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Equalising the Transmission Properties of Graph-Modelled Networks by Introducing the Control of the Resources Used to Transmit Information

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Abstract The object of the presented paper is to demonstrate the possibility of equalizing the transmission properties of networks described by graphs with an equal number of nodes of the same degree but differing in diameter and average path length. A node of a graph is understood to be a specialised device performing commutation and transmission functions, while an edge is understood to be a transmission link. It was assumed that this alignment can be achieved by controlling the transmission resources assigned to specific edges of a graph through which the network nodes communicate with each other. The concept of transmission resource, counted in contractual units, is understood as, for example, the number of time slots or the bandwidth of the links used to transmit information. The authors show that with the proposed method, the equalization of the mentioned network properties is achieved with a minimal increase in the global resources used for transmission.

Keywords ICT networks · Graph theory · Transmission properties

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1 Introduction

Nowadays, there is no doubt that the development of widely understood technologies related to communication networks is one of the most vital tasks the scientific community has to face. Without communication networks it would not be possible to develop any distributed technologies. So, it is well-timed to try to create a network with maximum efficiency and optimal and economical use of transmission resources. The performance of a communication network depends on how its connection topology is designed. These networks can be modelled using graphs, so graph theory plays an important role here. There are many network topologies that have been studied for their application to network modelling, such as ring, mesh, torus and hypercube. Each topology has some properties specific to it. One network topology suitable for telecommunication networks is the chordal ring, proposed by Arden and Lee in 1981 [1]. A chordal ring is a homogeneous, undirected cyclic graph. An important property of this type of network is the degree of nodes, understood as the number of edges of a graph incident to its vertex. It is an intermediate structure between a ring and a full graph, as it combines the properties of both these topologies. Due to the larger number of paths connecting the source node to the destination node, it is more resilient than a ring, while having a smaller diameter and average path length. The use of the chordal ring topology in large networks shows an advantage over full graphs, whose parameters mentioned above have lower values, but are characterised by an exponential increase in the number of connections as the number of nodes increases, which in turn is associated with their higher, economically unjustifiable cost. Since the implementation of the chordal ring topology was proposed in the early 1980s, many researchers have focused on improving and proposing new versions of this topology. This was due to its favourable parameters such as low latency in the transmission of information, fault tolerance and symmetry. The initial proposal to use third degree chordal rings based on a fixed chord length has consistently been developed and improved. One such improvement has been to increase the degree of classical topologies, for example to the fourth degree [2]; fifth degree [3] and even sixth degree [4]. Another way to further improve the aforementioned favourable parameters was to change the ways in which the chordal ring was connected, resulting in different types of nodes in the classes. Examples of such modified chordal rings were the modified sixth-degree ring presented in papers [5,6]. The properties of the proposed chordal ring topologies such as symmetry [7] and Hamiltonicity and asymmetry [8] have also been studied over the years. Researchers are also interested in topics related to the application of these topologies in compact routing [9], optimal free-table routing [10] and broadcasting [11,12]. New proposals to generate chordal ring structures continue to emerge, among others [13]. These constructs find applications in increasingly modern telecommunication technologies, including wireless networks [14], sensor networks [15]. Graph theory is used in solving problems that can be represented by graphs, such as algorithms used in network analysis or to describe phenomena related to social engineering, technology, computer engineering [16] and many other complex real-world systems [17]. As mentioned, a variety of problems can be described and analyzed using chordal ring structures, which makes them a useful tool. Graph theory can be said to be a versatile tool for describing and solving problems involving relationships between objects in various scientific fields.

2 Basic Information and Concepts

As it was proved in the work [18], transmission properties of networks that are described by graphs depend not only on their diameter and average path length, but also on the use of particular edges of these structures for data transmission, which is best illustrated by the example of Reference Graphs [19,20], which, despite having the same underlying parameters, may differ in the properties mentioned above.

Definition 1 Reference Graphs are regular structures with a predetermined number of nodes in which the diameter values and the average path lengths from any source node reach the same, theoretically calculated lower size limits.

Figure 1 shows examples of such graphs of the fourth degree formed by seven nodes with diameters of 2 and average path lengths of 1.333.

