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Error Analysis in Trifilar Inertia Measurements

J.L. du Bois · N.A.J. Lieven · S. Adhikari

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Abstract Accurate calculation of the moment of inertia of an irregular body is made difficult by the large number of quantities which must be measured. A popular method is to use a trifilar suspension system to measure the period of oscillation of the body in the horizontal plane. In this paper, some sources of error are discussed with particular attention given to the alignment of the test object's center of mass on the trifilar platform. The procedure is described, the necessary calculations are derived and the relative importance of accuracy in different measurements is assesed. It is determined that the accurate alignment of the centre of mass of the body being tested with the centre of the trifilar plate is insignificant compared to the accuracy of the other measurements required in the calculations.

Keywords Trifilar · Pendulum · Mass properties · Inertia · Measurement · Error

Nomenclature

D	displacement of body centre of mass from trifilar plate centre of mass along a line between the trifilar plate's centre of mass and a suspension wire attachment point at a corner
d	horizontal displacement of corners of trifilar plate
F	horizontal force applied to the corners of the trifilar plate by each suspension wire
F_S,F_L	force on corners nearest and furthest from the system centre of mass, respectively
g	gravitational constant
I_{zz}	moment of inertia of the entire system
I_{Pzz},I_{Bzz}	moment of inertia of the triangular plate and of the body, respectively
L	length of the suspension wires
m	mass of the triangular trifilar plate and the body combined
m_P	mass of the triangular trifilar plate
m_B	mass of the body being measured
R	distance from the centre of a trifilar plate to the wire attachment point in the corner
R_S,R_L	shortest and longest distances from system centre of mass to a plate corner, respectively
T_z	torsional force acting in horizontal plane
W	weight supported by each of the three suspension wires
W_S,W_L	weight carried by wires nearest and furthest from the system centre of mass, respectively
α	angle made between displaced suspension wire and its equilibrium position
α_S,α_L	angle between displaced suspension wire and its equilibrium position for the wires nearest and furthest from the system centre of mass, respectively
γ	ratio of ΔR to R
Δh	horizontal displacement of the trifilar plate
ΔR	change in centre of mass of entire system caused by displacement of the centre of mass of the body

J.L. du Bois (✉, SEM member) ·
N.A.J. Lieven (SEM member)
Department of Aerospace Engineering,
University of Bristol, Queens Building,
University Walk, Bristol BS8 1TR, UK
e-mail: jon.dubois@bristol.ac.uk

S. Adhikari (SEM member)
School of Engineering, University of Wales Swansea,
Singleton Park, Swansea SA2 8PP, UK

θ	rotation in horizontal plane
$\ddot{\theta}$	angular acceleration in horizontal plane
τ	period of oscillation of the entire system
ω	frequency of oscillation

Introduction

In dynamic analyses of solid components it is essential to know the moment of inertia. For simple geometry this can be calculated, but if the geometry is complicated or the exact specification unknown it is necessary to measure the inertia experimentally. With the advent of modern sensors and equipment a variety of dedicated machinery has been developed to address this purpose. For example, Witter et al [1] describe techniques using six degree of freedom (DOF) load cells and linear accelerometer arrays. With such machinery the majority of measurement error is accounted for as part of a regular calibration schedule, reducing the scope for measurement error in normal use. For one off applications and low budget operations, however, the use of these machines may be less appropriate and difficult to justify in comparison to more traditional methods. Genta and Delprete [2] analyse the different methods available and divide them into two broad categories: oscillatory and accelleratory. They conclude that the results from oscillatory methods are less affected by the presence of damping than those obtained using acceleration-based methods, and that torsional multifilar pendula are generally considered to be the most accurate. These are reported to be capable of producing results with errors of less than 1% [2, 3].

In the same paper they derive accurate expressions for the linear motion of a coupled three DOF pendulum (rotation, x- and y- translation), as well as non-linear motion of a one DOF pendulum (rotation) and infer that the non-linear nature of pendulum-based configurations has only a marginal effect on the results for the barimetric moment of inertia. Lyons [4] concurs that for plate rotations of less than 10° the non-linear component of the motion does not cause significant errors in the results.

