

# Error Estimation of INS Ground Alignment Through Observability Analysis

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**A systematic analysis of the observability of an inertial navigation system (INS) in ground alignment with Bar-Itzhack and Berman's error model is presented. It is shown that the unobservable states are separately contained in two decoupled subspaces. The constraints on the selection of unobservable states are discussed. An estimation algorithm which is derived fully from the horizontal velocity outputs for computing the misalignment angles is provided. It reveals that the azimuth error can be entirely estimated from the estimates of leveling error and leveling error rate, without using gyro output signals explicitly.**

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## I. INTRODUCTION

Initial misalignment is one of the major error sources of inertial navigation systems (INS). For terrestrial navigation, the initial alignment errors will affect the system error not only in the attitude indication but also in the velocity and position information [1-3]. Therefore, prior to normal navigation, alignment process must be performed to determine the orientation of the platform axes with respect to the navigation coordinate frame. One method for obtaining the initial angular orientation is through the use of an external reference by optical means. However, this method is very limited to operational environments. Alternatively, for most ground based applications, a self-contained alignment method known as gyrocompassing [2-7] provides another operational approach. As a general rule, gyrocompassing consists of two phases, that is, leveling and azimuth alignment. The basic principle of gyrocompassing consists of feeding signals proportional to the accelerometer outputs or/and velocity error outputs to the appropriate level gyros and azimuth gyro.

The purpose of initial alignment process is to drive the misalignment angles to zero. Unfortunately, this goal can never be achieved in a practical system. This drawback is deduced from the sensor errors which cannot be compensated ideally. From the control theoretical point of view, the basic difficulty associated with the self-alignment technique is that the system is not completely observable [8-10].

The determination of unobservable states during initial alignment process is very important in consideration of system performance. Generally, the system state variables can be transformed into the observable canonical form. It means that the observable part and unobservable part can be separated intentionally. However, the choice of observable (unobservable) states is certainly not unique since the number of transformations are innumerable. Early research workers showed that the determination of unobservable states was a part of physical judgments. This interesting problem of selection of the suitable unobservable states motivates us to investigate the observability of INS during ground alignment phase of operation.

We examine the observability of a linearized INS error model of a stationary vehicle. The horizontal velocity outputs of INS are the system measurements. We have found that the unobservable states which are distributed in two decoupled subspaces can be systematically determined. Also, the leveling errors can be estimated from the system measurements and their first derivatives. However, to estimate the azimuth error, the second derivative of system measurement is needed. It is equivalent to state that the estimation of azimuth error can be obtained from

the estimates of leveling error and leveling error rate. This phenomenon may facilitate the designing of filters for leveling and azimuth alignment simultaneously, without using gyro outputs explicitly.

## II. GROUND ALIGNMENT ERROR MODEL

In order to look into the behavior of an inertial navigator, a proper error model is now derived. It is well known that the description of the INS error propagation using a linearized error model is quite a good approximation. The characteristics of INSs can be derived from the linear error model.

Many different error models can be found in the literature. For the purpose of our analyses, we adopt the Bar-Itzhack and Berman's [10] derivation of the error model for INS in ground alignment. Assuming the navigation system coordinate frame is the local-level north (N), east (E), and down (D) coordinate system. Since the coupling of the vertical channel with the horizontal channels is very weak, the vertical channel can be ignored. When the initial alignment process is done at a fixed ground base where the geographic position is known precisely, then the gravity error and the position error state can be ruled out. Moreover, the system is nearly stationary and hence the coriolis acceleration can also be ignored. Under these assumptions, the error dynamics including the horizontal velocity errors and the attitude errors can be made considerably simple. In this case, the INS ground alignment error model can be written as [10]

$$\begin{bmatrix} \delta \dot{v}_N \\ \delta \dot{v}_E \\ \dot{\phi}_N \\ \dot{\phi}_E \\ \dot{\phi}_D \end{bmatrix} = \begin{bmatrix} 0 & 2\Omega_D & 0 & g & 0 \\ -2\Omega_D & 0 & -g & 0 & 0 \\ 0 & 0 & 0 & \Omega_D & 0 \\ 0 & 0 & -\Omega_D & 0 & \Omega_N \\ 0 & 0 & 0 & -\Omega_N & 0 \end{bmatrix} \begin{bmatrix} \delta v_N \\ \delta v_E \\ \phi_N \\ \phi_E \\ \phi_D \end{bmatrix} + \begin{bmatrix} \delta a_N \\ \delta a_E \\ \delta \omega_N \\ \delta \omega_E \\ \delta \omega_D \end{bmatrix} \quad (1)$$

