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# Erratum: The late jet in gamma-ray bursts and its interactions with a supernova ejecta and a cocoon

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The paper 'The late jet in gamma-ray bursts and its interactions with a supernova ejecta and a cocoon' was published in Mon. Not. R. Astron. Soc. **403,** 229–245 (2010).

In section 3.3 of the paper, the thermal emission luminosity of the 'late cocoon' at its breakout, $L_{\rm th}$, was incorrectly estimated as the blackbody luminosity for the internal temperature of the 'late cocoon'. This was an overestimate because it did not include the photon diffusion process which will certainly happen inside the extremely optically thick cocoon. The correct estimate is provided here.

We first consider the generalized problem of the surface luminosity from an expanding sphere of gas and radiation. Later we apply the obtained analytical results about $L(t)$ – the evolution of the surface luminosity – to the case of the late cocoon. We will consider a homologous, subrelativistic expansion, meaning $v(r) \propto r$ where $r$ is the radius inside the sphere. We further assume the internal energy of the sphere is dominated by the radiation, and the accumulated radiative loss up to a given time $t$ is very small compared with $E_{\rm int}(t)$ – the total internal energy at $t$. These two assumptions should hold at least during the early phase of the cocoon expansion. Therefore, the adiabatic law for a photon gas applies, i.e. $E_{\rm int}(t) \propto R(t)^{-1}$, where $R(t)$ is the outer boundary radius of the sphere. Let $E_0$, $R_0$, $T_0$ and $\tau_0$ be the initial internal energy, radius, central temperature and optical depth of the sphere at $t = 0$, respectively. The sphere expands as $R(t) = (t_0 + t)v$, where $v$ is a constant and $t_0 = R_0/v$.

Arnett (1980; also see Arnett 1996) has developed an analytical model based on the diffusion of radiation from a homologously expanding gas sphere. Utilizing the photon diffusion approximation throughout the sphere, i.e. $L \propto -r^2 \partial T^4/\partial r$, his model provides a solution of $L(t)$ as in $L_{\rm rela}(t) = L_{\rm rela}(0)\phi(t)$, where $L_{\rm rela}(0) = E_0/t_{0,d}$, and $t_{0,d} \simeq \tau_0 R_0/c$ is the diffusion time-scale of the whole sphere at $t = 0$; the temporal behaviour of $L_{\rm rela}$ is described by

$$\phi(t) = \exp\left[-\frac{t}{t_{0,d}}\left(1 + \frac{t}{2t_0}\right)\right]. \tag{1}$$

Since $t_0 \ll t_{0,d}$, the decline of $\phi(t)$ steepens from being flat towards a Gaussian at $t = \sqrt{2t_0 t_{0,d}}$, which sets the characteristic time-scale for $L_{\rm rela}(t)$. In this model, the density structure is one of a series of eigenfunctions, which in turn determines the temperature structure. Note that the luminosity temporal behaviour $\phi(t)$ (equation 1) is insensitive to the choice of the eigenvalue. Among all the eigenfunctions, the one corresponding to a uniform density structure is the most interesting one and has been argued to be the 'relaxed' case the system tends towards on a hydrodynamic time $t_0$ from any initial structure. This relaxation is probable because the large radiative flux during $t \lesssim t_0$ accelerates the outer layers and thus gives a very non-homologous expansion after $t_0$ (Arnett 1980).

Earlier than $t_0$, we usually should expect a 'transient' pulse. This is due to both the facts that photons at the skin layer can easily diffuse out and that the adiabatic expansion has not cooled the temperature down. This transient lasts for the hydrodynamical time $t_0$, after which it transitions to the 'relaxed' solution. Its amplitude generally differs from $L_{\rm rela}(0)$ because it is determined by the *initial* density and temperature structures, while the 'relaxed' solution is insensitive to the initial structures. We estimate the transient luminosity in the following.

Analogously to that of a shocked SN envelope (e.g., Chevalier 1992; Matzner & McKee 1999), we consider that the gas sphere initially has a power-law density structure as in

$$\rho(r) \propto (1 - r/R)^{\theta}, \tag{2}$$

where $\theta > 0$ corresponds to centrally condensed, and $\theta < 0$ to a shell-like structure. The temperature structure is found by applying the constant entropy throughout the sphere, thus, $T(r) \propto (1 - r/R)^{\theta/3}$. The total mass is given by $M = 8\pi R^3 \rho(r = 0)/[(1+\theta)(2+\theta)(3+\theta)]$. The temporal behaviour of the density is due to the expansion, i.e. $\rho(t) \propto (1 + t/t_0)^{-3}$. The temporal behaviour of the temperature is given by the photon gas adiabatic law, i.e. $T(t) \propto (1 + t/t_0)^{-1}$. The total internal energy is $E(t) = E_0(1 + t/t_0)^{-1}$.

