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Error Patterns in Addition and Subtraction of Fractions among Form Two Students

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This article discusses the types of errors and the pattern of systematic errors often made by students in the operation of addition and subtraction of fractions. The sample consisted of 80 Form Two students from a national secondary school in the Selangor state. The research instrument consists of a set of paper and pencil tests containing 40 items in addition and 40 items in the subtraction of fractional operations. The findings indicate that errors in addition operations are 29.8% careless errors, negligence errors 26.3% and 11.1% systematic random errors. In systematic errors, 50.6% of students have a problem converting to the lowest common denominator, 26.2% encounter problems in the process of understanding, and 14.9% have problems dealing with improper fractions. As for the subtraction of fractions, there are 26.4% systematic errors, 10.3% careless errors, and 2.5% random errors. In systematic errors, 47.9% of students faced problems in the process of understanding.

Background of Study

Research on students' ability to perform operations on fractions has shown disappointingly poor results. Researchers have consistently commented on the huge percentage of individuals lacking basic fraction skills. In Malaysia too our children face difficulties with fractions both in primary and secondary school (Nur Fazilah, 2002; Valarmathy, 2004; Wan, 2002).

An examination of fraction development in the mathematics text book recommended by the Ministry of Education Malaysia reveals that foundation work in fraction development begins early in the children's schooling, which is in Year Three. A recurring area of concern for classroom teachers is that students in the primary school face problems in fractions (Nur Fazilah, 2002;

Valarmathy, 2004). Most alarmingly, although they have been exposed to the computing of fractions, students in secondary school still make significant errors in the addition and subtraction of fractions (Wan, 2002). A survey done by Kim (2003) on topics that are difficult in school mathematics reveals that all the 329 schools in Penang have listed fractions as one of the most difficult topics at the primary and secondary level.

The Malaysian Mathematics curriculum (MOE, 2002) currently is in alignment with the National Council of Teachers of Mathematics (NTCM, 2000) current standards where students in grade 6-8 continue to refine their understanding of arithmetic operations on fractions and develop algorithms for computing with fractions.

A study by Yea-Ling (2005) revealed that fractions were more difficult than decimals or whole numbers for low ability students. Low ability students tended to use rule-based methods more often than high ability students. Furthermore, low ability students relied on standard written algorithms more than reflecting on number-sense based methods. Different difficulties occur when students use fractions and when they use decimals. Students in the low ability group misunderstood the concept that multiplication does not always make the answer larger.

Students' errors are often systematic and rule-based rather than random (Yetkin, 2003). In addition to student inventiveness, these errors may be caused by instruction that focuses on rote memorization. Students abstract or generalize procedures from following the steps in worked-out examples, but when their knowledge is rote or insufficient they might over generalize the rules and procedures. For example, students having difficulty in adding fractions may extrapolate erroneous algorithms from instruction on the representation of fractions. Students who are often presented fractions using pie graphs perform "1/2+1/3=2/5" and justify the solution as "adding one piece of a two piece pie and one piece of a three piece pie will result in two pieces out of five pieces altogether." Using appropriate representations will help students construct different characteristics of these concepts.

In the past decades formal arithmetic with fractions in primary schools generally resulted in the great majority of students having to follow meaningless rules of calculation. As a consequence Watanabe (2001) recommends shifting the subject of formal reasoning with fractions from primary to secondary education. The arguments were based on issues related to curriculum, development and instructional materials. On the contrary, Powel and Hunting (2003) feel that the fraction concepts are developmentally appropriate for primary age children and should not be delayed until the intermediate grades, but rather nurtured and built upon throughout the students' school careers. They emphasize that the mathematical power that these children will bring to higher education and discovery becomes more personal if the concept of fraction is taught earlier.

Afzal Ahmad et al. (2004) illustrated that one of the difficulties in learning fractions, decimals and percentages is that they have multiplicity of meanings. This means that any particular number, say 3/5 (or 0.6 or 60%) can be interpreted concretely in many ways, all of which occur in many real-life applications. This is in contrast to whole numbers, which are used mainly either for counting discrete objects or counting repetitions of measuring units as in working out lengths, and so on.

