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Essays in Asset Pricing and Volatility Risk

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Abstract

In the first chapter (``Good and Bad Uncertainty: Macroeconomic and Financial Market Implications" with Ivan Shaliastovich and Amir Yaron) we decompose aggregate uncertainty into `good' and `bad' volatility components, associated with positive and negative innovations to macroeconomic growth. We document that in line with our theoretical framework, these two uncertainties have opposite impact on aggregate growth and asset prices. Good uncertainty predicts an increase in future economic activity, such as consumption, and investment, and is positively related to valuation ratios, while bad uncertainty forecasts a decline in economic growth and depresses asset prices. The market price of risk and equity beta of good uncertainty are positive, while negative for bad uncertainty. Hence, both uncertainty risks contribute positively to risk premia.

In the second chapter (``A Tale of Two Volatilities: Sectoral Uncertainty, Growth, and Asset-Prices") I document several novel empirical facts: Technological volatility that originates from the consumption sector plays the ``traditional" role of depressing the real economy and stock prices, whereas volatility that originates from the investment sector boosts prices and growth; Investment (consumption) sector's technological volatility has a positive (negative) market-price of risk; Investment sector's technological volatility helps explain return spreads based on momentum, profitability, and Tobin's Q. I show that a standard DSGE two-sector model fails to fully explain these findings, while a model that features monopolistic power for firms and sticky prices, can quantitatively explain the differential impact of sectoral volatilities on real and financial variables.

In the third chapter (``From Private-Belief Formation to Aggregate-Vol Oscillation") I propose a model that relies on learning and informational asymmetry, for the endogenous amplification of the conditional volatility in macro aggregates and of cross-sectional dispersion during economic slowdowns. The model quantitatively matches the fluctuations in the conditional volatility of macroeconomic growth rates, while generating realistic real business-cycle moments. Consistently with the data, shifts in the correlation structure between firms are an important source of aggregate volatility fluctuations. Cross-firm correlations rise in downturns due to a higher weight that firms place on public information, which causes their beliefs and policies to comove more strongly.

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ABSTRACT

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Amir Yaron

In the first chapter ("Good and Bad Uncertainty: Macroeconomic and Financial Market Implications" with Ivan Shaliastovich and Amir Yaron) we decompose aggregate uncertainty into 'good' and 'bad' volatility components, associated with positive and negative innovations to macroeconomic growth. We document that in line with our theoretical framework, these two uncertainties have opposite impact on aggregate growth and asset prices. Good uncertainty predicts an increase in future economic activity, such as consumption, and investment, and is positively related to valuation ratios, while bad uncertainty forecasts a decline in economic growth and depresses asset prices. The market price of risk and equity beta of good uncertainty are positive, while negative for bad uncertainty. Hence, both uncertainty risks contribute positively to risk premia.

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CHAPTER 1 : Good and Bad Uncertainty: Macroeconomic and Financial Market Implications

(with Ivan Shaliastovich and Amir Yaron)[1]

1.1. Introduction

How do changes in economic uncertainty affect macroeconomic quantities and asset prices? We show that the answer to this question hinges on the type of uncertainty one considers. 'Bad' uncertainty is the volatility that is associated with negative innovations to macroeconomic quantities (e.g., output, consumption, earnings), and with lower prices and investment, while 'good' uncertainty is the volatility that is associated with positive shocks to these variables, and with higher asset prices and investment.

To illustrate these two types of uncertainties, it is instructive to consider two episodes: (i) the high-tech revolution of early-mid 1990s, and (ii) the recent collapse of Lehman Brothers in the fall of 2008. In the first case, and with the introduction of the world-wide-web, a common view was that this technology would provide many positive growth opportunities that would enhance the economy, yet it was unknown by *how much*? We refer to such a situation as 'good' uncertainty. Alternatively, the second case marked the beginning of the global financial crisis, and with many of the ensuing bankruptcy cases one knew that the state of economy was deteriorating—yet, again, it was not clear by *how much*? We consider this situation as a rise in 'bad' uncertainty. In both cases, uncertainty level rises relative to its long-run steady-state level, yet, the first case coincides with an optimistic view, and the second with a pessimistic one.

In this paper, we demonstrate that variations in good and bad uncertainty have separate and significant opposing impacts on the real economy and asset prices. We use an extended version of the long-run risks model of Bansal and Yaron (2004) to theoretically show conditions under which good and bad uncertainty have different impacts on prices. To make a meaningful distinction between good and bad uncertainty, we decompose, within the model, the overall shocks to consumption into two separate zero-mean components (e.g., jumps) which capture positive and negative growth innovations. The volatilities of these two shocks are time varying, and capture uncertainty fluctuations associated with the positive and negative parts of the distribution of consumption growth. Thus, in the model, valuation ratios are driven by three state variables: predictable consumption growth, good uncertainty, and bad uncertainty. Consequently, the stochastic discount factor, and therefore risk premia, are determined by three sources of risk: cash flow, good uncertainty, and bad uncertainty risks.

We show that with a preference for early resolution of uncertainty, the direct impact of both types of uncertainty shocks is to reduce prices, though, prices respond more to bad than to good uncertainty. For prices to rise in response to a good uncertainty shock there has to be an explicit positive link between good uncertainty and future growth prospects—a feature that we impose in our benchmark model.[2] We further show that the market price of good uncertainty risk and its equity beta have the same (positive) sign. Thus, even though prices can rise in response to good uncertainty, it commands a positive risk premium.

Overall, the model's key empirical implications include: (i) good uncertainty positively predicts future measures of economic activity, while bad uncertainty negatively forecasts future economic growth; (ii) good uncertainty fluctuations are positively related to asset valuations and to the real risk-free rate, while an increase in bad uncertainty depresses asset prices and the riskless yield; and (iii) the shocks to good and bad uncertainty carry positive and negative market prices of risk, respectively, yet both contribute positively to the risk premium.[3]

[2] Backus et al. (2010) also feature a direct feedback from volatility to future growth. However, they focus on total volatility and show the importance of this feedback for reconciling various lead-lag correlations between consumption growth and market returns.

[3] Although both uncertainties carry positive risk premium, their covariance, which may capture a common component, could contribute negatively to the risk premium.

We evaluate our model's empirical implications by utilizing a novel econometric approach to identify good and bad uncertainty from higher-frequency realized variation in the variables of interest (see Barndorff-Nielsen et al., 2010). Empirically, we use the exante predictable components of the positive and negative realized semivariances of industrial production growth rate as the respective proxies for good and bad uncertainty.[4] In its limiting behavior, positive (negative) semivariance captures one-half of the variation in any Gaussian symmetric movements in the growth rate of the variable of interest, as well as the variation of any non-Gaussian positive (negative) component in it. Thus, in our empirical work the positive (negative) semivariance captures the volatility component that is associated with the positive (negative) part of the total variation of industrial production growth, and its predictive component corresponds to the model concept for good (bad) uncertainty.

Consistent with the model, we document in the data that across various macroeconomic growth rates, and across various horizons, good economic uncertainty positively predicts future growth. This evidence includes growth for horizons of one to five years in consumption, output, investment, research and development (R&D), market earnings, and dividends. Similarly, we find a negative relationship between bad uncertainty and future growth rates of these macro variables. Together, these findings support the model feedback channel from macroeconomic uncertainty to future growth rates. Quantitatively, the impact of uncertainty has a large economic effect on the macro variables. For example, the private gross domestic product (GDP) growth increases by about 2.5% one year after a one standard deviation shock to good uncertainty, and this positive effect persists over the next three years. On the other hand, bad uncertainty shocks decrease output growth by about 1.3% one year after and their effects remain negative for several years. The responses of investment and R&D to these shocks are even stronger. Both capital and R&D investment significantly increase with good uncertainty and remain positive five years out, while they significantly drop with a shock to bad uncertainty. An implication of the offsetting re-

[4] We use industrial production because high-frequency real consumption data are not available for the long sample.

sponses to good and bad uncertainty is that the measured responses to overall uncertainty are going to be muted. Indeed, GDP growth declines only by about 0.25% after a shock to total uncertainty. The response to total uncertainty is significantly weaker than that to bad uncertainty, which underscores the potential importance of decomposing uncertainty into good and bad components.

The empirical evidence in the data is further consistent with the model's key asset-pricing implications. We document that the market price-dividend ratio and the risk-free rate appreciate with good uncertainty and decline with bad uncertainty. Quantitatively, the market log price-dividend ratio rises by about 0.07 one year out in response to a one standard deviation shock to good uncertainty and remains positive ten years afterward. Bad uncertainty shock depresses the log price-dividend ratio by 0.06 on impact and remains negative for ten years out. Similar to the macroeconomic growth rates, the response of the price-dividend ratio to total uncertainty is negative, but is understated relative to the response to bad uncertainty. The evidence for the response of the price-earnings ratio is very similar to that of the price-dividend ratio. In addition, consistent with the model, we show that both bad and good uncertainty positively predict future excess returns and their volatility.

