

PAPER • OPEN ACCESS

Error Prediction of CMM Using a Hybrid Model Based on Neural Network Quantile Regression and Kernel Density Estimation

To cite this article: Haiting Wu *et al* 2020 *J. Phys.: Conf. Ser.* **1605** 012102

View the [article online](#) for updates and enhancements.

You may also like

- [Real-time residual life prediction based on kernel density estimation considering abrupt change point detection](#)
Weizhen Zhang, Hui Shi, Jianchao Zeng et al.

- [Vulnerability of high-speed rail grid-connected system on branch potential energy transfer entropy](#)
Wen-Li Fan, Ye-Qi Xiao, Xiao-Feng He et al.

- [Kernel density estimation-based real-time prediction for respiratory motion](#)
Dan Ruan



242nd ECS Meeting

Oct 9 – 13, 2022 • Atlanta, GA, US

Early hotel & registration pricing ends September 12

Presenting more than 2,400 technical abstracts in 50 symposia

The meeting for industry & researchers in

BATTERIES
ENERGY TECHNOLOGY
SENSORS AND MORE!

 Register now!



ECS Plenary Lecture featuring M. Stanley Whittingham,
Binghamton University
Nobel Laureate –
2019 Nobel Prize in Chemistry



Error Prediction of CMM Using a Hybrid Model Based on Neural Network Quantile Regression and Kernel Density Estimation

Haiting Wu, Mei Zhang*, Guihua Li, Haifeng Zhao, Xiaoping Wu

Anhui University, Hefei, Anhui, 230601, China

94149942@qq.com

Abstract. The sources of dynamic measurement error of CMM are complex and influence each other and the traditional parameter modeling method is very difficult to model. In this paper, we propose a hybrid model which combines neural network quantile regression and kernel density estimation. The hybrid model realizes the advantages of multi-angle analysis, nonlinear fitting data and non-parametric error prediction. We use this model to analyze the complex relationship between dynamic measurement error value and 3d coordinates, positioning velocity, proximity distance and contact velocity. The results show that our model has good predictive performance and is superior to the least squares estimation model.

1. Introduction

At present, the main methods for modeling dynamic measurement error include multiple regression model, wavelet theory model, neural network model and so on [1-]. In practice, using the above single method requires the data to meet strict assumptions and conditions. In practice, the above model often encounters a series of problems so that it is not suitable for application. For example, it is difficult to determine the linear or nonlinear relationship between variables; the nonlinear relationship between variables is difficult to determine; the distribution of dependent variables is not easy to determine or does not meet the requirements; the use of a single method requires data to meet strict assumptions and conditions. In these cases, the above method is difficult to achieve the desired accuracy and effect. In this paper, a hybrid model based on quantile regression and kernel density estimation is proposed. Quantile regression of neural network can combine the strong nonlinear adaptive ability of neural network and the advantages of quantile regression to describe the independent variables in detail to obtain the nonlinear relationship between independent variables and dependent variables at different quantiles. Kernel density estimation combines the information of all quantiles to obtain the probability density curve of the dependent variable under the corresponding independent variable. Therefore, when the relationship between variables is complex and precise point prediction is needed, the hybrid model can get better results.

The dynamic measurement error of CMM is very complex and there are unknown interactions, so it is difficult to model with a single method [8-11]. It is well known that all error sources affect the final measurement result in the form of (x, y, z) coordinates. In addition, DCC(direct computer control) parameters, including positioning speed, proximity and contact speed, are easy to control and detect during measurement. As mentioned above, the independent variables (including the coordinates of measurement position and DCC parameters) and dependent variables (the dynamic measurement error)



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

can be acquired from experiments, but the relationship between them remains unknown. The hybrid model proposed in this paper has strong nonlinear adaptive ability and can obtain accurate point prediction by kernel density estimation. Therefore, this paper adopts a hybrid model based on neural network and kernel density estimation to analyze the relationship between the measurement error of CMM system and the three coordinates (x, y, z) and motion parameters of DCC.