One of the most frustrating subjects for teachers as well as their students is the study of fraction and more particularly, operations with fractions. Year after year, the students seem to learn to add, subtract, multiply and divide with fractions but it quickly becomes apparent that they tend to forget it all. An analysis of errors will pinpoint specific computational weaknesses in the addition and subtraction of fractions that are experienced by the majority of students.

Research on analysis of computational errors indicates that recognizing the error patterns of a child is the first step towards remediation (Cox, 1975). Ashlock's (1998) study on error patterns in computation also supports this. Ashlock noted that teachers need to be more sensitive to what pupils do if they want their pupils to compute successfully. To be able to do this, a purposeful diagnostic test has to be constructed; the pupil's written responses have to be analyzed and individual interview need to be conducted very carefully. As teachers identify and analyze pupils' errors, the procedures they used in their computations and the causes of their errors, teachers will gain understanding of the errors made. This will be very useful in helping them to develop effective instructional strategies.

The recent National Assessment of Educational Progress report shows that fractions are "exceedingly difficult for children to master (NAEP, 2001, p. 5). Additionally, students are frequently unable to remember prior experiences with fractions from lower grade levels (Groff, 1996). The NCTM states that just as students are struggling with learning fractions, so too are teachers feeling frustrated as they seek ways of teaching fractions effectively.

Early school failure can lead to a lack of self-confidence with subsequent detrimental effects on learning. Errors are seen as a basic and positive stage of the learning process. They are seen as a means to inquire into the nature of a subject. It is suggested that errors are a natural concomitant of students' attempts to integrate new material that they are taught with already established knowledge. Since erroneous rules cannot be avoided in instruction, educators are encouraged to use them as useful diagnostic tools to determine the nature of children's understanding of a mathematics topic (Emilie, 2004). The problems in diagnosis and remediation have been an important concern for both mathematics and special educators. As learning difficulties, in general, occur for a variety of reasons, similarly mathematics (quantitative and spatial) differences can occur in a variety of ways.

An analysis of our errors will lead to a better understanding of the topic in question. It is hoped that the findings highlighted in this article will help teachers in formulating teaching strategies and provide them with an idea of the error type and the reasons the errors in fractions have occurred. It is hoped that with this information, teachers will be able to improve on their instructional planning and pedagogical practices so that students will have a deeper conceptual understanding of fractions Furthermore, the instruments used would be beneficial to the teachers in diagnosing problems faced by their students in addition and subtraction of fractions and planning remedial work for them. The result of this study will be useful in various aspects to math educators.

Research Questions

To achieve its objectives, this study will concentrate on the following research questions:

1. What type of error in the addition of fractions is the most common among Form Two students?
2. What type of error in the subtraction of various fractions is most common among Form Two students?
3. What are the different types of systematic errors found in the addition of fractions among Form Two students?
4. What are the different types of systematic errors found in the subtraction of fractions among Form Two students?

Methodology

This study was carried out in one of the urban co-educational national secondary schools in Klang. The school is situated very near Klang town. The students come from a mixed socioeconomic status and the student composition is comprised 40% Chinese, 35% Malays, and 25% Indians. The school's co-curricula achievement is good and it has one of the best cricket teams in the state. In academic achievement the students fared poorly with a resulting 41.1% pass in the PMR 2006 and 7 students achieving 7A.

The target population of this study was 80 Form Two students currently studying in one of the national schools in Malaysia. These respondents were selected from a total of 273 Form Two students from varied socioeconomic backgrounds. A purposive sampling strategy was used in order to obtain a group of mixed ability students comprised of high, average and low achievers who were considered representative of the total population.

Table 1

Distribution of Subjects of the Study

Student Ability	Number of Students	Percentage
High achievers	25	31.25
Average	25	31.25
achievers		37.5
Low achievers	30	
Total	80	100

Table 1 represents the distribution of the subjects in this study. As shown in Table 1, out of the total sample of 80 students, 25 each were from the high ability group and average ability group, while the remaining 30 students were from the low ability group. Each of these students was tested on addition and subtraction of fractions.

The students in the sample were grouped into 3 different ability groupings. The division was based on their scores for the mathematics paper during the final year examination in Form One in 2006. The students with scores between 70 and 100 were classified as "high achievers". Those with scores between 45 and 69 were classified as "average achievers" and those scoring less than 45 were classified as "low achievers".

