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EQUIVARIANT STABLE HOMOTOPY AND FRAMED BORDISM

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ABSTRACT. This paper gives an *elementary* proof of the result that equivariant stable homotopy is the same as equivariant framed bordism.

1. Introduction. Let G be a finite group. The purpose of this paper is to give an *elementary* proof of the result that equivariant stable homotopy and equivariant framed bordism, as G homology theories, are the same.

The idea of the proof is as follows—full definitions and details will be found inside—let $\omega_V^G(X, A)$ denote equivariant framed bordism and let $\pi_V^{s,G}(X, A)$ denote equivariant stable homotopy. Then, for any subgroup H of G, we have the following commutative diagram

$$\begin{array}{ccc} \omega_V^G(X, A) & \xrightarrow{\Phi} & \pi_V^{s,G}(X, A) \\ \downarrow{\Psi_\omega} & & \downarrow{\Psi_\pi} \\ \omega_{VH}^{W(H)}(X^H, A^H) & \xrightarrow{\Phi^H} & \pi_{VH}^{s,W(H)}(X^H, A^H) \end{array}$$

where $W(H)$ denotes the quotient $N(H)/H$ and $N(H)$ is the normaliser of H in G. The maps Φ and Φ^H are the Pontrjagin-Thom maps, while Ψ_ω and Ψ_π denote taking fixed point sets with respect to H.

To each subgroup H of G, we can associate a pair of G spaces (EF, EF'). If, in the commutative diagram, we replace (X, A) by $(X, A) \times (EF, EF') = (X \times EF, X \times EF' \cup A \times EF)$ then we can show quite easily that

(1) Ψ_ω is an isomorphism,

(2) Φ^H is an isomorphism, and

(3) Ψ_π is injective.

It therefore follows that Φ (and Ψ_π) is an isomorphism.

Next, to the group G we can associate a finite sequence of G spaces $\emptyset = EF_1 \subset EF_2 \subset \cdots \subset EF_n$ with the following properties.

(1) For each i, Φ is an isomorphism for the space $(X, A) \times (EF_{i+1}, EF_i)$.

(2) For each G homology theory there is a long exact sequence involving the spaces $(X, A) \times (EF_j, EF_i)$, $(X, A) \times (EF_k, EF_i)$, $(X, A) \times (EF_k, EF_j)$ for any $i < j < k$.

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(3) EF_n is G contractible.

So, by induction and the five lemma, the result easily follows.

This result, for X a point, was first announced by G. Segal [12] with a proof to appear in J. J. O'Connor's thesis [9]. A proof by H. Hauschild was given in his thesis [6]. The result for X a point also appears as a corollary in the thesis of R. Rubinsztein [11].

2. **Equivariant framed bordism.** Let V be a G module—i.e. a finite dimensional real vector space on which G acts linearly, and let M be a G manifold.

If ξ is a G vector bundle on M, we say that ξ has a *V trivialization* if there exists some integer n and a G bundle isomorphism ϕ_M s.t.

$$\phi_M\colon \xi \oplus (\mathbf{R}^n \times M) \xrightarrow{\cong} (V \times M) \oplus (\mathbf{R}^n \times M)$$

where $\mathbf{R}^n$ denotes the trivial n dimensional G module. A *V framed G* manifold is a G manifold M together with a G homotopy equivalence class of V trivializations of the tangent bundle of M.

Notice that this definition of a V framed G manifold differs from that used in [7] and [12] where $\mathbf{R}^n$ is replaced by any G module U. However we do have the following result which will be needed later on.

LEMMA 2.1. *If M is a free G manifold then the above two notions of V framing are equivalent.*

PROOF. We need only show that if $TM \oplus (U \times M) \cong (V \times M) \oplus (U \times M)$ for some G module U then $TM \oplus (\mathbf{R}^n \times M) \cong (V \times M) \oplus (\mathbf{R}^n \times M)$ for some n. The G vector bundles over M are in a one-to-one correspondence with the vector bundles over M/G (see [1]), the correspondence being given by $E \longrightarrow E/G$, $\pi^*(E') \longleftarrow E'$ where $\pi\colon M \longrightarrow M/G$. So

$$TM/G \oplus (U \times M)/G \cong (V \times M)/G \oplus (U \times M)/G$$

as vector bundles over M/G. Now, there exists some bundle E over M/G such that $(U \times M)/G \oplus E$ is a trivial vector bundle over M/G—say $\mathbf{R}^n \times M/G$. Thus we have

$$TM/G \oplus (\mathbf{R}^n \times M)/G \cong (V \times M)/G \oplus (\mathbf{R}^n \times M)/G$$

and

$$TM \oplus (\mathbf{R}^n \times M) \cong (V \times M) \oplus (\mathbf{R}^n \times M).$$

