Erratum: Compact elastic objects in general relativity [Phys. Rev. D 105, 044025 (2022)]

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In Sec. IV of this paper, when considering causal wave propagation within the material, we defined four velocities in spherical symmetry, namely the *longitudinal* and *transverse* waves in the *radial* and *tangential* directions (the later oscillating in the radial direction), denoted by $c^2_{\mathrm{L,T}}$ and $\tilde{c}^2_{\mathrm{L,T}}$, respectively. For such velocities, explicit formulas were given in Eqs. (15)–(18), and we stated that "Due to spherical symmetry, there is a single transverse mode in both the radial and tangential directions, which vanishes in the perfect-fluid case, since $\hat{p}_{\tan} = \hat{p}_{\mathrm{rad}}$."

However, the correct statement is that there are five independent velocities. The missing velocity corresponds to transverse waves in the tangential direction oscillating in the tangential direction, which we now denote by $\tilde{c}^2_{\mathrm{TT}}$.

Moreover, as discussed by Karlovini and Samuelsson [1], in general, $\tilde{c}^2_{\mathrm{L}}$ and $\tilde{c}^2_{\mathrm{TT}}$ can only be obtained from a stored energy function w without any symmetry assumptions. This means that spherically symmetric materials are not simply determined by a spherically symmetric stored energy function $\hat{w}(\delta,\eta)$ [equivalently, $\hat{\rho}(\delta,\eta)$], but also by giving $\tilde{c}^2_{\mathrm{L}}(\delta,\eta)$ and $\tilde{c}^2_{\mathrm{TT}}(\delta,\eta)$. In fact, from these two velocities only one needs to be specified, since, as shown in [2], they are related by

$$\tilde{c}^2_{\mathrm{L}}(\delta,\eta) - \tilde{c}^2_{\mathrm{TT}}(\delta,\eta) = \frac{\delta^2\partial^2_{\delta}\hat{\rho}(\delta,\eta) + 3\delta\eta\partial^2_{\eta\delta}\hat{\rho}(\delta,\eta) + \frac{9}{4}\eta^2\partial^2_{\eta}\hat{\rho}(\delta,\eta) + \frac{3}{4}\eta\partial_{\eta}\hat{\rho}(\delta,\eta)}{\hat{\rho}(\delta,\eta) + \hat{p}_{\tan}(\delta,\eta)}. \tag{1}$$

Furthermore, it is also shown in [2] that the simplest way to reconstruct a viable spherically symmetric elastic matter model consists precisely of the *natural choice* given by Eq. (17) for $\tilde{c}^2_{\mathrm{L}}(\delta,\eta)$, and that the corresponding value of $\tilde{c}^2_{\mathrm{TT}}(\delta,\eta)$ is then given by

$$\tilde{c}^2_{\mathrm{TT}}(\delta,\eta) = \frac{\frac{3}{2}\eta\partial_{\eta}\hat{p}_{\tan}}{\hat{\rho} + \hat{p}_{\tan}}. \tag{2}$$

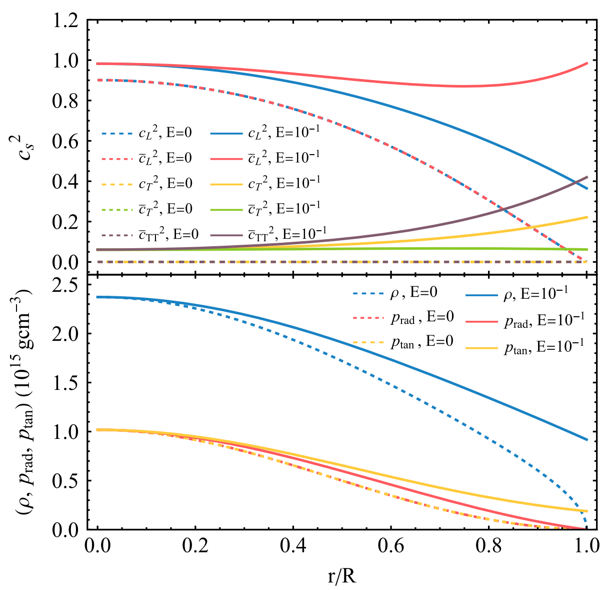

FIG. 1. Sound speeds (upper panel) and density and pressure profiles (bottom panel) for the quadratic elastic model with $n = 1/2$, $K = 6 \times 10^4 M^4_{\odot}$. We compare the perfect fluid case ($E = 0$, dashed lines) with an elastic configuration with $E = 10^{-1}$ (solid lines). The latter configuration features a light ring ($M/R \approx 0.35$) and the wave speeds are always subluminal. We include the previously missing sound speed velocity defined in Eq. (2).

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Overall, reality and causality require $0 \leq c^2_{\mathrm{L,T}} \leq 1$ and $0 \leq \tilde{c}^2_{\mathrm{L,T,TT}} \leq 1$.

Finally, it should be stressed that this correction in no way changes the conclusions of this article. In particular, the above conditions on the missing wave speed $\tilde{c}_{\mathrm{TT}}$ do not invalidate the highly compact elastic stars that were presented in this paper, as is shown in Fig. 1.

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[2] A. Alho, J. Natário, P. Pani, and G. Raposo, Spherically symmetric elastic bodies in general relativity (to be published).