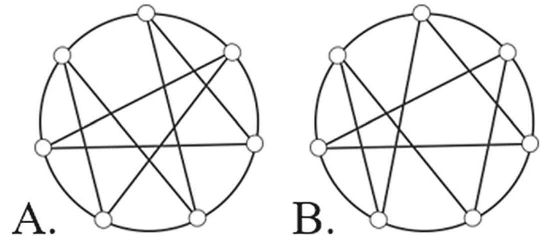


Fig. 1 Analysed reference graphs

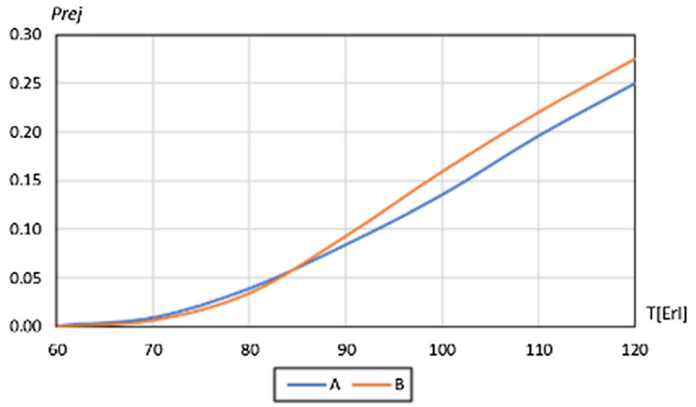


Fig. 2 Simulation results. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

These structures were subjected to simulation tests and the chart (Fig. 2) shows the results. To carry out the tests, simulation software developed by the authors of this article and described in detail in the publication [21] was used. As a parameter determining the transmission properties of the network, the probability of rejecting a service call in the function of changes in the traffic generated in the network nodes was chosen. In this case, the tests were carried out under the assumption that 64 transmission units are assigned to each edge and that the same amount of traffic is generated at each node.

As already mentioned, the factor causing differences in the transmission properties of the networks described by these graphs is the uneven use of individual edges of these structures, the measure of which is a parameter called the unevenness coefficient [22].

Definition 2 The unevenness coefficient w_{spi} is the parameter determining the number of uses of a given edge in sets of parallel paths (routes of the same length created by various configurations of these edges) connecting vertices of a graph.Individual edges can be a part of multiple paths, even those that connect the same nodes.

An unevenness coefficient is described by the formula:

$$w_{spi} = \sum_{i=1}^{D(G)} u_{io} \tag{1}$$

where $D(G)$ is the diameter of the graph and u_{io} values calculated the formula:

$$u_{io} = \frac{u_k}{k} \tag{2}$$

u_k means the number of uses of a particular edge in the sets of parallel paths of count k [23]. In the case under consideration (Fig. 1), the calculated w_{spi} coefficients have the values given in Table 1 (*ei* denotes the edge number of the graph).

The resource control principle is implemented according to the values of the calculated imbalance coefficients, which allows more efficient management of the global transmission resources of the analysed network. It involves

Table 1 Determined coefficient w_{spi} values

Graph A	*Edge*	*e0*	*e1*	*e2*	*e3*	*e4*	*e5*	*e6*
	w_{spi}	4.667	4.667	3.333	3.333	3.333	4.667	3.333
	Edge	*e7*	*e8*	*e9*	*e10*	*e11*	*e12*	*e13*
	w_{spi}	3.333	3.333	4.667	4.667	4.667	4.667	3.333
Graph B	*Edge*	*e0*	*e1*	*e2*	*e3*	*e4*	*e5*	*e6*
	w_{spi}	4.333	4.333	4.333	4.333	2.000	4.333	4.333
	Edge	*e7*	*e8*	*e9*	*e10*	*e11*	*e12*	*e13*
	w_{spi}	4.333	4.333	2.000	4.333	4.333	4.333	4.333

Table 2 Distribution of resources allocated to individual edges of graphs

Graph A	*Edge*	*e0*	*e1*	*e2*	*e3*	*e4*	*e5*	*e6*
	d_i	0.083	0.060	0.060	0.083	0.060	0.083	0.060
	RES_{ic}	75	53	53	75	53	75	53
	Edge	*e7*	*e8*	*e9*	*e10*	*e11*	*e12*	*e13*
	d_i	0.083	0.083	0.083	0.060	0.060	0.060	0.083
	RES_{ic}	75	75	75	53	53	53	75
Graph B	*Edge*	*e0*	*e1*	*e2*	*e3*	*e4*	*e5*	*e6*
	d_i0.077	0.077	0.077	0.077	0.036	0.077	0.077	
	RES_{ic}	69	69	69	69	32	69	69
	Edge	*e7*	*e8*	*e9*	*e10*	*e11*	*e12*	*e13*
	d_i	0.077	0.077	0.077	0.077	0.077	0.036	0.077
	RES_{ic}	69	69	69	69	69	32	69

RES_{ic} means the value of resources after rounding to integers

allocating more resources to edges that are used more frequently to transmit information, at the expense of reducing them for edges carrying less traffic. The size of network resources allocated to each edge is calculated using the rule:

$$RES_i = d_i \cdot RES_g, \quad where \quad d_i = \frac{w_{spi}}{\sum w_{spi}} \tag{3}$$

where RES_i is the resources used by the $i-th$ edge, RES_g is the total network resources, and $\sum w_{spi}$ is a sum of the coefficients calculated for all edges [24]. In the case under consideration RES_g = 896 contractual units, while in both cases the value of $\sum w_{spi}$ is 56. Table 2 shows the distribution of resources for the individual edges of the graphs.