The use of bifilar torsion pendula has been employed extensively for the measurement of the moment of inertia of aircraft and is considered to have advantages over simple compound pendula [5–7]. Some practical considerations are explored by Schwartz et al. [3]: notably the effect of torsional loading within the suspension wires. An obvious drawback of a bifilar pendulum is its unstable nature; only limited arrangements of an object can be accommodated. In contrast, three or more wires allow for almost any orientation of the body to be tested and also permit the suspension of a plate from the wires, upon which the test body may be rested. This may prove useful where direct attachment of wires to the body is difficult.

Genta and Delprete [2] and Lyons [4] both note that multifilar pendula with more than three suspension wires and undergoing transverse oscillations may suffer from irregular motion as some wires slacken, thereby adversely affecting the results. It is for this reason, and those discussed above, that the trifilar arrangement [8] is a popular choice in the measurement of moments of inertia.

Detailed investigations of many experimental considerations concerning this technique are reported by Genta and Delprete [2] and Lyons [4]; the subsequent sections of this paper will be confined to an idealised yet rigorous analysis in order to expand on this knowledge base.

In the "Methodology of the Trifilar Arrangement" section the trifilar method is summarised and simplified calculations are derived for the determination of moments of inertia. The "Centre of Mass Alignment" section examines how misalignment of the centre of mass, a common problem in simple experimental setups, can affect the computed moment of inertia. The significance of this effect is compared to that of other measurement errors in the "Error Analysis" section, and the analytical results are verified with respect to numerical data based on a typical experimental configuration in the "Example Calculations" section.

Methodology of the Trifilar Arrangement

A triangular plate is suspended from three vertical wires, one at each corner. The body whose inertia is to be determined is placed on the plate, with its centre of mass at the centre of the plate. The plate is then twisted in the horizontal plane and released so that it performs free rotational oscillations about the centre of mass. For the purposes of this paper, some simplifying assumptions are made: firstly, all damping is neglected and the pendulum is taken to be governed by simple harmonic motion; secondly, the mass of the wires is assumed to be negligible; thirdly, the wires are assumed to carry only axial loading (and specifically no torsional loading); and finally, the wires and plate are assumed to be inextesible and rigid, respectively, and only the rotational motion of the pendulum is taken into account to produce a one-DOF dynamic system.

To determine the properties of the system as a whole (the plate and the body), it is necessary to solve the dynamic equation formed by Newton's second law for rotation.

$$T_z = I_{zz}\ddot{\theta} \tag{1}$$

where T_z is the torque applied to the system in a horizontal plane by the suspension wires, I_{zz} is the moment of inertia about the vertical axis through the centre of mass (and geometric centre of the plate) and $\ddot{\theta}$ is the angular acceleration of the system.

The plate is shown viewed from above in Fig. 1(a), where its horizontal rotation can be seen. In Fig. 1(b) a view from the side shows the displacement of the suspending wires attached to the corners. This analysis will use linear theory, based on the assumption that the angles are small and

$$\begin{aligned} \sin\theta &\approx \theta \qquad & \sin\alpha &\approx \alpha \\ \cos\theta &\approx 1 \qquad & \cos\alpha &\approx 1 \end{aligned} \tag{2}$$

The two angles can be related by the horizontal displacement, d, of the corners, giving

$$\begin{aligned} L\sin\alpha &= d \\ R\sin\theta &= d \end{aligned} \tag{3}$$

so

$$L\alpha \approx R\theta \tag{4}$$

The vertical displacement of the plate is given by

$$\Delta h = L(1 - cos\alpha) \tag{5}$$

but using the assumptions in equation (2) it is found that

$$\Delta h \approx 0 \tag{6}$$

meaning that the height of the plate is assumed constant and the vertical acceleration is zero. This enables the weight supported by each of the three wires, W, to be computed easily using a static equilibrium equation in the vertical direction:

$$W = \frac{1}{3}mg(\cos\alpha)^{-1} \approx \frac{1}{3}mg \tag{7}$$

where m is the mass of the system and g is the gravitational acceleration. Following on from this, the horizontal force acting on one corner of the plate opposes the displacement by

$$F = -W\sin\alpha \approx -\frac{1}{3}mg\alpha \tag{8}$$

and the torque on the plate is defined in terms of the three corner forces and the distance from the centre, R, by

