# Equilibrium Structures of Three-, Four-, Five-, Six-, and SevenMembered Unsaturated N-Containing Heterocycles 

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## (S) Supporting Information


#### Abstract

Up to six different techniques are utilized to estimate the equilibrium structures $\left(r_{\mathrm{e}}\right)$ of a series of mostly unsaturated, N-containing heterocycles. Accurate Born-Oppenheimer $\left(r_{\mathrm{e}}^{\mathrm{BO}}\right)$ and, if allowed, semiexperimental $\left(r_{\mathrm{e}}^{\mathrm{SE}}\right)$, as well as empirical ( $r_{\mathrm{m}}$-type) estimates of the equilibrium structures of three-membered ( 1 H - and 2 H -azirine, aziridine), four-membered (azete), fivemembered (pyrrole, pyrazole, imidazole), six-membered (pyridine, pyrimidine, uracil), and seven-membered ( 1 H -azepine) rings, containing usually one but in some cases two N atoms, are determined. The agreement among the structural results of the different techniques is very satisfactory. It is shown that it is possible to use the $\operatorname{CCSD}(\mathrm{T})$ electronic structure method with the relatively small wCVTZ basis set, with all electrons correlated, and the effect of further basis set enlargement, wCVTZ $\rightarrow$ wCVQZ, computed at the MP2 level, to obtain reliable equilibrium structures for the semirigid molecules investigated. Extension to larger  $2 I I$-azirine  pyrrole 

1 H -azirine   pyrazole  pyrimidine  aziridine   imidazole  basis sets does not significantly improve the accuracy of the computed results. Although all molecules investigated are oblate, and their principal axis system is subject to large rotations upon isotopic substitution, the semiexperimental method, when applicable, provides accurate results, though in the difficult cases it must be augmented with the mixed regression method. Finally, it is noteworthy that the empirical mass-dependent $\left(r_{\mathrm{m}}\right)$ method also delivers surprisingly accurate structures for this class of compounds.


## 1. INTRODUCTION

The CN bond, along with the CC bond, has a central role in organic chemistry and biochemistry. The equilibrium length $\left(r_{\mathrm{e}}\right)$ of the $\mathrm{C} \equiv \mathrm{N}$ triple bond in nitriles is well-established at about $1.15 \AA,{ }^{1}$ and it does not vary much in different chemical environments. The variation of $r_{\mathrm{e}}(\mathrm{C} \equiv \mathrm{N})$ is between $1.153 \AA$ in $\mathrm{HCN}^{2}$ and $1.160 \AA$ in $\mathrm{HCC}-\mathrm{CN},{ }^{3}$ similarly constrained as the variation of the length of the $\mathrm{C} \equiv \mathrm{C}$ triple bond. To wit, compare $r_{\mathrm{e}}(\mathrm{C} \equiv \mathrm{N})$ in HCN with that in benzonitrile (1.158 $\AA)^{4}$ and $r_{\mathrm{e}}(\mathrm{C} \equiv \mathrm{C})$ in $\mathrm{C}_{2} \mathrm{H}_{2}(1.203 \AA)^{5}$ with that in phenylacetylene $(1.207 \AA) .{ }^{4}$ In clear contrast, the length of the $\mathrm{C}=\mathrm{N}$ double bond and especially that of the $\mathrm{C}-\mathrm{N}$ single bond is extremely variable, ranging from 1.35 to $1.55 \AA .{ }^{6}$ For example, the mean length of the $\mathrm{C}-\mathrm{N}$ single bond is $1.46 \AA,{ }^{7}$ but it varies from $1.348 \AA$ in $\mathrm{NH}_{2} \mathrm{CN}^{6}$ to $1.55 \AA$ in azete (see section 3 of this paper).

Structures of N-containing heterocycles of varying size and composition are interesting from several aspects. Heterocycles have determining roles in biochemistry, life sciences, and materials chemistry. Some of the interest also arises from structural underpinnings of chemical concepts like aromaticity, nonaromaticity, antiaromaticity, ${ }^{8}$ or homoaromaticity and
heteroaromaticity (as advocated by Schleyer et al., ${ }^{9}$ the structural "definition" of aromaticity is a "tendency toward bond length equalization and planarity (if applicable)"), of isomerization connected to N versus NH moieties, and the relation between the gas-phase acidity (or basicity) and the charge on nitrogen. Nevertheless, only few accurate $r_{\mathrm{e}}$ structure estimates are available for these heterocyclic molecules. ${ }^{6,10-12} \mathrm{~A}$ list of references detailing work executed prior to 2007 can be found in ref 6 . Though the situation improved since then, in an influential review published in 1980 Legon ${ }^{13}$ noted that "in principle, it is possible to determine the equilibrium geometry $\left(r_{\mathrm{e}}\right)$, but sufficient data are as yet available in the case of no ring". A large number of accurate equilibrium structures are available for many other, simple(r) semirigid molecules, ${ }^{14}$ but the situation is less than satisfactory for heterocyclic rings. One particular difficulty is that many heterocyclic rings are (nearly)

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2 H -azirine

pyrrole

pyrazole

aziridine

imidazole

azete

pyridine

pyrimidine

uracil



1H-azepine

Figure 1. Schematic structures and atom numbering of the molecules investigated.
oblate tops. Molecules of this shape experience a large rotation of the principal axis system upon certain isotopic substitutions. ${ }^{11}$ For such isotopologues it is difficult to obtain a good structural fit to the semiexperimental (SE) moments of inertia, $I_{a}^{\mathrm{SE}}$ and $I_{b}^{\mathrm{SE}}$, which may significantly reduce the accuracy of the semiexperimental equilibrium structural parameters, $r_{e}^{\text {SE }}$. While some knowledge has been accumulated for the three-, four-, five-, six-, and seven-membered unsaturated heterocycles studied (see Figure 1), a firm grip on structural trends is provided first in this paper for several related systems. To keep focused, the literature results are discussed separately for each molecule investigated.

While some of the techniques that can be used to determine $r_{\mathrm{e}}$ estimates of molecular structures are discussed even in textbooks, ${ }^{14}$ it is still highly beneficial to acquire further knowledge about the advantages and disadvantages of variants of these techniques (see Theoretical Background), especially for problematic cases. Thus, another goal of this work is to compare different methods, ab initio, semiexperimental, and empirical, to determine accurate equilibrium structures of medium-sized molecules.

## 2. THEORETICAL BACKGROUND

2.1. Born-Oppenheimer Equilibrium Structures. The highest level of electronic structure theory at which ab initio structures are optimized within this study is $\operatorname{CCSD}(\mathrm{T})$, the coupled cluster (CC) technique with single and double excitations ${ }^{15}$ as well as a perturbative treatment of connected triples. ${ }^{16}$ In the spirit of the focal-point analysis (FPA) technique, ${ }^{17,18}$ the second-order Møller-Plesset (MP2) level ${ }^{19}$ is also used extensively to estimate correction terms, whose computation at the $\operatorname{CCSD}(\mathrm{T})$ level seems unnecessarily expensive.

Different atom-centered, fixed-exponent Gaussian basis sets have been utilized during the optimizations, including the correlation-consistent basis sets cc-pVTZ and cc-pVQZ, ${ }^{20}$ abbreviated throughout this paper as VTZ and VQZ,
respectively. The augmented correlation-consistent basis set variants including diffuse functions, aug-cc-pVQZ and aug-cc$\mathrm{pV} 5 \mathrm{Z},{ }^{21}$ abbreviated here as AVQZ and AV5Z, respectively, were also employed. Optimizations were also performed with the correlation-consistent polarized weighted core-valence triple- $\zeta$, cc-pwCVTZ, and quadruple- $\zeta$, cc-pwCVQZ, basis sets, ${ }^{22,23}$ denoted here as wCVTZ, and wCVQZ, respectively. With the $w C V T Z$ and $w C V Q Z$ basis sets all of the electrons were correlated (AE), whereas with the VTZ, VQZ, AVQZ, and AV5Z basis sets the frozen core (FC) approximation is normally used in correlated-level computations.

Different schemes were employed to determine the best estimates of the Born-Oppenheimer equilibrium structure, $r_{\mathrm{e}}^{\mathrm{BO}}$, of the molecules investigated. First, when possible, that is, for molecules not too large, the wCVQZ basis set was used and all electrons were correlated; consequently,

$$
\begin{equation*}
r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})=\mathrm{CCSD}(\mathrm{~T}) \_\mathrm{AE} / \mathrm{wCVQZ} \tag{1}
\end{equation*}
$$

The $r_{\mathrm{e}}^{\mathrm{BO}}$ structure was also estimated in the FC approximation using the VQZ basis set and the CCSD(T) method. The core-core and core-valence correlation is missing from the $\operatorname{CCSD}(\mathrm{T})$ _FC/VQZ treatment; the corresponding structural corrections ${ }^{24}$ were computed at the MP2 level using the wCVQZ basis set, resulting in another $r_{e}^{\mathrm{BO}}$ estimate:

$$
\begin{align*}
r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{II})= & \mathrm{CCSD}(\mathrm{~T}) \_\mathrm{FC} / \mathrm{VQZ}+\mathrm{MP2}(\mathrm{AE}) / \mathrm{wCVQZ} \\
& -\mathrm{MP} 2(\mathrm{FC}) / \mathrm{wCVQZ} \tag{2}
\end{align*}
$$

To check whether convergence is achieved at this level, the effect of further basis set enlargement was occasionally computed at the MP2 level using the AVQZ and AV5Z basis sets.

A third scheme was also utilized. For larger molecules, the structure was computed with the smaller wCVTZ basis set and the $\operatorname{CCSD}(\mathrm{T}) \_A E$ method, and the effect of further basis set enlargement, wCVTZ $\rightarrow$ wCVQZ, was estimated at the MP2
level. In other words, the $r_{\mathrm{e}}^{\mathrm{BO}}$ parameters are obtained using the following equation:

$$
\begin{align*}
r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{III})= & \mathrm{CCSD}(\mathrm{~T}) \_\mathrm{AE} / \mathrm{wCVTZ}+\mathrm{MP} 2(\mathrm{AE}) / \mathrm{wCVQZ} \\
& -\mathrm{MP} 2(\mathrm{AE}) / \mathrm{wCVTZ} \tag{3}
\end{align*}
$$

The basic assumption of eq 3 is that the correction due to basis set enlargement, wCVTZ $\rightarrow$ wCVQZ, is small and can, therefore, be estimated at the MP2 level. This approximation was investigated in several previous studies by us ${ }^{4,11,82}$ and was found to be reliable. The approximation was further checked in the present work. For the smallest molecules, each of the three schemes were used, and the results were compared.

The $\operatorname{CCSD}(\mathrm{T})$ optimizations were performed with the CFOUR program package, ${ }^{25}$ while the MP2 optimizations utilized the Gaussian09 suite of codes. ${ }^{26}$
2.2. Semiexperimental Equilibrium Structures. While the semiexperimental method has been employed in many studies and by many authors (see ref 14 for details), it is worth pointing out here that the fundamentals of this technique, yielding an $r_{\mathrm{e}}^{\mathrm{SE}}$ equilibrium structure, were laid down in 1978 in ref 27 . This paper is considered by many as a pioneering and significant contribution to structural chemistry by Peter Pulay, Wilfried Meyer, and Jim Boggs, not reflected correctly even by the close to 150 citations this paper received up to 2014.

To determine the semiexperimental equilibrium rotational constants, the published experimental ground-state rotational constants were employed first. However, as in the case of benzonitrile and phenylacetylene, ${ }^{4}$ it became obvious that some of the fitted experimental rotational constants are not accurate enough for our purposes. For this reason, we redid the spectral fits in the same way as in refs 4 and 28: using the method of predicate observations, in which theoretical centrifugal distortion constants, derived from a quadratic B3LYP/6$311+G(3 d f, 2 p d)$ force field, are used as supplementary data in a weighted least-squares fit to the measured transitions. ${ }^{29,30}$

To correct the effective experimental rotational constants for each isotopologue and to obtain their equilibrium counterparts, cubic force field computations ${ }^{31}$ were performed at the optimized structures with two different methods: (i) the Kohn-Sham density functional theory (DFT) ${ }^{32}$ using Becke's three-parameter hybrid exchange functional ${ }^{33}$ and the Lee-Yang-Parr correlation functional, ${ }^{34}$ together denoted as B3LYP, with the $6-311+G(3 d f, 2 p d)$ basis set, and (ii) the MP2 (FC) method with the standard VTZ basis. For pyrrole, the all-electron MP2 method was also used with the wCVTZ basis set. Although many DFT functionals could be used to compute anharmonic force fields, the B3LYP method was preferred as it is widely used and it provides accurate underlying equilibrium structures contributing to the fact that it is known to yield satisfactory force fields. ${ }^{35,36}$ To avoid the nonzero force dilemma, ${ }^{37}$ all force fields were evaluated at the corresponding optimized geometries. All anharmonic force field computations were performed with the electronic structure package Gaussian09. ${ }^{26}$

The semiexperimental equilibrium rotational constants, $B_{\mathrm{e}}^{\mathrm{SE}}$, were calculated from the experimental ground-state rotational constants, $B_{0}$, of the particular molecule using the equation

$$
\begin{equation*}
B_{\mathrm{e}}^{\mathrm{SE}}=B_{0}+\Delta B_{\mathrm{vib}}+\Delta B_{\mathrm{el}} \tag{4}
\end{equation*}
$$

where $\Delta B_{\text {vib }}$ is the rovibrational correction ${ }^{38}$ calculated from the cubic force field, and $\Delta B_{\mathrm{el}}$ is the electronic correction, which may be obtained from the rotational $g$ tensor. ${ }^{39}$

The equilibrium structural parameters were determined by a weighted least-squares fit based on the semiexperimental equilibrium moments of inertia. The weights of the semiexperimental rotational constants were determined iteratively. At each step, an analysis of the residuals permitted checking the appropriateness of the weights and the compatibility of the rotational constants and the predicate observations. It is possible to automate, at least partly, this procedure by using the Iteratively Reweighted Least Squares (IRLS) method, whereby data with large residuals are weighted down. ${ }^{29,40}$ Two weighting schemes are used: Huber weighting, where the weight linearly decreases as the residual increases, and biweight weighting, where, in addition, data with large residuals are eliminated.