After adjusting the transmission resources, simulation studies were carried out again and the result is shown in Fig. 3.

Based on the results obtained, it can be concluded that by introducing a network resource control that uses the results of the calculation of the unevenness coefficients, an alignment of the characteristics of the Reference Graphs with equal number and degree of nodes can be achieved, while the size of the global resources does not change.

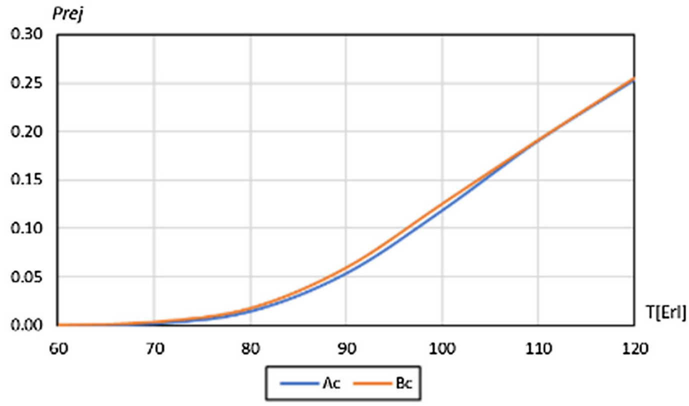


Fig. 3 Simulation results after resource adjustment. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

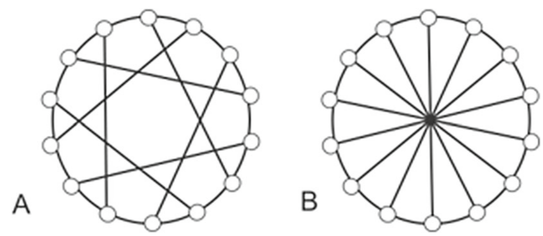


Fig. 4 The graphs describing the studied networks. $D(G)_A = 3, d_{avA} = 2.0769$; $D(G)_B = 4, d_{avB} = 2.3846$

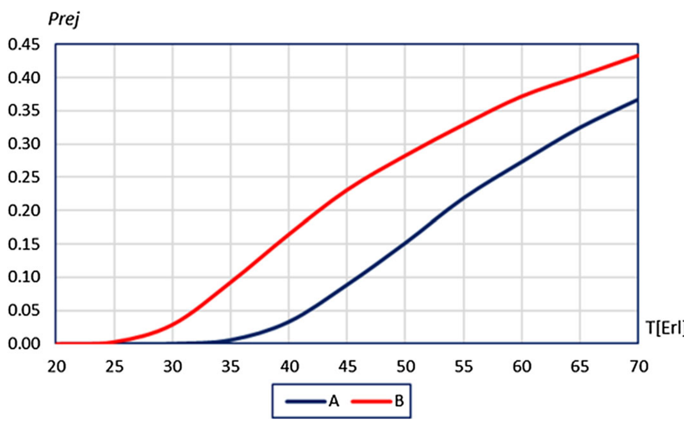


Fig. 5 Results of simulation tests of networks described by 14-node third-degree graphs A, B (Fig. 4). T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

3 Equalization of Transmission Characteristics of Networks Described by Regular Graphs with Different Diameters and Average Path Lengths

The reference graphs discussed in the previous part of the article are characterised by the same basic parameters, while, as it was specified in the introduction, the analysis and research aimed to check whether it is possible to equalise the characteristics of networks described by graphs whose basic parameters differ [25]. The following example is used to explain the principle and the procedure. Simulation tests of networks described by chord rings of degree three with 14 nodes (Fig. 4) were carried out, and the graph (Fig. 5) shows the obtained results assuming that 64 units are assigned to each edge.

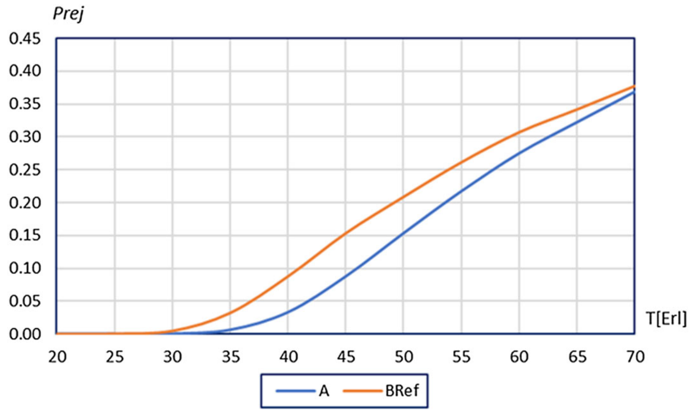


Fig. 6 The results of simulation tests of networks described by 14-node graphs of the third degree after the modification of the resources of graph B resulting from the inclusion of Ref_{dav} values T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 3 The determined values of the parameters w_{spi} (e_i stands for the edge number)

e_i	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
$w_{spi\,B}$	24.4	13.2	24.4	24.4	13.2	24.4	13.2	24.4	13.2	24.4	13.2
e_i	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	
$w_{spi\,B}$	24.4	13.2	24.4	13.2	24.4	24.4	24.4	24.4	24.4	24.4	