Consider a surface layer of thickness $s$ whose photon diffusion time is $t$, i.e.

$$\frac{s\tau(s)}{\lambda c} = t, \tag{3}$$

where $\tau(s) = \int_{R-s}^{R} \rho\kappa\, {\rm d}r = \tau_0 (s/R)^{(\theta+1)}$ is the optical depth of this layer. It can be solved to give

$$s = \left(\frac{ct R^{3+\theta}}{\tau_0 R_0^2}\right)^{\frac{1}{2+\theta}}. \tag{4}$$

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The internal energy contained in this surface layer is $E_{\rm rad}(t) = \int_{R-s}^{R} 4\pi r^2 a T(r,t)^4 {\rm d}r$. The integral can be carried out to give

$$E_{\rm rad}(t) = \frac{E_0}{18}\left(\frac{R}{R_0}\right)^{-1}\left(\frac{s}{R}\right)^{4\theta/3+1} \times\left[(4\theta+6)(4\theta+9) - 2(4\theta+3)(4\theta+9)\left(\frac{s}{R}\right) + (4\theta+3)(4\theta+6)\left(\frac{s}{R}\right)^2\right]. \quad (5)$$

For early times that we are interested in, $s/R \ll 1$, so

$$E_{\rm rad}(t) \simeq \frac{(4\theta+6)(4\theta+9)}{18} E_0 \left(\frac{R}{R_0}\right)^{-1}\left(\frac{s}{R}\right)^{4\theta/3+1}. \quad (6)$$

This portion of energy will diffuse out of the sphere during time $t$, so the diffusive luminosity is $L(t) \simeq E_{\rm rad}(t)/t$. Plugging in the expression for $s$ and the temporal dependence in $R(t)$, it gives

$$L(t) = \frac{(4\theta+6)(4\theta+9)}{18}\frac{E_0}{t_0}\left(\frac{c/v}{\tau_0}\right)^{\frac{4\theta+3}{3(\theta+2)}} \times\left[\frac{t}{t_0}\left(1+\frac{t}{t_0}\right)\right]^{-\frac{(3-\theta)}{3(\theta+2)}}. \quad (7)$$

In particular, the luminosity at $t = t_0$ is

$$L_{\rm tran}(t_0) = 2^{\frac{\theta-3}{3(\theta+2)}} \times \frac{(4\theta+6)(4\theta+9)}{18}\frac{E_0}{t_0}\left(\frac{c/v}{\tau_0}\right)^{\frac{4\theta+3}{3(\theta+2)}}, \quad (8)$$

which sets the amplitude of the early 'transient' signal. After $t_0$, the system adjusts hydrodynamically toward the 'relaxed' state corresponding to Arnett's solution, and the luminosity falls from the peak value $L_{\rm tran}(t_0)$ to the 'relaxed' value $L_{\rm rela}(0)$. This adjustment of the sphere structure occurs in a time-scale between $t_0$ and $t_{0,d}$. Fig. 1 collectively illustrates the overall evolution of $L$.

Note that the assumed initial structure of the sphere, most interestingly $\theta$, affects the 'transient' peak luminosity. For instance, as $\theta$ increases, $L_{\rm tran}(t_0)$ drops. This is because a more steeply decreasing density structure means less energy is contained in the outer layers of the sphere and can diffuse out within time $t_0$.

The most interesting case of the sphere's initial structure is the one in which both the density and the temperature are uniform in the sphere, not only because it has the simplest form but also because

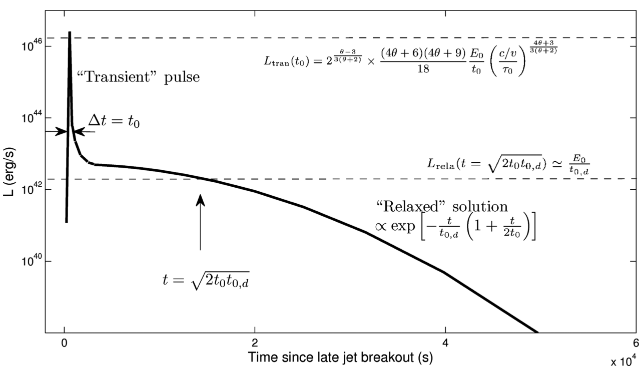


**Figure 1.** Illustration of the evolution of the bolometric luminosity from a hot, expanding cocoon. It is jointly described by a 'transient' solution for $t \lesssim t_0$ and a 'relaxed' solution afterwards. The parameter values that correspond to the curve shown here are $\tau_0 = 2 \times 10^7$, $E_0 = 1 \times 10^{50}$ erg, $R_0 = 10^{11}$ cm, $v = c/\sqrt{3}$ and $\theta = 0$. As $\theta$ increases, the peak luminosity of the 'transient' drops; when $\theta \approx 3$, it drops to the luminosity level of the 'relaxed' solution, thus becoming observationally indiscernible from the latter.

the 'transient' pulse can precede and easily stand out against the later, 'relaxed', diffusive emission. As regards to the late cocoon, the isothermal condition is probably viable because the cocoon was heated by the passage of the jet, hence the heating was done throughout the cocoon, from the rear end to the front end, which is different from the heating in a star where the heating source is at the centre and a temperature gradient is required by the diffusion and by the hydrostatic equilibrium equation.