When the number of isotopologues is not sufficient to calculate a structure, there is still another way to obtain an accurate structure estimate: extrapolating the computed $a b$ initio rotational constants to infinite basis set size. ${ }^{41,42}$ The $a b$ initio equilibrium values of the rotational constants of the parent species $X_{e}(\mathrm{X}=A, B, C)$ are calculated first. For this, the ab initio structure was optimized using the $\operatorname{CCSD}(\mathrm{T})$ method with the cc-pwCVnZ basis sets ( $n=\mathrm{T}, \mathrm{Q}$ ), with all electrons correlated. The rovibrational correction $\Delta X_{0}=X_{0}-X_{e}$ is added to these $X_{\mathrm{e}}(\mathrm{T})$ and $X_{\mathrm{e}}(\mathrm{Q})$ values, yielding $X_{0}(\mathrm{~T})$ and $X_{0}(\mathrm{Q})$. The assumption that the $\Delta X_{0}$ rovibrational corrections are almost constant, a generally excellent assumption, ${ }^{43}$ is further verified here. Then, the rotational constants $X_{0}(\mathrm{~T})$ and $X_{0}(\mathrm{Q})$ are extrapolated as a function of the computed structural parameters $r_{\mathrm{e}}(\mathrm{T})$ and $r_{\mathrm{e}}(\mathrm{Q})$. The intersection of the line with the experimental $X_{0}$ of the parent species gives the extrapolated $r_{\mathrm{e}}$. With these $r_{\mathrm{e}}$ and the appropriate set of atomic masses the rotational constants of any missing isotopologue can be calculated, although with the increased computational effort described above.
2.3. Mass-Dependent Experimental Structures, $r_{\mathrm{m}}$. In the particular case of oblate molecules it is often useful to determine the structure by the empirical mass-dependent method, $r_{\mathrm{m}}$, proposed by Watson et al. ${ }^{44}$ This technique takes into account the variation of the rovibrational correction upon isotopic substitution in an approximate manner. In particular, it takes care of the rotation of the principal axis system (PAS) and of the presence of small coordinates. In principle, it allows the determination of structures close to the assumed equilibrium structure.

In the PAS, $\xi=a, b, c$ of the parent isotopologue $(i=1)$, the three components of the ground-state moment of inertia of the parent are written as

$$
\begin{equation*}
I_{\xi}^{0}(1)=I_{\xi}^{\mathrm{m}}+c_{\xi} \sqrt{I_{\xi}^{\mathrm{m}}(1)}+d_{\xi} \hat{M}(1) \tag{5}
\end{equation*}
$$

The last two terms of eq 5 provide the rovibrational contribution. $\hat{M}(1)$ is a known, coordinate-independent function of the atomic masses of a particular isotopologue, here of the parent $(i=1)$. This last term is useful when small Cartesian coordinates are present. In a center-of-mass coordinate system parallel to that of the parent, the elements of the $3 \times 3$ ground-state moment of inertia tensor of an isotopologue $i$ must be written as

$$
\begin{equation*}
I_{\xi \eta}^{0}(i)=I_{\xi \eta}^{\mathrm{m}}(i)+c_{\xi \eta} \sqrt{I_{\xi \eta}^{\mathrm{m}}(i)}+\delta_{\xi \eta} d_{\xi} \cdot \hat{M}(i) \tag{6}
\end{equation*}
$$

with rovibrational parameters $c_{\xi \eta}$ and $d_{\xi}$. Equation 6 defines elements of a nondiagonal $3 \times 3$ inertia tensor. The
nondiagonal terms $c_{\xi \eta}$ are useful when the rotation of the PAS upon isotopic substitution is large.

The model described above is named $r_{\mathrm{m}}^{(2 \mathrm{r})}$. When the nondiagonal terms $c_{\xi \eta}$ are neglected, we obtain the $r_{m}^{(2)}$ model. Setting $d_{\xi}=0$ in eqs 5 and 6 , the models $r_{\mathrm{m}}^{(2 \mathrm{r})}$ and $r_{m}^{(2)}$ reduce to models $r_{\mathrm{m}}^{(\mathrm{Ir})}$ and $r_{\mathrm{m}}^{(\mathrm{I})}$, respectively. The accuracy is supposed to increase with the level of refinement of the model in the direction $r_{m}^{(1)} \rightarrow r_{m}^{(2 r)}$. However, as the number of parameters to be fitted increases, the least-squares regression tends to become ill-conditioned, and the accuracy may actually get worse.

## 3. THREE-MEMBERED RINGS

The most stable form of the molecules with a $\mathrm{C}_{2} \mathrm{H}_{3} \mathrm{~N}$ molecular formula and on the ground-state potential energy surface is methyl cyanide, $\mathrm{CH}_{3} \mathrm{CN}$. Next in the order of stability come 2 H -azirine and ketenimine, $\mathrm{H}_{2} \mathrm{C}=\mathrm{C}=\mathrm{NH}$. The stability of 2 H -azirine is increased by the electron-rich N atom. Nevertheless, the total ring-strain energy of 2 H -azirine is substantial, on the order of $180 \mathrm{~kJ} \mathrm{~mol}^{-1}$. 1 H -azirine lies $\sim 170 \mathrm{~kJ} \mathrm{~mol}^{-1}$ above 2 H -azirine. ${ }^{45}$ The least stable form on the singlet $\mathrm{C}_{2} \mathrm{H}_{3} \mathrm{~N}$ PES, iminocarbene, is predicted to be less stable than 1 H azirine by some $270 \mathrm{~kJ} \mathrm{~mol}^{-1}$; in fact, this molecule has a triplet ground electronic state.
3.1. Structure of 2 H -Azirine. 2 H -azirine (often called 1 azirine, where the number refers to the position of the double bond), $c-\mathrm{CH}_{2} \mathrm{CHN}$, is a three-membered heterocyclic molecule with an equilibrium structure of $C_{s}$ point-group symmetry, containing a nitrogen atom and a $\mathrm{C}=\mathrm{N}$ double bond (see Figure 1).

The microwave (MW) spectrum of 2 H -azirine was measured first in 1976 by Ford. ${ }^{46}$ That study was later extended to the millimeterwave (MMW) range permitting the determination of accurate ground-state spectroscopic constants for the parent species. ${ }^{47}$ While numerous $a b$ initio computations have been performed on $\mathrm{C}_{2} \mathrm{H}_{3} \mathrm{~N}$ species to determine the relative stability of the different structural isomers, ${ }^{43}$ up to now no accurate equilibrium structures have been determined for the cyclic 1 H and 2 H -azirine molecules.

The $r_{\mathrm{e}}^{\mathrm{BO}}$ structure of 2 H -azirine was estimated via eqs $1-3$, and the results obtained are listed in Table 1. The three equations yield almost identical structural parameters. Furthermore, an attempt to improve the accuracy of the $r_{\mathrm{e}}^{\mathrm{BO}}$ estimate by using the larger V5Z basis set and by taking into account the effect of diffuse functions by utilizing the AV5Z basis shows that these two effects almost fully compensate each other at the MP2 level and can thus be safely neglected even at the level of accuracy sought in this study.

As only the rotational constants of the parent species are known, it is not possible to determine an $r_{\mathrm{e}}^{\mathrm{SE}}$ structure for this molecule. However, the extrapolation method described at the end of Section 2.2 can be utilized. This procedure was tried with the B3LYP $/ 6-311+G(3 d f, 2 \mathrm{pd})$ anharmonic force field and was repeated for each rotational constant $X=A, B$, and $C$. The overall agreement between the $r_{\mathrm{e}}^{\mathrm{BO}}$ and $r_{\mathrm{e}}^{\mathrm{SE}}$ structures is extremely good, as can be seen in Table S1 of the Supporting Information, and the mean of the results is given in the last column of Table 1 . What is even more convincing is that if we change the rovibrational corrections by $5 \%$, the structural results do not change significantly, indicating that they are quite precise and accurate.
It is worth noting that the single $\mathrm{N}_{1}-\mathrm{C}_{2}$ bond of 2 H -azirine at $r_{\mathrm{e}}^{\mathrm{SE}}=r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})=1.546 \AA$ is substantially longer than the $\mathrm{C}-\mathrm{N}$ bond in acyclic methylamine, $\mathrm{CH}_{3} \mathrm{NH}_{2}$, whose $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$ value is
Table 1. Equilibrium Structural Parameters of 2 H -Azirine ${ }^{\boldsymbol{a}}$

| method | $\operatorname{CCSD}(\mathrm{T})$ | MP2 | MP2 | $\operatorname{CCSD}(\mathrm{T})$ | MP2 | MP2 | MP2 | diff. ${ }^{\text {b }}$ | $\operatorname{CCSD}(\mathrm{T})^{c}$ | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{II})^{d}$ | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{III})^{e}$ | $r_{\mathrm{e}}^{\text {SE } f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| approx. ${ }^{g}$ | FC | AE | FC | AE | AE | FC | FC |  | AE |  |  |  |
| basis set | VQZ | wCVQZ | wCVQZ | wCVTZ | wCVTZ | VQZ | AV5Z |  | wCVQZ |  |  |  |
| $\mathrm{N}_{1}=\mathrm{C}_{3}$ | 1.2575 | 1.2576 | 1.2609 | 1.2568 | 1.2598 | 1.2614 | 1.2610 | -0.0004 | 1.2542 | 1.2542 | 1.2546 | 1.2546 |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.5503 | 1.5481 | 1.553 | 1.5517 | 1.5534 | 1.5536 | 1.5530 | -0.0006 | 1.5455 | 1.5454 | 1.5464 | 1.5463 |
| $\mathrm{C}_{2}-\mathrm{C}_{3}$ | 1.4510 | 1.4399 | 1.4431 | 1.4496 | 1.4416 | 1.4435 | 1.4435 | 0.0000 | 1.4476 | 1.4478 | 1.4479 | 1.4479 |
| $\mathrm{C}_{3}-\mathrm{H}$ | 1.0791 | 1.0754 | 1.0769 | 1.0786 | 1.0765 | 1.0769 | 1.0768 | -0.0001 | 1.0776 | 1.0776 | 1.0775 | 1.0778 |
| $\mathrm{C}_{2}-\mathrm{H}$ | 1.0824 | 1.0779 | 1.0794 | 1.0815 | 1.0787 | 1.0795 | 1.0794 | -0.0001 | 1.0809 | 1.0809 | 1.0807 | 1.0809 |
| $\angle\left(\mathrm{N}_{1} \mathrm{C}_{3} \mathrm{H}\right)$ | 139.04 | 138.92 | 138.88 | 139.08 | 138.92 | 138.88 | 138.83 | -0.055 | 139.06 | 139.07 | 139.07 | 139.06 |
| $\angle\left(\mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}\right)$ | 151.55 | 151.42 | 151.39 | 151.37 | 151.24 | 151.38 | 151.46 | 0.080 | 151.58 | 151.57 | 151.55 | 151.55 |
| $\angle\left(\mathrm{N}_{1} \mathrm{C}_{2} \mathrm{H}\right)$ | 115.82 | 115.46 | 115.40 | 115.83 | 115.44 | 115.40 | 115.36 | -0.043 | 115.87 | 115.88 | 115.85 | 115.86 |
| $\angle\left(\mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}\right)$ | 120.40 | 120.38 | 120.37 | 120.44 | 120.40 | 120.37 | 120.33 | -0.037 | 120.42 | 120.41 | 120.41 | 120.42 |
| $\angle\left(\mathrm{HC}_{2} \mathrm{H}\right)$ | 116.65 | 116.91 | 116.96 | 116.61 | 116.89 | 116.97 | 117.05 | 0.086 | 116.59 | 116.61 | 116.62 | 116.59 |
| $\mathrm{H}_{3} \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}_{2}$ | -80.66 | -81.05 | -81.13 | -80.69 | -81.11 | -81.13 | -81.16 | -0.032 | -80.61 | -80.59 | -80.63 | -80.62 |

Table 2. Equilibrium Structural Parameters of $1 H$-Azirine ${ }^{a}$

| parameter | $\mathrm{FC}-\operatorname{CCSD}(\mathrm{T})^{\boldsymbol{b}}$ | AE-CCSD (T) ${ }^{\text {b }}$ | $\mathrm{FC}-\mathrm{CCSD}(\mathrm{T})^{\text {b,c }}$ | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{II})^{\text {d }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | VQZ | wCVQZ | wCVQZ |  |
| NC | 1.5219 | 1.5174 | 1.5214 | 1.5179 |
| NH | 1.0233 | 1.0221 | 1.0234 | 1.0220 |
| $\mathrm{C}=\mathrm{C}$ | 1.2794 | 1.2762 | 1.2789 | 1.2767 |
| CH | 1.0716 | 1.0702 | 1.0715 | 1.0703 |
| $\angle(\mathrm{CNH})$ | 106.64 | 106.80 | 106.67 | 106.78 |
| $\angle(\mathrm{NCH})$ | 138.11 | 138.17 | 138.13 | 138.15 |
| $\angle$ (CCH) | 156.74 | 156.70 | 156.73 | 156.71 |
| HNCH | 82.04 | 81.95 | 82.03 | 81.97 |
| ${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b} \mathrm{AE}=$ all electrons correlated; $\mathrm{FC}=$ frozen core approximation. ${ }^{c} r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$, see eq $1 .{ }^{d}$ See eq 2. |  |  |  |  |

