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RESEARCH ARTICLE

Equality and responsibility: ex ante and ex post redistribution mechanisms

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Abstract
We study redistribution in a setting where individual responsibility and circumstance characteristics determine pre-tax income. We distinguish between ex ante and ex post versions of the key principles of compensation and reward. Furthermore, we distinguish between absolute and relative versions of reward. On the basis of these axioms, we provide characterizations of five familiar and two new redistribution mechanisms.

Keywords Compensation · Reward · Ex ante · Ex post · Responsibility

JEL Classification D63

1 Introduction

We study income redistribution among individuals who differ in the characteristics that determine pre-tax income. These characteristics comprise circumstances that are ethically arbitrary (e.g., parental background) and responsibility characteristics that are ethically significant (e.g., work effort). The different ethical status of circumstance and responsibility characteristics gives rise to two key principles. The principle of compensation calls for the elimination of the pre-tax income inequalities caused by circumstances. The principle of reward, on the other hand, requires the preservation of the pre-tax income inequalities that are due to the exercise of responsibility. The

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challenge is to design redistribution mechanisms that combine these two key principles.[1]

The compensation and reward principles each exist in two versions: an "ex ante" version that only uses information on circumstance characteristics, and an "ex post" version that only uses information on responsibility characteristics.[2] The different versions of the principles appeared gradually in the literature. Initially, Bossert (1995) and Bossert and Fleurbaey (1996) focused on ex post compensation (equal income for equal responsibility) and ex ante reward (equal transfer for equal circumstances). Later, Ooghe et al. (2007), Checchi and Peragine (2010) and Fleurbaey and Peragine (2013) introduced ex ante compensation (equal average income across circumstance groups).[3] Finally, Trannoy (2017) proposed ex post reward (equal average transfer across responsibility groups).

We make two contributions to the literature on redistribution. Our first contribution is to identify the redistribution mechanisms that satisfy combinations of ex ante and ex post versions of compensation and reward. As we will see, several of the possible combinations have heretofore not been explored.[4]

Our second contribution is to introduce and study relative versions of reward. Such relative versions require preserving relative pre-tax income differentials, whereas the literature so far has focused on absolute differentials. It is surprising that relative reward has not appeared before in the literature on redistribution mechanisms, as fairness in taxation is often cast in terms of relative concepts such as proportionality and progressivity.

Table 1 summarizes our results. We provide new characterizations of five familiar redistribution mechanisms: the observable average egalitarian equivalent (OAEE) and observable average conditionally egalitarian (OACE) mechanisms (Bossert et al. 1999), the average egalitarian equivalent (AEE) and average conditionally egalitarian (ACE) mechanisms (Bossert and Fleurbaey 1996), and the relative average egalitarian equivalent (RAEE) mechanism (Bossert 1995). These new characterizations are simple and the proofs straightforward. Moreover, the introduction of relative reward yields two new mechanisms to which we refer as the relative observable average conditionally egalitarian (ROACE) and relative average conditionally egalitarian (RACE) mechanisms.

A key feature of our approach is that each of our characterizations combines compensation and reward axioms that share the same informational basis. That is, we either combine ex post compensation with ex post reward or ex ante compensation with ex ante reward. This emphasizes an interpretation of the egalitarian equivalent type mech-

[1] See Fleurbaey (2008), Fleurbaey and Maniquet (2011) and Roemer and Trannoy (2015, 2016) for surveys.

[2] Roemer and Trannoy (2016, footnote 20) criticize the terminology of "ex ante" and "ex post" on the basis that the timing of the genesis of circumstance and responsibility characteristics is not explicitly modeled. This is a valid point, but we nevertheless adopt the terminology because it is rapidly becoming standard and no obvious alternative terminology is available.

[3] A circumstance group collects all individuals in society whose circumstance characteristics are identical. A responsibility group is defined similarly.

[4] Ooghe et al. (2007) and Checchi and Peragine (2010) combine ex ante and ex post compensation with an alternative version of reward, the so-called utilitarian reward principle. Neither Fleurbaey and Peragine (2013) nor Trannoy (2017) characterize mechanisms, but, rather, are interested in the (in)compatibility of the various axioms.

Table 1 Results

Axiom	Ex post compensation	Ex ante compensation	Weak ex ante compensation
Ex ante reward	✗	OACE	ACE*
Ex post reward	OAEE	✗	✓
Weak ex post reward	AEE*	✓	✓
Relative ex ante reward	✗	ROACE	RACE*
Relative ex post reward	OAEE	✗	✓
Weak relative ex post reward	RAEE*	✓	✓

We write ✗ if the axioms are incompatible and ✓ if the axioms are compatible and allow for many mechanisms (Appendix A). Mechanisms without * are characterized by the two axioms, whereas mechanisms with * require in addition an inequality preservation axiom

anisms as embodying the ex post perspective and the conditionally egalitarian type mechanisms as embodying the ex ante perspective.

In the next section we provide the notation for our analysis. Section 3 presents the axioms of compensation, reward and inequality preservation. In Sects. 4 and 5 we provide the characterization results pertaining to the ex post and ex ante perspectives. Section 6 discusses the alternative axioms used by Bossert and Fleurbaey (1996), Bossert et al. (1999) and Fleurbaey and Peragine (2013). Section 7 concludes.

2 Notation

Let $\mathcal{N}$ be the collection of all finite subsets of the set of positive integers. Each individual i in a society N in $\mathcal{N}$ is characterized by a pair (r_i, s_i) in $R \times S$, where r_i and s_i are i's responsibility and circumstance characteristics. For a society N, we let $r_N = (r_i)_{i \in N}$ and $s_N = (s_i)_{i \in N}$. We call (r_N, s_N) a characteristics profile. The set D collects all characteristics profiles, i.e., $D = \bigcup_{N \in \mathcal{N}} R^n \times S^n$, where n is the cardinality of N. For a profile (r_N, s_N) in D, we use $R(r_N, s_N)$ and $S(r_N, s_N)$ to denote, respectively, the set of responsibility characteristics and the set of circumstance characteristics observed in the profile. That is, $R(r_N, s_N) = \{r_i : i \in N\}$ and $S(r_N, s_N) = \{s_i : i \in N\}$. A responsibility group is a set $N(r)$, with cardinality $n(r)$, that collects all individuals i in N such that r_i equals r. Similarly, a circumstance group is a set $N(s)$, with cardinality $n(s)$, that collects all individuals i in N such that s_i equals s. We use $n(r, s)$ to denote the number of individuals in N with characteristics (r, s).

An income function $f : R \times S \to \mathbb{R}_{++}$ assigns to each combination of characteristics (r, s) in $R \times S$ a pre-tax income $f(r, s)$.[5] A redistribution mechanism is a function F that assigns to each characteristics profile (r_N, s_N) in D a post-tax income distribution $F(r_N, s_N) = (F_i(r_N, s_N))_{i \in N}$ in $\mathbb{R}^n$. We focus only on budget-balanced mechanisms, i.e., mechanisms such that $\sum_{i \in N} F_i(r_N, s_N) = \sum_{i \in N} f(r_i, s_i)$ for each (r_N, s_N) in D.

[5] The assumption of positive pre-tax income is only necessary for the results involving relative reward axioms. Zero or negative pre-tax income is allowed in all other results.

3 Axioms

3.1 Compensation

Axioms of compensation say that income inequalities due to differences in circumstances ought to be eliminated. We distinguish between ex post and ex ante versions of compensation. The terminology of "ex ante" and "ex post" can be understood as follows. Imagine that an individual's circumstance characteristics are determined temporally before his responsibility characteristics. Ex ante axioms use only the information on circumstance characteristics, whereas ex post axioms use only the information on responsibility characteristics.[6]

Ex post compensation says that two individuals with the same responsibility characteristics should be assigned the same post-tax income.

Ex post compensation. For each (r_N, s_N) in D and all i and j in N, if $r_i = r_j$, then $F_i(r_N, s_N) = F_j(r_N, s_N)$.

Ex ante compensation says that all circumstance groups should receive the same average post-tax income.

Ex ante compensation. For each (r_N, s_N) in D and all s and s' in $S(r_N, s_N)$, we have

$$\frac{1}{n(s)} \sum_{i \in N(s)} F_i(r_N, s_N) = \frac{1}{n(s')} \sum_{i \in N(s')} F_i(r_N, s_N).$$

Ex post and ex ante compensation are compatible. For example, the egalitarian mechanism that assigns to each individual the average pre-tax income in society satisfies both axioms.[7] In contrast, Fleurbaey and Peragine (2013) find an incompatibility between their versions of ex post and ex ante compensation in a related setting. We further discuss their alternative versions of ex ante compensation in Sect. 6.2.

A prominent motivation for ex ante compensation is that it requires to equalize the value of the opportunities of individuals.[8] This motivation assumes that an individual's circumstance characteristics are predetermined, whereas his responsibility characteristics are open to choice. The value of the individual's opportunities is then measured as the average income computed across the responsibility characteristics observed in his circumstance group, that is, the average income of the circumstance group.

[6] The distinction between the informational bases corresponding to the ex ante and ex post perspectives has proven to be of great relevance in the empirical literature. See Ramos and Van de gaer (2016) for a particularly thorough account of this point.