It was assumed that to equalize the transmission characteristics of the studied graphs, it is sufficient to calculate the ratio of their average path lengths and multiply the resources corresponding to the edges of the graph with the larger average path length by a ratio factor called Ref, which is calculated from the formula:

$$Ref_{G1/G2} = \frac{d_{avG1}}{d_{avG2}} \quad (4)$$

d_{avG1} is the average path length of the graph under the study, d_{avG2} is the average path length of the reference graph.

The adjusted values of the resources, which should make it possible to achieve similar transmission properties of both networks described by the above-mentioned graphs, were calculated using the following formula:

$$RES_{icG1} = Ref_{G1/G2} \cdot RES_{iG1} \quad (5)$$

In the analysed example, the value of $Ref_{B/A}$ for graph A is 1.148153. After taking this value into account, the edges of graph B were assigned 73 units, which means that the global resources would increase by 189 units. The graph (Fig. 6) illustrates the obtained results of the tests.

The accompanying diagram shows that this way of using the $Ref_{G1/G2}$ coefficient does not guarantee an equalisation of the transmission properties of the tested networks. In the search for a method that would result in achieving the assumed objective, i.e., similar transmission characteristics of the network, the method used for aligning the transmission properties of Reference Graphs was applied.

Using the determined values of the unevenness coefficients given in Table 3, the resource values for the individual edges of graph B were adjusted (Table 4), which, as mentioned earlier, results in more efficient use of the transmission properties of the network. Sum of unevenness coefficients $\sum w_{spi\,B} = 434$. In the case of graph A, the values of all unevenness coefficients are equal to 18 and therefore the resources assigned to its edges have not changed and are equal to 64 units and $\sum w_{spi\,A} = 378$. The results for graph B described above are shown in Fig. 7.

Table 4 Adjusted and rounded resource values for individual edges of graph B

e_i	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
RES_{icB}	76	41	76	76	41	76	41	76	41	76	41
e_i	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	
RES_{icB}	76	41	76	41	76	76	76	76	76	76	

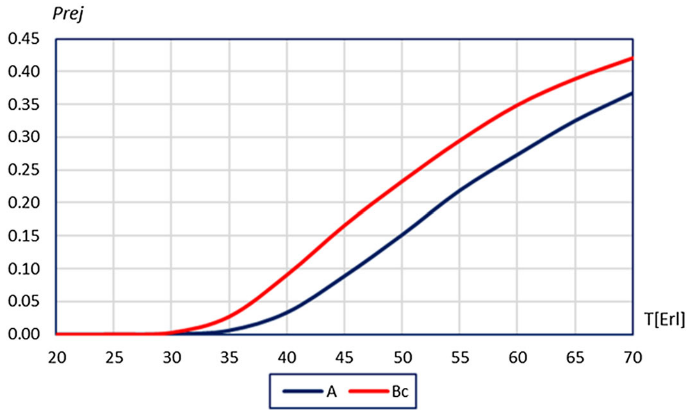


Fig. 7 Results of the study of analysed graphs after resource modification. Bc denotes the test results of graph B after introducing the correction resulting from the calculated values of the w_{spi} parameters. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 5 Results of resource calculations after taking $Ref_{B/A}$ into account

e_i	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10
RES_{icc}	87	47	87	87	47	87	47	87	47	87	47
e_i	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	
RES_{icc}	87	47	87	47	87	87	87	87	87	87	

Adjusted values of RES_{icc} resources have been calculated, which should make it possible to obtain similar transmission properties of the networks described by the above-mentioned graphs:

$$RES_{icc} = Ref_{G1/G2} \cdot RES_{ic} \tag{6}$$

Table 5 gives the results of the calculations of the resources assigned to the edges of the graphs after considering the values of the $Ref_{B/A}$ parameter

The chart (Fig. 8) shows the simulation results.

To achieve the assumed goal of the undertaken activities, i.e., to equalise the characteristics of data transmission in networks described by graphs with different basic parameters (diameter, average path length), it is necessary to increase the total transmission resources for graph B by 203 units. Further, to verify that the minimum value of resources necessary to obtain a probability Prej close to the reference graph was achieved, simulation tests were carried out on the network described by graph B, with changes in the value of the parameter $Ref_{B/A}$ and in the volume of generated traffic (30–70) [Erl]. The results of the calculations are presented in Table 6.

Figure 9 shows the differences in the values of the probability of obtaining correct transmission in relation to graph A.

It has been checked whether the proposed mode is also effective when the networks are described by graphs with different average path lengths but the same diameter. For this purpose, the graphs shown in Fig. 10 were examined.

