheme	Conditional Error Probability $P_S(\varepsilon\vert\gamma)$
Coherent binary signalling:	
(a) Coherent PSK	$0.5\,\mathrm{erfc}(\sqrt{\gamma})$
(b) Coherent detection of differentially encoded PSK	$\mathrm{erfc}(\sqrt{\gamma})-\frac{1}{2}\mathrm{erfc}^2(\sqrt{\gamma})$
(c) Coherent FSK	$0.5\,\mathrm{erfc}(\sqrt{\gamma/2})$
Noncoherent binary signalling:	
(a) DPSK	$0.5\exp(-\gamma)$
(b) Noncoherent FSK	$0.5\exp(-\gamma/2)$
Quadrature signalling:	
(a) QPSK	$\mathrm{erfc}(\sqrt{\gamma/2})-0.25\,\mathrm{erfc}^2(\sqrt{\gamma/2})$
(b) MSK	$\mathrm{erfc}(\sqrt{\gamma/2})-0.25\,\mathrm{erfc}^2(\sqrt{\gamma/2})$
(c) π/4-DQPSK with Gray coding	$\frac{1}{2\pi}\int_0^{\pi}\frac{\exp(-\gamma(2-\sqrt{2}\cos\theta))}{\sqrt{2}-\cos\theta}d\theta=\frac{1}{2\pi}\int_0^{\pi}\exp\left(\frac{-2\gamma}{2-\sqrt{2}\cos\theta}\right)d\theta$
Multilevel signalling:	
(a) Square QAM	$2q\,\mathrm{erfc}(\sqrt{p\gamma})-q^2\mathrm{erfc}^2(\sqrt{p\gamma})$ where $q=1-1/\sqrt{M}$ and $p=1.5/(M-1)$
(b) MPSK	$\frac{1}{\pi}\int_0^{\pi-\pi/M}\exp\left(\frac{-\gamma\sin^2(\pi/M)}{\sin^2\theta}\right)d\theta$
(c) MDPSK [23]	$\frac{\sin(\pi/M)}{\pi}\int_0^{\pi/2}\frac{\exp(-\gamma[1-\cos(\pi/M)\cos\theta])}{1-\cos(\pi/M)\cos\theta}d\theta$ or $\frac{1}{\pi}\int_0^{\pi-\pi/M}\exp\left(\frac{-\gamma\sin^2(\pi/M)}{1+\cos(\pi/M)\cos\theta}\right)d\theta$
(d) Two-dimension M-ary signal constellations [11, 12]	$\frac{1}{2\pi}\sum_{u=1}^{N}Pr(S_u)\int_0^{\eta_u}\exp\left(\frac{-\gamma\kappa_u\sin^2(\Psi_u)}{\sin^2(\theta+\Psi_u)}\right)d\theta$ where N is the number of signal points, and $Pr(S_u)$ is the *a priori* probability that the uth signal point is transmitted.

$p_x(\cdot)$ into frequency domain, since the CHF of x (i.e., sum of L fading amplitudes) is simply the product of the individual CHFs. However, it is difficult (in general) to invert $\phi_x(\cdot)$ in order to get the pdf of x in a closed form. Previously, it was for this reason that a Fourier series approach was used [8], [9].

A. CHF Method with Parseval's Theorem

By contrast, applying Parseval's theorem [15, p. 371] to the product integral (3) directly leads to

$$P_s=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{FT}[P_s(\varepsilon|x)]\phi_x^*(\omega)\,d\omega \tag{5}$$

where notation $\phi_x^*(\cdot)$ denotes the complex conjugate of the CHF of x. In fact, (5) can also be obtained by using the inverse FT representation of the pdf, then rearranging the order of integration. It should be pointed out that the use of (5) circumvents the need to find the pdf of x. But, that being the case, we also need $\phi_x(\cdot)$ and the FT of $P_s(\varepsilon|x)$. Fortunately, this turns out to be very easily computed. The CHF of x is given by

$$\phi_x(\omega)=\prod_{l=1}^{L}\phi_{\alpha_l}\left(\omega\sqrt{\frac{E_s}{LN_0}}\right)=\prod_{l=1}^{L}\phi_{\upsilon_l}\left(\frac{\omega}{\sqrt{L}}\right) \tag{6}$$

where $\upsilon_l=\alpha_l\sqrt{E_s/N_0}$ and $\phi_\upsilon(\cdot)$ for all common fading environments are tabulated in Table II. Alternative expressions for the CHF in Rician and Nakagami-q channels are derived in Appendix A. Next, we will focus on the derivation of the FT of the CEP for various modulation formats. The FT of the generic CEP [i.e., (2)] is given by [8], [16]

$$\begin{aligned}G(\omega)&=\sum_u\int_o^{\eta_u}a_u(\theta)\int_0^{\infty}\exp\{-x^2b_u(\theta)+j\omega x\}\,dx\,d\theta\\&=\frac{1}{2}\sum_u\int_o^{\eta_u}\frac{a_u(\theta)}{b_u(\theta)}\left\{\sqrt{\pi b_u(\theta)}\exp\left(\frac{-\omega^2}{4b_u(\theta)}\right)\right.\\&\qquad\left.+j\omega\Phi\left(1,\,\frac{3}{2};\,\frac{-\omega^2}{4b_u(\theta)}\right)\right\}d\theta\end{aligned} \tag{7}$$

where $\Phi(a,\,b;\,c)$ is the confluent hypergeometric function of the first kind, defined as

$$\Phi(a,\,b;\,c)=\sum_{n=1}^{\infty}\frac{(a)_n}{(b)_n}\frac{c^n}{n!} \tag{8}$$

TABLE II
PDF AND CHF OF FADING AMPLITUDE FOR SEVERAL FADING CHANNEL MODELS

Channel Model	PDF and CHF of fading amplitude of the l-th branch $\upsilon_l = \alpha_l\sqrt{E_s/N_0}$
Rayleigh	PDF: $f_{\upsilon_l}(z) = \frac{2z}{\bar{\gamma}_l}\exp\left(\frac{-z^2}{\bar{\gamma}_l}\right)$, $z \geq 0$ where $\bar{\gamma}_l = \frac{E_s}{N_0}E[\alpha_l^2]$ = average SNR per symbol CHF: $\phi_{\upsilon_l}(\omega) = \Phi\left(1;\frac{1}{2};\frac{-\bar{\gamma}_l\omega^2}{4}\right) + j\omega\sqrt{\frac{\pi\bar{\gamma}_l}{4}}\exp\left(\frac{-\bar{\gamma}_l\omega^2}{4}\right)$ [2][5]
Rician	PDF: $f_{\upsilon_l}(z) = \frac{2(1+K_l)z}{\bar{\gamma}_l}\exp\left(-K_l - \frac{(1+K_l)z^2}{\bar{\gamma}_l}\right)I_0\left(2z\sqrt{\frac{K_l(K_l+1)}{\bar{\gamma}_l}}\right)$, $z \geq 0$ where Rice factor $K_l \geq 0$ CHF: $\phi_{\upsilon_l}(\omega) = \exp(-K_l)\sum_{i=0}^{\infty}\frac{K_l^i}{i!}\Phi\left(i+1;\frac{1}{2};\frac{-\bar{\gamma}_l\omega^2}{4(1+K_l)}\right)$ $+ j\omega\sqrt{\frac{\bar{\gamma}_l}{1+K_l}}\exp(-K_l)\sum_{i=0}^{\infty}\frac{\Gamma(i+3/2)K_l^i}{(i!)^2}\Phi\left(i+\frac{3}{2};\frac{3}{2};\frac{-\bar{\gamma}_l\omega^2}{4(1+K_l)}\right)$ [9]
Nakagami-q $(-1 \leq b \leq 1)$	PDF: $f_{\upsilon_l}(z) = \frac{2z}{\bar{\gamma}_l\sqrt{1-b_l^2}}\exp\left(\frac{-z^2}{[1-b_l^2]\bar{\gamma}_l}\right)I_0\left(\frac{b_l z^2}{[1-b_l^2]\bar{\gamma}_l}\right)$, $z \geq 0$ [24] where $b_l = (1-q_l^2)/(1+q_l^2)$ and the fading parameter $0 \leq q_l \leq \infty$ CHF: $\phi_{\upsilon_l}(\omega) = \sqrt{1-b_l^2}\sum_{i=0}^{\infty}\frac{(b_l/2)^{2i}\Gamma(2i+1)}{(i!)^2}\Phi\left(2i+1;\frac{1}{2};\frac{-\bar{\gamma}_l[1-b_l^2]\omega^2}{4}\right)$ $+ j\omega\sqrt{\bar{\gamma}_l}[1-b_l^2]\sum_{i=0}^{\infty}\frac{(b_l/2)^{2i}\Gamma(2i+3/2)}{(i!)^2}\Phi\left(2i+\frac{3}{2};\frac{3}{2};\frac{-\bar{\gamma}_l[1-b_l^2]\omega^2}{4}\right)$
Nakagami-m	PDF: $f_{\upsilon_l}(z) = \frac{2}{\Gamma(m_l)}\left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l}z^{2m_l-1}\exp\left(\frac{-m_l z^2}{\bar{\gamma}_l}\right)$, $z \geq 0$ [21] where Nakagami-m fading parameter $m_l \geq 0.5$ CHF: $\phi_{\upsilon_l}(\omega) = \Phi\left(m_l, \frac{1}{2}; \frac{-\bar{\gamma}_l\omega^2}{4m_l}\right) + j\omega\frac{\Gamma(m_l+1/2)}{\Gamma(m_l)}\sqrt{\frac{\bar{\gamma}_l}{m_l}}\Phi\left(m_l+\frac{1}{2};\frac{3}{2};\frac{-\bar{\gamma}_l\omega^2}{4m_l}\right)$ [4][8]

and $(a)_n = a(a+1)\cdots(a+n+1) = \Gamma(a+n)/\Gamma(a)$ is the Pochhammer symbol. The confluent hypergeometric function can be computed efficiently using a convergent series for small arguments and via a divergent expansion for large arguments [4].