$$T_z = 3RF = -Rmg\alpha \tag{9}$$

Using equation (4) produces

$$T_z = -\frac{R^2mg}{L}\theta \tag{10}$$

and substitution into equation (1) gives the dynamic equation of motion

$$\frac{R^2mg}{L}\theta + I_{zz}\ddot{\theta} = 0 \tag{11}$$

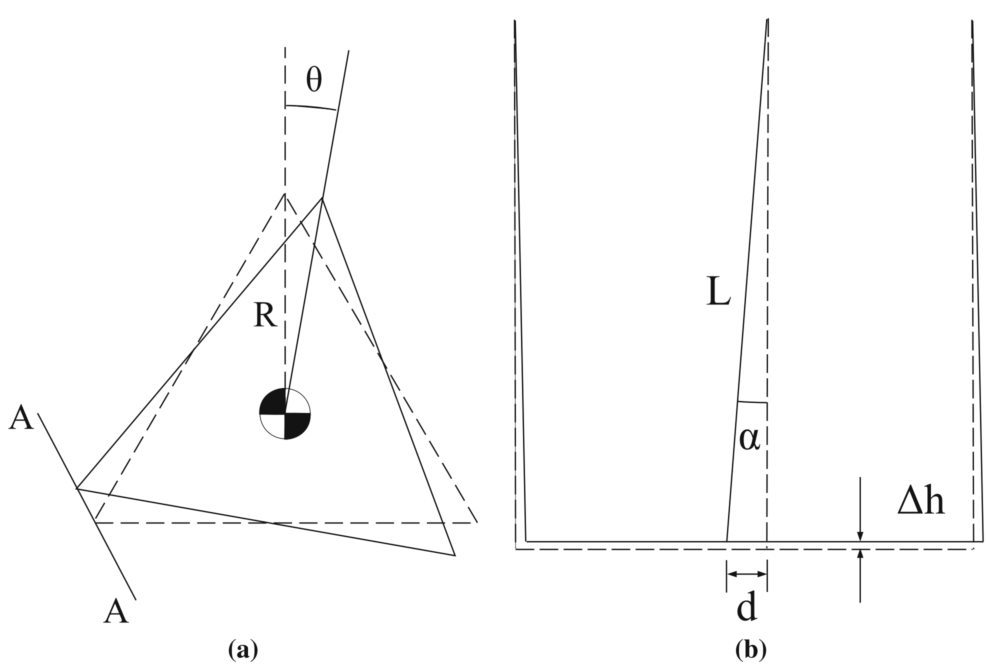

Fig. 1 Triangular plate rotated from rest by an angle θ about the vertical axis through its centre. The wires suspending the plate form an angle α with their vertical equilibrium positions. The change in the plate's vertical position, Δh, has been exagerated. (**a**) Top view. (**b**) Side view through section AA

Assuming simple harmonic motion, the solution can be found in terms of the angular frequency of ocsillation, ω, from

$$\frac{R^2mg}{L} - \omega^2 I_{zz} = 0 \tag{12}$$

and substituting $\omega = 2\pi/\tau$ gives

$$I_{zz} = \frac{R^2mg\tau^2}{4\pi^2 L} \tag{13}$$

where τ is the period of oscillation. Finally, the mass and inertia are split into components belonging to the plate and the body:

$$m = m_P + m_B \tag{14}$$

$$I_{zz} = I_{Pzz} + I_{Bzz} \tag{15}$$

where the subscripts P and B refer to the plate and body respectively so that the moment of inertia of the body is found using

$$I_{Bzz} = \frac{R^2 g\tau^2}{4\pi^2 L}(m_P + m_B) - I_{Pzz} \tag{16}$$

I_{Pzz} can be obtained by applying equation (13) with the period of the unladen plate, so that six measurements must be made in order to calculate the moment of inertia of the body using equation (16). In addition to this it is necessary to deduce the position of the centre of mass of the body being measured in order that it can be lined up with the centre of the plate. This is sometimes difficult and in the next section, the effect of misaligning the centres is discussed.

