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ERRATUM

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Erratum to: Size-dependent effective properties of a heterogeneous material with interface energy effect: from finite deformation theory to infinitesimal strain analysis

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The corresponding author of the above paper regrets that in the expressions of the first kind Piola–Kirchhoff surface stress $\mathbf{S}_s$ and the Lagrangian description of the Young–Laplace equations some terms associated with the "out-plane term" of surface gradient $\mathbf{F}_s$ and "out-plane term" of $\mathbf{S}_s$ were missing. In fact, $\mathbf{F}_s$ and $\mathbf{S}_s$ are "two-point" tensors, with base vectors both on the tangent planes $\mathcal{J}_{\mathrm{Y}}^0$ and $\mathcal{J}_{\mathrm{y}}$, and are not tensors in the two-dimensional tangent plane $\mathcal{J}_{\mathrm{Y}}^0$ only. The correct forms of $\mathbf{F}_s$, $\mathbf{S}_s$ and the Lagrangian description of the Young–Laplace equations are given as follows.

(i) If the displacement on the surface $\mathbf{u}$ is decomposed into $\mathbf{u}_{0s} = u_0^{\alpha}\mathbf{A}_{\alpha}$ in the tangent plane $\mathcal{J}_{\mathrm{Y}}^0$ and $\mathbf{u}_{0n} = u_0^n\mathbf{A}_3$ along the normal direction of $\mathcal{J}_{\mathrm{Y}}^0$, then the surface gradient can be written as

$$\mathbf{F}_s = \mathbf{a}_{\alpha} \otimes \mathbf{A}^{\alpha} = \mathbf{i}_0 + \mathbf{u}\nabla_{0s} + d_{0\alpha}\mathbf{A}_3 \otimes \mathbf{A}^{\alpha} \tag{c1}$$

where $\mathbf{i}_0$ is the second-order identity tensor on $\mathcal{J}_{\mathrm{Y}}^0$, $\mathbf{u}\nabla_{0s}$ is defined as

$$\mathbf{u}\nabla_{0s} = \mathbf{u}_{0s}\nabla_{0s} - u_0^n\mathbf{b}_0 = u_0^{\lambda}|_{\alpha}\mathbf{A}_{\lambda} \otimes \mathbf{A}^{\alpha} - u_0^n\mathbf{b}_0 \tag{c2}$$

and $d_{0\alpha} = u_0^{\lambda}b_{0\lambda\alpha} + u_0^n|_{\alpha}$, $\mathbf{b}_0 = b_{0\lambda\alpha}\mathbf{A}^{\lambda} \otimes \mathbf{A}^{\alpha}$ is the curvature tensor of the surface in the reference configuration.

Equation (c1) can be expressed as the sum of an "in-plane term" $\overset{(in)}{\mathbf{F}_s} = \mathbf{i}_0 + \mathbf{u}\nabla_{0s}$ and an "out-plane term" $\overset{(ou)}{\mathbf{F}_s} = d_{0\alpha}\mathbf{A}_3 \otimes \mathbf{A}^{\alpha}$. Hence the first kind Piola–Kirchhoff surface stress can also be decomposed into an "in-plane term" $\overset{(in)}{\mathbf{S}_s} = \overset{(in)}{\mathbf{F}_s} \cdot \overset{(1)}{\mathbf{T}_s}$, and an "out-plane term" $\overset{(ou)}{\mathbf{S}_s} = \overset{(ou)}{\mathbf{F}_s} \cdot \overset{(1)}{\mathbf{T}_s}$.

(ii) The "out-plane term" $\mathbf{S}_s^{(ou)}$ in the above paper is missing. In Eq. (4), this missing term is $\mathbf{S}_s^{(ou)} = J_2\left(\partial\gamma/\partial J_1 + J_2\partial\gamma/\partial J_2 + \gamma\right)\overset{(ou)}{\mathbf{F}_s}$, and in Eqs. (12) and (15), this missing term is $\gamma_0^*\overset{(ou)}{\mathbf{F}_s}$, and the discussion on Eq. (16) is inappropriate due to this missing term.

In Eq. (6), the missing term is $\left(\partial\gamma/\partial J_1 + J_2\partial\gamma/\partial J_2 + \gamma\right)\left(\overset{(ou)}{\mathbf{F}_s} + \overset{(ou)}{\mathbf{F}}{}_s^{\mathrm{T}}\right)$, and therefore, the missing term in Eq.(13) is $\gamma_0^*\left(\overset{(ou)}{\mathbf{F}_s} + \overset{(ou)}{\mathbf{F}}{}_s^{\mathrm{T}}\right)$. It should be mentioned that, for small deformation, $\mathbf{i}_0 + \left(\overset{(ou)}{\mathbf{F}_s} + \overset{(ou)}{\mathbf{F}}{}_s^{\mathrm{T}}\right)$ is

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equal to the second-order identity tensor **i** in the tangent plane $\mathcal{J}_y$, and the Cauchy surface stress is a tensor in tangent plane $\mathcal{J}_y$ of the deformed surface.