where  $\delta v$  and  $\phi$  represent the velocity error and attitude error, respectively;  $\delta a$  and  $\delta \omega$  represent, respectively, the generalized accelerometer error and the generalized gyro drift rate [11];  $g$  is the local gravity;  $\Omega$  represents the Earth rate; finally, the subscripts N, E, and D denote the corresponding components in the North-East-Down navigation coordinate system. Note that a form of this error model can be used for gimbal systems as well as strapdown systems [11, 12].

Rewrite (1) in compact notation

$$\dot{\mathbf{x}}' = \mathbf{A}'\mathbf{x}' + \mathbf{b} \quad (2)$$

where the variables  $\mathbf{x}'$ ,  $\mathbf{A}'$ , and  $\mathbf{b}$  are identified with its counterpart in (1). It is reasonable and practical to assume the generalized accelerometer errors and gyro drift rates as constant in ground alignment phase. Hence, the sensor errors can be modeled as

$$\dot{\mathbf{b}} = \begin{bmatrix} \delta \mathbf{a} \\ \delta \dot{\omega} \end{bmatrix} = 0 \quad (3)$$

where  $\delta \mathbf{a} = [\delta a_N, \delta a_E]^T$  represents the generalized accelerometer error vector and  $\delta \omega = [\delta \omega_N, \delta \omega_E, \delta \omega_D]^T$  represents the generalized gyro drift rate vector.

Combing (2) and (3), yields

$$\begin{bmatrix} \dot{\mathbf{x}}' \\ \dot{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}' & \mathbf{I} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}' \\ \mathbf{b} \end{bmatrix} \quad (4)$$

where  $\mathbf{I}$  is the identity matrix. This linear dynamic error model can also be expressed in compact notation as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (5)$$

where  $\mathbf{x}^T = [\mathbf{x}'^T, \delta \mathbf{a}^T, \delta \omega^T]$  and the definition of  $\mathbf{A}$  is obvious from (4).

Finally, we consider the outputs of INS horizontal velocity components as the system measurements, namely,

$$\mathbf{z} = [z_1, z_2]^T = [\delta v_N, \delta v_E]^T. \quad (6)$$

Then the relationship between the measurements and the system  $\mathbf{A}'$  in (2) can be written as

$$\mathbf{z} = \mathbf{C}'\mathbf{x}' \quad (7)$$

where

$$\mathbf{C}' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \quad (8)$$

Similarly, the relationship between the measurements and the system  $\mathbf{A}$  in (5) can be written as

$$\mathbf{z} = \mathbf{C}\mathbf{x} \quad (9)$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

Now, the observability of the system  $\mathbf{A}'$  and the system  $\mathbf{A}$  can be analyzed.

## III. OBSERVABILITY EXAMINATION

Consider a linear time-invariant system. If the rank of the observability test matrix is equal to the order of the system, then the system is completely observable. On the contrary, if the system is not completely observable, the number of unobservable states is the difference between the order of the system and the rank of the observability test matrix. In general, the

observability test matrix for the system matrix  $A$  with measurement matrix  $C$  can be expressed as [8]

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (11)$$

where  $n$  is the order of the system. Therefore, the necessary condition for the observable system is that the observability test matrix is full rank.

Now, the observability test matrix for the system  $A'$  with measurement matrix  $C'$  can be written as

$$Q' = \begin{bmatrix} C' \\ C'A' \\ C'A'^2 \end{bmatrix}. \quad (12)$$

After some explicit calculations by substituting (8) and the definition of  $A'$  (see (2)) into (12), it is easily seen that the rank of  $Q'$  is 5 which is equal to the order of the system  $A'$ . Thus the matrix  $Q'$  is full rank. So, the system  $A'$  is completely observable. It implies that if the sensor errors are fully compensated, the estimation problem during alignment process can be automatically solved.