Centre of Mass Alignment

Precise determination of the centre of mass of irregular bodies can prove difficult, and aligning the center of mass with the centre of a trifilar plate can be even more troublesome. The importance of this issue is highlighted by Pal and Gaberson [9, 10]. Derriman [11] describes a method of compensating for such misalignments in a torsional "pendulum" which could be adapted to a trifilar pendulum, however his method requires somewhat complicated apparatus and will only give the moment of inertia of the body about the point at the centre of the rotation and not its baricentric moment of inertia. Lyons [4] briefly states a formula for determining moments of inertia with offset centers of mass, as an alternative to that derived here.

If the body is placed on the plate with its centre of mass misaligned, the value calculated for the moment of inertia is affected in two ways:

1. The inertia of the system is no longer simply a sum of the two component inertias, but must be calculated taking into account the distance of each component centre of mass from the system centre of mass.
2. The centre of mass (and therefore the centre of rotation) is no longer located at the geometric centre of the plate. This will redefine the weight distribution between the wires and also the angle, α, through which each wire is displaced for a given plate rotation, θ. This in turn redefines the equation of motion.

To examine the extent of these effects, it is assumed that the centre of mass of the body lies somewhere on a line directly between the centre and a corner of the triangle. It is displaced a distance D from the centre, as depicted in Fig. 2. The displacement of the overall system centre of mass, ΔR, is then defined by the ratio of the component masses:

$$\Delta R = \frac{m_B}{m_P + m_B} D \tag{17}$$

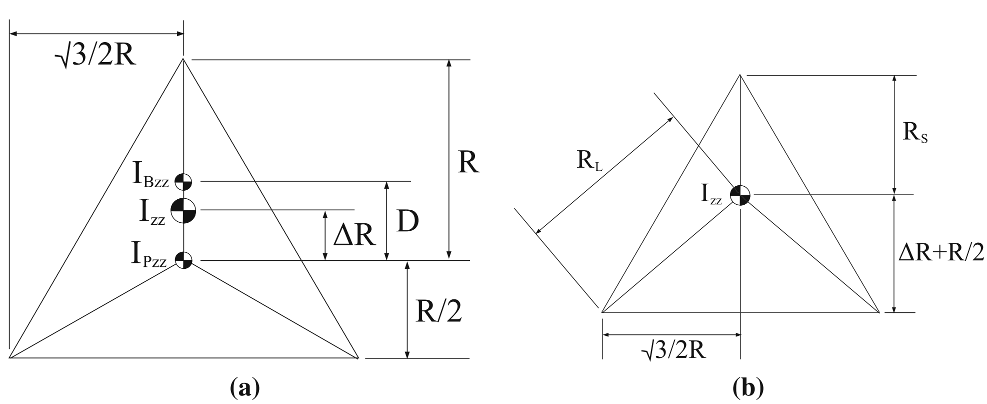

Fig. 2 Geometry of the system when the centres of mass of the plate and the body are misaligned. (**a**) Centres of mass. (**b**) New shorter and longer R values, R_S and R_L

Addressing the first item above, equation (14) must now be replaced by

$$I_{zz} = \left(I_{Pzz} + m_P \Delta R^2\right) + \left(I_{Bzz} + m_B (D - \Delta R)^2\right)$$
$$= I_{Pzz} + I_{Bzz} + \frac{m_P m_B D^2}{m_P + m_B} \tag{18}$$

Substituting back into equation (13) produces

$$I_{Bzz} = \frac{R^2 g \tau^2}{4\pi^2 L}(m_P + m_B) - I_{Pzz} - \frac{m_P m_B D^2}{m_P + m_B} \tag{19}$$

where the last term accounts for the change in moment of inertia of the system due to misalignment.

Addressing the second item above, it is necessary to reformulate the equation of motion to correspond with the new weight distribution and centre of rotation. The distances of the corners from the centre of mass are shown in Fig. 2 and are given by

$$R_S = R - \Delta R$$
$$R_L = \sqrt{\left(\Delta R + \frac{R}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} R\right)^2} \tag{20}$$

or

$$\frac{R_S}{R} = 1 - \gamma$$
$$\frac{R_L}{R} = \sqrt{1 + \gamma + \gamma^2} \qquad \text{where } \gamma = \frac{\Delta R}{R} \tag{21}$$