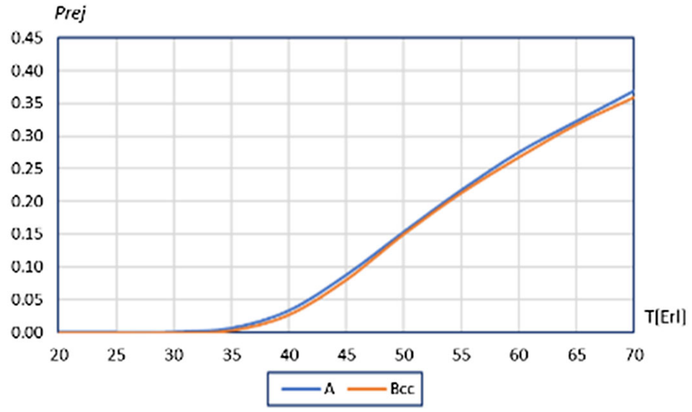


Fig. 8 Results of simulation tests after the adjustment of transmission resources taking into account the value of the $REF_{B/A}$ parameter. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 6 Prej values in the function of Ref_B parameter changes

T[Erl]	0.95 $Ref_{B/A}$	1.00 $Ref_{B/A}$	1.02 $Ref_{B/A}$	Graph A
30	0.00048	0.00016	9E-05	0.00025
40	0.04120	0.02749	0.01515	0.03316
50	0.17246	0.15095	0.12682	0.15386
60	0.29011	0.26836	0.24442	0.27574
70	0.37815	0.35958	0.34085	0.36953

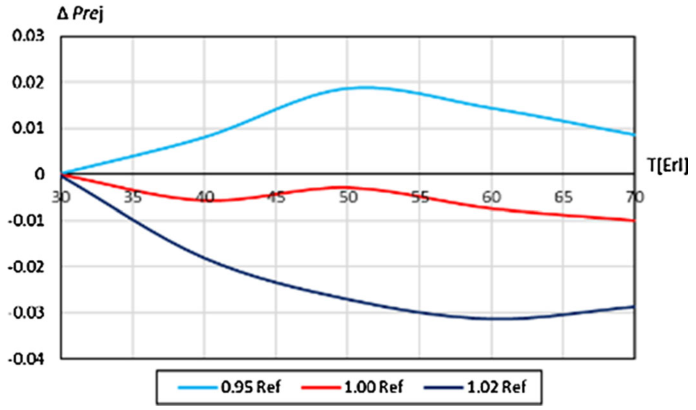
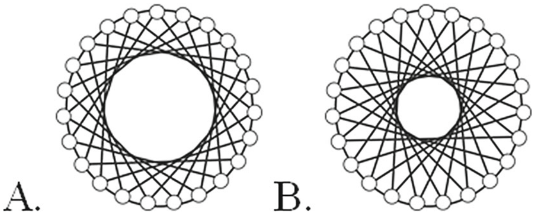


Fig. 9 Variations in the probability values of rejecting a service call Δ Prej with respect to the characteristic values for graph A, as a function of changes in the generated traffic T measured in Erlangs

Fig. 10 Analysed graphs with twenty-three nodes



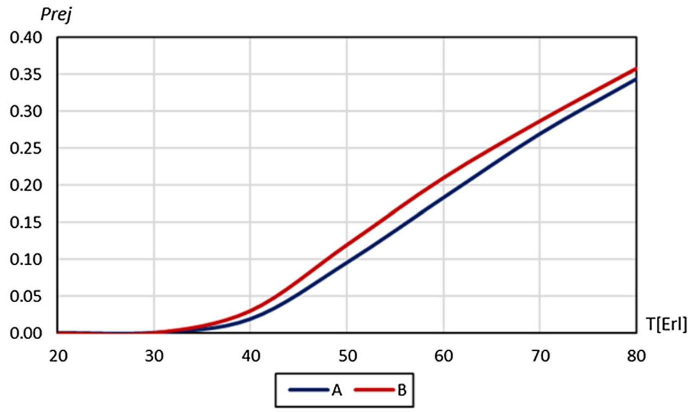


Fig. 11 Test results of the networks described by the graphs shown in Fig. 10. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

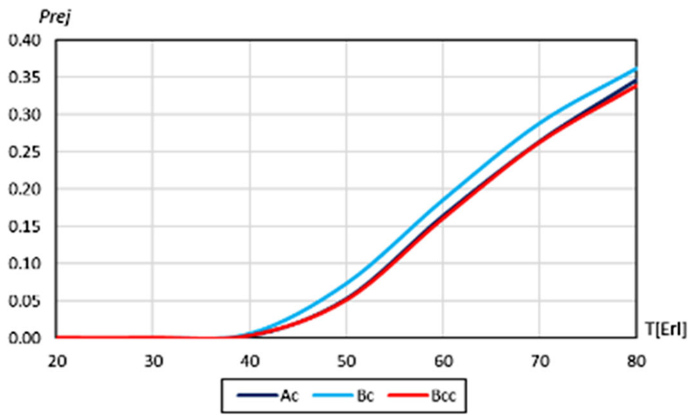


Fig. 12 Network test results: Ac and Bc: after the initial resource adjustment, Bcc: after taking into account the value of the Ref_B parameter. T: the intensity of traffic generated in nodes measured in Erlangs, Prej: the probability of rejecting a service call