Table 3. Structure of Aziridine ${ }^{a}$

| parameter ${ }^{\text {b }}$ | $r_{\mathrm{e}}{ }^{\text {BO } c}$ | MP2/VQZ ${ }^{\text {d }}$ | MP2/AV5Z ${ }^{\text {d }}$ | $r_{\mathrm{e}} \mathrm{BO} e$ | $r_{\text {e }}^{\text {SE }}$ | $r_{\mathrm{m}}^{(2)}$ | $r_{\text {s }}{ }^{\text {f }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CN | 1.47024 | 1.4702 | 1.4703 | 1.4703 | 1.47013(6) | 1.4698(7) | 1.475 |
| CC | 1.47736 | 1.4758 | 1.4756 | 1.4772 | 1.47703(8) | 1.4771(9) | 1.481 |
| NH | 1.01244 | 1.0124 | 1.0124 | 1.0124 | 1.01279(13) | 1.0133(4) | 1.016 |
| $\mathrm{CH}_{\text {c }}$ | 1.08082 | 1.0794 | 1.0793 | 1.0807 | 1.08099(13) | 1.0821(3) | 1.084 |
| $\mathrm{CH}_{\text {t }}$ | 1.07956 | 1.0781 | 1.0780 | 1.0795 | 1.07971(13) | 1.0810(3) | 1.083 |
| $\angle$ (HNC) | 109.17 | 109.00 | 109.20 | 109.37 | 109.376(9) | 109.38(3) |  |
| $\angle(\mathrm{CNC})$ | 60.32 | 60.25 | 60.24 | 60.31 | 60.311(6) | 60.33(3) | 60.25 |
| $\angle(\mathrm{NCC})$ | 59.84 | 59.87 | 59.88 | 59.85 | 59.845(3) | 59.84(2) |  |
| $\angle\left(\mathrm{H}_{\mathrm{c}} \mathrm{CH}_{\mathrm{t}}\right)$ | 115.33 | 115.60 | 115.69 | 115.42 | 115.424(9) | 115.59(8) | 115.72 |
| $\angle\left(\mathrm{NCH}_{c}\right)$ | 118.35 | 118.32 | 118.23 | 118.26 | 118.28(2) | 118.34(7) | 118.26 |
| $\angle\left(\mathrm{NCH}_{\mathrm{t}}\right)$ | 114.51 | 114.31 | 114.24 | 114.44 | 114.46(2) | 114.32(7) | 114.27 |
| $\angle\left(\mathrm{CCH}_{\mathrm{c}}\right)$ | 117.82 | 117.68 | 117.71 | 117.85 | 117.829(14) | 117.80(4) | 117.75 |
| $\angle\left(\mathrm{CCH}_{\mathrm{t}}\right)$ | 119.59 | 119.50 | 119.46 | 119.55 | 119.538(14) | 119.37(5) | 119.32 |

${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b}$ The carbon-bonded cis hydrogens (subscript c) are situated on the same side of the plane of the triangle $\mathrm{C}-$ $\mathrm{N}-\mathrm{C}$ as the nitrogen-bonded hydrogen. ${ }^{c} r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$, corresponding to an all-electron $\operatorname{CCSD}(\mathrm{T}) / \mathrm{cc}-\mathrm{pwCVQZ}$ optimization. ${ }^{d}$ Frozen core approximation. ${ }^{e}$ Correction MP2(FC)/AV5Z -MP2(FC)/VQZ included. ${ }^{f}$ Reference 52.
$1.461 \AA$. Furthermore, the $C_{2}-C_{3}$ bond, at $1.448 \AA$, is substantially shorter than it is in acyclic molecules. These structural results are in clear agreement with the antiaromatic nature of 2 H -azirine. The long $\mathrm{C}_{2}-\mathrm{N}$ bond is also consistent with the major thermal reaction of 2 H -azirines, as via cleavage of this bond one can form vinyl nitrene intermediates from them.
3.2. Structure of 1 H -Azirine. 2 H -azirine, whose equilibrium structure was determined in the previous section, has been computed ${ }^{43}$ to be considerably more stable than its tautomeric form 1 H -azirine (2-azirine using the other common nomenclature): 1 H -azirine is inherently antiaromatic (its planar form would have four $\pi$ electrons), its ring strain is substantial, and thus the molecule has high reactivity.

Since the $1 H$-azirine molecule is rather unstable, no experimental results concerning its MW and MMW spectra are available. Nevertheless, an accurate $r_{\mathrm{e}}^{\mathrm{BO}}$ structure can be computed for this simple triatomic heterocycle. The $r_{\mathrm{e}}^{\mathrm{BO}}$ structure of 1 H -azirine was estimated via eqs 1 and 2 ; however, for the latter equation the core correlation was computed at the $\operatorname{CCSD}(\mathrm{T})$ level. The results obtained are listed in Table 2. Just like $2 H$-azirine, $1 H$-azirine has a very long NC $(1.518 \AA)$ and a short $\mathrm{C}=\mathrm{C}(1.277 \AA)$ bond. To relieve strain due to antiaromaticity, the NH structural unit of 1 H -azirine is strongly out of the plane defined by the three heavy atoms.
3.3. Structure of Aziridine. Aziridine, $c-\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{NH}$, also called ethylene imine, is a nonaromatic three-membered saturated N -heterocycle. It is the only saturated N -heterocycle whose equilibrium structure is investigated in this study.

Aziridine is the saturated equivalent of 1 H -azirine, and it is the nitrogen analogue of epoxide.

The equilibrium structure of aziridine exhibits $C_{s}$ point-group symmetry. The barrier for the pyramidal inversion around N is large, some $80 \mathrm{~kJ} \mathrm{~mol}^{-1}$, while the ring-strain energy of aziridine is similar to that of cyclopropane, about $110 \mathrm{~kJ} \mathrm{~mol}^{-1}$. The rotational spectrum of aziridine has been studied in great detail, see refs $48-50$ for a review of earlier work. It is a molecule worth studying in detail because of its potential astrophysical interest. ${ }^{48}$ Furthermore, it is an oblate top (Ray's asymmetry parameter, $\kappa=+0.67$ ) not too far from the spherical top limit, making its centrifugal distortion analysis and the determination of its structure particularly interesting. Its infrared spectrum was recently recorded at high resolution and analyzed. ${ }^{51}$ Particularly relevant to the present study is the work of Bak and Skaarup, ${ }^{52}$ who measured the spectra of the monoisotopically substituted species and who determined a complete substitution structure, $r_{s}$, for this molecule.

The $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$ structural parameters of aziridine are reported in Table 3. An attempt to improve the accuracy of the $r_{\mathrm{e}}^{\mathrm{BO}}$ estimate by using the larger V5Z basis set and by taking into account the effect of diffuse functions via utilizing the AV5Z basis shows that these two effects almost fully compensate each other at the MP2 level, except for the $\angle(\mathrm{HNC})$ bond angle, which increases by $0.2^{\circ}$ when going from MP2(FC)/VQZ to MP2/(FC)/AV5Z. For this particular structural parameter, the increase is due both to the diffuse functions and to the larger V5Z basis set, the increase being $0.14^{\circ}$ when going from MP2(FC)/VQZ to MP2(FC)/AVQZ.

Table 4. Structure of Azete ${ }^{a}$

| method | $\operatorname{CCSD}(\mathrm{T})$ | MP2 | MP2 | $\operatorname{CCSD}(\mathrm{T})$ | MP2 | MP2 | MP2 | diff. ${ }^{\text {b }}$ | $\operatorname{CCSD}(\mathrm{T})^{\text {c }}$ | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{II})^{d}$ | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{III})^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| approx. ${ }^{f}$ | FC | AE | FC | AE | AE | FC | FC |  | AE |  |  |
| basis set | VQZ | wCVQZ | wCVQZ | wCVTZ | wCVTZ | VQZ | AV5Z |  | wCVQZ |  |  |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.2913 | 1.2847 | 1.2877 | 1.2908 | 1.2865 | 1.2881 | 1.2881 | 0.0000 | 1.2882 | 1.2883 | 1.2890 |
| $\mathrm{C}_{2}-\mathrm{C}_{3}$ | 1.5454 | 1.5354 | 1.5388 | 1.5440 | 1.5377 | 1.5393 | 1.5385 | -0.0008 | 1.5419 | 1.5420 | 1.5417 |
| $\mathrm{C}_{3}-\mathrm{C}_{4}$ | 1.3397 | 1.3291 | 1.3324 | 1.3386 | 1.331 | 1.3329 | 1.3325 | -0.0004 | 1.3362 | 1.3364 | 1.3367 |
| $\mathrm{N}_{1}-\mathrm{C}_{4}$ | 1.5549 | 1.5558 | 1.5598 | 1.5569 | 1.5612 | 1.5604 | 1.5593 | -0.0011 | 1.5509 | 1.5509 | 1.5515 |
| $\mathrm{C}_{2}-\mathrm{H}_{2}$ | 1.0846 | 1.0816 | 1.0831 | 1.0839 | 1.0825 | 1.0832 | 1.0831 | -0.0001 | 1.0831 | 1.0831 | 1.0830 |
| $\mathrm{C}_{3}-\mathrm{H}_{3}$ | 1.0786 | 1.0756 | 1.0772 | 1.0781 | 1.0767 | 1.0772 | 1.0772 | 0.0000 | 1.0771 | 1.0770 | 1.0770 |
| $\mathrm{C}_{4}-\mathrm{H}_{4}$ | 1.0791 | 1.0759 | 1.0774 | 1.0785 | 1.0768 | 1.0774 | 1.0774 | 0.0000 | 1.0776 | 1.0776 | 1.0776 |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{C}_{3}$ | 96.05 | 96.38 | 96.42 | 96.16 | 96.52 | 96.43 | 96.36 | -0.06 | 96.01 | 96.07 | 96.02 |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 84.54 | 84.69 | 84.67 | 84.58 | 84.69 | 84.67 | 84.72 | 0.05 | 84.56 | 84.57 | 84.58 |
| $\angle \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}_{2}$ | 136.62 | 136.25 | 136.23 | 136.50 | 136.11 | 136.22 | 136.31 | 0.09 | 136.63 | 136.61 | 136.64 |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}_{3}$ | 138.59 | 138.97 | 138.98 | 138.59 | 138.99 | 138.97 | 138.95 | -0.02 | 138.60 | 138.55 | 138.57 |
| $\angle \mathrm{H}_{4} \mathrm{C}_{4} \mathrm{C}_{3}$ | 139.14 | 139.59 | 139.63 | 139.13 | 139.61 | 139.65 | 139.68 | 0.03 | 139.11 | 139.17 | 139.11 |

${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b} \mathrm{MP2}(\mathrm{FC}) / \mathrm{AVSZ}-\mathrm{MP2}(\mathrm{FC}) / \mathrm{VQZ} .{ }^{c} r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$, see eq $1 .{ }^{d}$ See eq $2 .{ }^{e}$ See eq $3 .{ }^{f} \mathrm{AE}=$ all electrons correlated; FC = frozen core approximation.

The anharmonic force fields of the eight isotopologues for which measured rotational constants are available were computed at the MP2(FC)/VTZ level of electronic structure theory. The derived $\alpha$-constants were combined with the experimental ground-state rotational constants to yield estimates of semiexperimental equilibrium rotational constants, which were also corrected for the electronic contribution. The experimental ground state and semiexperimental equilibrium rotational constants are given in Table S2 of the Supporting Information. The experimental values of the molecular $g$-tensor are taken from ref 53. The semiexperimental equilibrium structure is determined by a weighted least-squares fit using an uncertainty of 0.2 MHz for $A$ and $B$ and 0.1 MHz for $C$. The fit is extremely well-conditioned, indicating that the derived structure is likely to be reliable. This $r_{\mathrm{e}}^{\mathrm{SE}}$ structure is given in Table 3 and can be compared to the $r_{\mathrm{e}}^{\mathrm{BO}}$ estimate. The agreement is almost perfect. Inspection of the residuals of the fit, also given in Supporting Information, Table S2, shows that there are a few large residuals, in particular for the $A$ and $B$ constants of the $c-\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{ND}$ species. These disagreements may be explained by the fact that the angle of rotation of the PAS upon substitution is very large, $\varphi=177^{\circ}$, and further confirmed by the large residuals of the $A$ and $B$ rotational constants, which have about the same absolute value but of opposite sign. However, these difficulties do not significantly affect the quality of the fit. The $r_{\mathrm{s}}$ and the empirical $r_{\mathrm{m}}^{(2)}$ structures are also given in Table 3. It is interesting to note that the empirical $r_{\mathrm{m}}^{(2)}$ structure is in surprisingly good agreement with the equilibrium structures. This may be explained by the favorable conditioning of the least-squares system.

The structure of aziridine is close to those of cyclopropane ${ }^{54}$ and oxirane. ${ }^{11}$ The $\angle C N C$ angle at $60.3^{\circ}$ is close to the $\angle C C C$ angle in cyclopropane, $60^{\circ}$, and considerably smaller than the analogous $\angle \mathrm{COC}$ angle in oxirane, $61.9^{\circ}$. The CC bond length in aziridine at $1.477 \AA$ is intermediate between the values found in cyclopropane, $1.504 \AA$, and in oxirane, $1.461 \AA$. This indicates a steady increase of the double bond character when going from cyclopropane to oxirane through aziridine. The $\angle \mathrm{HCH}$ angle in aziridine at $115.4^{\circ}$ is also intermediate between the values found in cyclopropane, $114.8^{\circ}$, and in oxirane, $116.1^{\circ}$. Likewise, the CH bond length in aziridine at $\sim 1.080-$ $1.081 \AA$ is also intermediate between the values found in cyclopropane, $1.079 \AA$, and in oxirane, $1.082 \AA$. As a
consequence, the nonbonded $d(\mathrm{H} \cdots \mathrm{H})$ distance is not constant in these molecules.

## 4. FOUR-MEMBERED RINGS: AZETE

Azete, $c-\mathrm{C}_{3} \mathrm{H}_{3} \mathrm{~N}$, is an unsaturated four-membered ring with three carbon atoms, one nitrogen atom, and two formal double bonds (Figure 1). It is extremely unstable, as would follow immediately from its alternate name azacyclobutadiene and its molecular orbital level structure. Azete has conjugated double bonds that form a system of four $\pi$-electrons. Therefore, it behaves as an extremely reactive diradical. The MW spectrum of azete has not yet been analyzed.