Substituting (6) and (7) into (5), while realizing that the imaginary part of this integral is zero (since the ASER is real), we get an analytical ASER expression for binary and M-ary modulation formats with predetection EGC

$$\begin{aligned} P_s &= \frac{1}{\pi}\int_0^\infty \mathrm{Real}\{G(\omega)\phi_x^*(\omega)\}\,d\omega \\ &= \frac{1}{\pi}\int_0^\infty \mathrm{Real}\{G^*(\omega)\phi_x(\omega)\}\,d\omega \\ &= \frac{2}{\pi}\int_0^{\pi/2}\frac{\Psi(\tan\zeta)}{\sin(2\zeta)}\,d\zeta \end{aligned} \tag{9}$$

where $\Psi(\omega) = \mathrm{Real}\{\omega G(\omega)\phi_x^*(\omega)\}$. The representation of (9) is exact. The last integral in (9) can be estimated accurately by using a Gauss–Chebyshev quadrature (GCQ) formula [13, eq. (25.4.38)] to obtain a rapidly converging series expression for the EGC receiver performance over generalized fading channels [4]. Also, note that the evaluation of (9) for the most general case involves two-fold integrals. Now, we will identify four special cases of the CEP $P_s(\varepsilon|x)$ that allow the evaluation of the generic expression given in (9) to be further simplified into a single finite-range integral. This simplification is attributable to the availability of closed-form formulas for the FT of $P_s(\varepsilon|x)$.

1) Exponential Form: $P_s(\varepsilon|x) = a\exp(-bx^2)$: The instantaneous BER of some noncoherent binary modulation schemes (e.g., DPSK and NCFSK) can be expressed in the exponential form. In this case, the FT of $a\exp(-bx^2)$ is given by

$$G(\omega) = \frac{a}{\sqrt{b}}\left[\frac{\sqrt{\pi}}{2}\exp\left(\frac{-\omega^2}{4b}\right) + jF\left(\frac{w}{2\sqrt{b}}\right)\right] \tag{10}$$

where $F(\cdot)$ denotes the Dawson's integral:

$$F(x) = \exp(-x^2)\int_0^x \exp(t^2)\,dt = x\Phi\left(1, \frac{3}{2}, -x^2\right). \tag{11}$$

The corresponding ASER is obtained by substituting (10) into (9).

2) Complementary Error Function: $P_s(\varepsilon|x) = a\mathrm{erfc}(\sqrt{b}x)$: The CEP of some coherent binary modulation schemes (e.g., CPSK and CFSK) are in the form of a complementary error function. The FT of $a\mathrm{erfc}(\sqrt{b}x)$ can be shown as

$$G(\omega) = \frac{a}{\omega}\left\{\frac{2}{\sqrt{\pi}}F\left(\frac{\omega}{2\sqrt{b}}\right) + j\left[1 - \exp\left(\frac{-\omega^2}{4b}\right)\right]\right\}. \tag{12}$$

TABLE III
PARAMETERS a AND b FOR DIFFERENT BINARY MODULATION FORMATS AND DETECTION SCHEMES

b \ a	1/2	1
1/2	Orthogonal CFSK	Orthogonal NCFSK
1	Antipodal CPSK	Antipodal DPSK

Therefore, the ASER can be computed efficiently by substituting (12) into (9).

3) $P_s(\varepsilon|x) = a\mathrm{erfc}(\sqrt{b}x) - c\mathrm{erfc}^2(\sqrt{b}x)$*:* The instantaneous SER for QPSK, MQAM, and coherent detection of differentially encoded PSK can be expressed in the form $a\mathrm{erfc}(\sqrt{b}x) - c\mathrm{erfc}^2(\sqrt{b}x)$. Utilizing the results of (12) and recognizing that the FT of the $c\mathrm{erfc}^2(\sqrt{b}x)$ term can be derived using integration by parts, we get [4]

$$G(\omega) = \frac{2a}{\omega\sqrt{\pi}} F\left(\frac{\omega}{2\sqrt{b}}\right) - \frac{4c}{\omega\sqrt{\pi}} \times \left[F\left(\frac{\omega}{2\sqrt{b}}\right) - F\left(\frac{\omega}{2\sqrt{2b}}\right)\exp\left(\frac{-\omega^2}{8b}\right)\right] + j\left\{\frac{a}{\omega}\left[1-\exp\left(\frac{-\omega^2}{4b}\right)\right] - \frac{c}{\omega}\left[1-\exp\left(\frac{-\omega^2}{4b}\right) - \frac{4}{\pi}F^2\left(\frac{\omega}{2\sqrt{2b}}\right)\right]\right\}. \tag{13}$$

4) $P_s(\varepsilon|x) = \Gamma(a, bx^2)/2\Gamma(a)$*:* A unified BER expression for coherent, differentially coherent, and noncoherent detection of binary signals transmitted over an AWGN channel is presented in [18] (see Table III)

$$P_b(\varepsilon|x) = \frac{\Gamma(a, bx^2)}{2\Gamma(a)} \tag{14}$$

where $\Gamma(\cdot, \cdot)$ denotes the complementary incomplete Gamma function and $\Gamma(\cdot)$ is the Gamma function. Using identity [19, eq. (6.452.1)], with some additional algebraic manipulations, the FT of $P_b(\varepsilon|x)$ can be shown as

$$G(\omega) = \frac{j}{\omega}\left[\frac{1}{2} - \frac{\Gamma(2a)}{\Gamma(a)2^a}\exp\left(\frac{-\omega^2}{8b}\right)D_{-2a}\left(\frac{-j\omega}{\sqrt{2b}}\right)\right] \tag{15}$$

where $D_{-v}(\,)$ denotes the parabolic cylinder function, or alternatively

$$G(\omega) = \frac{\Gamma(2a)}{[\Gamma(a)]^2 2^{2a}}\sqrt{\frac{\pi}{b}}\,\Phi\left(\frac{1}{2}+a,\, \frac{3}{2};\, \frac{-\omega^2}{4b}\right) + \frac{j}{\omega}\left[\frac{1}{2} - \frac{\Gamma(2a)\sqrt{\pi}}{\Gamma(a)\Gamma(a+1/2)2^{2a}}\,\Phi\left(a,\, \frac{1}{2};\, \frac{-\omega^2}{4b}\right)\right]. \tag{16}$$

For the special cases of $a = 1$ and $a = 1/2$, (16) agrees with (10) and (12), respectively. Thus, substituting (16) into (9), we obtain a unified ABER expression for all the binary modulation schemes employing EGC treated in [8] and [9].

In addition to the forms listed in Sections II-A-1–II-A-4, we can also increase the computational efficiency of (9) if $a_u(\theta) = \kappa_1$ and $b_u(\theta)$ in (7) is either in the form of $\kappa_2/\sin^2(\theta+\kappa_3)$ or $\kappa_2/\cos^2(\theta+\kappa_3)$, where κ_1, κ_2 and κ_3 are constants (independent of θ). This is because $\mathrm{Real}\{G(\omega)\}$ can be expressed in closed form (see Appendix B). For instance, the performance of MPSK with predetection EGC can be evaluated using (17) in conjunction with (9), instead of directly applying (7)

$$G(\omega) = \frac{1}{\sqrt{\pi}\omega}\left\{F\left(\frac{\omega}{2\sqrt{\kappa_2}}\right) - \exp\left(\frac{-\omega^2}{4\kappa_2}\sin^2(\eta)\right) \times F\left(\frac{\omega\cos(\eta)}{2\sqrt{\kappa_2}}\right)\right\} + j\omega\frac{1}{2\pi}\int_0^{\eta}\frac{\sin^2(\theta)}{\kappa_2} \times \Phi\left(1,\, \frac{3}{2};\, \frac{-\omega^2\sin^2(\theta)}{4\kappa_2}\right)d\theta \tag{17}$$

where $\eta = \pi - \pi/M$ and $\kappa_2 = \sin^2(\pi/M)$. For $M = 2$, (17) reduces to (12). By equating the imaginary part, we get the following integral identity:

$$\int_0^{\pi/2}\sin^2(\theta)\Phi\left(1,\, \frac{3}{2};\, -\beta\sin^2(\theta)\right)d\theta = \frac{\pi}{4\beta}[1-\exp(-\beta)].$$

B. CHF Method with a Desirable Exponential Integral Form for $Q(x)$

In the preceding section, the average error rate calculations always involve a two-step procedure [see (9)]. First, the FT of the CEP need to be determined. Subsequently, replacing the complex conjugate of the CHF into (9), we get the desired results in terms of a single finite-range integral. However, for the special case of coherent binary signaling, the requirement to find the FT of the CEP is no longer necessary because the Gaussian probability integral (or the complementary error function) has a desirable exponential integral form.