The Research Instrument

For this study the researchers adapted the instrument from Kallom (1924). As revised, it comprised 40 test items which fall into eight levels for the subtraction of fractions. For the classification of systematic errors in addition and subtraction of fractions, the researchers used the classification by Brueckner (1930). The fractions used in the tests for addition of fractions were restricted to those having a denominator not larger than 21 and a common denominator not larger than 36. For the subtraction of fractions, the test restricted the fractions used to those having a denominator not larger than 15 and a common denominator not larger than 30. The computations in the test were kept as simple as possible so that errors due to faulty handling of fractions would be revealed rather than errors due to difficult computation. Test items were also obtained from the Form Two textbook used in Malaysian secondary schools. Table 2 and Table 3 show the description of item types according to the levels for addition and subtraction respectively.

Table 2

Description of Item Types in Addition of Fractions

Item level	Description of item level	Example
Level 1	Addition of proper fractions with unrelated denominators. No reduction	¼+1/3=7/12
Level 2	Addition of proper fractions with related denominators. Reduction.	½+1/6=4/6=2/3
Level 3	Addition of proper fractions with unrelated denominators. No reduction. Answer change from improper fraction to mixed numbers.	2/3+3/7=23/12=12/ 21
Level 4	Addition of proper fractions with unrelated denominators. No reduction. Answer change from improper fraction to mixed numbers.	1/3+5/6=7/6=11/6

Level 5	Addition of proper fractions with unrelated denominators. Reduction.	1/3+2/8=14/24=7/12
Level 6	Addition of mixed number and proper fractions with related denominators. Reduction.	5/12+11/3=19/12=13/4
Level 7	Addition of mixed numbers with related denominators. No reduction. Answer change from improper fraction to mixed number.	73/4+½=75/4=81/4
Level 8	Addition of mixed numbers with unrelated denominators. No reduction.	41/5+11/4=59/20

Table 3

Description of Item Types in Subtraction of Fractions

Item Level	Description of item level	Example
Level 1	Addition of proper fractions having common denominators. Reduction	¾-1/4=2/4=½
Level 2	Whole numbers subtracted from mixed numbers.	6½-2=4½
Level 3	Proper fractions subtracted from mixed numbers. Fractions with similar denominators. Reduction.	75/6-1/6=74/6=72/3
Level 4	Mixed numbers subtracted from whole numbers	5-11/3=32/3
Level 5	Proper fractions subtracted from mixed numbers with similar denominators. Value of subtrahend bigger than minuend. No reduction.	31/7-4/7=24/7
Level 6	Proper fraction subtracted from whole number	3-½=2½
Level 7	Proper fraction subtracted from mixed number. Unrelated denominators.	11/3-¼=1½

Level 8	Mixed numbers subtracted from mixed numbers. Number of subtrahend bigger than minuend. Reduction.	41/8-17/8=22/8=2¼

Findings of the Study

Types of Errors in Addition of Fractions

Table 4 shows the distribution of errors in the addition and subtraction of fractions according to percentage. Table 4 shows that almost 26.3% of errors are systematic errors for both the addition and subtraction of fractions. Students have manifested more careless errors and random errors in addition compared to subtraction of fractions. The percentage of students not making any errors in subtraction is 60.8% compared to 32.8% in addition.

Table 4

Distribution of Errors in Addition and Subtraction of Fractions

Type of error	Addition of fraction	Subtraction of fraction
Systematic error	168 (26.3%)	169 (26.4%)
Random error	71 (11.1%)	16 (2.5%)
Careless error	191 (29.8%)	66 (10.3%)
No error	210 (32.8%)	389 (60.8%)

Table 5

Distribution of Students Manifesting Errors in Addition of Fractions

Item	Systematic Error n (%)	Careless Error n (%)	Random Error n (%)	No Error n (%)	Total Error for Each Item n (%)	Total N (%)
1	7 (8.8)	26 (32.5)	6 (7.5)	41 (51.3)	39 (48.7)	80 (100)
2	39 (48.8)	25 (31.3)	8 (10)	8 (10)	72 (90)	80 (100)
3	15	21	7	37	43	80

