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EQUILIBRIUM FLOW AREAS OF INLETS ON SANDY COASTS[a]

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INTRODUCTION

In the summer of 1930, the writer made a reconnaissance survey of the beaches and harbors of the Pacific Coast of the United States for the U.S. Beach Erosion Board. In the preceding years many tidal inlets on the North Pacific Coast had been improved for navigation by constructing jetties, and the progress of these inlets towards stabilization was being followed through frequent hydrographic surveys which were available for study. The obvious fact that large inlets were found at large bays and small inlets at small bays suggested the possibility that there might be a unique relationship between entrance area and tidal prism. The data then available (2)[2] showed good agreement with

$$A = 4.69 \times 10^{-4} P^{0.85} \quad \ldots\ldots (1)$$

in which A = the minimum flow cross section of the entrance channel (throat) measured below mean sea level, in square feet; and P = the tidal prism corresponding to the diurnal or spring range of tide, in cubic feet. [The tide tables of the U.S. Coast and Geodetic Survey list either the diurnal or the spring range of tide but not both. Along the Pacific Coast of the U.S., the tide shows a diurnal inequality with the long runout following mean higher high water (ebb tide). The 1931 paper (2) was based with one exception on data pertaining to inlets on the Pacific Coast and the diurnal range was used in the correlation. Within the accuracy with which tidal prisms may be calculated, either the diurnal range or the spring range may be used, depending on which range is listed in the Tide Tables.]

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[2] Numerals in parentheses refer to corresponding items in Appendix I.—References.

The data then available agreed closely with Eq. 1, but the agreement was regarded as fortuitous for the following reasons:

1. The tidal prism was computed as the product of the tidal area at high water shown on the USC&GS charts times the diurnal or spring range in the ocean at the inlet.
2. There was no apparent effect of the size of bottom material in the inlet channel on the flow area.
3. Jettied and unjettied entrances followed the same curve.
4. The data pertained, with one exception, to the Pacific Coast where the tide shows a marked diurnal inequality and only a small variation in range with location.

The writer believed at the time that precise and extensive data would demonstrate the influence of the factors just mentioned and that Eq. 1 was merely an approximation of a relationship which would depend upon material size, degree of exposure to wave action, jetty protection, and possibly other quantities as parameters.

FLOW AREA—TIDAL PRISM DATA

Casual comparison by a number of writers on other inlets has shown a surprisingly small deviation from Eq. 1 for large and small inlets, with and without jetties, on the Atlantic, Gulf and Pacific coasts of the United States, Mexico, Southeast Asia, and Australia. The phenomena involved seemed too complex to yield so simple a relationship and the present study was undertaken to eliminate data of uncertain accuracy and to discover any consistent influence of the factors mentioned previously. Clearly, the nature of such data makes an appraisal of the accuracy a matter of judgment, but a few considerations could be applied, namely:

1. If the tidal range is approximately constant around the shores of the bay and the low water area is 75% or more of the high water area, the prism can be computed with an accuracy of $\pm$ 10%.
2. When the tidal range in the bay is markedly less than at the entrance, as in the case of Fire Island Inlet, accurate determination of the tidal prism must be based upon a detailed summation of the surface area, range, and phase relationships or upon flow measurements at the entrance.
3. The high water area is usually deineated accurately on the charts, but the low water is frequently ill defined.
4. Surveys are usually made for navigation purposes and are incomplete outside the navigable areas.

Appraisal of the available data in the light of these considerations yielded the data shown in Table 1, believed to be accurate within $\pm$ 10% in flow area and $\pm$ 15% in tidal prism.

Inlets without jetties, ranging from Delaware Bay with a tidal prism of 1.2×10^{11} cu ft to Estero Punta Banda with 3.0×10^{8} cu ft, follow the linear relationship

$$A = 2.0 \times 10^{-5} P \qquad (2)$$

Reliable data on smaller inlets have not yet been obtained. With the exception of Delaware Bay, these inlets without jetties also agree with Eq 1, down to a tidal prism of 1.1×10^7 cu ft.

