PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

THIRD SERIES, VOLUME 54, NUMBER 3 **SEPTEMBER 1996**

RAPID COMMUNICATIONS

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication should be no longer than 4 printed pages and must be accompanied by an abstract. Page proofs are sent to authors.

Error prevention scheme with four particles

Lev Vaidman, Lior Goldenberg, and Stephen Wiesner

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel

(Received 29 March 1996)

It is shown that a simplified version of the error correction code recently suggested by Shor [Phys. Rev. A **52**, R2493 (1995)] exhibits manifestation of the quantum Zeno effect. Thus, under certain conditions, protection of an unknown quantum state is achieved. Error prevention procedures based on four-particle and two-particle encoding are proposed and it is argued that they have feasible practical implementations. [S1050-2947(96)51308-9]

PACS number(s): 03.65.Bz, 89.70.+c

Recently, Shor [1] obtained a surprising and even seemingly paradoxical result: an unknown quantum state can be kept unchanged in a noisy environment. His procedure was optimized by others [2–5], all trying to solve the problem of decoherence in quantum computers. However, at the present stage these results have more theoretical than practical importance, since they assume the existence of a fairly sophisticated quantum computer which has not been built yet. Shor's idea has another application which is closer to practical applications of today. Storing an unknown quantum state for some period of time (a ''quantum memory'') is an essential ingredient of various quantum cryptographic schemes [6,7]. It also can improve the reliability of the transmission of a quantum state [8]. The present technology is very close to the practical realization of these ideas [9–13].

The reason why one can see Shor's result as paradoxical is the following. The expectation value of an operator can be measured using weak adiabatic measurements [14,15]. Even if the coupling is very weak it can lead to a definite result, provided it is applied for a long enough time. This weak interaction with the measuring device can be considered as an action of a noisy environment, and therefore Shor's procedure performed frequently during the measurement should apparently keep the quantum state unchanged. Consider two eigenstates of an operator A with the eigenvalues a_1, and a_2. If the initial state of the measured system is $|a_1\rangle$, then the outcome of the measurement will be $\langle A\rangle = a_1$; if, instead, the initial state is $|a_2\rangle$, then the outcome of the measurement will be $\langle A\rangle = a_2$; and if the initial state is $1/\sqrt{2}(|a_1\rangle + |a_2\rangle)$, then the outcome of the measurement will be $\langle A\rangle = 1/2(a_1 + a_2)$. However, a pointer showing $1/2(a_1 + a_2)$ is a physical situation which is different from the mixture of situations in which the pointer shows a_1 and a_2. Therefore, we face a contradiction with the linearity of quantum theory.

The solution to this apparent paradox is that the coupling necessary for the adiabatic measurement of A, even if it is very weak, is different from the noise that can be dealt with using Shor's method. In the Shor procedure a qubit is encoded in nine particles, and the noise acting on each particle is assumed to be independent. However, in general, the variable A is related to several particles, such that the measurement requires coupling to some of them simultaneously. If we bring the particles to the same location and perform the adiabatic coupling with the measuring device, the independence requirement is not fulfilled explicitly. For some nonlocal variables there are measurement methods which can be applied without moving the particles to the same location [16], but then the parts of the measuring device which interact with the various particles must be in a correlated state prior to the interaction. Again, this corresponds to a correlated noise, and therefore Shor's procedure is not applicable.

There is another aspect of Shor's method which did not get enough attention. Usually, Shor's procedure and its modifications are considered as error *correction* schemes. Indeed, these methods correct the state completely if only one particle has decohered. But, Shor's procedure is also a scheme for *preventing* errors. For any system and any finite time T, frequent enough performance of Shor's procedure will ''freeze'' the state, exhibiting manifestation of the quantum Zeno effect. The use of the Zeno effect for correcting errors in quantum computers was first suggested by Zurek [17], and it is a part of a scheme recently proposed by Barenco *et al.* [18].