2. Neural network quantile regression and kernel density estimation

2.1. Neural network quantile regression

Quantile regression can yield more complete information about explanatory variables by calculating the regression line at several percentiles of the distribution. It can overcome the situation of asymmetric distribution and large dispersion of data, describe the relationship between explained variables and explanatory variables in more detail, and fully investigate the complete conditional distribution of explained variables. Its linear regression model is

$$y_\tau = x^T \beta(\tau) + \varepsilon(\tau), \quad t = 1, 2, 3, \dots, n \quad (1)$$

Where τ ($0 < \tau < 1$) is quantile, x_t is the matrix of dependent variables, $\beta(\tau)$ is the regression coefficient vector. $\varepsilon(\tau)$ is the error term. By solving linear programming problem $\min_{\beta} \sum_{i=1}^n \rho(y_i - x_i^T \beta)$, we get $\beta(\tau)$. The neural network quantile regression uses the neural network to obtain the appropriate $\beta(\tau)$ and the activation function to realize the nonlinear fitting. Its regression model is

$$y_\tau = f(\beta_{ij}(\tau)g(v_{ij}(\tau)) + \beta_{0j}(\tau)) + \varepsilon(\tau) \quad (2)$$

Where $f(x)$ and $g(x)$ are the activation functions, $v(\tau)$ is the linear quantile regression with respect to X under the τ quantile. In this paper, we use a three-layer neural network, in which the activation function $g(x)$ uses the Relu function and $f(x)$ uses the Sigmoid function.

$$g(x) = \max(0, x) \quad (3)$$

$$f(x) = \frac{1}{1 + e^{-z}} \quad (4)$$

2.2. Kernel density estimation

Although the neural network quantile regression of neural network can obtain more information about the relationship between independent variables and dependent variables, when the data is predicted, only the corresponding prediction interval can be obtained, but the precise point prediction value cannot be obtained. Kernel density estimation can use the results of neural network quantile regression to obtain the probability density curve of the predicted value and then achieve the point prediction.

Since the kernel density estimation does not need to make any assumptions about the prior distribution of random variables, when the input variables, kernel functions and optimal window width are determined, the kernel density estimation method can be used to obtain its continuous probability density curve. Suppose $e_1, e_2, e_3, \dots, e_n$ are N samples of prediction error, and its probability density function is $f(x)$, then $f(x)$ is

$$f(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x-e_i}{h}\right) = \frac{1}{n} \sum_{i=1}^n K_h(x-e_i) \quad (5)$$

In the formula, h is the window width, n is the sample size, and $K(x)$ is the kernel function. After the kernel function is determined, we usually minimize

$$\begin{aligned} MISE(h) &= E \int (f'_h(x) - f(x))^2 dx \\ &= AMISE(h) + o(1/nh + h^4) \end{aligned} \quad (6)$$

So we just have to minimize

$$AMISE(h) = \frac{\int K(x)^2 dx}{nh} + \frac{m_2(x)^2 h^4 R(f'')}{4} \quad (7)$$

Wherein,

$$R(g) = \int g(x)^2 dx, m_2(K) = \int x^2 K(x) dx \quad (8)$$

The optimal window width h is the extreme point of the above formula. So, solve for

$$\frac{\partial}{\partial h} AMISE(h) = -\frac{R(K)}{nh^2} + m_2(K)^2 h^3 R(f'') = 0 \quad (9)$$

We get

$$h_{AMISE} = \frac{R(K)^{1/5}}{m_2(K)^{2/5} R(f'')^{1/5} n^{1/5}} \quad (10)$$

When the kernel function is determined, R, m, f'' are determined. But $R(f'')$ is unknown. Silverman proposed the rule of thumb to replace f with the normal density of variance matching the estimated variance, that is, estimate $R(f'')$ with $R(\phi)/\sigma^5$ to get the optimal window width

$$h = \left(\frac{4}{3n}\right)^{1/5} \sigma \quad (11)$$

Where ϕ is the standard normal density function, σ is the sample variance.

3. Measurement error analysis and prediction of CMM

3.1. Data collection

In order to establish a more accurate and practical dynamic error model of measuring machine, an experimental program of dynamic measurement error acquisition of measuring machine is designed in this paper. In this experiment, the dynamic measurement process of the measured part is realized by touching the probe. Some early studies focused on the positioning error of typical moving bridge CMM without actual contact operation[1,2,3,8]. The validity of the method is verified under different measuring positions and different DCC parameters, including the whole process of actual measurement. In this experiment, all error caused by the main engine, guide rail, environment and touch probe can be considered. Therefore, this method can be used to analyze the dynamic error of moving CMM.