TABLE 1.—VALUES OF *P* AND *A*

Inlet (1)	Location (2)	Tidal prism on spring of diurnal tide, *P*, in cubic feet (3)	Minimum flow area at entrance channel below mean sea level, *A*, in square feet (4)
		(*a*) Without Jetty	
Delaware Bay	Atlantic	1.25×10^{11}	2.5×10^6
Golden Gate	Pacific	5.1×10^{10}	9.38×10^5
Willapa	Pacific	2.50×10^{10}	3.94×10^5
North Edisto River	Atlantic	4.58×10^9	9.95×10^4
Tomales Bay	Pacific	1.58×10^9	3.6×10^4
Fire Island[a]	Atlantic	2.18×10^9	3.56×10^4
Jones Inlet[a]	Atlantic	1.5×10^9	2.89×10^4
Punta Banda	Pacific	2.99×10^8	5.46×10^3
		(*b*) One Jetty	
Rockaway	Atlantic	3.7×10^9	8.6×10^4
Tillamook	Pacific	2.11×10^9	3.69×10^4
East Rockaway	Atlantic	7.6×10^8	1.15×10^4
		(*c*) Two Jetties	
Columbia	Pacific	3.82×10^{10}	5.08×10^5
Grays Harbor	Pacific	2.43×10^{10}	2.85×10^5
Galveston[b]	Gulf of Mexico	1.59×10^{10}	2.2×10^5
Charleston	Atlantic	5.75×10^9	1.44×10^5
Humboldt	Pacific	4.38×10^9	7.55×10^4
San Diego	Pacific	3.38×10^9	6.17×10^4
Coos Bay	Pacific	2.84×10^9	6.11×10^4
Umpqua	Pacific	2.20×10^9	4.62×10^4
Absecon	Atlantic	1.48×10^9	3.13×10^4
Morichee	Atlantic	1.57×10^9	2.04×10^4
Yaquina	Pacific	7.73×10^8	1.98×10^4
Nahalem	Pacific	6.0×10^8	1.12×10^4
Siuslaw	Pacific	4.64×10^8	1.10×10^4
Mission Bay[c]	Pacific	4.2×10^8	1.04×10^4
Coquelle	Pacific	3.89×10^8	9.02×10^3
Newport Beach	Pacific	1.98×10^8	5.89×10^3
Pendleton Boat Basin[c]	Pacific	1.14×10^7	4.64×10^2

[a] Data by Saville—before jetties.
[b] Includes flow area and prism of San Luis Pass.
[c] Data by D. Inman.

The three inlets with single jetties (shown by triangles in Fig. 1), considered separately, would yield a relationship between tidal prism and flow area differing slightly from Eqs. 1 and 2, but all three points fall close to both of these curves.

The data on inlets with two jetties in equilibrium agreed closely with Eq. 1; there was no reason to modify this equation to represent the data subsequently obtained. The range of tidal prism covered is from 3.8×10^{10} cu ft (Columbia River Entrance) to 1.1×10^{7} cu ft (Pendleton Boat Basin).

There is no obvious reason that the tidal prism-entrance area relationship in equilibrium should have any particular functional form such as Eqs. 1 or 2. Fig. 2 shows the same data as Fig. 1, but the curve shown there has been faired through the points; greater precision in establishing the functional relationship of Fig. 2 seems unjustified in view of the scatter of the data, but it should be