The $r_{\mathrm{e}}^{\mathrm{BO}}$ structure of azete was determined using eqs $1-3$, and the results are presented in Table 4. As for 2 H -azirine, these three equations give almost identical results. Furthermore, an attempt to improve the accuracy of the $r_{\mathrm{e}}^{\mathrm{BO}}$ estimate by using the larger V5Z basis set and by taking into account the effect of diffuse functions at the AV5Z level shows that these two effects almost compensate each other and can thus be neglected.

It is interesting to note that the length of the single bond $\mathrm{N}_{1}-\mathrm{C}_{4}$ at $1.551 \AA$ is significantly longer than the $\mathrm{C}-\mathrm{N}$ bond length in methylamine, $\mathrm{CH}_{3} \mathrm{NH}_{2}$, whose corresponding $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$ value is $1.461 \AA$. This is in agreement with the pronounced antiaromatic nature of azete.

## 5. FIVE-MEMBERED RINGS

The MW spectra of the five-membered N -heterocycles investigated here, namely, pyrrole, pyrazole, and imidazole (Figure 1), have all been investigated experimentally. Therefore, for these molecules not only $r_{\mathrm{e}}^{\mathrm{BO}}$ but also dependable $r_{\mathrm{e}}^{\text {SE }}$ structures can be determined.
5.1. Structure of Pyrrole. Pyrrole is a nonbasic N heterocyclic aromatic compound of formula $\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{NH}$. It is a planar five-membered ring with an equilibrium structure of point-group symmetry $C_{2 v}$ (Figure 1).

The MW spectrum of this oblate molecule $(\kappa=+0.94)$ and its six monosubstituted isotopic species was studied by Nygaard et al., ${ }^{55}$ and a substitution structure was derived. More recently, the MMW spectrum of the parent species was measured. ${ }^{56}$ The ground-state rotational constants were further improved by a combined fit of rotational transitions and ground-state combination differences. ${ }^{57}$ The analysis of the vibrational

Table 5. Structure of Pyrrole ${ }^{a}$

| method | MP2(AE) |  | $\operatorname{CCSD}(\mathrm{T})$ | $r_{\mathrm{e}}{ }^{\text {BO } ~} b$ | $r_{\mathrm{e}}{ }^{\mathrm{SE}}$ |  | $r_{\mathrm{m}}^{(1)}$ | $r_{\text {s }}{ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| basis set | CVTZ | wCVQZ | CVTZ AE |  | B3LYP ${ }^{\text {d }}$ | MP2 ${ }^{\text {e }}$ |  |  |
| CN | 1.3653 | 1.3628 | 1.37173 | 1.3692 | 1.3694(2) | 1.36940(17) | 1.3764(10) | 1.370 |
| NH | 1.0027 | 1.0022 | 1.00175 | 1.0013 | 1.00062(17) | 1.00086(14) | 0.9943(8) | 0.996 |
| $\mathrm{C}_{2} \mathrm{C}_{3}$ | 1.3787 | 1.3760 | 1.37625 | 1.3736 | 1.3728(3) | 1.3723(2) | 1.3783(15) | 1.382 |
| $\mathrm{C}_{2} \mathrm{H}$ | 1.0742 | 1.0731 | 1.07589 | 1.0748 | 1.07452(15) | 1.07532(13) | 1.0740(8) | 1.076 |
| $\mathrm{C}_{3} \mathrm{C}_{4}$ | 1.4130 | 1.4105 | 1.42502 | 1.4225 | $1.4232(5)$ | 1.4231(4) | 1.430(3) | 1.417 |
| $\mathrm{C}_{3} \mathrm{H}$ | 1.0749 | 1.0738 | 1.07667 | 1.0756 | 1.07436(17) | 1.07527(16) | 1.0747(9) | 1.077 |
| $\angle \mathrm{CNC}$ | 110.18 | 110.19 | 109.81 | 109.82 | 109.82(2) | 109.809(16) | 109.67(9) | 109.8 |
| $\angle \mathrm{CNH}$ | 124.91 | 124.90 | 125.10 | 125.09 | 125.089(10) | 125.096(8) | 125.16(5) | 121.5 |
| $\angle \mathrm{NC}_{2} \mathrm{C}_{3}$ | 107.48 | 107.46 | 107.77 | 107.76 | 107.76(2) | 107.762(15) | 107.85(9) | 107.7 |
| $\angle \mathrm{NC}_{2} \mathrm{H}$ | 121.31 | 121.32 | 121.18 | 121.19 | 121.04(9) | 120.99(7) | 121.5(5) |  |
| $\angle \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}$ | 131.22 | 131.21 | 131.05 | 131.05 | 131.20(9) | 131.25(7) | 130.7(5) |  |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 107.43 | 107.44 | 107.33 | 107.33 | 107.329(13) | 107.334(12) | 107.32(7) | 107.4 |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}$ | 125.50 | 125.50 | 125.70 | 125.70 | 125.98(9) | 125.94(6) | 126.0(5) |  |
| $\angle \mathrm{C}_{4} \mathrm{C}_{3} \mathrm{H}$ | 127.06 | 127.06 | 126.98 | 126.97 | 126.69(8) | 126.73(5) | 126.6(5) | 127.1 |

${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b}$ From eq 3. ${ }^{c}$ Reference 55 . ${ }^{d}$ Rovibrational corrections from the B3LYP/6-311+G(3df,2pd) cubic force field. ${ }^{e}$ Rovibrational corrections from the $\operatorname{MP2}(\mathrm{AE}) / \mathrm{wCVTZ}$ cubic force field.
spectrum has been reviewed by Hegelund et al. ${ }^{58}$ These authors also computed ab initio the rotational, centrifugal distortion, and vibration-rotation interaction constants. ${ }^{59}$ There is also a determination of the structure of pyrrole- ${ }^{15} \mathrm{~N}$ in nematic liquid crystals. ${ }^{60}$ However, a complex formation was observed between pyrrole and the liquid-crystal molecules hampering an accurate structure determination. Finally, note that the electronic spectrum of pyrrole has been investigated by using a hierarchy of coupled-cluster methods. ${ }^{61}$

An estimate to $r_{\mathrm{e}}^{\mathrm{BO}}$ of pyrrole was computed using eq 3. The structural parameters obtained are given in Table 5.

To determine $r_{\mathrm{e}}^{\mathrm{SE}}$ of pyrrole, two cubic force fields were computed, at the B3LYP/6-311+G(3df,2pd) and MP2(AE)/ wCVTZ levels of theory. The ground-state rotational constants of the parent species were taken from ref 56 . The ground-state rotational spectra of the isotopic species were refitted using as predicates the quartic centrifugal distortion constants derived from the harmonic force field computed at the B3LYP/6$311+G(3 \mathrm{df}, 2 \mathrm{pd})$ level of theory. The experimental frequencies were taken from ref 55 . The experimental values of the molecular $g$-tensor are taken from ref 53. The experimental ground-state and semiexperimental equilibrium rotational constants, obtained with the help of a MP2(AE)/wCVTZ cubic force field, as well as the inertial defects, are given in Table S3 of the Supporting Information.

The semiexperimental structure, given in Table 5, is determined by an iteratively reweighted least-squares fit using the biweight weighting scheme. The $r_{\mathrm{e}}^{\mathrm{SE}}$ structures derived with the MP2 and B3LYP force fields are in good agreement with the $r_{\mathrm{e}}^{\mathrm{BO}}$ structure; the largest differences can be observed for the $\angle \mathrm{CCH}$ angles, up to $0.2^{\circ}$ for $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}$. The $r_{\mathrm{e}}^{\mathrm{SE}}$ structure derived from the B3LYP/ $6-311+G(3 \mathrm{df}, 2 \mathrm{pd})$ cubic force field is marginally less accurate, but the difference may be considered negligible. To estimate the quality of the derived structure, it is informative to have a look at the equilibrium inertial defect, which should be exactly zero for a planar molecule. The experimental ground-state inertial defect is $\Delta_{0}=+0.0162 \mathrm{u} \AA^{2}$ for the parent species; after the rovibrational correction, based on the MP2/wCVTZ force field, it becomes $-0.0087 \mathrm{u}^{2}$, and after the electronic correction it is $\Delta_{e}=0.0004 u \AA^{2}$, that is, practically zero. This confirms that the rovibrational correction is likely to be accurate and that the electronic correction,
although small, is not negligible. The B3LYP/6-311+G(3df, 2 pd ) force field gives similar results, $\Delta_{\mathrm{e}}=-0.0007 \mathrm{u} \AA^{2}$ after inclusion of the electronic correction. Similar values are obtained for the isotopic species; however, the values for the deuterated species are slightly worse, in particular for the ND species. This observation may be because the ground-state rotational constants of this isotopologue are less accurate, but it could also be explained by a small deficiency of the force field. This is confirmed by the fact that the residual for the $A$ constant of the ND species, shown in Supporting Information, Table S3, is large. The angle of rotation of the PAS upon isotopic substitution, also shown in Supporting Information, Table S3, is large; in particular, there is an $a / b$ axis switching from the ${ }^{13} \mathrm{C}_{2}$ and $\mathrm{D}_{2}$ species. However, this switching does not seem to affect the quality of the fit. This stability is probably the consequence of the good conditioning of the fit: thanks to the $C_{2 v}$ point-group symmetry, the number of parameters to be determined is small, compared to the number of independent rotational constants, 14.

As expected, the structure of pyrrole is close to that of furane. ${ }^{11}$ Nevertheless, there are a couple of noteworthy differences. The $\mathrm{C}_{2} \mathrm{C}_{3}$ bond in pyrrole at $1.374 \AA$ is slightly longer than that in furane, $1.355 \AA$, while the $C_{3} C_{4}$ bond at $1.423 \AA$ is slightly shorter than in furane, $1.434 \AA$. Furthermore, $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ at $107.3^{\circ}$ in pyrrole is slightly larger than it is in furane, where it is $106^{\circ}$. Also as expected, the differences between the equilibrium structures of pyrrole and cyclopentadiene ${ }^{62}$ are significant: the geometrical parameters for this last molecule are $\mathrm{C}_{2} \mathrm{C}_{3}=1.346 \AA, \mathrm{C}_{3} \mathrm{C}_{4}=1.466 \AA$, and $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}=109.13^{\circ}$.
5.2. Structure of Pyrazole. Pyrazole is a planar, nearoblate $(\kappa=+0.915)$ N-heterocyclic aromatic compound with the formula $\mathrm{C}_{3} \mathrm{H}_{3} \mathrm{~N}_{2} \mathrm{H}$ (Figure 1). It is a five-membered ring with two adjacent nitrogen atoms. The derivatives of pyrazole have many medicinal applications.

The MW spectrum of pyrazole was first measured by Kirchhoff. ${ }^{63}$ Later, Nygaard et al. ${ }^{64}$ have investigated the MW spectra of the complete set of monosubstituted isotopic pyrazoles and have determined a complete $r_{\mathrm{s}}$ structure. The rotational Zeeman spectrum of the parent species has been studied by Stolze and Sutter. ${ }^{65}$ The rotational spectrum of 1-Dpyrazole was studied by microwave Fourier transform

Table 6. Structure of Pyrazole ${ }^{a}$

| method | MP2(AE) |  | $\operatorname{CCSD}(\mathrm{T}) \mathrm{AE}$ | $r_{\mathrm{e}}^{\mathrm{BO}}{ }^{\text {b }}$ | $r_{\text {e }}^{\text {SE }}$ | $r_{\text {m }}^{(1)}$ | $r_{\text {s }}{ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| basis set | CVTZ | wCVQZ | CVTZ |  |  |  |  |
| $\mathrm{N}_{1}-\mathrm{H}$ | 1.0040 | 1.0034 | 1.0025 | 1.0020 | 1.0014(4) | 0.9956(4) | 0.9978(4) |
| $\mathrm{N}_{1}-\mathrm{N}_{2}$ | 1.3362 | 1.3334 | 1.3465 | 1.3437 | 1.3431(6) | 1.348(1) | 1.3488(6) |
| $\mathrm{N}_{2}=\mathrm{C}_{3}$ | 1.3407 | 1.3378 | 1.3323 | 1.3293 | 1.3286(7) | 1.329 (1) | 1.3306(5) |
| $\mathrm{N}_{1}-\mathrm{C}_{5}$ | 1.3528 | 1.3500 | 1.3547 | 1.3518 | 1.3523(6) | 1.354(1) | 1.3591(1) |
| $\mathrm{C}_{3}-\mathrm{C}_{4}$ | 1.4005 | 1.3979 | 1.4122 | 1.4096 | 1.4093(6) | 1.411(1) | 1.4162(2) |
| $\mathrm{C}_{4}=\mathrm{C}_{5}$ | 1.3802 | 1.3774 | 1.3792 | 1.3765 | 1.3771(8) | 1.376(1) | 1.3724(4) |
| $\mathrm{C}_{3}-\mathrm{H}$ | 1.0749 | 1.0739 | 1.0770 | 1.0759 | 1.0755(4) | 1.0763(4) | 1.0784(4) |
| $\mathrm{C}_{4}-\mathrm{H}$ | 1.0736 | 1.0725 | 1.0752 | 1.0741 | 1.0736(4) | 1.0738(4) | 1.0756(5) |
| $\mathrm{C}_{5}-\mathrm{H}$ | 1.0743 | 1.0732 | 1.0761 | 1.0750 | 1.0740(5) | 1.0751(4) | 1.0774(5) |
| $\angle \mathrm{HN}_{1} \mathrm{~N}_{2}$ | 118.69 | 118.74 | 118.92 | 118.97 | 118.97(11) | 118.61(12) | 118.40(3) |
| $\angle \mathrm{HN}_{1} \mathrm{C}_{5}$ | 127.47 | 127.52 | 127.70 | 127.75 | 127.79(12) | 128.32(13) |  |
| $\angle \mathrm{N}_{2} \mathrm{~N}_{1} \mathrm{C}_{5}$ | 113.84 | 113.74 | 113.38 | 113.28 | 113.24(5) | 113.07(5) | 113.07(3) |
| $\angle \mathrm{N}_{1} \mathrm{~N}_{2} \mathrm{C}_{3}$ | 103.74 | 103.89 | 103.93 | 104.08 | 104.18(3) | 104.12(4) | 104.07(1) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{C}_{4}$ | 105.80 | 105.83 | 106.24 | 106.27 | 106.23(4) | 106.36(4) | 106.42(2) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{H}$ | 121.90 | 121.89 | 121.78 | 121.77 | 121.84(11) | 121.47(10) | 121.47(4) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{H}$ | 132.30 | 132.28 | 131.98 | 131.96 | 131.93(10) | 132.16(9) |  |
| $\angle \mathrm{N}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 111.88 | 111.75 | 112.07 | 111.94 | 111.90(5) | 111.97(5) | 111.94(3) |
| $\angle \mathrm{N}_{2} \mathrm{C}_{3} \mathrm{H}$ | 119.21 | 119.26 | 119.41 | 119.46 | 119.49(14) | 119.59(14) | 119.31(4) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{3} \mathrm{H}$ | 128.91 | 128.99 | 128.53 | 128.60 | 128.62(15) | 128.44(15) |  |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{C}_{3}$ | 104.74 | 104.80 | 104.38 | 104.43 | 104.46(4) | 104.48(3) | 104.50(2) |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{H}$ | 126.95 | 126.93 | 127.29 | 127.27 | 127.23(13) | 127.67(11) |  |
| $\angle \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{H}$ | 128.31 | 128.27 | 128.33 | 128.30 | 128.32(13) | 127.85(12) | 127.90(5) |
| istances in | angles in | ${ }^{b}$ From eq | eference 64 . |  |  |  |  |