The only assumptions required for freezing a state are the boundness of the energy uncertainty of the system and the environment, and the independence of the noise disturbing each particle. If the energy uncertainty is bounded, then the rate of change of the quantum state of the system plus the environment is bounded. Given N, the number of Shor's tests performed during the time T, the evolution of a qubit and its environment $|e\rangle$ during a short period of time T/N can be written in the following way:

$$|0\rangle|e\rangle \rightarrow \gamma_1|0\rangle|e\rangle + \delta_1|0\rangle|e_1\rangle + \delta_2|1\rangle|e_2\rangle,$$

$$|1\rangle|e\rangle \rightarrow \gamma_2|1\rangle|e\rangle + \delta_3|1\rangle|e_3\rangle + \delta_4|0\rangle|e_4\rangle, \qquad (1)$$

where $\langle e|e_1\rangle = \langle e|e_3\rangle = 0$, $\gamma_i = 1 - O(1/N^2)$, and $\delta_j = O(1/N)$. This evolution differs from what was considered before in that that no change takes place in the zero order in $1/N$. We consider the situation in which all the particles evolve (independently) according Eq. (1) with different coefficients and different final states of their local environments. We claim that if Shor's tests are performed frequently enough they will always yield the outcome ''no error,'' and, in the same limit of large number of tests N, the state of the system will not be changed during the finite time T (and this is what led us see the apparent paradox of Shor's scheme described above).

We propose to simplify Shor's procedure. We keep the encoding step and the measurement step, but we suggest that we omit the correction step. Moreover, the last part of the measurement step, the observation, can be omitted, too—the coupling with the measuring device is enough. Clearly, a protective device that only tests a system is significantly simpler than one that also makes corrections. However, the most important point is that we can reduce the number of particles involved in the encoding of a quantum state. In the original Shor procedure three triplets are used for determining which particle was damaged: three particles or three triplets are compared, and if one is different from the other two the state is corrected. In the present procedure we do not have to perform corrections, so two doublets are enough. Therefore, we need only four particles instead of nine. The encoding is given by the following transformation:

$$|0\rangle \rightarrow |0_E\rangle \equiv \tfrac{1}{2}(|00\rangle + |11\rangle)(|00\rangle + |11\rangle),$$

$$|1\rangle \rightarrow |1_E\rangle \equiv \tfrac{1}{2}(|00\rangle - |11\rangle)(|00\rangle - |11\rangle), \qquad (2)$$

such that a qubit $\alpha|0\rangle + \beta|1\rangle$ is encoded as $\alpha|0_E\rangle + \beta|1_E\rangle$.

The protection procedure consists of frequent tests that the four-particle system has not left the subspace of the encoded states given by Eq. (2). In order to see that an arbitrary encoded state $\alpha|0_E\rangle + \beta|1_E\rangle$ is indeed frozen due to the quantum Zeno effect we note that the two following conditions are fulfilled: First, after the evolution for a short time T/N the amplitude of the state outside the subspace of the encoded states is of the order of $1/N$. The probability for getting the result ''out of subspace'' is of the order of $1/N^2$, and therefore, the probability of obtaining such an outcome during the time T is proportional to $1/N$. Taking N, the number of tests during the time T, large enough we can decrease the probability of such an error below any desired level. Second, after the evolution of time T/N the amplitude of the state $\beta^*|0_E\rangle - \alpha^*|1_E\rangle$, which is the state inside the subspace of the encoded states orthogonal to the initial encoded state, is of the order of $1/N^2$. Therefore, given that all N projections yielded ''inside the subspace,'' the difference between the final and the initial states is of the order of $1/N$ and can be neglected for large N.

The required projection can be performed in several steps which are, at least conceptually, simple. Each step is a certain *nonlocal* measurement in the sense that we measure a nonlocal variable related to two or more particles. We can do it without bringing the particles of the system together using correlated particles of the measuring device [14], but in fact, since the simultaneity of the coupling with different particles of the system is not crucial for our purpose, we can use even single-particle measuring devices.

The first step is to test that there are no terms which include the states $|01\rangle$ and $|10\rangle$ for the first two particles. The method is as follows: A test particle, prepared in a certain state, interacts with the first and then with the second particle of the system. The interaction ''flips'' the state of the test particle if the particle of the system is in the state $|1\rangle$ and does not flip it if the particle is in the state $|0\rangle$. The states $|0\rangle$ and $|1\rangle$ of the particles of the system remain unchanged by this procedure. Then we measure the final state of the test particle. If this state is identical to the initial state we know that the system has only terms of the form $|00\rangle$ and $|11\rangle$. If we perform our tests frequently enough the probability of finding ''wrong'' terms during all the tests goes to zero. In this case we can omit the last part of the procedure since the quantum Zeno effect requires only correlation with some external system, and therefore, the observation of the state of the test particle is not necessary. It is interesting to note that it is also not necessary to prepare a well-defined initial state of the test particle. Although there are certain initial states of the test particle which end up uncorrelated to the system, several test particles emerging from a truly random source will work, too.