For the system  $A$  with measurement matrix  $C$ , the observability test matrix can be written as

$$Q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^4 \end{bmatrix}. \quad (13)$$

After some explicit calculations by substituting (10) and the definition of  $A$  (see (5)) into (13), it can be found that the rank of  $Q$  is 7 which is smaller than the system order 10. Thus the matrix  $Q$  is not full rank. So, the system  $A$  is not completely observable and the estimation becomes an unsolvable problem with three unobservable states. It is clear that the observability loss in system  $A$  is generated by the augmentation from (2)–(5). It implies that the system can be made observable if the dynamics of the sensor errors are ignored. However, this assumption is practically weak. The determination of the unobservable states is now a key problem for estimation in alignment and calibration phase of operation.

Since the system measurements are observable by definition, states  $\delta v_N$  and  $\delta v_E$  are undoubtedly observable. For convenience, let's define the following:

$$\mathbf{x}_1 = [\delta v_N, \delta v_E]^T \quad (14)$$

$$\mathbf{x}_2 = [\phi_E, \delta a_N, \delta \omega_N, \delta \omega_D]^T \quad (15)$$

$$\mathbf{x}_3 = [\phi_N, \phi_D, \delta a_E, \delta \omega_E]^T \quad (16)$$

and

$$\mathbf{y}_1 = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (17)$$

$$\mathbf{y}_2 = \begin{bmatrix} \dot{z}_1 - 2\Omega_D z_2 \\ \ddot{z}_2 + 4\Omega_D^2 z_2 \\ z_1^{(iii)} + 8\Omega_D^3 z_2 \\ z_2^{(iv)} - 16\Omega_D^4 z_2 \end{bmatrix} \quad (18)$$

$$\mathbf{y}_3 = \begin{bmatrix} \dot{z}_2 + 2\Omega_D z_1 \\ \ddot{z}_1 + 4\Omega_D^2 z_1 \\ z_2^{(iii)} - 8\Omega_D^3 z_1 \\ z_1^{(iv)} - 16\Omega_D^4 z_1 \end{bmatrix}. \quad (19)$$

Since the rank of a matrix is invariant under elementary row operations, the observability associated with the matrix  $Q$ , (13), is equivalent to the solvability of the following equation

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} \quad (20)$$

where  $I$  is the identity matrix

$$Q_2 = \begin{bmatrix} g & 1 & 0 & 0 \\ -3g\Omega_D & -2\Omega_D & -g & 0 \\ -7g\Omega_D^2 - g\Omega_N^2 & -4\Omega_D^2 & -3g\Omega_D & g\Omega_N \\ 15g\Omega_D^3 + 3g\Omega_N^2\Omega_D & 8\Omega_D^3 & 7g\Omega_D^2 & -3g\Omega_N\Omega_D \end{bmatrix} \quad (21)$$

and

$$Q_3 = \begin{bmatrix} -g & 0 & 1 & 0 \\ -3g\Omega_D & g\Omega_N & 2\Omega_D & g \\ 7g\Omega_D^2 & -3g\Omega_N\Omega_D & -4\Omega_D^2 & -3g\Omega_D \\ 15g\Omega_D^3 + g\Omega_N^2\Omega_D & -7g\Omega_N\Omega_D^2 - g\Omega_N^3 & -8\Omega_D^3 & -7g\Omega_D^2 - g\Omega_N^2 \end{bmatrix}. \quad (22)$$

Equation (20) shows that the observability of the system can be determined by the solvability of three decoupled matrix equations. It is obvious that  $\mathbf{x}_1$  is observable. Hence the three unobservable states must reside in  $\mathbf{x}_2$  and  $\mathbf{x}_3$  which are, respectively, governed by

$$\mathbf{y}_2 = Q_2 \mathbf{x}_2 \quad (23)$$

and

$$\mathbf{y}_3 = Q_3 \mathbf{x}_3. \quad (24)$$

It is easily found that when the system is not at the Earth pole,  $\Omega_N \neq 0$ . Notice that the first column of  $Q_2$  is a linear combination of other three columns. So, from (21), the rank of  $Q_2$  is 3 which is one less than the order of  $Q_2$ . Therefore, only one unobservable state can be chosen from the components of  $\mathbf{x}_2$ .