A mobile bridge CMM MC850 (equipped with RenishawTP20 probe, needle length 20mm, tip ball diameter 4mm) is used to verify the hybrid model. The error sampling experiment is arranged as follows, in which the spatial coordinate parameters: x (is 0mm, 150mm, 300mm, 450mm, 600mm, 750mm), y (is 150mm, 300mm, 450mm, 550mm), z (is -473mm, -324mm, 581), and DCC parameters are: positioning velocity v1 (is 20mm/s, 60mm/s, 100mm/s), and the value of distance a (is 1 mm, 2 mm, 5 mm, 8 mm), and contacting velocity v2 (is 2mm/s, 4mm/s, 6mm/s, 8). There are totally 3456 combinations of the variables. Each combination was sampled 5 times and the error data was deleted, so a total of 17088 sets of data were obtained.

3.2. Model data and its quantification

Since neural network quantile regression requires a large amount of training data, we selected 17,000 sets of data as the training set, and the remaining 88 sets of data as the test set. In order to eliminate the influence of different scales of independent variables, the normalized data were used for the quantile regression of neural network. Represents data that can be divided into -1 and 1 by the following formula. In error prediction, the predicted value of the original data is obtained by inverse normalization.

$$x_i = \frac{x_i - \bar{x}}{\max(x) - \min(x)} \quad (12)$$

x_i is a multidimensional matrix, \bar{x} is the average of the matrix x . $\max(x)$ and $\min(x)$ is the maximum and minimum values of the matrix x .

In this paper, the neural network quantile regression model is a single hidden layer network, in which the number of iterations is 2000, the input layer is 6, the hidden layer is 10, and the output layer is 1. In the kernel density estimation, the kernel function selects Gauss function, and the window width can be calculated to get $h=0.38$. All programs are implemented in Python using two dependent libraries, keras and tensorflow.

$$K_{\text{Gauss}}(u) = (2\pi)^{-1/2} \exp(-u^2/2) \quad (13)$$

3.3. Analysis of dynamic error modeling

We took the training set as the input to train our neural network, and visualized the whole training process to get Figure 1. Loss stands for training Loss, making it as small as possible. According to Figure 1, we can see that loss tends to stabilize after 2000 iterations, which proves that our network has been trained. After that, we used the trained neural network to process the test set to obtain 9 conditional quantiles and put them into the kernel density estimation method for point prediction. We randomly selected a group of data from the test set and performed the above steps to obtain the probability distribution curve shown in Figure 2, where the dotted line represents the true value. Take the highest point in Figure 2 as our point prediction. We used the above method and the least-squares estimation model trained with the same training set to process all the data in the test set, and obtained Figure 3.

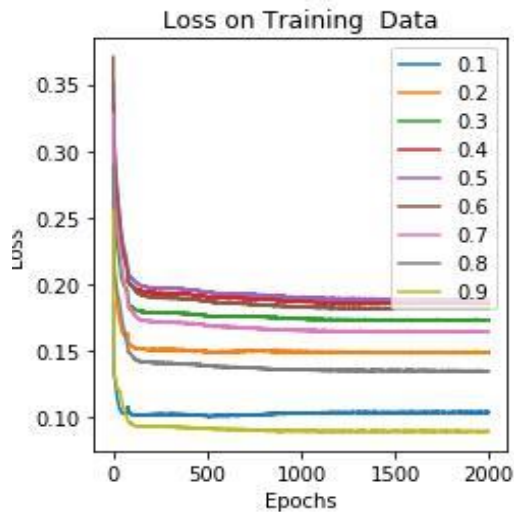


Figure 1. Visualization of the training process.

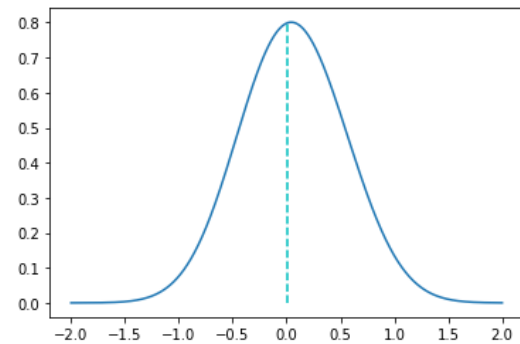


Figure 2. Probability density curve obtained from a certain set of data.

In order to further verify the effectiveness of this method, we use MSE as the measurement index, so as to obtain the least square error of the least square estimation model is 14.4, and the hybrid model is 4.3. The results show that the hybrid model based on neural network quantile regression and kernel density estimation is superior to the least squares estimation model.