Finally, we estimate the market prices of good and bad volatility risks using the cross-section of asset returns that includes the market return, 25 equity portfolios sorted on book-to-market ratio and size, and two bond portfolios (Credit and Term premium portfolios). We show that the market price of risk is positive for good uncertainty, while it is negative for bad uncertainty. Moreover, asset returns have a positive exposure (beta) to good uncertainty risk, and a negative exposure to bad uncertainty risk. Consequently, both good and bad uncertainty command a positive risk premium, although the interaction of their shocks can contribute negatively to the total risk compensation, since the good and bad uncertainty shocks are positively correlated. The market risk premium is 7.2% in the data relative to 8.2% in the model. In the data, the value spread is 4.38%, which is comparable to 3.34% in the model. The size spread is 4.39%, relative to 5.21% in the model. For the Credit premium portfolio the risk premium is 1.98% in the data and 2.15% in the model, and the Term premium is 1.82% in the data relative to 0.64% in the model.

1.1.1. Related literature

Our paper is related to a growing theoretical and empirical literature that documents the connection between economic uncertainty, aggregate quantities, and asset prices. Our concept of economic uncertainty refers to the time series volatility of shocks to economic quantity variables of interest (e.g., consumption and GDP growth). This is distinct from other aspects of uncertainty, such as parameter uncertainty, learning, robust-control, and ambiguity (see discussions in Pastor and Veronesi, 2009a; Hansen and Sargent, 2010; Epstein and Schneider, 2010). While there is a long-standing and voluminous literature on the time-varying second moments in asset returns, the evidence for time variation in the second moments of macro aggregates, such as consumption, dividends, earnings, investment, and output, is more limited and recent. Kandel and Stambaugh (1991) is an early paper providing evidence for stochastic volatility in consumption growth. More recently, McConnell and Perez-Quiros (2000), Stock and Watson (2003), and Bansal, Khatchatrian, and Yaron (2005b) provide supporting evidence that volatility measures based on macro aggregates feature persistent predictable variation.

The evidence on time-varying volatility of macro aggregates has also instilled recent interest in examining the role of uncertainty in dynamic stochastic general equilibrium (DSGE) production models. Bloom (2009) shows that increased volatility, measured via VIX, leads to an immediate drop in consumption and output growth rates as firms delay their investment decisions. Generally, the literature has emphasized a negative relationship between growth and uncertainty—see Ramey and Ramey (1995), Gilchrist et al. (2014), Fernandez-Villaverde et al. (2011), and Basu and Bundick (2012), to name a few. Other papers, such as Gilchrist and Williams (2005), Jones et al. (2005), Malkhozov (2014), and Kung and Schmid (2014) feature alternative economic channels which can generate a positive relationship between uncertainty and investment and thus growth. In addition, Croce et al.

(2012) and Pastor and Veronesi (2012) highlight the negative impact of government policy uncertainty on prices and growth.

In terms of asset prices, Bansal and Yaron (2004) show that with Epstein and Zin (1989) recursive preferences and an intertemporal elasticity of substitution (IES) larger than one, economic uncertainty is a priced risk, and is negatively related to price-dividend ratios. More recently, Bansal et al. (2014) examine the implications of macroeconomic volatility for the time variation in risk premia, for the return on human capital, and for the cross-section of returns. They develop a dynamic capital asset-pricing model (CAPM) framework for which one of the factors, in addition to the standard cash flow and discount rate risks, is aggregate volatility. Campbell et al. (2012) also analyze the role of uncertainty in an extended version of the intertemporal capital asset-pricing model (ICAPM). While both papers document a significant role for uncertainty, Bansal et al. (2014) find both the betas and market price of uncertainty risk to be negative, and thus uncertainty to positively contribute to equity risk premia, whereas the evidence in Campbell et al. (2012) is more mixed in terms of whether assets have negative or positive exposure (beta) to volatility. The empirical framework in this paper, allowing for two types of uncertainties, can in principle accommodate several of these uncertainty effects.

Our framework features two types of macroeconomic uncertainties. In terms of estimating two types of uncertainties, the literature has mainly focused on return-based measures. Patton and Sheppard (2015), Feunou et al. (2013), and Bekaert et al. (2015) use return data to capture fluctuations in good and bad volatilities, and study their effects on the dynamics of equity returns. Specifically, Patton and Sheppard (2015) and Feunou et al. (2013) use realized semivariance measures to construct the two volatilities, whereas we construct bad and good uncertainty measures directly from the macro aggregates.

Our framework is also related to a recent literature which highlights non-Gaussian shocks in the fundamentals. One analytically convenient specification that our framework accommodates and which is widely used features Poisson jumps in consumption dynamics (see, e.g., Eraker and Shaliastovich, 2008; Benzoni et al., 2011; Drechsler and Yaron, 2011; and Tsai and Wachter, 2014 for recent examples). In another specification, which again can be accommodated within our framework, the cash flow shocks are drawn from a Gamma distribution with a time-varying shape parameter, in which case the consumption shock dynamics follow the good and bad environment specification in Bekaert and Engstrom (2009). Finally, an alternative approach for generating time variation in higher-order moments is provided in Colacito et al. (2013). They model shocks to expected consumption as drawn from a skew-normal distribution with time-varying parameters and allow for a separate process for stochastic volatility. Our modeling approach focuses on bad and good volatility as the key driving forces for time variation in consumption growth distribution, and is largely motivated by our empirical analysis.

There is also a voluminous literature on the implications of time-varying higher-order moments of returns for risk pricing. For example, Bansal and Viswanathan (1993) develop a nonlinear pricing kernel framework and show its improvement in explaining asset prices relative to a linear arbitrage pricing theory (APT) model, while Chabi-Yo (2012) develops an intertemporal capital asset pricing model in which innovations in higher moments are priced. The empirical literature identifies these risks based on financial market data, and generally finds that left-tail risk is important for explaining the time series and cross-section of returns above and beyond the market volatility risk; see, e.g., Kapadia (2006), Adrian and Rosenberg (2008), Harvey and Siddique (2000), Chang et al. (2013), and Conrad et al. (2013).[5]

The rest of this paper is organized as follows. In Section 1.2 we provide a theoretical framework for good and bad uncertainty and highlight their role for future growth and asset prices. Section 1.3 discusses our empirical approach to construct good and bad uncertainty in the macroeconomic data. In Section 1.4 we show our empirical results for the effect of good and bad uncertainties on aggregate macro quantities and aggregate asset prices,

[5]See also a related literature on market downside risk, e.g., Ang et al. (2006) and Lettau et al. (2014), which emphasizes the importance of market left-tail risk.

and the role of uncertainty risks for the market return and the cross-section of risk premia. Section 1.5 discusses the robustness of our key empirical results, and the last section provides concluding comments.

1.2. Economic model

To provide an economic structure for our empirical analysis, in this section we lay out a version of the long-run risks model that incorporates fluctuations in good and bad macroeconomic uncertainties. We use our economic model to highlight the roles of the good and bad uncertainties for future growth and the equilibrium asset prices.

1.2.1. Preferences

We consider a discrete-time endowment economy. The preferences of the representative agent over the future consumption stream are characterized by the Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989) and Weil (1989):

$$U_t = \left[(1-\beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{1.1}$$

where C_t is consumption, β is the subjective discount factor, γ is the risk-aversion coefficient, and ψ is the elasticity of intertemporal substitution (IES). For ease of notation, the parameter θ is defined as $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the recursive preferences collapse to the standard case of expected power utility, in which case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds the reciprocal of IES ($\gamma > 1/\psi$), the agent prefers early resolution of uncertainty of consumption path, otherwise, the agent has a preference for late resolution of uncertainty.

As is shown in Epstein and Zin (1989), the logarithm of the intertemporal marginal rate of substitution implied by these preferences is given by:

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{1.2}$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log growth rate of aggregate consumption, and $r_{c,t}$ is a log return on the asset which delivers aggregate consumption as dividends (the wealth portfolio). This return is different from the observed return on the market portfolio as the levels of market dividends and consumption are not the same. We solve for the endogenous wealth return and the equilibrium stochastic discount factor in (1.2) using the dynamics for the endowment process and the standard Euler equation,

$$E_t \left[exp\{m_{t+1}\} R_{i,t+1} \right] = 1, \tag{1.3}$$

which hold for the return on any asset in the economy, $R_{i,t+1}$, including the wealth portfolio.

1.2.2. Consumption dynamics

Our specification of the endowment dynamics incorporates the underlying channels of the long-run risks model of Bansal and Yaron (2004), such as the persistent fluctuations in expected growth and the volatility of consumption process. The novel ingredients of our model include: (i) the decomposition of the total macroeconomic volatility into good and bad components associated with good and bad consumption shocks, respectively, and (ii) the direct effect of macroeconomic volatilities on future economic growth. We show that these new model features are well-motivated empirically and help us interpret the relation between the good and bad uncertainties, the economic growth, and the asset prices in the data.