Since $Q(x)$ is the tail probability of a zero-mean, unit variance of the Gaussian random variable (GRV) exceeds x and the CHF of GRV is $\phi(t) = \exp(-t^2/2)$, we immediately get[4]

$$Q(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty}\frac{1}{t}e^{-t^2/2}\sin(tx)\,dt \tag{18}$$

by invoking the Gil–Pelaez inversion theorem (Fourier inversion formula) [25]. This form is suitable (i.e., in a desirable exponential form) for performing an average over the distribution of the fading amplitudes[5] because the integrand can be written in a product form, which facilitates averaging over the individual statistical distributions of the α_l's and then perform integration over t. Hence, an exact analytical expression for the coherent binary signaling schemes with EGC is given by

$$\begin{aligned} P_b &= \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} t^{-1}e^{-t^2/2}\mathrm{Imag}\{\phi_x(2\sqrt{a}t\}\,dt \\ &= \frac{1}{2} - \frac{1}{2\pi}\int_0^{\infty} t^{-1}e^{-t}\mathrm{Imag}\{\phi_x(2\sqrt{at}\}\,dt \end{aligned} \tag{19}$$

where $a = 1$ for CPSK and $a = 1/2$ for CFSK.

[4]This particular form may be obtained using identities [19, eqs. (3.952.6), (8.253.1)] after some algebraic manipulations.

[5]Using $\phi(t)$ for the GRV in conjunction with the Laplace inversion formula, we can derive Craig's representation of $Q(x)$, which is in a desirable exponential form suitable for the analysis of MRC systems [26].

Since $\phi_x(2\sqrt{a}t) = \int_0^\infty e^{j2\sqrt{a}t\omega} p_x(\omega)\, d\omega$ (by definition), we can show that

$$\lim_{t\to 0} \frac{1}{t}\,\mathrm{Imag}\{\phi_x(2\sqrt{a}t)\} = \lim_{t\to 0} \int_0^\infty \frac{\sin(2\sqrt{a}t\omega)}{t}\, p_x(\omega)\, d\omega = \int_0^\infty 2\sqrt{a}\omega p_x(\omega)\, d\omega = 2\sqrt{a}E[x]$$

where $E[x]$ is the mean value of x. The above observation suggests that the first integral in (19) is well behaved, even as $t \to 0$ (i.e., no singularity at $t = 0$), and is therefore suitable for numerical integration. Using variable substitution $t = \tan\theta$, we obtain

$$P_b = \frac{1}{2} - \frac{2}{\pi} \int_0^{\pi/2} \frac{\exp(-0.5\tan^2\theta)}{\sin 2\theta} \times \mathrm{Imag}\{\phi_x(2\sqrt{a}\tan\theta)\}\, d\theta = \frac{1}{2} - \frac{1}{\pi} \int_0^{\pi/2} \frac{\exp(-\tan\theta)}{\sin 2\theta}\,\mathrm{Imag}\{\phi_x(2\sqrt{a\tan\theta})\}\, d\theta. \quad (20)$$

Since (20) is considerably simpler than (9), it is recommended for the error-rate calculations of coherent binary modulation schemes (CPSK and CFSK).

III. Closed-Form Solutions in Rayleigh and Nakagami Fading

In a related work, [3] formulated the problem of computing the ABER for coherent binary PSK with EGC diversity in the framework of statistical decision theory, and was then able to obtain closed-form solutions for $L \leq 3$ in a Rayleigh fading channel. His solution to binary noncoherent FSK and DPSK systems in Rayleigh fading is restricted to a second-order diversity. By contrast, in this section, we derive some closed-form formulas for the ABER with EGC in a Nakagami-m fading channel via the CHF methods (Sections II-A and II-B). By utilizing identity (C.1), it is possible to derive closed-form solutions for coherent, differentially coherent, and noncoherent binary modulation schemes for any $L \geq 1$ in terms of the Appell hypergeometric function. However, in this paper, we shall restrict our attention to the practically important cases of second- and third-order diversity. Our results are sufficiently general to handle arbitrary fading parameters and dissimilar branch powers. In some cases (if the fading severity indexes $m_l$are restricted to positive integers or are multiples of half of odd integers), the Appell function can be further simplified into either a finite sum of Gauss hypergeometric series or a finite polynomial.

The results in Section III- A are based on the following observation. When evaluating (19) for an L-order diversity system in Nakagami-m fading, the imaginary part of $\phi_x(\cdot)$ is a sum of 2^{L-1} terms, each of which is a product of L confluent hypergeometric functions (see the last entry in Table II). Therefore, we need to solve the generic integral

$$I = \int_0^\infty x^{\nu-1} e^{-bx} \prod_{k=1}^{n} \Phi(a_k;\, b_k;\, c_k x)\, dx. \quad (21)$$

Some solutions to this are provided in Appendix C.

A. Coherent Detection

Although closed-form solutions for coherent BPSK in Nakagami-m and Rayleigh fading channels can be obtained using the CHF methods outlined in either Section II-A or II-B, the derivation using the latter approach appears to be simpler. For instance, substituting (6) (using entry 4 of Table II) into (19) and using identity [19, eq. (7.621.4)], we immediately get an exact closed-form solution for the performance of binary CPSK and CFSK without diversity[6] (i.e., $L = 1$) in Nakagami-m fading with an arbitrary m

$$P_b = \frac{1}{2} - \sqrt{\frac{\overline{\gamma}}{(m\delta + \overline{\gamma})\pi}}\, \frac{\Gamma(m+1/2)}{\Gamma(m)} \times {}_2F_1\left(1-m,\, \frac{1}{2};\, \frac{3}{2};\, \frac{\overline{\gamma}}{m\delta+\overline{\gamma}}\right) \quad (22)$$

where $\delta = 1$ for BPSK, $\delta = 2$ for BFSK, and ${}_2F_1(\cdot,\,\cdot;\,\cdot;\,\cdot)$ is the Gauss hypergeometric function

$${}_2F_1(a;\, b;\, c;\, z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n}\, \frac{z^n}{n!}, \qquad |z| < 1. \quad (23)$$

Furthermore, if a or b (or both) in (23) is either zero or a negative integer, the series is finite and thus converges for all z. Consequently, (22) reduces to a finite polynomial for integer m since

$${}_2F_1(-w; b; c; z) = \sum_{n=0}^{w} \frac{(-w)_n (b)_n}{(c)_n}\, \frac{z^n}{n!}, \qquad w = 0,\, 1,\, 2\cdots. \quad (24)$$

The simplicity of our derivation should be compared to a rather long derivation, of an equivalent result, found in Appendix A of [20]. In addition to this, when $m = 1/2$, (22) may be simplified into

$$P_b = \frac{1}{2} - \frac{1}{\pi}\sin^{-1}\left(\sqrt{\frac{2\overline{\gamma}}{\delta + 2\overline{\gamma}}}\right) \quad (25)$$

since ${}_2F_1(1/2,\, 1/2;\, 3/2;\, z^2) = (\sin^{-1} z)/z$.

1) Case $L = 2$: Similarly, for $L = 2$, we need to evaluate

$$P_b = \frac{1}{2} - \frac{1}{2\pi} \int_0^\infty t^{-1} e^{-t} \times \left[A_1\left(2\sqrt{t/\delta},\, 2\right) B_2\left(2\sqrt{t/\delta},\, 2\right) + A_2\left(2\sqrt{t/\delta},\, 2\right) B_1\left(2\sqrt{t/\delta},\, 2\right)\right] dt \quad (26)$$

where $A_k(\omega,\, L) = \Phi(m_k,\, (1/2);\, (-\omega^2\overline{\gamma}_k/4Lm_k))$ and

$$B_k(\omega,\, L) = \omega \frac{\Gamma(m_k + 1/2)}{\Gamma(m_k)} \sqrt{\frac{\overline{\gamma}_k}{Lm_k}}\, \Phi\left(m_k + \frac{1}{2},\, \frac{3}{2};\, \frac{-\omega^2\overline{\gamma}_k}{4Lm_k}\right).$$

[6]Equation (22) can also be used to analyze EGC receiver performance (approximate analysis) with independent and identically distributed (i.i.d.) diversity branches by replacing and $m \to mL$ and $\overline{\gamma} \to \overline{\gamma} + (L-1)\overline{\gamma}\Gamma^2(m + 1/2)/m\Gamma^2(m) \approx L\overline{\gamma}(1 - 1/5m)$ as suggested by Nakagami [21]. However, we would like to highlight that the application of this approximate analysis is quite limited because in practice i.i.d. branches are rarely available and the accuracy of the approximation is inadequate for small values of m. Details of the EGC approximate analysis can be found in [27], which also treats the unequal signal strength scenario.