The diameter of both graphs is equal to 4, while the average path lengths are d_{avA} = 2.3636 and d_{avB} = 2.4545. Figure 11 shows the comparative results of the simulation tests without the introduction of corrections to the network resources assigned to individual edges (each edge was assigned 64 units), while Fig. 12 shows the results of the tests after the introduction of corrections to the resources, also taking into account the impact of the value of the determined parameter Ref_B equal to 1.0385.

Based on the obtained results, it can be concluded that the average length of the paths plays a decisive role in the alignment of the characteristics of the studied networks, while the size of the diameter of the graphs describing these networks is virtually irrelevant, of course under the condition of equal number and degree of nodes describing these networks.

4 Equalization of Transmission Characteristics of Networks Described by Regular Graphs with More Complex Structure

This part of the article describes the results of the examination of structures that are closer in their topology to real conditions, but while maintaining the regularity condition. Figure 13 shows an example of a virtual network described by a third-degree regular graph with 26 nodes.

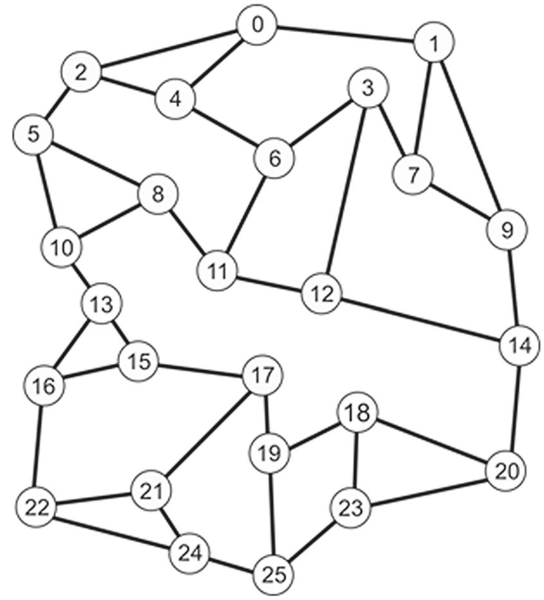


Fig. 13 Tested network structure described by a regular graph of the third degree

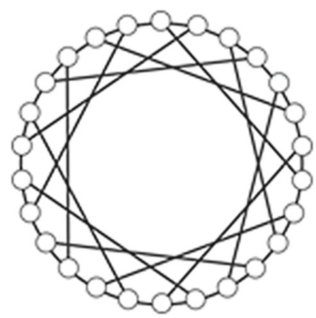


Fig. 14 Reference graph of the third degree with 26 nodes

The values of the basic parameters of the graph shown in Fig. 13 are as follows: the diameter is 7 and the average path length is 3.6954.

The network described by the chord graph shown in Fig. 14 was chosen as a reference structure, with a diameter of 5 and an average path length of $d_{av} = 2.84$.

Figure 15 shows the results of simulation tests performed assuming that each edge (there are 39 of them) is assigned 64 units of transmission resources, which means that the global transmission resources of these two networks are 2496 units.

It was checked what results would be obtained by applying the Ref_A factor to correct the resources assigned to the edges. The value of this coefficient is 1.3012, so each edge of the graph is assigned 83 transmission units. Figure 16 illustrates the obtained results of the simulation.

As can be seen from the graph, the obtained transmission characteristics of network A improved slightly, but deviated from those of network B, so a method using unevenness coefficients was used. Their values for graph A have been calculated and are shown in Table 7. All w_{spi} values of the graph B are the same and equal 47.333.

To make more rational use of the network capacity, the sizes of the network resources allocated to each edge were calculated (Table 8) using the results in Table 7.

The values of the resources assigned to the edges of graph B, due to equal w_{spi} values, have not changed, hence they are 64 units. The original modification of the resources assigned to the edges of graph A was made, and the simulation tests were repeated, the results of which are shown in Fig. 17.