[7] Moreover, the two axioms are independent. Indeed, F^{OACE} satisfies ex ante compensation, but violates ex post compensation, whereas F^{OAEE} satisfies ex post compensation, but violates ex ante compensation. To see these violations, consider $N = \{1, 2, 3\}$ where $(r_N, s_N) = ((r_1, s_1), (r_2, s_2), (r_3, s_3)) = ((\rho, \sigma), (\rho, \sigma'), (\rho', \sigma'))$. Assume that $f(\rho, \sigma) = 10$, $f(\rho, \sigma') = 20$ and $f(\rho', \sigma') = 30$. We have $F^{OACE}(r_N, s_N) = (20, 15, 25)$. Thus, F^{OACE} violates ex post compensation since individuals 1 and 2 do not receive the same post-tax income. We have $F^{OAEE}(r_N, s_N) = (15, 15, 30)$. Thus, F^{OAEE} violates ex ante compensation since the average post-tax income of individuals 1 and 2 is not equal to the average post-tax income of individual 3.

[8] On this motivation, see Ooghe et al. (2007).

However, this motivation loses some of its force if the distribution of responsibility characteristics varies across circumstance groups. For example, average income may be greater in one circumstance group than in another due to the mere coincidence that the former circumstance group has a relatively greater share of individuals with high work effort (which we assume to be a responsibility characteristic for this example) than the latter. Then the difference in average income between the two circumstance groups need not indicate unequal opportunities.

In order to address the above critique, we consider a weaker version of ex ante compensation that applies only to cases in which the distribution of responsibility characteristics is the same in each circumstance group. For a characteristics profile (r_N, s_N) in D, the distribution of responsibility characteristics is the same in each circumstance group if $n(r, s)/n(s) = n(r, s')/n(s')$ for all s and s' in $S(r_N, s_N)$ and each r in $R(r_N, s_N)$. Let $D^* \subset D$ collect all such characteristics profiles. Weak ex ante compensation requires ex ante compensation only for the characteristics profiles in D^*.

Weak ex ante compensation. For each (r_N, s_N) in D^* and all s and s' in $S(r_N, s_N)$, we have

$$\frac{1}{n(s)} \sum_{i \in N(s)} F_i(r_N, s_N) = \frac{1}{n(s')} \sum_{i \in N(s')} F_i(r_N, s_N).$$

Note that ex post compensation implies weak ex ante compensation. Indeed, on the domain D^*, equality of income within each responsibility group implies equality of average post-tax income across all circumstance groups.

3.2 Reward

Axioms of reward justify the inequalities arising from the exercise of individual responsibility. We make two distinctions. First, between ex ante and ex post versions of reward, that is, versions that only use information on circumstance characteristics and versions that only use information on responsibility characteristics.[9] Second, between reward axioms that justify absolute income differentials and those that justify relative ones.

Ex ante reward says that the absolute pre-tax income difference between two individuals with the same circumstance characteristics should be preserved. In other words, both individuals should receive the same transfer, where a transfer is defined as the difference between post-tax and pre-tax income.

Ex ante reward. For each (r_N, s_N) in D and all i and j in N, if $s_i = s_j$, then $F_i(r_N, s_N) - f(r_i, s_i) = F_j(r_N, s_N) - f(r_j, s_j)$.

[9] It is more common to use the terms “ex ante” and “ex post” for compensation axioms than for reward axioms. However, both Fleurbaey and Peragine (2013) and Trannoy (2017) discuss the informational basis of reward axioms and link them to the ex ante and ex post perspectives. We choose to make the informational basis explicit in the names of the reward axioms.

Ex post reward says that all responsibility groups should receive the same average transfer.

Ex post reward. For each (r_N, s_N) in D and all r and r' in $R(r_N, s_N)$, we have

$$\frac{1}{n(r)} \sum_{i \in N(r)} [F_i(r_N, s_N) - f(r_i, s_i)] = \frac{1}{n(r')} \sum_{i \in N(r')} [F_i(r_N, s_N) - f(r_i, s_i)].$$

Ex post reward was recently introduced by Trannoy (2017). In Trannoy's setting, a characteristics profile has exactly one individual for each pair of responsibility and circumstance characteristics. On this restricted domain, ex ante reward implies ex post reward. However, on the full domain, the two axioms are independent.[10]

Weak ex post reward requires ex post reward only for the characteristics profiles in which the distribution of circumstance characteristics is the same for each responsibility group. That is, $n(r, s)/n(r) = n(r', s)/n(r')$ for all r and r' in $R(r_N, s_N)$ and each s in $S(r_N, s_N)$. The subset of D that collects all such characteristics profiles coincides with the set D^* defined above. Indeed, the distribution of circumstance characteristics is the same for each responsibility group if and only if the distribution of responsibility characteristics is the same for each circumstance group.[11]

Weak ex post reward. For each (r_N, s_N) in D^* and all r and r' in $R(r_N, s_N)$, we have

$$\frac{1}{n(r)} \sum_{i \in N(r)} [F_i(r_N, s_N) - f(r_i, s_i)] = \frac{1}{n(r')} \sum_{i \in N(r')} [F_i(r_N, s_N) - f(r_i, s_i)].$$

Note that ex ante reward implies weak ex post reward. On the domain D^*, equality of transfers within each circumstance group implies equality of average transfer across all responsibility groups.

The next three reward axioms are relative variants of the previous three axioms. Relative ex ante reward requires that the relative income differential between two individuals with the same circumstance characteristics be preserved.

Relative ex ante reward. For each (r_N, s_N) in D and all i and j in N, if $s_i = s_j$, then $F_i(r_N, s_N)/f(r_i, s_i) = F_j(r_N, s_N)/f(s_j, r_j)$.

[10] Indeed, F^{OACE} satisfies ex ante reward, but violates ex post reward, whereas F^{OAEE} satisfies ex post reward, but violates ex ante reward. To see the violations, consider again the example in footnote 7. We have $F^{OACE}(r_N, s_N) = (20, 15, 25)$. Since individuals 1 and 2 receive an average transfer of 2.5 and individual 3 receives an average transfer of -5, F^{OACE} violates ex post reward. We have $F^{OAEE}(r_N, s_N) = (15, 15, 30)$. Since the post-tax income difference between individuals 2 and 3 is -15, whereas their pre-tax income difference is -10, F^{OAEE} violates ex ante reward. Moreover, the two axioms are compatible. Indeed, the laissez-faire mechanism that assigns to each individual his pre-tax income satisfies both ex ante reward and ex post reward.

[11] The condition $n(r, s)/n(s) = n(r, s')/n(s')$ is equivalent to $n(r, s)/n(s) = n(r)/n$. The latter is equivalent to $n(r, s)/n(r) = n(s)/n$, which in turn is equivalent to the condition $n(r, s)/n(r) = n(r', s)/n(r')$.

Relative ex post reward says that all responsibility groups should have the same ratio of average post-tax income to average pre-tax income.

Relative ex post reward. For each (r_N, s_N) in D and all r and r' in $R(r_N, s_N)$, we have

$$\frac{\sum_{i \in N(r)} F_i(r_N, s_N)}{\sum_{i \in N(r)} f(r_i, s_i)} = \frac{\sum_{i \in N(r')} F_i(r_N, s_N)}{\sum_{i \in N(r')} f(r_i, s_i)}.$$

Weak relative ex post reward requires relative ex post reward only for the characteristics profiles in D^*.

Weak relative ex post reward. For each (r_N, s_N) in D^* and all r and r' in $R(r_N, s_N)$, we have

$$\frac{\sum_{i \in N(r)} F_i(r_N, s_N)}{\sum_{i \in N(r)} f(r_i, s_i)} = \frac{\sum_{i \in N(r')} F_i(r_N, s_N)}{\sum_{i \in N(r')} f(r_i, s_i)}.$$

3.3 Inequality preservation

The role of compensation and reward axioms is to specify which income inequalities ought to exist between the individuals in society. Compensation focuses on the inequalities between individuals with different circumstance characteristics, whereas reward addresses the inequalities between individuals with different responsibility characteristics. We now consider axioms of inequality preservation, which require these inequalities to remain the same as the population varies.[12]

We consider the case of a shrinking population. Suppose that M is the original society and that N is the society that remains after some individuals have left, taking along their pre-tax incomes. Inequality preservation requires that the income inequalities among the remaining individuals in N are the same as they were in M. To avoid impossibilities, we need to impose that the characteristics profiles (r_N, s_N) and (r_M, s_M) have equal marginal distributions of characteristics.[13] We say that two characteristics profiles (r_N, s_N) and (r_M, s_M) in D have equal marginal distributions of characteristics if the following two conditions are met. First, $R(r_N, s_N) = R(r_M, s_M)$ and $S(r_N, s_N) = S(r_M, s_M)$. Second, $n(r)/n = m(r)/m$ for each r in $R(r_N, s_N)$ and $n(s)/n = m(s)/m$ for each s in $S(r_N, s_N)$.

Figure 1 presents an example of two characteristics profiles with equal marginal distributions of characteristics. There are two distinct responsibility characteristics r^1 and r^2 and three distinct circumstance characteristics s^1, s^2 and s^3. Each cell reports the number of individuals with the corresponding responsibility and circumstance characteristics.

[12] Cappelen and Tungodden (2009) study axioms that require the preservation of inequalities under changes in the characteristics profile rather than under population variations.

[13] All seven characterized redistribution mechanisms violate the strengthenings of the axioms absolute and relative inequality preservation that are obtained by dropping the restriction of equal marginal distributions of characteristics.