Let (X, A) be a G topological pair; then a V framed bordism element of (X, A) is a pair (M, f) where

(i) M is a V framed G manifold, and

(ii) $f: M \longrightarrow X$ is an equivariant map with $f(\partial M) \subset A$. (∂M denotes the boundary of M.)

If M is a V framed G manifold, then we have a trivialization

$$\phi_M: TM \oplus (\mathbf{R}^n \times M) \cong (V \times M) \oplus (\mathbf{R}^n \times M).$$

Let $-\phi_M$ denote the trivialization

$$-\phi_M = \phi_M \oplus (-\mathrm{id}): TM \oplus (\mathbf{R}^n \times M) \oplus (\mathbf{R} \times M)$$
$$\cong (V \times M) \oplus (\mathbf{R}^n \times M) \oplus (\mathbf{R} \times M)$$

where $-\mathrm{id}$: $\mathbf{R} \times M \longrightarrow \mathbf{R} \times M$ is the map defined by sending (t, m) to $(-t, m)$. The manifold with this framing will be denoted by $-M$.

Two V framed bordism elements (M, f), (M, f') of (X, A) are said to be equivalent if there exists a pair (N, q) where

(i) N is a $V \oplus \mathbf{R}$ framed G manifold,

(ii) $M \cup (-M') \subset \partial N$, the induced V framings on ∂N restricted to M, $-M'$ agreeing with that on M, $-M'$ respectively,

(iii) $q: N \longrightarrow X$ is an equivariant map with $q|M = f$, $q|(-M') = f'$ and $q(\partial N \backslash (M \cup (-M'))) \subset A$.

The set of V framed bordism elements of (X, A) under this equivalence relationship forms an abelian group denoted by $\omega_V^G(X, A)$.

Let $V_0, V_1, \ldots, V_r$ be a complete set of irreducible nonisomorphic G modules–with V_0 being the trivial one dimensional G module, i.e. $\mathbf{R}$. Thus any G module V may be represented uniquely as a sum $V = \Sigma_{i=0}^r n_i V_i$ where the n_i are integers ($n_i \geqslant 0$) and $n_i V_i$ means the direct sum of n_i copies of V_i.

An element $\alpha \in RO(G)$–the real representation ring of G–may be written as $\alpha = \Sigma_{i=0}^r \alpha_i V_i$ where the α_i are integers. Let α^+ be the sum $\Sigma\, \alpha_j V_j$ where $\alpha_j > 0$ and let α^- be the sum $\Sigma -\alpha_k V_k$ where $\alpha_k < 0$. Then $\alpha = \alpha^+ - \alpha^-$ and each of α^+ and α^- are G modules. We define

$$\omega_\alpha^G(X, A) \equiv \omega_{\alpha^+}^G(D(\alpha^-) \times X, S(\alpha^-) \times X \cup D(\alpha^-) \times A)$$

where D, S stand for the unit disc and unit sphere respectively.

The set $\{\omega_\alpha^G(X, A); \alpha \in RO(G)\}$ forms a G homology theory indexed by elements $\alpha \in RO(G)$. (This theory has suspension isomorphisms for trivial G modules $\mathbf{R}^n$, although later on it will follow that we have suspension isomorphisms for all G modules.)

3. **Equivariant stable homotopy.** We recall [7] the definition of equivariant stable homotopy. If V is a G module, let S^V denote its one point compactification, in which ∞ is regarded as base point. We say that a G module W is

admissible if W contains at least one copy of each irreducible G module—for example, W may be $\Sigma_{i=0}^{r} V_i$ where the V_i are as defined in §2.