Similarly, because the first column of  $Q_3$  is a linear combination of the third and fourth columns, the second column of  $Q_3$  is equal to the fourth column times  $\Omega_N$ . From (22), the rank of  $Q_3$  is 2. It is thus obvious that there are two unobservable states contained in  $\mathbf{x}_3$ . By observation,  $\phi_D$  and  $\delta\omega_E/\Omega_N$  have the same effect on the measurement derivatives  $\mathbf{y}_3$ . Therefore, (24) can be rewritten as

$$\mathbf{y}_3 = \begin{bmatrix} -g & 0 & 1 \\ -3g\Omega_D & g\Omega_N & 2\Omega_D \\ 7g\Omega_D^2 & -3g\Omega_N\Omega_D & -4\Omega_D^2 \\ 15g\Omega_D^3 + g\Omega_N^2\Omega_D & -7g\Omega_N\Omega_D^2 - g\Omega_N^3 & -8\Omega_D^3 \end{bmatrix} \times \begin{bmatrix} \phi_N \\ \phi_D + \frac{\delta\omega_E}{\Omega_N} \\ \delta a_E \end{bmatrix} \quad (25)$$

which shows that only one of  $\phi_D$  and  $\delta\omega_E$  can be observed at a time. However, they can be chosen as unobservable states simultaneously. Thus  $\phi_N$  and  $\delta a_E$  must be observable.

Note that when the system is located at the Earth pole,  $\Omega_N = 0$ , both the fourth column of  $Q_2$  and the second column of  $Q_3$  are zero. Then both  $\delta\omega_D$  and  $\phi_D$  are definitely unobservable. That is why the INS cannot be self-aligned at the pole. In this case, the rank of  $Q_2$  is reduced to 2 and the rank of  $Q_3$  is unchanged.

#### IV. ESTIMATION OF MISALIGNMENT ANGLES

The objective of initial alignment is to drive the misalignment angles  $\phi_N$ ,  $\phi_E$ , and  $\phi_D$  to zero or as small as possible. It is necessary that these states be all observable. Following the discussions given in Section III,  $\delta a_E$  and  $\delta\omega_E$  in  $\mathbf{x}_3$  are inevitably unobservable. Then, only one unobservable state can be chosen from the components of  $\mathbf{x}_2$ . Theoretically, the choice is arbitrary except  $\phi_E$ . However, in order to achieve better accuracy, the unobservable state must be selected carefully. Intuitively, from the first two columns of  $Q_2$ , it is obvious that  $\phi_E$  and  $\delta a_N/g$  are strongly coupled. Besides, if we do not choose  $\delta a_N$  as the unobservable state, more time derivatives of measurements are needed to compute the estimate of  $\phi_E$ . That causes poor estimation which should be avoided in practice. Therefore, the best choice of the unobservable states are  $\delta a_N$ ,  $\delta a_E$ , and  $\delta\omega_E$  for the system in ground alignment process. In this case, both  $\delta\omega_N$  and  $\delta\omega_D$  can also be estimated for the purpose of calibration.

Once the unobservable states have been selected, we can engage in designing an estimation algorithm for computing the estimates of misalignment angles. Combing the first equation of (23) and the first two

equations of (25), we have

$$\dot{z}_1 - 2\Omega_D z_2 = g\phi_E + \delta a_N \quad (26)$$

$$z_2 + 2\Omega_D z_1 = -g\phi_N + \delta a_E \quad (27)$$

$$\ddot{z}_1 + 4\Omega_D^2 z_1 = -3g\Omega_D \phi_N + g\Omega_N \left( \phi_D + \frac{\delta\omega_E}{\Omega_N} \right) + 2\Omega_D \delta a_E. \quad (28)$$

Substituting (6) into the above equations and solving for misalignment angles, yields

$$\phi_N = -\frac{1}{g}(\delta\dot{v}_E + 2\Omega_D \delta v_N - \delta a_E) \quad (29)$$

$$\phi_E = \frac{1}{g}(\delta\dot{v}_N - 2\Omega_D \delta v_E - \delta a_N) \quad (30)$$

$$\phi_D = \frac{1}{g\Omega_N}(\delta\ddot{v}_N - 3\Omega_D \delta\dot{v}_E - 2\Omega_D^2 \delta v_N + \Omega_D \delta a_E) - \frac{\delta\omega_E}{\Omega_N}. \quad (31)$$

Since we have chosen  $\delta a_E$ ,  $\delta a_N$ , and  $\delta\omega_E$  as unobservable states, the best estimates can be obtained by setting these unobservable states to zero, i.e.,

$$\hat{\phi}_N = -\frac{1}{g}(\delta\dot{v}_E + 2\Omega_D \delta v_N) \quad (32)$$

$$\hat{\phi}_E = \frac{1}{g}(\delta\dot{v}_N - 2\Omega_D \delta v_E) \quad (33)$$

$$\hat{\phi}_D = \frac{1}{g\Omega_N}(\delta\ddot{v}_N - 3\Omega_D \delta\dot{v}_E - 2\Omega_D^2 \delta v_N). \quad (34)$$

These equations show that the leveling errors,  $\phi_N$  and  $\phi_E$ , can be estimated from the system measurements and their first time derivatives, and that the azimuth error,  $\phi_D$ , can be estimated from the measurements and their time derivatives up to second order.