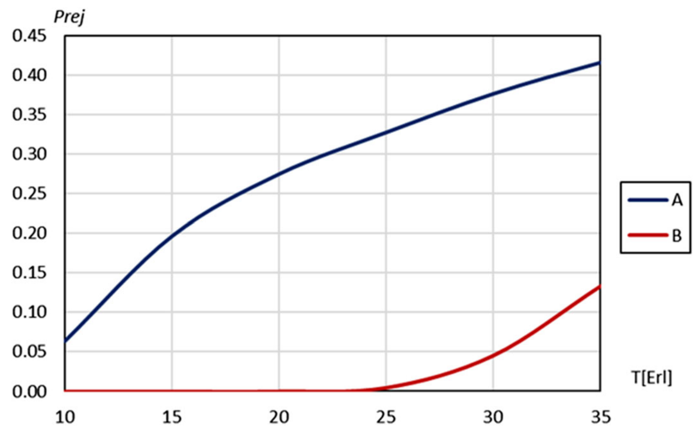


Fig. 15 Test results of the simulation networks described by the graphs shown in Figs. 15a and 16b. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

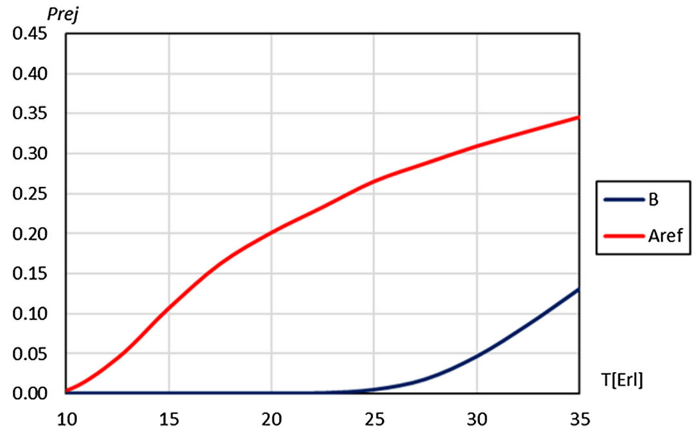


Fig. 16 Results of the simulation tests of the networks described by the analysed graphs after taking into account the value of the Ref_A parameter ($Aref$). T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 7 Unevenness coefficient values

$Edge$	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9
w_{spi}	68.00	57.67	18.33	18.00	59.00	29.50	87.17	39.67	26.50	50.17
$Edge$	e10	e11	e12	e13	e14	e15	e16	e17	e18	e19
w_{spi}	33.17	24.00	85.83	52.17	26.50	82.00	99.33	98.50	160.83	84.50
$Edge$	e20	e21	e22	e23	e24	e25	e26	e27	e28	e29
w_{spi}	105.00	76.67	78.17	175.17	13.17	78.83	65.67	79.50	30.67	89.00
$Edge$	e30	e31	e32	e33	e34	e35	e36	e37	e38	
w_{spi}	8.83	78.50	75.67	20.17	80.17	16.67	16.00	48.33	65.00	

In this case a sum of the coefficients calculated for all edges $\sum w_{spi\,A} = 2402$, $\sum w_{spi\,B} = 1846$. Using the formula (5), the adjusted resource values were calculated taking into account the value of the parameter $Ref_A = 1.3012$. (Table 9).

The results of the simulations carried out are shown in Fig. 18 (RES_{icc} values are rounded to integer values).

Table 8 Set of network resources assigned to the edges of a graph (after rounding)

$Edge$	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9
RES_i	71	60	19	19	61	31	91	41	28	52
$Edge$	e10	e11	e12	e13	e14	e15	e16	e17	e18	e19
RES_i	34	25	89	54	28	85	103	102	167	88
$Edge$	e20	e21	e22	e23	e24	e25	e26	e27	e28	e29
RES_i	109	80	81	182	14	82	68	83	32	92
$Edge$	e30	e31	e32	e33	e34	e35	e36	e37	e38	
RES_i	9	82	79	21	83	17	17	50	68	

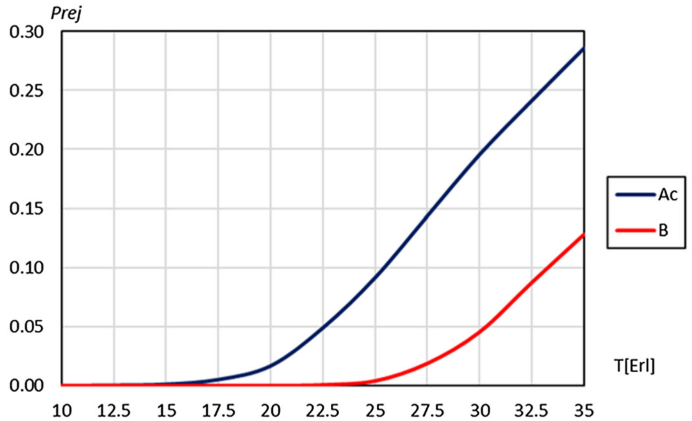


Fig. 17 Simulation results after modifying the resources of graph$A(Ac)$. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 9 Resources assigned to the edges of graph A after taking into account the Ref_A parameter value

