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# ERICSON FLUCTUATIONS AND THE BOHR MODEL IN HADRON PHYSICS

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## A B S T R A C T

Analogies between the statistical bootstrap model for hadrons and the familiar statistical model for nuclei are pointed out, and used as a guide for suggesting new statistical treatments of hadron reactions :

i) the Fermi statistical model is modernized by including the full Hagedorn spectrum of resonances, and brought into correspondence with the Bohr model by assuming that the reaction proceeds via an incoherent sum over direct channel resonances ;

ii) a definite prescription is given, predicting which hadron reactions should exhibit Ericson fluctuations ; it is shown that the peaks and dips in $\pi N$ elastic scattering between p = 1.5 and 5 GeV/c can be interpreted as Ericson fluctuations, although further experiments are needed to establish this interpretation definitively;

iii) the rapidly falling cross-sections found in exotic exchange reactions such as backward $K^-p \to K^-p$ are interpreted as the incoherent part of the sum over a Hagedorn spectrum of direct channel resonances ; especially large Ericson fluctuations are predicted for such cases.

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## 1. INTRODUCTION

Although statistical models for hadron physics have a history going back more than 20 years [1], they are still controversial and incompletely developed. By contrast, statistical models for nuclear physics are generally accepted as the best available explanation for large bodies of data, and have undergone a rather comprehensive and sophisticated development.

In Section 2 of the present paper, we briefly remind the reader what is done with the statistical approach in nuclear physics, and indicate certain respects in which the statistical approach to hadron physics [especially the statistical bootstrap model of Hagedorn [2] and Frautschi [3]] is analogous. The aim here is partly pedagogical : to put the unfamiliar hadron models in place alongside the familiar, well-understood nuclear statistical models. But the comparison also serves as a guide for suggesting new treatments of hadron physics; Sections 3-6 are devoted to this use of the nuclear analogy.

Specifically we suggest in Section 4 that the Fermi statistical model be modernized by including the full Hagedorn spectrum of resonances, and brought into closer correspondence with the Bohr statistical model by assuming that the reaction proceeds via an incoherent sum over direct channel resonances. This program has been applied to $N\bar{N}$ annihilation at rest by Hamer [4].

Another feature of the statistical approach to nuclei is that level spacings and partial widths fluctuate; the Bohr model only gives the behaviour averaged over such fluctuations. In Section 3, we discuss fluctuations of level spacings and partial widths in the separable resonance region. Unfortunately it appears that the region of separable hadron resonances is too small to provide an adequate statistical sample for studies of this kind. In Section 5 we review the effect of fluctuations in level spacings and partial widths at higher energies where resonances overlap, and describe how the resulting "Ericson fluctuations" of the cross-section are analyzed in nuclear physics [5]. The same method of analysis is applied to hadron physics in Section 6, and used to suggest new interpretations of a number of familiar phenomena such as the peaks and dips in intermediate energy $\pi N$ scattering, and the rapidly decreasing cross-sections for exotic exchange reactions. A definite prescription is given, predicting the magnitude of Ericson fluctuations in various reactions; the predictions range from large fluctuations in exotic exchange reactions to no fluctuations in reactions such as pp scattering which lack direct channel resonances.

## 2. THE ANALOGY BETWEEN NUCLEAR AND HADRON STATISTICAL MODELS

Statistical models for the density of excited nuclear levels go back to Bethe's work [6] of 1936. Bethe put $Z$ protons and $A-Z$ neutrons into a box with the normal nuclear radius, and considered a free fermion gas. He showed that for energies such that most of the fermions are still degenerate, the density of states

$$\rho(E) \equiv \frac{dn}{dE} \tag{1}$$

rises with excitation energy $E$ as

$$\rho(E) \propto \exp\left(\sqrt{\frac{AE}{c}}\right) \tag{2}$$

where $c$ is a constant of order 2.5 MeV. Experimentally, excited nuclear levels show up as resonances. At high excitations one deduces from nuclear measurements that a rapid rise occurs, qualitatively consistent with Bethe's formula (to achieve quantitative agreement with specific nuclei, especially at low excitations, the model must be refined by adding effects of the potential which distinguish between even and odd nuclei, put in some shell model effects, etc.).

To treat <u>reactions</u> statistically, further assumptions are needed. In the popular model of Bohr, reactions proceed via an <u>incoherent</u> sum over direct channel resonances. If an average resonance decays into various final states at a rate proportional to phase space, it follows that reaction rates are proportional to the phase space of the final state.

Of course, there are many cases in nuclear physics where statistical ideas do not work and one uses a "<u>direct reaction</u>" model. There is no clean-cut rule for when direct reactions dominate, but they are more likely to be important if the initial and final states are closely related. Also when any reaction is followed to sufficiently high kinetic energies, direct reactions eventually dominate even if the Bohr model successfully described the low kinetic energy region. The nomenclature "direct" requires a word of explanation. What the nuclear physicist calls a "direct reaction" is none other what a hadron physicist calls an "exchange reaction", particular examples being identifiable as meson exchange, nucleon exchange, Pomeron exchange, and photon exchange [7]. The term "direct" refers here to time : the final state emerges quickly in a direct reaction, whereas the particles in a Bohr reaction are supposed to spend a longer time in the intermediate resonant state. From another point of view, the difference is that many direct channel resonances add coherently in a "direct reaction" amplitude [8),7)], whereas they add incoherently in a "Bohr reaction" amplitude.

Finally, nuclear physics provides examples where a mixed description is most useful. For example at energies above 20 MeV, the production of various numbers of neutrons can be treated as a direct reaction which knocks out one or two neutrons, leaving an excited nucleus which boils off further neutrons with a thermodynamic distribution [9)-11)].

It is interesting to note that while Bethe's model for the level density depends on fewer assumptions than the models for reactions, it cannot be fully tested without recourse to reaction models [12)]. To be sure, at low excitations, a direct count of levels with full information on the quantum numbers and degeneracy can be made, analogous to the Rosenfeld Tables of hadron physics. But this is practicable only up to a certain energy, which is too low to test Bethe's asymptotic expression very well. Good information is again available even in heavy nuclei for excitations of about 7 MeV, just above the single neutron threshold, where the average resonance width is still less than the average spacing, and the excellent energy resolution attainable with slow neutrons allows detection of the individual levels. At higher energies the resonances overlap and the level density can be deduced only with the aid of reaction models such as the "boiling off" picture of neutron emission, or Ericson fluctuations.

Table I contains a summary of the models we have just reviewed. To the right of each nuclear model is listed its analogue in hadron physics.

The most familiar hadron analogue is of course the non-statistical case of exchange reactions.

Another well-known case is the pure statistical model for hadron reactions, popularized by Fermi [1)]. The two incoming particles were assumed to coalesce in an interaction volume where thermodynamic equilibrium at a uniform temperature was reached, followed by emission proportional to phase space. This picture explains many features of $N\bar{N}$ annihilation. However, at higher kinetic energies it fails to produce sufficient forward peaking [by imposing angular momentum conservation, one does find peaking in the model; for example in a spinless reaction $\ell_z = 0$ and the peaking is due to the fact that Legendre polynomials have a smaller envelope at $90^\circ$ than at $0^\circ$. But this peaking is forward-backward symmetric and is anyway much too small. To fit the data, it is absolutely essential to introduce coherence between different partial waves, which takes us outside the statistical picture].

Hagedorn noted this problem, and also noticed that it was not adequate to count just the phase space for free $\pi$, K, and N. One should also include the effects of resonances. But how many resonances ? Hagedorn attacked this problem and the reaction problem simultaneously, applying a bootstrap hypothesis.

As far as the level density is concerned, Hagedorn's result was [2)]

$$\rho(m) \xrightarrow[m \to \infty]{} c\, m^{a}\, e^{bm} \tag{3}$$

an even faster increase than in nuclear physics.

As far as reactions are concerned, Hagedorn produced a two-step model [13)]. The first step has a non-statistical element : the two colliding bodies lose part of their longitudinal velocity, with the lost kinetic energy going into internal excitation. At the conclusion of this step, there are two excited states proceeding along the original line of flight, each with a definite internal distribution of temperatures. The second step is purely statistical : each excited state "boils off" hadrons with distributions controlled by the local temperature. We see that Hagedorn's model is an analogue of the mixed description used at high energies in nuclear physics - a non-statistical collision (which introduces the necessary distinction between longitudinal and transverse momenta) followed by a boiling off of particles from the resulting excited state [14)].

Recently the present author [3)] has reformulated Hagedorn's treatment of the level density, introducing some technical modifications that make it possible to pin down the level density more precisely [3),15),16)]. The reformulation also makes more explicit a point that was already implicit in Hagedorn's work : the hadron level density can be derived from just two conditions (a statistical condition and a bootstrap condition on the constituents), without direct reference to scattering or to the assumption that local thermodynamic equilibrium is achieved in scattering.

Thus we have a situation displaying a considerable analogy to nuclear physics.

i) The level density of excited states is determined on the basis of a simple statistical assumption, plus one assumption concerning the constituents.

ii) Some, but not all, low energy reactions can be understood statistically. An extra assumption concerning coherence is needed in this case.

iii) Some high energy reactions can be understood in terms of a more complicated model involving both dynamical and statistical assumptions.

iv) As in nuclear physics, the predicted $\rho(m)$ cannot be established conclusively by direct count of levels [although the existing spectrum is quite compatible with Hagedorn's distribution as far as it goes [17),15)]]. In fact, the detailed experimental analysis of resonances in the $\pi N$ channel has already been pushed up near the energy (around $E_{CM} = 2$ GeV for low $J$, higher for high $J$) where levels with <u>the same $J^P$</u> are predicted to start overlapping, making further disentangling of individual levels prohibitively difficult. Conventional phase shift analysis will simply miss most of the levels in this region. The best evidence for the Hagedorn spectrum has been obtained from a different source - by assuming the Hagedorn model for high energy reactions, and comparing the Boltzmann factor

$$\exp\left[-\sqrt{m^2+p_{\parallel}^2+p_{\perp}^2}/kT\right],$$

which occurs when particles are boiled off in that model [18)], with experimental distributions for large transverse momenta $p_{\perp}$, or for the production of heavy pairs with large mass $m$.