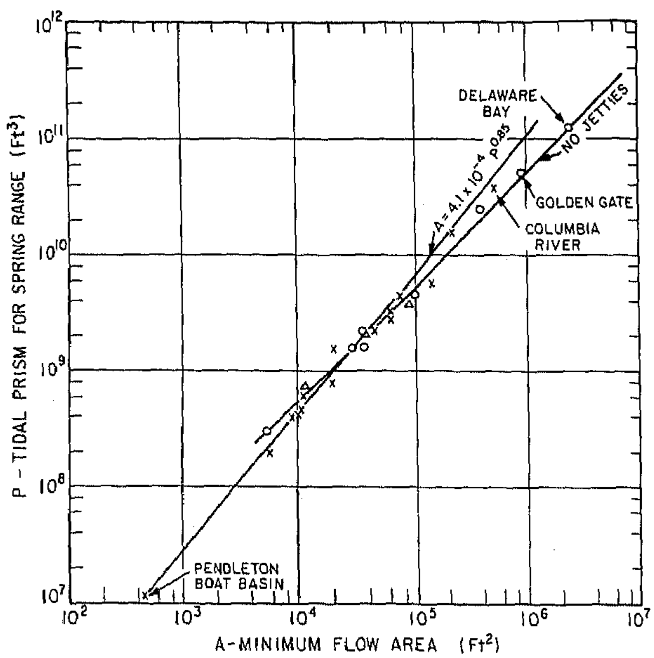

FIG. 1.—MINIMUM FLOW AREA AND TIDAL PRISM COMPARED WITH EQ. 1 AND 2 (DATA FROM TABLE 1)

noted that nearly all of the points agree with the curve within the probable accuracy of the data.

Estuaries of rivers follow the same area-prism relationship as the inlets to tidal bays and lagoons. Maximum river flows are small compared with tidal flows and persist for only short periods.

INLET DYNAMICS AND SAND MOVEMENT

Fig. 3 shows the gross configuration of a tidal inlet which has very nearly ideal proportions; a crescent-shaped bar seaward having a center of curvature near the throat section, a swash channel alongshore at each end of the bar, and a controlling depth over the bar much smaller than at the throat section. The currents on the flood tide are shown schematically in Fig 3(*a*), with the flow converging from all seaward directions towards the entrance. Fig. 3(*b*) shows schematically the currents existing seaward of the entrance during the ebb

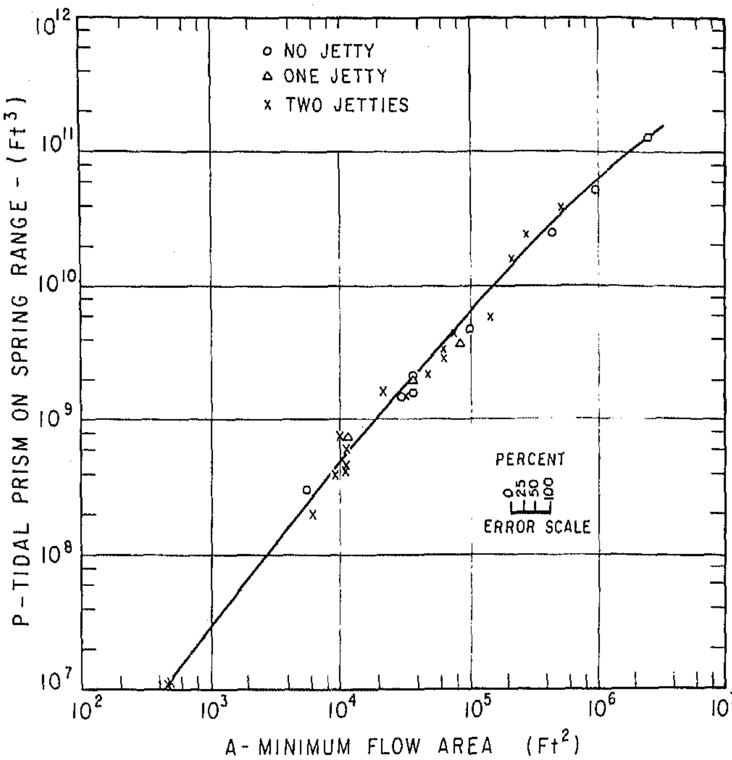

FIG. 2.—MINIMUM FLOW AREA AND TIDAL PRISM (DATA FROM TABLE 1)