Equation (4) is replaced by

$$\alpha_S = \frac{R_S}{L}\theta \qquad \alpha_L = \frac{R_L}{L}\theta \tag{22}$$

and equations (7–9) become

$$W_S = \frac{1}{3} mg\,(1 + 2\gamma) \qquad W_L = \frac{1}{3} mg\,(1 - \gamma) \tag{23}$$
$$F_S = -W_S \alpha_S \qquad F_L = -W_L \alpha_L \tag{24}$$
$$T_z = R_S F_S + 2 R_L F_L \tag{25}$$

Combining equations (22–25),

$$T_z = -\frac{1}{3} mg\theta \left[(1 + 2\gamma)\,\frac{R_S{}^2}{L} + (2 - 2\gamma)\,\frac{R_L{}^2}{L}\right] \tag{26}$$

Substituting equation (21) and simplifying gives

$$T_z = -\left(1 - \gamma^2\right) \frac{R^2 mg}{L}\theta \tag{27}$$

Putting this into equation (1) produces

$$\left(1 - \gamma^2\right) \frac{R^2 mg}{L}\theta + I_{zz}\ddot{\theta} = 0 \tag{28}$$

and assuming simple harmonic motion,

$$I_{zz} = \left(1 - \gamma^2\right) \frac{R^2 mg \tau^2}{4\pi^2 L} \tag{29}$$

Finally, equation (18) is substituted to give

$$I_{Bzz} = \left(1 - \gamma^2\right) \frac{R^2 g \tau^2}{4\pi^2 L}\,(m_P + m_B) - I_{Pzz} - \frac{m_P m_B D^2}{m_P + m_B}$$
$$= \frac{R^2 g \tau^2}{4\pi^2 L}\,(m_P + m_B) - I_{Pzz} - \frac{m_B D^2}{m_P + m_B}$$
$$\times \left(m_P + \frac{m_B g \tau^2}{4\pi^2 L}\right) \tag{30}$$

The final term is the error in the moment of inertia measurement caused by misalignment of the inertial centres:

$$\varepsilon_D = -\frac{m_B D^2}{m_P + m_B}\left(m_P + \frac{m_B g \tau^2}{4\pi^2 L}\right) \tag{31}$$

There are two components to the error, represented by the two bracketed terms. The first is caused by the increased inertia of the system (plate and body combined); the second is caused by the change in weight distribution and centre of rotation due to the new centre of mass position. If the mass of the plate, m_P, is small compared to that of the body, m_B, such that

$$\frac{m_P}{m_B} \ll \frac{I_{zz}}{R^2 m} \tag{32}$$

then the second component has the greatest effect on the measurement so that

$$\varepsilon_D \approx -\frac{D^2 m_B g \tau^2}{4\pi^2 L}. \tag{33}$$

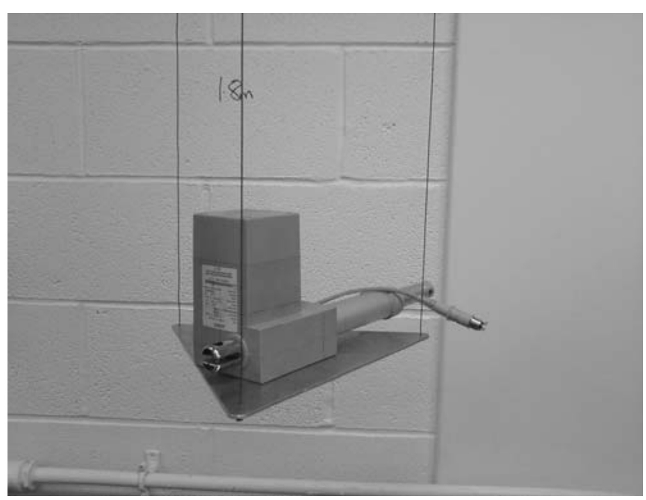

Fig. 3 The actuator *in situ* on the trifilar suspension plate

Table 1 Measured properties of the actuator and trifilar suspension arrangement

Quantity	Measurement
R	0.174 m
L	1.832 m
I_{Pzz}	4.035×10^{-3} kg m^2
m_P	0.422 kg
m_B	2.750 kg
τ	1.615 s
D	0.000 m

