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Error bounds for linear complementarity problems of weakly chained diagonally dominant B-matrices

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Abstract
In this paper, new error bounds for the linear complementarity problem are obtained when the involved matrix is a weakly chained diagonally dominant B-matrix. The proposed error bounds are better than some existing results. The advantages of the results obtained are illustrated by numerical examples.

MSC: 90C33; 60G50; 65F35

Keywords: error bound; linear complementarity problem; weakly chained diagonally dominant matrix; B-matrix

1 Introduction

A linear complementarity problem (LCP) is to find a vector $x \in \mathbb{R}^{n\times 1}$ such that

$$(Mx+q)^T x = 0, \qquad Mx+q \geq 0, \quad x \geq 0,$$

where $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^{n\times 1}$. The LCP has various applications in the free boundary problems for journal bearing, the contact problem, and the Nash equilibrium point of a bimatrix game [1–3].

The LCP has a unique solution for any $q \in \mathbb{R}^{n\times 1}$ if and only if M is a P-matrix [4]. In [5], Chen *et al.* gave the following error bound for the LCP when M is a P-matrix:

$$\|x - x^*\|_\infty \leq \max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty \|r(x)\|_\infty,$$

where x^* is the solution of the LCP, $r(x) = \min\{x, Mx + q\}$, $D = \operatorname{diag}(d_i)$ with $0 \leq d_i \leq 1$, and the min operator $r(x)$ denotes the componentwise minimum of two vectors. If M satisfies special structures, then some bounds of $\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty$ can be derived [6–11].

Definition 1 ([4]) A matrix $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ is called a B-matrix if for any $i, j \in \mathbb{N} = \{1, 2, \dots, n\}$,

$$\sum_{k\in N} m_{ik} > 0, \qquad \frac{1}{n}\left(\sum_{k\in N} m_{ik}\right) > m_{ij}, \quad j \neq i.$$

Definition 2 ([12]) A matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is called a weakly chained diagonally dominant (*wcdd*) matrix if A is diagonally dominant, *i.e.*,

$$|a_{ii}| \geq r_i(A) = \sum_{j=1,\neq i}^{n} |a_{ij}|, \quad \forall i \in \mathbb{N},$$

and for each $i \notin J(A) = \{i \in \mathbb{N} : |a_{ii}| > r_i(A)\} \neq \emptyset$, there is a sequence of nonzero elements of A of the form $a_{ii_1}, a_{i_1 i_2}, \ldots, a_{i_r j}$ with $j \in J(A)$.

Definition 3 ([13]) A matrix $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ is called a weakly chained diagonally dominant (*wcdd*) B-matrix if it can be written in the form $M = B^+ + C$ with B^+ a *wcdd* matrix whose diagonal entries are all positive.

García-Esnaola *et al.* [8] gave the upper bound for $\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty$ when M is a B-matrix: Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a B-matrix with the form

$$M = B^+ + C,$$

where

$$B^+ = [b_{ij}] = \begin{bmatrix} m_{11} - r_1^+ & \cdots & m_{1n} - r_1^+ \\ \vdots & & \vdots \\ m_{n1} - r_n^+ & \cdots & m_{nn} - r_n^+ \end{bmatrix}, \tag{1}$$

and $r_i^+ = \max\{0, m_{ij} | j \neq i\}$. Then

$$\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty \leq \frac{n-1}{\min\{\beta, 1\}}, \tag{2}$$

where $\beta = \min_{i\in\mathbb{N}}\{\beta_i\}$ and $\beta_i = b_{ii} - \sum_{j\neq i} |b_{ij}|$.