	(18.8)	(26.3)	(8.8)	(46.3)	(53.7)	(100)
4	14	24	10	32	48	80
	(17.5)	(30)	(12.5)	(40)	(60)	(100)
5	39	20	11	10	70	80
	(48.8)	(25)	(13.8)	(12.5)	(87.5)	(100)
6	34	28	9	9	71	80
	(42.5)	(35)	(11.3)	(11.3)	(88.7)	(100)
7	15	25	13	27	53	80
	(18.8)	(31.3)	(16.3)	(33.8)	(66.2)	(100)
8	5	22	7	46	34	80
	(6.3)	(27.5)	(8.8)	(57.5)	(42.5)	(100)
Total	168	191	71	210	430	80
	(26.3%)	(29.8%)	(11.1%)	(32.8%)	(67.2%)	(100%)

Table 5 shows the distribution of errors in the addition of fractions. From Table 5 it can be seen that 67.2% of the students have manifested errors in the addition of fractions. This shows that more than two thirds of the students have manifested errors in the addition of fractions. In comparison with the different types of errors, it can be deduced that careless errors constitute the most errors.

Table 6

Comparison of the Number and Percentage of Errors for Each Item in the Addition of Fractions

	Error			No Error
Item	Systematic n (%)	Careless n (%)	Random n (%)	n (%)
1	7	26	6	41
	(4.2)	(13.6)	(8.5)	(19.5)
2	39	25	8	8
	(23.3)	(13.1)	(11.3)	(3.8)
3	15	21	7	37
	(8.9)	(11)	(9.9)	(17.6)
4	14	24	10	32
	(8.3)	(12.6)	(14.1)	(15.2)

5	39	20	11	10
	(23.2)	(10.5)	(15.5)	(4.8)
6	34	28	9	9
	(20.2)	(14.7)	(12.7)	(4.3)
7	15	25	13	27
	(8.9)	(13.1)	(18.3)	(12.9)
8	5	22	7	46
	(3.0)	(11.5)	(9.9)	(21.9)
Total	168	191	71	210
	(100)	(100)	(100)	(100)

Table 6 shows the comparison and percentage of errors for each item in the addition of fractions. From Table 6 it can be seen that the second item, the fifth, and the sixth have the most systematic errors compared with the other items. These items require the students to reduce the fraction result. Item 6 has the most number of errors in the careless error category. The item with the least number of students manifesting careless errors is item 5.

In the random error category, the seventh item has the most random error type. This item requires the skill of adding mixed numbers and proper fractions with unrelated denominators. The fraction result does not involve reduction. Item 8 has the most number of students manifesting no error. This item involves the addition of mixed numbers with unrelated denominators. Item 2, however, has the least number of students manifesting no error. The ratio of the number of errors to the number of no errors for this item is almost 2:1 which denotes that for every two incorrect answers there is one correct answer produced by the sample of students.

Types of Errors in Subtraction of Fractions

Table 7 shows that 40.2% of students have manifested errors in the subtraction of fractions. This shows that almost 2/5 of the students have manifested errors in the subtraction of fractions. Among the different types of errors, systematic errors constitute the most number of errors in comparison to careless error and random error.

Table 7

Distribution of Students Manifesting Errors in Subtraction of Fractions

Item	Systematic Error n (%)	Careless Error n (%)	Random Error n (%)	No Error n (%)	Total Error for Each Item n (%)	Total n (%)
1	34 (42.5)	4 (5)	0 (0)	42 (52.5)	38 (47.5)	80 (100)
2	33 (41.3)	15 (18.8)	1 (1.3)	31 (38.9)	49 (61.3)	80 (100)
3	5 (6.3)	10 (12.5)	0 (0)	65 (81.3)	15 (18.7)	80 (100)
4	30 (37.5)	4 (5.0)	1 (1.3)	45 (56.3)	35 (43.8)	80 (100)
5	21 (26.3)	7 (8.8)	1 (1.3)	51 (63.8)	29 (36.3)	80 (100)
6	11 (13.8)	9 (11.3)	1 (1.3)	59 (73.8)	21 (23.8) (100)	80
7	3 (3.80	11 (13.8)	5 (6.3)	56 (76.3)	19 (23.8) (100)	80
8	32 (40)	6 (7.5)	7 (8.8)	35 (43.8)	45 (56.3) (100)	80
Total	169 (26.4)	66 (10.3)	16 (2.5)	389 (60.8)	251 640 (39.2) (100)	