The mass moment of inertia of a triangular prism about its longitudinal axis is given by

$$I_{zz} = \frac{5\sqrt{3}}{36} R^2 m \approx \frac{1}{4} R^2 m \tag{34}$$

so for a body with similar mass distribution to that of the trifilar plate, a useful rule of thumb is obtained by simplifying equation (32) to

$$\frac{m_P}{m_B} \ll \frac{1}{4}. \tag{35}$$

Error Analysis

An error analysis will be undertaken here to assess the relative importance of the accuracy of different variables. In order to present a clear analysis it will be assumed that the mass of the plate is much less than that of the body being measured, such that m_P and I_{Pzz} are both negligible, $m = m_B$, $I_{zz} = I_{Bzz}$ and equation (32) holds true. It can be shown using the same approach given here that the final result obtained in equation (45) is not affected by these assumptions but the derivation becomes far more protracted. Accordingly, applying these simplifications to equation (30) gives

$$I_{zz} = \frac{mg\tau^2}{4\pi^2 L}\left(R^2 - D^2\right) \tag{36}$$

The misalignment error, D, is replaced by a dimensionless quantity, D^+, to produce

$$I_{zz} = \frac{R^2 mg\tau^2}{4\pi^2 L}\left(1 - D^{+2}\right) \qquad \text{where } D^+ = \frac{D}{R} \tag{37}$$

The sensitivity of I_{zz} to D^+ is given by

$$\frac{\partial I_{zz}}{\partial D^+} = -\frac{R^2 mg\tau^2}{2\pi^2 L} D^+ \tag{38}$$

Expressing this as a fraction of the total moment of inertia,

$$\frac{\frac{\partial I_{zz}}{\partial D^+}}{I_{zz}} = -\frac{2D^+}{1 - D^{+2}} \tag{39}$$

Similar equations can be derived for the other parameters. In preparation for integration the total error in I_{zz} caused by small changes in the measured variables is expressed as

$$\delta I_{zz} = \frac{\partial I_{zz}}{\partial D^+}\delta D^+ + \frac{\partial I_{zz}}{\partial R}\delta R + \frac{\partial I_{zz}}{\partial m}\delta m + \frac{\partial I_{zz}}{\partial \tau}\delta\tau + \frac{\partial I_{zz}}{\partial L}\delta L \tag{40}$$

and the fractional error in I_{zz} is then

$$\frac{\delta I_{zz}}{I_{zz}} = -\frac{2D^+}{1 - D^{+2}}\delta D^+ + \frac{2}{R}\delta R + \frac{1}{m}\delta m + \frac{2}{\tau}\delta\tau - \frac{1}{L}\delta L \tag{41}$$

Integrating gives the fractional error as

$$\ln\frac{I_{zz} + \Delta I_{zz}}{I_{zz}} = \ln\frac{1 - \left(D^+ + \Delta D^+\right)^2}{1 - D^{+2}} + 2\ln\frac{R + \Delta R}{R} + \ln\frac{m + \Delta m}{m} + 2\ln\frac{\tau + \Delta\tau}{\tau} - \ln\frac{L + \Delta L}{L} \tag{42}$$

or

$$\frac{I_{zz} + \Delta I_{zz}}{I_{zz}} = \left(\frac{1 - \left(D^+ + \Delta D^+\right)^2}{1 - D^{+2}}\right)\left(\frac{R + \Delta R}{R}\right)^2 \times \left(\frac{m + \Delta m}{m}\right)\left(\frac{\tau + \Delta\tau}{\tau}\right)^2\left(\frac{L}{L + \Delta L}\right) \tag{43}$$

Making the assumption that the error variables are small compared to their nominal parameter values,

Table 2 Estimated and calculated effect of measurement error on computed second moment of inertia of the plate and body combined: $I_{zz} = I_{Bzz} + I_{Pzz}$

Quantity	Effect of 1% error		Effect of 10% error	
	Expected (%)	Calculated (%)	Expected (%)	Calculated (%)
R	2.00	2.01	20.0	21.0
L	−0.990	−0.990	−9.09	−9.09
m	1.00	1.00	10.0	10.0
τ	2.00	2.01	20.0	21.0
D	0	−0.0108	0	−1.08

Table 3 Estimated and calculated effect of measurement error on computed second moment of inertia of the body alone: I_{Bzz}

