J. Geomag. Geoelectr., 35, 119-130, 1983

Equivalent Source Modeling of the Core

Magnetic Field Using Magsat Data

M. A. MAYHEW1 and R. H. ESTES

BTS, Inc. Seabrook, Maryland 20706, U.S.A.

(Received August 30, 1982; Revised February 13, 1983)

An iterative least squares estimation algorithm with the capability for including a priori statistical information has been implemented to recover multiple magnetic dipole models of the Earth's main magnetic field. The dipoles are fixed to a specified radius at or below the core-mantle boundary and centered on equal area blocks. The algorithm can solve for dipole magnitudes only (fixed orientations), or allow full freedom of orientation and solve for vector components. First-order external field parameters can be estimated simultaneously, as well as magnetic observatory biases when observatory annual means data is used. Secular variation is modeled by making the dipole components linear functions of time. Single-epoch and time dependent dipole models are derived using Magsat and observatory annual means data. Equivalent spherical harmonic representations are computed in closed form from the dipole models for comparison with truncated spherical harmonic models estimated in the standard way from the same data sets. A model consisting of 93 dipoles arranged
21° apart was computed based on observatory annual means data from 1974 through

1977 and a selected Magsat data set and is compared with candidate IGRF 1975 models and their 1984 secular variation. The equivalent dipole source representation is shown to be comparable to the standard spherical harmonic approach in accuracy and, for high resolution models, to be superior in computational efficiency for field model evaluation when three degree-of-freedom dipoles (i.e., unconstrained by direction) are utilized. Fixing of the dipole positions results in rapid convergence of the dipole solutions for single-epoch models. For time-dependent models, a sufficiently long time interval of data or a priori values and statistics must be available for the derivatives to achieve convergence. In contrast to spherical harmonic models incorporating secular variation parameters based on the same data, the correlation structure of dipole magnetic moment derivatives with the constant magnetic moment parameters is small, indicating the ability to strongly separate the constant field and the secular variation.

1. Introduction

The equivalent source technique is a conceptually simple means of describing

potential fields which has been widely applied in exploration geophysics (e.g.

1Present address: U. S. National Science Foundation, Washington, D. C. 20550, U.S.A.

119

120 M. A. MAYHEW and R. H. ESTES

DAMPNEY, 1969), in modeling long wavelength satellite magnetic anomalies (MAYHEW, 1979), and in modeling the core field (see review by STERNS and ALLDREDGE, 1973). The method consists of setting up an internal arrangement of dipoles and adjusting their magnetic moments (and in some applications their positions) such that the synthetic field arising from them collectively best fits a set of observations of the field. A number of authors have attempted to apply the equivalent source technique to modeling of the main field and its secular change. MCNISH (1940) and LOWES and RUNCORN (1951) used graphical approximation techniques for this purpose. ALLDREDGE and HURWITZ (1964) and ALLDREDGE and STERNS (1969) treated the problem using inverse theory and with the benefit of computers. With this approach, by examination of the behavior of modeled sources, one can hope to gain more insight into the physics of core field generation than can be gotten by classical spherical harmonic analysis (ALLDREDGE and HURWITZ, 1964; COX, 1968; HARRISON and CARLE, 1982). For example, the fields of radial dipoles approximate the fields of horizontal current loops. PEDDLE (1979) used expressions for current loops directly. Further, if one wishes to separate core from crustal fields, the equivalent source method appears to hold more promise than spherical harmonic analysis because of the very considerable spectral overlap expected for the two fields. CARLE and HARRISON (1982) and HARRISON and CARLE (1982) have shown that high order harmonics with which one would model the crustal field contain significant power in the very long wavelengths which characterize the core field.

In most previous studies, the angular coordinates and radial distance of the dipoles have been treated as variables in addition to the dipole moments; the difficulty here is that the function expressing the dipole field is highly non-linear in the geographic variables, and this leads to a non-linear inverse problem. Many iterations through a given data set are required for convergence, which is not achieved at all unless good starting parameter estimates are available. We have experimented with fitting the main field using different numbers of equivalent sources arranged in equal area at fixed radii at and inside the core-mantle boundary. By fixing the radius for a given series of runs, we avoid the convergence problems which result from the extreme non-linearity of the problem when dipole positions are allowed to vary. We report here the results of a comparison between this approach and the standard spherical harmonic approach for modeling the main field in terms of accuracy and com-putational efficiency.