Table 7. Structure of Imidazole ${ }^{a}$

|  | $\operatorname{CCSD}(\mathrm{T})$ | MP2 (AE) |  | $r_{\text {e }}{ }^{\mathrm{BO}} \mathrm{b}$ | $r_{\mathrm{e}}^{\mathrm{SE}} \boldsymbol{c}$ | $r_{\mathrm{m}}^{(2 \mathrm{r})}$ | $r_{\text {e }}^{\text {EE } d}$ | $r_{\text {s }}{ }^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CVTZ AE | CVTZ | wCVQZ |  |  |  |  |  |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.36216 | 1.3586 | 1.3561 | 1.3597 | 1.3612(9) | 1.3612(10) | 1.3594(3) | 1.369 |
| $\mathrm{N}_{1}-\mathrm{C}_{5}$ | 1.3774 | 1.3678 | 1.3652 | 1.3748 | 1.3738(9) | $1.3735(10)$ | 1.3758(3) | 1.3777 |
| $\mathrm{N}_{1}-\mathrm{H}$ | 1.00316 | 1.0038 | 1.0033 | 1.0027 | 1.0008(5) | 0.9952(4) | 1.0013(1) | 0.995 |
| $\mathrm{C}_{2}=\mathrm{N}_{3}$ | 1.3140 | 1.3182 | 1.3154 | 1.3112 | 1.3111(8) | 1.3093(8) | 1.3114(2) | 1.3164 |
| $\mathrm{C}_{2}-\mathrm{H}$ | 1.07731 | 1.0749 | 1.0738 | 1.0762 | 1.0759(6) | 1.0769(4) | 1.0761(1) | 1.075 |
| $\mathrm{N}_{3}-\mathrm{C}_{4}$ | 1.38035 | 1.3692 | 1.3663 | 1.3775 | 1.3797(8) | 1.3788(9) | 1.3774(3) | 1.388 |
| $\mathrm{C}_{4}=\mathrm{C}_{5}$ | 1.36952 | 1.3729 | 1.3702 | 1.3668 | 1.3627(8) | $1.3605(9)$ | 1.3663(3) | 1.3672 |
| $\mathrm{C}_{4}-\mathrm{H}$ | 1.07658 | 1.0746 | 1.0735 | 1.0755 | 1.0747(6) | 1.0756(4) | 1.0749(2) | 1.0733 |
| $\mathrm{C}_{5}-\mathrm{H}$ | 1.07522 | 1.0733 | 1.0722 | 1.0741 | 1.0764(5) | 1.0763(4) | 1.0734(3) | 1.0766 |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{C}_{5}$ | 107.17 | 107.58 | 107.63 | 107.22 | 107.02(5) | 106.90(4) | 107.18(1) | 106.96 |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{H}_{1}$ | 126.42 | 126.24 | 126.20 | 126.38 | 126.23(16) | 126.27(11) | 126.44(4) |  |
| $\angle \mathrm{C}_{5} \mathrm{~N}_{1} \mathrm{H}_{1}$ | 126.42 | 126.18 | 126.17 | 126.40 | 126.75(15) | 126.83(11) | 126.38(3) |  |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{~N}_{3}$ | 112.01 | 111.56 | 111.41 | 111.87 | 111.91(6) | 111.97(4) | 111.90(2) | 111.87 |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{H}_{2}$ | 122.25 | 122.45 | 122.53 | 122.32 | 122.53(12) | 122.44(9) | 122.39(3) |  |
| $\angle \mathrm{N}_{3} \mathrm{C}_{2} \mathrm{H}_{2}$ | 125.74 | 126.00 | 126.06 | 125.81 | 125.57(12) | 125.59(9) | 125.71(3) |  |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{3} \mathrm{C}_{4}$ | 104.92 | 105.07 | 105.25 | 105.10 | 105.02(5) | 104.99(4) | 105.08(2) | 105.04 |
| $\angle \mathrm{N}_{3} \mathrm{C}_{4} \mathrm{C}_{5}$ | 110.78 | 110.82 | 110.69 | 110.66 | 110.60(6) | 110.64(4) | 110.69(2) | 110.45 |
| $\angle \mathrm{N}_{3} \mathrm{C}_{4} \mathrm{H}_{4}$ | 121.39 | 121.56 | 121.61 | 121.43 | 121.51(11) | 121.46(8) | 121.48(3) |  |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{H}_{4}$ | 127.83 | 127.62 | 127.70 | 127.91 | 127.89(11) | 127.90(8) | 127.83(4) |  |
| $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{C}_{4}$ | 105.12 | 104.98 | 105.02 | 105.16 | 105.45(6) | 105.50(4) | 105.16(3) | 105.68 |
| $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{H}_{5}$ | 122.27 | 122.40 | 122.40 | 122.27 | 121.92(12) | 121.95(9) | 122.29(3) |  |
| $\angle \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{H}_{5}$ | 132.61 | 132.62 | 132.57 | 132.57 | 132.62(12) | 132.55(9) | 132.56(4) |  |

${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b}$ From eq $1 .{ }^{c}$ Simple weighted fit. ${ }^{d}$ Fit with predicates, see text. ${ }^{e}$ Reference 70.
(MWFT) spectroscopy and by Zeeman spectroscopy by Böttcher and Sutter. ${ }^{66}$ The MMW spectrum of the parent species was measured, and a complete ab initio quadratic force field was calculated by Wlodaraczak et al. ${ }^{67}$ There are also ab initio estimates of the equilibrium structure of pyrazole determined by electronic structure computations up to the QCISD level of theory. ${ }^{68}$

In the present study, an estimate to $r_{\mathrm{e}}^{\mathrm{BO}}$ of pyrazole was computed using eq 3 . The structural results obtained are given in Table 6.

To determine the semiexperimental structure, the cubic force field was computed at the B3LYP/6-311+G(3df,2pd) level of theory. The derived $\alpha$-constants were combined with the experimental ground-state rotational constants to yield
estimates of semiexperimental equilibrium rotational constants, which were also corrected for the electronic contribution. The experimental values of the molecular $g$-tensor were taken from refs 65 and 66. The experimental ground-state inertial defect is $\Delta_{0}=+0.0316 u \AA^{2}$ for the parent species; after the rovibrational correction it becomes $-0.0104 \mathrm{u}^{2}$, and after the electronic correction it is $\Delta_{\mathrm{e}}=-0.0007 \mathrm{u}^{2}$, that is, practically zero. This confirms that the rovibrational correction is likely to be accurate and that the electronic correction, although small, is once more not negligible. Similar values to those reported above are obtained for the isotopic species, with a very narrow range of $0.0008 \mathrm{u}^{2}$ for the equilibrium inertial defect. The ground-state and semiexperimental equilibrium rotational constants are given in Table S4 of the Supporting Information.

A weighted structural fit employing uncertainties of 0.05 MHz for $A_{\mathrm{e}}$ and $B_{\mathrm{e}}$ and 0.02 MHz for $C_{e}$ provides satisfactory results. The derived structural parameters are given in Table 6, and the residuals of the fit are given in Table S 4 of the Supporting Information. Inspection of these residuals shows that for the species ${ }^{15} \mathrm{~N}_{1}$ and ${ }^{13} \mathrm{C}_{3}$ the values are large for $A$ and $B$ and of opposite sign, which is typical of an oblate molecule with large rotation of the PAS upon isotopic substitution. ${ }^{11}$ However, the standard deviations of the structural parameters remain small, and it is also telling that the $r_{\mathrm{e}}^{\mathrm{SE}}$ structure is in very good agreement with the $r_{\mathrm{e}}^{\mathrm{BO}}$ one.

The $r_{\mathrm{s}}$ and the empirical $r_{\mathrm{m}}^{(1)}$ structures are also given in Table 6. It is interesting to note that the empirical $r_{\mathrm{m}}^{(1)}$ structure is in fair agreement with the equilibrium structure.
5.3. Structure of Imidazole. Imidazole is a structural isomer of pyrazole with two nonadjacent nitrogen atoms. It exists in two equivalent tautomeric forms, and the molecule is amphoteric: the $\mathrm{N}_{1}-\mathrm{H}$ group is a proton donor, and the $\mathrm{N}_{3}$ atom is a proton acceptor. Imidazole is incorporated in many biological molecules, for instance, histidine. Its equilibrium structure on the ground electronic state has $C_{s}$ point-group symmetry, and the molecule is an oblate top with $\kappa=+0.86$.

The MW spectrum of imidazole was first analyzed by Blackman et al., ${ }^{69}$ who paid attention to the complex quadrupole hyperfine splitting. Christen et al. ${ }^{70}$ measured the MW spectra of 11 isotopic species, including single substitution for each atom, and they determined a complete $r_{s}$ structure of imidazole.

The best estimate to $r_{\mathrm{e}}^{\mathrm{BO}}$ of imidazole was computed using eq 3, with structural parameters given in Table 7. Addition of diffuse functions to the wCVQZ basis set (aug-cc-pwCVQZ) has a very small effect on the structure: the CN bond lengths are lengthened by less than $0.0007 \AA$, this lengthening being partly compensated by enlargement of the basis from wCVQZ to wCV5Z, while the effect on the angles is also smaller than $0.07^{\circ}$.

To improve the quality of the rotational constants employed during the structural fit, the ground-state rotational spectra of all species were refitted using as predicates the quartic centrifugal distortion constants derived from the harmonic force field computed at the B3LYP/6-311+G(3df,2pd) level. The experimental rotational frequencies were taken from ref 70 . The anharmonic force fields of the 11 isotopologues were computed at the B3LYP/6-311+G(3df,2pd) level. The derived $\alpha$-constants were combined with the experimental ground-state rotational constants to yield estimates of semiexperimental equilibrium rotational constants, which were also corrected for the electronic contribution. The experimental values of the molecular $g$-tensor were taken from ref 65 . The ground-state
and semiexperimental equilibrium rotational constants, as well as the inertial defects, are given in Table S5 of the Supporting Information. The experimental ground-state inertial defect is $\Delta_{0}$ $=+0.0287 \mathrm{u}^{2}$ for the parent species; after the rovibrational correction it becomes $-0.0107 \mathrm{u}^{2}$, and after the electronic correction it is $\Delta_{\mathrm{e}}=-0.0016 \mathrm{u}^{2}$, that is, practically zero. Similar results are obtained for the isotopic species. This indicates that the rovibrational corrections are probably accurate and that the electronic correction, although small, is not negligible, as found repeatedly in this study.

The $r_{\mathrm{e}}^{\mathrm{SE}}$ structure of imidazole was first determined by a weighted least-squares fit. The structural parameters obtained, given in Table 7 , especially those related to $\mathrm{C}_{5}$, are not satisfactory; the $r_{\mathrm{e}}^{\mathrm{SE}}$ value for the $\mathrm{C}_{4}-\mathrm{C}_{5}$ bond length is too short by $0.004 \AA$, and the $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{C}_{4}$ and $\angle \mathrm{N}_{1} \mathrm{C}_{5} \mathrm{H}_{5}$ angles deviate by $0.3^{\circ}$ from their $r_{\mathrm{e}}^{\mathrm{BO}}$ counterparts. This is due to two difficulties. First, imidazole is an oblate molecule, and there are large rotations of the PAS upon isotopic substitution, see Table S5 of the Supporting Information. In particular, when the $C_{5}$ carbon atom is substituted, the axes $a$ and $b$ are switched. Then, there are several small Cartesian coordinates, especially $b\left(\mathrm{~N}_{3}\right)=$ $0.202 \AA, a\left(\mathrm{C}_{2}\right)=0.220 \AA$, and $a\left(\mathrm{H}_{2}\right)=0.348 \AA$. This is enough to hamper an accurate determination of $r_{\mathrm{e}}^{\mathrm{SE}}$ of this molecule.

