

Erratum to “Feller Semigroups Obtained by Variable Order Subordination”

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Walter Hoh pointed out to us an obvious omission in the construction of the intermediate space $H^{p_{\lambda_0}}(\mathbb{R}^n)$ on page 299. In general, of course, the sesquilinear form $B_{\lambda_0}(u, v)$ need not be symmetric. Hence in order to construct the space $H^{p_{\lambda_0}}(\mathbb{R}^n)$ we need to replace B_{λ_0} by its symmetric part

$$\tilde{B}_{\lambda_0}(u, v) := \frac{1}{2}(B(u, v) + \overline{B(v, u)}) + \lambda_0(u, v)_0.$$

Clearly \tilde{B}_{λ_0} is a scalar product and we must define $H^{p_{\lambda_0}}(\mathbb{R}^n)$ as the completion of $S(\mathbb{R}^n)$ (or $H^{\psi_1, 1}(\mathbb{R}^n)$) with respect to \tilde{B}_{λ_0} . In order to apply the Lax-Milgram theorem later on, we need now

Lemma A. *The sesquilinear form B_{λ_0} is continuous on $H^{p_{\lambda_0}}(\mathbb{R}^n)$.*

Proof. Using [12, Corollary 2.4.23] it follows that

$$\begin{aligned} \frac{1}{2}(p_{\lambda_0}(x, D) + p_{\lambda_0}^*(x, D)) &= \frac{1}{2}(p_{\lambda_0}(x, D) + \bar{p}_{\lambda_0}(x, D)) + r_1(x, D) \\ &= p_{\lambda_0}(x, D) + r_1(x, D), \end{aligned}$$

where $r_1 \in S_0^{1+\tau_1, \psi_1}(\mathbb{R}^n)$ and we used that $p(x, \xi)$ is real-valued. Consider

$$\begin{aligned} |B_{\lambda_0}(u, v)| &= |(p_{\lambda_0}(x, D)u, v)_0| \\ &\leq \frac{1}{2} |((p_{\lambda_0}(x, D) + p_{\lambda_0}^*(x, D))u, v)_0| + |(r_1(x, D)u, v)_0| \\ &= |\tilde{B}_{\lambda_0}(u, v)| + |(r_1(x, D)u, v)_0|. \end{aligned}$$

We know that $\tilde{B}_{\lambda_0}(u, v)$ is continuous on $H^{p_{\lambda_0}}(\mathbb{R}^n)$. Therefore our calculations are reduced to estimate $|(r_1(x, D)u, v)_0|$. We know that $r_1 \in S_0^{\tau_1+1, \psi_1}(\mathbb{R}^n)$, and so $r_1 \in S_0^{1+\tau_1+\sigma+\tau_1\sigma, \psi_0}(\mathbb{R}^n)$. By Theorem 1.4 this implies that

$$|(r_1(x, D)u, v)_0| \leq c \|u\|_{\psi_0, \frac{1+\tau_1+\sigma+\tau_1\sigma}{2}} \|v\|_{\psi_0, \frac{1+\tau_1+\sigma+\tau_1\sigma}{2}}.$$

If $\tau_1 + \sigma + \tau_1\sigma < 1$ we get

$$\|u\|_{\psi_0, \frac{1+\tau_1+\sigma+\tau_1\sigma}{2}} \leq \|u\|_{\psi_0, 1} \leq c \|u\|_{p_{\lambda_0}},$$

implying the result by (16). □

Since $\tilde{B}_{\lambda_0}(u, u) = B_{\lambda_0}(u, u)$ it is obvious that B_{λ_0} is also coercive in $H^{p_{\lambda_0}}(\mathbb{R}^n)$.

With these changes the rest of the paper remains unchanged. We apologise for any inconvenience our omission caused to any reader.

We would like to take the opportunity to correct the following misprints:

- page 295: read \mathbb{N}_0^n instead of \mathbb{N}_0^n ;
- page 299, formula (15): read $\lambda_0 \geq 0$ instead of $f\lambda_0 \geq 0$;
- page 305, line 12 from above: read [11,(2.7)] instead of [12,(2.7)].