The next step is the same procedure performed with particles 3 and 4. After completing this test we know that the state of the four-particle system has the form

$$\begin{aligned} &a(|00\rangle + |11\rangle)(|00\rangle + |11\rangle) \\ &\quad + b(|00\rangle - |11\rangle)(|00\rangle - |11\rangle) \\ &\quad + c(|00\rangle + |11\rangle)(|00\rangle - |11\rangle) \\ &\quad + d(|00\rangle - |11\rangle)(|00\rangle + |11\rangle). \end{aligned} \qquad (3)$$

For completing the projection on the subspace of the encoded states in Eq. (2) we have to show that the coefficients c and d vanish. In order to see how this can be achieved we rewrite the state in Eq. (3) using new local bases for all particles, $|\tilde{0}\rangle \equiv 1/\sqrt{2}(|0\rangle + |1\rangle)$ and $|\tilde{1}\rangle \equiv 1/\sqrt{2}(|0\rangle - |1\rangle)$,

$$a(|\tilde{0}\tilde{0}\rangle + |\tilde{1}\tilde{1}\rangle)(|\tilde{0}\tilde{0}\rangle + |\tilde{1}\tilde{1}\rangle) + b(|\tilde{0}\tilde{1}\rangle + |\tilde{1}\tilde{0}\rangle)(|\tilde{0}\tilde{1}\rangle + |\tilde{1}\tilde{0}\rangle) + c(|\tilde{0}\tilde{0}\rangle + |\tilde{1}\tilde{1}\rangle)(|\tilde{0}\tilde{1}\rangle + |\tilde{1}\tilde{0}\rangle) + d(|\tilde{0}\tilde{1}\rangle + |\tilde{1}\tilde{0}\rangle)(|\tilde{0}\tilde{0}\rangle + |\tilde{1}\tilde{1}\rangle). \quad (4)$$

The final step of the procedure is similar to the first two steps, but it involves interaction with all four particles of the system. The test particle, prepared in a certain state, interacts with all four particles one after the other in such a way that it ''flips'' if the particle is in the state $|\tilde{1}\rangle$ and does not ''flip'' if the particle is in the state $|\tilde{0}\rangle$. We ''look'' on the test particle only after it has interacted with all four particles. If the final state of the test particle is identical to the initial one, we know that the state of the system belongs to the desired subspace. Again, observing the test particle at the end of the process is not necessary for the Zeno effect to occur.

The Shor error correcting method was optimized such that only five particles instead of nine are used [4,5]. Thus, one might expect that the ideas of five-particle encoding procedures can be used to reduce the number of particles necessary for the Zeno-type error prevention method. However, the simple counting argument used to show that five is the minimal number of particles required for error correction [4,8] suggests that three qubits are not enough for error prevention. A necessary requirement for an error prevention code is that at the first order, i.e., via one-particle decoherence, the state $|0_E\rangle$ cannot evolve to the state $|1_E\rangle$. If we assume that each type of decoherence (flip, sign change, and flip together with sign change) moves an encoded ''0'' to different orthogonal states, then there are $2^3 - 1 - 3\times 3 = -2$ available states for the encoded ''1.'' This, of course, is a meaningless statement since the assumption of the orthogonality of the states created after various one-particle decoherence actions is wrong. Nevertheless, the general statement remains true and can be checked by a straightforward analysis: all possible three-particle encodings of one state generate, via single-particle decoherence, enough states to cover the whole Hilbert space of the three-particle system. Note that for four-particle encoding this naive counting shows no problem; we have $2^4 - 1 - 4\times 3 = 3$ orthogonal states for encoding ''1.''