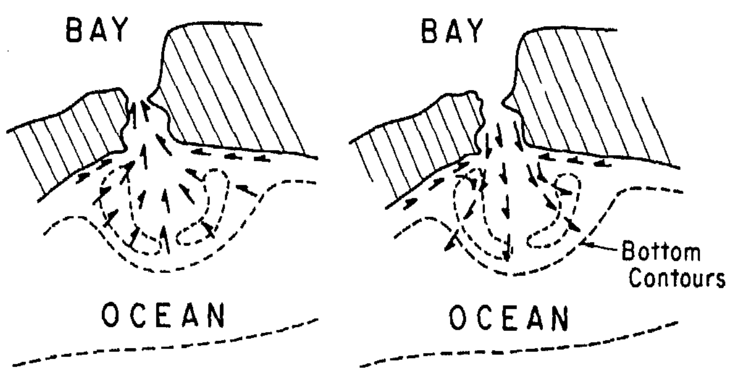

FIG. 3.—IDEALIZED CURRENT DIAGRAM AT TIDAL INLET ON FLOOD AND EBB TIDES

tide; here, the momentum of the flow through the entrance forms a jet directed seaward, and the lateral mixing of this jet induces an eddy on each side. These idealized diagrams show that the currents near the shore are directed toward the entrance from both sides, on both the flood and ebb tides. Figs. 3(*a*) and 3(*b*) show the current situation resulting from tide alone without the effect of currents induced by local winds, by wave action, or by oceanic circulation; these effects would be superimposed on the pattern shown in Fig. 3.

Added to this pattern of tidal currents in Fig. 3 is the effect of refraction of the waves by the crescent-shaped bar and by the tidal currents. Refraction tends to bend the wave crests to become parallel to the bottom contours, thus focusing them on the entrance and inducing currents towards the entrance from each side in the surf zone. The ebb current, running against the wave crests adds somewhat to this focusing action while the flood currents tend to counteract it. In addition to inducing currents in the surf zone, the breakers throw sand in suspension to be transported by whatever current exists there. Thus it appears that there is, under the action of waves approaching perpendicular to the shore, sand movement from both sides along the shore towards the entrance on both the flood and ebb phases of the tide. Tidal currents through the inlet must sweep this littoral drift away if the channel is to remain open, moving the sand either into the bay or seaward to the bar or in both directions. When the tide exhibits a diurnal inequality with the long-runout following higher high water, as on the Pacific Coast of the United States, the ebb currents probably predominate and move the littoral drift seaward. Along the Gulf Coast, the diurnal inequality produces flood currents which are predominant, and these currents may cause the littoral drift to accumulate inside the bay, a situation which may account for the instability of these entrances prior to stabilization by jetties and dredging.

The transportation of sand by currents alone is characterized by a critical bottom velocity below which no motion occurs, by a rate of bed motion which increases exponentially with velocity above the critical value, and by a higher critical velocity above which saltation or suspension develops. Near a tidal entrance, sand movement at the bottom is further complicated by the effect of oscillatory currents due to waves and by irregularity of the bottom. Quantitative prediction of the capacity of the tidal currents to move sand away from the inlet and of the predominance of either the flood or ebb currents would be extremely tedious if not impossible to accomplish. In this study it was assumed that the higher ranges of the tide would dominate. Either the diurnal range or the spring range was used in computing the tidal prism, for the practical reason that one or the other of these ranges is readily available. The agreement shown in Fig. 1 indicates that this range of tide is reasonably representative of the capacity of the tidal currents to maintain the channel.

The capacity of the tidal currents to maintain an inlet open is perhaps best represented by the maximum rate of flow, $A\ V_{max}$. However, this quantity has not often been measured directly because extensive current measurements would be required to obtain a representative average value. Apparently, quoted values of maximum flow have frequently been calculated from the tidal prism and not measured.

The flow phenomena described, which control the equilibrium configuration of an inlet, are of such complexity as to make the use of a single tractive force on the bottom of doubtful value in forecasting inlet area. The primary variable involved is the tidal prism, P, and the prime dependent variable desired is the throat area, A. Computation of the tractive force must assume a bottom roughness and selection of representative depths and velocities. It is, at best, an intermediate quantity to be used in the process of forecasting area A and is then really redundant, if this area may be obtained directly from the tidal prism.