We define $\pi_\alpha^{s,G}(X, A)$ to be the direct limit (over $k \in \mathbf{Z}$) of the G homotopy classes of base point preserving G maps from $S^{kW\oplus\alpha^+}$ to $S^{kW\oplus\alpha^-} \wedge (X^+/A^+)$. In symbols

$$\pi_\alpha^{s,G}(X, A) = \lim_{k\longrightarrow\infty} [S^{kW\oplus\alpha^+}; S^{kW\oplus\alpha^-} \wedge (X^+/A^+)]_G^0.$$

The maps

$$[S^{kW\oplus\alpha^+}; S^{kW\oplus\alpha^-} \wedge (X^+/A^+)]_G^0 \longrightarrow [S^{(k+1)W\oplus\alpha^+}; S^{(k+1)W\oplus\alpha^-} \wedge (X^+/A^+)]_G^0$$

are given by suspending with S^W.

This definition is independent of the choice of W—so long as W contains at least one copy of each irreducible G module—see [7].

The set $\{\pi_\alpha^{s,G}(X, A); \alpha \in RO(G)\}$ forms a G homology theory and has suspension isomorphisms for all G modules V, i.e.

$$\pi_\alpha^{s,G}(X, A) \cong \pi_{\alpha\oplus V}^{s,G}(D(V) \times X, S(V) \times X \cup D(V) \times A),$$

in other words it forms a G homology theory in the sense of [7].

4. **The commutative diagram.** Since

$$h_\alpha^G(X, A) = h_{\alpha^+}^G(D(\alpha^-) \times X, S(\alpha^-) \times X \cup D(\alpha^-) \times A),$$

in the case that $h = \omega$ or $h = \pi$, we shall henceforth only look at $h_V^G(X, A)$ where V is some G module.

The aim of this section is to show the existence of the following commutative diagram.

$$\begin{array}{ccc} \omega_V^G(X, A) & \xrightarrow{\Phi} & \pi_V^{s,G}(X, A) \\ \downarrow \Psi_\omega & & \downarrow \Psi_\pi \\ \omega_{VH}^{W(H)}(X^H, A^H) & \xrightarrow{\Phi^H} & \pi_{VH}^{s,W(H)}(X^H, A^H) \end{array}$$

(a) The map Φ is given by the Pontrjagin-Thom construction, which we proceed to describe.

Let (M, f) be a V framed bordism element of (X, A). We know that $TM \oplus (\mathbf{R}^n \times M) \cong (V \oplus \mathbf{R}^n) \times M$, also we may embed M in $(k-n)W$ where k is some large number and W is admissible in the sense of §3. If $\nu(M, (k-n)W)$ denotes the normal bundle of M in $(k-n)W$ then we have the following bundle isomorphisms:

$$TM \oplus \nu(M, (k-n)W) \cong (k-n)W \times M,$$

$$(V \times M) \oplus (\mathbf{R}^n \times M) \oplus \nu(M, (k-n)W) \cong (\mathbf{R}^n \times M) \oplus ((k-n)W \times M)$$

so

$$\nu(M, (k-n)W \oplus \mathbf{R}^n \oplus V) \cong (\mathbf{R}^n \times M) \oplus ((k-n)W \times M)$$

and thus

$$\nu(M, kW \oplus V) \cong kW \times M.$$

In other words we can embed M in $kW \oplus V$ for some large k, such that the normal bundle is $kW \times M$. Consider the following sequence of maps:

$$\begin{aligned} S^{kW\oplus V} &= D(kW \oplus V)/S(kW \oplus V) \longrightarrow D(\nu)/(D(\nu|\partial M) \cup S(\nu)) \\ &\xrightarrow{\cong} (M \times D(kW))/((\partial M \times D(kW)) \cup (M \times S(kW))) \\ &\xrightarrow{f \times \mathrm{id}} (X \times D(kW))/((A \times D(kW)) \cup (X \times S(kW))). \end{aligned}$$

The composite defines an element of $\pi_V^{s,G}(X, A)$ and so defines the map Φ.

(b) If (M, f) is a V framed bordism element of (X, A) then M^H is a V^H framed $W(H)$ manifold and $f|M^H$: $M^H \longrightarrow X^H$ is a $W(H)$ equivariant map with $f(\partial M^H) \subset A^H$. We therefore define $\psi_\omega(M, f)$ to be $(M^H, f|M^H)$.

(c) The map Φ^H is the Pontrjagin-Thom construction as in (a).