Equation (7) should be written as

$$\begin{aligned} \mathbf{N}\cdot[\![\mathbf{S}^0]\!]\cdot\mathbf{N} &= -\left(\mathbf{S}_s^{(in)}\right):\mathbf{b}_0 - \left[\mathbf{N}\cdot\left(\mathbf{S}_s^{(ou)}\right)\right]\cdot\nabla_{0s}, \\ \mathbf{P}_0\cdot[\![\mathbf{S}^0]\!]\cdot\mathbf{N} &= -\left(\mathbf{S}_s^{(in)}\right)\cdot\nabla_{0s} + \left[\mathbf{N}\cdot\left(\mathbf{S}_s^{(ou)}\right)\cdot\mathbf{b}_0\right] \end{aligned} \tag{c3}$$

and accordingly Eq. (8) should be

$$\begin{aligned} \mathbf{N}\cdot[\![\Delta\mathbf{S}^0]\!]\cdot\mathbf{N} &= -\left(\Delta\mathbf{S}_s^{(in)}\right):\mathbf{b}_0 - \left[\mathbf{N}\cdot\left(\Delta\mathbf{S}_s^{(ou)}\right)\right]\cdot\nabla_{0s}, \\ \mathbf{P}_0\cdot[\![\Delta\mathbf{S}^0]\!]\cdot\mathbf{N} &= -\left(\Delta\mathbf{S}_s^{(in)}\right)\cdot\nabla_{0s} + \left[\mathbf{N}\cdot\left(\Delta\mathbf{S}_s^{(ou)}\right)\cdot\mathbf{b}_0\right] \end{aligned} \tag{c4}$$

Equation (27) should be

$$\begin{aligned} [\![\sigma_{rr}]\!]|_{r=a} &= \frac{1}{a^2}\left(\gamma_0^* + 2\gamma_1^* + \gamma_1\right)\left(2u_r + u_\theta\cot\theta + \frac{\partial u_\theta}{\partial\theta}\right)\Bigg|_{r=a} \\ &\quad - \frac{1}{a^2}\gamma_0^*\left(\frac{\partial^2 u_r}{\partial\theta^2} - \frac{\partial u_\theta}{\partial\theta} + \frac{\partial u_r}{\partial\theta}\cot\theta - u_\theta\cot\theta\right)\Bigg|_{r=a}, \\ [\![\sigma_{r\theta}]\!]|_{r=a} &= \frac{1}{a^2}\left[\left(2\gamma_0^* + \gamma_1^*\right)u_\theta + \left(\gamma_1^* + \gamma_1\right)\left(u_\theta\cot^2\theta - \frac{\partial u_\theta}{\partial\theta}\cot\theta - \frac{\partial^2 u_\theta}{\partial\theta^2}\right)\right]\Bigg|_{r=a} \\ &\quad - \frac{1}{a^2}\left[\left(2\gamma_0^* + 2\gamma_1^* + \gamma_1\right)\frac{\partial u_r}{\partial\theta}\right]\Bigg|_{r=a}. \end{aligned} \tag{c5}$$

Equation (34) should be

$$\mu_* = \mu_1 + \frac{L_1 + L_2}{L_3 + L_4} \tag{c6}$$

where

$$\begin{aligned} &L_0 = 2\gamma_0^{*2} - \gamma_1\left(\gamma_1 + 2\gamma_1^*\right) - \gamma_0^*\left(5\gamma_1 + 2\gamma_1^*\right), \quad L_1 = 10\,(7 - 10\nu_1)\,L_0, \\ &L_2 = -\frac{1}{10a}\left(2\gamma_0^* - 7\gamma_1 - 2\gamma_1^*\right)L_4, \quad L_3 = 4\,(-7 + 10\nu_1)\,a\left(13\gamma_0^* + 7\gamma_1 + 12\gamma_1^*\right), \\ &L_4 = 10a^2\left[4\,(-7 + 10\nu_1)\,\mu_0 - (7 + 5\nu_1)\,\mu_1\right]. \end{aligned}$$

Based on (c6), Eq. (36) should be written as

$$\bar{\mu} = \mu_0 + \frac{15\mu_0 f\,(1 - \nu_0)\left[-L_1 - L_2 + (L_3 + L_4)(\mu_0 - \mu_1)\right]}{L_5 + L_6} \tag{c7}$$

where

$$\begin{aligned} &L_5 = 2\,(1 - f)\,(-4 + 5\nu_0)\,(L_1 + L_2), \\ &L_6 = (L_3 + L_4)\left[(-7 + 5\nu_0)\,\mu_0 + 2\,(-4 + 5\nu_0)\,(f\mu_0 + \mu_1 - f\mu_1)\right]. \end{aligned}$$

Therefore, Eq. (38) should be

$$\bar{\mu}_{\text{void}} = \mu_0 + 15\mu_0 f\,(1 - \nu_0)\,\frac{L_0 + M_1 + M_2}{N_0 + N_1 + N_2} \tag{c8}$$

where

$$\begin{aligned} &M_1 = 2\left(3\gamma_0^* + 2\gamma_1^*\right)\mu_0 a, \;\; M_2 = 4\mu_0^2 a^2, \;\; N_0 = 2\,(1 - f)\,(4 - 5\nu_0)\,L_0, \\ &N_1 = 2\left[3N\gamma_0^* + 21\,(-1 + \nu_0)\,\gamma_1 + 2\,(N - 5 + 5\nu_0)\,\gamma_1^*\right]\mu_0 a, \\ &N_2 = 4a^2\,(N - 2 + 2\nu_0)\,\mu_0^2, \;\; N = -5 + 3\nu_0 + 2f\,(-4 + 5\nu_0). \end{aligned}$$

From (c7) with $\gamma_1 = \gamma_1^* = 0$, Eq. (40) can be corrected easily.

It is noted that the basic idea, theoretical framework and main results presented in the paper are not affected by these missing terms.

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