Table 8

Comparison of the Number and Percentage of Errors for Each Item in the Subtraction of Fractions

Error

Item	Systematic n (%)	Careless n (%)	Random n (%)	No Error n (%)
1	34 (20.1)	4 (6.1)	0 (0)	42 (10.8)
2	39 (23.3)	25 (13.1)	8 (11.3)	8 (3.8)
3	15 (8.9)	21 (11)	7 (9.9)	37 (17.6)
4	14 (8.3)	24 (12.6)	10 (14.1)	32 (15.2)
5	39 (23.2)	20 (10.5)	11 (15.5)	10 (4.8)
6	34 (20.2)	28 (14.7)	9 (12.7)	9 (4.3)
7	15 (8.9)	25 (13.1)	13 (18.3)	27 (12.9)
8	5 (3.0)	22 (11.5)	7 (9.9)	46 (21.9)
Total	169 (100)	66 (100)	16 (100)	389 (100)

Table 9

Systematic Errors in Lack of Comprehension of Process Involved (Subtraction of Fractions)

Systematic Errors	High achievers n (%)	Average Achievers n (%)	Weak Achievers n (%)	Total n (%)
1.Lack of comprehension of process involved	-	7 (4.1)	74 (43.8)	81 (47.9)
a) Subtracted numerators and multiplied denominators	-	-	3 (1.8)	3 (1.8)
b) Subtracted fraction in	-	3	42	45

minuend from fraction in subtrahend		(1.8)	(24.9)	(26.6)
c) Subtracted whole numbers and placed fraction in result	-	4 (4.1)	27 (16.0)	31 (18.3)
d) Reduced whole number by 1 and placed fraction in the result fraction	-	-	2 (1.2)	2 (1.2)

Table 9 shows the systematic errors manifested by the average and weak achievers in the lack of the comprehension of process involved in the subtraction of fractions. A total of 81 (47.9%) of the systematic errors belong in this category. Subtracting the proper fraction in minuend from fraction in the subtrahend constituted 26.6%. This shows that students lacked complete understanding of the process involved in the subtraction of fractions.

Analysis of Systematic Errors in the Addition of Fractions

Table 10

Systematic Errors in Lack of Comprehension of Process Involved (Addition of Fractions)

Systematic Errors	High achievers n (%)	Average Achievers n (%)	Weak Achievers n (%)	Total n (%)
1.Lack of comprehension of process involved	-	-	44 (26.2)	44 (26.2)
a) Added numerators and denominators	-	-	33 (19.6)	33 (19.6)
b) Added numerators, used one denominator for denominator sum	-	-	6 (3.6)	6 (3.6)
c) Added numerators and multiplied denominators	-	-	3 (1.8)	3 (1.8)
d) Added whole number to numerator in fraction	-	-	1 (0.6)	1 (0.6)

e) Added whole number to numerator and denominator of fraction	-	-	1 (0.6)	1 (0.6)

Table 10 shows the systematic errors manifested by the high achievers, average achievers and weak achievers in the lack of the comprehension of process involved in the addition of fractions. Some 26.2% of the systematic errors belong in this category and it is manifested by only the weak achievers. Adding numerators and denominators constituted 19.6% of this category. This shows that the students lacked complete understanding of the process involved.

Table 11

Systematic Errors in Difficulty with Improper Fractions

Systematic Error	High achievers	Average achievers	Weak achievers	Total
2. Difficulty with improper fractions	5 (3.0)	7 (4.2)	13 (17.7)	25 (14.9)
a) Did not change improper fractions to mixed numbers	5 (3.0) 7	7 (4.2)	12 (7.1)	24 (14.3)
b) Error in changing to improper fractions	-	-	1 (0.6)	1 (0.6)

Table 8 shows the systematic errors with improper fractions which constituted about 14.9% of the total errors. About 14.3% of the errors were due to pupils not changing improper fractions to mixed numbers. The weak achievers constituted 7.1% of the total while the average and weak achievers constituted 4.2% and 3.0% respectively for errors in this category.