(d) Ψ_π is defined by taking fixed point sets with respect to H, i.e.

$$[S^{kW\oplus V}; S^{kW} \wedge X/A]^0_G \longrightarrow [S^{kW^H \oplus V^H}; S^{kW^H} \wedge X^H/A^H]^0_{W(H)}.$$

(W^H is clearly admissible for the group $W(H)$.)

The diagram is clearly commutative.

5. **Families.** Recall that a *family* F in G is a collection of subgroups of G such that

(a) if $H \in F$ and $K \subset H$, then $K \in F$ and

(b) if $H \in F$ and $g \in G$, then $gHg^{-1} \in F$.

Following Palais [10], see also Bredon [2], we define universal spaces EF as follows. If H is a subgroup of G, let $EW(H)$ denote the universal $W(H)$ space (i.e. a contractible free $W(H)$ space such that $EW(H) \longrightarrow EW(H)/W(H) = BW(H)$ is a numerable $W(H)$ principal bundle). Let EF be defined by

$$EF = * (G \times_{N(H)} EW(H))$$

where the join is taken over a complete set of conjugacy classes of subgroups H

in F. (*Note*. The join used here is not the Palais join as in [10] and [2]–this is needed only if F is not a family.)

We have, in particular, if X is a G space all of whose isotropy subgroups belong to F then there is a unique map (up to G homotopy) $X \longrightarrow EF$.

Given a G homology theory h^G we define a new G homology theory $h^G[F, F']$ for pairs $F' \subset F$ of families in G by

$$h^G_*[F, F'](X, A) = h^G_*(X \times EF, A \times EF \cup X \times EF').$$

This idea, in this form, comes from tom Dieck [5]. That $h^G[F, F']$ is indeed a G homology theory is not too difficult to prove. For example, the long exact sequence

$$(5.1) \cdots \longrightarrow h^G_*[F, F'](A) \longrightarrow h^G_*[F, F'](X) \longrightarrow h^G_*[F, F'](X, A) \longrightarrow \cdots$$

is obtained by looking at the associated h^G theory long exact sequence of the triple $(X \times EF, X \times EF' \cup A \times EF, X \times EF')$ and using the fact that

$$\begin{aligned} h^G_*(X \times EF' \cup A \times EF, X \times EF') &= h^G_*(A \times EF, A \times EF') \quad \text{(by excision)} \\ &= h^G_*[F, F'](A). \end{aligned}$$

Let $F'' \subset F' \subset F$ be families in G and consider the triple $(X \times EF, X \times EF' \cup A \times EF, X \times EF'' \cup A \times EF)$. Looking at the associated h^G theory long exact sequence and using the fact that

$$\begin{aligned} &h^G_*(X \times EF' \cup A \times EF, X \times EF'' \cup A \times EF) \\ &\qquad \cong h^G_*(X \times EF', X \times EF'' \cup A \times EF') \quad \text{(by excision)} \\ &\qquad = h^G_*[F', F''](X, A) \end{aligned}$$

we obtain the following long exact sequence

$$\begin{aligned} &\cdots \longrightarrow h^G_*[F', F''](X, A) \longrightarrow h^G_*[F, F''](X, A) \longrightarrow h^G_*[F, F'](X, A) \\ (5.2) \qquad &\qquad \longrightarrow h^G_{*-1}[F', F''](X, A) \longrightarrow \cdots \end{aligned}$$

Note. For bordism type theories we can also define $h^G_*[F, F'](X, A)$ along the lines of Conner and Floyd [4] and Stong [13]–the resulting theory agrees with the one defined above, see the paper of tom Dieck [5].

If $F' = \emptyset$ then we write $h^G_*[F, F'](X, A)$ as $h^G_*[F](X, A)$, if furthermore $F = \{1\}$, the family consisting of just the trivial subgroup, then we write it as $h^G_*[\text{free}](X, A)$.

If $F = \text{All}$, the family consisting of all subgroups then $h^G_*[\text{All}](X, A) = h^G_*(X, A)$.

6. Proof of main theorem. Recall that two families $F' \subset F$ in G are said to be adjacent if $F \backslash F'$ only contains the conjugates of some single group, say H.