It is apparent that we need to find solution to the integrals in the form

$$I[m_1, a_1, m_2, a_2] = \int_0^\infty t^{-1/2} e^{-(1+a_1+a_2)t} \Phi\left(\frac{1}{2} - m_1, \frac{1}{2}; a_1 t\right) \times \Phi\left(1 - m_2, \frac{3}{2}; a_2 t\right) dt \qquad (27)$$

after applying Kummer's transformation formula [19, eq. (9.212.1)] to $A_k(\omega, L)$ and $B_k(\omega, L)$. This integral can be evaluated using (C.1), i.e.,

$$I[m_1, a_1, m_2, a_2] = \sqrt{\frac{\pi}{1+a_1+a_2}} F_2\left(\frac{1}{2}; \frac{1}{2} - m_1, 1 - m_2; \frac{1}{2}, \frac{3}{2}; \frac{a_1}{1+a_1+a_2}, \frac{a_2}{1+a_1+a_2}\right) \qquad (28)$$

which holds for any real $m_k \geq 1/2$, $k \in \{1, 2\}$. Therefore, an exact closed-form solution to (26) (expressed in terms of the Appell hypergeometric function of the second kind) is given by

$$P_b = \frac{1}{2} - \sum_{k=1, l \neq k}^{2} \frac{\Gamma(m_l + 1/2)}{\pi \Gamma(m_l)} \sqrt{\frac{\overline{\gamma}_l}{2 m_l \delta}} \times I\left[m_k, \frac{\overline{\gamma}_k}{2 m_k \delta}, m_l, \frac{\overline{\gamma}_l}{2 m_l \delta}\right]. \qquad (29)$$

Now, we will show that (29) can be further simplified into finite polynomials, if both $m_k (k = 1, 2)$ are assumed to be either positive integers or alternatively, multiples of halved odd integers. This is because $F_2(\cdot; (1/2) - m_i, 1 - m_j; \cdot, \cdot; \cdot, \cdot)$ reduces to a finite series if $m_i = (1/2), (3/2), \ldots,$ or m_j is a positive integer (or both). So, the development of (31) and (33) relies on these observations.

a) If both m_1 and m_2 are assumed to be positive integers, then (29) reduces to

$$P_b = \frac{1}{2} - I_2(1, 2, \delta) - I_2(2, 1, \delta) \qquad (30)$$

where

$$I_2(x, y, \delta) = \frac{\Gamma(m_y + 1/2)}{\pi \Gamma(m_y)} \sum_{k=0}^{m_y - 1} \binom{m_y - 1}{k} \frac{\Gamma(k + 1/2)}{(2k+1)!!} (-2)^k \times \left[\frac{\overline{\gamma}_y m_x}{\overline{\gamma}_y m_x + \overline{\gamma}_x m_y + 2 m_x m_y \delta}\right]^{k + (1/2)} \times {}_2F_1\left(\frac{1}{2} - m_x, k + \frac{1}{2}; \frac{1}{2}; \frac{\overline{\gamma}_x m_y}{\overline{\gamma}_y m_x + \overline{\gamma}_x m_y + 2 m_x m_y \delta}\right). \qquad (31)$$

Also, note that ${}_2F_1(\cdots)$ in (31) can be replaced by a finite polynomial by applying Kummer's transformation formula [19, eq. (9.131.1)] and then using (24). Furthermore, by substituting $m_1 = m_2 = 1$ in (30) and recognizing that ${}_2F_1(a, b; b; x) = (1 - x)^{-a}$, we get

$$P_b = \frac{1}{2} - \frac{1}{2}\left[\frac{\sqrt{\overline{\gamma}_1(\overline{\gamma}_1 + 2\delta)} + \sqrt{\overline{\gamma}_2(\overline{\gamma}_2 + 2\delta)}}{\overline{\gamma}_1 + \overline{\gamma}_2 + 2\delta}\right] \qquad (32)$$

which is identical to [3, eq. (23)].

b) If both m_1 and m_2 are assumed to be multiples of halved odd integers, then (29) can also be expressed in a closed form similar to (30). As a result, $I_2(x, y, \delta)$ is now given by

$$I_2(x, y, \delta) = \frac{\Gamma(m_y + 1/2)}{\pi \Gamma(m_y)} \sqrt{\frac{\overline{\gamma}_y}{2 m_y \delta}} \sum_{k=0}^{m_x - 1/2} \times \binom{m_x - 1/2}{k} \frac{\Gamma(k + 1/2)}{(2k-1)!!} \left(\frac{-\overline{\gamma}_x}{m_x \delta}\right)^k \times \left[\frac{2 m_x m_y \delta}{\overline{\gamma}_y m_x + \overline{\gamma}_x m_y + 2 m_x m_y \delta}\right]^{k + (1/2)} \times {}_2F_1\left(1 - m_y, k + \frac{1}{2}; \frac{3}{2}; \frac{\overline{\gamma}_y m_x}{\overline{\gamma}_y m_x + \overline{\gamma}_x m_y + 2 m_x m_y \delta}\right). \qquad (33)$$

It is apparent that ${}_2F_1(\cdot, \cdot; \cdot;,)$ in (33) reduces to a finite polynomial for $k \geq 1$ from the definition of Gauss hypergeometric function. For the particular case of $m_1 = m_2 = 1/2$, we can show that the ABER for binary CPSK and CFSK with second-order EGC diversity is given by

$$P_b = \frac{1}{2} - \frac{1}{\pi} \sin^{-1}\left(\sqrt{\frac{\overline{\gamma}_x}{\overline{\gamma}_x + \overline{\gamma}_y + \delta}}\right) - \frac{1}{\pi} \sin^{-1}\left(\sqrt{\frac{\overline{\gamma}_y}{\overline{\gamma}_x + \overline{\gamma}_y + \delta}}\right). \qquad (34)$$

2) Case $L = 3$: Using (19) and (C.1), the ABER of binary CPSK and CFSK with third-order EGC diversity is given by

$$P_b = \frac{1}{2} + I_{3a}(1, 2, 3, \delta) - [I_{3b}(1, 2, 3, \delta) + I_{3b}(1, 3, 2, \delta) + I_{3b}(2, 3, 1, \delta)] \qquad (35)$$

where

$$I_{3a}(x, y, z, \delta) = \frac{1}{2\pi} \int_0^\infty t^{-1} e^{-t} B_x\left(2\sqrt{t/\delta}, 3\right) \times B_y\left(2\sqrt{t/\delta}, 3\right) B_z\left(2\sqrt{t/\delta}, 3\right) dt = \frac{2\Gamma(m_x + 1/2)\Gamma(m_y + 1/2)\Gamma(m_z + 1/2)}{\sqrt{\pi}\Gamma(m_x)\Gamma(m_y)\Gamma(m_z)\lambda^{3/2}} \sqrt{\zeta_x \zeta_y \zeta_z} \times F_A\left(\frac{3}{2};, 1 - m_x, 1 - m_y, 1 - m_z; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; \frac{\zeta_x}{\lambda}, \frac{\zeta_y}{\lambda}, \frac{\zeta_z}{\lambda}\right) \qquad (36)$$

and

$$I_{3b}(x, y, z, \delta) = \frac{1}{2\pi}\int_0^\infty t^{-1}e^{-t}A_x\left(2\sqrt{t/\delta}, 3\right) \times A_y\left(2\sqrt{t/\delta}, 3\right)B_z\left(2\sqrt{t/\delta}, 3\right)dt$$
$$= \frac{\Gamma(m_z+1/2)}{\Gamma(m_z)}\sqrt{\frac{\zeta_z}{\pi\lambda}} \times F_A\left(\frac{1}{2}; \frac{1}{2}-m_x, \frac{1}{2}-m_y, 1-m_z; \frac{1}{2}, \frac{1}{2}, \frac{3}{2}; \frac{\zeta_x}{\lambda}, \frac{\zeta_y}{\lambda}, \frac{\zeta_z}{\lambda}\right) \quad (37)$$

which are valid for any real $m_k \geq 1/2$, $k \in \{1, 2, 3\}$, where $\lambda = 1+\zeta_x+\zeta_y+\zeta_z$, for which $\zeta_k = \overline{\gamma}_k/(3m_k\delta)$.

Using the above idea, one may also derive closed-form formulas for EGC in Nakagami-m fading for any value of L using (C.1). Besides binary modulation schemes, the method also applies to QPSK and M-ary QAM, given that $G(\omega)$ can be expressed in terms of the confluent hypergeometric series. In Appendix D, we show that the Appell function in (36) and (37) may be replaced by a finite sum of the Gauss hypergeometric series or a finite polynomial for the positive integer fading severity index. However, the number of terms grows exponentially with the increasing value of m_k, $k \in \{1, 2, 3\}$.

