



Equivalence of Non-Equilibrium Ensembles and Representation of Friction in Turbulent Flows: The Lorenz 96 Model

Valerio Lucarini (1,2) and Giovanni Gallavotti (3)

(1) University of Hamburg, Institute of Meteorology, Klimacampus, Hamburg, Germany (valerio.lucarini@zmaw.de), (2) University of Reading, Department of Mathematics and Statistics, (3) Sapienza University of Rome, Department of Physics

We construct different equivalent non-equilibrium statistical ensembles in a simple yet instructive N -degrees of freedom model of atmospheric turbulence, introduced by Lorenz in 1996. The vector field can be decomposed into an energy-conserving, time-reversible part, plus a non-time reversible part, including forcing and dissipation. We construct a modified version of the model where viscosity varies with time, in such a way that energy is conserved, and the resulting dynamics is fully time-reversible. For each value of the forcing, the statistical properties of the irreversible and reversible model are in excellent agreement, if in the latter the energy is kept constant at a value equal to the time-average realized with the irreversible model. In particular, the average contraction rate of the phase space of the time-reversible model agrees with that of the irreversible model, where instead it is constant by construction. We also show that the phase space contraction rate obeys the fluctuation relation, and we relate its finite time corrections to the characteristic time scales of the system. A local version of the fluctuation relation is explored and successfully checked. The equivalence between the two non-equilibrium ensembles extends to dynamical properties such as the Lyapunov exponents, which are shown to obey to a good degree of approximation a pairing rule. These results have relevance in motivating the importance of the chaotic hypothesis. In explaining that we have the freedom to model non-equilibrium systems using different but equivalent approaches, and, in particular, that using a model of a fluid where viscosity is kept constant is just one option, and not necessarily the only option, for describing accurately its statistical and dynamical properties.