N	r^1	r^2	Total
s^1	0	2	2
s^2	1	1	2
s^3	1	3	4
Total	2	6	8

(a) Profile (r_N, s_N)

M	r^1	r^2	Total
s^1	1	2	3
s^2	1	2	3
s^3	1	5	6
Total	3	9	12

(b) Profile (r_M, s_M)

Fig. 1 Equal marginal distributions of characteristics

Absolute inequality preservation requires the absolute income differentials between individuals to be preserved as M shrinks into N.

Absolute inequality preservation. For all (r_N, s_N) and (r_M, s_M) in D with equal marginal distributions of characteristics and with $N \subset M$ and for all i and j in N, we have

$$F_i(r_N, s_N) - F_j(r_N, s_N) = F_i(r_M, s_M) - F_j(r_M, s_M).$$

Relative inequality preservation requires the relative income differentials between individuals to be preserved.

Relative inequality preservation. For all (r_N, s_N) and (r_M, s_M) in D with equal marginal distributions of characteristics and with $N \subset M$ and for all i and j in N, we have

$$\frac{F_i(r_N, s_N)}{F_j(r_N, s_N)} = \frac{F_i(r_M, s_M)}{F_j(r_M, s_M)}.$$

Two remarks are in order. First, the inequality preservation axioms ensure not only the preservation of inequalities between the remaining individuals in N, but also the preservation of their incomes whenever this possibility arises. Formally, if F is a redistribution mechanism that satisfies either absolute or relative inequality preservation, then, for all (r_N, s_N) and (r_M, s_M) in D with equal marginal distributions of characteristics and with $N \subset M$, we have that $\sum_{i \in N} F_i(r_M, s_M) = \sum_{i \in N} f(r_i, s_i)$—which is required for income preservation to be a possibility—implies $F_i(r_N, s_N) = F_i(r_M, s_M)$ for each i in N. This implication reveals a connection between the inequality preservation axioms and the so-called consistency axiom, which has been studied intensively in a variety of resource allocation settings.[14]

Second, given anonymity, the inequality preservation axioms imply that the post-tax income distribution is invariant under replication of the population. Anonymity requires that individuals with the same characteristics receive the same post-tax income.[15]

Anonymity. For each (r_N, s_N) in D and all i and j in N, if $r_i = r_j$ and $s_i = s_j$, then $F_i(r_N, s_N) = F_j(r_N, s_N)$.

[14] See Thomson (2011) for an overview.

[15] Ex post compensation, ex ante reward and relative ex ante reward each imply anonymity.

We next define replication invariance. Given a characteristics profile (r_N, s_N) in D, we say that (r_M, s_M) is a replica of (r_N, s_N) if $N \subset M$ and there exists a natural number $k > 1$ such that $m = k \times n$ and $m(r, s) = k \times n(r, s)$ for each (r, s) in $R \times S$. Replication invariance says the following: for all (r_N, s_N) and (r_M, s_M) in D, if (r_M, s_M) is a replica of (r_N, s_N), then, for each i in N and j in M such that $r_i = r_j$ and $s_i = s_j$, we have that $F_i(r_N, s_N) = F_j(r_M, s_M)$. If F is a redistribution mechanism that satisfies anonymity and either absolute or relative inequality preservation, then F satisfies replication invariance. This implication follows easily from the observation that if (r_M, s_M) is a replica of (r_N, s_N), then (r_N, s_N) and (r_M, s_M) have equal marginal distributions of characteristics.[16]

In the next two sections, we use the compensation, reward and inequality preservation axioms defined above to characterize redistribution mechanisms.

4 Ex post redistribution mechanisms

As shown by Bossert (1995), ex post compensation clashes with ex ante reward.[17] Similarly, ex ante versions of compensation clash with ex post versions of reward (see Appendix A). A natural way to deal with this incompatibility is to stick either to the ex post perspective or to the ex ante perspective in combining compensation and reward. We start in this section with the ex post perspective.

We provide four new characterizations of three familiar mechanisms: the observable average egalitarian equivalent mechanism (Bossert, Fleurbaey and Van de gaer, 1999), the average egalitarian equivalent mechanism (Bossert and Fleurbaey 1996) and the relative average egalitarian equivalent mechanism (Bossert 1995).[18]

First, consider the observable average egalitarian equivalent mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{OAEE}(r_N, s_N) = \frac{1}{n(r_i)} \sum_{j \in N(r_i)} f(r_i, s_j).$$

The mechanism assigns to each individual the average pre-tax income of the individual's responsibility group.

The observable average egalitarian equivalent mechanism is characterized by ex post compensation and ex post reward. Alternatively, it is characterized by ex post compensation and relative ex post reward.

Theorem 1 *(a) A redistribution mechanism F satisfies ex post compensation and ex post reward if and only if $F = F^{OAEE}$.*

[16] Note that all seven redistribution mechanisms that we characterize satisfy replication invariance.

[17] Bossert (1995) shows that the axioms can be combined only if the income function is additively separable. The income function f is said to be additively separable if there exist functions $g : R \to \mathbb{R}$ and $h : S \to \mathbb{R}$ such that $f(r, s) = g(r)+h(s)$ for each (r, s) in $R \times S$. Similarly, it can be shown that ex post compensation and relative ex ante reward can be combined only if the income function f is multiplicatively separable. That is, if there exist $g : R \to \mathbb{R}$ and $h : S \to \mathbb{R}$ such that $f(r, s) = g(r)h(s)$ for each (r, s) in $R \times S$.

[18] What we call the relative average egalitarian equivalent mechanism is referred to as F^* by Bossert (1995) and as the generalized proportionality principle by Cappelen and Tungodden (2017).

(b) A redistribution mechanism F satisfies ex post compensation and relative ex post reward if and only if $F = F^{OAEE}$.

Proof (a) We focus on the "only if" part. Let F be a redistribution mechanism that satisfies ex post compensation and ex post reward. Let (r_N, s_N) be a characteristics profile in D. Let $N(r^1), N(r^2), \dots, N(r^m)$ be the partition of N into responsibility groups.

By ex post compensation, for each k in $\{1, 2, \dots, m\}$ and all i and j in $N(r^k)$, we have $F_i(r_N, s_N) = F_j(r_N, s_N)$. Let $F(N(r^k))$ denote this common value. By ex post reward, there exists a real number α such that, for each k in $\{1, 2, \dots, m\}$,

$$\alpha = F(N(r^k)) - \frac{1}{n(r^k)} \sum_{i \in N(r^k)} f(r_i, s_i),$$

i.e.,

$$F(N(r^k)) = \frac{1}{n(r^k)} \sum_{i \in N(r^k)} f(r_i, s_i) + \alpha. \tag{1}$$

Budget-balancedness and (1) imply

$$\begin{aligned}\sum_{i \in N} f(r_i, s_i) &= n(r^1) F(N(r^1)) + \cdots + n(r^m) F(N(r^m)) \\ &= \sum_{i \in N(r^1)} f(r_i, s_i) + \cdots + \sum_{i \in N(r^m)} f(r_i, s_i) + \alpha \sum_{i=1}^{m} n(r^i) \\ &= \sum_{i \in N} f(r_i, s_i) + \alpha \sum_{i=1}^{m} n(r^i).\end{aligned}$$

Hence, $\alpha = 0$ and (1) becomes

$$F(N(r^k)) = \frac{1}{n(r^k)} \sum_{i \in N(r^k)} f(r_i, s_i).$$

(b) We focus on the "only if" part. Let F be a redistribution mechanism that satisfies ex post compensation and relative ex post reward. Let (r_N, s_N) be a characteristics profile in D. Let $N(r^1), N(r^2), \dots, N(r^m)$ be the partition of N into responsibility groups.

By ex post compensation, for each k in $\{1, 2, \dots, m\}$ and all i and j in $N(r^k)$, we have $F_i(r_N, s_N) = F_j(r_N, s_N)$. Let $F(N(r^k))$ denote this common value. By relative ex post reward, there exists a real number α such that, for each k in $\{1, 2, \dots, m\}$,

$$\alpha = \frac{n(r^k) F(N(r^k))}{\sum_{i \in N(r^k)} f(r_i, s_i)},$$

i.e.,

$$n(r^k)F(N(r^k)) = \alpha \sum_{i \in N(r^k)} f(r_i, s_i). \tag{2}$$

By budget-balancedness and (2), we obtain

$$\begin{aligned}\sum_{i\in N} f(r_i, s_i) &= n(r^1)F(N(r^1)) + \cdots + n(r^m)F(N(r^m)) \\ &= \alpha \left[\sum_{i \in N(r^1)} f(r_i, s_i) + \cdots + \sum_{i \in N(r^m)} f(r_i, s_i) \right] \\ &= \alpha \sum_{i \in N} f(r_i, s_i).\end{aligned}$$

Hence, $\alpha = 1$ and (2) becomes

$$F(N(r^k)) = \frac{1}{n(r^k)} \sum_{i \in N(r^k)} f(r_i, s_i).$$

□

Note that Trannoy (2017) uses the observable average egalitarian equivalent mechanism, adapted to his setting, as an example of a mechanism that satisfies ex post compensation and ex post reward. His interest does not lie in providing a characterization, but rather in proving the compatibility of the two axioms.

Next, we turn to the average egalitarian equivalent mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{AEE}(r_N, s_N) = \frac{1}{n} \sum_{j \in N} f(r_i, s_j) + \frac{1}{n} \sum_{j \in N} f(r_j, s_j) - \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j).$$

The mechanism assigns to each individual i the average pre-tax income in a hypothetical society where all individuals have the same responsibility characteristics as i, plus a uniform constant to balance the budget.