To improve the bound in (2), Li *et al.* [14] presented the following result: Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is defined as (1). Then

$$\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty \leq \sum_{i=1}^{n} \frac{n-1}{\min\{\bar{\beta}_i, 1\}} \prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}|\right), \tag{3}$$

where $\bar{\beta}_i = b_{ii} - \sum_{j=i+1}^{n} |b_{ij}| l_i(B^+)$, $l_k(B^+) = \max_{k\leq i\leq n}\{\frac{1}{|b_{ii}|} \sum_{j=k,\neq i}^{n} |b_{ij}|\}$ and

$$\prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}|\right) = 1, \quad \text{if } i = 1.$$

Recently, when M is a weakly chained diagonally dominant (*wcdd*) B-matrix, Li *et al.* [13] gave a bound for $\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty$: Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a *wcdd* B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is defined as (1). Then

$$\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty \leq \sum_{i=1}^{n} \left(\frac{n-1}{\min\{\tilde{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j}\right), \tag{4}$$

where $\tilde{\beta}_i = b_{ii} - \sum_{j=i+1}^{n} |b_{ij}| > 0$ and $\prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j} = 1$ if $i = 1$.

This bound in (4) holds when M is a B-matrix since a B-matrix is a weakly chained diagonally dominant B-matrix [13].

Now, some notation is given, which will be used in the sequel. Let $A = [a_{ij}] \in \mathbb{R}^{n\times n}$. For $i, j, k \in \mathbb{N}$, denote

$$u_i(A) = \frac{1}{|a_{ii}|} \sum_{j=i+1}^{n} |a_{ij}|, \qquad u_n(A) = 0,$$

$$b_k(A) = \max_{k+1 \le i \le n} \left\{ \frac{\sum_{j=k,\neq i}^{n} |a_{ij}|}{|a_{ii}|} \right\}, \qquad b_n(A) = 1,$$

$$p_k(A) = \max_{k+1 \le i \le n} \left\{ \frac{|a_{ik}| + \sum_{j=k+1,\neq i}^{n} |a_{ij}| b_k(A)}{|a_{ii}|} \right\}, \qquad p_n(A) = 1.$$

The rest of this paper is organized as follows: In Section 2, we present some new bounds for $\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty$ when M is a *wcdd* B-matrix. Numerical examples are given to verify the corresponding results in Section 3.

2 Main results

In this section, some new upper bounds for $\max_{d\in[0,1]^n} \|(I - D + DM)^{-1}\|_\infty$ are provided when M is a *wcdd* B-matrix. Firstly, several lemmas, which will be used later, are given.

Lemma 1 ([13]) *Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a wcdd B-matrix with the form $M = B^+ + C$, where B^+ is defined as* (1). *Then*

$$\left\| \left(I + \left(B_D^+\right)^{-1} C_D\right)^{-1} \right\|_\infty \le n - 1,$$

where $B_D^+ = I - D + DB^+$ and $C_D = DC$.

Lemma 2 ([15]) *Let $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ be a wcdd M-matrix with $u_k(A)p_k(A) < 1$ ($\forall k \in \mathbb{N}$). Then*

$$\|A^{-1}\|_\infty \le \max\left\{ \sum_{i=1}^{n} \left(\frac{1}{a_{ii}(1 - u_i(A)p_i(A))} \prod_{j=1}^{i-1} \frac{u_j(A)}{1 - u_j(A)p_j(A)} \right), \right.$$
$$\left. \sum_{i=1}^{n} \left(\frac{p_i(A)}{a_{ii}(1 - u_i(A)p_i(A))} \prod_{j=1}^{i-1} \frac{1}{1 - u_j(A)p_j(A)} \right) \right\},$$

where

$$\prod_{j=1}^{i-1} \frac{u_j(A)}{1 - u_j(A)p_j(A)} = 1, \qquad \prod_{j=1}^{i-1} \frac{1}{1 - u_j(A)p_j(A)} = 1, \quad \textit{if } i = 1.$$

Lemma 3 ([14]) *Let $\gamma > 0$ and $\eta \ge 0$. Then, for any $x \in [0,1]$,*

$$\frac{1}{1 - x + \gamma x} \le \frac{1}{\min\{\gamma, 1\}}, \qquad \frac{\eta x}{1 - x + \gamma x} \le \frac{\eta}{\gamma}.$$