## 3. <u>FLUCTUATIONS IN THE SEPARABLE-RESONANCE REGION</u>

We have indicated in the previous section that useful information is provided by nuclear levels with excitation energies of order 7 MeV, just above the single-neutron threshold. What is done with this information, and is there an analogous region for hadrons ?

The nuclear levels in this region are closely spaced in medium and heavy nuclei. Yet they are separable, because of the small average level width $\Gamma$ and the excellent energy resolution attainable with slow neutrons, for the band of energies between single-neutron threshold and a few keV above. The large sample of resonances thus obtained supports statistical models in several respects [12)] :

i) the level density $\rho(E)$ is well-determined in this band of energies. The form of Eq. (2) is roughly verified as far as $A$ dependence and the prediction of a large level density are concerned, although the precise nature of the energy dependence cannot be verified over such a narrow band of energies.

ii) the level spacings are not uniform, but have a statistical distribution consistent with the distribution of eigenvalues of a matrix whose elements are independent random variables [19].

iii) The total and partial widths also follow statistical distributions [19]. Partial widths for decay into a particular channel exhibit large fluctuations from one resonance to the next, whereas the sum over partial widths for decay into $N$ electromagnetic channels fluctuates only as $N^{-\frac{1}{2}}$.

All these tests of statistical theory require good data on a sizeable sample of resonances. For the fluctuation studies ii) and iii), the sample should ideally be from a region where the average level density and number of open channels remain essentially constant, to ensure that changes in these parameters are not responsible for variations in level spacing and width. This condition is met in nuclear physics : the average resonance width $\Gamma$ is much less than the MeV scale which characterizes changes in level density [Eq. (2)], single-particle optical model level spacings, etc. $\Gamma$ is so small because the number of nuclear states at these energies is much larger than the number of open channels (excluding the weakly coupled channels reached by electromagnetic or weak decays). An average physical state finds itself in communication with an open channel only a small fraction of the time.

None of these conditions applies in hadron physics. Right from the beginning of the resonance spectrum the $\Delta(1235)$ and $\rho$ have widths $\Gamma \approx m_\pi$, i.e., on the same scale which characterizes hadrons generally and changes in the level density in particular [$b \approx m_\pi^{-1}$ in Eq. (3)]. Statistical studies of level spacings and partial widths should ideally employ samples from a range of masses $\Delta m < m_\pi$, since only in such a range is the level density and the number of open channels approximately constant. But at present the Rosenfeld Tables contain at most one or two levels for a given $J^P$, B, S, I, Q ... in each interval $\Delta m = m_\pi$ - not enough for statistical studies [20]. This is no accident; as soon as the number of levels becomes greater than one or two in each partial wave, the levels will overlap (because $\Gamma \gtrsim m_\pi$) and become impossible to disentangle. The beautiful checks of statistical theory performed by nuclear physicists in the separable resonance region cannot be repeated in hadron physics.

## 4. MODERNIZATION OF THE FERMI MODEL

The Fermi statistical model, in its original form [1], did not represent a perfect analogy to Bohr's statistical model :

i) only the phase space corresponding to free metastable particles such as $\pi$'s, K's, and N's was counted. However, Fermi himself [1] recognized that if additional particles were discovered, they should be included. After the discovery of the first few mesons and baryon resonances, the phase space for channels involving these new particles was added [21],[22], with salutory results. Our proposal is to complete this process by counting the phase space for all open channels involving any particles in the entire Hagedorn spectrum.

ii) Fermi made no mention of intermediate direct channel resonances, which play such an important role in Bohr's model. However, the Fermi model was meant to apply at energies of about $2M_N$ or higher, where an appreciable number of open channels was present. According to the statistical bootstrap model [2],[3], or the related [23],[24] ideas of duality models, this is exactly the condition for many direct channel resonances to be present (at least for non-exotic channels). Thus it is natural to complete the analogy with Bohr's model by writing the amplitude as a sum over uncorrelated direct channel resonances (for amplitudes which are non-exotic in the direct channel and not dual to Pomeron exchange).

An immediate consequence of ii) is that the branching ratios into different channels are the same as for the decay of an average resonance with the appropriate mass and quantum numbers. These branching ratios for an average resonance, including the volume factor needed to compare two-body and n - body channels, also appear in building up the statistical bootstrap spectrum [3]. Thus the volume used in our modified Fermi model must be the <u>same</u> as the volume used in determining the parameters of the statistical bootstrap spectrum [15], such as the coefficient b of the exponent in Eq. (3).

The modified Fermi model as described thus far carries one only through the first generation decay of the intermediate resonant states. Most of the channels at this stage include unstable resonances from the Hagedorn spectrum, which will subsequently undergo further decay in a chain down to the metastable particles such as $\pi$, K, and N. To obtain predictions allowing direct comparison with data, all generations in the decay chain must be treated by the same statistical methods as the first generation.

To which reactions can the modified Fermi model apply ? It cannot give the dominant term in elastic reactions (or in a reaction such as $\pi^- p \to \pi^0 n$ whose amplitude can be written as a linear combination of elastic amplitudes), for the direct channel resonance terms add coherently in this case. Likewise it cannot give the main term in any reaction well above threshold if that reaction exhibits forward or backward peaks, for such peaks again imply a coherence among contributions of different direct channel resonances which lies outside the purely statistical approach. In addition, we shall not attempt to apply it at total centre-of-mass energies less than, say, 1 GeV for $B=0$ channels or 1.5 GeV for $B=1$ channels, where the small number of states does not fully merit statistical treatment.

There remains the possibility of application at sufficiently high $E_{CM}$ to inelastic reactions near their threshold (i.e., up to the onset of forward or backward peaks). A necessary criterion for the Bohr model to give the dominant term in nuclear physics is that the compound nuclear lifetime must be greater than or at least equal to the nuclear relaxation time (i.e., the collision time for a nucleon inside the nucleus). Otherwise the independence of resonance decay from resonance formation can hardly be established. In hadron physics both times are of order $10^{-23}$ sec; one is just at the edge of the region of applicability. Thus the justification of the Bohr model cannot be as cleancut as it was for nuclei. The same difficulty, stated in the form $\Gamma \approx m_\pi$ , plagues attempts to study fluctuations in the hadronic statistical model, by making it difficult to obtain a good statistical sample of non-overlapping resonances (Section 3) or to separate Ericson fluctuations from other effects (Section 6). Nevertheless, it seems worth while to try the model in view of the simple predictions it provides for reaction rates near threshold (where most other popular models fail) and the fresh viewpoint it brings to the study of fluctuations at intermediate energies. It is likely that the statistical mechanism contains a large share of truth even if the long-lived states which make it clearly distinguishable from competing effects in nuclei are lacking for hadrons.

At first sight the list of reactions to which the Fermi model could apply - mainly inelastic reactions near threshold - may not seem especially impressive. However, in the statistical bootstrap model most of the reactions at any given centre-of-mass energy are only about $E_{kin} \approx m_\pi c^2$ above their threshold. Thus the Fermi model may apply to a large fraction of reactions at each energy, even though few of these reactions happen to be well-explored experimentally, except for $\bar{N}N$ annihilation.

A limitation to our approach is that only amplitudes which are non-exotic in the direct channel and not dual to Pomeron exchange can be represented as sums over direct channel resonances. One expects intuitively that the Fermi model, if valid at all, should also apply to exotic channels. Of course the model can simply be applied without using direct channel resonances, but then one cannot relate the "interaction volume" to the parameters of the spectrum as we did for non-exotic channels, and the mechanism (if any) for fluctuations would also seem to be different.

Another kind of limitation on the predictive capacity of the modified Fermi model is introduced by the fluctuations discussed in Sections 3, 5 and 6. If we consider a number of reactions at a given energy (in the overlapping resonance region, let us say) the various amplitudes will differ somewhat from the average value given by the Fermi model, due to the Ericson fluctuations at that particular energy. When only a single energy is observed, as in $N\bar{N}$ annihilation at rest, the fluctuations are not directly observable, but are nonetheless expected to be present and serve as a criterion for what accuracy can be expected in fitting the Fermi model to experiment. The scale of the fluctuations expected for decay of a resonance into a particular final state i (such as $\pi^+\pi^-$) is set by the density of states $dn_i/dE \equiv \rho_i$ in that channel, multiplied by the natural width of the decaying resonance :

$$N_i = \Gamma \rho_i \tag{4}$$

The fluctuations in an amplitude are of order $N_i^{-\frac{1}{2}}$; the more probable the final state the less the fluctuations.

$N\bar{N}$ annihilation at low energies is the reaction offering the most extensive and accurate data in a region where the modified Fermi model may apply. Hamer [4] has applied the model in detail to the case of annihilation at rest; the reader is referred to his paper for a fuller account of methods of calculation and interpretation of the effects of fluctuations.

## 5. ERICSON FLUCTUATIONS IN NUCLEAR PHYSICS

Since Ericson fluctuations are not particularly well known among high energy physicists, we shall begin by describing what they look like and how they are analyzed in nuclear physics [5].

The general idea is that at energies high enough for resonances to overlap, one still sees peaks and dips. These are attributed not to individual resonances, but to fluctuations in the number and coupling strength of the overlapping resonances. This is a natural suggestion in view of the fact that fluctuations in partial width and level spacing were observed directly in the lower energy region of separable resonances (Section 3).

The idea is most simply applied to reactions like those discussed in Section 4, for which the Bohr model is valid. But it can be applied even in the presence of "direct reactions" where the main part of the amplitude is not given by the Bohr model, and we shall look into this more general case in preparation for some of the applications we have in mind for hadrons.