It is evident, from (32)–(34) and (29)–(31), that the errors in the estimation are

$$\begin{bmatrix} \delta\phi_N \\ \delta\phi_E \\ \delta\phi_D \end{bmatrix} = \begin{bmatrix} \hat{\phi}_N - \phi_N \\ \hat{\phi}_E - \phi_E \\ \hat{\phi}_D - \phi_D \end{bmatrix} = \begin{bmatrix} -\frac{\delta a_E}{g} \\ \frac{\delta a_N}{g} \\ \frac{\delta\omega_E}{\Omega_N} - \frac{\delta a_E}{g} \tan L \end{bmatrix} \quad (35)$$

where  $L$  is the geographic latitude at the system location. This result is identical with the accuracy that is often shown in the self-alignment schemes [2, 7, 10]. Obviously, this is consistent with physical interpretations. The leveling estimation errors are caused by the accelerometer errors. The azimuth estimation error is caused by the east gyro drift rate and the north leveling error. Both of them are latitude dependent. For example, the leveling estimation error due to 1 mg accelerometer error is 3.4 arc-min. The azimuth estimation error due to 0.015 deg/h east gyro drift rate is 3.4 sec  $L$  arc-min; and due to 1 mg east accelerometer error is  $-3.4 \tan L$  arc-min.

Finally, differentiating (33), yields

$$\dot{\hat{\phi}}_E = \frac{1}{g}(\delta\ddot{v}_N - 2\Omega_D\delta\dot{v}_E). \quad (36)$$

Substituting (32) and (36) into (34), it can be found that

$$\dot{\hat{\phi}}_D = \frac{1}{\Omega_N}(\dot{\hat{\phi}}_E + \Omega_D\hat{\phi}_N) \quad (37)$$

which shows that the azimuth error can be computed from the estimates of the leveling error about north axis and the leveling error rate about east axis. Note that the estimation of azimuth error does not explicitly depend upon gyro output signals. This phenomenon can be used in an alternate filter design for leveling and azimuth alignment simultaneously.

It is also easy to modify the estimation algorithm for aligning an arbitrary wander-azimuth mechanization system.

## V. CONCLUSIONS

Based upon the Bar-Itzhack and Berman's INS error model, the observability of INS operating in ground alignment phase is analyzed. It is realized that the unobservable states are induced by the augmentation of sensor errors. In general, there are three unobservable states with one contained in  $\mathbf{x}_2 = [\phi_E, \delta a_N, \delta\omega_N, \delta\omega_D]^T$  and two contained in  $\mathbf{x}_3 = [\phi_N, \phi_D, \delta a_E, \delta\omega_E]^T$ . The selection of unobservable state from  $\mathbf{x}_2$  is arbitrary. However, the unobservable states in  $\mathbf{x}_3$  cannot be arbitrarily chosen in that  $\phi_D$  and  $\delta\omega_E$  cannot be observed at the same time.

When the system is located at the Earth pole, the number of unobservable states becomes 4 among which  $\phi_D$  and  $\delta\omega_D$  are definitely unobservable states.

The determination of the unobservable states is dependent upon mission requirements. For the purpose of alignment, the best choice of unobservable states are  $\delta a_N$ ,  $\delta a_E$ , and  $\delta\omega_E$ . An estimation algorithm has been derived for aligning the INS on ground stationary base. It is shown that the leveling errors can be estimated from the measured velocity outputs and their first time derivatives. While the second time derivative of north-velocity component is needed in estimating the azimuth misalignment angle. Furthermore, the estimated azimuth misalignment has been found proportional to the estimates of the leveling error about the north axis and the leveling error rate about the east axis. This property is useful for designing alternate filter algorithms for leveling and azimuth alignment.

Though this paper focuses on analyzing the local-level north-slaved system, it can be easily extended to cover arbitrary wander-azimuth mechanization systems.

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