B. Noncoherent Detection

Let us denote $C(\omega) = (1/4)\sqrt{\pi\delta}\exp((-\omega^2\delta/4))$ and $D(\omega) = (\omega\delta/4)\Phi(1, 3/2; (-\omega^2/4)$. Using the CHF method based on Parseval's theorem, we can show that the performance of binary NCFSK ($\delta = 2$) and DPSK ($\delta = 1$), without diversity reception, in Nakagami-m fading with an arbitrary m is given by

$$P_b = \frac{1}{\pi}\int_0^\infty [A_1(\omega, 1)C(\omega) + B_1(\omega, 1)D(\omega)]\,d\omega = \frac{1}{2}\left(\frac{m\delta}{m\delta+\overline{\gamma}}\right)^m \quad (38)$$

with the aid of [19, eq. (7.621.4)] and (C.4). Equation (38) is identical to [18, eq. (11)].

Now, for $L = 2$, the ABER with predetection EGC may be evaluated as

$$P_b = I_{2a}(1,2,\delta) - I_{2b}(1,2,\delta) + I_{2c}(1,2,\delta) + I_{2c}(2,1,\delta) \quad (39)$$

where

$$I_{2a}(x, y, \delta) = \frac{1}{\pi}\int_0^\infty A_x(\omega, 2)A_y(\omega, 2)C(\omega)\,d\omega = \frac{1}{2}\sqrt{\frac{\delta}{2}}m_x^{m_x}m_y^{m_y} \times \frac{(2m_y\delta+\overline{\gamma}_y)^{m_x-1/2}(2m_x\delta+\overline{\gamma}_x)^{m_y-1/2}}{(2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)^{m_x+m_y-1/2}} \times {}_2F_1\left(\frac{1}{2}-m_x, \frac{1}{2}-m_y; \frac{1}{2}; \frac{\overline{\gamma}_x\overline{\gamma}_y}{(2m_y\delta+\overline{\gamma}_y)(2m_x\delta+\overline{\gamma}_x)}\right) \quad (40)$$

see also (41), shown at the bottom of the page, and

$$I_{2c}(x, y, \delta) = \frac{1}{\pi}\int_0^\infty A_x(\omega, 2)B_y(\omega, 2)D(\omega)d\omega \quad (42)$$

with $A_i(\omega, L)$ and $B_i(\omega, L)$ defined as they are in (26). Using (C.1), we can show that

$$I_{2c}(x, y, \delta) = \frac{\delta\Gamma(m_y+1/2)}{16\Upsilon^{3/2}\Gamma(m_y)}\sqrt{\frac{\overline{\gamma}_y}{2\pi m_y}} \times F_A\left(\frac{3}{2}; \frac{1}{2}-m_x, 1-m_y, 1; \frac{1}{2}, \frac{3}{2}, \frac{3}{2}; \frac{\overline{\gamma}_x}{8m_x\Upsilon}, \frac{\overline{\gamma}_y}{8m_y\Upsilon}, \frac{-\delta}{4\Upsilon}\right) \quad (43)$$

which holds for any real $m \geq 1/2$ and $\Upsilon = (\overline{\gamma}_x/8m_x) + (\overline{\gamma}_y/8m_y)$. If m_y is a positive integer, then (42) can be restated as (using identity [8, eq. (14)])

$$I_{2c}(x, y, \delta) = \frac{\delta\Gamma(m_y+1/2)}{8\pi\Gamma(m_y)}\sqrt{\frac{\overline{\gamma}_y}{2m_y}}\sum_{k=0}^{m_y-1}\frac{1}{(2k+1)!!} \times \binom{m_y-1}{k}\left(\frac{-\overline{\gamma}_y}{4m_y}\right)^k \times I(x, y, \delta, k) \quad (44)$$

where

$$I(x, y, \delta, k) = \int_0^\infty t^{k+1/2}e^{-t\Upsilon}\Phi\left(\frac{1}{2}-m_x, \frac{1}{2}; \frac{\overline{\gamma}_xt}{8m_x}\right) \times \Phi\left(1, \frac{3}{2}; \frac{-\delta t}{4}\right)dt = \frac{\Gamma(k+3/2)}{\Upsilon^{k+3/2}} \times F_2\left(k+\frac{3}{2}; \frac{1}{2}-m_x, 1; \frac{1}{2}, \frac{3}{2}; \frac{\overline{\gamma}_x}{8m_x\Upsilon}, \frac{-\delta}{4\Upsilon}\right). \quad (45)$$

$$I_{2b}(x, y, \delta) = \frac{1}{\pi}\int_0^\infty B_x(\omega, 2)B_y(\omega, 2)C(\omega)\,d\omega = \frac{\Gamma(m_x+1/2)\Gamma(m_y+1/2)(2m_y\delta+\overline{\gamma}_y)^{m_x-1}(2m_x\delta+\overline{\gamma}_x)^{m_y-1}}{2\Gamma(m_x)\Gamma(m_y)(2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)^{m_x+m_y-1/2}} \times m_x^{m_x}m_y^{m_y}\sqrt{2\delta\overline{\gamma}_x\overline{\gamma}_y}\,{}_2F_1\left(1-m_x, 1-m_y; \frac{3}{2}; \frac{\overline{\gamma}_x\overline{\gamma}_y}{(2m_y\delta+\overline{\gamma}_y)(2m_x\delta+\overline{\gamma}_x)}\right) \quad (41)$$

The integral in (45) can be expressed as a finite sum using the procedure suggested in Appendix D. For the case $k=0$, using (C.4), we obtain (46), shown at the bottom of the page.

It should also be pointed out that for $k \geq 1$, the final result from (45) will be a sum of 2×4^k terms. Letting $m_x = m_y = 1$ (Rayleigh fading) in (44), we obtain an equivalent expression to that derived in [3] but in a slightly different form. Alternatively, by replacing $\Phi(1, 3/2; z) = [\Phi(1, 1/2; z) - 1]/2z$ [19, eq. (9.212.2)] and $k=0$ in (45), we get (47), shown at the bottom of the page, after simplification of the resultant integrals using [19, eq. (7.621.4)] and (C.5). Now, it is easy to verify that (44) reduces to the familiar expression in [3] for Rayleigh fading ($m_x = m_y = 1$). If $m_x = (1/2), (3/2), \ldots$, then (42) may be rewritten as

$$\begin{aligned} I_{2c}(x, y, \delta) &= \frac{\delta\Gamma(m_y+1/2)}{8\pi\Gamma(m_y)}\sqrt{\frac{\overline{\gamma}_y}{2m_y}}\sum_{k=0}^{m_x-1}\frac{1}{(2k-1)!!} \\ &\quad\times\binom{m_x-1/2}{k}\left(\frac{-\overline{\gamma}_x}{4m_x}\right)^k \\ &\quad\times\int_0^\infty t^{k+1/2}e^{-t((2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)/(8m_xm_y))} \\ &\quad\times\Phi\left(1-m_y, \frac{3}{2}; \frac{\overline{\gamma}_yt}{8m_y}\right)\Phi\left(\frac{1}{2}, \frac{3}{2}; \frac{\delta t}{4}\right)dt \end{aligned} \tag{48}$$

with the aid of the identity

$$\Phi\left(m, \frac{1}{2}; -a\right) = e^{-a}\sum_{k=0}^{m-1/2}\frac{(-2)^k}{(2k-1)!!}\binom{m-1/2}{k}a^k, \quad m = \frac{1}{2}, \frac{3}{2}, \ldots. \tag{49}$$

Once again, the closed-form solution for (48) may be attained using the iterative method outlined in Appendix D. In this case, the integral in (48) can be replaced by a sum of 4^k terms. In particular, (48) reduces to a strikingly simple solution for $m_x = m_y = 1/2$

$$I_{2c}(x, y, \delta) = \frac{1}{2\pi}\sqrt{\frac{\delta}{\overline{\gamma}_x+\overline{\gamma}_y+\delta}} \times\sin^{-1}\left(\sqrt{\frac{\overline{\gamma}_y\delta}{(\overline{\gamma}_x+\overline{\gamma}_y)(\overline{\gamma}_x+\delta)}}\right). \tag{50}$$

At this juncture, it is also worth noting that the ASER expression for M-ary modulation formats (e.g., MPSK) with dual-branch EGC can be reduced into a single finite-range integral in a Rayleigh fading environment and some special cases of Nakagami-m fading, owing to the availability of the closed-form solutions found in Section III-B.

IV. Numerical Results

In this section, we provide several representative numerical curves illustrating the EGC receiver performance over Rayleigh, Rician, and Nakagami-m fading channels using the analytical results derived in the preceding sections. These results are validated using a quasi-analytical (QA) simulation technique. In our simulations, each data point is obtained by generating 10^9 random samples of x, defined in (1).