In fact, some single-particle errors create identical vectors such that we have only six mutually orthogonal states obtained from the state $|0_E\rangle$. Moreover, one error yields only four new states from the state $|1_E\rangle$. So we still have $2^4 - 1 - 6 - 1 - 4 = 4$ states available. As pointed out by Shor, [19] this allows us to encode one more qubit using the same four particles. The states of a system composed by two qubits can be encoded as follows:

$$|00\rangle \rightarrow |0_E\rangle \equiv \tfrac{1}{2}(|00\rangle + |11\rangle)(|00\rangle + |11\rangle),$$
$$|01\rangle \rightarrow |1_E\rangle \equiv \tfrac{1}{2}(|00\rangle - |11\rangle)(|00\rangle - |11\rangle),$$
$$|10\rangle \rightarrow |2_E\rangle \equiv \tfrac{1}{2}(|01\rangle + |10\rangle)(|01\rangle + |10\rangle),$$
$$|11\rangle \rightarrow |3_E\rangle \equiv \tfrac{1}{2}(|01\rangle - |10\rangle)(|01\rangle - |10\rangle). \quad (5)$$

There is no single-particle error which can bring from one encoded state $|i_E\rangle$ $(i=0,1,2,3)$ to another. Thus, frequent projections on the subspace generated by the four states $|i_E\rangle$ should protect an unknown state in four-dimensional Hilbert space, i.e., two qubits. The realization of this projection is only slightly more difficult than the projection on the two-state subspace. The last step (the four-particle measurement) remains the same, but instead of the two two-particle tests we need to perform one four-particle measurement similar to that of the second step, but in the original basis.

We have shown that encoding one or two qubits in four qubits is in principle enough for the error prevention procedure. However, it is important to examine the type of noise in our system. Our method relies on the Zeno effect so it can deal only with ''slow'' noise. The characteristic time of the noise coupling has to be larger than the time interval between the projection measurements. If the realistic model of the noise is that molecules of the environment cause very fast finite uncertain changes during rare collisions with the particles, then our method is not applicable [20]. It also cannot help if the main cause of the decoherence is some spontaneous decay process, since the quantum Zeno effect does not take place when the time interval between the measurements is larger than the characteristic time for which the exponential decay approximation is applicable. However, if the appropriate model is that the environment becomes slowly entangled with the system, then our method works.

Even in this case the error correction codes have some advantages over error prevention codes. The frequency of the required procedures is significantly smaller in the error correction code; the error correction procedure has to be performed before the time that the second-order disturbance becomes large, while the error prevention procedure has to be performed before the time that the first-order disturbance becomes large. Thus, if we want to slow down the decoherence by a factor of N, we have to perform our error prevention procedure by the same factor N more frequently than an error correction code. However, since our procedure is much more simple, it is very plausible that it will be more practical in some cases. In particular, since the technology of handling several qubits is just developing, it is most probable that the first experiments will be performed with a minimal number of entangled particles.

There are quantum systems for which the noise leads mainly to dephasing, leaving the amplitude unchanged. This happens when the orthogonal states in a *particular* basis become entangled with the environment. It has been shown [8,21] that for this restricted type of decoherence there exist three-particle error correction codes. The quantum Zeno effect can help in this case too: a two-particle error prevention scheme exists [22]. The encoding is given by

$$|0\rangle \rightarrow |0_E\rangle \equiv 1/\sqrt{2}(|00\rangle + |11\rangle),$$
$$|1\rangle \rightarrow |1_E\rangle \equiv 1/\sqrt{2}(|01\rangle + |10\rangle). \quad (6)$$

The error prevention procedure is especially simple in this case. We have to test that our state belongs to the subspace generated by the two encoded states $|0_E\rangle$ and $|1_E\rangle$. This can be implemented using just a single step of the type described above. Again, the particle of the measuring device is prepared in a certain state, then it interacts with the two particles of the system, one after the other. The interaction is such that the state of the test particle flips if the particle is in the state $|\tilde{1}\rangle$ and does not flip if the particle is in the state $|\tilde{0}\rangle$. If the state of the system belongs to the subspace of the encoded states, which can be written in the form $a|\tilde{0}\tilde{0}\rangle+b|\tilde{1}\tilde{1}\rangle$, the state of the test particle will not be flipped after the two interactions, while it will be flipped if the state of the two particles does not belong to this subspace. This correlation leads to the quantum Zeno effect, which effectively prevents the system leaving the subspace of encoded states. Moreover, the state does not change significantly inside the subspace. The phase error after one such operation is of the order of $1/N^2$, and therefore, for a large number of tests N during the period of time T the total error can be neglected.