At this stage, it is useful to determine an $r_{\mathrm{m}}^{(2 \mathrm{r})}$ structure. Usually it is hoped that the $r_{\mathrm{m}}^{(2 \mathrm{r})}$ structure determination would help to better analyze the origin of problems associated with the $r_{\mathrm{e}}^{\mathrm{SE}}$ structure. As shown in Table 7, the $r_{\mathrm{m}}^{(2 \mathrm{r})}$ structure has exactly the same shortcomings as the $r_{\mathrm{e}}^{\mathrm{SE}}$ structure; in particular, the $\mathrm{C}_{4}-\mathrm{C}_{5}$ bond length is much too short.

Nevertheless, it is possible to improve the semiexperimental fit. It is known that the computed rovibrational corrections are affected mainly by a systematic error, which is a few percent of the rovibrational correction. ${ }^{43}$ As a consequence, the semiexperimental inertial defects are different from zero. It is worth correcting for this error by multiplying all the rovibrational corrections by a constant factor, $f=0.960$ in this case; with this choice the equilibrium inertial defect of the parent species becomes zero. This correction reduces the standard deviations of the parameters but does not significantly affect their values. The next step forward is to use the mixed regression method. ${ }^{29,71}$ In this method, some $r_{\mathrm{e}}^{\mathrm{BO}}$ internal coordinates (called predicate observations, entered with an appropriate uncertainty) are fitted simultaneously with the semiexperimental moments of inertia. The uncertainties chosen for the rotational constant were (in kHz ): 35 for $B$ and 40 for $C$, and we increased the uncertainties of the $B$ constants for isotopologues with a large PAS rotation. The uncertainty of the predicates was $0.002 \AA$ for the bond lengths and $0.2^{\circ}$ for the angles. This fit gives very satisfactory results and is given in the second-to-last column of Table 7. These parameters are considered to be the ones providing the best representation of the equilibrium structure of imidazole.

The lone pair of the $\mathrm{N}_{1}$ nitrogen atom (pyrrolic nitrogen) in imidazole and pyrazole can contribute two electrons to the aromatic sextet. On the contrary, the second nitrogen atom $\left(\mathrm{N}_{2}\right.$ for pyrazole or $\mathrm{N}_{3}$ for imidazole) has its lone pair localized in $\mathrm{sp}^{2}$ orbitals. As a consequence, imidazole and pyrazole have structural properties intermediate between pyrrole and pyridine. For example, the $\mathrm{C}_{2}=\mathrm{N}_{3}$ bond length in imidazole at $1.311 \AA$ and especially the $\mathrm{N}_{2}=\mathrm{C}_{3}$ bond length in pyrazole at $1.329 \AA$ are close to the value found in pyridine, $1.336 \AA$ (see below, section 6.1). Furthermore, the $\mathrm{N}_{1} \mathrm{C}_{5}$ bond length in pyrazole at $1.352 \AA$ or the $\mathrm{N}_{1} \mathrm{C}_{2}$ and $\mathrm{N}_{1} \mathrm{C}_{5}$ bond lengths in

Table 8. Structure of Pyridine ${ }^{a}$

|  | MP2(AE) |  | $\operatorname{CCSD}(\mathrm{T}) \mathrm{AE}$ | $r_{\text {e }}{ }^{\text {BO } b}$ | $r_{\mathrm{e}}^{\text {SE }}$ | $r_{\text {m }}^{(2)}$ | $r_{\text {s }}{ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CVTZ | wCVQZ | CVTZ |  |  |  |  |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.3367 | 1.3338 | 1.3391 | 1.3362 | 1.3362(5) | 1.3368(8) | 1.3376(4) |
| $\mathrm{C}_{2}-\mathrm{C}_{3}$ | 1.3891 | 1.3865 | 1.3936 | 1.3910 | 1.3902(4) | $1.3915(13)$ | 1.3938(6) |
| $\mathrm{C}_{3}-\mathrm{C}_{4}$ | 1.3875 | 1.3848 | 1.3915 | 1.3889 | 1.3890(4) | 1.3906(7) | 1.3916(4) |
| $\mathrm{C}_{2}-\mathrm{H}$ | 1.0818 | 1.0807 | 1.0839 | 1.0828 | 1.0816(4) | 1.0839(6) | 1.0865(4) |
| $\mathrm{C}_{3}-\mathrm{H}$ | 1.0796 | 1.0785 | 1.0816 | 1.0805 | 1.0795(4) | 1.0800(6) | 1.0826(4) |
| $\mathrm{C}_{4}-\mathrm{H}$ | 1.0801 | 1.0790 | 1.0822 | 1.0811 | 1.0803(4) | 1.0805(5) | 1.0818(2) |
| $\angle \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{H}$ | 120.85 | 120.83 | 120.80 | 120.78 | 120.71(2) | 120.78(3) | 120.80(2) |
| $\angle \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5}$ | 118.29 | 118.33 | 118.40 | 118.44 | 118.42(4) | 118.44(6) | 118.40(3) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{3} \mathrm{H}$ | 121.23 | 121.21 | 121.35 | 121.33 | 121.34(5) | 121.27(6) | 121.36(4) |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 118.69 | 118.70 | 118.51 | 118.53 | 118.54(4) | 118.51(6) | 118.53(3) |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}$ | 120.08 | 120.08 | 120.14 | 120.14 | 120.11(6) | 120.22(7) | 120.12(3) |
| $\angle \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}$ | 120.28 | 120.37 | 120.18 | 120.27 | 120.30(6) | 120.31(7) | 120.19(3) |
| $\angle \mathrm{NC}_{2} \mathrm{C}_{3}$ | 123.76 | 123.60 | 123.96 | 123.80 | 123.80(4) | 123.78(6) | 123.80(3) |
| $\angle \mathrm{NC}_{2} \mathrm{H}$ | 115.96 | 116.03 | 115.86 | 115.93 | 115.90(5) | 115.91(6) | 116.01(3) |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{C}_{6}$ | 116.82 | 117.06 | 116.66 | 116.90 | 116.90(4) | 116.97(6) | 116.94(3) |
| istances in | gles in | ${ }^{b}$ From | erence 73 . |  |  |  |  |

imidazole at 1.360 and $1.375 \AA$, respectively, are close to the value found in pyrrole, $1.369 \AA$.

## 6. SIX-MEMBERED RINGS

6.1. Structure of Pyridine. Pyridine is a basic N heterocyclic compound of formula $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{~N}$ (Figure 1). It is an important solvent and reagent, and its ring occurs in many compounds of importance for life.

Pyridine is a planar oblate top $(\kappa=+0.848)$ with an equilibrium structure of point-group symmetry $C_{2 v}$. Its rotational spectrum has been analyzed many times, results published prior to 2005 are reviewed by Ye et al. ${ }^{72}$ Particularly relevant for the present study is the work of Mata et al., ${ }^{73}$ who investigated the spectra of the deuterated species between 14 and 27 GHz and determined a precise $r_{\mathrm{s}}$ structure. As a result of the analysis of FTIR spectra, rotational constants of excited vibrational states are also available for pyridine. ${ }^{74}$ Nevertheless, these constants are not used during the present structural analysis. There is also an early gas electron diffraction (GED) study for the structure of this molecule from Pyckhout et al., ${ }^{75}$ where information from infrared data and $a b$ initio computations was also utilized. In an influential paper from 1984, Pongor, Pulay, and $\mathrm{Boggs}^{76}$ determined accurate $r_{\mathrm{e}}^{\mathrm{BO}}$ estimates and a scaled quantum mechanical (SQM) force field ${ }^{77,78}$ for pyridine, aiding the assignment of the vibrational spectrum of the molecule. Later, several higher-level ab initio and DFT computations were reported on the structure and the vibrational spectrum of pyridine. ${ }^{79}$ All these studies agree that the main characteristic of the structure of pyridine, as compared to benzene, is that the $\angle \mathrm{CNC}$ angle is significantly smaller than $120^{\circ}$, its value being slightly smaller than $117^{\circ}$.

A new estimate to $r_{\mathrm{e}}^{\mathrm{BO}}$ of pyridine was computed in this study using eq 3. The $r_{\mathrm{e}}^{\mathrm{BO}}$ (III) structural parameters obtained are given in Table 8.

The ground-state rotational constants of the parent species were taken from ref 70 . The ground-state rotational spectra of the isotopic species were refitted using as predicates the quartic centrifugal distortion constants derived from the harmonic force field computed at the B3LYP $/ 6-311+G(3 d f, 2 p d)$ level of theory. The experimental frequencies were taken from ref 71 , except for the $\mathrm{D}_{5}$ species, which are from ref 80 , and for the $4-{ }^{13} \mathrm{C}$ and ${ }^{15} \mathrm{~N}$ species, which are from ref 81 .

The anharmonic force fields of ten isotopologues were computed at the B3LYP/6-311+G(3df,2pd) level. The derived $\alpha$-constants were combined with the experimental ground-state rotational constants to yield estimates of semiexperimental equilibrium rotational constants, which were also corrected for the electronic contribution. The experimental values of the molecular $g$-tensor are taken from ref 82 for the parent species and from ref 83 for the ${ }^{15} \mathrm{~N}$ species. The ground-state and semiexperimental equilibrium rotational constants as well as the inertial defects are given in Table S6 of the Supporting Information. The experimental ground-state inertial defect is $\Delta_{0}$ $=+0.039 \mathrm{u} \AA^{2}$ for the parent species; after the rovibrational correction it becomes $-0.015 \mathrm{u}^{2}$, and after the electronic correction it is $\Delta_{e}=0.0031 \mathrm{u}^{2}$, that is, close to zero. Similar results are obtained for the isotopic species; the range of the equilibrium inertial defects is only $0.0014 \mathrm{u}^{2}$. This confirms that the rovibrational correction is likely to be accurate and that the electronic correction, although small, is not negligible.

The semiexperimental structure of pyridine is determined by an iteratively reweighted least-squares fit using the biweight weighting scheme. The structural parameters obtained are given in Table 8. The weight assigned to each datum in the final iteration is very similar and close to one, the only exceptions being the $A$ rotational constants of the species $\mathrm{D}_{2}, \mathrm{D}_{3}$, and ${ }^{13} \mathrm{C}_{3}$, which have a slightly lower weight. Agreement between the $r_{e}^{3 \mathrm{EE}}$ and $r_{\mathrm{e}}^{\mathrm{BO}}$ structures is almost perfect. Again, this is related to the good conditioning of the least-squares system. The $r_{\mathrm{s}}$ structure of Mata et al. ${ }^{73}$ is also given in Table 8. It is worth emphasizing the surprisingly high consistency of the angle estimates. The empirical $r_{\mathrm{m}}^{(2)}$ structure is also given in Table 8. It is interesting to note that it is in surprisingly good agreement with the equilibrium structure, with the exception of the CH bond lengths.

Comparison of the structure of pyridine with that of benzene, for which $r_{\mathrm{e}}(\mathrm{CC})=1.392 \AA$ and $r_{\mathrm{e}}(\mathrm{CH})=1.081$ $\AA,{ }^{84}$ shows that neither the CC nor the CH bond lengths are affected much by the single $\mathrm{CH} \rightarrow \mathrm{N}$ substitution. It is still interesting to note that the decreased $\pi$-electron density of pyridine as compared to benzene leads to a very slight contraction, instead of elongation, of $r_{\mathrm{e}}(\mathrm{CC})$, to $1.390 \AA$ on average. The ring is strongly distorted at the ipso angle; $\angle \mathrm{CNC}$ is smaller than $120^{\circ}$ by $3.1^{\circ}$, and the ortho inner-ring angle,

Table 9. Equilibrium Structural Parameters of Pyrimidine ${ }^{a}$

|  | $\operatorname{CCSD}(\mathrm{T})$ |  |  |  | MP2 | diff. ${ }^{\text {b }}$ |  | $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{II})^{c}$ | $r_{\mathrm{e}}^{\text {SE } d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\text { approx. }{ }^{e}$ | FC | AE | FC | FC | FC |  | $\mathrm{AE}^{f}$ |  |  |
| basis set | VQZ | wCVQZ | wCVQZ | VQZ | AV5Z |  | wCVQZ |  |  |
| $\mathrm{N}_{1}-\mathrm{C}_{2}=\mathrm{N}_{3}-\mathrm{C}_{2}$ | 1.3369 | 1.3310 | 1.334 | 1.3343 | 1.3341 | -0.0002 | 1.3338 | 1.3339 | 1.3331(3) |
| $\mathrm{N}_{1}-\mathrm{C}_{6}=\mathrm{N}_{3}-\mathrm{C}_{4}$ | 1.3380 | 1.3322 | 1.3353 | 1.3356 | 1.3354 | -0.0002 | 1.3349 | 1.3349 | 1.3349(6) |
| $\mathrm{C}_{4}-\mathrm{C}_{5}$ | 1.3907 | 1.3828 | 1.3861 | 1.3865 | 1.3863 | -0.0002 | 1.3873 | 1.3874 | 1.3874(4) |
| $\mathrm{C}_{2}-\mathrm{H}_{2}$ | 1.0833 | 1.0800 | 1.0814 | 1.0815 | 1.0815 | 0.0000 | 1.0818 | 1.0819 | 1.0820(23) |
| $\mathrm{C}_{4}-\mathrm{H}_{4}=\mathrm{C}_{6}-\mathrm{H}_{6}$ | 1.0839 | 1.0805 | 1.0820 | 1.082 | 1.0821 | 0.0001 | 1.0825 | 1.0824 | 1.0843(17) |
| $\mathrm{C}_{5}-\mathrm{H}_{5}$ | 1.0808 | 1.0776 | 1.0791 | 1.0792 | 1.0793 | 0.0001 | 1.0793 | 1.0793 | 1.0799(23) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{~N}_{3}$ | 127.50 | 127.05 | 127.16 | 127.16 | 127.07 | -0.087 | 127.393 | 127.393 | 127.364(38) |
| $\angle \mathrm{C}_{6} \mathrm{~N}_{1} \mathrm{C}_{2}$ | 115.58 | 115.93 | 115.86 | 115.86 | 115.93 | 0.068 | 115.655 | 115.656 | 115.712(21) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{6} \mathrm{C}_{5}$ | 122.36 | 122.06 | 122.09 | 122.09 | 122.00 | -0.032 | 122.330 | 122.332 | 122.276(19) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{C}_{6}$ | 116.62 | 116.96 | 116.94 | 116.94 | 116.96 | 0.015 | 116.637 | 116.632 | 116.661(36) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{H}_{2}$ | 116.25 | 116.47 | 116.42 | 116.42 | 116.46 | 0.043 | 116.304 | 116.303 | 116.318(19) |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{H}_{4}$ | 121.22 | 121.35 | 121.36 | 121.36 | 121.39 | 0.032 | 121.205 | 121.207 | 121.24(20) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{H}_{5}$ | 121.69 | 121.52 | 121.53 | 121.53 | 121.52 | -0.007 | 121.682 | 121.684 | 121.670(18) |