The average egalitarian equivalent mechanism is characterized by absolute inequality preservation, ex post compensation and weak ex post reward.

For the proof, we define the concept of an induced profile. For a characteristics profile (r_N, s_N) in D, the induced profile $(r_{\bar{N}}, s_{\bar{N}})$ is the characteristics profile that meets the following three conditions. First, $N \subset \bar{N}$. Second, $R(r_N, s_N) = R(r_{\bar{N}}, s_{\bar{N}})$ and $S(r_N, s_N) = S(r_{\bar{N}}, s_{\bar{N}})$. Third, $\bar{n}(r, s) = n(r) \times n(s)$ for each r in $R(r_N, s_N)$ and each s in $S(r_N, s_N)$. Note that $\bar{n} = n^2$ as well as $\bar{n}(r) = n(r) \times n$ and $\bar{n}(s) = n(s) \times n$ for each r in $R(r_N, s_N)$ and each s in $S(r_N, s_N)$. It is straightforward to see that the characteristics profiles (r_N, s_N) and $(r_{\bar{N}}, s_{\bar{N}})$ have equal marginal distributions of characteristics and that $(r_{\bar{N}}, s_{\bar{N}})$ is an element of D^*. Figure 2 gives an example of a characteristics profile (r_N, s_N) and the corresponding induced profile $(r_{\bar{N}}, s_{\bar{N}})$.

N	r^1	r^2	r^3	Total
s^1	0	2	1	3
s^2	1	0	3	4
Total	1	2	4	7

(a) Profile (r_N, s_N)

$\bar{N}$	r^1	r^2	r^3	Total
s^1	3	6	12	21
s^2	4	8	16	28
Total	7	14	28	49

(b) Profile $(r_{\bar{N}}, s_{\bar{N}})$

Fig. 2 A characteristics profile and the corresponding induced profile

Theorem 2 *A redistribution mechanism F satisfies absolute inequality preservation, ex post compensation and weak ex post reward if and only if $F = F^{AEE}$.*

Proof We prove the "only if" part. Let F be a redistribution mechanism that satisfies absolute inequality preservation, ex post compensation and weak ex post reward. Let (r_N, s_N) be a characteristics profile in D and let $(r_{\bar{N}}, s_{\bar{N}})$ be the induced characteristics profile in D^*. Let $N(r^1), N(r^2), \ldots, N(r^m)$ be the partition of N into responsibility groups.

Two observations that follow from the definition of $(r_{\bar{N}}, s_{\bar{N}})$ are that, for each k in $\{1, 2, \ldots, m\}$,

$$\bar{n}(r^k) = n(r^k) \times n \tag{3}$$

and

$$\sum_{i \in \bar{N}(r^k)} f(r_i, s_i) = n(r^k) \sum_{i \in N} f(r^k, s_i). \tag{4}$$

Since weak ex post reward and ex post reward coincide on the domain D^* and since the profile $(r_{\bar{N}}, s_{\bar{N}})$ is in D^*, Theorem 1 implies that, for each i in $\bar{N}$,

$$\begin{aligned} F_i(r_{\bar{N}}, s_{\bar{N}}) &= \frac{1}{\bar{n}(r_i)} \sum_{j \in \bar{N}(r_i)} f(r_i, s_j) \\ &= \frac{1}{n(r_i) \times n} \left[n(r_i) \sum_{j \in N} f(r_i, s_j) \right] \\ &= \frac{1}{n} \sum_{j \in N} f(r_i, s_j), \end{aligned} \tag{5}$$

where the second equality follows from (3) and (4). By absolute inequality preservation, there exists a real number α such that, for each i in N,

$$F_i(r_N, s_N) = F_i(r_{\bar{N}}, s_{\bar{N}}) + \alpha. \tag{6}$$

Combining (5) and (6), we get

$$F_i(r_N, s_N) = \frac{1}{n} \sum_{j \in N} f(r_i, s_j) + \alpha. \tag{7}$$

By budget-balancedness and (7),

$$\sum_{i \in N} f(r_i, s_i) = \sum_{i \in N} F_i(r_N, s_N)$$
$$= \frac{1}{n} \sum_{i \in N} \sum_{j \in N} f(r_i, s_j) + n\alpha.$$

Hence,

$$\alpha = \frac{1}{n} \sum_{i \in N} f(r_i, s_i) - \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j).$$

Plugging this into (7), we obtain

$$F_i(r_N, s_N) = \frac{1}{n} \sum_{j \in N} f(r_i, s_j) + \frac{1}{n} \sum_{j \in N} f(r_j, s_j) - \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j).$$

□

Finally, we define the relative average egalitarian equivalent mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{RAEE}(r_N, s_N) = \frac{\sum_{j \in N} f(r_i, s_j)}{\sum_{k \in N} \sum_{j \in N} f(r_k, s_j)} \sum_{j \in N} f(r_j, s_j).$$

The mechanism assigns to each individual i the average pre-tax income in a hypothetical society where all individuals have the same responsibility characteristics as i, times a uniform constant to balance the budget.

The relative average egalitarian equivalent mechanism is characterized by relative inequality preservation, ex post compensation and weak relative ex post reward.

Theorem 3 *A redistribution mechanism F satisfies relative inequality preservation, ex post compensation and weak relative ex post reward if and only if $F = F^{RAEE}$.*

Proof We focus on the "only if" part. Let F be a redistribution mechanism that satisfies relative inequality preservation, ex post compensation and relative weak ex post reward. Let (r_N, s_N) be a characteristics profile in D and let $(r_{\bar{N}}, s_{\bar{N}})$ be the induced characteristics profile in D^*. Let $N(r^1), N(r^2), \ldots, N(r^m)$ be the partition of N into responsibility groups.

Since weak relative ex post reward and relative ex post reward coincide on the domain D^* and since $(r_{\bar{N}}, s_{\bar{N}})$ is in D^*, Theorem 1 implies that, for each i in $\bar{N}$,

$$\begin{aligned} F_i(r_{\bar{N}}, s_{\bar{N}}) &= \frac{1}{\bar{n}(r_i)} \sum_{j \in \bar{N}(r_i)} f(r_i, s_j) \\ &= \frac{1}{n(r_i) \times n} \left[n(r_i) \sum_{j \in N} f(r_i, s_j) \right] \\ &= \frac{1}{n} \sum_{j \in N} f(r_i, s_j), \end{aligned} \tag{8}$$

where the second equation follows from (3) and (4). By relative inequality preservation, there exists a real number α such that, for each i in N,

$$F_i(r_N, s_N) = F_i(r_{\bar{N}}, s_{\bar{N}}) \times \alpha. \tag{9}$$

Combining (8) and (9), we get

$$F_i(r_N, s_N) = \alpha \frac{1}{n} \sum_{j \in N} f(r_i, s_j). \tag{10}$$

By budget-balancedness and (10),

$$\begin{aligned} \sum_{i \in N} f(r_i, s_i) &= \sum_{i \in N} F_i(r_N, s_N) \\ &= \alpha \frac{1}{n} \sum_{i \in N} \sum_{j \in N} f(r_i, s_j). \end{aligned}$$

Hence,

$$\alpha = \frac{\sum_{i \in N} f(r_i, s_i)}{\frac{1}{n} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j)}.$$

Plugging this into (10) gives

$$F_i(r_N, s_N) = \frac{\sum_{i \in N} f(r_i, s_i)}{\sum_{k \in N} \sum_{j \in N} f(r_k, s_j)} \sum_{j \in N} f(r_i, s_j).$$

□

5 Ex ante redistribution mechanisms

We now turn to the ex ante perspective. We provide new characterizations of two familiar mechanisms: the observable average conditionally egalitarian mechanism (Bossert et al. 1999) and the average conditionally egalitarian mechanism (Bossert and Fleurbaey 1996). Moreover, we introduce and characterize two new mechanisms: the relative observable average conditionally egalitarian mechanism and the relative average conditionally egalitarian mechanism.

First, consider the observable average conditionally egalitarian mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{OACE}(r_N, s_N) = f(r_i, s_i) + \frac{1}{n}\sum_{j \in N} f(r_j, s_j) - \frac{1}{n(s_i)} \sum_{j \in N(s_i)} f(r_j, s_j).$$

The mechanism assigns to each individual i his pre-tax income plus the difference between the average pre-tax income in society and the average pre-tax income in i's circumstance group.

The observable average conditionally egalitarian mechanism is characterized by ex ante compensation and ex ante reward. The idea for this characterization is due to Bossert et al. (1999, p. 50). We provide the easy proof for the sake of completeness.

Theorem 4 *A redistribution mechanism F satisfies ex ante compensation and ex ante reward if and only if $F = F^{OACE}$.*

Proof We focus on the "only if" part. Let F be a redistribution mechanism that satisfies ex ante compensation and liberal reward. Let (r_N, s_N) be a characteristics profile in D. Let $N(s^1), N(s^2), \ldots, N(s^m)$ be the partition of N into circumstance group.

