Erratum to "Topology of Hopf surfaces"

[This JOURNAL, Vol. 27 (1975), 222-238]

By Masahide KATO

(Received March 25, 1988)

The author was kindly informed by M. Ue that the statement of Theorem 9 was incomplete and a certain subcase was missing. As was pointed out by Ue, this error had its origin in Lemma 4 of the paper which claimed that $N_{SL(2,C)}(B_n)=B_{2n}$ for $n\geq 2$. In fact, this equality holds for $n\geq 3$, but for n=2, we have $N_{SL(2,C)}(B_2)=D$. By this mistake, we must correct Lemmas 4, 5, 6, 7, Proposition 8, and Theorem 9, though they hold true under the condition that $K\neq B_2$. The other results are OK without change. We correct these errors as follows. In Lemma 4, $N_{SL(2,C)}(B_n)=B_{2n}$ $(n\geq 3)$, $N_{SL(2,C)}(B_2)=D$. In Lemma 5, $N_{GL(2,C)}(B_n)=C^*I\cdot B_{2n}$ $(n\geq 3)$, $N_{GL(2,C)}(B_2)=C^*I\cdot D$. In Lemma 6, Case 2, for $K=B_n$ $(n\geq 3)$, $u=\begin{pmatrix} \rho_{2n} & 0\\ 0 & \rho_{2n}^{-1} \end{pmatrix}$, and for $K=B_2$, $u=u_1:=\begin{pmatrix} \rho_4 & 0\\ 0 & \rho_4^{-1} \end{pmatrix}$ or $u=u_2:=\frac{1}{\sqrt{2}}\begin{pmatrix} \rho_4^3 & \rho_4^3\\ \rho_4 & -\rho_4 \end{pmatrix}$. In case $K=B_2$, Lemma 7 should be replaced by

LEMMA 7'. If $(K=B_2 \text{ and } if) G$ is indecomposable, then G can be expressed as either

(a) $G = G_0 \cup gG_0$, where $G_0 = \{c^2I\} \times H$, $c \in C^*$, |c| < 1, $g = cu_1$, or (b) $G = G_0 \cup gG_0 \cup g^2G_0$,

where $G_0 = \{c^3I\} \times H$, $c \in C^*$, |c| < 1, $g = cu_2$.

In case $K=B_2$, Proposition 8 should be replaced also by

PROPOSITION 8'. If $K=B_2$, then G is conjugate to one of the following three groups;

(a) $G = \{c^2I\} \times H \cup (cu_1)(\{c^2I\} \times H),$

(b) $G = \{c^3I\} \times H \cup (cu_2)(\{c^3I\} \times H) \cup (cu_2)^2(\{c^3I\} \times H),$

(c)
$$G = \{cI\} \times H$$
,

where $c \in C^*$, |c| < 1.

The statement (2) in Theorem 9 should read as follows;

(2)' (S^{3}/H) -bundle over S^{1} whose transition function $u: S^{3}/H \rightarrow S^{3}/H$ is of order 2 or 3 as an element of the diffeomorphism group of S^{3}/H .

The correction of Lemma 6 is essential, and can be done by elementary group theoretic calculations. Here we use the fact that $S_3 := N_{SL(2,C)}(B_2)/B_2$ is the symmetric group of order 6 and that S_3 has only three conjugacy classes represented by the identity and the images of u_1 and u_2 . The proof of Theorem 10 works also in the case $K=B_2$ though u is not determined uniquely, since any two groups belonging to distinct cases of (a), (b) and (c) in Proposition 8' are not isomorphic to one another.

We note that, in case $K=B_2$, G is indecomposable if and only if G is conjugate in $GL(2, \mathbb{C})$ to one of the following three groups $\{H, g\}$; (1) $H=\{\lambda I, K\}$ and $g=cu_1$, (2) $H=\{\lambda I, K\}$ and $g=cu_2$, (3) $H=\{\lambda u_2, K\}$ and $g=cu_1$, where $c\in\mathbb{C}^*$, |c|<1, and λ is a root of unity. The corresponding transition functions of the (S^3/H) -bundle structures are of order 2 in cases (1) and (3), and of order 3 in case (2).

Masahide KATO

Department of Mathematics Sophia University Kioi-cho, Chiyoda-ku 102 Tokyo Japan