Consider for example an elastic reaction such as

$$p + Z \to p + Z \,. \tag{5}$$

The imaginary part of the non-flip amplitude is expected to dominate. Write it as a sum over direct channel resonance contributions :

$$\begin{aligned} \mathrm{Im}\, A_J^{el} &= \mathrm{Im} \sum_n \frac{\gamma_n \gamma_n}{-E + E_n - i\frac{\Gamma_n}{2}} \\ &\simeq \frac{2}{\langle\Gamma\rangle} \sum_n^{N_J} \gamma_n^2 \end{aligned} \tag{6}$$

where $N_J$ is the number of "overlapping resonances", i.e., the resonances in an interval $\Delta E \simeq \Gamma$. Since we have chosen an elastic amplitude, the contributions of individual resonances all add up with the same sign. Nevertheless, from one energy interval $\Delta E \simeq \Gamma$ to the next there will be statistical fluctuations in the strength of $\sum \gamma^2$, of relative order $1/\sqrt{N_J}$. Thus $A_J^{el}$ can be expressed as the sum of a dominant (mainly imaginary) coherent term and a smaller fluctuation term,

$$A_J^{el} = A^C(J) + A^F(J) \,. \tag{7}$$

From its relative magnitude one can show that the average of $A^F(J)$ over fluctuations is just the Bohr amplitude ("compound elastic scattering") for this particular partial wave. Adding all partial waves, one obtains

$$A^{el} = A^C(\theta) + A^F(\theta) \,. \tag{8}$$

Again the average over energy $\langle A^F(\theta)\rangle$ is the Bohr amplitude - not dominant in elastic scattering, but still unavoidably present. At $\theta = 0^\circ$ where all partial waves add coherently, $A^F/A^C$ is expected to be of order $1/\sqrt{N}$, with [25]

$$N = \sum_J N_J \simeq \Gamma\rho . \tag{9}$$

The dominant term $A^C(\theta)$ will exhibit the usual sharp diffraction peak, possibly with diffraction minima at angles determined by the nuclear radius. This structure varies only slowly as a function of energy. By contrast $A^F$ varies rapidly, on a scale $\Delta E \simeq \Gamma$. One looks for these fluctuations in <u>interference</u> with the dominant coherent term. The search is aided by the angular dependence of $A^F$, which is on the average weak and symmetric about $90^\circ$ because partial waves do not add coherently in $A^F$ (recall our discussion of the Fermi model in Section 2). What angular dependence $\langle A^F\rangle$ does have comes essentially from the envelope of the Legendre polynomials which is less at $90^\circ$. Thus $A^F$ is relatively easier to see at large angles where $A^C$ has fallen far below its peak value.

A classic example is the reaction

$$p + {}^{56}Fe \longrightarrow p + {}^{56}Fe \tag{10}$$

which has been studied [26] over the range $E_p = 9.3$ to $9.6$ MeV at intervals of 2 to 5 keV, at each of several angles between $63^\circ$ and $171^\circ$. The very close spacing of points in energy is necessary to resolve the Ericson fluctuations, which have a width comparable to the average resonance width $\Gamma \approx 3$ keV. The data on $d\sigma/d\Omega$ are shown in Fig. 1. At fixed energy, the dependence on angle is characteristic of a diffraction peak with minima. But at fixed angle, the cross-section exhibits rapid Ericson fluctuations as the energy is varied.

This example illustrates several points :

i) not every dip or peak is an Ericson fluctuation. In the example, the two large dips in the angular distribution are identified as diffraction minima rather than Ericson fluctuations because they persist at all energies.

ii) it is in fact hard to distinguish Ericson fluctuations in the angular dependence at a single energy. The average width $\Delta\theta$ of a fluctuation peak or dip cannot be much less than $\Delta\theta \simeq 180^\circ/\ell_{max}$ where

$\ell_{max} \simeq pR$ is the largest strongly scattered partial wave. In many cases this is not much smaller than the spacing between diffraction dips in $A^C$.

iii) at fixed $\theta$ or $t$ one sees the Ericson fluctuations more easily. Their typical width $\Delta E$ is of order (is in fact a measure of) $\Gamma$; in the example above an interval of $\simeq 100\,\Gamma$ was surveyed, allowing many fluctuations to be seen.

As we have just seen, neither the $\Delta\theta$ nor the $\Delta E$ of a fluctuation depends on the strength of $A^F$. It is the relative height of the fluctuations which is sensitive to $|A^F(\theta)|$, and thus to $N = \Gamma\rho$. Nuclear physicists have devised the following quantitative measure [5], to be applied at fixed $\theta$ or $t$. One measures the differential cross-section, denoted by $\sigma(E)$, at evenly spaced intervals over a range $E_1 \leq E \leq E_2$. The average $\langle\sigma\rangle$ is formed over this range. Then one forms the normalized correlation function

$$C_{exp} = \left\langle \frac{(\sigma(E) - \langle\sigma\rangle)^2}{\langle\sigma\rangle^2} \right\rangle \tag{11}$$

as a measure of fluctuations. Theoretically, if $A^C(\theta)$ is essentially constant over the range of energies considered, $\langle\sigma\rangle$ can be expressed as

$$\begin{aligned} \langle\sigma\rangle = \langle |A^C(\theta) + A^F(\theta)|^2\rangle &= |A^C(\theta)|^2 + \langle |A^F(\theta)|^2\rangle \\ &\equiv \sigma^C + \sigma^F \end{aligned} \tag{12}$$

since the interference term averages to zero. In terms of these quantities, one can show that [5]

$$C_{th} = \frac{(\sigma^F)^2 + 2\sigma^F\sigma^C}{(\sigma^F + \sigma^C)^2} \tag{13}$$

(note the limits $C = 2\sigma^F/\sigma^C$ for large $\sigma^C$ and $C = 1$ for vanishing $\sigma^C$). Comparing Eqs. (11) and (13), one deduces $\sigma^F/\sigma^C$ from the data. With the aid of Eq. (12), $\sigma^F$ and $\sigma^C$ can also be determined separately. From the determination of $\sigma^F/\sigma^C$ at $\theta = 0^\circ$ one deduces

$$N^{-1} \simeq \sigma^F(0^\circ)/\sigma^C(0^\circ) \tag{14}$$

[if $\sigma^F/\sigma^C$ is small and hard to find directly at $\theta = 0^\circ$, it suffices to make the easier determination at large $\theta$ and use the slow variation of $\sigma^F$ with $\theta$ and its symmetry about $90^\circ$ to estimate $\sigma^F$ at $0^\circ$].

A further analysis [5] of the width of the fluctuations also determines $\Gamma$. The level density is given [Eq. (9)] by $\rho \approx N/\Gamma$. Thus Ericson fluctuations provide an estimate of the level density at rather high energies where a direct count of levels is not possible.

Although our example involved elastic scattering, the analysis of Eqs. (11)-(14) is general and also applies to other types of direct reaction such as meson exchange, baryon exchange, etc., as well as to cases where no coherent term is present at all ($\sigma^C = 0$). The analysis can be simplified if $\sigma^C = 0$ [5]. Note that while $\langle\sigma\rangle$ can be greater than or equal to $\sigma^F$ depending on the relative strength of direct reactions, it cannot be less.

## 6. ERICSON FLUCTUATIONS IN HADRON PHYSICS

Historically, Ericson fluctuations were searched for by Allaby et al. [27], who studied

$$p + p \rightarrow p + p \tag{15}$$

at 16.9 GeV/c at centre-of-mass angles $67^\circ$ to $90^\circ$. Their result was negative : the differential cross-section was very smooth. Another relevant experiment was that of Akerlof et al. [28], who studied reaction (15) at fixed $\theta_{CM} = 90^\circ$, varying E in small steps. Once again, no evidence for fluctuations was found : the cross-section fell in a very smooth exponential fashion, with occasional changes in slope which appear to be systematic features extending to other angles as well [28]. These negative results discouraged further searches for Ericson fluctuations in hadron physics.

On the other hand, the motivation given in Section 5 for Ericson fluctuations is very general and should apply also to hadrons. But not necessarily to all hadron reactions; if the pp channel lacks resonances, no fluctuations need occur there ! So the early searches may simply have had the bad luck to look in the wrong place.

It is important then to continue the search for Ericson fluctuations, because :

i) in principle if one believes in a statistical model at all, it must be possible to find fluctuations. Ericson [5] has stressed that fluctuations occur in a huge range of phenomena from radio-emission by galaxies to nuclear reactions; why should hadron reactions be different ?

ii) the study of fluctuations could provide a practical tool giving information on $\rho(E)$ at energies where resonances overlap and a direct count of level densities cannot be made. This approach would supplement the only existing method - Hagedorn's interpretation of the $p_\perp$ and heavy pair distributions in terms of the boiling off of hadrons.

Thus motivated, we proceed to look for fluctuations in channels where resonances exist. We shall borrow the whole mathematical apparatus of the nuclear analysis, always remembering that it will work much more badly for hadrons because whereas $\Gamma_{\text{nuclear}}$ was very small on the scale on which the nuclear level density varies ($\approx$ 1 MeV), $\Gamma_{\text{hadron}} \approx m_\pi$ is the same as the scale on which the hadron level density varies.

As a practical matter, one has the choice of studying reactions such as backward $K^-p \to K^-p$ where $A^C$ is small (here we expect large fluctuations but the data is usually rather poor), or reactions such as $\pi p \to \pi p$ where $A^C$ is large (smaller fluctuations, but better data). We shall begin with the case of large $A^C$.

## 6.1 $A^F$ in interference with large coherent terms

The best studied reaction with abundant direct channel resonances is

$$\pi + p \longrightarrow \pi + p \, . \tag{16}$$

The coherent term $|A^C|^2$ is dominant, at least near the forward and backward directions, and is therefore easy to estimate from the experimental cross-section.

To obtain a crude theoretical estimate of $A^F$ we need to know the number of overlapping resonances [25]

$$N(m) \simeq \Gamma(m)\, \rho(m) \, . \tag{17}$$

For $\Gamma$ we take

$$\Gamma(m) = m_\pi , \tag{18}$$

which while not exact is certainly of the right order of magnitude. For the $m$ dependence of $\rho$ we take :

$$\rho(m) = \frac{c}{m^{7/2}} e^{m/kT_0} . \tag{19}$$

This is the Hagedorn form [Eq. (3)] with $a = -3$ [15)-16)] and with an extra $m^{-\frac{1}{2}}$ which occurs [29)] because while $\rho^{tot}$ involves a sum over all $J_z$, only that fraction of intermediate states with $J_z = J_z(\text{initial})$ contributes to the amplitude. For the numerical parameters in (19) we take Hagedorn's value [2)]

$$kT_0 = 160 \text{ MeV} \tag{20}$$

and

$$c = \frac{m_\pi^{5/2}}{3} \tag{21}$$

which is roughly consistent with the numerical studies of Hamer and Frautschi [15)].