For $\theta = 0$, i.e. uniform density structure, the peak luminosity of the 'transient' pulse is $L_{\rm tran}(t_0) \approx \sqrt{c/(\tau_0 v)}E_0/t_0$. When compared with the blackbody luminosity, using $E_0 = 4\pi R_0^3 a T_0^4/3$, one has $L_{\rm tran}(t_0) \approx 4\pi R_0^2 \sigma T_0^4/\sqrt{\tau_0 c/v}$. Therefore, the 'transient' peak luminosity is about $\sqrt{\tau_0 c/v}$ times smaller than the blackbody luminosity for the internal temperature of the sphere.

In the case of the late cocoon, assuming uniform density, the thermal emission luminosity at the late cocoon breakout is $L_{\rm tran}(t_0) \sim \sqrt{c_{\rm s} c/\tau_{\rm c}} E_{\rm c}/r_{\rm SN}$, where $c_{\rm s} \approx c/\sqrt{3}$ is the sound speed in the cocoon and is also the speed at which the broken-out late cocoon expands, $E_{\rm c}$ is the total energy of the late cocoon and is equal to the late jet luminosity times the jet penetration time of the supernova ejecta, $r_{\rm SN}$ is the supernova ejecta shell radius which is of the same order as the late cocoon size, and $\tau_{\rm c}$ is the late cocoon's initial optical depth. $\tau_{\rm c}$ depends on the mass of the late cocoon, $M_{\rm c}$, of which we do not have a firm knowledge; we can only infer $M_{\rm c}$ to be a small fraction of the SN ejecta mass. As a crude, order-of-magnitude, estimation we take $M_{\rm c} \sim 10^{-3}$–$10^{-1}{\rm M}_{\odot}$, which implies $\tau_{\rm c} \sim 10^7$–$10^{10}$. Plugging in numbers we find

$$L_{\rm tran}(t_0) \approx 3.3 \times 10^{45}\, L_{\rm j,49}^{1/2}\theta_{\rm j,-1} M_{\rm SN,1}^{1/2} M_{\rm c,-1}^{-1/2} r_{\perp,11} \times \Delta_{\rm SN,11}^{1/2} r_{\rm SN,11}^{-1}\ {\rm erg\ s^{-1}}. \quad (9)$$

Explanations of additional parameter symbols are provided in Shen, Kumar & Piran (2010).

The non-spherical shape of the late cocoon, e.g. elongated along the late jet axis, may change the number slightly but not by an order of magnitude. A radially decreasing density distribution in the cocoon will result in a lower $L_{\rm tran}(t_0)$. For a very steeply declining density distribution (e.g. $\theta > 3$), $L_{\rm tran}(t_0)$ may be so low that the transient signal would be observationally indiscernible from that of the 'relaxed' diffusive solution, and the latter would dominate the long time-scale signal with a luminosity of

$$L_{\rm rela}\left(t \leq \sqrt{2t_0 t_{0,d}}\right) \approx 2 \times 10^{41}\, L_{\rm j,49}^{1/2}\theta_{\rm j,-1}\Delta_{\rm SN,11}^{1/2} M_{\rm SN,1}^{1/2} \times r_{\rm SN,11} M_{\rm c,-1}^{-1}\ {\rm erg\ s^{-1}} \quad (10)$$

up to $t = \sqrt{2t_0 t_{0,d}} \approx 3 \times 10^5\ M_{\rm c,-1}^{1/2}$ s, after which it drops in a Gaussian.

Our previous estimation of the duration of this thermal transient, i.e. the time it takes for the bulk of the late cocoon to escape the ejecta, is still valid since that time is numerically equivalent to the time that the late cocoon takes to double its size.

In summary, while our conclusion that the late jet–SN ejecta interaction produces a thermal transient which peaks at X-ray band and lasts for $\sim$10 s is not changed, the bolometric luminosity of this transient is corrected to be $\sim 10^{45}$–$10^{46}$ erg s$^{-1}$. For a typical gamma-ray burst redshift of $z$ = 1–2, this luminosity translates to an observed flux of $\sim 10^{-13}$–$10^{-11}$ erg s$^{-1}$ cm$^{-2}$. With a sensitivity of $2 \times 10^{-14}$ erg s$^{-1}$ cm$^{-2}$ in $10^4$ s (Gehrels et al. 2004), the *Swift* X-Ray Telescope (XRT) may not be able to detect this thermal transient since the predicted flux is only marginally above the sensitivity whereas the duration of the transient is much shorter than the sensitivity-required integration time. The slow-varying signal of the 'relaxed' diffusive stage is less luminous ($L_{\rm rela} \sim 10^{41}$–$10^{43}$ erg s$^{-1}$)

and lasts longer ($t \sim 10^4$–$10^5$ s). This signal would have a typical flux $\leq 10^{-14}$ erg s$^{-1}$ cm$^{-2}$, thus has very little chance to be detected by XRT. These conclusions are consistent with the non-detection of a thermal emission component by XRT in early afterglows so far, particularly in those showing X-ray flares.

Our conclusions about the late jet–cocoon interaction are not affected by this correction.

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