Our benchmark specification for the consumption dynamics is written as follows:

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_c(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}), && (1.4)\\
x_{t+1} &= \rho x_t + \tau_g V_{gt} - \tau_b V_{bt} \\
&\qquad + \sigma_x(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}), && (1.5)
\end{aligned}
$$

where x_t is the predictable component of next-period consumption growth, and ε_{gt+1} and ε_{bt+1} are two mean-zero consumption shocks which for parsimony affect both the realized and expected consumption growth.[6] The shocks ε_{gt+1} and ε_{bt+1} separately capture positive and negative shocks in consumption dynamics, respectively, and are modeled as,

$$
\varepsilon_{i,t+1} = \tilde{\varepsilon}_{i,t+1} - E_t \tilde{\varepsilon}_{i,t+1}, \quad \text{for } i = \{g, b\}, \tag{1.6}
$$

where the underlying shocks $\tilde{\varepsilon}_{i,t+1}$ have a positive support, namely, $\tilde{\varepsilon}_{i,t+1} > 0$ for $i = \{g, b\}$. This ensures that the consumption shocks ε_{gt+1} and ε_{bt+1} are conditionally mean zero, and are driven by positive and negative shocks to consumption growth, respectively.

We assume that the volatilities of consumption shocks are time varying and driven by the state variables V_{gt} and V_{bt}; in particular,

$$
\begin{aligned}
Var_t \varepsilon_{g,t+1} &= Var_t \tilde{\varepsilon}_{g,t+1} \equiv V_{gt}, \\
Var_t \varepsilon_{b,t+1} &= Var_t \tilde{\varepsilon}_{b,t+1} \equiv V_{bt}.
\end{aligned}
$$

This allows us to interpret V_{gt} and V_{bt} as good and bad macroeconomic uncertainties, that is, uncertainties regarding the right and left tail movements in consumption growth. In our

[6] It is straightforward to extend the specification to allow for separate shocks in realized and expected consumption growth rates and break the perfect correlation of the two. This does not affect our key results, and so we do not entertain this case to ease the exposition.

specification, the good and bad uncertainties follow separate AR(1) processes,

$$V_{g,t+1} = (1-\nu_g)V_{g0} + \nu_g V_{gt} + \sigma_{gw} w_{g,t+1}, \tag{1.7}$$

$$V_{b,t+1} = (1-\nu_b)V_{b0} + \nu_b V_{bt} + \sigma_{bw} w_{b,t+1}, \tag{1.8}$$

where for $i = \{g, b\}$, V_{i0} is the level, ν_i the persistence, and $w_{i,t+1}$ the shock in the uncertainty. For simplicity, the volatility shocks are Normally distributed, and we let α denote the correlation between the good and bad volatility shocks.

By construction, the macro volatilities govern the magnitude of the good and bad consumption innovation. In addition to that, our feedback specification in (1.5) also allows for a direct effect of good and bad macro uncertainty on future levels of economic growth. Backus et al. (2010) use a similar feedback specification from a single (total) volatility to future growth. Our specification features two volatilities (good and bad), and for $\tau_g > 0$ and $\tau_b > 0$, an increase in good volatility raises future consumption growth rates, while an increase in bad volatility dampens future economic growth. The two-volatility specification captures, in a reduced-form way, an economic intuition that good uncertainty, through the positive impact of new innovation on growth opportunities, would increase investment and hence future economic growth, while bad uncertainty, due to the unknown magnitude of adverse news and its impact on investment, would result in lower growth in the future. While we do not provide the primitive micro-foundation for this channel, we show direct empirical evidence to support our volatility feedback specification. Further, we show that the volatility feedback for future cash flows also leads to testable implications for the asset prices which are supported in the data.

It is important to note that our specification for consumption growth displays non-Gaussian dynamics with time-varying mean, volatility, and higher-order moments. Specifically, total consumption volatility is equal to the sum of the good and bad uncertainties, $V_{gt} + V_{bt}$, whereas skewness, kurtosis, and all other higher moments are functions of the underlying volatility variables V_{gt} and V_{bt}. The specific way in which V_{gt} and V_{bt} affect those higher moments depends on the underlying distribution for $\tilde{\epsilon}_{i,t+1},\ i = \{b, g\}$. One specification that is analytically convenient and widely used features Poisson jumps in the consumption fundamentals, in which case, skewness is directly related to fluctuations in the intensity of jumps. In this case, the time variation in jump intensity affects separately the left and right tails of the consumption distribution, and hence the movements in good and bad volatility and higher-order moments. Another specification is one in which $\tilde{\epsilon}_{i,t+1}$ are drawn from a Gamma distribution with a scale parameter 1 and a time-varying shape parameter, in which case the consumption shocks dynamics follow the good and bad environment specification in Bekaert and Engstrom (2009). The time-varying shape parameters governing the Gamma distribution drive the variance and higher-order moments of consumption growth distribution. An alternative approach for generating time variation in higher-order moments is given in Colacito et al. (2013). They model shocks to expected consumption as drawn from a skew-normal distribution with time-varying parameters and a separate process for stochastic volatility which leads to separate movements in consumption volatility and skewness. Our modeling approach focuses on bad and good volatility as the key driving forces for time variation in consumption growth distribution, which is largely motivated by our empirical analysis.

1.2.3. Equilibrium asset prices

To get closed-form expressions for the equilibrium asset prices, we consider the consumption shock distribution for which the log moment-generating function is linear in the underlying variances $V_{g,t}$ and $V_{b,t}$. That is,

$$\log E_t e^{u\varepsilon_{i,t+1}} = f(u)V_{i,t}, \quad \text{for } i = \{g, b\}, \tag{1.9}$$

and the function $f(u)$ captures the shape of the moment-generating function of the underlying consumption shocks. As discussed earlier, prominent examples of such distributions include compound Poisson jump distribution and Gamma distribution. As shown in A.1.2, for this class of distributions the function $f(.)$ is non-negative, convex, and asymmetric, that is, $f(u) > f(-u)$ for $u > 0$.

We use a standard log-linearization approach to obtain analytical solutions to our equilibrium model. Below we show a summary of our key results, and all the additional details are provided in A.1.2.

In equilibrium, the solution to the log price-consumption ratio on the wealth portfolio is linear in the expected growth and the good and bad uncertainty states:

$$pc_t = A_0 + A_x x_t + A_{gv} V_{gt} + A_{bv} V_{bt}. \tag{1.10}$$

The slope coefficients are given by:

$$\begin{aligned} A_x &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}, \\ A_{gv} &= \tilde{A}_{gv} + \tau_g \frac{\kappa_1 A_x}{1-\kappa_1\nu_g}, \qquad \tilde{A}_{gv} = \frac{f(\theta((1-\frac{1}{\psi})\sigma_c + \kappa_1 A_x \sigma_x))}{\theta(1-\kappa_1\nu_g)}, \\ A_{bv} &= \tilde{A}_{bv} - \tau_b \frac{\kappa_1 A_x}{1-\kappa_1\nu_b}, \qquad \tilde{A}_{bv} = \frac{f(-\theta((1-\frac{1}{\psi})\sigma_c + \kappa_1 A_x \sigma_x))}{\theta(1-\kappa_1\nu_g)}, \end{aligned} \tag{1.11}$$

where the parameter $\kappa_1 \in (0,1)$ is the log-linearization coefficient, and the $\tilde{A}$s are the uncertainty loadings on the price-consumption ratio that would be obtained if the consumption dynamics did not include a direct feedback from uncertainty to growth prospects, namely, if $\tau_b = \tau_g = 0$.

As can be seen from the above equations, the response of the asset valuations to the underlying macroeconomic states is pinned down by the preference parameters and model parameters which govern the consumption dynamics. The solution to the expected growth loading A_x is identical to Bansal and Yaron (2004), and implies that when the substitution effect dominates the wealth effect ($\psi > 1$), asset prices rise with positive growth prospects: $A_x > 0$.

The expressions for the uncertainty loadings are more general than the ones in the literature and take into account our assumptions on the volatility dynamics. First, our specification separates positive and negative consumption innovations which have their own good and bad volatility, respectively. The impact of this pure volatility channel on asset prices is captured by the first components of the volatility loadings in (1.11), $\tilde{A}_{gv}$ and $\tilde{A}_{bv}$. In particular, when both γ and ψ are above one, these two loadings are negative: $\tilde{A}_{gv}, \tilde{A}_{bv} < 0$. That is, with a strong preference for early resolution of uncertainty, the agent dislikes volatility, good or bad, so the direct effect of an increase in uncertainty about either positive or negative tail of consumption dynamics is to decrease equilibrium equity prices. In the absence of the cash flow effect, both good and bad uncertainties depress asset valuations, albeit by a different amount. Indeed, due to a positive skewness of underlying consumption shocks, an increase in good (bad) uncertainty asymmetrically raises the right (left) tail of the future consumption growth distribution, and this asymmetry leads to a quantitatively larger negative response of the asset prices to bad uncertainty than to good uncertainty: $|\tilde{A}_{bv}| > |\tilde{A}_{gv}|$.