If the elastic amplitude were made up entirely of direct channel resonances, one would estimate [25)]

$$|A^F(0°)| \simeq \frac{\text{Im}\, A^C(0°)}{\sqrt{N}} \tag{22}$$

in which case

$$\frac{d\sigma^F}{dt}(0°) = \frac{1}{N} \frac{d\sigma^C}{dt}(0°) \ x^2 \tag{23}$$

with $x^2$ of order 1. However, according to Freund [30)] and Harari [31)], only a fraction

$$y = \frac{\sigma^{tot}(E) - \sigma^{tot}(\infty)}{\sigma^{tot}(E)} \tag{24}$$

of the forward amplitude couples to direct channel resonances. So the amplitude $A^F$ must be multiplied by this small fraction and, in what follows, we shall use $x^2 = y^2$. Note that $y$ is essentially zero (flat $\sigma^{tot}$) for exotic channels such as pp and $K^+p$, leading to the expectation that no fluctuations occur in $pp \to pp$ and $K^+p \to K^+p$.

There is accurate elastic $\pi N$ data of the type we require ($E$ varied over a considerable range in small, equally spaced steps at fixed $\theta$ or momentum transfer) only at $0^o$ and $180^o$. At $0^o$ the data are in the form of total cross-section measurements; for the purposes of our analysis we express these data as $\mathrm{Im}\, A(0^o)$ by means of the optical theorem and square to form an effective $d\sigma/dt$ :

$$\text{"}\frac{d\sigma}{dt}(0^\circ)\text{"} \equiv |\mathrm{Im}\, A(0^\circ)|^2 = \frac{(\sigma^{tot}(E))^2}{16\pi} . \tag{25}$$

We use only the data for $E_{CM} \geq 2$ GeV; our analysis in terms of resonances overlapping in each $J^P$ state cannot be applied at energies much lower than this. The data [32)-36)] for $\pi^- p \to \pi^- p$ and $\pi^+ p \to \pi^+ p$ are displayed in Figs. 2-4.

Also displayed is $d\sigma^F/dt$, calculated at $0^o$ by means of Eqs. (17)-(24). We recall that $d\sigma^F/dt$ takes the same value at $180^o$ as $0^o$, and we give it the same value for $\pi^+ p$ and $\pi^- p$ elastic scattering [37)]. Thus $d\sigma^F/dt$ is the same for all cases in Figs. 2-4.

The peaks and dips in the data of Figs. 2-4 are traditionally interpreted in terms of individual (high $J$) resonances interfering with a smooth background. We are proposing a somewhat different interpretation in terms of numerous overlapping resonances (quite possibly including the ones usually suggested) with fluctuating level density and coupling strength. To show whether the peaks and dips are commonly due to fluctuations in more than one $J^P$ state, as required by our interpretation, comparable data at angles other than $0^o$ and $180^o$ will be needed. Lacking such information, the best we can do in the present paper is to show that our interpretation is consistent with the existing data.

It is immediately clear from Figs. 2-4 that our interpretation is qualitatively consistent with the data. Through interference terms, the fluctuations in $d\sigma/dt$ are expected to be of order 10% when $d\sigma/dt = 10^2 d\sigma^F/dt$, and of order 1% when $d\sigma/dt = 10^4 d\sigma^F/dt$. The experimental fluctuations are indeed of this order of magnitude; they are greater for the smaller cross-sections and die away more slowly for $d\sigma(180^{\circ})/dt \sim s^{-2.5}$ than for "$d\sigma(0^{\circ})/dt$" $\sim$ constant. To cite a further example, $\sigma^{tot}$ appeared smooth above lab. momentum $p = 3$ GeV/c (as expected from the $10^4 d\sigma^F/dt$ curve in Figs. 2 and 3) when measured with 1% statistical accuracy [38], but exhibited additional small peaks in $\sigma^{tot}(\pi^+ p)$ at $p = 3.77$ GeV/c and $\sigma^{tot}(\pi^- p)$ at $p = 3.24$ GeV/c when measured with 0.1% statistical accuracy [39]. This example illustrates an important point : even though fluctuations are especially small in $\sigma^{tot}$, it is a good place to look for them because it can be measured far more accurately than any differential cross-section.

To show that our interpretation is quantitatively consistent with the data, we study the correlation function. We need a slight generalization of Eqs. (11)-(13) to the case where the non-fluctuating part of the cross-section varies strongly with E. The appropriate generalization taken from nuclear physics [40)-42)] is

$$C_{exp} = \left\langle \left( \frac{\sigma(E) - \sigma^S(E)}{\sigma^S(E)} \right)^2 \right\rangle \tag{26}$$

where $\sigma^S(E)$ is a smooth curve drawn through the data. The curve is subjective, but constrained to have the same average as $\sigma(E)$. In practice we always use a simple monotonically decreasing form for $\sigma^S(E)$. The resulting values of $C_{exp}$ for elastic $\pi^- p$ and $\pi^+ p$ scattering, backward and forward, are displayed in Table II.

Also displayed in Table II are theoretical values for the correlation function. These are computed using the appropriate generalizations of Eqs. (12) and (13),

$$\sigma^S(E) = \sigma^C(E) + \sigma^F(E) \tag{27}$$

and

$$C_{th} = \left\langle \frac{(\sigma^F(E))^2 + 2\,\sigma^F(E)\,\sigma^C(E)}{(\sigma^F(E) + \sigma^C(E))^2} \right\rangle . \tag{28}$$

As input we take the theoretical estimate of $\sigma^F(E)$ given by Eqs. (17)-(24) and the experimentally determined $\sigma^S(E)$.

The agreement in Table II between $C_{th}$ thus computed and $C_{exp}$ is quite good; certainly as good as could be expected in the available energy range [43]. The important result here is not the agreement in over-all magnitude, which could easily be adjusted by varying the imperfectly known parameters c, $T_o$ and $\Gamma$. Rather it is the agreement with the non-adjustable features of $C_{th}$ - that $C_{th}$ is less for larger $d\sigma/dt$, and decreases rapidly with energy - which establishes the consistency of our interpretation.

The average width $\Gamma$ can also be estimated from the data, either by a correlation function method [5], or by the number of maxima per energy interval [5], or crudely from the width of an average fluctuation. The results are not consistent enough to yield a definite value for $\Gamma$, but agree on placing it in the range $\Gamma \lesssim 175$ MeV.

To distinguish our interpretation from one in terms of individual resonances, comparable data are needed at intermediate angles. An individual resonance interfering with a smooth background gives a definite angular dependence, whereas in the fluctuation picture the peaks and dips in the energy dependence at intermediate angles are not simply correlated to those at $0^o$ and $180^o$. With regard to repeating the fluctuation analysis at intermediate angles, the following practical points are worth noting :

i) data at fixed t or u are preferable to data at fixed $\theta$ because $A^C$ varies much less rapidly at fixed t or u.

ii) to analyze the data, one needs the dependence of $d\sigma^F(\theta)/dt$ on $\theta$, which is related to the J dependence of the level density. The J dependence in the statistical bootstrap model is not known uniquely at present [44], but we can obtain a crude estimate of $d\sigma^F(\theta)/dt$ by assuming the fluctuation amplitude is the same for all $\ell < Rp_{CM}$ and zero for $\ell > Rp_{CM}$ where R is the interaction radius. Neglecting spin, we write [45]

$$\frac{d\sigma^F}{dt}(\theta) = \frac{\sum_{\ell=0}^{\ell_{max}} P_\ell^2(\cos\theta)}{\sum_{\ell=0}^{\ell_{max}} 1}\,\frac{d\sigma^F}{dt}(0^o) \quad . \tag{29}$$

For example with the choice $\ell_{max} = p_{CM}/200$ MeV/c evaluated at laboratory momentum 5 GeV/c, we find the curve $d\sigma^F/dt$ shown in Fig. 5. Comparing this curve with the experimental points [46], we note that fluctuations should be especially large at intermediate angles where the experimental $d\sigma/dt$ has its minimum [47].

Another possible interpretation of the data is that the peaks and dips are the result of exciting "giant resonances" - resonances which have wave functions closely related to the ground state wave function, and are therefore connected to the ground state by especially large matrix elements. In nuclear physics, giant resonances are readily distinguished by several criteria :

i) they have the full single-particle width of order 1 MeV, instead of the much narrower width characteristic of most nuclear levels (Section 3).

ii) when studied with good resolution they break up into many narrow peaks; this is interpreted as a sharing of the mathematical state which has the large matrix element amongst many physical states.

Neither of these criteria would be likely to apply to giant hadron resonances, since normal hadron resonances already have a width of order $m_\pi$ [48]. Thus giant hadron resonances would look much like other resonances, except for a specially large branching ratio into the ground state, and one might interpret the peaks and dips in $\pi N$ scattering in terms of giant resonances. This interpretation can again be distinguished from the statistical picture by its simple prediction for the angular distribution (even if the "giant state" is shared among several physical resonances, they will all have the same $J^P$ and thus the same angular dependence). Another test is provided by the branching ratio into the ground state; here too the angular dependence comes in since interpretation of the branching ratio is sensitive to J.

Let us close this subsection by reviewing to what extent we have accomplished our original goals :

a) One goal was to use Ericson fluctuations as a practical tool for determining $\rho(E)$. It turns out that this tool is not very precise, even if the alternative interpretations of the data can be eliminated. The underlying difficulty is that not more than one fluctuation occurs in an energy interval over which $\rho(E)$ changes by 2 (i.e., $\Gamma \approx kT_0$, the same condition which made it difficult to study statistical fluctuations in the separable resonance region). A second problem is that the predicted $d\sigma^F/dt$ falls so fast that the fluctuations can be followed only through a relatively narrow energy range [for example, we estimate that the search of Allaby et al. [27] in pp scattering at the relatively high momentum 16.9 GeV/c would have found no fluctuations even if the pp channel had the same $d\sigma^F/dt$ as the $\pi p$ channel [].

b) The other objective, of establishing that Ericson fluctuations occur where required by the statistical model, has been partially attained. We have given a precise criterion for where they should be, and in the best available case the data are roughly consistent with our picture. There are competing interpretations of that data, but further tests are possible.