Throughout this section let $F' \subset F$ be adjacent families in G with $H \in F \backslash F'$. Returning to the commutative diagram (§4), replacing (X, A) by the pair $(EF \times X, EF \times A \cup EF' \times X)$ gives the following commutative diagram.

$$\begin{array}{ccc} \omega_V^G[F, F'](X, A) & \xrightarrow{\Phi} & \pi_V^{s,G}[F, F'](X, A) \\ \downarrow \Psi_\omega & & \downarrow \Psi_\pi \\ \omega_{VH}^{W(H)}[\text{free}](X^H, A^H) & \xrightarrow{\Phi^H} & \pi_{VH}^{s,W(H)}[\text{free}](X^H, A^H) \end{array}$$

(Observe that $(EF \times X)^H = EW(H) \times X^H$ and $(EF \times A \cup EF' \times X)^H = EW(H) \times A^H$.)

THEOREM 6.1. *Ψ_ω is an isomorphism.*

PROOF. We shall first define a map

$$\Theta: \omega_{VH}^{W(H)}[\text{free}](X^H, A^H) \longrightarrow \omega_V^G[F, F'](X, A).$$

Let $(N, t) \in \omega_{VH}^{W(H)}[\text{free}](X^H, A^H)$, so N is a V^H framed $W(H)$ manifold and $t: N \longrightarrow EW(H) \times X^H$ is a $W(H)$ equivariant map with $t(\partial N) \subset EW(H) \times A^H$. It follows that N must be a free $W(H)$ manifold. Let $(V^H)^\perp$ denote the orthogonal complement of V^H in V and consider the following manifold

$$Q = G \times_{N(H)} (N \times D((V^H)^\perp))$$

which is easily seen to be a V framed G manifold. The isotropy subgroups in Q are contained in the family F, hence there is a unique (up to G homotopy) equivariant map $q_1: Q \longrightarrow EF$. (The map $q_1^H: Q^H \longrightarrow (EF)^H = EW(H)$ agrees with pt where $p: EW(H) \times X^H \longrightarrow EW(H)$ is the projection map.)

We thus obtain a map

$$q = q_1 \times (G \times_{N(H)} t): Q \longrightarrow EF \times (G \times_{N(H)} X^H) \hookrightarrow EF \times X.$$

Since

$$\partial Q = G \times_{N(H)} (\partial N \times D((V^H)^\perp)) \cup G \times_{N(H)} (N \times S((V^H)^\perp))$$

it follows that $q(\partial Q) \subset EF \times A \cup EF' \times X$ and so (Q, q) determines an element of $\omega_V^G[F, F'](X, A)$. We define Θ by $\Theta(N, T) = (Q, q)$. Clearly $\Psi_\omega \Theta = \text{id}$. The fact that $\Theta \Psi_\omega = \text{id}$ follows from the next two lemmas.

LEMMA 6.2. *If M is a V framed G manifold then the normal bundle of M^H in M is trivial and is given by $M^H \times (V^H)^\perp$.*

PROOF. $TM \oplus (\mathbf{R}^n \times M) \cong (V \times M) \oplus (\mathbf{R}^n \times M)$ so

$$TM^H \oplus (\mathbf{R}^n \times M^H) \cong (V^H \times M^H) \oplus (\mathbf{R}^n \times M^H)$$

and

$$\begin{aligned} TM^H \oplus \nu(M^H, M) \oplus (\mathbf{R}^n \times M^H) &\cong (TM \oplus (\mathbf{R}^n \times M))|M^H \\ &\cong ((V \times M) \oplus (\mathbf{R}^n \times M))|M^H \cong (V \times M^H) \oplus (\mathbf{R}^n \times M^H) \\ &\cong (V^H \times M^H) \oplus ((V^H)^\perp \times M^H) \oplus (\mathbf{R}^n \times M^H), \end{aligned}$$

which implies that $\nu(M^H, M)$ is $M^H \times (V^H)^\perp$ since the H representation in $TM^H \oplus (\mathbf{R}^n \times M^H)$ is H trivial but not so in $\nu(M^H, M)$.

The next lemma is the analogue of Lemma 5.1 in [13].