$Edge$	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9
RES_{icc}	92	78	25	24	80	40	118	54	36	68
$Edge$	e10	e11	e12	e13	e14	e15	e16	e17	e18	e19
RES_{icc}	45	32	116	71	36	111	134	133	217	114
$Edge$	e20	e21	e22	e23	e24	e25	e26	e27	e28	e29
RES_{icc}	142	104	106	237	18	107	89	107	41	120
$Edge$	e30	e31	e32	e33	e34	e35	e36	e37	e38	
RES_{icc}	12	106	102	27	108	23	22	65	88	

By introducing the resource control, similar transmission characteristics of both networks were obtained after increasing the overall transmission resources of graph A by 724 units. Additional tests were carried out by modifying the allocated network resources in a function of changes in the Ref_A parameter. Table 10 presents the calculation results obtained and Fig. 19 shows the corresponding simulation results.

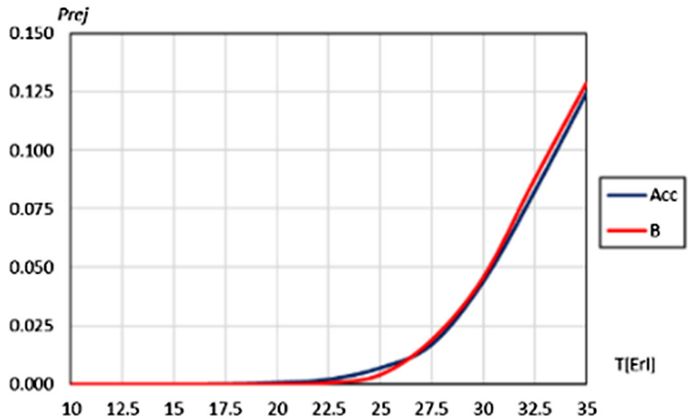


Fig. 18 Simulation results after modifying the resources taking into consideration the value of the $Ref(Acc)$ parameter. T: traffic volume generated in nodes measured in Erlangs, Prej: probability of rejecting a service call

Table 10 Prej values with changes in the generated traffic in the range of 15–35 [Erl] at different values of the parameter Ref

T[Erl]	Graph A				Graph B
	Ref_A	0.95 Ref_A	1.025 Ref_A	1.05 Ref_A	
15.0	0.000	0.000	0.000	0.000	0.000
20.0	0.001	0.001	0.001	0.000	0.000
25.0	0.007	0.013	0.005	0.004	0.004
30.0	0.044	0.069	0.035	0.024	0.046
35.0	0.124	0.152	0.100	0.091	0.129

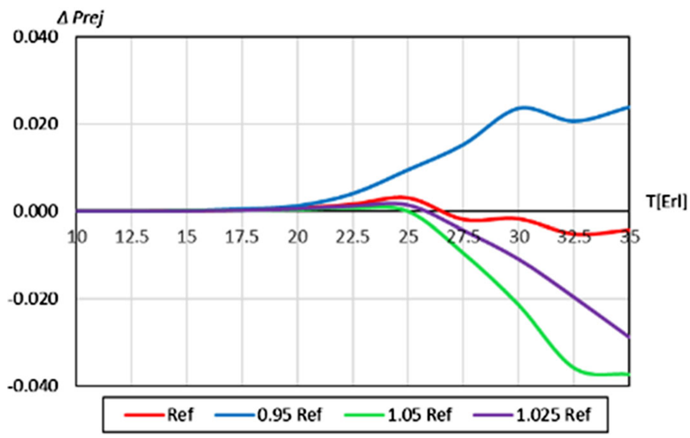


Fig. 19 Variations in the probability values of rejecting a service call Δ Prej for the characteristic values for graph A, as a function of changes in the generated traffic T measured in Erlangs

5 Summary and Conclusions

Based on the analysis of the obtained results it has been stated that at the cost of increasing the transmission resources assigned to particular network edges, calculated with the use of the Ref parameter, it is possible to obtain similarity of transmission properties of networks described by regular graphs of equal degree and with the same number of nodes but different diameter sizes and average path lengths, without the necessity of changing the topology of internodal connections and with minimal increase in the network resources used for information transmission.

The result of the considerations contained in the paper is the following algorithm, aimed at equalizing the transmission properties of networks described by graphs with the same number of nodes and the same number of nodes of a given degree.

1. Preliminary operations:
   - Selecting from the analysed networks the one described by a graph with shorter average path length.
   - Running a preliminary simulation to compare the transmission properties of the network assuming equal resources for each edge.
   - Calculation of unevenness coefficients for both studied networks.
   - Calculation of the value of transmission resources after an adjustment to make more rational use of these resources.
   - Performing a simulation after the adjustment.
2. Equalisation of transmission characteristics:
   - Determining the aggregate coefficient wspi values.
   - Calculation of the RefG1/G2 coefficient.
   - Adjusting the network resources taking into account the RefG1/G2 value.
   - Performing tests after the adjustment.
   - Determination of modification costs (size of additional transmission units).

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