${ }^{a}$ Distances in $\AA$ and angles in degrees. ${ }^{b} \mathrm{MP} 2(\mathrm{FC}) / \mathrm{AV} 5 \mathrm{Z}$ - MP2(FC)/VQZ. ${ }^{c}$ See eq 2. ${ }^{d}$ See text and Table S7 of the Supporting Information. ${ }^{e}$ AE $=$ all electrons correlated; $\mathrm{FC}=$ frozen core approximation. ${ }_{r_{\mathrm{e}} \mathrm{BO}}(\mathrm{I})$, see eq 1 .
$\angle \mathrm{NCC}$, is larger than the reference $120^{\circ}$ value by $3.8^{\circ}$. It is somewhat unusual that the substitution effect at the ortho position exceeds that at the ipso one.
6.2. Structure of Pyrimidine. Pyrimidine, $c-\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{~N}_{2}$, is an aromatic N -heterocyclic compound with its two nitrogen atoms at meta position. The other two possible diazines are pyrazine and pyridazine, whose structures have been investigated before, ${ }^{10,85}$ though not to the extent provided here. Because of a double $\mathrm{CH} \rightarrow \mathrm{N}$ substitution, the $\pi$-electron density in pyrimidine decreases further as compared to pyridine, resulting in much lower basicity.

The MW spectrum of pyrimidine was first analyzed by Blackman et al. ${ }^{86}$ Later, Caminati and Damiani ${ }^{87}$ measured the spectra of the ${ }^{15} \mathrm{~N}$ and one ${ }^{13} \mathrm{C}$ isotopic species. More recently, Kisiel et al. ${ }^{88}$ extended these measurements and obtained the rotational constants of all singly substituted ${ }^{13} \mathrm{C}$ - and ${ }^{15} \mathrm{~N}$ isotopic species in natural abundance. They determined a heavy-atom empirical $r_{0}$ structure. The structure of pyrimidine was also determined by combined analysis of data obtained by GED, rotational spectroscopy, and liquid-crystal NMR. ${ }^{89}$ As a result of the analysis of FTIR spectra, rotational constants of excited vibrational states are also available for pyrimidine. ${ }^{90}$ These constants are not used during the present structural analysis.

Estimates to $r_{\mathrm{e}}^{\mathrm{BO}}$ of pyrimidine were computed using eqs 1 and 2. The corresponding $r_{\mathrm{e}}^{\mathrm{BO}}(\mathrm{I})$ and $r_{\mathrm{e}}^{\mathrm{BO}}$ (II) structural parameters are given in Table 9. The anharmonic force fields of the five isotopologues analyzed in ref 86 were computed at the B3LYP $/ 6-311+\mathrm{G}(3 \mathrm{df}, 2 \mathrm{pd})$ level. The derived $\alpha$-constants were combined with the experimental ground-state rotational constants to yield estimates of semiexperimental equilibrium rotational constants, which were also corrected for the electronic contribution. As there are no experimental values for the molecular $g$-tensor, they were calculated at the B3LYP/ $6-311+G(3 d f, 2 \mathrm{pd})$ level of theory with Gaussian09. The results are $g_{a a}=-0.100, g_{b b}=-0.122$, and $g_{c c}=0.031$. The groundstate and semiexperimental equilibrium rotational constants as well as the inertial defects are given in Table S7 of the Supporting Information. The experimental ground-state inertial defect is $\Delta_{0}=+0.0350 u \AA^{2}$ for the parent species; after the rovibrational correction it becomes $-0.0141 \mathrm{u} \AA^{2}$, and after the electronic correction it is $\Delta_{e}=-0.0016 u \AA^{2}$, that is, practically
zero once again. This suggests that in the present case the electronic correction is not negligible. As the rotational constants of the deuterated species are not known, it is not possible to determine a complete semiexperimental structure. However, as for imidazole, the mixed regression method may be used. The predicate observations are the $r_{\mathrm{e}}^{\mathrm{BO}}$ internal coordinates from Table 9 with an uncertainty of $0.002 \AA$ for the bond lengths and $0.2^{\circ}$ for the angles. The final uncertainty retained for the rotational constant was (in kHz ): 50 for $\mathrm{A}, 20$ for $B$, and 40 for $C$. Several trials were made starting with a minimum set of predicates, that is, the four internal coordinates locating the hydrogen atoms: $\mathrm{C}_{2} \mathrm{H}_{2}, \mathrm{C}_{4} \mathrm{H}_{4}=\mathrm{C}_{6} \mathrm{H}_{6}, \mathrm{C}_{5} \mathrm{H}_{5}$, and $\angle \mathrm{N}_{3} \mathrm{C}_{4} \mathrm{H}_{4}=\angle \mathrm{N}_{1} \mathrm{C}_{6} \mathrm{H}_{6}$. Then, the number of predicates was increased to the maximum number of nine. All the fits give compatible results, which are in good agreement with the $r_{\mathrm{e}}^{\mathrm{BO}}$ structure. The results presented in the last column of Table 9 correspond to the fit with the complete set of predicates.
6.3. Structure of Uracil. Uracil, $\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{~N}_{2} \mathrm{O}_{2}$ (Figure 1), is a pyrimidine derivative that is one of the four nucleobases forming RNAs. While it is not a simple N-heterocyclic molecule like the others investigated in this study, we decided to include it in this study as it is a molecule whose structure and vibrational fundamentals have been investigated by Boggs et al., ${ }^{91}$ and knowledge of the accurate structure and spectra of uracil is indeed very important.

The MW spectrum of uracil was first recorded by Brown et al. ${ }^{92}$ using a Stark spectrometer. More recently, the jet-cooled rotational spectrum has been measured using laser ablation molecular beam FTMW spectroscopy. ${ }^{93}$ The spectra of the ${ }^{13} \mathrm{C}$, ${ }^{15} \mathrm{~N}$, and ${ }^{18} \mathrm{O}$ isotopic species were also measured permitting the determination of an empirical $r_{s}$ structure. A partial $r_{\mathrm{e}}^{\mathrm{SE}}$ structure was calculated with the help of a cubic force field computed at the B3LYP/N07D level of theory. ${ }^{94}$ In the same work, the structure was also computed at the $\operatorname{CCSD}(\mathrm{T})$ level, in conjunction with correlation-consistent basis sets (cc-pVnZ, with $n=3(T)$ and $4(\mathrm{Q})$ ), taking also into account the corevalence correlation effects with the wCVTZ basis and the corrections due to diffuse functions using the AVTZ basis. Recently, the accuracy of the $r_{\mathrm{e}}^{\mathrm{BO}}$ structure estimate was significantly improved using the larger wCVQZ basis set for the estimation of the core correlation and the AwCVQZ basis set

Table 10. Structure of Uracil ${ }^{\boldsymbol{a}}$

| method | $r_{\mathrm{e}}^{\mathrm{BO}}$ |  |  | GED | $r_{\mathrm{e}}^{\text {SE }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ref 94 | ref 95 | this work | ref 95 | ref 94 | this work |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.3785 | 1.3796 | 1.3807 | 1.381(2) | 1.3818(5) | 1.3810(6) |
| $\mathrm{C}_{2}-\mathrm{N}_{3}$ | 1.3756 | 1.3775 | 1.3774 | 1.379 (2) |  | 1.3749 (7) |
| $\mathrm{N}_{3}-\mathrm{C}_{4}$ | 1.3974 | 1.4005 | 1.3993 | 1.402(2) | 1.3979(4) | 1.3991(6) |
| $\mathrm{C}_{4}-\mathrm{C}_{5}$ | 1.4539 | 1.4578 | 1.4558 | 1.454(8) | 1.4550(6) | 1.4548(6) |
| $\mathrm{C}_{5}-\mathrm{C}_{6}$ | 1.3433 | 1.3446 | 1.3435 | 1.339(18) | 1.3450(6) | 1.3429(7) |
| $\mathrm{N}_{1}-\mathrm{C}_{6}$ | 1.3723 | 1.3729 | 1.3726 | 1.374(2) | 1.3720(6) | 1.3722(7) |
| $\mathrm{C}_{2}-\mathrm{O}_{7}$ | 1.2112 | 1.2111 | 1.2099 | 1.210(1) | 1.2103(2) | 1.2101(4) |
| $\mathrm{C}_{4}-\mathrm{O}_{8}$ | 1.2138 | 1.2131 | 1.2126 | 1.212(1) | 1.2128(2) | 1.2186(4) |
| $\mathrm{N}_{1}-\mathrm{H}$ | 1.0046 | 1.0048 | 1.0045 | $1.005(10)$ | 1.0043(12) |  |
| $\mathrm{N}_{3}-\mathrm{H}$ | 1.0090 | 1.0090 | 1.0088 | 1.009(10) | 1.0083(12) |  |
| $\mathrm{C}_{5}-\mathrm{H}$ | 1.0766 | 1.0764 | 1.0761 | 1.076(13) | 1.0757(12) |  |
| $\mathrm{C}_{6}-\mathrm{H}$ | 1.0793 | 1.0794 | 1.0792 | 1.079(3) |  | 1.0781(12) |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{C}_{6}$ | 123.38 | 123.60 | 123.45 | 123.8(3) | 123.37(2) | 123.39(4) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{6} \mathrm{C}_{5}$ | 121.91 | 121.68 | 121.93 | 121.6(4) | 121.92(1) | 121.90(2) |
| $\angle \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{C}_{6}$ | 119.49 | 119.66 | 119.58 | 120.0(3) | 119.52(2) | 119.61(3) |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{~N}_{3}$ | 113.97 | 113.82 | 113.76 | 113.6(3) | 113.86(2) | 113.75(4) |
| $\angle \mathrm{C}_{4} \mathrm{~N}_{3} \mathrm{C}_{2}$ | 127.75 | 127.73 | 127.97 | 128.0(3) | 127.942 | 127.97(4) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{~N}_{3}$ | 113.51 | 113.51 | 113.32 | 113.0(2) | 113.383 | 113.38(4) |
| $\angle \mathrm{N}_{3} \mathrm{C}_{2} \mathrm{O}_{7}$ | 123.62 | 123.68 | 123.76 | 124.4(5) | 123.88(4) | 123.84(5) |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{O}_{8}$ | 125.83 | 125.77 | 125.85 | 126.5(6) | 125.77(5) | 125.84(4) |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{H}$ | 115.22 | 115.12 | 115.25 | 115.2(6) |  | 115.14(12) |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{3} \mathrm{H}$ | 115.7 | 115.70 | 115.60 | 114(7) |  | 115.62(13) |
| $\angle \mathrm{C}_{6} \mathrm{C}_{5} \mathrm{H}$ | 122.11 | 122.01 | 122.00 | 122 |  | 122.12(11) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{6} \mathrm{H}$ | 115.34 | 115.50 |  | 112(4) |  | 115.39(6) |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{O}_{7}$ |  | 122.81 | 122.92 | 122.6(5) |  | 122.78(5) |
| tances in | in degre |  |  |  |  |  |

for the effects of the diffuse functions. ${ }^{95}$ The correction due to the diffuse functions was found in this study negligibly small.

To extend previous work and further improve the accuracy of the $r_{\mathrm{e}}$ structure estimate of uracil, we computed the effect of basis set enlargement VQZ $\rightarrow$ AV5Z at the MP2 level. The correction is quite small, always smaller than $0.001 \AA$ for the bond lengths; the largest change is a decrease of the angle $\angle \mathrm{C}_{2} \mathrm{~N}_{3} \mathrm{C}_{4}$ by $0.12^{\circ}$. Finally, the $r_{\mathrm{e}}^{\mathrm{BO}}$ structure is obtained using the equation

$$
\begin{align*}
r_{\mathrm{e}}^{\mathrm{BO}}= & \mathrm{CCSD}(\mathrm{~T}) / \mathrm{CVTZ} \_\mathrm{AE}+\mathrm{MP2}(\mathrm{AE}) \\
& /[\mathrm{wCVQZ}-\mathrm{CVTZ}]+\mathrm{MP2}(\mathrm{FC}) \\
& /[\mathrm{AV} 5 \mathrm{Z}-\mathrm{VQZ}] \tag{7}
\end{align*}
$$

The results are given in Table 10, where they are compared with those of previous determinations.

The structure of uracil is defined by 21 independent internal coordinates. The rotational constants are available for 10 isotopologues; this only gives 20 independent rotational constants, because the molecule is planar at equilibrium. Furthermore, no rotational constants are available for the deuterated species. In their fits, Puzzarini and Barone ${ }^{94}$ had to fix several parameters. In similar situations it is advantageous to use the mixed regression method, and this was done here. First, we redid the fits of the rotational transitions ${ }^{93}$ using the method of predicate observations, with the quartic centrifugal distortion constants derived from a quadratic B3LYP/6-311+G(3df,2pd) force field. As there are no experimental values for the molecular $g$-tensor, they were also calculated at the B3LYP/6$311+\mathrm{G}(3 \mathrm{df}, 2 \mathrm{pd})$ level of theory with Gaussian09. The results are $g_{a a}=-0.054, g_{b b}=-0.048$, and $g_{c c}=-0.013$. The
uncertainties chosen for the rotational constants were (in kHz ): 30 for $A, 20$ for $B$, and 10 for $C$. The uncertainties of the predicates were chosen to be $0.002 \AA$ for the bond lengths and $0.2^{\circ}$ for the angles. This set of data and uncertainties gives a good fit, whose results are also given in Table 10. The experimental ground-state and semiexperimental equilibrium rotational constants, as well as the inertial defects and the residuals of the fit, are given in Table S8 of the Supporting Information. This semiexperimental structure is in perfect agreement with our $r_{\mathrm{e}}^{\mathrm{BO}}$ structure; it is also in good agreement with the GED structure, especially for the angles, but it is significantly more accurate.