Fig. 2 depicts the average error rate performance of coherent BPSK employing EGC that combines two and three i.i.d. diversity branches in a Nakagami-m environment, for several positive integer fading several indexes. Since the curves obtained using (30) (for $L=2$) and (35) (for $L=3$) coincide with their QA counterpart, this observation validates the accuracy of our derivations. Similarly, the performance of QPSK with second- and fourth-order EGC diversity in Nakagami-m fading (for real fading severity index) is plotted in Fig. 3. These curves are generated using (9), (13), and the fourth entry of

$$\begin{aligned} I(x, y, \delta, 0) &= 8m_y\sqrt{\frac{2\pi m_xm_y}{\overline{\gamma}_xm_y+\overline{\gamma}_ym_x}}\left[\frac{m_x(2m_y\delta+\overline{\gamma}_y)}{2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x}\right]^{m_x-(1/2)} \\ &\quad\times\left\{\left[\frac{2m_x^2}{2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x}\right]{}_2F_1\left(\frac{1}{2}, \frac{1}{2}-m_x; \frac{3}{2}; \frac{2\overline{\gamma}_xm_y^2\delta}{(2m_y\delta+\overline{\gamma}_y)(\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)}\right)\right. \\ &\quad\left.+\frac{1-2m_x}{2m_y\delta+\overline{\gamma}_y}{}_2F_1\left(\frac{1}{2}, \frac{3}{2}-m_x; \frac{3}{2}; \frac{2\overline{\gamma}_xm_y^2\delta}{(2m_y\delta+\overline{\gamma}_y)(\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)}\right)\right\} \end{aligned} \tag{46}$$

$$\begin{aligned} I(x, y, \delta, 0) &= \frac{4}{\delta}m_y\sqrt{\frac{2\pi m_y}{\overline{\gamma}_y}}\left(\frac{\overline{\gamma}_ym_x}{\overline{\gamma}_xm_y+\overline{\gamma}_ym_x}\right)^{m_x} - \frac{4}{\delta}\sqrt{\frac{2\pi m_y(\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)}{(2m_y\delta+\overline{\gamma}_y)(2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)}} \\ &\quad\times\left[\frac{2m_xm_y\delta+\overline{\gamma}_ym_x}{2m_xm_y\delta+\overline{\gamma}_xm_y+\overline{\gamma}_ym_x}\right]^{m_x}{}_2F_1\left(\frac{1}{2}-m_x, -\frac{1}{2}; \frac{1}{2}; \frac{2\overline{\gamma}_xm_y^2\delta}{(2m_y\delta+\overline{\gamma}_y)(\overline{\gamma}_xm_y+\overline{\gamma}_ym_x)}\right) \end{aligned} \tag{47}$$

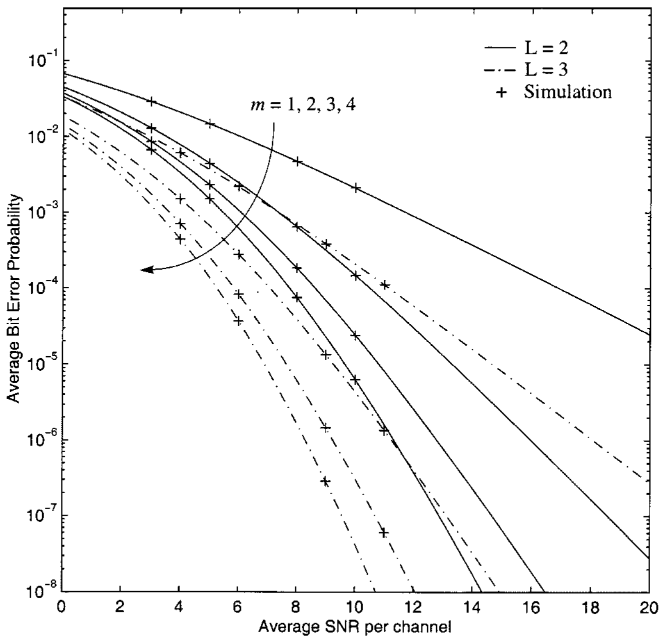

Fig. 2. Error rates for coherent BPSK employing second- and third-order EGC diversity over Nakagami-m channels.

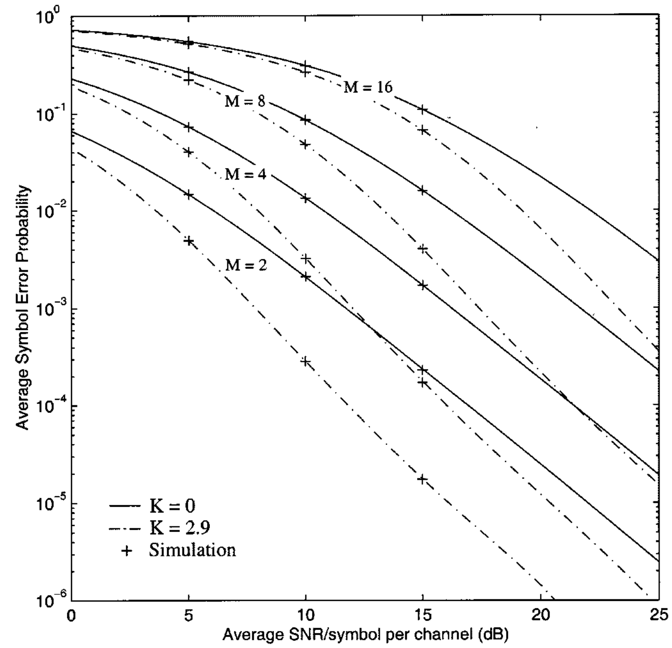

Fig. 4. Error probability performance of a dual-diversity EGC receiver for MPSK in Rayleigh ($K = 0$) and Rician ($K = 2.9$) fading channels.

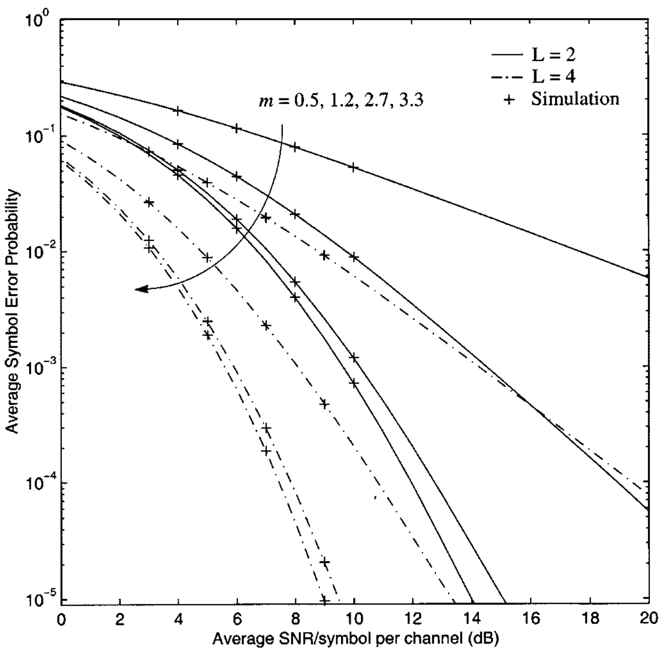

Fig. 3. Second- and fourth-order EGC receiver performance for QPSK over Nakagami-m fading channels.

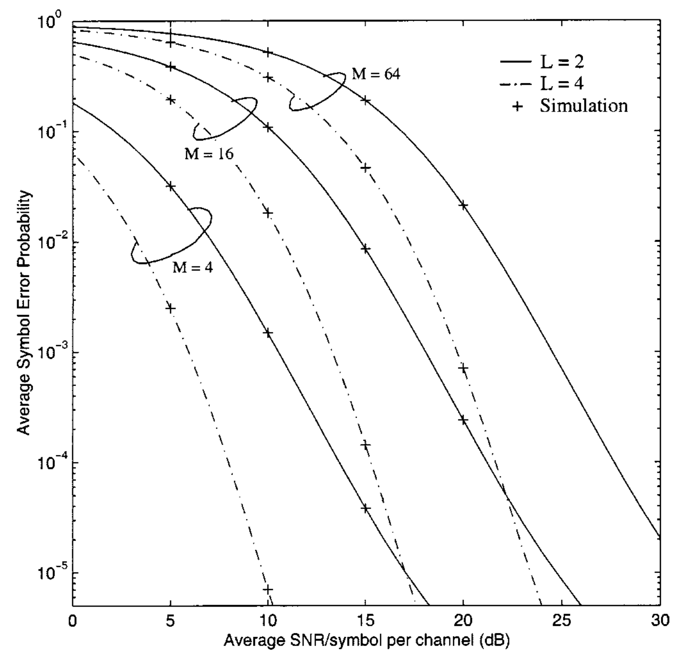

Fig. 5. Error probability performance of MQAM employing EGC diversity in a Rician fading environment ($K = 4.5$).

Table II. QA simulations are also performed to serve as a "sanity check" on the analytical results. In Fig. 4, we plot the error performance of MPSK employing second-order EGC diversity in Rayleigh and Rician fading channels. We have also validated that the curves obtained using (9) and (17) yield identical results when (17) is replaced by (12) (for BPSK) and (13) (for QPSK). Finally, Fig. 5 presents the average error probability performance of MQAM employing predetection EGC in Rician fading channels. Once again, the results obtained using (9), (13), and the second entry of Table II agree with those obtained via QA simulation.