2. Methodology

MAYHEW (1978) and MAYHEW and ESTES (1981) have given the expressions for the anomaly components and the anomaly in the total field due to a dipole having arbitrary position and orientation at an arbitrary external point. The magnetic

potential ofadipole of moment M is V=-M・ ∇(1/l), where l is the distance between the dipole and the external point;the associated vector field is F=-∇V.

The goal of the analysis is to estimate the magnetic moments of all dipoles from observations of the total field and its components, and a computer program has been

Equivalent Source Modeling of the Gore Magnetic Field Using Magsat Data 121

developed for this purpose. It uses an iterative least squares algorithm with the capability for introducing a priori information which has been described in some detail by MAYHEW and ESTES (1981). The dipoles are set to a specified radius within the earth, and are centered on nearly equal area blocks. The program has the option to solve for either the dipole magnitudes only (with the dipole orientations constrained to a preselected direction), or to allow full freedom of orientation for the dipoles and solve for the three components. When the dipole directions are unconstrained there are three parameters to be estimated per dipole, but only one per dipole when the direction is constrained radially. Secular variation is modeled by making each dipole magnetic moment component a linear function of time. This doubles the number of parameters for a given problem at a single epoch. While the inversion problem is linear for vector component data, an iterative solution is required when scalar magnetometer data is utilized due to the non-linearity introducted for this data type. The non-linearity is mild, however, and when the model parameters are well determined by the data only two iterations are generally required.
The set of simultaneously estimated parameters has been further extended in two ways. First, a set of three external field parameters expressed as spherical harmonic
coefficients g10, g11, h11 can be computed Second, when the secular variation

parameters are included in the solution, constant biases due to local crustal magnetic anomalies can be estimated at each observatory for which annual means data is used (LANGEL et al., 1982). The inclusion of the additional modeling parameters to represent the influence of external and crustal sources in the data allows the dipole sources to more accurately model the core field.
The spherical harmonic representation of a given dipole distribution may be expressed in closed form for comparison with standard spherical harmonic field models. Consider the set of {j} dipoles, where the lth dipole is described by a source

vector

M=(Mri,Mθi,Mφi)

and a spatial position

(γi, θi, φm),

where γ, θ, and φ represent radial distance, colatitude, and longitude, respectively.
The spherical harmonic coefficients are given (in Schmidt normalized form) by

(1)

122 M. A. MAYHEW and R. H. ESTES

Similar expressions were presented by HURWITZ (1960). The coefficients may be calculated up to any order. The spatial power spectral content of the expansion is given by the relation (LOWEs, 1966)

(2)

A procedure was developed for establishing global dipole arrays in which the dipoles occupy nearly equal areas and are also approximately equi-distant. It makes use of an icosahedron, a regular polyhedron with 20 equilateral triangular faces, inscribed in a sphere, with the edges radially projected on the sphere. The spherical icosahedron is the division of the sphere having the greatest number of regular pieces. Further discrete subdivision of the spherical triangles can be made, forming the base on which a nearly uniform distribution of points on the sphere may be defined. Computer
runs were made for 12, 42, 92, and 162 dipoles, having angular separations of 63°, 32°, 21°, and 16°, respectively.

Several main field solutions have been derived for various numbers of dipoles arranged on equal area projections at the core-mantle boundary. In these solutions, the dipoles have been either fixed radially or allowed complete freedom of orientation, and have been modeled with and without time dependence and with and without the addition of a geocentric dipole (giving three additional parameters). Some results are given in the next sections.

3. Single-Epoch Models

Several dipole models were made using the set of quiet Magsat data collected during November 5 and 6, 1979 (LANGEL et al., 1980). The results of some of these are compared in Table 1. A selection of parameter values is given in MAYHEW and ESTES (1982). The models for which the dipoles are constrained radially have a single degree of freedom per dipole, while the unconstrained models have three degrees of freedom per dipole. The inclusion of the geocentric dipole significantly improved the rms error
of the 32° dipole solution. The direction of the geocentric dipole was within 10° of that

calculated from the first degree spherical harmonic terms of MGST (6/80) (LANGEL et al., 1980), while the magnitude was greater by approximately 4%. The magnitude of the geocentric dipole dominated those of the core-mantle boundary by an order of
magnitude. For the 21° spacing models, the fit to the data was the same with and without a geocentric dipole. The geocentric dipole showed no closer agreement to the first degree terms of MGST (6/80) than did that of the 32° dipole solution. All parame-ters in these solutions were strongly observable mathematically in the least squares