By ex ante compensation, for each k in $\{1, 2, \ldots, m\}$,

$$\frac{1}{n}\sum_{i \in N} f(r_i, s_i) = \frac{1}{n(s^k)} \sum_{i \in N(s^k)} F_i(r_N, s_N). \tag{11}$$

By ex ante reward, for each k in $\{1, 2, \ldots, m\}$, there exists a real number α_k such that, for each i in $N(s^k)$,

$$F_i(r_N, s_N) = f(r_i, s^k) + \alpha_k. \tag{12}$$

Plugging (12) into (11) yields

$$\frac{1}{n}\sum_{i \in N} f(r_i, s_i) = \frac{1}{n(s^k)} \sum_{i \in N(s^k)} f(r_i, s^k) + \alpha_k,$$

i.e.,

$$\alpha_k = \frac{1}{n}\sum_{i \in N} f(r_i, s_i) - \frac{1}{n(s^k)} \sum_{i \in N(s^k)} f(r_i, s^k).$$

Plugging this back into (12) yields

$$F_i(r_N, s_N) = f(r_i, s^k) + \frac{1}{n}\sum_{i \in N} f(r_i, s_i) - \frac{1}{n(s^k)} \sum_{i \in N(s^k)} f(r_i, s^k).$$

□

Second, consider the average conditionally egalitarian mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{ACE}(r_N, s_N) = f(r_i, s_i) - \frac{1}{n} \sum_{j \in N} f(r_j, s_i) + \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j).$$

The mechanism assigns to each individual i the difference between i's pre-tax income and the average pre-tax income in a hypothetical society where all individuals have the same circumstance characteristics as i, plus a uniform constant to balance the budget.

The average conditionally egalitarian mechanism is characterized by absolute inequality preservation, weak ex ante compensation and ex ante reward.

Theorem 5 *A redistribution mechanism F satisfies absolute inequality preservation, weak ex ante compensation and ex ante reward if and only if $F = F^{ACE}$.*

Proof We focus on the "only if" part. Let F be a redistribution mechanism that satisfies absolute inequality preservation, weak ex ante compensation and ex ante reward. Let (r_N, s_N) be a characteristics profile in D and let $(r_{\bar{N}}, s_{\bar{N}})$ be the induced characteristics profile in D^*. Let $N(s^1), N(s^2), \ldots, N(s^m)$ be the partition of N into circumstance groups.

Two observations that follow from the definition of $(r_{\bar{N}}, s_{\bar{N}})$ are that, for each k in $\{1, 2, \ldots, m\}$,

$$\bar{n}(s^k) = n(s^k) \times n \tag{13}$$

and

$$\sum_{i \in \bar{N}(s^k)} f(r_i, s_i) = n(s^k) \sum_{i \in N} f(r_i, s^k). \tag{14}$$

Since weak ex ante compensation and ex ante compensation coincide on the domain D^* and since $(r_{\bar{N}}, s_{\bar{N}})$ is in D^*, Theorem 4 implies that, for each i in $\bar{N}$,

$$\begin{aligned} F_i(r_{\bar{N}}, s_{\bar{N}}) &= f(r_i, s_i) + \frac{1}{\bar{n}} \sum_{j \in \bar{N}} f(r_j, s_j) - \frac{1}{\bar{n}(s_i)} \sum_{j \in \bar{N}(s_i)} f(r_j, s_j) \\ &= f(r_i, s_i) + \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j) - \frac{1}{n(s_i) \times n} \left[n(s_i) \sum_{j \in N} f(r_j, s_i) \right] \\ &= f(r_i, s_i) + \frac{1}{n^2} \sum_{k \in N} \sum_{j \in N} f(r_k, s_j) - \frac{1}{n} \sum_{j \in N} f(r_j, s_i), \end{aligned} \tag{15}$$

where the second equality follows from (13) and (14).

By absolute inequality preservation, there exists a real number α such that, for each i in N,

$$F_i(r_N, s_N) = F_i(r_{\bar{N}}, s_{\bar{N}}) + \alpha. \tag{16}$$

Combining (15) and (16), we get

$$F_i(r_N, s_N) = f(r_i, s_i) + \frac{1}{n^2}\sum_{k\in N}\sum_{j\in N} f(r_k, s_j) - \frac{1}{n}\sum_{j\in N} f(r_j, s_i) + \alpha. \tag{17}$$

By budget-balancedness and (17),

$$\begin{aligned}\sum_{i\in N} f(r_i, s_i) &= \sum_{i\in N} F_i(r_N, s_N)\\ &= \sum_{i\in N} f(r_i, s_i) + \frac{1}{n}\sum_{k\in N}\sum_{j\in N} f(r_k, s_j) - \frac{1}{n}\sum_{k\in N}\sum_{j\in N} f(r_k, s_j) + n\alpha.\end{aligned}$$

Hence, $\alpha = 0$ and (17) becomes

$$F_i(r_N, s_N) = f(r_i, s_i) + \frac{1}{n^2}\sum_{k\in N}\sum_{j\in N} f(r_k, s_j) - \frac{1}{n}\sum_{j\in N} f(r_j, s_i).$$

□

Next, we define the relative observable average conditionally egalitarian mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{ROACE}(r_N, s_N) = \frac{f(r_i, s_i)}{\frac{1}{n(s_i)}\sum_{j\in N(s_i)} f(r_j, s_j)}\left[\frac{\sum_{j\in N} f(r_j, s_j)}{n}\right].$$

The mechanism assigns to each individual the average pre-tax income times the ratio of his pre-tax income to the average pre-tax income of his circumstance group.

The relative observable average conditionally egalitarian mechanism is characterized by ex ante compensation and relative ex ante reward.

Theorem 6 *A redistribution mechanism F satisfies ex ante compensation and relative ex ante reward if and only if $F = F^{ROACE}$.*

Proof We focus on the "only if" part. Let F be a redistribution mechanism that satisfies ex ante compensation and relative ex ante reward. Let (r_N, s_N) be a characteristics profile in D. Let $N(s^1), N(s^2), \ldots, N(s^m)$ be the partition of N into circumstance groups.

By ex ante compensation, for each k in $\{1, 2, \ldots, m\}$,

$$\frac{1}{n(s^k)}\sum_{i\in N(s^k)} F_i(r_N, s_N) = \frac{1}{n}\sum_{i\in N} f(r_i, s_i). \tag{18}$$

By relative ex ante reward, for each k in $\{1, 2, \ldots, m\}$ and all i and j in $N(s^k)$,

$$\frac{F_i(r_N, s_N)}{f(r_i, s^k)} = \frac{F_j(r_N, s_N)}{f(r_j, s^k)},$$

i.e.,

$$F_j(r_N, s_N) = F_i(r_N, s_N)\frac{f(r_j, s^k)}{f(r_i, s^k)}.$$

Thus,

$$\sum_{j\in N(s^k)} F_j(r_N, s_N) = \sum_{j\in N(s^k)} F_i(r_N, s_N)\frac{f(r_j, s^k)}{f(r_i, s^k)} = \frac{F_i(r_N, s_N)}{f(r_i, s^k)} \sum_{j\in N(s^k)} f(r_j, s^k).$$

Plugging this into (18) yields

$$\frac{1}{n(s^k)}\frac{F_i(r_N, s_N)}{f(r_i, s^k)}\left[\sum_{j\in N(s^k)} f(r_j, s^k)\right] = \frac{1}{n}\sum_{j\in N} f(r_j, s_j),$$

i.e.,

$$F_i(r_N, s_N) = \frac{f(r_i, s_i)}{\frac{1}{n(s_i)}\sum_{j\in N(s_i)} f(r_j, s_j)}\left[\frac{\sum_{j\in N} f(r_j, s_j)}{n}\right].$$

□

Finally, we define the relative average conditionally egalitarian mechanism: for each characteristics profile (r_N, s_N) in D and each i in N, we have

$$F_i^{RACE}(r_N, s_N) = \frac{f(r_i, s_i)}{\sum_{k\in N} f(r_k, s_i)}\left[\frac{\sum_{k\in N} f(r_k, s_k)}{\sum_{k\in N}\left[\frac{f(r_k, s_k)}{\sum_{l\in N} f(r_l, s_k)}\right]}\right].$$

The mechanism assigns to each individual i the average pre-tax income times the ratio of i's pre-tax income to the average pre-tax income in a hypothetical society where all individuals have the same circumstance characteristics as i, times a uniform constant to balance the budget.

The relative average conditionally egalitarian mechanism is characterized by relative inequality preservation, weak ex ante compensation and relative ex ante reward.

Theorem 7 *A redistribution mechanism F satisfies relative inequality preservation, weak ex ante compensation and relative ex ante reward if and only if $F = F^{RACE}$.*

Proof We focus on the "only if" part. Let F be a mechanism that satisfies relative inequality preservation, weak ex ante compensation and relative ex ante reward. Let (r_N, s_N) be a characteristics profile in D and let $(r_{\bar{N}}, s_{\bar{N}})$ be the induced characteristics profile in D^*. Let $N(s^1), N(s^2), \ldots, N(s^m)$ be the partition of N into circumstance groups.

Since weak ex ante compensation and ex ante compensation coincide on the domain D^* and since $(r_{\bar{N}}, s_{\bar{N}})$ is in D^*, Theorem 6 implies that, for each i in $\bar{N}$,

$$\begin{aligned} F_i(r_{\bar{N}}, s_{\bar{N}}) &= \frac{f(r_i, s_i)}{\frac{1}{\bar{n}(s_i)} \sum_{j \in \bar{N}(s_i)} f(r_j, s_j)} \left[\frac{\sum_{j \in \bar{N}} f(r_j, s_j)}{\bar{n}} \right] \\ &= \frac{f(r_i, s_i)}{\frac{1}{n(s_i) \times n} n(s_i) \sum_{j \in N} f(r_j, s_i)} \left[\frac{\sum_{j \in N} \sum_{k \in N} f(r_j, s_k)}{n^2} \right] \\ &= \frac{f(r_i, s_i)}{\sum_{j \in N} f(r_j, s_i)} \left[\frac{\sum_{j \in N} \sum_{k \in N} f(r_j, s_k)}{n} \right], \end{aligned} \tag{19}$$

where the second equality follows from (13) and (14).

