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ERRATUM

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Erratum to: Testing the Concept of Quark–Hadron Duality with the ALEPH τ Decay Data

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The original article has been published with errors in some numerical calculations. The author inadvertently used in all orders of perturbation theory the N^2LO value for the coefficient c_L, $c_L|_{N^2LO} = 0.555401$. The values of the coefficient obtained in various orders read[1]

Perturbative orders	LO	NLO	N^2LO	N^3LO	N^4LO
c_L	0.301262,	0.453421,	0.555401,	0.651373,	0.721687.

Equations (45), (47) and (52) of the article now become

$$\begin{aligned}
\alpha_s(m_\tau^2)|_{\rm NLO} &= 0.337 \pm 0.016_{\rm exp} \pm 0.032_{\rm th} \\
\alpha_s(m_\tau^2)|_{\rm N^2LO} &= 0.321 \pm 0.016_{\rm exp} \pm 0.008_{\rm th} \\
\alpha_s(m_\tau^2)|_{\rm N^3LO} &= 0.313 \pm 0.014_{\rm exp} \pm 0.004_{\rm th} \\
\alpha_s(m_\tau^2)|_{\rm N^4LO} &= 0.308 \pm 0.014_{\rm exp} \pm 0.002_{\rm th},
\end{aligned} \tag{45}$$

$$\begin{aligned}
s_{\rm p}|_{\rm NLO} &= 1.710 \pm 0.054_{\rm exp} \pm 0.002_{\rm th}\ {\rm GeV}^2 \\
s_{\rm p}|_{\rm N^2LO} &= 1.709 \pm 0.054_{\rm exp} \pm 0.001_{\rm th}\ {\rm GeV}^2 \\
s_{\rm p}|_{\rm N^3LO} &= 1.707 \pm 0.054_{\rm exp} \pm 0.001_{\rm th}\ {\rm GeV}^2 \\
s_{\rm p}|_{\rm N^4LO} &= 1.705 \pm 0.054_{\rm exp} \pm 0.001_{\rm th}\ {\rm GeV}^2.
\end{aligned} \tag{47}$$

$$\begin{aligned}
\hat{R}^{\rm pert.}_{\tau,\rm V}|_{s>s_{\rm p}} &= 0.3747 \cdot 10^{-1} + 0.3275 \cdot 10^{-2} + 0.3937 \cdot 10^{-3} + 0.9270 \cdot 10^{-4} \\
&\quad + 0.3304 \cdot 10^{-4} + (0.6047 \cdot 10^{-5}) \approx 0.04127.
\end{aligned} \tag{52}$$

As seen above from formula (47) that the extracted value of $s_{\rm p}$ monotonically decreases as the perturbative order increases, as it should be. The numbers in Tables 3, 4, 5 and 7 are corrected (see below).

[1] These numbers correspond to the exact (numeric) four-loop order running coupling.

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Table 3 Numerical values for the parameters in the $\overline{\text{MS}}$ scheme extracted from the τ data order-by-order within the modified procedure based on APT$^+$

Observable	Approximation to the Adler function				
	LO	NLO	N^{2}LO	N^{3}LO	N^{4}LO
s_{p} GeV^2	1.707	1.710	1.709	1.707	1.705
Λ GeV	0.486	0.378	0.348	0.332	0.323
$\alpha_s(m_\tau^2)$	0.401	0.337	0.321	0.313	0.308