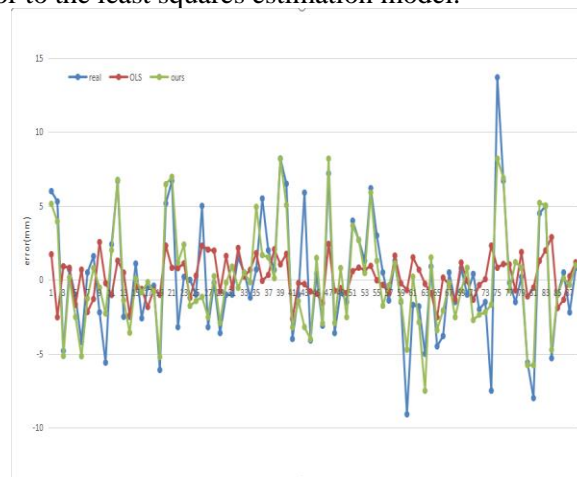


Figure 3. Comparison between predicted values and true values

4. Conclusion

Because of the complexity and interactivity of measuring error of CMM, it is difficult to analyze and predict the error. In this paper, a hybrid model based on quantile regression and kernel density estimation based on neural network is proposed to analyze the measurement error by taking 6 original variables of position coordinates x , y , z , positioning velocity, proximity distance and contact velocity as independent variables. The results show that the prediction accuracy of this method is improved, and its effect is better than that of least squares regression.

Acknowledgments

This research is supported by Anhui Natural Science Foundation : 1908085ME172, and National Natural Science Foundation of China (No. 61876002).

References

- [1] Bai E W, Liu Y. Recursive direct weight optimization in nonlinear system identification: A minimal probability approach[J]. *IEEE Trans Automat Control*, 2007, 52: 1218–1231
- [2] Pillonetto G, Quang M, Chiuso A. A new kernel-based approach for nonlinear system identification[J]. *IEEE Trans Automat Control*, 2011, 56: 2825–2840
- [3] Li K, Peng J, Bai E W. A two-stage algorithm for identification of nonlinear dynamic systems. *Automatica*, 2006, 42:1187–1196
- [4] Li K, Peng J, Bai E W. A two-stage algorithm for identification of nonlinear dynamic systems. *Automatica*, 2006, 42:1187–1196
- [5] HE F Y,ZHANG Z S. Image blur kernel estimation based on Markov random field learning model[J].*JOURNAL OF SOUTHEAST UNIVERSITY(Natural Science Edition)*, 2016 , 46(6): 1143-1148(in Chinese)
- [6] Roll J, Nazin A, Ljung L. Nonlinear system identification via direct weight optimization[J]. *Automatica*, 2005, 41:475–490
- [7] Zhang Mei. Analysis of CMM dynamic measurement error based on kernel estimation[C]// *Automation*. IEEE, 2017.
- [8] BAI E W, LI K, ZHAO W X et al..Variable selection in nonlinear non-parametric system identification[J].*SCIENTIA SINICA Mathematic*, 2016, 46(10): 1383-1400(in Chinese)
- [9] FEI Y T, ZHAO J, WANG H T et al..A Review of Research on Dynamic error of Coordinate Measuring Machines[J].*CHINESE JOURNAL OF SCIENTIFIC INSTRUMENT*, 2004 , (4): 773-776(in Chinese)
- [10] YANG H T. Research on Error Model Building and Error Correcting Technique of Coordinate Measuring Machines[D].Hefei: Hefei University of Technology
- [11] DONG C S, MU Y H, ZHANG G X. ASSESSING THE DYNAMIC CHARACTERISTICS OF CMMS WITH A LASER THRFEROMETER[J].*JOYRNAL OF TIANJIN UNIVERSITY*, 1998, 31(5): 621-626(in Chinese)
- [12] JI Y, LIU J Z. Research on the CMM Compensation error Models Based on Finite Element Simulations[J].*Technology and Test*, 2009, 6:78-80
- [13] ZHANG S Y, GE X Q, WANG B. Nonparametric Regression and its Application[J].*The Journal of Quantitative & Technical Economics*, 1997, 10: 60-65, 87(in Chinese)
- [14] JIANG J K. Application of Nonparametric Regression in the Evaluation Method of Commercial Housing Characteristic Price[D].Zhejiang: Zhejiang University,2006
- [15] JING Y C. Kernel Principal Component Coefficients in the Multiple Linear Model Estimation and Selection[D].Hebei: Hebei United University,2015(in Chinese)
- [16] P. B. Dhanish, J. Mathew, Effect of CMM point coordinate uncertainty on uncertainties in determination of circular features[J].*Measurement*, 2006, 39:522-531