Theorem 1 *Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a wcdd B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is defined as* (1). *If, for each $i \in \mathbb{N}$,*

$$\hat{\beta}_i = b_{ii} - \sum_{j=i+1}^{n} |b_{ij}| p_i\big(B^+\big) > 0,$$

then

$$\max_{d\in[0,1]^n} \big\| (I - D + DM)^{-1} \big\|_\infty \le \max\left\{ \sum_{i=1}^{n} \frac{n-1}{\min\{\hat{\beta}_i, 1\}} \prod_{j=1}^{i-1} \left(\frac{1}{\hat{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}| \right), \sum_{i=1}^{n} \frac{(n-1)p_i(B^+)}{\min\{\hat{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\hat{\beta}_j} \right\}, \tag{5}$$

where

$$\prod_{j=1}^{i-1} \left(\frac{1}{\hat{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}| \right) = 1, \qquad \prod_{j=1}^{i-1} \frac{b_{jj}}{\hat{\beta}_j} = 1, \quad \text{if } i = 1.$$

Proof Let $M_D = I - D + DM$. Then

$$M_D = I - D + DM = I - D + D\big(B^+ + C\big) = B_D^+ + C_D,$$

where $B_D^+ = I - D + DB^+$. Similar to the proof of Theorem 2 in [13], we see that B_D^+ is a *wcdd* M-matrix with positive diagonal elements and $C_D = DC$, and, by Lemma 1,

$$\big\| M_D^{-1} \big\|_\infty \le \big\| \big(I + \big(B_D^+\big)^{-1} C_D\big)^{-1} \big\|_\infty \big\| \big(B_D^+\big)^{-1} \big\|_\infty \le (n-1) \big\| \big(B_D^+\big)^{-1} \big\|_\infty. \tag{6}$$

By Lemma 2, we have

$$\begin{aligned} \big\| \big(B_D^+\big)^{-1} \big\|_\infty \le \max\Bigg\{ & \sum_{i=1}^{n} \frac{1}{(1 - d_i + b_{ii}d_i)(1 - u_i(B_D^+)p_i(B_D^+))} \prod_{j=1}^{i-1} \frac{u_j((B_D^+))}{1 - u_j((B_D^+))p_j(B_D^+)}, \\ & \sum_{i=1}^{n} \frac{p_i(B_D^+)}{(1 - d_i + b_{ii}d_i)(1 - u_i((B_D^+))p_i(B_D^+))} \prod_{j=1}^{i-1} \frac{1}{1 - u_j(B_D^+)p_j(B_D^+)} \Bigg\}. \end{aligned}$$

By Lemma 3, we can easily get the following results: for each $i, j, k \in \mathbb{N}$,

$$b_k\big(B_D^+\big) = \max_{k+1\le i\le n} \left\{ \frac{\sum_{j=k,\ne i}^{n} |b_{ij}| d_i}{1 - d_i + b_{ii}d_i} \right\} \le \max_{k+1\le i\le n} \left\{ \frac{\sum_{j=k,\ne i}^{n} |b_{ij}|}{b_{ii}} \right\} = b_k\big(B^+\big),$$

$$\begin{aligned} p_k\big(B_D^+\big) &= \max_{k+1\le i\le n} \left\{ \frac{|b_{ik}| d_i + \sum_{j=k+1,\ne i}^{n} |b_{ij}| d_i b_k(B_D^+)}{1 - d_i + b_{ii}d_i} \right\} \\ &\le \max_{k+1\le i\le n} \left\{ \frac{|b_{ik}| + \sum_{j=k+1,\ne i}^{n} |b_{ij}| b_k(B_D^+)}{b_{ii}} \right\} \\ &\le \max_{k+1\le i\le n} \left\{ \frac{|b_{ik}| + \sum_{j=k+1,\ne i}^{n} |b_{ij}| b_k(B^+)}{b_{ii}} \right\} \\ &= p_k\big(B^+\big), \end{aligned}$$

and

$$\begin{aligned}\frac{1}{(1-d_i+b_{ii}d_i)(1-u_i(B_D^+)p_i(B_D^+))} &= \frac{1}{1-d_i+b_{ii}d_i-\sum_{j=i+1}^{n}|b_{ij}|d_ip_i(B_D^+)} \\ &\le \frac{1}{\min\{b_{ii}-\sum_{j=i+1}^{n}|b_{ij}|p_i(B^+),1\}} \\ &= \frac{1}{\min\{\hat{\beta}_i,1\}}. \end{aligned} \tag{7}$$