The flow areas of Jones Inlet and Fire Island Inlet were obtained by T. Saville from surveys made by the Long Island State Park Commission prior to the construction of jetties. The figures quoted were the averages over the flow areas at the same cross section taken from several surveys. The littoral drift here is from the east and the easterly side of both inlets overlaps the west side. Heavy wave action at these locations would drive sand across the eastern spit towards the channel and would probably leave a reduced flow area after each storm. Surveys are made normally in the summer season of relatively calm wave action when the flow area would approximate its equilibrium value. The flow areas were measured at the same cross section, which may not have been the minimum area at the time of the survey. Considering these circumstances, it is believed that the data on these inlets, quoted in Table 1, were within the accuracy criteria stated previously. Noteworthy is the fact that the flow area after the construction of the jetty at Fire Island Inlet differs by less than 5% from the area shown in Table 1, before the jetty was built.

The system of littoral currents near an entrance shown in Fig. 3 tends to close an inlet—and this tendency would increase with an increase in the severity and duration of wave action, except that under very severe storm conditions the bar may be scoured away and the entrance enlarged. For each size of inlet, there may be some severity and duration of wave attack which will close the entrance despite the scouring effect of the tidal currents. Data on this point are scarce, but two locations not far apart on the Pacific Coast give an indication of this effect. Lake Earl, north of Crescent City, Calif., has an area of 1.4×10^8 sq ft; the tide diurnal range at this point is 6.9 ft and the potential tidal prism is 9.4×10^8 cu ft. Lake Earl is separated from the ocean by a narrow beach; a channel to the ocean is normally opened during the winter rainy season but closed in the summer. The beach separating Lake Earl from the ocean runs north-south and is exposed to the full intensity of wave action. The inlet to Drake's Estero, on an east-west beach in the lee of Point Reyes, is open continuously; its tidal prism is approximately 7.1×10^8 cu ft, less than the potential tidal prism of Lake Earl. Wave action at Drake's Estero is normally light, consisting of long swells refracted around Point Reyes. At times, however, this inlet is subjected to storm waves of short duration from the south which widen the entrance and alter the entrance channels. (Drake's Estero was not included in the tabulated data because the flow area was not known.)

The Boat Basin at Camp Pendleton has a tidal prism of 1.14×10^7 sq ft. This tiny inlet is an entrance within an entrance, being located within the area protected by the converging jetties of Oceanside Harbor and subjected to the mild but continuous action of long, low waves diffracted and refracted inside the jetties.

Galveston Entrance on the Gulf Coast shares a tidal prism with San Luis Pass, which has a flow area 25% as large as Galveston. In Table 1 the flow area shown is the summation of the areas of the two channels and the prism is the total tributary to both.

The data presented in Table 1 pertain to inlets which are believed to have reached a state of equilibrium at the time of the survey. During periods of abnormal wave action, the increased littoral sand movement towards the entrance tends to reduce the flow area, but the counter balancing scour of the tidal currents, being controlled by the tidal cycle, remains unchanged and reduced areas following storms would be expected. Conversely, long, high jetties, which extend seaward beyond the zone of active bottom sand movement, cut off the alongshore drift and should tend to maintain a flow area larger than that corresponding to Fig. 2, once the larger area has been dredged. In any event, jetties should reduce the rate of approach to equilibrium. Undoubtedly, some of the scatter of the data in Table 1 is due to nonequilibrium conditions, which would tend to make the plotted flow areas too small, if only storm wave action is effective, and too large, if the deviation from equilibrium results from dredging.

CURRENTS IN ENTRANCE CHANNELS

The volume of water accumulated in a tidal bay or estuary during a flood tide, or discharged on the ebb is the integral of the product of the flow area and the average velocity, expressed as

$$\text{volume} = \int^{T/2} a\, v\, dt \quad \ldots\ldots (3)$$

in which $a = f_a\ (z)$; $v = f_v\ (h, z, t)$; T = period of full tidal cycle; a = the instantaneous flow area; v = velocity averaged over area A; h = range of tide; and z = surface elevation. The flow area varies with the stage of the tide; the average velocity over this area is a function of the range and stage of the tide and of the flow area.