Table 4 Comparison of the expansion functions $\mathfrak{A}_k(m_\tau^2, s_{\text{p}})$, $\mathcal{A}_k(m_\tau^2)$ and the powers of the "couplant" $a_s(m_\tau^2)$

k	$a_s^k(m_\tau^2)$	$\mathcal{A}_k(m_\tau^2)$	$\mathfrak{A}_k(m_\tau^2, s_{\text{p}})$
1	$0.9797 \cdot 10^{-1}$	0.1511	$0.3275 \cdot 10^{-2}$
2	$0.9599 \cdot 10^{-2}$	$0.1876 \cdot 10^{-1}$	$0.2400 \cdot 10^{-3}$
3	$0.9405 \cdot 10^{-3}$	$0.2000 \cdot 10^{-2}$	$0.1455 \cdot 10^{-4}$
4	$0.9214 \cdot 10^{-4}$	$0.1834 \cdot 10^{-3}$	$0.6733 \cdot 10^{-6}$
5	$0.9028 \cdot 10^{-5}$	$0.1383 \cdot 10^{-4}$	$0.1599 \cdot 10^{-7}$

The four-loop "couplant" is calculated using the value $\Lambda = 0.3225$ GeV. To calculate the functions $\mathfrak{A}_k(m_\tau^2, s_{\text{p}})$, we have used the values $\Lambda|_{\text{N}^4\text{LO}} = 0.3225$ GeV and $s_{\text{p}}|_{\text{N}^4\text{LO}} = 1.7053$ GeV obtained within APT$^+$. To calculate the functions $\mathcal{A}_k(m_\tau^2)$ we have used the value $\Lambda|_{\text{N}^4\text{LO}} = 0.395$ GeV extracted from the ALEPH data within CIPT

Table 5 Estimates for $\alpha_s(M_z^2)$ obtained from the ALEPH τ lepton decay vector data order-by-order in perturbation theory

Perturbative order	$\alpha_s(M_z^2)\vert_{\text{APT}^+}$	$\alpha_s(M_z^2)\vert_{\text{CIPT}}$
N^{2}LO	$0.1187 \pm 0.0019 \pm 0.0005$	$0.1238 \pm 0.0009 \pm 0.0005$
N^{3}LO	$0.1176 \pm 0.0018 \pm 0.0005$	$0.1224 \pm 0.0009 \pm 0.0005$
N^{4}LO	$0.1170 \pm 0.0018 \pm 0.0005$	$0.1217 \pm 0.0009 \pm 0.0005$

The results obtained within APT$^+$ and CIPT are compared. Two errors are given, the experimental (first number) and the error from the evolution procedure (second number)

Table 7 Different approximations to the "experimental" Adler function as a function of the scale

Q GeV	$D^{(1)}_{\text{"exp"}}(Q^2)$	$D^{(2)}_{\text{"exp"}}(Q^2)$	$D^{(3)}_{\text{"exp"}}(Q^2)$	$D^{(4)}_{\text{"exp"}}(Q^2)$	$D^{(5)}_{\text{"exp"}}(Q^2)$
0.1	0.06494	0.06494	0.06494	0.06494	0.06494
0.2	0.23003	0.23005	0.23004	0.23003	0.23002
0.3	0.43541	0.43546	0.43544	0.43541	0.43539
0.4	0.63196	0.63205	0.63201	0.63196	0.63192
0.5	0.79431	0.79444	0.79438	0.79430	0.79424
0.6	0.91613	0.91631	0.91623	0.91612	0.91604
0.7	1.0015	1.0017	1.0016	1.0015	1.0014
0.8	1.0582	1.0585	1.0583	1.0582	1.0580
0.9	1.0940	1.0943	1.0942	1.0940	1.0938
1.0	1.1154	1.1158	1.1157	1.1154	1.1152
1.5	1.1321	1.1327	1.1324	1.1320	1.1317

The function $D^{(k)}_{\text{"exp"}}(Q^2)$ has the pQCD component evaluated within APT$^+$ at N$^{(k-1)}$LO. To construct this component, we employ the four-loop order running coupling

The central value of $\alpha_s(M_z^2)$ obtained at N^{4}LO reproduces the lattice determination of the strong coupling constant [1]. Beyond NLO, we have found good agreement with the renormalization scheme invariant determination of the strong coupling constant [2,3]. Main conclusions reached in the article are not changed.

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