In addition to the direct volatility effect, in our model the good and bad uncertainties can also impact asset prices through their feedback on future cash flows (see Eq. (1.5)). For $\tau_b > 0$, the negative effect of bad uncertainty on future expected growth further dampens asset valuations, and as shown in (1.11), the bad volatility coefficient A_{bv} becomes even more negative. On the other hand, when good uncertainty has a positive and large impact on future growth, the cash flow effect of the good uncertainty can exceed its direct volatility effect, and as a result the total asset-price response to good uncertainty becomes positive: $A_{gv} > 0$. Hence, in our framework, good and bad uncertainties can have opposite impact on equity prices, with bad uncertainty shocks decreasing and good uncertainty shocks increasing asset valuations, which we show is an important aspect of the economic data.[7]

The aforementioned effect of uncertainty on asset valuations is related to several recent

[7] Note that in our simple endowment economy, welfare is increasing in the value of the consumption claim. When A_{gv} is positive, the implication is that good uncertainty shock increases welfare. This is not surprising since for A_{gv} to be positive there must be a significant positive feedback from this uncertainty to future growth. The bad uncertainty, as in Bansal and Yaron (2004), unambiguously reduces welfare.

studies. In the context of long-run risks models with preferences for early resolution of uncertainty, Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011) entertain jumps in cash flows and show that asset valuations drop with increase in jump intensity, and in particular, are sensitive to jumps which affect the left tail of consumption distribution. This effect on prices is also reflected in Colacito et al. (2013) who show that asset valuations decline when skewness becomes more negative. Tsai and Wachter (2014) consider a specification that incorporates time-varying rare disasters and booms. As their Poisson jump shocks are uncompensated, the intensities of booms and disasters have a direct impact on expected growth and thus capture the differential τ effects highlighted above, which leads to a differential impact of jump intensity on prices. Finally, in the context of the habits model in Bekaert and Engstrom (2009), prices decline at times of high expected growth and increase at times of good or bad variance of Gamma-distributed consumption growth shocks. The difference in the response of prices to uncertainty relative to our specification is due to the preference structure, and in particular, the preference for early resolution of uncertainty.

In the model, the good and bad uncertainty can also have different implications on equilibrium risk-free rates. Using a standard Euler equation (1.3), the solutions to equilibrium yields on $n-$period real bonds are linear in the underlying state variables:

$$y_{t,n} = \frac{1}{n}(B_{0,n} + B_{x,n}x_t + B_{gv,n}V_{gt} + B_{bv,n}V_{bt}), \tag{1.12}$$

where $B_{x,n}$, $B_{gv,n}$, and $B_{bv,n}$ are the bond loadings to expected growth, good, and bad uncertainty factors, whose solutions are provided in A.1.2. As shown in the literature, real bond yields increase at times of high expected growth, and the bond loading $B_{x,n}$ is positive. Further, an increase in either good and bad uncertainty raises the precautionary savings motive for the representative agent, so the direct impact of either uncertainty on risk-free rates is negative. However, in addition to the direct volatility effect, in our framework good and bad uncertainties also have an impact on future economic growth. Similar to the discussion of the consumption claim, bad uncertainty reduces future growth rates which further dampens real rates, so $B_{bv,n}$ becomes more negative. On the other hand, the positive cash flow impact of good volatility can offset the precautionary savings motive at longer maturities and can lead to a positive response of interest rates to good uncertainty. Thus, due to the volatility feedback, in our framework good and bad uncertainties can have opposite effects on the risk-free rates, which we show is consistent with the data.

1.2.4. Risk compensation

Using the model solution to the price-consumption ratio in (1.10), we can provide the equilibrium solution to the stochastic discount factor in terms of the fundamental states and the model and preference parameters. The innovation in the stochastic discount factor, which characterizes the sources and magnitudes of the underlying risk in the economy, is given by:

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= -\lambda_x \sigma_x (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ &\quad -\lambda_{gv} \sigma_{gw} w_{g,t+1} \\ &\quad -\lambda_{bv} \sigma_{bw} w_{b,t+1}, \end{aligned} \tag{1.13}$$

and λ_x, λ_{gv}, and λ_{bv} are the market prices of risk of growth, good volatility, and bad volatility risks. Their solutions are given by:

$$\lambda_x = (1-\theta)\kappa_1 A_x + \gamma \frac{\sigma_c}{\sigma_x} \tag{1.14}$$

$$\lambda_{gv} = (1-\theta)\kappa_1 A_{gv}, \tag{1.15}$$

$$\lambda_{bv} = (1-\theta)\kappa_1 A_{bv}. \tag{1.16}$$

When the agent has a preference for early resolution of uncertainty, the market price of consumption growth risk λ_x is positive: $\lambda_x > 0$. Consistent with our discussion of the price-consumption coefficients, the market prices of the volatility risks depend on the strength of the volatility feedback for future cash flow. When the good and bad uncertainties have no impact on future growth ($\tau_g = \tau_b = 0$), the market prices of both volatility risks are negative. Indeed, with preference for early resolution of uncertainty, the agent dislikes volatility, good or bad, and thus high uncertainties represent high risk states for the investor. The market prices of uncertainty risks change when we introduce volatility feedback for future growth. When bad volatility predicts lower future growth, it makes bad volatility fluctuations even riskier, which increase, in absolute value, the market price of bad uncertainty risk, so $\lambda_{bv} < 0$. On the other hand, when good uncertainty positively impacts future economic growth, the market price of good uncertainty can become positive: $\lambda_{gv} > 0$. Thus, in our framework, bad and good uncertainty can have opposite market prices of risk.

To derive the implications for the risk premium, we consider an equity claim whose dividends represent a levered claim on total consumption, similar to Abel (1990) and Bansal and Yaron (2004). Specifically, we model the dividend growth dynamics as follows,

$$\Delta d_{t+1} = \mu_d + \phi_x x_t + \sigma_d u_{d,t+1}, \tag{1.17}$$

where $\phi_x > 0$ is the dividend leverage parameter which captures the exposure of equity cash flows to expected consumption risks, and $u_{d,t+1}$ is a Normal dividend-specific shock which for simplicity is homoskedastic and independent from other economic innovations.[8] Using the dividend dynamics, we solve for the equilibrium return on the equity claim, $r_{d,t+1}$, in an analogous way to the consumption asset. The return dynamics satisfies,

$$\begin{aligned} r_{d,t+1} \quad = \quad & E_t[r_{d,t+1}] + \beta_x \sigma_x (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ & + \beta_{gv} \sigma_{gw} w_{g,t+1} + \beta_{bv} \sigma_{bw} \\ & + \sigma_d u_{d,t+1}, \end{aligned} \tag{1.18}$$

[8] It is straightforward to generalize the dividend dynamics to incorporate stochastic volatility of dividend shocks, correlation with consumption shocks, and the feedback effect of volatility to expected dividends (see, e.g., Bansal et al., 2012; and Schorfheide et al., 2013). As our focus is on aggregate macroeconomic uncertainty, these extensions do not affect our key results, and for simplicity are not entertained. However, it is worth noting that, by convexity, separate idiosyncratic dividend volatility can be positively related to equity prices (see, e.g., Pastor and Veronesi, 2006; Ai and Kiku, 2012; and Johnson and Lee, 2014).

where β_x, β_{gv}, and β_{bv} are the equity betas which reflect the response of the asset valuations to the underlying expected growth, good, and bad volatility risks, respectively. Similar to the consumption asset case, the equity betas to growth risks and good volatility risks are positive, while the equity beta to bad uncertainty risks is negative: $\beta_x > 0, \beta_{gv} > 0, \beta_{bv} < 0$. Further, since the volatilities of $\epsilon_{b,t+1}$ and $\epsilon_{g,t+1}$ are driven by $V_{b,t}$ and $V_{g,t}$, it immediately follows from Eq. (1.18) that the conditional variance of returns is time varying and increasing in good and bad uncertainties (see A.1.2 for details).