V. Conclusions

This paper presents a concise, frequency-domain approach for evaluating the performance of a broad class of modulation schemes employing predetection equal-gain diversity receivers in a variety of fading environments. The results are sufficiently general to allow for arbitrary fading parameters, as well as dissimilar mean signal strengths across all the diversity branches. The generality and computational efficiency of the new results presented in this paper rendering themselves as powerful means for both theoretical analysis and practical applications. Some new closed-form solutions for EGC receiver performance in a Nakagami fading environment are also derived.

APPENDIX A

In this appendix, we derive finite-range integral expressions for the CHF of Rician and Nakagami-q RVs. This representation has an advantage over its infinite series expression because it circumvents the need to compute the confluent hypergeometric series recursively (i.e., in [9, eqs. (12)–(14)]).

Different from [9], we replace the modified Bessel function of the first kind $I_0(\cdot)$ with its exponential integral representation [19, eq. (8.431.3)] to derive the CHF of v_l in Rician fading

$$\phi_{v_l}(\omega) = \frac{\exp(-K_l)}{\pi} \int_0^{\pi} \exp[\zeta_l^2(\theta)/4] D_{-2}[-\zeta_l(\theta)]\, d\theta \tag{A.1}$$

where $\zeta_l(\theta) = j\omega\sqrt{\overline{\gamma}_l/2(1+K_l)} + \sqrt{2k_l}\cos\theta$ and the second-order function of the parabolic cylinder $D_{-2}(\cdot)$ may be expressed as

$$D_{-2}(z) = \exp(-z^2/4) - z\exp(z^2/4)\sqrt{\frac{\pi}{2}}\,\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right). \tag{A.2}$$

Now, using variable substitution $t = \cos\theta$ and then applying the GCQ formula [13, eq. (25.4.38)], we have yet another series expression for $\phi_{v_l}(\cdot)$ in a Rician fading environment

$$\phi_{v_l}(\omega) \approx \exp(-K_l)\left\{1 + \frac{1}{n}\sqrt{\frac{\pi}{2}}\sum_{i=1}^{n}\zeta_l(\theta_i) \times \exp[\zeta_l^2(\theta_i)/2]\mathrm{erfc}\left[\frac{-\zeta_l(\theta_i)}{\sqrt{2}}\right]\right\} \tag{A.3}$$

where $\theta_i = (2i-1)\pi/2n$. Note that there is a tradeoff involved when selecting a value for n: greater accuracy may be obtained using a larger n value, but at the expense of increased number of terms. A bound on the approximation error for the truncated series can be derived using the procedure outlined in [17] and [22].

In a similar fashion to our development of (A.1), the CHF of v_l for Nakagami-q fading derived in Table II (entry 3) may be restated as a single finite-range integral

$$\begin{aligned}\phi_{v_l}(\omega) &= \frac{\sqrt{1-b_l^2}}{\pi}\int_0^{\pi}\frac{1}{(1-b_l\cos\theta)} \times \exp[\xi_l^2(\theta)/4]D_{-2}[-\xi_l(\theta)]\,d\theta \\ &= \frac{\sqrt{1-b_l^2}}{\pi}\int_0^{\pi}\frac{1}{(1-b_l\cos\theta)} \times \left[1 + \xi_l(\theta)\sqrt{\frac{\pi}{2}}\exp[\xi_l^2(\theta)/2]\mathrm{erfc}[-\xi_l(\theta)/\sqrt{2}]\right]d\theta\end{aligned} \tag{A.4}$$

where $\xi_l(\theta) = j\omega\sqrt{(\overline{\gamma}_l(1-b_l^2)/2(1-b_l\cos\theta))}$. For the special cases of $b_l = 0$ and $b_l = 1$, the Nakagami-q distribution reverts to the Rayleigh and the one-sided normal distribution for the respective values. Using (A.2) and the GCQ formula, we can also derive an expression similar to (A.3) for the Nakagami-q RV. To the best of the authors' knowledge, all the results derived in this Appendix are new.

APPENDIX B

In this appendix, closed-form solutions for the real part of $G(\omega)$ in (7) are derived for two special cases as follows: 1) $b_u(\theta) = \kappa_2/\sin^2(\theta+\kappa_3)$ and 2) $b_u(\theta) = \kappa_2/\cos^2(\theta+\kappa_3)$. This is achieved by assuming that $a_u(\theta) = \kappa_1$ and κ_1, κ_2 and κ_3 are constants. Initially, from (7), we have

$$\mathrm{Real}\{G(\omega)\} = \frac{\sqrt{\pi}\kappa_1}{2}\sum_u\int_0^{\eta_u}\frac{1}{\sqrt{b_u(\theta)}}\exp\left(\frac{-\omega^2}{4b_u(\theta)}\right)d\theta. \tag{B.1}$$

For case 1), substituting $b_u(\theta) = \kappa_2/\sin^2(\theta+\kappa_3)$ in (B.1) and then using variable transformation $\alpha = \theta + \kappa_3$, we obtain

$$\mathrm{Real}\{G(\omega)\} = \frac{\sqrt{\pi}\kappa_1}{2\sqrt{\kappa_2}}\exp\left(\frac{-\omega^2}{4\kappa_2}\right)\sum_u\int_{\kappa_3}^{\eta_u+\kappa_3} \times \exp\left(\frac{\omega^2}{4\kappa_2}\cos^2\alpha\right)\sin\alpha\, d\alpha. \tag{B.2}$$

Now, using variable substitution $t = (\omega\cos\alpha/2\sqrt{\kappa_2})$, (B.2) reduces to

$$\mathrm{Real}\{G(\omega)\} = \frac{\sqrt{\pi}\kappa_1}{\omega}\exp\left(\frac{-\omega^2}{4\kappa_2}\right)\sum_u\int_{(\omega\cos(\eta_u+\kappa_3)/2\sqrt{\kappa_2})}^{(\omega\cos\kappa_3/2\sqrt{\kappa_2})}\exp(t^2)\,dt. \tag{B.3}$$

This can be written in a closed form as

$$\begin{aligned}\mathrm{Real}\{G(\omega)\} = \frac{\sqrt{\pi}\kappa_1}{\omega}\exp\left(\frac{-\omega^2}{4\kappa_2}\right) &\times \sum_u\left\{\exp\left(\frac{\omega^2}{4\kappa_2}\cos^2\kappa_3\right)F\left(\frac{\omega\cos\kappa_3}{2\sqrt{\kappa_2}}\right)\right. \\ &- \exp\left(\frac{\omega^2}{4\kappa_2}\cos^2(\eta_u+\kappa_3)\right) \\ &\left.\times F\left(\frac{\omega\cos(\eta_u+\kappa_3)}{2\sqrt{\kappa_2}}\right)\right\}\end{aligned} \tag{B.4}$$

where $F(\cdot)$ corresponds to Dawson's integral [see (11)].

For case 2), where $b_u(\theta) = \kappa_2/\cos^2(\theta+\kappa_3)$, while following the development of (B.2)–(B.4), we can show that $\mathrm{Real}\{G(\omega)\}$

is given by

$$\begin{aligned}&\mathrm{Real}\{G(\omega)\}\\&=\frac{\sqrt{\pi}\kappa_1}{\omega}\exp\left(\frac{-\omega^2}{4\kappa_2}\right)\sum_u\\&\quad\times\left\{-\exp\left(\frac{\omega^2}{4\kappa_2}\sin^2\kappa_3\right)F\left(\frac{\omega\sin\kappa_3}{2\sqrt{\kappa_2}}\right)\right.\\&\quad\left.+\exp\left(\frac{\omega^2}{4\kappa_2}\sin^2(\eta_u+\kappa_3)\right)F\left(\frac{\omega\sin(\eta_u+\kappa_3)}{2\sqrt{\kappa_2}}\right)\right\}.\end{aligned}\tag{B.5}$$

The above formulas can be used to speed up the computation of (9) for MPSK and other two-dimensional signal constellations (e.g., Craig's formula [12]) given that they collapse one of the double integrals into a single integral.