Quantity	Effect of 1% error		Effect of 10% error	
	Expected (%)	Calculated (%)	Expected (%)	Calculated (%)
R	2.00	2.28	20.0	23.8
L	−0.990	−1.12	−9.09	−10.3
m	1.00	1.13	10.0	11.3
τ	2.00	2.28	20.0	23.8
D	0	−0.0122	0	−1.22

multiplying out equation (43) and ignoring numerator terms above second order in the error variables gives

$$\frac{\Delta I_{zz}}{I_{zz}} = -2\frac{\Delta D^{+}D^{+}}{1-D^{+2}} + 2\frac{\Delta R}{R} + \frac{\Delta m}{m} + 2\frac{\Delta\tau}{\tau} - \frac{\Delta L}{L+\Delta L} + O\left(\Delta D^{+2}, \Delta R^2, \Delta m^2, \Delta\tau^2, \Delta L^2\right) \tag{44}$$

If the nominal value of D^{+} is zero, corresponding to the assumption that the centres of mass are intended to be aligned, the equation can be simplified further:

$$\frac{\Delta I_{zz}}{I_{zz}} = 2\frac{\Delta R}{R} + \frac{\Delta m}{m} + 2\frac{\Delta\tau}{\tau} - \frac{\Delta L}{L+\Delta L} + O\left(\Delta D^{+2}, \Delta R^2, \Delta m^2, \Delta\tau^2, \Delta L^2\right) \tag{45}$$

From this it is seen that, to first order, the alignment of the body centre of mass on the trifilar plate is not important. As explained at the start of this section, the same result is obtained using the full equation for the moment of inertia given in equation (30). In this case I_{zz} and m are defined as before by equations (14) and (15), and refer to the *combined* properties of the plate and body.

Example Calculations

To examine the strength of these conclusions, some example calculations are now presented, based on real numerical data. The body to be used is a linear ball screw actuator, shown resting on a trifilar plate in Fig. 3. The measured quantities for this arrangement are listed in Table 1.

From these data it is possible to calculate the mass ratio: $m_P/m_B = 0.153$. This is not significantly below the value of 1/4 given in equation (35). As such the mass and inertia of the plate can not be entirely neglected in error calculations; equation (45) is valid for the combined properties of the plate and body but not for the body on its own.

Using equation (16) with the measured values, and assuming the mass centres to be aligned, the combined second moment of inertia is calculated as 33.98×10^{-3} kg m^2. This means the second moment of inertia of the actuator is 29.94×10^{-3} kg m^2.

Two tables are now presented, showing the expected percentage error, according to equation (45), in the moment of inertia for 1% and 10% changes in each of the measured variables compared to the actual change calculated by putting the perturbed values into equation (30). Table 2 shows the error in the combined second moment of inertia of the body and plate, $I_{zz} = I_{Bzz} + I_{Pzz}$, while Table 3 shows the error in the second moment of inertia of the body considered in isolation, I_{Bzz}. The first data set indicates that the estimated and calculated errors for the combined system are similar (and in fact identical for the linear terms). The accuracy of the estimated values is seen to decrease with the magnitude of the error. The second data set illustrates the difference between the errors calculated for the body alone and those estimated for the combined plate and body system. These results demonstrate the inadequacy of equation (45) in predicting the errors in the body inertia computation when equation (32) is invalidated: the estimated error does not correspond as well with the calculated values. Nonetheless, it is seen that the relative importance of the measured variables is similar in both cases. Most significantly, the alignment of the centres of mass has a very small effect on the moment of inertia computation, requiring an alignment error of around 10% of the radius circumscribed by the plate to produce a 1% error in the result.

Conclusion

A formula for computing the moment of inertia of a body using a trifilar pendulum has been derived, using some simplifying assumptions but taking account of misalignment of the body center of mass with the plate center of mass, and the corresponding changes in body motion and weight distribution between the wires. The effects of measurement errors are calculated and shown to correspond well with first order approximations, and this is substantiated using data from a typical experimental configuration. These findings have been used to assess the relative importance of each measurement, the most significant observation being that precise alignment of the body's center of mass on the plate is less important than accuracy in the other measurements: a misalignment of approximately 10% of the radius of a circle circumscribed about the trifilar plate is required to produce only a 1% error in the calculation. This is significant because accurate alignment of irregular bodies on trifilar plates is arguably the most difficult operation of the procedure, and subject to the greatest error.

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