In equilibrium, the risk compensation on equities depends on the exposure of the asset to the underlying sources of risk, the market prices of risks, and the quantity of risk. Specifically, the equity risk premium is given by,

$$\begin{aligned} E_t R_{d,t+1} - R_{f,t} \approx log \quad & E_r e^{r_{d,t+1} - r_{f,t}} \\ & = \left[f(-\lambda_x \sigma_x) - f((\beta_x - \lambda_x)\sigma_x) + f(\beta_x \sigma_x) \right] V_{gt} \\ & + \left[f(\lambda_x \sigma_x) - f((\lambda_x - \beta_x)\sigma_x) + f(-\beta_x \sigma_x) \right] V_{bt} \\ & + \beta_{gv} \lambda_{gv} \sigma_{gw}^2 + \beta_{gv} \lambda_{bv} \sigma_{bw}^2 \\ & + \alpha \sigma_{bw} \sigma_{gw} (\beta_{gv} \lambda_{bv} + \beta_{bv} \lambda_{gv}). \end{aligned} \tag{1.19}$$

In our model, all three sources of risks contribute to the risk premia, and the direct contribution of each risk to the equity risk premium is positive. The first two components of the equity premium above capture the contribution of the non-Gaussian growth risk, which is time varying and driven by the good and bad volatilities. When $\gamma > 1$ and $\psi > 1$, the market price of growth risk λ_x and the equity exposure to growth risk β_x are both positive. As we show in A.1.2, this implies that the equity premium loadings on both good and bad volatilities are positive, so that the growth risks receive positive risk compensation unconditionally, and this risk compensation increases at times of high good or bad volatility. The remaining constant components in the equity risk premia equation capture the contributions of the Gaussian volatility shocks. As the market prices of volatility risks and equity exposure to volatility risks have the same sign, the volatility risks receive positive risk compensation in equities. The last term in the decomposition above captures the covariance between good and bad uncertainty risk, and is negative when the two uncertainties have positive correlation ($\alpha > 0$).

To get further intuition for the nature of the risk compensation, we consider a Taylor expansion of the equity risk premium:

$$\begin{aligned} E_t R_{d,t+1} - R_{f,t} \approx const + \beta_x \lambda_x \sigma_x^2 (V_{gt} + V_{bt}) \\ - \lambda_x \beta_x \sigma_x^3 (\lambda_x - \beta_x)(V_{gt} - V_{bt}) + \ldots \end{aligned} \tag{1.20}$$

The constant in this equation captures constant contribution of volatility risks to the risk premia. Subsequent terms pick out the second- and third-order components in the decomposition of the non-Gaussian growth risk premia; for simplicity, we omit higher-order terms. The second-order component is standard, and is equal to the negative of the covariance of log returns and log stochastic discount factor. This component is driven by the quantity of total consumption variance, $V_{gt} + V_{bt}$. An increase in either good or bad volatility directly raises total consumption variance, and hence increases equity risk premia (β_x and λ_x are both positive). The third-order component is driven by the quantity of consumption skewness, $V_{gt} - V_{bt}$. Under typical parameter calibration of the model, $\lambda_x > \beta_x$. [9] This implies that when V_{bt} increases relative to V_{gt} and the skewness of consumption shocks decreases (becomes more negative), the equity premium goes up. Hence, the total risk premium increases at times of high good or bad volatility, but the bad volatility has a larger effect capturing the importance of the left tails.

The quantities of total consumption variance and skewness risk are time varying themselves, and directly contribute to the equity risk premium. In our model, the total variance and skewness are linearly related to the good and bad volatilities, so that the risk compensation

[9] In the model, $\lambda_x = (1-\theta)\kappa_1 A_x + \gamma\sigma_c/\sigma_x$, and $\beta_x = \kappa_{1,d} H_x$. The term $(1-\theta)$ is positive under early resolution of uncertainty, and amounts to 28 under a typical calibration of $\gamma = 10$, $\psi = 1.5$. The equity price response to growth news H_x is magnified relative to consumption asset-price response A_x by the leverage of the dividend stream ϕ_x, so that H_x/A_x is around 3–5. The log-linearization parameters $\kappa_1 \approx \kappa_{1,d} \approx 1$. In all, this provides, $\lambda_x > \beta_x$.

for the variance and skewness risk are components of the constant risk compensation for good and bad volatility risks in (1.19)–(1.20). We show the implied market prices and equity betas to variance and skewness risk in A.1.2. In particular, in our framework agents dislike states with low consumption growth skewness (larger left tails), thus leading asset prices to fall in those states.

1.3. Data and uncertainty measures

1.3.1. Data

In our benchmark analysis we use annual data from 1930 to 2012. Consumption and output data come from the Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) tables. Consumption corresponds to the real per capita expenditures on non-durable goods and services and output is real per capita gross domestic product minus government consumption. Capital investment data are from the NIPA tables; R&D investment is available at the National Science Foundation (NSF) for the 1953 to 2008 period, and the R&D stock data are taken from the BEA Research and Development Satellite Account for the 1959 to 2007 period. To measure the fluctuations in macroeconomic volatility, we use monthly data on industrial production from the Federal Reserve Bank of St. Louis.

Our aggregate asset-price data include 3-month Treasury bill rate, the stock price and dividend on the broad market portfolio from the Center for Research in Security Prices (CRSP), and aggregate earnings data from Robert Shiller's website. We adjust nominal short-term rate by the expected inflation to obtain a proxy for the real risk-free rate. Additionally, we collect data on equity portfolios sorted on key characteristics, such as book-to-market ratio and size, from the Fama-French Data Library. Our bond portfolios, as in Ferson et al. (2013), include the excess returns of low- over high-grade corporate bonds (Credit premium portfolio), and the excess returns of long- over short-term Treasury bonds (Term premium portfolio).[10] To measure the default spread, we use the difference between the BAA and

[10] We thank Wayne Ferson for providing us data on these bond portfolios which we extend till 2012 using long-term government data and corporate bond data from Barclays.

AAA corporate yields from the Federal Reserve Bank of St. Louis.

The summary statistics for the key macroeconomic variables are shown in Panel A of Table 1. Over the 1930 to 2012 sample period the average consumption growth is 1.8% and its volatility is 2.2%. The average growth rates in output, capital investment, market dividends, and earnings are similar to that in consumption, and it is larger for the R&D investment (3.5%) over the 1954 to 2008 period. As shown in the table, many of the macroeconomic variables are quite volatile relative to consumption: the standard deviation of earnings growth is 26%, of capital investment growth is almost 15%, and of the market dividend growth is 11%. Most of the macroeconomic series are quite persistent with an AR(1) coefficient of about 0.5.

Panel B of Table 1.1 shows the summary statistics for the key asset-price variables. The average real log market return of 5.8% exceeds the average real rate of 0.3%, which implies an equity premium (in logs) of 5.5% over the sample. The market return is also quite volatile relative to the risk-free rate, with a standard deviation of almost 20% compared to 2.5% for the risk-free rate. The corporate yield on BAA firms is on average 1.2% above that for the AAA firms, and the default spread fluctuates significantly over time. The default spread, real risk-free rate, and the market price-dividend ratio are very persistent in the sample, and their AR(1) coefficients range from 0.72 to 0.88.

1.3.2. Measurement of good and bad uncertainties

To measure good and bad uncertainty in the data, we follow the approach in Barndorff-Nielsen et al. (2010) to decompose the usual realized variance into two components that separately capture positive and negative (hence, "good" and "bad") movements in the underlying variable, respectively. While we focus on the variation in the aggregate macroeconomic variables, Feunou et al. (2013) and Patton and Sheppard (2015) entertain a similar type of semivariance measures in the context of stock market variation.[11]

[11] The use of semivariance in finance goes back to at least Markowitz (1959), and more recent applications include, for example, Hogan and Warren (1974) and Lewis (1990).

Specifically, consider an aggregate macroeconomic variable y (e.g., industrial production, earnings, consumption), and let Δy stand for the demeaned growth rate in y. Then, we define the positive and negative realized semivariances, RV_p and RV_n, as follows:

$$RV_{p,t+1} = \sum_{i=1}^{N} \mathbb{I}(\Delta y_{t+\frac{i}{N}} \geq 0)\Delta y^2_{t+\frac{i}{N}}, \tag{1.21}$$

$$RV_{n,t+1} = \sum_{i=1}^{N} \mathbb{I}(\Delta y_{t+\frac{i}{N}} < 0)\Delta y^2_{t+\frac{i}{N}}, \tag{1.22}$$

where $\mathbb{I}(.)$ is the indicator function and N represents the number of observations of y available during one period (a year in our case). It is worth noting that RV_p and RV_n add up to the standard realized variance measure, RV, that is,

$$RV_{t+1} = \sum_{i=1}^{N} \Delta y^2_{t+\frac{i}{N}} = RV_{n,t+1} + RV_{p,t+1}.$$

Barndorff-Nielsen et al. (2010) show that in the limit the positive (negative) semivariance captures one-half of the variation of any Gaussian symmetric shifts in Δy, plus the variation of non-Gaussian positive (negative) fluctuations; see A.1.1 for further details. Notably, the result in this paper implies that asymptotically, the semivariances are unaffected by movements in the conditional mean; however, given the finite-sample considerations, we confirm the robustness of our results removing the fluctuations in conditional mean.