Table 12

Systematic Errors in Difficulty in Borrowing

Systematic Errors	High achievers n (%)	Average Achievers n (%)	Weak Achievers n (%)	Total n (%)
1. Difficulty in borrowing	-	3 (1.8)	3 (1.8)	6 (3.6)
a) Disregarded having borrowed from whole number	-	2	1 (1.8)	3 (1.8)
b) Prefixed number borrowed to numerator	-		2 (1.2)	2 (1.2)
c) Borrowed but disregarded fraction in minuend	-	1 (0.6)	-	1 (0.6)

Table 12 shows the systematic errors manifested by the average achievers and weak achievers in the systematic errors in difficulty in borrowing involved in the subtraction of fractions. A total of 6 (3.6%) of the systematic errors belong in this category. Some 3 students have disregarded having borrowed from the whole number while 2 other students had prefixed number borrowed to the numerator, and 1 student had borrowed but disregarded the fraction in the minuend.

Discussion and Conclusion

This study raises implications for teachers of secondary school mathematics. The findings show that Form Two students have received instruction in fractions since Standard Four, but many, especially among the weak achievers, encounter difficulties in the addition and subtraction of fractions. These pupils have been neglected; therefore, the errors are still persistent when they are in secondary school. Looking at the questions in diagnostic tests, we cannot deny that the students have received instruction in them since

Standard Four. Teaching of fractions must be done in meaningful ways so that formal mathematics is seen as a subject for whole class discussion in primary school.

Conceptual oriented instruction enables students to achieve a level of computation competence they would not have achieved had they been in a procedurally oriented math class (Madsen, 1995). Conceptually oriented instruction enhances students' ability to understand, and through these understandings computational competence is achieved. Experiences in problem solving, estimation, mental arithmetic and calculator activities provide opportunities for students to explore arithmetic concepts in many different ways. The traditional drill and practice curriculum and instruction provides students with only one way to solve a computational problem – using a memorized algorithm.

Research on math instruction supports the use of math manipulatives, especially with low achieving students (Weaver & Suydam, 1972). If children are given the freedom to choose among mathematical models to solve a particular problem, they will select the model that makes the idea most meaningful to them.

The popularity of print material also suggests that children spend much of their class time working in symbolic settings rather than experiencing problem solving in a variety of more appropriate physical settings. The teachers' tendency to assign individual work suggests that children have little opportunity to express their thoughts and ideas and to interact with teachers and other students. Consequently this lack of interaction may have an adverse effect on the child's understanding of fractions because as Fuson (1987) has argued, verbalization plays a major role in concept and skill acquisition.

One possible consequence of limiting physical settings to promote understanding is what has been termed "representation rigidity" (Silver, 1983). Fifteen out of twenty community college students interviewed in Silver's study reported seeing three shaded parts of a pie subdivided into four segments into four congruent parts when asked to provide an image of the fraction three-fourths. So dominant was the "pie" image that 10 subjects could not report any secondary images when asked to think of alternative ways to visualize this fraction. Silver found that those students who demonstrated a better mastery of fractions were able to report a wider variety of fraction images than less capable students.

Clements and Del Campo (1989) believe that teachers should consciously create mathematics learning environments which enable children to link their verbal knowledge, their visual imagery and relevant episodes in which they have previously engaged in mathematics. Ng (1998) stresses that when teachers are unfamiliar with the content they are teaching, they would not have much time to explore ways to teach mathematics meaningfully, especially when they have so little time to do research on their own. It is not sufficient to determine that a student has difficulty with adding fractions. The diagnosis should reveal whether the deficiency lies in finding common denominators, whether certain fractions are more difficult than others, and whether the process is understood, among others.

The situation revealed by this study should be made a matter of serious concern. It should lead school authorities to investigate the administrative and teaching techniques which may be responsible for the low competency of Form Two students in the addition and subtraction of fractions. The current instruction and illustrations of fractions in text books should be further examined because the findings of this study are similar to the findings in Yap (1982) and Wan (2002) which indicate a high incidence of systematic errors made by the pupils especially those from the weaker classes. It is suggested that a longitudinal study be done to examine if systematic errors are persistent. Future research should also include the error analysis of low and average achievers among primary and secondary students after the usage of learning materials in the learning of fractions.

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