Equivalent Source Modeling of the Core Magnetic Field Using Magsat Data 123

Table 1. Characteristics of models 1-5. "Fit" is RMS to quiet Magsat data set of Nov. 5 and 6, 1979. External field parameters are solved for in each case.

estimation, with standard errors from three to four orders of magnitude smaller than the magnitude of the parameters. The coefficients of the spherical harmonic repre-sentation given by Eq (1) for models 3, 4, and 5 from Table 1 showed differences of less than 1nT (nanotesla) when compared with MGST (6/80) through degree and order ten. The spatial power spectra computed from spherical harmonic expansions of the dipole model by Eq (2) were compared with the spectrum of MGST (10/81), a spherical harmonic model based on Magsat data through degree and order 23 (LANGEL and EsTES, 1982a). Figure 1 shows an example (model 4). The spectrum of MGST (10/81) descends nearly linearly from second order down to about order 12, where it flattens out, becoming more "white". The dipole spectrum follows the MGST (10/81) spectrum down, and begins to flatten, but then falls off again at higher order, leading to a "bulge" in the spectrum.
The cause of the bulge was investigated by means of a simulation which consisted
of generating synthetic data at exactly the same spatial locations as the MGST(6/80) data set used in the equivalent dipole source models (a "pseudo-data set"), with the "data" consisting of measurements due to a crustal field plus a geocentric dipole only

.

The crustal field model used was a global equivalent source model based on POGO data (R. A. Langel, personal communication) which was expanded to degree and order 40 in spherical harmonics to analyze its spectral character, as well as to generated the synthetic data. The spectrum of the synthetic crustal field is undoubtedly biased at low order, since ultra-long wavelength trends were removed from the POGO data in generating the crustal equivalent source model. The work of J. Meyer and others (personal communication) predicts a white spectrum down to very low order. The distortion of the equivalent source spectrum does not, however, invalidate certain inferences which can be made from the results of the simulation. Least squares fits were
made to the pseudo-data set with a,geocentric dipole plus dipoles of 32° and 21°

spacing at the core/mantle boundary. The results for cases with and without Gaussian noise added to the synthetic data are given in Table 2, and the spectra of selected results are given in Figure 2. The results suggest that the beginning of the bump in the spectrum of model 4 in Fig. 1 is due to crustal influence, while the departure of the spectrum from that of MGST (10/81) around expansion order 17 is due to the
resolution limits of the dipole density (i.e., 360°/21°=17). The principal inference is that the core dipole array is able to model the long wavelength part of the crustal

124 M. A. MAYHEW and R. H. EaTEs

○ MGST (10/81), degree and order 23

・ Dipole Model #4, Table 1

Fig. 1. Power spectrum Sn of field model MGST (10/81) compared with spectrum of Dipole Model 4.

Equivalent Source Modeling of the Core Magnetic Field Using Magsat Data 125

Table 2. Simulations using POGO crustal anomaly field model. Synthesized data set is the vector sum of a centered dipole and the crustal field. "Fit" is RMS to synthesized data.

spectrum, so that even by placing dipoles within the core to model the main field one cannot achieve a clean core-crustal field separation. However, we suggest that a good way to achieve the core-crustal separation goal would be to simultaneously model both core and crustal fields (at Magsat altitudes) using dipole arrays both within the crust and within the core. This would require a powerful computer, since something on the order of 12,000 parameters would need to be estimated for crustal sources separated by about 200km, the limit of crustal source resolution achievable with Magsat data (MAYHEW and GALLIHER, 1982).

4. Time-Dependent Models

Models have been obtained with both 32° and 21° dipole spacings using a quiet
(Kp<1-) Magsat data set extending over four months (November 5, 1979-March 15,