LEMMA 6.3. *Let (M, f) and (M', f') be elements of $\omega_V^G[F, F'](X, A)$ and suppose that M' is a regularly embedded submanifold of M with $f|M' = f'$. If every point of $M\setminus M'$ has isotropy group belonging to F' then these elements represent the same class in $\omega_V^G[F, F'](X, A)$.*

PROOF. Consider $M \times I$ where I is the unit interval, with $r: M \times I \longrightarrow EF \times X$ given by $r(m, t) = f(m)$. We have

(i) $M \times I$ is a $V \oplus R$ framed G manifold,

(ii) $M \cup (-M') \subset \partial(M \times I)$, with the induced V trivializations on ∂N restricted to $M, -M'$ agreeing with that on $M, -M'$ respectively,

(iii) $r: M \times I \longrightarrow EF \times X$ is an equivariant map with

$$\begin{aligned} r(\partial(M \times I)\setminus(M \cup (-M'))) &= r((\partial M \times I \cup M \cup (-M))\setminus(M \cup (-M'))) \\ &= r((\partial M \times I) \cup (-(M\setminus M'))) \subset EF \times A \cup EF' \times X. \end{aligned}$$

THEOREM 6.4. *Φ^H is an isomorphism.*

PROOF. This result follows from Lemma 2.1 and the fact that transversality works for G maps between free G spaces—see for example [8].

COROLLARY 6.5. *Φ is injective and Ψ_π is surjective.*

THEOREM 6.6. *If $A = \varnothing$ then Ψ_π is injective.*

PROOF. Suppose $f \in \pi_V^{s,G}[F, F'](X)$ and that $\Psi_\pi(f) = 0$, i.e. that $\Psi_\pi(f) \in \pi_{VH}^{s,W(H)}[\text{free}](X^H)$ is $W(H)$ null homotopic.

Since EF is the join of $G \times_{N(H)} EW(H)$ with EF' we consider $G \times_{N(H)} EW(H)$ as being a subspace of EF. Also $X \times (G \times_{N(H)} EW(H))$ is a subspace of $D(kW) \times X \times EF$ in the obvious way, and hence a subspace of $S^{kW} \wedge X \wedge (EF/EF')$.

Let $L = f^{-1}(X \times (G \times_{N(H)} EW(H)))$ and let $*$ denote the base point of $S^{kW} \wedge X \wedge (EF/EF')$; then $L \cap f^{-1}(*) = \emptyset$. We shall show that $L = \emptyset$. Let a be some point of L and let G_a denote the isotropy subgroup at a, then either

(I) $G_a \notin F$,

(II) $G_a \in F \backslash F'$, or

(III) $G_a \in F'$.

In case (I) $G_a \notin F$ then $G_{f(a)} \notin F$ since $G_a \subset G_{f(a)} \lesssim H \in F$, and hence this case does not arise. In case (II) $G_a \in F\backslash F'$ means that G_a is conjugate to H and $f(a) \in (S^{kW} \wedge X \wedge (EF/EF'))^{G_a}$ which by assumption on $\Psi_\pi(f)$ may be assumed to be the base point and hence $a \notin L$, so this case does not arise. Finally in case (III) $G_a \in F'$, but f restricted to such points factors through $(D(kW) \times X \times EF')$ which is in the base point of $(S^{kW} \wedge X \wedge (EF/EF'))$, thus this case also does not arise.

It follows that $L = \emptyset$, in other words $f^{-1}(X \times (G \times_{N(H)} EW(H))) = \emptyset$. Since the complement of $X \times (G \times_{N(H)} EW(H))$ in $S^{kW} \wedge X \wedge (EF/EF')$ is G contractible it follows that f is G null homotopic.

COROLLARY 6.7. *Φ is an isomorphism.*

PROOF. If $A = \emptyset$, then this follows immediately from Theorems 6.1, 6.4 and 6.6. In general the long exact sequence 5.1 and the five lemma provide a proof.

We have shown that Φ is an isomorphism for every pair of adjacent families. Since G is a finite group we can find families $\emptyset = F_1 \subset F_2 \subset \cdots \subset F_n =$ All, such that $F_i \subset F_{i+1}$ are adjacent families in G. So by induction and the five lemma on the long exact sequence 5.2 we can show that Φ is an isomorphism for all pairs $F' \subset F$ of families in G. In particular for $\emptyset \subset$ All. This completes the proof of the result that equivariant framed bordism is the same as equivariant stable homotopy.

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