The positive and negative semivariances are informative about the realized variation associated with movements in the right and left tail, respectively, of the underlying variable. Positive (negative) semivariance therefore corresponds to good (bad) realized variance states of the underlying variable and thus, we use the predictable component of this measure as the empirical proxy for exante good (bad) uncertainty. To construct the predictive components, we project the logarithm of the future average $h-$period realized semivariance on the set of time t predictors X_t :

$$\log\left(\frac{1}{h}\sum_{i=1}^{h} RV_{j,t+i}\right) = \text{const}_j + \nu_j' X_t + \text{error}, \quad j = \{p, n\}, \tag{1.23}$$

and take as the proxies for the exante good and bad uncertainty V_g and V_b the exponentiated fitted values of the projection above:

$$V_{g,t} = exp\left(\text{const}_p + \nu_p' X_t\right), \qquad V_{b,t} = exp\left(\text{const}_n + \nu_n' X_t\right). \tag{1.24}$$

The log transformation ensures that our exante uncertainty measures remain strictly positive.

In addition to measuring the exante uncertainties, we use a similar approach to construct a proxy for the expected consumption growth rate, x_t which corresponds to the fitted value of the projection of future consumption growth on the same predictor vector X_t :

$$\frac{1}{h}\sum_{i=1}^{h} \Delta c_{j,t+i} = \text{const}_c + \nu_c' X_t + \text{error},$$

$$x_t = \text{const}_c + \nu_c' X_t.$$

In our empirical applications we let y be industrial production, which is available at monthly frequency, and use that to construct realized variance at the annual frequency. As there are 12 observations of industrial production within a year, our measurement approach is consistent with the model setup which allows for multiple good and bad shocks within a period (a year). To reduce measurement noise in constructing the uncertainties, in our benchmark empirical implementation we set the forecast window h to three years. Finally, the set of the benchmark predictors X_t includes positive and negative realized semivariances RV_p, RV_n, consumption growth Δc, the real-market return r_d, the market price-dividend ratio pd, the real risk-free rate r_f, and the default spread def.[12]

[12] As shown in Section 1.5, our results are robust to using standard ordinary least squares (OLS) regression

Panel C of Table 1 reports the key summary statistics for our realized variance measures. The positive and negative semivariances contribute about equally to the level of the total variation in the economic series, and the positive semivariance is more volatile than the negative one. The realized variation measures co-move strongly together: the contemporaneous correlation between total and negative realized variances is 80%, and the correlation between the positive and negative realized variance measures is economically significant, and amounts to 40%.

Fig. 1.1 shows the plot of the total realized variance, smoothed over the three-year window to reduce measurement noise. As can be seen from the graph, the overall macroeconomic volatility gradually declines over time, consistent with the evidence in McConnell and Perez-Quiros (2000) and Stock and Watson (2003), as well as Bansal, Khatchatrian, and Yaron (2005b), Lettau et al. (2008), and Bansal et al. (2014). Further, the realized variance is strongly countercyclical: indeed, its average value in recessions is twice as large as in expansions. The most prominent increases in the realized variance occur in the recessions of the early and late 1930s, the recession in 1945, and more recently, in the Great Recession in the late 2000s. Not surprisingly, the countercyclicality of the total variance is driven mostly by the negative component of the realized variance. To highlight the difference between the positive and negative variances, we show in Fig. 1.2 the residual positive variance (smoothed over the three-year window) which is orthogonal to the negative variance. This residual is computed from the projection of the positive realized variance onto the negative one. As shown on the graph, the residual positive variance sharply declines in recessions, and the largest post-war drop in the residual positive variance occurs in the recession of 2008–2009.

We project the logarithms of the future three-year realized variances and the future three-year consumption growth rates on the benchmark predictor variables to construct the exante uncertainty and expected growth measures. It is hard to interpret individual slope coefficients due to the correlation among the predictive variables, so for brevity we do not report

instead of the log, the use of alternative predictors, different forecast windows h, removing the conditional mean in constructing the semivariance measures, and using other measures for y.

them in the paper; typically, the market variables, such as the market price-dividend ratio, the market return, the risk-free rate, and the default spread, are significant in the regression, in addition to the lags of the realized variance measures themselves. The R^2 in these predictive regressions ranges from 30% for the negative variance and consumption growth to 60% for the positive variance.

We show the fitted values from these projections alongside the realized variance measures on Fig. 1.3. The logs of the realized variances are much smoother than the realized variances themselves (see Fig. 1.1), and the fitted values track well both the persistent declines and the business-cycle movements in the underlying uncertainty. We exponentiate the fitted values to obtain the proxies for the good and bad exante uncertainties. Figs. 1.4 and 1.5 show the total uncertainty and the residual exante good uncertainty which is obtained from the projection of the good uncertainty on the bad uncertainty. Consistent with our discussion for the realized quantities, the total uncertainty gradually decreases over time, and the residual good uncertainty generally goes down in bad times. Indeed, in the recent period, the residual good uncertainty increases in the 1990s, and then sharply declines in 2008. Notably, the exante uncertainties are much more persistent than the realized ones: the AR(1) coefficients for good and bad uncertainties are about 0.5, relative to 0.2–0.3 for the realized variances.

1.4. Empirical results

In this section we empirically analyze the implications of good and bad uncertainty along several key dimensions. In Section 1.4.1 we analyze the effects of uncertainty on aggregate macro quantities such as output, consumption, and investment. In Section 1.4.2 we consider the impact of uncertainties on aggregate asset prices such as the market price-dividend ratio, the risk-free rate, and the default spread. In Section 1.4.3 we examine the role of uncertainty for the market and cross-section of risk premia. Our benchmark analysis is based on the full sample from 1930–2012 and in the robustness section we show that the key results are maintained for the post-war period.

1.4.1. Macroeconomic uncertainties and growth

Using our empirical proxies for good and bad uncertainty, V_{gt} and V_{bt}, we show empirical support that good uncertainty is associated with an increase in future output growth, consumption growth, and investment, while bad uncertainty is associated with lower growth rates for these macro quantities. This is consistent with our cash flow dynamics in the economic model specification shown in Eq. (1.5).

To document our predictability evidence, we regress future growth rate for horizon h years on the current proxies for good and bad uncertainty and the expected growth, that is, we run a predictive regression

$$\frac{1}{h}\sum_{j=1}^{h} \Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + error,$$

for the key macroeconomic variables of interest y and forecast horizons h from one to five years. Table 1.2 reports the slope coefficients and the R^2 for the regressions of consumption growth, private GDP, corporate earnings, and market dividend growth, and Table 1.3 shows the evidence for capital investment and R&D measures.

It is evident from these two tables that across the various macroeconomic growth rates and across all the horizons, the slope coefficient on good uncertainty is always positive. This is consistent with the underlying premise of the feedback channel of good uncertainty on macro growth rates. Further, except for the three-year horizon for earnings, all slope coefficients for bad uncertainty are negative, which implies, consistently with the theory, that a rise in bad uncertainty would lead to a reduction in macro growth rates. Finally, in line with our economic model, the expected growth channel always has a positive effect on the macro growth rates as demonstrated by the positive slope coefficients across all the predicted variables and horizons.

The slope coefficients for all three predictive variables are economically large and in many cases are also statistically significant. All our tables include the usual Newey-West $t-$statistics for all the estimated coefficients. Additionally, to facilitate the comparison of the empirical results with our economic model, we also indicate the significance of the coefficients against the economically motivated alternative one-sided hypotheses. For the growth predictability regressions, our hypotheses are that growth and good uncertainty have a positive impact, while bad uncertainty has a negative impact, respectively. As shown in Tables 1.2 and 1.3, the expected growth (cash flow) channel is almost always significant, while the significance of good and bad uncertainty varies across predicted variables and maturities, although they tend to be significant in one-sided tests. Because, the uncertainty measures are quite correlated, the evaluation of individual significance may be difficult to assess. Therefore, in the last column of these tables we report the $p-$value of a Wald test for the joint significance of good and bad uncertainty. For the most part the tests reject the joint hypothesis that the loadings on good and bad uncertainty are zero. In particular, at the five-year horizon, all of the $p-$values are below 5%, and they are below 1% for all the investment series at all the horizons.

It is worth noting that the adjusted R^2s for predicting most of the future aggregate growth series are quite substantial. For example, the consumption growth R^2 is 50% at the one-year horizon, and the R^2 for the market dividends reaches 40%, while it is about 10% for earnings and private GDP. For the investment and R&D series the R^2s at the one-year horizon are also substantial and range from 28% to 55%. The R^2s generally decline with the forecast horizon but for many variables, such as consumption and investment, they remain quite large even at five years.

To further illustrate the economic impact of uncertainty, Figs. 1.6–1.8 provide impulse responses of the key economic variables to good and bad uncertainty shocks. The impulse response functions are computed from a first-order vector autoregression (VAR(1)) that includes bad uncertainty, good uncertainty, predictable consumption growth, and the macroeconomic variable of interest. Each figure provides three panels containing the responses to a one standard deviation shock in good, bad, and total uncertainty, respectively.