1980). Secular variation was modeled using first time derivatives for the dipole magnetization vector components, which doubles the total number of parameters in the solution. The solutions displayed a very slow convergence in the time derivatives, although at each iteration the conversion of the dipole parameters to spherical harmonic coefficients gnm, hnm, and gnm, hnm by Eqs. 1 and 2 showed close agreement with a standard spherical harmonic model to degree and order 13 in the constant terms and degree and order 7 in the first derivative terms based on the same data set. The slow convergence was due to the use of scalar magnetometer data together with poor mathematical observability of the time variation of the individual dipoles over the short time period. However, while the observability of the time variation of the individual dipoles over the four month period was very poor, the information content to the extent of determining the global time variation to degree 7 in spherical harmonics was strong. A priori values for the geocentric dipole derivatives were obtained from g10, g11 and h11 of the standard spherical harmonic model and the solution was then statistically constrained for the geocentric dipole using these values. The convergence using this technique showed much improvement.
To further improve the observability of the time variation, observatory annual means data for a selected set of magnetic observatories from 1974-1977 was added to
the above Magsat data set, and an unconstrained epoch 1980 dipole solution for 21°

126 M. A. MAYHEW and R. H. ESTES

○ MGST (10/81), degree and order 23

● Dipole Model #4, Table 1

° "Crustal" Spectrum

△ 21° dipole array to model "crustal" field

▲ 32° dipole array to model "crustal" field

Fig. 2. Power spectra Sn of synthetic "crustal" field and two dipole array models lit to this held compared with spectra of field model MGST (10/81) and Dipole Model 4.

Equivalent Source Modeling of the Core Magnetic Field Using Magsat Data 127

Table 3. Statistical comparison (σ) of candidate IGRF models and 1974-1980 Dipole Model with selected observatory annual means (nT).

Table 4. Statistical comparison of candidate IGRF models and 1974-1980 Dipole Model with selected Magsat data (nT) at epoch 1980.

(with a geocentric dipole) was obtained solving simultaneously for the observatory anomaly bias vectors (LANGEL et aL., 1982). This solution is referred to as the 1974-1980 Dipole Model. The solution converged in three iterations, showing small correlations among the constant parameters, among the time deviative parameters, and between the constant and time deviation parameters (typically on the order of. 01), although a few correlations reached values of approximately. 4 between some nearest neighbor dipole parameters. This correlation structure is in stark contrast to that observed in spherical harmonic models which incorporate secular variation para.. meters based on the same data set where the constant and derivative terms for a given degree and order may show correlations as high as. 99. The numerical values of the dipole parameters, the observatory anomaly bias vectors, and the coefficients of the

128 M. A. MAYHEW and R. H. FSTES

Table 5. Low degree and order spherical harmonic coefficients at Epoch 1980 derived from Equivalent
Dipole Model of 21° resolution using Magsat data and observatory Annual Means from 1974-1977.

equivalent spherical harmonic expansion of the 93 dipoles through degree and order 24 are given by MAYHEW and ESTES (1982).
The 1974-1980 Dipole Model is compared with candidate models for the 1975 IGRF presented by NASA, USGS and IGS in Tables 3 through 5 (PEDDIE, 1982). Table 3 evaluates the fit to selected observatory annual means (LANGEL and ESTES,
1982b) using the statistic σ defined by LANGEL et al. (1980), while Table 4 compares the

models with Magsat data. The dipole model performs well with respect to the candidate IGRF models. Table 5 displays the low degree and order spherical harmonic coefficients of the Dipole Model and the USGS candidate 1980 IGRF Secular Variation model, which is based on conventional techniques using more recent observatory annual means. The trends in the secular variation coefficients are very similar, although the magnitude of the first degree dipole model terms are larger by a few nT than the USGS model. This is a result of the dipole model heavily utilizing Magsat data at epoch 1980. Field models based only on Magsat data yield g10 on the order of 28nT/yr and H11 on the order of -20nT/yr. The availability of ample observatory annual means data for years later than 1980 are required to more fully evaluate the different secular variation terms.

Comparison of the number of arithmetic operations (multiplication, addition,
and trigonometric and square root evaluation) required to evaluate the magnetic field from dipole models and spherical harmonic models show the dipole models to be

Equivalent Source Modeling of the Core Magnetic Field Using Magsat Data 129

superior in computational efficiency for spatial resolution better than 21° when three

degrees-of-freedom unconstrained dipoles are utilized. The computation burden of field model evaluation is comparable to the standard spherical harmonic approach when the dipoles are constrained radially.