Maybe in a somewhat pessimistic tone we want to conclude by saying that the real problem with error preventing or correcting codes is the noise introduced by the procedure itself. As was explained at the beginning, the type of interaction involved in the prevention-correction measurements requires either bringing the particles of the system together, or letting them interact with correlated particles or with a single particle as proposed here. In all these cases the noise, if present, cannot be considered independent, and therefore the error correction or prevention effects of all the discussed methods do not occur. This does not mean that the result of Shor is not important—even the reduction of decoherence between the measurements is an extremely important and surprising effect.

Taking into account the price in the noise which we will probably have to pay in every projection procedure, the fact that we have to perform them more frequently is a significant disadvantage of error prevention schemes over error correction schemes. But again, it is compensated for by the fact that we need a smaller number of steps for each projection procedure. Since our code seems to be the simplest code proposed so far, it has a good chance to be the first implemented in a real laboratory. The experimental observation of the quantum Zeno effect reported by Itano *et al.* [23] contributes to our optimism. Thus, our scheme might serve as an effective testbed for the robustness of quantum computers and other quantum communication devices.

One of us (L.V.) would like to acknowledge the hospitality of the Institut für Experimentalphysik, Universität Innsbruck, where a part of this work was done. This research was supported in part by Grant No. 614/95 of the Basic Research Foundation (administered by the Israel Academy of Sciences and Humanities).

[1] P. Shor, Phys. Rev. A **52**, R2493 (1995).
[2] A.R. Calderbank and P.W. Shor, AT&T, Report No. quant-ph/9512032, 1995 (unpublished).
[3] A. Steane, Proc. R. Soc. London (to be published).
[4] R. Laflamme, C. Miquel, J.P. Paz, and W.H. Zurek, Los Alamos National Laboratory, Report No. quant-ph/9602019, 1996 (unpublished).
[5] C. Bennett, D. DiVincenzo, J. Smolin, and W. Wootters (unpublished).
[6] E. Biham, B. Huttner, and T. Mor, Technion, Report No. quant-ph/9604021, 1996 (unpublished).
[7] L. Goldenberg and L. Vaidman, Phys. Rev. Lett. **75**, 1239 (1995).
[8] A. Ekert and C. Macchiavello, Oxford University, Report No. quant-ph/9602022, 1996 (unpublished).
[9] P.D. Townsend, Electron. Lett. **30**, 809 (1994).
[10] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, and H.J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).
[11] J.I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[12] T. Pellizzari, S.A. Gardiner, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. **75**, 3788 (1995).
[13] C. Monroe, D.M. Meekhof, B.E. King, W.M. Itano, and D.J. Wineland, Phys. Rev. Lett. **75,** 4714 (1995).
[14] Y. Aharonov and L. Vaidman, Phys. Lett. A **178**, 38 (1993).
[15] Y. Aharonov, J. Anandan, and L. Vaidman, Phys. Rev. A **47**, 4616 (1993).
[16] Y. Aharonov, D. Albert, and L. Vaidman, Phys. Rev. D **34**, 1805 (1986).
[17] W. Zurek, Phys. Rev. Lett. **53**, 391 (1984).
[18] A. Barenco, A. Berthiaume, D. Deutsch, A. Ekert, R. Jozsa, and C. Macchiavello, Oxford University, Report No. quant-ph/9604028, 1996 (unpublished).
[19] P. Shor (private communication).
[20] Recently a computer simulation of such noise was done by S. Braunstein, Ulm University, Report No. quant-ph/9604036, 1996 (unpulbished).
[21] S. Braunstein, Ulm University, Report No. quant-ph/9603024, 1996 (unpublished).
[22] I. L. Chuang and R. Laflamme, Stanford University, Report No. quant-ph/9511003, 1995 (unpublished).
[23] W.M. Itano, D.J. Heinzen, J.J. Bollinger, and D.J. Wineland, Phys. Rev. A **41**, 2295 (1990).