### 6.2 $A^F$ in the absence of coherent terms

We now turn to reactions with small $A^C$. Less data are available for such reactions but they are nevertheless important because of the prospect of especially large fluctuations when $\langle d\sigma/dt\rangle$ is near its minimum possible value $d\sigma^F/dt$.

To find examples of small $A^C$, we look at reactions where only exotic exchanges are present, such as

$$K^- + p \to K^- + p \tag{30}$$

and

$$\bar{p} + p \to \bar{p} + p \tag{31}$$

at small u. Here $A^C$ consists of Regge cut contributions plus possible exotic Regge pole exchanges, both of which may be quite small. Indeed, $d\sigma/du$ typically falls like $s^{-10}$ (or, as we shall suggest below, $e^{-b\sqrt{s}}$) in such cases.

For the particular case $K^-p \to K^-p$ at $u = 0$, Michael [49] has estimated that the Regge cut contribution is less than the experimental $d\sigma/du$ at laboratory momenta less than $\approx 5$ GeV/c. Above 5 GeV/c, the Regge cut term, falling only as $s^{-3}$ compared to the more rapid decrease of the lower energy cross-section, is expected to dominate. The existing data [50)-52] and Michael's estimate for the cut contribution are shown in Fig. 6.

Michael's interest was in finding the Regge cut term above 5 GeV/c. Our main interest is in the region below 5 GeV/c where this form of coherent term is relatively very small. Of course the data may still be dominated by $|A^C|^2 \sim s^{-10}$ corresponding to an exotic Regge pole exchange with $\alpha \approx -4$. But it is also possible that the dominant term is $|A^F|^2 \sim e^{-b\sqrt{s}}$, i.e., "compound elastic scattering" (the incoherent part of the sum over direct channel resonances) with a Hagedorn spectrum.

This possibility can be tested in several ways :

i) one can predict $d\sigma^F(180^\circ)/dt$ in the same manner as for $\pi N$ scattering, and compare it with the data. To carry out the prediction, we use the statistical model estimate of Hamer and Frautschi [15] that the level density is about the same for strange baryons as for nucleons,

$$\rho^{(m)}_{B=1,S=-1,Q=0} \simeq \rho^{(m)}_{B=1,S=0,Q=0} \quad . \tag{32}$$

The expected fluctuation level is somewhat different because the part of the total cross-section coupled to direct channel resonances $[\sigma^{tot}(E) - \sigma^{tot}(\infty)]$ and the kinematic relation between laboratory momentum and $E_{CM}$ change from $\pi^- p$ to $K^- p$ scattering. Calculating these well-defined effects, we estimate that

$$\frac{d\sigma^F}{dt}(K^- p \to K^- p) = 1.2 \frac{d\sigma^F}{dt}(\pi^- p \to \pi^- p) \tag{33}$$

at each value of $\theta$ and laboratory momentum. This estimate, as shown in Fig. 6, is quite close to the data.

ii) At a given energy, the angular dependence of $d\sigma^F(\theta)/dt$ [as estimated from Eqs. (29) and (33)] can be compared with the data to find the range of $u$ over which $d\sigma^F/dt$ is important. For example at 5 GeV/c (Fig. 7), $d\sigma^F(\theta)/dt$ is a large part of the experimental cross-section [52] from $u = 0$ to $u = -3$ GeV$^2$. At $p \leq 2.5$ GeV/c [50] the near equality of $d\sigma^F/dt$ with the data holds only at $180^\circ$; away from $180^\circ$ the experimental cross-section generally increases whereas $d\sigma^F/dt$ decreases.

iii) The crucial test would be to measure $d\sigma(180^\circ)/dt$ with good accuracy at small intervals over a broad range of energies, as in the backward $\pi^- p$ measurements of Kormanyos et al. [32], and look for fluctuations. Large fluctuations are expected in our theory since $d\sigma^F/dt$ is so close to $d\sigma/dt$. It must be admitted that no fluctuations are visible in the present data below 2.5 GeV/c, but closely spaced measurements over a broader range of momenta are needed to settle the question [43].

Similar comments should apply to a number of other exotic reactions such as backward $\bar{p}p \to \bar{p}p$ and backward $\bar{p}p \to \bar{K}K$. It would also be interesting to study in this spirit the low energy behaviour of certain non-exotic reactions such as $\bar{p}p \to \pi^+\pi^-$ and forward $\bar{p}p \to \bar{K}K$ which behave like $d\sigma/dt \sim s^{-10}$ below a certain $s$, after which $d\sigma/dt$ takes on the weaker $s$ dependence characteristic of Reggeized baryon exchange [53]. Note that if rapid variation of $d\sigma/dt$ over a range of low $s$ in these reactions indicates the dominance of $|A^F|^2$, this tends to justify Hamer's use of purely statistical analysis for $N\bar{N}$ annihilation at threshold, and suggests the extent of the energy range over which such an analysis would continue to give a good description.

## 6.3 Overview

Our general picture of reactions possessing direct channel resonances is :

i) isolated peaks at low centre-of-mass energies (single resonance region);

ii) fluctuations at intermediate energies (overlapping resonance region);

iii) smooth energy dependence of $d\sigma/dt$ at high energies (overlapping resonances so numerous that fluctuations are small). For channels with a high threshold, region i) will of course be absent.

Qualitatively this picture is quite conventional. What we have added is a definite quantitative criterion for the onset of region ii) in a partial wave amplitude (resonances with the same B,S,Q,J,... start to overlap in the Hagedorn spectrum) and a definite quantitative criterion for the dying away of fluctuations.

These criteria depend on a specific model for the level density - that of Hagedorn. Is it possible that our general ideas about fluctuations are correct but nature has chosen a different, more slowly growing level density ? The partial answer we can give at present is that the reactions we have analyzed in detail, $\pi^{\pm}p$ and backward $K^-p$ elastic scattering, cannot be understood in terms of fluctuations on a smaller number of overlapping resonances $N$. If alternative explanations apply (e.g., giant resonances in $\pi^{\pm}p$ scattering, exotic exchange in $K^-p \to pK^-$) then $d\sigma^F/dt$ is even lower and $N$ is even greater than our estimate.

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TABLE I : STATISTICAL MODELS

| Application | Nuclear | Hadron |
|---|---|---|
| 1) Level density | Bethe<br>$\rho(E) \propto \exp(\sqrt{AE/c})$ | Hagedorn, Frautschi<br>$\rho(m) \propto \exp(bm)$ |
| 2) Reactions (additional assumptions required) | | |
| a) pure statistical – rate $\propto$ phase space | Bohr | Fermi, Hamer ($N\bar{N}$ annihilation near threshold) |
| b) mixed – at higher energies, dynamical reaction followed by evaporation $\propto$ phase space | Serber, Le Couteur, Jackson | Hagedorn |
| c) purely dynamical – | "direct reaction" | "exchange reaction" |
| 3) Empirical study of level density at high energy | requires use of statistical models of reactions (boiling off of neutrons, Ericson fluctuations) | requires use of statistical models of reactions (boiling off of hadrons, perhaps Ericson fluctuations) |

TABLE II : ANALYSIS OF FLUCTUATIONS IN $\pi N$ SCATTERING

| Reaction | Momentum Range (p in GeV/c) | $C_{exp}$ | $C_{th}$ |
|---|---|---|---|
| $\pi^- p \to \pi^- p \quad (180^o)$ | 1.7 - 3.4<br>3.5 - 5.3<br>1.7 - 5.3 | 0.31<br>0.04<br>0.17 | 0.48<br>0.04<br>0.27 |
| $\pi^+ p \to \pi^+ p \quad (u=0)$ | 1.75 - 5.25 | 0.15 | 0.23 *) |
| $\frac{\sigma^{tot}(\pi^+ p) - \sigma^{tot}(\pi^- p)}{2}$ | 1.5 - 3.4<br>3.5 - 5.3<br>1.5 - 5.3 | 0.41<br>0.00<br>0.23 | 0.44<br>0.00<br>0.24 |
| $\frac{\sigma^{tot}(\pi^+ p) + \sigma^{tot}(\pi^- p) - 2\sigma^{tot}(\infty)}{2}$ | 1.5 - 3.9 | 0.02 | 0.02 |

*) One should use $d\sigma^F(u=0)/dt$ in estimating $C_{th}$ for this case, but in the absence of firm information on the J dependence of the spectrum we have used $d\sigma^F(180^o)/dt$. Thus our estimate is somewhat too high.

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FIGURE CAPTIONS

Figure 1 : $d\sigma/d\Omega$ for $p+{}^{56}Fe \rightarrow p+{}^{56}Fe$ measured in steps of 2-5 keV around 9.4 MeV [taken from Ref. 26)].

Figure 2 : Comparison of data on $\pi^- p \rightarrow \pi^- p$ at $180^\circ$ and $0^\circ$ with the theoretical cross-section $d\sigma^F/dt$ at which fluctuations would reach 100%. The experimental values for $d\sigma(180^\circ)/dt$ are from Ref. 32), "$d\sigma(0^\circ)/dt$" is calculated from data of Refs. 33) and 34) with the aid of Eq. (25), and $d\sigma^F/dt$ is calculated from Eqs. (17)-(24) of the text.