Fig. 1.6 provides the impulse response of private GDP growth to uncertainty. Panel A of the figure demonstrates that output growth increases by about 2.5% after one year due to a good uncertainty shock, and this positive effect persists over the next three years. Panel B shows that bad uncertainty decreases output growth by about 1.3% after one year, and remains negative even ten years out. Panel C shows that output response to overall uncertainty mimics that of bad uncertainty but the magnitude of the response is significantly smaller—output growth is reduced by about 0.25% one year after the shock, and becomes positive after the second year. Recall that good and bad uncertainty have opposite effects on output yet they tend to comove, and therefore the response to total uncertainty becomes less pronounced.

Fig. 1.7 provides the impulse response of capital investment to bad, good, and total uncertainty, while Fig. 1.8 shows the response of R&D investment to these respective shocks. The evidence is even sharper than that for GDP. Both investment measures significantly increase with good uncertainty and remain positive till about five years out. These investment measures significantly decrease with a shock to bad uncertainty and total uncertainty several years out. Total uncertainty is a muted version of the impulse response to bad uncertainty and is consistent with the finding in Bloom (2009) who shows a significant short-run reduction of total output in response to uncertainty shock, followed by a recovery and overshoot. Comparing Panels B and C of the figures highlights a potential bias in the magnitude of the decline in investment (and other macro quantities) in response to uncertainty when total uncertainty is used rather than bad uncertainty. For example, for capital investment the maximal decline is about 2.3% for total uncertainty and 3% for bad uncertainty, and for the R&D investment the maximal response is 0.6% for total uncertainty while it is 1.1% for bad uncertainty, which indicates that the response differences are economically significant. Thus, decomposing uncertainty to good and bad components allows for a cleaner and sharper identification of the impact of uncertainty on growth.

Finally, we have also considered the impact of good and bad uncertainty on aggregate employment measures. Consistent with our findings for economic growth rates, we find that high good uncertainty predicts an increase in future aggregate employment and hours worked and a reduction in future unemployment rates, while high bad uncertainty is associated with a decline in future employment and an increase in unemployment rates. In the interest of space, we do not report these results in the tables.

1.4.2. Macroeconomic uncertainties and aggregate prices

We next use our good and bad uncertainty measures to provide empirical evidence that good uncertainty is associated with an increase in stock market valuations and decrease in the risk-free rates and the default spreads, while bad uncertainty has an opposite effect on these asset prices. This is consistent with the equilibrium asset-price implications in the model specification in Section 1.2.

To document the link between asset prices and uncertainties, we consider contemporaneous projections of the market variables on the expected growth and good and bad uncertainties, which we run both in levels and in first differences, that is:[13]

$$y_t = a + b'[x_t, V_{gt}, V_{bt}] + error,$$
$$\Delta y_t = a + b'[\Delta x_t, \Delta V_{gt}, \Delta V_{bt}] + error,$$

where now, y refers to the dividend yield, risk-free rate, or default spread.

Table 1.4 shows the slope coefficients and the R^2s in these regressions for the market price-dividend ratio, the real risk-free rate, and the default spread. As is evident from the table, the slope coefficients on bad uncertainty are negative for the market price-dividend ratio and the real risk-free rate, and they are positive for the default spread. The slope coefficients are of the opposite sign for the good uncertainty, and indicate that market valuations and interest rates go up and the default spread falls at times of high good uncertainty. Finally,

[13] Instead of the first difference, we have also run the regression on the innovations into the variables, and the results are very similar.

the price-dividend ratio and the risk-free rates increase, while the default spread falls at times of high expected growth. Importantly, all these empirical findings are consistent with the implication of our model, outlined in Section 1.2, that high expected growth, high good volatility, and low bad volatility are good economic states.

The slope coefficients for our three state variables are economically large. In most cases, the volatility slope coefficients are statistically significant using economically motivated one-sided tests. These tests specify that good (bad) volatility has a positive (negative) impact on price-dividend ratio and risk-free rate, and opposite for the default spreads. Jointly, the two uncertainty variables are always significant with a $p-$value of 1% or below. The statistical significance is especially pronounced for the first difference projections. Recall that the asset-price variables that we use are very persistent and may contain slow-moving near-unit root components which can impact statistical inference. First difference (or alternatively, using the innovations into the variables) substantially reduces the autocorrelation of the series and allows us to more accurately measure the response of the asset prices to the underlying shocks in macroeconomic variables.

It is also worth noting that our three macroeconomic factors can explain a significant portion of the variation in asset prices. The R^2 in the regressions is 20% for the level of the price-dividend ratio and 60% for the first difference. For the real rate, the R^2s are about 20%, and it is 50% for the level of the default spread and 30% for the first difference.

Figs. 1.9 and 1.10 further illustrate the impact of uncertainties on asset prices and show the impulse responses of the price-dividend and price-earnings ratio to a one-standard deviation uncertainty shock from the VAR(1). Panel A of Fig. 1.9 documents that the price-dividend ratio increases by 0.07 one year after a good uncertainty shock and remains positive ten years out. Similarly, the price-earnings ratio increases to about 0.04 in the first two years and its response is also positive at ten years, as depicted in Panel A of Fig. 1.10. Bad uncertainty shocks depress both immediate and future asset valuations. Price-dividend ratio drops by 0.06 on the impact, while price-earnings ratio declines by about 0.04 one year after, and all the impulse responses are negative ten years after the shock. The response of the asset prices to the total uncertainty shock is significantly less pronounced than the response to bad uncertainty: the price-dividend ratio decreases immediately by only 0.04 on the impact of the total uncertainty shock, and the response reaches a positive level of 0.01 at one year and goes to zero after three years. Similarly, price-earnings ratio decreases by 0.01 one year after the impact, and the response becomes positive after three years. This weaker response of prices to total uncertainty is consistent with the analysis in Section 1.2, where it is shown that asset prices' react less to good uncertainty than they do to bad uncertainty even when there was no feedback effect from good uncertainty to expected growth and asset prices reaction to both uncertainties were negative. In the model and in the data, total uncertainty is a combination of the correlated bad and good uncertainty components, which have opposite effects on the asset prices, and it therefore immediately follows that the response of asset prices to the total uncertainty shock is less pronounced. This muted response of asset prices to the total uncertainty masks the significant but opposite effects that different uncertainty components can have on asset valuations, and motivates our decomposition of the total uncertainty into the good and bad parts.

As a final assessment of the model implications for the market return, we evaluate the impact of our macroeconomic uncertainty measures on future level and realized variation in excess returns. In our framework V_{gt} and V_{bt} are the key state variables which drive fluctuations in risk premia and volatility of returns, and in particular, the model-implied loadings of the risk premia and volatility on both V_{gt} and V_{bt} are positive. Panel A in Table 1.5 provides the regression results for predicting excess returns for one, three, and five years. At one- and three-year horizons, the loading coefficients are positive on both measures of exante uncertainty and jointly statistically significant. At the five-year horizon the loading on V_{bt} is positive while that on V_{gt} is negative although both coefficients are statistically insignificant. The R^2s for the three- and five-year horizons are non-negligible at about 10%. Similarly, Panel B of Table 1.5 shows that return volatility loadings on good and bad uncertainty are positive and jointly statistically significant at all horizons with economically significant R^2s of 15–30%. These findings are in line with the model implications. It is also interesting to note that the coefficient on V_{gt} is smaller than that of V_{bt}, consistent with the notion that the effect of bad uncertainty is more important for asset pricing than that of good uncertainty. This is also consistent with the findings in Colacito et al. (2013) for the importance of time variation in skewness and left tail.

1.4.3. Macroeconomic uncertainties and cross-section of returns

Using our empirical measures in the data, we show the implications of macroeconomic growth and good and bad uncertainties for the market and a cross-section of asset returns. Our empirical analysis yields the following key results. First, the risk exposures (betas) to bad uncertainty are negative, and the risk exposures to good uncertainty and expected growth are positive for the market and across the considered asset portfolio returns. This is consistent with our empirical evidence on the impact of growth and uncertainty fluctuations for the market valuations in Section 1.4.2, and with the equilibrium implications of the model in Section 1.2. Second, in line with the theoretical model, we document that bad uncertainty has a negative market price of risk, while the market prices of good uncertainty and expected growth risks are positive in the data. Hence, the high-risk states for the investors are those associated with low expected growth, low good uncertainty, and high bad uncertainty. We show that the risk premia for all the three macroeconomic risk factors are positive, and the uncertainty risk premia help explain the cross-section of expected returns beyond the cash flow channel.