It is tempting to compare the structure of uracil with that of pyrimidine, because they share the same ring skeleton. However, the two molecules have rather different structures because only pyrimidine is aromatic, with inner-ring bond lengths being intermediate between single and double bonds. For instance, the length of the $\mathrm{N}_{1} \mathrm{C}_{2}$ bond in pyrimidine is $1.334 \AA$, which is definitely shorter than a single bond, whereas in uracil it is $1.381 \AA$, which is a value typical of a single bond albeit shortened by the presence of the adjacent $\mathrm{C}=\mathrm{O}$ bond. Likewise, the $\mathrm{C}_{5} \mathrm{C}_{6}$ bond length at $1.343 \AA$ in uracil is close to a double bond value, whereas in pyrimidine it is much longer, its value being $1.387 \AA$.

## 7. SEVEN-MEMBERED RING: 1H-AZEPINE

There are four possible structural isomers $(1 H-, 2 H-, 3 H-$, and 4 H -azepine), but only 1 H -azepine (called azepine hereafter) is investigated here. Planar $1 H$-azepine would be antiaromatic, and thus the molecule distorts into a boat form and assumes a geometry with a plane of symmetry perpendicular to the

Table 11. Equilibrium Structural Parameters of Azepine ${ }^{a}$

| method <br> basis set | $\begin{gathered} \mathrm{CCSD}(\mathrm{~T})_{\mathrm{C}} \mathrm{FC} \\ \mathrm{VTZ} \end{gathered}$ | $\begin{gathered} \text { MP2(FC) } \\ \text { VTZ } \end{gathered}$ | MP2(FC) <br> AVQZ | $\begin{gathered} \text { MP2(FC) } \\ \text { wCVQZ } \end{gathered}$ | MP2(AE) <br> wCVQZ | $r_{\text {e }} \mathrm{BO} b$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}_{1}-\mathrm{C}_{2}$ | 1.4216 | 1.4187 | 1.4154 | 1.4149 | 1.4113 | 1.4147 |
| $\mathrm{N}_{1}-\mathrm{H}$ | 1.0077 | 1.0079 | 1.0070 | 1.0066 | 1.0053 | 1.0055 |
| $\mathrm{C}_{2}-\mathrm{C}_{3}$ | 1.3441 | 1.3444 | 1.3428 | 1.3416 | 1.3383 | 1.3392 |
| $\mathrm{C}_{2}-\mathrm{H}$ | 1.0854 | 1.0845 | 1.0841 | 1.0836 | 1.0821 | 1.0835 |
| $\mathrm{C}_{3}-\mathrm{C}_{4}$ | 1.4673 | 1.4520 | 1.4500 | 1.4495 | 1.4466 | 1.4624 |
| $\mathrm{C}_{3}-\mathrm{H}$ | 1.0835 | 1.0818 | 1.0817 | 1.0811 | 1.0796 | 1.0819 |
| $\mathrm{C}_{4}-\mathrm{C}_{5}$ | 1.3501 | 1.3533 | 1.3515 | 1.3503 | 1.3467 | 1.3447 |
| $\mathrm{C}_{4}-\mathrm{H}$ | 1.0852 | 1.0835 | 1.0832 | 1.0827 | 1.0812 | 1.0834 |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{C}_{7}$ | 116.16 | 113.36 | 114.01 | 113.83 | 114.12 | 117.10 |
| $\angle \mathrm{C}_{2} \mathrm{~N}_{1} \mathrm{H}$ | 110.88 | 110.86 | 111.29 | 111.20 | 111.38 | 111.49 |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{C}_{3}$ | 124.27 | 122.80 | 122.94 | 122.90 | 123.01 | 124.53 |
| $\angle \mathrm{N}_{1} \mathrm{C}_{2} \mathrm{H}$ | 115.02 | 116.93 | 116.70 | 116.79 | 116.73 | 114.74 |
| $\angle \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{H}$ | 120.02 | 120.27 | 120.35 | 120.31 | 120.26 | 120.05 |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 125.24 | 124.65 | 124.79 | 124.75 | 124.78 | 125.41 |
| $\angle \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}$ | 117.03 | 117.08 | 116.93 | 116.98 | 116.97 | 116.87 |
| $\angle \mathrm{C}_{4} \mathrm{C}_{3} \mathrm{H}$ | 117.72 | 118.27 | 118.27 | 118.27 | 118.25 | 117.71 |
| $\angle \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5}$ | 125.17 | 124.80 | 124.85 | 124.83 | 124.85 | 125.25 |
| $\angle \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{H}$ | 116.50 | 117.05 | 117.02 | 117.02 | 117.00 | 116.45 |
| $\angle \mathrm{C}_{5} \mathrm{C}_{4} \mathrm{H}$ | 118.27 | 117.98 | 117.97 | 117.98 | 117.99 | 118.26 |
| $\mathrm{C}_{7} \mathrm{~N}_{1} \mathrm{C}_{2} \mathrm{C}_{3}$ | 55.94 | 62.29 | 61.25 | 61.54 | 60.97 | 54.33 |
| $\mathrm{C}_{7} \mathrm{~N}_{1} \mathrm{C}_{2} \mathrm{H}$ | -125.31 | -118.78 | -120.01 | -119.53 | -120.02 | -127.02 |
| HN1C2C3 | -176.30 | -172.28 | -171.88 | -171.93 | -171.85 | -175.83 |
| $\mathrm{HN}_{1} \mathrm{C}_{2} \mathrm{H}$ | 2.45 | 6.65 | 6.86 | 6.99 | 7.16 | 2.82 |
| $\mathrm{N}_{1} \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | -0.26 | -2.12 | -1.67 | -1.76 | -1.48 | 0.47 |
| $\mathrm{N}_{1} \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{H}$ | 178.27 | 177.32 | 177.43 | 177.45 | 177.60 | 178.53 |
| $\mathrm{HC}_{2} \mathrm{C}_{3} \mathrm{C}_{4}$ | 178.95 | 178.99 | 179.63 | 179.34 | 179.54 | 179.78 |
| $\mathrm{HC}_{2} \mathrm{C}_{3} \mathrm{H}$ | -0.43 | -1.57 | -1.27 | -1.44 | -1.37 | -0.06 |
| $\mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5}$ | -32.08 | -33.64 | -33.46 | -33.55 | -33.54 | -31.89 |
| $\mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{H}$ | 150.67 | 151.24 | 151.42 | 151.30 | 151.18 | 150.73 |
| $\mathrm{HC}_{3} \mathrm{C}_{4} \mathrm{C}_{5}$ | 149.40 | 146.93 | 147.45 | 147.25 | 147.39 | 150.06 |
| $\mathrm{HC}_{3} \mathrm{C}_{4} \mathrm{H}$ | -27.84 | -28.19 | -27.68 | -27.91 | -27.89 | -27.31 |
| $\mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{C}_{6}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\mathrm{HC}_{4} \mathrm{C}_{5} \mathrm{C}_{6}$ | 177.20 | 175.07 | 175.08 | 175.11 | 175.24 | 177.33 |
| $\mathrm{HC}_{4} \mathrm{C}_{5} \mathrm{H}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

assumed plane of the carbon atoms. Note that stable derivatives of azepine have been shown to be nonplanar.

We are not aware of any experimental structural results for azepine, a ring with $C_{s}$ point-group symmetry, where the symmetry plane is "perpendicular" to the assumed plane of the heavy atoms of the ring.

Geometry optimizations have been reported for azepine at the MCSCF/6-31G(d,p), the B3LYP/6-311++G**, and the MP2/6-311++G** levels of electronic structure theory. ${ }^{96,97}$ As azepine is significantly larger than the other molecules of this work, its $r_{\mathrm{e}}^{B O}$ structure was first estimated in the FC approximation using the VTZ basis set and the $\operatorname{CCSD}(\mathrm{T})$ method. The core-core and core-valence correlation corrections were computed at the MP2 level using the wCVQZ basis set, and the small effect of further basis set enlargement was also computed at the MP2 level using the AVQZ basis set. The final $r_{\mathrm{e}}^{\mathrm{BO}}$ structure estimate is given by the equation

$$
\begin{align*}
r_{\mathrm{e}}^{\mathrm{BO}}= & \mathrm{CCSD}(\mathrm{~T}) \_\mathrm{FC} / \mathrm{VQZ}+\mathrm{MP} 2(\mathrm{AE}) / \mathrm{wCVQZ} \\
& -\mathrm{MP2} 2(\mathrm{FC}) / \mathrm{wCVQZ}+\mathrm{MP2}(\mathrm{FC}) / \mathrm{AVQZ} \\
& -\mathrm{MP2} 2(\mathrm{FC}) / \mathrm{VTZ} \tag{8}
\end{align*}
$$

The most important structural parameters obtained are given in Table 11. The first obvious conclusion is that azepine is definitely nonplanar; most of the torsional angles deviate significantly from 0 or $180^{\circ}$. Furthermore, the $\mathrm{C}-\mathrm{C}$ single bonds $r\left(\mathrm{C}_{3}-\mathrm{C}_{4}\right) \equiv r\left(\mathrm{C}_{5}-\mathrm{C}_{6}\right)$ at $1.467 \AA$ are much longer than the $\mathrm{C}=\mathrm{C}$ double bonds $r\left(\mathrm{C}_{2}=\mathrm{C}_{3}\right) \equiv r\left(\mathrm{C}_{6}=\mathrm{C}_{7}\right)$ at $1.344 \AA$ and $r\left(\mathrm{C}_{4}=\mathrm{C}_{5}\right)$ at $1.350 \AA$. The value found for the $r(\mathrm{C}-\mathrm{N})$ bond length at $1.415 \AA$ is typical for a single bond. It is also worth noting that the MP2 method does not seem to be able to predict the torsional angles with the usual accuracy; the deviations may approach $7^{\circ}$.

## 8. CONCLUSIONS

In this study highly accurate equilibrium structures have been derived for the following N -heterocycles: 1 H - and 2 H -azirine, aziridine, azete, pyrrole, pyrazole, imidazole, pyridine, pyrimidine, uracil, and 1 H -azepine. For several of these molecules not only $r_{\mathrm{e}}^{\mathrm{BO}}$ but also $r_{\mathrm{e}}^{\mathrm{SE}}$ estimates of equilibrium structures could be determined. All equilibrium structures agree nicely with each other confirming the accuracy of their determination.

The present investigation resulted in, as well as confirmed, a number of methodological findings significant for structural determinations of semirigid molecules. These can be
summarized as follows: (1) The all-electron $\operatorname{CCSD}(\mathrm{T})$ method with the wCVQZ basis set yields highly satisfactory structural results for all the molecules investigated. It is found that the extension to larger basis sets does not significantly improve the accuracy of the results, because the small correction when going from VQZ to V5Z basis is almost completely compensated by the effect of diffuse functions when going from V5Z to AV5Z basis. The $\operatorname{CCSD}(\mathrm{T})$ method with the VQZ basis set in the frozen-core approximation and a core correction calculated at the MP2/wCVQZ level gives results of comparable quality. Of importance for later structural studies of even larger systems is the fact that it is also possible to use the $\operatorname{CCSD}(\mathrm{T})$ method with the smaller wCVTZ basis set, all electrons correlated, and the effect of further basis set enlargement, $\mathrm{wCVTZ} \rightarrow \mathrm{wCVQZ}$, calculated at the MP2 level. This cheaper method is almost as accurate as the previous ones. (2) The semiexperimental method gives results of at least the same accuracy provided a full data set of accurate ground-state rotational constants is available. This study confirms that it is highly desirable to have accurate ground-state rotational constants to determine an accurate structure, although this may often imply refitting the spectroscopic constants to measured spectra to obtain more reliable rotational constants. (3) It must be noted that, at least for this class of compounds, the empirical mass-dependent method $\left(r_{\mathrm{m}}\right)$ also delivers rather accurate estimates of equilibrium structures. This useful observation may be connected to the fact that the least-squares systems used for the structural refinements are well-conditioned.

The present study also offers several chemical insights. Although it has been utilized for a long time, the geometry of the molecule may not be a good indicator of its aromaticity. ${ }^{98,99}$ However, it is interesting to have a look at the bond lengths of adjacent bonds because aromaticity often implies a tendency to bond length equalization. The antiaromatic molecules 2 H azirine and azete have extremely different CN bond lengths; in particular, both molecules have a very long single bond. The nonaromatic molecule aziridine has a CN bond length of 1.470 $\AA$, close to the value found in methylamine, $1.462 \AA$. The aromatic compounds show very small differences between the lengths of the adjacent bonds, which are intermediate between single and double bonds. Finally, the case of uracil is interesting because, contrary to pyrimidine, it is not aromatic. Nevertheless, the $\mathrm{N}_{1} \mathrm{C}_{2}$ and $\mathrm{N}_{1} \mathrm{C}_{6}$ bond lengths differ by only $0.008 \AA$.

## - ASSOCIATED CONTENT

## (s) Supporting Information

Eight tables, separated by molecules. They include ground state and semiexperimental rotational constants, rovibrational corrections, residuals of the structural fits, equilibrium inertial defects $\Delta_{e}$, and angles of rotation of the principal axis system upon isotopic substitution for the molecules of this study. This material is available free of charge via the Internet at http:// pubs.acs.org.

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## Notes

The authors declare no competing financial interest.

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