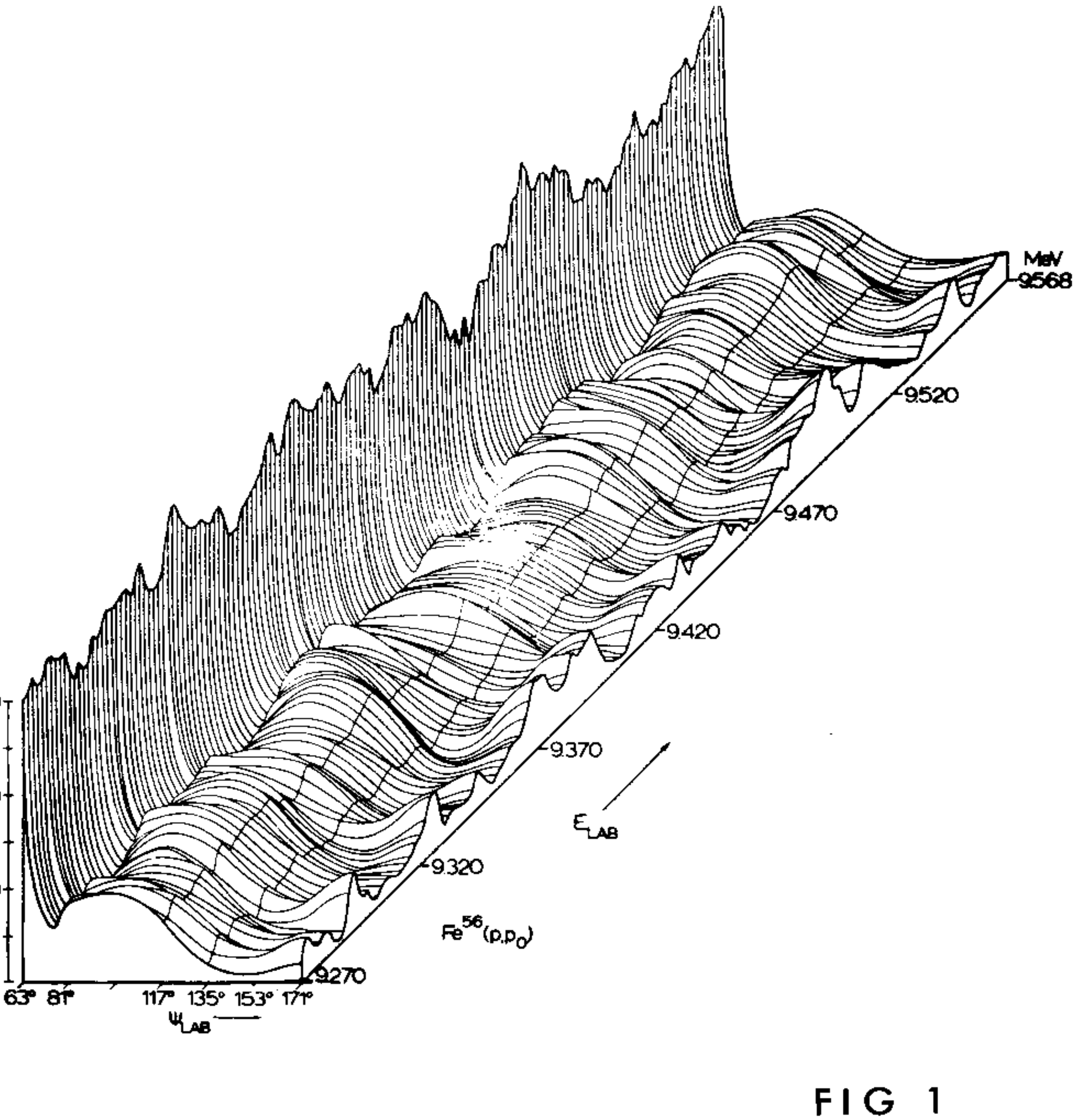
Figure 3 : Comparison of data on $\pi^+ p \rightarrow \pi^+ p$ at $u=0$ and $0^\circ$ with $d\sigma^F(0^\circ)/dt$. The experimental values for $d\sigma(u=0)/dt$ are from Refs. 35) and 36), and "$d\sigma(0^\circ)/dt$" is calculated from data of Refs. 33) and 34) with the aid of Eq. (25).

Figure 4 : $\pi N$ elastic scattering : comparison of "$d\sigma(0^\circ)/dt$" for $I=1$ exchange and $I=0$ Regge exchange [calculated from data of Refs. 33) and 34)] with $d\sigma^F/dt$.

Figure 5 : Comparison of the differential cross-sections for $\pi^- p \rightarrow \pi^- p$ and $\pi^+ p \rightarrow \pi^+ p$ at 5 GeV/c [Ref. 45)] with $d\sigma^F(\theta)/dt$ [estimated according to Eq. (29)].

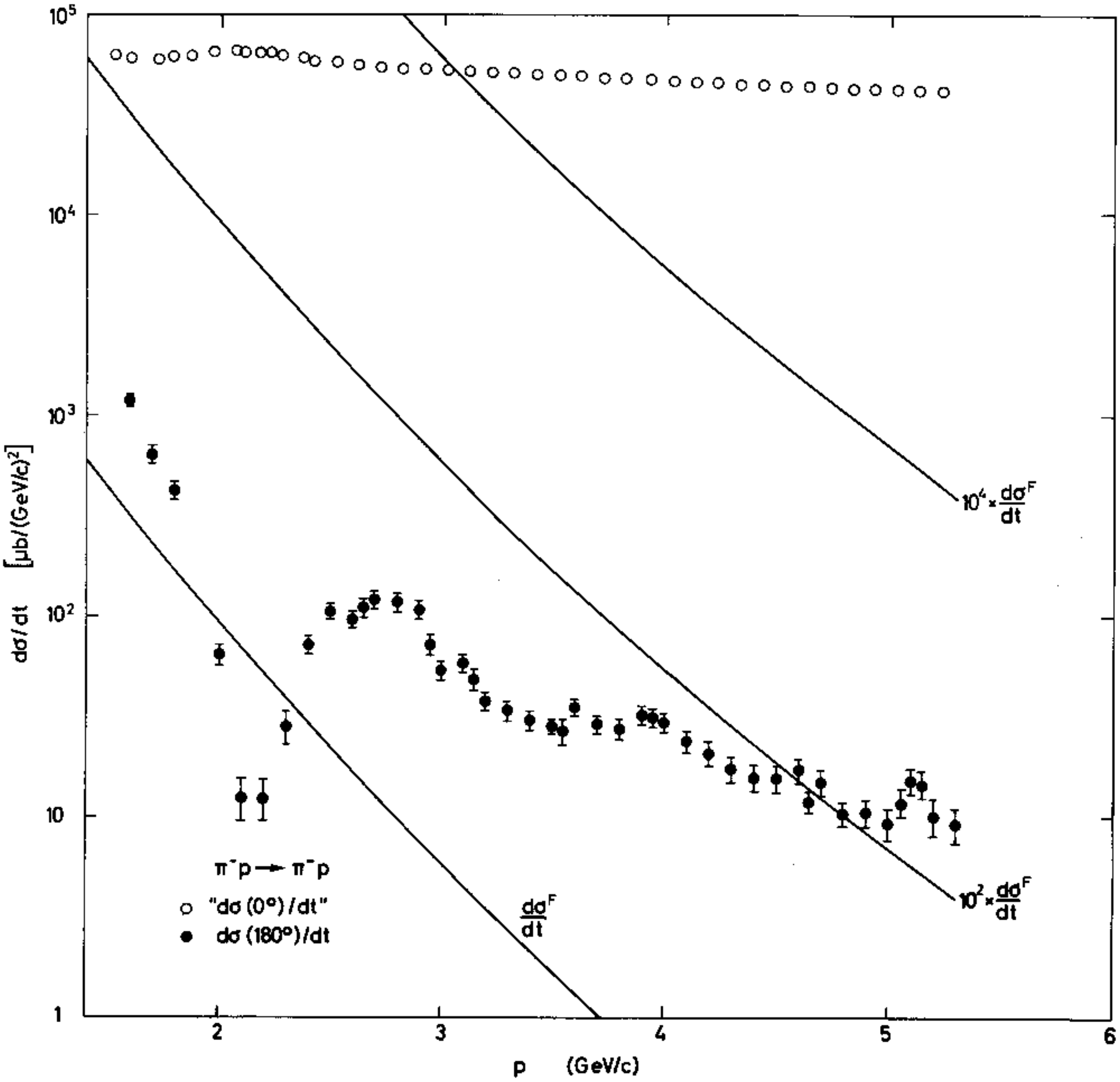
Figure 6 : Comparison of data on $K^- p \rightarrow K^- p$ at $180^\circ$ with $d\sigma^F/dt$. The data are from Refs. 49) ($p \leq 2.44$ GeV/c); 50) (3.55 GeV/c); and 51) (5.0 GeV/c). $d\sigma^F/dt$ is calculated from Eq. (33) of the text.

Figure 7 : Comparison of the differential cross-sections for $K^- p \rightarrow K^- p$ and $K^+ p \rightarrow K^+ p$ at 5 GeV/c [Ref. 51)] with $d\sigma^F(\theta)/dt$ [estimated according to Eq. (33)].