By relative inequality preservation, there exists a real number α such that, for each i in N,

$$F_i(r_N, s_N) = F_i(r_{\bar{N}}, s_{\bar{N}}) \times \alpha. \tag{20}$$

Combining (19) and (20), we get

$$F_i(r_N, s_N) = \alpha \frac{f(r_i, s_i)}{\sum_{j \in N} f(r_j, s_i)} \left[\frac{\sum_{j \in N} \sum_{k \in N} f(r_j, s_k)}{n} \right]. \tag{21}$$

By budget-balancedness and (21),

$$\begin{aligned} \sum_{i \in N} f(r_i, s_i) &= \sum_{i \in N} F_i(r_N, s_N) \\ &= \alpha \left[\frac{\sum_{j \in N} \sum_{k \in N} f(r_j, s_k)}{n} \right] \sum_{i \in N} \left[\frac{f(r_i, s_i)}{\sum_{j \in N} f(r_j, s_i)} \right]. \end{aligned}$$

Hence,

$$\alpha = \frac{\sum_{i \in N} f(r_i, s_i) \left[\frac{n}{\sum_{k \in N} \sum_{j \in N} f(r_k, s_j)} \right]}{\sum_{i \in N} \left[\frac{f(r_i, s_i)}{\sum_{j \in N} f(r_j, s_i)} \right]}.$$

Plugging this into (21) gives

$$F_i(r_N, s_N) = \frac{f(r_i, s_i)}{\sum_{j \in N} f(r_j, s_i)} \frac{\sum_{j \in N} f(r_j, s_j)}{\sum_{k \in N} \left[\frac{f(r_k, s_k)}{\sum_{j \in N} f(r_j, s_k)} \right]}.$$

□

Table 2 Characterizations by Bossert and Fleurbaey (1996) and Bossert et al. (1999)

Mechanism	Compensation axiom	Reward axiom
AEE	Weak group solidarity in s	Equal transfer for uniform s
ACE	Equal income for uniform r	Weak individual monotonicity in r
OAEE	Conditional weak group solidarity in s	Equal transfer for partially uniform s
OACE	Equal income for partially uniform r	Conditional weak individual monotonicity in r

Each of these four characterizations assumes that the redistribution mechanism satisfies anonymity

6 Connections to the literature

6.1 Alternative compensation and reward axioms

Table 2 presents the alternative characterizations of the average egalitarian equivalent and average conditionally egalitarian mechanisms by Bossert and Fleurbaey (1996) and of the observable average egalitarian equivalent and observable average conditionally egalitarian mechanisms by Bossert et al. (1999). Each characterization uses one compensation and one reward axiom. In this subsection, we discuss the relationship between these axioms and ours.[19]

We begin with the compensation axioms. First, group solidarity in s says that each individual should gain or lose the same amount of post-tax income if some individual's circumstance characteristics change (Bossert and Fleurbaey 1996). Second, weak group solidarity in s says that the change in the post-tax income of an individual should not depend on whether this individual is the one whose circumstance characteristics have changed (Bossert and Fleurbaey 1996). Third, conditional weak group solidarity in s says that the change in the post-tax income of an individual should not depend on whether this individual is the one whose circumstance characteristics have changed, but can depend on whether this individual has the same responsibility characteristics as the one whose circumstance characteristics have changed (Bossert et al. 1999).

Group solidarity in s (GSS). For all (r_N, s_N) and (r_N, s'_N) in D, if there exists a j in N such that $s_j \neq s'_j$ with (r_N, s_N) and (r_N, s'_N) coinciding everywhere else, then, for each i in N, we have

$$F_i(r_N, s'_N) - F_i(r_N, s_N) = \frac{1}{n}[f(r_j, s'_j) - f(r_j, s_j)].$$

Weak group solidarity in s (WGSS). There exists a function $\phi : R \times S^3 \times \cup_{N \in \mathcal{N}} R^n \rightarrow \mathbb{R}$ such that, for all (r_N, s_N) and (r_N, s'_N) in D, if there exists a j in N such that $s_j \neq s'_j$ with (r_N, s_N) and (r_N, s'_N) coinciding everywhere else, then, for each i in N, we have

[19] We rephrase the axioms of Bossert and Fleurbaey (1996) and Bossert et al. (1999) to fit into our variable-population setting.

$$F_i(r_N, s'_N) - F_i(r_N, s_N) = \frac{1}{n}[f(r_j, s'_j) - f(r_j, s_j)] + \phi(r_i, s_i, s_j, s'_j, r_N).$$

Conditional weak group solidarity in s **(CWGSS).** There exists a function $\phi : R \times S^3 \times \cup_{N \in \mathcal{N}} R^n \times \{0, 1\} \to \mathbb{R}$ such that, for all (r_N, s_N) and (r_N, s'_N) in D, if there exists a j in N such that $s_j \neq s'_j$ with (r_N, s_N) and (r_N, s'_N) coinciding everywhere else, then, for each i in N, we have

$$F_i(r_N, s'_N) - F_i(r_N, s_N) = \frac{1}{n}[f(r_j, s'_j) - f(r_j, s_j)] + \phi(r_i, s_i, s_j, s'_j, r_N, I),$$

where $I = 0$ if $r_i = r_j$ and $I = 1$ if $r_i \neq r_j$.

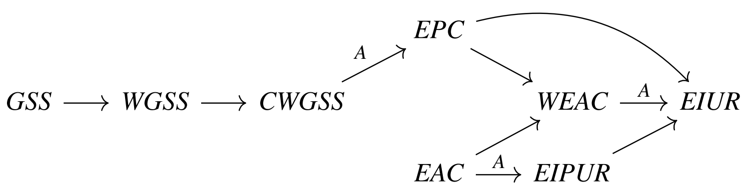
Next, equal income for partially uniform r says that if there is a uniform responsibility characteristic within each circumstance group, then all individuals should receive the same post-tax income (Bossert et al. 1999). Finally, equal income for uniform responsibility says that if there is a uniform responsibility characteristic in society, then all individuals should receive the same post-tax income (Bossert and Fleurbaey 1996).

Equal income for partially uniform r **(EIPUR).** For each (r_N, s_N) in D, if $r_i = r_j$ for all i and j in $N(s)$ for each s in $S(r_N, s_N)$, then $F_i(r_N, s_N) = F_j(r_N, s_N)$ for all i and j in N.

Equal income for uniform r **(EIUR).** For each (r_N, s_N) in D, if $r_i = r_j$ for all i and j in N, then $F_i(r_N, s_N) = F_j(r_N, s_N)$ for all i and j in N.

Proposition 1 presents the relationships between the five compensation axioms defined in this subsection and ex post compensation (EPC), ex ante compensation (EAC) and weak ex ante compensation (WEAC). The proof of the proposition is in Appendix B.

Proposition 1 *The relationships between the compensation axioms are as follows:*



An arrow from axiom X to axiom X′ signifies that X implies X′. An A indicates that the implication holds only for anonymous redistribution mechanisms.

We now turn to the reward axioms used by Bossert and Fleurbaey (1996) and Bossert et al. (1999). First, individual monotonicity in r says that a change in an individual's responsibility characteristics should only affect this individual's post-tax income (Bossert and Fleurbaey 1996). In other words, the change should not affect any individual's transfer. Second, weak individual monotonicity in r says that the change in the transfer that an individual receives should not depend on whether this individual is the one whose responsibility characteristics have changed (Bossert and Fleurbaey 1996). Third, conditional weak individual monotonicity in r says that the change in the transfer that an individual receives should not depend on whether this individual is the one whose responsibility characteristics have changed, but can depend on whether this individual has the same circumstance characteristics as the one whose responsibility characteristics have changed (Bossert et al. 1999).

Individual monotonicity in r (IMR). For all (r_N, s_N) and (r'_N, s_N) in D, if there exists a j in N such that $r_j \neq r'_j$ with (r_N, s_N) and (r'_N, s_N) coinciding everywhere else, then, for each i in N, we have

$$F_i(r'_N, s_N) - f(r'_i, s_i) = F_i(r_N, s_N) - f(r_i, s_i).$$

Weak individual monotonicity in r (WIMR). There exists a function $\xi : R \times S \times R^2 \times \cup_{N \in \mathcal{N}} S^n \to \mathbb{R}$ such that, for all (r_N, s_N) and (r'_N, s_N) in D, if there exists a j in N such that $r_j \neq r'_j$ with (r_N, s_N) and (r'_N, s_N) coinciding everywhere else, then, for each i in N, we have

$$F_i(r'_N, s_N) - f(r'_i, s_i) = F_i(r_N, s_N) - f(r_i, s_i) + \xi(r_i, s_i, r_j, r'_j, s_N).$$

Conditional weak individual monotonicity in r (CWIMR). There exists a function $\xi : R \times S \times R^2 \times \cup_{N \in \mathcal{N}} S^n \times \{0, 1\} \to \mathbb{R}$ such that, for all (r_N, s_N) and (r'_N, s_N) in D, if there exists a j in N such that $r_j \neq r'_j$ with (r_N, s_N) and (r'_N, s_N) coinciding everywhere else, then, for each i in N, we have

$$F_i(r'_N, s_N) - f(r'_i, s_i) = F_i(r_N, s_N) - f(r_i, s_i) + \xi(r_i, s_i, r_j, r'_j, s_N, I),$$

where $I = 0$ if $s_i = s_j$ and $I = 1$ if $s_i \neq s_j$.