APPENDIX C

In this appendix, we provide some useful formulas that facilitate the derivation of closed-form solutions for the EGC receiver in Rayleigh and Nakagami-m environments (see Section III). Substituting $M_{x,\mu}(z)=e^{-z/2}z^{\mu+1/2}\Phi(\mu-x+1/2,\,1+2\mu;\,z)$ in [19, eq. (7.622.3)], and after some algebraic manipulations, we obtain an integral identity involving product of Kummer functions

$$\begin{aligned}&\int_0^\infty x^{\nu-1}e^{-bx}\prod_{k=1}^{n}\Phi(a_k;\,b_k;\,c_kx)\,dx\\&=b^{-v}\Gamma(v)F_A\left(v;\,a_1,\ldots,a_n;\,b_1,\ldots,b_n;\,\frac{c_1}{b},\,\ldots,\,\frac{c_n}{b}\right)\\&\quad\left[b_k>0,\,v>0,\,\sum c_k<b\right]\end{aligned}\tag{C.1}$$

where $M_{x,\mu}(z)$ is Whittaker's function of the first kind and $F_A(\cdots)$ denotes the Appell hypergeometric function defined as [19, eq. (9.19)], *viz.*,

$$\begin{aligned}&F_A(\alpha;\,\beta_1,\,\ldots,\,\beta_n;\,\gamma_1,\,\ldots,\,\gamma_n;\,z_1,\,\ldots,\,z_n)\\&=\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\cdots\sum_{m_n=0}^{\infty}\\&\quad\times\frac{(\alpha)_{m_1+\cdots+m_n}(\beta_1)_{m_1}\cdots(\beta_n)_{m_n}}{(\gamma_1)_{m_1}\cdots(\gamma_n)_{m_n}m_1!\cdots m_n!}\,z_1^{m_1}\cdots z_n^{m_n}.\end{aligned}\tag{C.2}$$

If all β_i, $i\in\{1,\,2,\,\ldots,\,n\}$, are either zero or negative integer, then it becomes apparent that (C.2) reduces to a finite series because $(\beta_i)_{m_i}=0$ for $m_i>-\beta_i$. Furthermore, for the particular case of $n=2$, the Appell function in (C.1) can be replaced by the series

$$\begin{aligned}&F_2(v;\,a_1,\,a_2;\,b_1,\,b_2;\,c_1,\,c_2)\\&=\sum_{n=0}^{\infty}\frac{(v)_n(a_1)_n}{(b_1)_n n!}\,c_1^n{}_2F_1(v+n,\,a_2;b_2;c_2)\\&=\sum_{n=0}^{\infty}\frac{(v)_n(a_2)_n}{(b_2)_n n!}\,c_2^n{}_2F_1(v+n,\,a_1;b_1;c_1)\\&\quad[|c_1|+|c_2|<1,\,b_k>0]\end{aligned}\tag{C.3}$$

which reduces to a finite series, if v or a_1 or a_2 (or all of them) in (C.3) is either zero or a negative integer. Some functional relations between hypergeometric functions of two variables (transformation formulas) are given in [19, eq. (9.183.2)]. In addition to this, the Appell hypergeometric series of the second kind has the following property:

$$\begin{aligned}&F_2(b;\,a_1,\,a_2;\,b,\,b;\,x,\,y)\\&=(1-x)^{-a_1}(1-y)^{-a_2}{}_2F_1\left(a_1,\,a_2;\,b;\,\frac{xy}{(1-x)(1-y)}\right)\end{aligned}\tag{C.4}$$

As a check, we find that (C.4) yields [19, eq. (7.622.1)]. Also, using (C.1) and (C.4), we can show that

$$\begin{aligned}&\int_0^\infty t^{c-1}e^{-st}\Phi(a,\,c;\,\mu t)\Phi(\alpha,\,c;\,\lambda t)\,dt\\&=\frac{\Gamma(c)s^{a+\alpha-c}}{(s-\mu)^a(s-\lambda)^\alpha}\times{}_2F_1\left(a,\,\alpha;\,c;\,\frac{\lambda\mu}{(s-\lambda)(s-\mu)}\right).\end{aligned}\tag{C.5}$$

APPENDIX D

For integer values of $m_k(k=1,\,2,\,3)$, $I_{3a}(x,\,y,\,z,\,\delta)$ can be evaluated in a closed form

$$\begin{aligned}&I_{3a}(x,\,y,\,z,\,\delta)\\&=\frac{4\sqrt{\zeta_x\zeta_y\zeta_z}}{\pi}\,\frac{\Gamma(m_x+1/2)\Gamma(m_y+1/2)\Gamma(m_z+1/2)}{\Gamma(m_x)\Gamma(m_y)\Gamma(m_z)}\\&\quad\times\sum_{k_1=0}^{m_x-1}\frac{(-2\zeta_x)^{k_1}}{(2k_1+1)!!}\binom{m_x-1}{k_1}\sum_{k_2=0}^{m_y-1}\frac{(-2\zeta_y)^{k_2}}{(2k_2+1)!!}\binom{m_y-1}{k_2}\\&\quad\times\sum_{k_3=0}^{m_z-1}\frac{(-2\zeta_z)^{k_3}}{(2k_3+1)!!}\binom{m_z-1}{k_3}\frac{\Gamma(k_1+k_2+k_3+3/2)}{\lambda^{k_1+k_2+k_3+3/2}}\end{aligned}\tag{D.1}$$

which follows directly from the definition of (C.2) because $(\beta)_n=0$ for $n>-\beta$ if β is a negative integer. Similarly, by assuming that m_z is an integer, $I_{3b}(x,\,y,\,z,\,\delta)$ can be restated as

$$\begin{aligned}&I_{3b}(x,\,y,\,z,\,\delta)\\&=\frac{\Gamma(m_z+1/2)}{\pi\Gamma(m_z)}\sum_{k=0}^{m_z-1}\frac{(-2)^k}{(2k+1)!!}\binom{m_z-1}{k}\zeta_z^{k+1/2}\\&\quad\times I(x,\,y,\,z,\,\delta,\,k)\end{aligned}\tag{D.2}$$

where

$$\begin{aligned}I(x,\,y,\,z,\,\delta,\,k)=&\int_0^\infty t^{k-1/2}e^{-\lambda t}\Phi\left(\frac{1}{2}-m_x,\,\frac{1}{2};\,\zeta_x t\right)\\&\times\Phi\left(\frac{1}{2}-m_y,\,\frac{1}{2};\,\zeta_y t\right)dt.\end{aligned}\tag{D.3}$$

It is obvious that for our subsequent developments, we need to evaluate the integral in (D.3) in closed form. For $k=0$, (D.3) can be evaluated using (C.5)

$$\begin{aligned} I(x, y, z, \delta, 0) &= \sqrt{\pi}(1+\zeta_x+\zeta_y+\zeta_z)^{(1/2)-m_x-m_y} \\ &\times (1+\zeta_y+\zeta_z)^{m_x-(1/2)}(1+\zeta_x+\zeta_z)^{m_y-(1/2)} \\ &\times {}_2F_1\left(\frac{1}{2}-m_x, \frac{1}{2}-m_y; \frac{1}{2}; \frac{\zeta_x\zeta_y}{(1+\zeta_y+\zeta_z)(1+\zeta_x+\zeta_z)}\right). \end{aligned} \tag{D.4}$$

Thus, for the particular case of Rayleigh fading ($m_1 = m_2 = m_3 = 1$) and $L = 3$, it can be shown that (35) reduces to [3, eq. (21)], as expected.

For $k = 1$, we utilize Gauss's contiguous relation [19, eq. (9.212.3)]

$$\Phi(a, c; x) = \frac{c-a}{c}\Phi(a, c+1; x) + \frac{a}{c}\Phi(a+1, c+1; x) \tag{D.5}$$

to expand $\Phi(\cdots)$ in (D.3). This results in a sum of four integrals. Now, applying (C.5), $I(x, y, z, \delta, 1)$ can be evaluated as a finite sum of the Gauss hypergeometric series

$$\begin{aligned} I(x, y, z, \delta, 1) &= \frac{\sqrt{\pi}\chi_2^{m_x-1/2}\chi_3^{m_y-1/2}}{\chi_1^{m_x+m_y+1/2}} \\ &\times \left[2m_x m_y {}_2F_1\left(\frac{1}{2}-m_x, \frac{1}{2}-m_y; \frac{3}{2}; \chi_4\right)\right. \\ &+ m_x(1-2m_y)\frac{\chi_1}{\chi_3} {}_2F_1\left(\frac{1}{2}-m_x, \frac{3}{2}-m_y; \frac{3}{2}; \chi_4\right) \\ &+ m_y(1-2m_x)\frac{\chi_1}{\chi_2} {}_2F_1\left(\frac{1}{2}-m_y, \frac{3}{2}-m_x; \frac{3}{2}; \chi_4\right) \\ &+ \frac{1}{2}(1-2m_x)(1-2m_y)\frac{\chi_1^2}{\chi_2\chi_3} \\ &\times {}_2F_1\left(\frac{3}{2}-m_y, \frac{3}{2}-m_x; \frac{3}{2}; \chi_4\right) \end{aligned} \tag{D.6}$$

where $\chi_1 = 1+\zeta_x+\zeta_y+\zeta_z$, $\chi_2 = 1+\zeta_y+\zeta_z$, $\chi_3 = 1+\zeta_x+\zeta_z$ and $\chi_4 = \zeta_x\zeta_y/\chi_2\chi_3$. By performing the above steps recursively (i.e., using (D.5) then applying (C.5), as we have done for the case $k=1$), one can derive closed-form solutions for the integral (D.3) when $k = 2, 3, \ldots,$ and so on. However, the number of terms (sum of integrals) grows exponentially (4^k).

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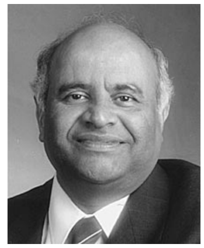

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