Furthermore, by Lemma 3, we have

$$\begin{aligned}\frac{u_i(B_D^+)}{1-u_i(B_D^+)p_i(B_D^+)} &= \frac{\sum_{j=i+1}^{n}|b_{ij}|d_i}{1-d_i+b_{ii}d_i-\sum_{j=i+1}^{n}|b_{ij}|d_ip_i(B_D^+)} \\ &\le \frac{\sum_{j=i+1}^{n}|b_{ij}|}{b_{ii}-\sum_{j=i+1}^{n}|b_{ij}|p_i(B^+)} \\ &= \frac{1}{\hat{\beta}_i}\sum_{j=i+1}^{n}|b_{ij}| \end{aligned} \tag{8}$$

and

$$\begin{aligned}\frac{1}{1-u_i(B_D^+)p_i(B_D^+)} &= \frac{1-d_i+b_{ii}d_i}{1-d_i+b_{ii}d_i-\sum_{j=i+1}^{n}|b_{ij}|d_ip_i(B_D^+)} \\ &\le \frac{1-d_i+b_{ii}d_i}{b_{ii}-\sum_{j=i+1}^{n}|b_{ij}|p_i(B^+)} \\ &= \frac{b_{ii}}{\hat{\beta}_i}. \end{aligned} \tag{9}$$

By (7), (8), and (9), we obtain

$$\left\|\left(B_D^+\right)^{-1}\right\|_\infty \le \max\left\{\sum_{i=1}^{n}\frac{1}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\left(\frac{1}{\hat{\beta}_j}\sum_{k=j+1}^{n}|b_{jk}|\right),\sum_{i=1}^{n}\frac{p_i(B^+)}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\hat{\beta}_j}\right\}. \tag{10}$$

Therefore, the result in (5) holds from (6) and (10). □

Since a B-matrix is also a *wcdd* B-matrix, then by Theorem 1, we find the following result.

Corollary 1 *Let* $M=[m_{ij}]\in\mathbb{R}^{n\times n}$ *be a* B*-matrix with the form* $M=B^++C$*, where* $B^+=[b_{ij}]$ *is defined as* (1). *Then*

$$\begin{aligned}&\max_{d\in[0,1]^n}\left\|(I-D+DM)^{-1}\right\|_\infty \\ &\quad\le \max\left\{\sum_{i=1}^{n}\frac{n-1}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\left(\frac{1}{\hat{\beta}_j}\sum_{k=j+1}^{n}|b_{jk}|\right),\sum_{i=1}^{n}\frac{(n-1)p_i(B^+)}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\hat{\beta}_j}\right\}, \end{aligned} \tag{11}$$

where $\hat{\beta}_i$ *is defined as in Theorem* 1.

We next give a comparison of the bounds in (4) and (5) as follows.

Theorem 2 *Let $M = [m_{ij}] \in \mathbb{R}^{n\times n}$ be a wcdd B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is defined as* (1). *Let $\bar{\beta}_i$, $\tilde{\beta}_i$, and $\hat{\beta}_i$ be defined as in* (3), (4), *and* (5), *respectively. Then*

$$\max\left\{\sum_{i=1}^{n}\frac{n-1}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\left(\frac{1}{\hat{\beta}_j}\sum_{k=j+1}^{n}|b_{jk}|\right),\sum_{i=1}^{n}\frac{(n-1)p_i(B^+)}{\min\{\hat{\beta}_i,1\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\hat{\beta}_j}\right\}$$
$$\leq\sum_{i=1}^{n}\left(\frac{n-1}{\min\{\tilde{\beta}_i,1\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\tilde{\beta}_j}\right). \tag{12}$$

Proof Since B^+ is a *wcdd* matrix with positive diagonal elements, for any $i \in \mathbb{N}$,

$$0 \leq p_i(B^+) \leq 1, \qquad \tilde{\beta}_i \leq \hat{\beta}_i. \tag{13}$$

By (13), for each $i \in \mathbb{N}$,

$$\frac{1}{\hat{\beta}_i} \leq \frac{1}{\tilde{\beta}_i}, \qquad \frac{1}{\min\{\hat{\beta}_i,1\}} \leq \frac{1}{\min\{\tilde{\beta}_i,1\}}. \tag{14}$$

The result in (12) follows by (13) and (14). □

Remark 1
(i) Theorem 2 shows that the bound in (5) is better than that in (4).
(ii) When n is very large, one needs more computations to obtain these upper bounds by (5) than by (4).