5. Conclusions

The modeling of the main field with an equivalent dipole representation has been found to be comparable to the standard spherical harmonic approach in accuracy. The

32° dipole density (42 dipoles) corresponds approximately to an eleventh degree/order
spherical harmonic expansion (143 parameters) while the 21° dipole density (92

dipoles) corresponds to approximately a seventeenth degree and order expansion (323 parameters).
Fixing of the dipole positions results in rapid convergence of the dipole solutions for single-epoch models. For time dependent models, a sufficiently long time interval of data or a priori values and statistics must be available for the derivatives to achieve convergence. In contrast to standard spherical harmonic models based on the same data, the correlation structure of dipole magnetic moment derivatives with the constant magnetic moment parameters is small, indicating the ability to strongly
separate the constant field and the secular variation. The dipole model of 21° spatial

resolution using observatory annual means data from 1974-1977 together with Magsat data performs very well when compared to candidate IGRF models of the main field and its secular variation based on conventional spherical harmonic techniques.

This work was supported by NASA Contract NAS 5-26047. R. A. Langel supplied the

global equivalent source crustal anomaly model. J. Meyer kindly showed us a draft of a forthcoming paper.

REFERENCES

ALLDREDGE, L. R. and L. HURWITZ, Radial dipoles as the sources of the Earth's main magnetic field, J. Geophys. Res., 69, 2631-2640, 1964.
ALLDREDGE, L. R. and C. O. STEARNS, Dipole model of the sources of the Earth's magnetic field and secular change, J. Geophys. Res., 74, 6583-6593, 1969.
CARLE, H. M. and C. G. A. HARRISON, A problem in representing the core magnetic field of the Earth, Geophys. Res. Lett., 9, 265-268, 1982.
COX, A., Lengths of geomagnetic polarity intervals, J. Geophys. Res., 73, 3247-3260, 1968.
DAMPNEY, C. N. G., The equivalent source technique, Geophysics, 34, 39-53, 1969.
HARRISON, C. G. A. and H. M. CARLE, Modeling the core magnetic field of the Earth, Phil. Trans. R. Soc. Lond. A., 306, 179-191, 1982.
HURWITZ, L., Eccentric dipoles and spherical harmonic analysis, J. Geophys. Res., 65, 2555-2556, 1960. LANGEL, R. A. and R. H. ESTES, A geomagnetic field spectrum, Geophys. Res. Lett., 9, 250-253, 1982a. LANGEL, R. A, and R. H. ESTES, A comparison of IGRF candidate models, J. Geomag. Geoelectr., 34, 365-381, 1982b.

130 M. A. MAYHEW and R. H. ESTES

LANGEL, R. A., R. H. ESTES, G. D. MEAD, E. B. FABIANO, and E. R. LANCASTER, Initial geomagnetic field model from Magsat vector data, Geophys. Res. Lett., 7, 793-796, 1980.
LANGEL, R. A., R. H. ESTES, and G. D. MEAD, Some new methods in geomagnetic field modeling applied to the 1960-1980 epoch, J. Geomag. Geoelectr., 34, 327-349, 1982.
LowES, F. J., Mean square values on the sphere of spherical harmonic vector fields, J. Geophys. Res., 71, 2179, 1966.
LowES, F. J. and S. K. RUNCORN, The analysis of the geomagnetic secular variation, Phil Trans. R. Astron. Soc. A., 243, 525-546, 1951.
MAYHEW, M. A., Inversion of satellite magnetic anomaly data, J. Geophys., 45, 119-128, 1979. MAYHEW, M. A. and R. H. ESTES, Constrained inversion of satellite magnetic anomaly data, Final Report, NASA/Goddard Space Flight Center Contract NAS 5-25698, Volume 3, 68 pp., 1981.
MAYHEW, M. A. and R. H. ESTES, Equivalent source modeling of the main field using Magsat data, Final Report, NASA f Goddard Space Flight Center Contract NAS 5-26047, 14 pp., 1982.
MAYHEW, M. A. and S. C. GALLIHER, An equivalent layer magnetization model for the United States derived from Magsat data, Geophys. Res. Lett., 9, 311-313, 1982.
MCNISH, A. G., Physical representations of the geomagnetic field, Trans. Am. Geoph. Union, 21, 287-291, 1940.
PEDDIE, N. W., Current loop models of the Earth's magnetic field, J. Geophys. Res., 84, 4517-4523, 1979. PEDDIE, N. W., International Geomagnetic Reference Field :the third generation, J. Geomag. Geoelectr., 34, 309-326, 1982.
STEARNS, C. O. and L. R. ALLDREDGE, Models of sources of the Earth's magnetic field, in Methods of Computational Physics, 13, 61-92, 1973.