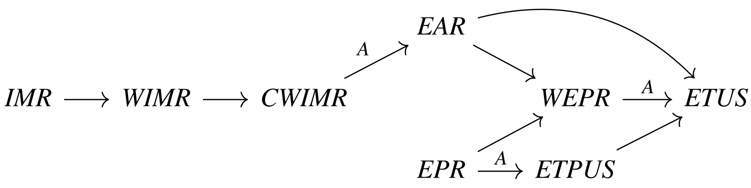
The next axiom, equal transfer for partially uniform s, says that if there is a uniform circumstance characteristic within each responsibility group, then all individuals should receive the same transfer, which by budget-balancedness must be zero (Bossert et al. 1999). Finally, equal transfer for uniform circumstance characteristics says that if there is a uniform circumstance characteristic in society, then all individuals should receive the same transfer (Bossert and Fleurbaey 1996).

Equal transfer for partially uniform s (ETPUS). For each (r_N, s_N) in D, if $s_i = s_j$ for all i and j in $N(r)$ for each r in $R(r_N, s_N)$, then $F_i(r_N, s_N) = f(r_i, s_i)$ for each i in N.

Equal transfer for uniform s (ETUS). For each (r_N, s_N) in D, if $s_i = s_j$ for all i and j in N, then $F_i(r_N, s_N) = f(r_i, s_i)$ for each i in N.

Proposition 2 summarizes the relationships between the five reward axioms defined in this subsection and ex ante reward (EAR), ex post reward (EPR) and weak ex post reward (WEPR). The proof is in Appendix B.

Proposition 2 *The relationships between the reward axioms are as follows:*



An arrow from axiom X to axiom X′ signifies that X implies X′. An A indicates that the implication holds only for anonymous redistribution mechanisms.

6.2 Alternatives to ex ante compensation

Ex ante compensation requires that there are no two circumstance groups such that one is better off than the other in terms of average post-tax income. Many alternative expressions of this requirement can be obtained by varying the criterion used to compare circumstance groups. We discuss two such alternative axioms, which are based on axioms proposed by Fleurbaey and Peragine (2013) in a related setting.[20]

No dominance says that there should be no two circumstance groups such that each post-tax income in one group is strictly greater than each post-tax income in the other group.

No dominance. For each (r_N, s_N) in D, there exist no s and s' in $S(r_N, s_N)$ such that $\min\{F_i(r_N, s_N) : i \in N(s)\} > \max\{F_i(r_N, s_N) : i \in N(s')\}$.

The next axiom strengthens no dominance. Strong no dominance says that there should be no two circumstance groups such that, for the individuals in these groups sharing the same responsibility characteristics, those in one group have a strictly greater post-tax income than those in the other.

Strong no dominance. For each (r_N, s_N) in D, there exist no s and s' in $S(r_N, s_N)$ such that, for each i in $N(s)$ and each j in $N(s')$, we have that $r_i = r_j$ implies $F_i(r_N, s_N) > F_j(r_N, s_N)$.

Ex ante compensation implies no dominance, but is independent of strong no dominance.

We examine the compatibility of each of the two alternative axioms with ex ante reward. First, we show that there are many mechanisms that satisfy no dominance and ex ante reward. Consider a mechanism F that everywhere coincides with F^{OACE},

[20] There are important differences between our setting and that of Fleurbaey and Peragine (2013). First, they study rankings of all post-tax income distributions rather than mechanisms that select the best distribution. In Bosmans and Öztürk (forthcoming) we discuss the ex post and ex ante perspectives in the setting of rankings. Second, they restrict the domain of characteristics profiles to those in which each responsibility characteristic in R occurs in each circumstance group. The alternatives to ex ante compensation that we present here remain meaningful also without this domain restriction.

except for the following characteristics profile:

$$(r_N, s_N) = ((r_1, s_1), (r_2, s_2), (r_3, s_3), (r_4, s_4)) = ((\rho, \sigma), (\rho', \sigma), (\rho, \sigma'), (\rho', \sigma')).$$

Assume that $f(\rho, \sigma) = 10$, $f(\rho', \sigma) = 30$ and $f(\rho, \sigma') = f(\rho', \sigma') = 20$. We have $F^{OACE}(r_N, s_N) = (10, 30, 20, 20)$. Let

$$F(r_N, s_N) = (10 + \delta, 30 + \delta, 20 - \delta, 20 - \delta).$$

The mechanism F satisfies no dominance and ex ante reward for each real number δ in the interval $[-5, 5]$. Hence, there are many mechanisms that satisfy no dominance and ex ante reward.

Second, we show that there are no redistribution mechanisms that satisfy strong no dominance and ex ante reward.[21] Consider the following characteristics profile:

$$\begin{aligned}(r_N, s_N) &= ((r_1, s_1), (r_2, s_2), (r_3, s_3), (r_4, s_4), (r_5, s_5), (r_6, s_6)) \\ &= ((\rho, \sigma), (\rho, \sigma'), (\rho', \sigma'), (\rho', \sigma''), (\rho, \sigma'''), (\rho', \sigma''')).\end{aligned}$$

Assume that $f(\rho, \sigma) = f(\rho, \sigma') = 4$ and $f(\rho', \sigma') = f(\rho', \sigma'') = f(\rho, \sigma''') = f(\rho', \sigma''') = 8$. By ex ante reward, there exist real numbers δ, δ', δ'' and δ''' such that

$$\begin{aligned}\delta &= F_1(r_N, s_N) - f(\rho, \sigma), \\ \delta' &= F_2(r_N, s_N) - f(\rho, \sigma') = F_3(r_N, s_N) - f(\rho', \sigma'), \\ \delta'' &= F_4(r_N, s_N) - f(\rho', \sigma''), \text{ and} \\ \delta''' &= F_5(r_N, s_N) - f(\rho, \sigma''') = F_6(r_N, s_N) - f(\rho', \sigma''').\end{aligned}$$

Strong no dominance requires that $\delta = \delta'$, $\delta' = \delta''$ and $\delta'' = \delta'''$, but also that $\delta = \delta''' + 4$. Hence, ex ante reward and strong no dominance cannot be jointly satisfied.

Several further alternatives to ex ante compensation are worth examining. For example, Lefranc et al. (2008) suggest first order stochastic dominance as a criterion to compare circumstance groups. We leave the examination of this and other alternatives for further research.

7 Conclusion

We have identified the redistribution mechanisms that satisfy ex post versions of compensation and reward, as well as those that satisfy ex ante versions of these axioms. Our results identify the mechanisms of the egalitarian equivalent type as representing the ex post perspective and those of the conditionally egalitarian type as representing the ex ante perspective.

[21] The two axioms are compatible if we restrict the domain, as do Fleurbaey and Peragine (2013), to characteristics profiles in which each responsibility characteristic in R occurs in each circumstance group.

Moreover, we have introduced relative versions of reward and studied the implication of substituting these for the standard absolute versions. This has given rise to two new redistribution mechanisms, both of which belong to the ex ante perspective.

Several of our characterizations have made use of variable-population axioms of inequality preservation, a first for this setting. We believe it would be interesting to further explore the extension to variable populations.

Appendix A

We address the (in)compatibilities between the axioms reported in Table 1. We start by showing that there are no mechanisms that satisfy ex ante compensation and ex post reward. Consider

$$(r_N, s_N) = ((r_1, s_1), (r_2, s_2), (r_3, s_3)) = ((\rho, \sigma), (\rho, \sigma'), (\rho', \sigma'')).$$

Assume that $f(\rho, \sigma) = 6$, $f(\rho, \sigma') = 4$ and $f(\rho', \sigma'') = 8$. Ex ante compensation requires a transfer of 2 from individual 3 to individual 2. Then, the average transfer in $N(\rho)$ would be 1, whereas the average transfer in $N(\rho')$ would be -2, which violates ex post reward.

Next, we show that there are multiple mechanisms that satisfy each of the following three combinations: weak ex ante compensation and ex post reward, ex ante compensation and weak ex post reward, and weak ex ante compensation and weak ex post reward.

First, we consider the combination of weak ex ante compensation and ex post reward. Consider a mechanism F^1 that coincides with F^{OAEE} everywhere except for the following characteristics profile:

$$(r_N, s_N) = ((r_1, s_1), (r_2, s_2), (r_3, s_3), (r_4, s_4)) = ((\rho, \sigma), (\rho, \sigma'), (\rho', \sigma), (\rho', \sigma')).$$

Assume that $f(\rho, \sigma) = 10$, $f(\rho, \sigma') = 20$, $f(\rho', \sigma) = 30$ and $f(\rho', \sigma') = 40$. We have $F^{OAEE}(r_N, s_N) = (15, 15, 35, 35)$. Assume that we have

$$F^1(r_N, s_N) = (15 - \delta, 15 + \delta, 35 + \delta, 35 - \delta).$$

The mechanism F^1 satisfies weak ex ante compensation and ex post reward for any real number δ. Hence, many mechanisms satisfy weak ex ante compensation and ex post reward.