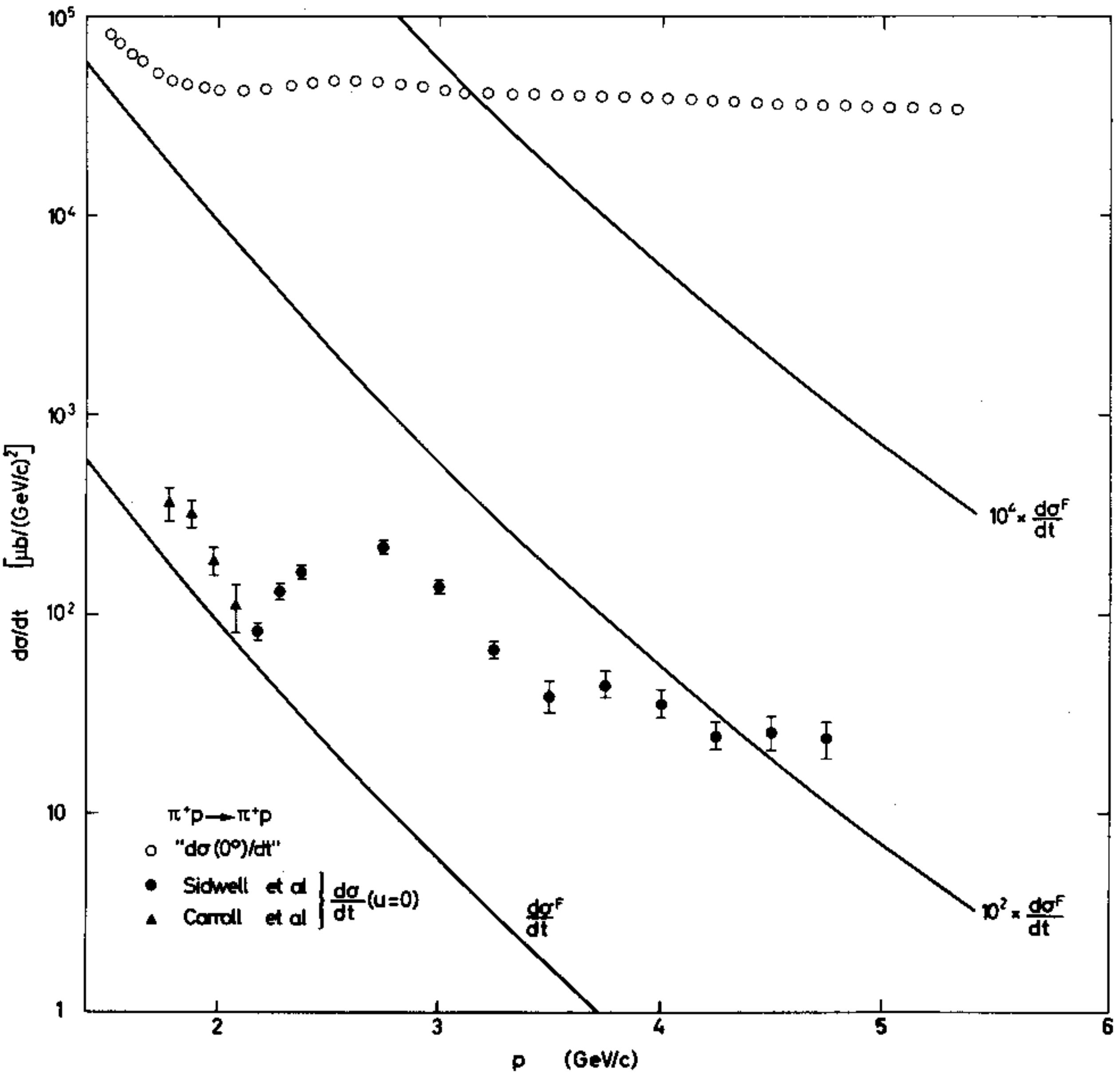
90
60
30
mb/sr
63° 81° 117° 135° 153° 171°
$\psi_{LAB}$
9.270
9.320
9.370
9.420
9.470
9.520
9.568
MeV
$Fe^{56}(p,p_0)$
$E_{LAB}$


FIG 1



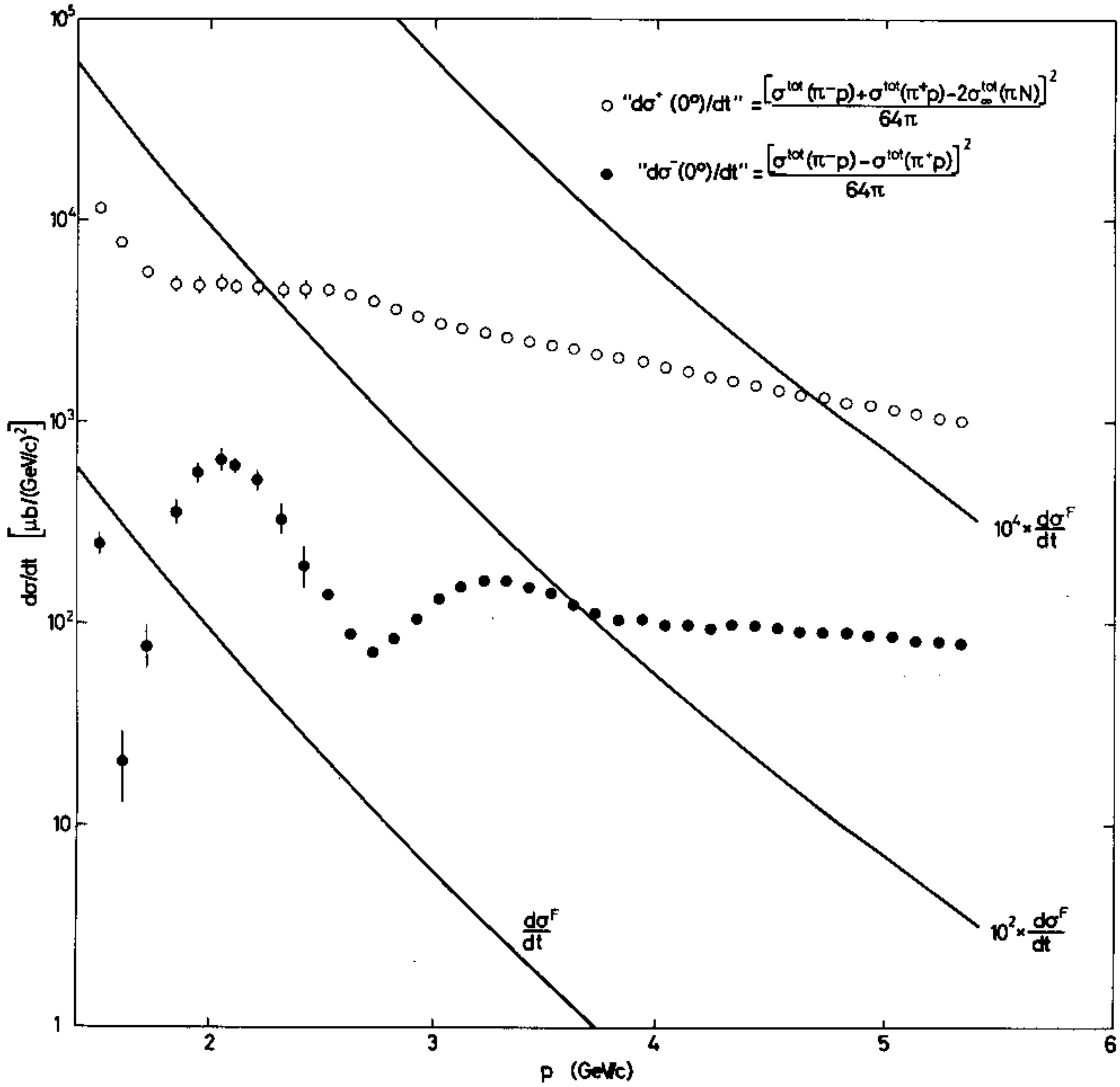
$10^5$
$10^4$
$10^3$
$10^2$
10
1
$d\sigma/dt$ $[\mu b/(GeV/c)^2]$
2
3
4
5
6
p (GeV/c)
$10^4 \times \frac{d\sigma^F}{dt}$
$10^2 \times \frac{d\sigma^F}{dt}$
$\frac{d\sigma^F}{dt}$
$\pi^- p \rightarrow \pi^- p$
○ "$d\sigma$ (0°)/dt"
● $d\sigma$ (180°)/dt


FIG 2



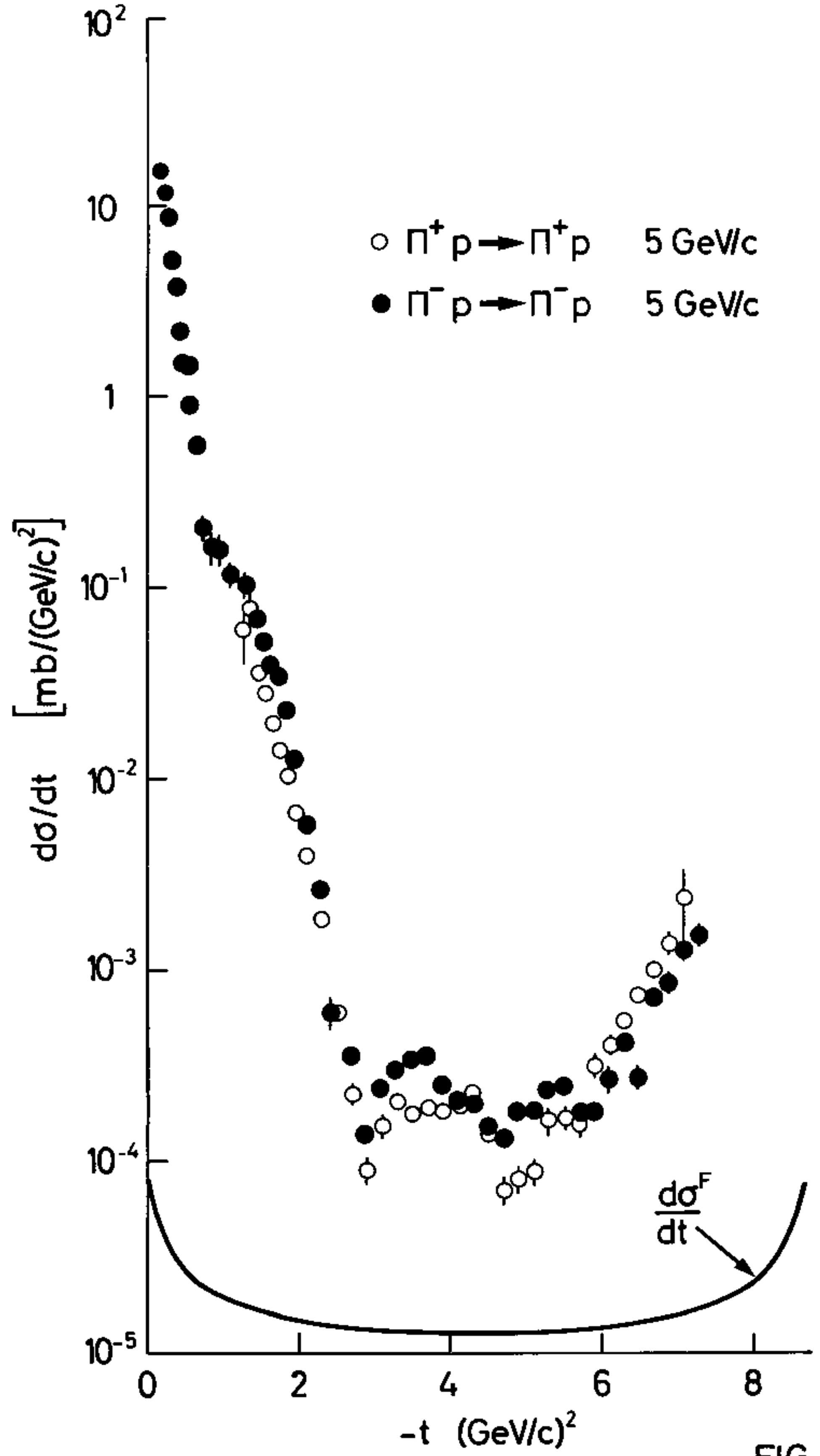
10^5
10^4
10^3
10^2
10
1
2
3
4
5
6
dσ/dt [μb/(GeV/c)^2]
p (GeV/c)
π+p → π+p
○ "dσ(0°)/dt"
● Sidwell et al
▲ Carroll et al
dσ/dt (u=0)
dσ^F/dt
10^2 × dσ^F/dt
10^4 × dσ^F/dt