If it is assumed that: (1) The flow area is constant and equal to A, the minimum area below mean sea level; and (2) the duration of both flood and ebb are equal to $T/2$, then the average velocity is a sinusoidal function of time

$$v = V_{\max} \sin \frac{2\pi t}{T} \quad \ldots\ldots (4)$$

Applying these relationships to Eq. 3 the tidal prism would be

$$P = \frac{A\, V_{\max}\, T}{\pi} \quad \ldots\ldots (5)$$

Comparing Eqs. 2 and 5 with T = 44,700 sec, the maximum average flow area over area A = $V_{\max}$ = 3.5 fps.

Not many inlets are large enough to correspond to the assumptions made in deriving Eq. 5, but these assumptions do apply approximately to the entrance to Delaware Bay between the Virginia Capes. The velocity at strength of flow reported by the USC&GS at this entrance is 3.55 fps. This agreement is not as good as the quoted figures would seem to indicate because the USC&GS data apply to the main navigation channel where velocity should exceed the average.

The flow conditions at smaller entrances depart appreciably from the assumptions underlying Eq. 5. The percentage change in flow area from high to low tide increases as the inlet size decreases, the flow area tide-stage relationship probably distorts the variation of velocity with time, and the maximum average velocity probably does not occur at mean sea level. For these reasons, a comparison of Eq. 1 and 5 to obtain an expression for $V_{\max}$ as a function of tidal prism is not justified from hydraulic considerations; however, the result does correspond approximately with velocities observed at small inlets. Using the value of A from Eq. 1 and $V_{\max}$ from Eq. 4

$$V_{max} = 0.15P^{0.15} \quad (6)$$

Caldwell (1) has studied the velocities at tidal inlets, as reported by the USC &GS, as affected by the shape of the interior water surface. Comparing Caldwell's data on velocities with Eq. 6, at inlets where the tidal prism is known, yielded fairly good agreement with Eq 6. The writer has not quoted these data because the theoretical basis for Eq. 6 is hydraulically not very sound. A thorough study of the hydraulic flow conditions in tidal inlets is needed.

CONCLUSIONS

The data cited pertain to inlets in equilibrium under tidal currents on the mainland coasts of the United States. Conclusions drawn from these facts are:

1. The equilibrium minimum flow area of an inlet, with or without jetties, is controlled by the tidal prism. A reduction of the tidal prism by sedimentation, vegetation, or artificial fill will reduce the flow area.
2. If the tidal area is connected to the sea through two or more inlets, the sum of the flow areas corresponds to the curve in Fig. 2; closure of one or more of these channels will enlarge the flow area of the others.
3. Jetties not only stabilize the position of an inlet but also protect it against closure under wave action.
4. Very small inlets can be kept open by tidal currents, if they are protected against strong surf and littoral drift.
5. The equilibrium flow area of an inlet depends to a minor extent, if at all, on bed material size.
6. Tractive force does not appear to provide a meaningful criterion for the equilibrium conditions of tidal inlets.
7. Estuaries of large rivers follow the same flow area-tidal prism relationship as tidal lagoons and bays.

APPENDIX I.—REFERENCES

1. Caldwell, J. M., "Tidal Currents at Inlets in the United States," *Proceedings* of the ASCE, Vol. 81, Separate No. 716, June, 1955.
2. O'Brien, M. P., "Estuary Tidal Prism Related to Entrance Areas," *Civil Engineering*, Vol. I, No. 8, 1931, p. 738.

APPENDIX II.—NOTATION

The following symbols are used in this paper:

a = instantaneous flow area;
h = range of tide;

P = volume of tidal prism;
T = period of full tidal cycle;
v = velocity averaged over area a;
V_{max} = maximum value of v; and
z = surface elevation.