Second, we consider the combination of ex ante compensation and weak ex post reward. Consider a mechanism F^2 that coincides with F^{OACE} everywhere except for the characteristics profile defined in the previous paragraph. We have $F^{OACE}(r_N, s_N) = (15, 15, 35, 35)$. Assume that we have

$$F^2(r_N, s_N) = (15 - \delta, 15 + \delta, 35 + \delta, 35 - \delta).$$

The mechanism F^2 satisfies ex ante compensation and weak ex post reward for any real number δ. Hence, many mechanisms satisfy ex ante compensation and weak ex post reward.

Third, we consider the combination of weak ex ante compensation and weak ex post reward. The mechanisms F^1 and F^2 defined above satisfy these axioms for any real number δ.

We omit the demonstrations of the remaining incompatibility (of ex ante compensation and relative ex post reward) and compatibilities (of weak ex ante compensation and relative ex post reward, of ex ante compensation and weak relative ex post reward, and of weak ex ante compensation and weak relative ex post reward) in Table 1. These are easily established using examples similar to those above.

Appendix B

Proof of Proposition 1 Bossert and Fleurbaey (1996) have established the following implications: $GSS \to WGSS$, $WGSS \overset{A}{\to} EPC$ and $EPC \to EIUR$. We consider the remaining implications.

(a) $WGSS \to CWGS$. The statement is trivial and we omit the proof.

(b) $CWGSS \overset{A}{\to} EPC$. First, we show that the implication does not hold without anonymity. Consider the mechanism F: there exist k and ℓ in $\mathcal{N}$ such that, for each profile (r_N, s_N) in D for which both k and ℓ are in N, we have, for each i in N,

$$F_i(r_N, s_N) = \begin{cases} \frac{1}{n}\sum_{j\in N} f(r_j, s_j) + 1 & \text{if } i = k, \\ \frac{1}{n}\sum_{j\in N} f(r_j, s_j) - 1 & \text{if } i = \ell, \\ \frac{1}{n}\sum_{j\in N} f(r_j, s_j) & \text{if } i \neq k, \ell, \end{cases}$$

and, for every other profile (r_N, s_N) in D, we have $F_i(r_N, s_N) = \sum_{j\in N} f(r_j, s_j)/n$ for each i in N.

The mechanism F satisfies group solidarity in s, and hence conditional weak group solidarity in s, but violates anonymity and ex post compensation.

Next, we show that conditional weak group solidarity in s and anonymity imply ex post compensation. We follow the reasoning used by Bossert and Fleurbaey (1996) when they show that weak group solidarity in s implies ex post compensation.

Let F be a mechanism that satisfies anonymity and conditional weak group solidarity in s. Let (r_N, s_N) in D be such that there exist i and j in N for which $r_i = r_j$ and $s_i \neq s_j$. We have to show that $F_i(r_N, s_N) = F_j(r_N, s_N)$.

Let s'_N in S^n be such that $s'_j = s_i$ with s_N and s'_N coinciding everywhere else. Since $(r_i, s_i) = (r_j, s'_j)$, we have $F_i(r_N, s'_N) = F_j(r_N, s'_N)$ by anonymity. Conditional weak group solidarity in s requires

$$F_i(r_N, s_N) - F_i(r_N, s'_N) = \frac{1}{n}[f(r_j, s_j) - f(r_j, s'_j)] + \phi(r_i, s'_i, s'_j, s_j, r_N, 0)$$

and

$$F_j(r_N, s_N) - F_j(r_N, s'_N) = \frac{1}{n}[f(r_j, s_j) - f(r_j, s'_j)] + \phi(r_i, s'_j, s'_j, s_j, r_N, 0).$$

We have $\phi(r_i, s'_i, s'_j, s_j, r_N, 0) = \phi(r_i, s'_j, s'_j, s_j, r_N, 0)$ since $s'_i = s_i = s'_j$. Hence, $F_i(r_N, s_N) = F_j(r_N, s_N)$.

(c) $EPC \to WEAC$ *and* $WEAC \stackrel{A}{\to} EIUR$. Both statements are trivial.

(d) $EAC \to WEAC$, $EAC \stackrel{A}{\to} EIPUR$ *and* $EIPUR \to EIUR$. Only the second statement is non-trivial and will be proven.

Let F be a redistribution mechanism that satisfies ex ante compensation and anonymity. Let (r_N, s_N) in D be such that $r_i = r_j$ for all i and j in $N(s)$ for each s in $S(r_N, s_N)$. Let $N(s^1), N(s^2), \ldots, N(s^m)$ be the partition of N into circumstance groups. By anonymity, for each k in $\{1, 2, \ldots, m\}$ and all i and j in $N(s^k)$, we have $F_i(r_N, s_N) = F_j(r_N, s_N)$. Let $F(N(s^k))$ denote this common value and note that this common value is also the average income of circumstance group k. Thus, by ex ante compensation, for all k and ℓ in $\{1, 2, \ldots, m\}$, we have $F(N(s^k)) = F(N(s^\ell))$. That is, for all i and j in N, we have $F_i(r_N, s_N) = F_j(r_N, s_N)$. We omit the straightforward demonstration that ex ante compensation without anonymity does not imply equal income for partially uniform r. □

Proof of Proposition 2. Bossert and Fleurbaey (1996) have established the following implications: $IMR \to WIMR$, $WIMR \stackrel{A}{\to} EAR$ and $EAR \to ETUS$. We consider the remaining implications.

(a) $WIMR \to CWIMR$. The statement is trivial and we omit the proof.

(b) $CWIMR \stackrel{A}{\to} EAR$. First, we show that the implication does not hold without anonymity. Consider the mechanism F: there exist k and ℓ in $\mathcal{N}$ such that, for each profile (r_N, s_N) in D for which both k and ℓ are in N, we have, for each i in N,

$$F_i(r_N, s_N) = \begin{cases} f(r_i, s_i) + 1 & \text{if } i = k, \\ f(r_i, s_i) - 1 & \text{if } i = \ell, \\ f(r_i, s_i) & \text{if } i \neq k, \ell, \end{cases}$$

and, for every other profile (r_N, s_N) in D, we have $F_i(r_N, s_N) = f(r_i, s_i)$ for each i in N.

The mechanism F satisfies individual monotonicity in r, and hence conditional weak individual monotonicity in r, but violates anonymity and ex ante reward.

Next, we show that anonymity and conditional weak individual monotonicity in r imply ex ante reward. We follow the reasoning used by Bossert and Fleurbaey (1996) when they show that weak individual monotonicity in r implies ex ante reward.

Let F be a mechanism that satisfies anonymity and conditional weak individual monotonicity in r. Let (r_N, s_N) in D be such that there exist i and j in N for which $s_i = s_j$ and $r_i \neq r_j$. We have to show that $F_i(r_N, s_N) - f(r_i, s_i) = F_j(r_N, s_N) - f(r_j, s_j)$.

Let r'_N in R^n be such that $r'_j = r_i$ with r_N and r'_N coinciding everywhere else. Since $(r_i, s_i) = (r'_j, s_j)$, we have $F_i(r'_N, s_N) = F_j(r'_N, s_N)$ by anonymity. Conditional weak individual monotonicity in r requires

$$F_i(r_N, s_N) - f(r_i, s_i) = F_i(r'_N, s_N) - f(r'_i, s_i) + \xi(r'_i, s_i, r'_j, r_j, s_N, 0)$$

and

$$F_j(r_N, s_N) - f(r_j, s_j) = F_j(r'_N, s_N) - f(r'_j, s_i) + \xi(r'_j, s_i, r'_j, r_j, s_N, 0).$$

We have $f(r'_i, s_i) = f(r'_j, s_i)$ and $\xi(r'_i, s_i, r'_j, r_j, s_N, 0) = \xi(r'_j, s_i, r'_j, r_j, s_N, 0)$ since $r'_i = r_i = r'_j$. It follows that $F_i(r_N, s_N) - f(r_i, s_i) = F_j(r_N, s_N) - f(r_j, s_j)$.

(c) $EAR \rightarrow WEPR$ *and* $WEPR \overset{A}{\rightarrow} ETUS$. Both statements are trivial.

(d) $EPR \rightarrow WEPR$, $EPR \overset{A}{\rightarrow} ETPUS$ *and* $ETPUS \rightarrow ETUS$. Only the second statement is non-trivial and will be proven.

Let F be a mechanism that satisfies ex post reward and anonymity. Let (r_N, s_N) in D be such that $s_i = s_j$ for all i and j in $N(r)$ for each r in $R(r_N, s_N)$. Let $N(r^1), N(r^2), \ldots, N(r^m)$ be the partition of N into responsibility groups. By anonymity, for each k in $\{1, 2, \ldots, m\}$ and all i and j in $N(r^k)$, we have $F_i(r_N, s_N) = F_j(r_N, s_N)$. Let $F(N(r^k))$ and $f(N(r^k))$ denote the common values of the post-tax and pre-tax income of the responsibility group k. Note that $F(N(r^k)) - f(N(r^k))$ is the average transfer of responsibility group k. Thus, by ex post reward, for all k and ℓ in $\{1, 2, \ldots, m\}$, we have $F(N(r^k)) - f(N(r^k)) = F(N(r^\ell)) - f(N(r^\ell))$. By budget-balancedness, for each i in N, we have $F_i(r_N, s_N) = f(r_i, s_i)$. We omit the straightforward demonstration that ex post reward without anonymity does not imply equal transfer for partially uniform s. ◻

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