3 Numerical examples

In this section, we present numerical examples to illustrate the advantages of our derived results.

Example 1 Consider the family of B-matrices in [14]:

$$M_k = \begin{bmatrix} 1.5 & 0.5 & 0.4 & 0.5 \\ -0.1 & 1.7 & 0.7 & 0.6 \\ 0.8 & -0.1\frac{k}{k+1} & 1.8 & 0.7 \\ 0 & 0.7 & 0.8 & 1.8 \end{bmatrix},$$

where $k \geq 1$. Then $M_k = B_k^+ + C_k$, where

$$B_k^+ = \begin{bmatrix} 1 & 0 & -0.1 & 0 \\ -0.8 & 1 & 0 & -0.1 \\ 0 & -0.1\frac{k}{k+1} - 0.8 & 1 & -0.1 \\ -0.8 & -0.1 & 0 & 1 \end{bmatrix}.$$

By (2), we have

$$\max_{d\in[0,1]^4}\left\|(I - D + DM_k)^{-1}\right\|_\infty \leq \frac{4-1}{\min\{\beta,1\}} = 30(k+1).$$

It is obvious that

$$30(k+1) \to +\infty, \quad \text{if } k \to +\infty.$$

By (3), we get

$$\max_{d\in[0,1]^4} \left\|(I-D+DM_k)^{-1}\right\|_\infty \le 15.2675.$$

By Theorem 7 of [11], we have

$$\max_{d\in[0,1]^4} \left\|(I-D+DM_k)^{-1}\right\|_\infty \le 13.6777.$$

By Corollary 1 of [13], we have

$$\max_{d\in[0,1]^4} \left\|(I-D+DM_k)^{-1}\right\|_\infty \le \sum_{i=1}^{4}\left(\frac{3}{\min\{\tilde{\beta}_i,1\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\tilde{\beta}_j}\right) \approx 15.2675.$$

By (11), we obtain

$$\max_{d\in[0,1]^4} \left\|(I-D+DM_k)^{-1}\right\|_\infty \le 9.9683.$$

In these two cases, the bounds in (2) are equal to 60 ($k=1$) and 90 ($k=2$), respectively.

Example 2 Consider the *wcdd B*-matrix in [13]:

$$M = \begin{bmatrix} 1.5 & 0.2 & 0.4 & 0.5 \\ -0.1 & 1.5 & 0.5 & 0.1 \\ 0.5 & -0.1 & 1.5 & 0.1 \\ 0.4 & 0.4 & 0.8 & 1.8 \end{bmatrix}.$$

Then $M = B^+ + C$, where

$$B^+ = \begin{bmatrix} 1 & -0.3 & -0.1 & 0 \\ -0.6 & 1 & 0 & -0.4 \\ 0 & -0.6 & 1 & -0.4 \\ -0.4 & -0.4 & 0 & 1 \end{bmatrix}.$$

By (4), we get

$$\max_{d\in[0,1]^4} \left\|(I-D+DM)^{-1}\right\|_\infty \le 41.1111.$$

By (5), we have

$$\max_{d\in[0,1]^4} \left\|(I-D+DM)^{-1}\right\|_\infty \le 21.6667.$$

4 Conclusions

In this paper, we present some new upper bounds for $\max_{d\in[0,1]^n} \|(I-D+DM)^{-1}\|_\infty$ when M is a weakly chained diagonally dominant B-matrix, which improve some existing results. A numerical example shows that the given bounds are efficient.

Competing interests
The author declares that he has no competing interests.

Author's contributions
Only the author contributed to this work. The author read and approved the final manuscript.

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