FIG 3



$10^5$
$10^4$
$10^3$
$10^2$
10
1
2
3
4
5
6
p (GeV/c)
$d\sigma/dt$ $[\mu b/(GeV/c)^2]$
○ "$d\sigma^+(0°)/dt$" $= \frac{[\sigma^{tot}(\pi^- p)+\sigma^{tot}(\pi^+ p)-2\sigma^{tot}_{\infty}(\pi N)]^2}{64\pi}$
● "$d\sigma^-(0°)/dt$" $= \frac{[\sigma^{tot}(\pi^- p)-\sigma^{tot}(\pi^+ p)]^2}{64\pi}$
$10^4 \times \frac{d\sigma^F}{dt}$
$10^2 \times \frac{d\sigma^F}{dt}$
$\frac{d\sigma^F}{dt}$


FIG 4



$10^2$
10
1
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$d\sigma/dt$ $[mb/(GeV/c)^2]$
○ $\pi^+ p \rightarrow \pi^+ p$ 5 GeV/c
● $\pi^- p \rightarrow \pi^- p$ 5 GeV/c
$\frac{d\sigma^F}{dt}$
0
2
4
6
8
$-t$ $(GeV/c)^2$


FIG. 5

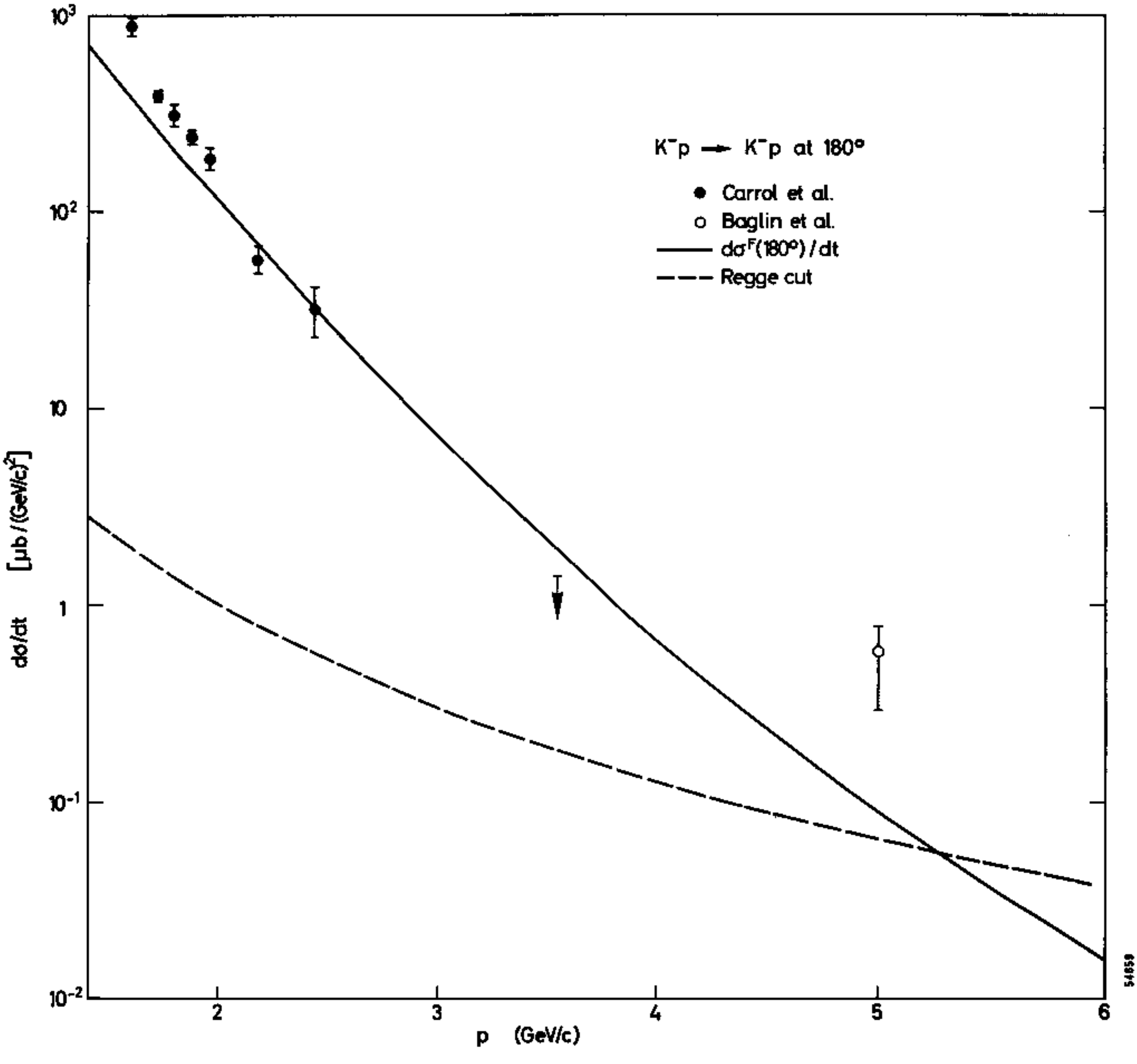
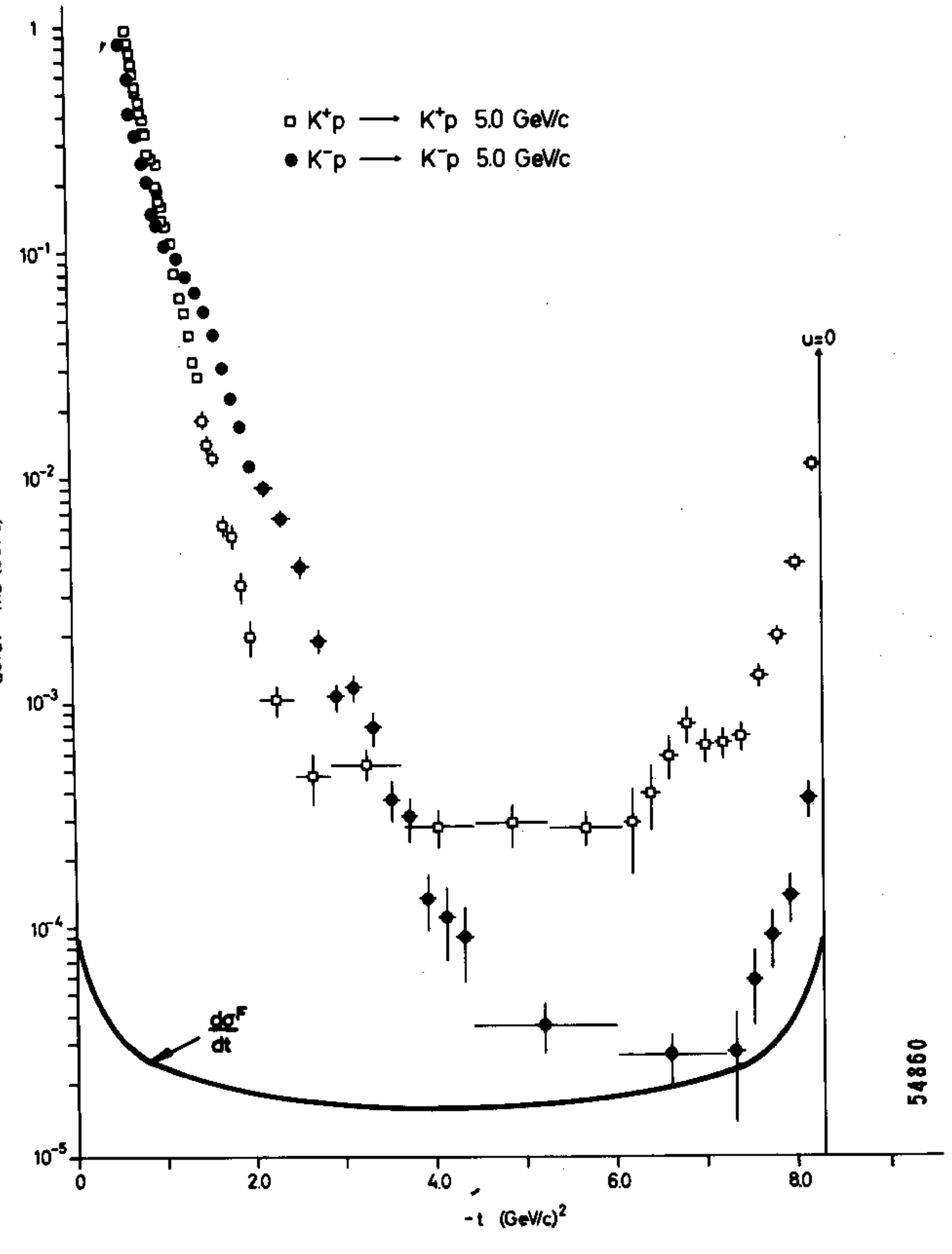

K⁻p → K⁻p at 180°
Carrol et al.
Baglin et al.
$d\sigma^F(180°)/dt$
Regge cut
$d\sigma/dt$ [μb/(GeV/c)$^2$]
p (GeV/c)
$10^3$
$10^2$
10
1
$10^{-1}$
$10^{-2}$
2
3
4
5
6
54859

FIG 6



□ K⁺p ⟶ K⁺p 5.0 GeV/c
● K⁻p ⟶ K⁻p 5.0 GeV/c
u=0
dσ/dt mb/(GeV/c)$^2$
$d\sigma^F/dt$
-t (GeV/c)$^2$
54860


FIG 7