Specifically, following our theoretical model, the portfolio risk premium is given by the product of the market prices of fundamental risks $\Lambda = (\lambda_x, \lambda_{gv}, \lambda_{bv})$, the variance-covariance matrix Ω which captures the quantity of risk, and the exposure of the assets to the underlying macroeconomic risk β_i:[14]

$$E[R_{i,t+1} - R_{f,t}] = \Lambda'\Omega\beta_i. \tag{1.25}$$

Given the innovations to the portfolio returns and to our aggregate risk factors, we can estimate the equity exposures and the market prices of expected growth and bad and good uncertainty risks using a standard Fama and MacBeth (1973) procedure.[15] We first obtain the return betas by running a multivariate regression of each portfolio return innovation on the innovations to the three factors:

$$\begin{aligned} r_{i,t+1} - E_t r_{i,t+1} &= \text{const} + \beta_{i,x}(x_{t+1} - E_t[x_{t+1}]) \\ &\quad + \beta_{i,gv}(V_{g,t+1} - E_t[V_{g,t+1}]) \\ &\quad + \beta_{i,bv}(V_{b,t+1} - E_t[V_{b,t+1}]) \\ &\quad + \text{error}. \end{aligned} \tag{1.26}$$

The slope coefficients in the above projection, $\beta_{i,x}$, $\beta_{i,gv}$, and $\beta_{i,bv}$, represent the portfolio exposures to expected growth, good uncertainty, and bad uncertainty risk, respectively. Next we obtain the factor risk premia $\tilde{\Lambda}$ by running a cross-sectional regression of average returns on the estimated betas:

$$\overline{R_i - R_f} = \tilde{\lambda}_x\beta_{i,x} + \tilde{\lambda}_{gv}\beta_{i,gv} + \tilde{\lambda}_{bv}\beta_{i,bv} + \text{error}. \tag{1.27}$$

We impose a zero-beta restriction in the estimation and thus run the regression without an intercept. The implied factor risk premia, $\tilde{\Lambda} = (\tilde{\lambda}_x, \tilde{\lambda}_{gv}, \tilde{\lambda}_{bv})$, encompass both the vector of

[14] In our model growth shocks are non-Gaussian and therefore the risk premia may include higher-order terms associated with expected growth risk. The volatility risk premia are still linear in the volatility risk exposures. As the focus of our paper is on volatility risk, we maintain a standard linear framework for cross-section evaluation.

[15] We have also considered an alternative econometric approach to measure return innovations similar to Bansal, Dittmar, and Lundblad (2005a), Hansen et al. (2008), and Bansal et al. (2014). The results are similar to our benchmark specification.

the underlying prices of risks Λ and the quantity of risks Ω :

$$\tilde{\Lambda} = \Omega\Lambda.$$

To calculate the underlying prices of expected growth, good and bad uncertainty risk Λ, we pre-multiply the factor risk premia $\tilde{\Lambda}$ by the inverse of the quantity of risk Ω, which corresponds to the estimate of the unconditional variance of the factor innovation in the data. To compute standard errors, we embed the two-state procedure into Generalized Method of Moments (GMM), which allows us to capture statistical uncertainty in estimating jointly asset exposures and market prices of risk.

In our benchmark implementation, we use the market return, the cross-section of 25 equity portfolios sorted on book-to-market ratio and size, as well as two bond portfolios (Credit premium and Term premium portfolios).[16] Table 1.6 shows our key evidence concerning the estimated exposures of these portfolios to expected growth and uncertainty risks and the market prices of risks. Panel A of the table documents that our macroeconomic risk factors are priced in the cross-section, and the market prices of expected growth and good uncertainty risks are positive, while the market price of bad uncertainty risk is negative. This indicates that the adverse economic states for the investor are those with low growth, high bad uncertainty, and low good uncertainty, consistent with the theoretical model. Using one-sided tests against these economically motivated alternatives, the market price of the expected growth risk is significant at a 1% level, while the market prices of volatility risks are significant at a 5% level.

Panel B of the table shows that the equity and bond returns are exposed to these three sources of risks. In particular, all assets have a positive exposure to expected growth risk. The estimated exposures to bad uncertainty risks are negative, while the betas to good uncertainty risks are positive for all the considered asset portfolios. Thus, consistent with

[16] In a related literature, Chen (2010), Bhamra et al. (2010), and McQuade (2014) develop economic models to study defaults and corporate bond spreads, and Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2007) consider model implications for nominal bond yields.

our economic model, our evidence indicates that asset returns increase at times of high expected growth and high good uncertainty and decrease at times of high bad uncertainty, and the magnitudes of the response vary in the cross-section. Nearly all of the estimated exposures are significant at a 1% level.

We combine the estimated market prices of risk, quantity of risk, and the equity and bond betas to evaluate the cross-sectional risk premia implications of our model, and report these empirical results in Table 1.7. As shown in the table, our estimated model can match quite well the level and the dispersion of the risk premia in the cross-section of assets. The market risk premium is 7.2% in the data relative to 8.2% in the model. To help compare the implications of the model to the data, we aggregate the reported average returns into value and size spreads. We define the value spread as the difference between the weighted average returns on the highest and lowest book-to-market portfolios across the five sizes. Similarly, the size spread is defined as the difference between the weighted average returns on the biggest and smallest size portfolios across the book-to-market sorts. In the data, the value spread is 4.38%, which is comparable to 3.34% in the model. The size spread is 4.39%, relative to 5.21% in the model. For the Credit premium portfolio the risk premium is 1.98% in the data and 2.15% in the model. The Term premium is 1.82% in the data relative to 0.64% in the model.[17] We further use the risk premium condition (1.25) to decompose the model risk premia into the various risk contributions. Because our risk factors are correlated, in addition to the own risk compensations for individual shocks (i.e., terms involving the variances on the diagonal of Ω) we also include the risk components due to the interaction of different shocks (i.e., the covariance elements off the diagonal). As shown in the table, the direct risk compensations for the expected growth and good and bad uncertainty shocks are positive for all the portfolios. This is an immediate consequence of our empirical finding that the equity and bond betas and the market prices of risks are of the same sign, so the direct contribution of each source of risk to the total risk premium is positive. On the other

[17] The model excludes an inflation factor which is well known to be important for explaining the term premia.

hand, the risk premia interaction terms can be negative and quite large, e.g., the risk premia due to the covariance of good and bad uncertainty. While it is hard to assess separate risk contributions of each risk factor due to the non-negligible covariance interactions, our results suggest that both good and bad uncertainties have considerable impact on the level and the cross-section of returns.

Overall, our findings for the expected growth risk channel are in line with Bansal, Dittmar, and Lundblad (2005a), Hansen et al. (2008), and Bansal et al. (2014) who show the importance of growth risk for the cross-section of expected returns. Our evidence for the bad uncertainty is further consistent with Bansal et al. (2014), who document that total macroeconomic volatility has a negative market price of risk and depresses asset valuations in the cross-section. On the other hand, our finding for the separate role of the good uncertainty for the stock returns, above and beyond the expected growth and total uncertainty channel, is a novel contribution of this paper.

1.5. Robustness

Our benchmark empirical results are based on the predictive uncertainty measures which are constructed from industrial production data, and which span the full sample period from 1930 to 2012. In this section, we show that our main conclusions are not mechanical and are robust to alternative proxies for the realized variation measures, the construction of the exante uncertainties, and using the post-war period.

1.5.1. Simulation analysis

We use a calibrated model to conduct a Monte-Carlo simulation analysis of the realized semivariances and verify that our empirical results are not driven by the mechanics of the constructed estimators. Specifically, we consider a long-run risks model of Bansal and Yaron (2004) which features conditionally Gaussian consumption shocks, a single stochastic volatility process, and no volatility feedback into the expected growth. Hence, under the null of the model, there are no separate movements in good and bad volatilities and no effect of volatility on future growth. This allows us to assess whether the mechanics of the construction of the semivariances or the finite-sample considerations can spuriously generate our empirical findings. The model setup and calibration are described in A.1.3 and follow Bansal, Kiku, and Yaron (2012). We simulate the model on monthly frequency, and use the same approach as in Section 1.3.2 to construct realized positive and negative variances based on the simulated consumption data.The exante expectations of the quantities are determined from the projections on the model predictive variables, which include positive and negative semivariances, consumption growth, market return, the market price-dividend ratio, and the risk-free rate.

Tables 1.8–1.9 show the model evidence for the projections of consumption and dividend growth rates, for horizons of one to five years, on the extracted expected growth, and the good and bad uncertainties. We report model evidence for finite samples of 83 years each, and population values based on a long simulation of 1,000,000 years. The top panels in the tables report the findings under the benchmark specification. Consistent with our empirical robustness analysis (see below), we also consider two modifications of the benchmark specification, where the predictive uncertainties are based on straight OLS rather than log of the variances, and where we use the AR(1) fit to the monthly consumption growth to remove the fluctuations in the conditional mean.

Table 1.8 reports the slopes and the R^2s for the consumption and dividend projections. As shown in the table, the slope coefficients on bad (good) volatility are generally positive (negative), at least for one- and three-year horizons, and these coefficients decrease (increase) with the horizon of the regressions. The evidence is especially pronounced in the population; indeed, in benchmark simulation specifications all the bad (good) volatility slopes from one to five years to maturity are positive (negative). This is opposite of what we find in the data, where the coefficients on bad (good) volatility are generally most negative (positive) at short horizons, and tend to increase (decrease) with the horizon. The table also shows a considerable amount of noise in estimating the exante uncertainties in small samples, as all the small-sample volatility loadings are insignificant.

In Table 1.9 we assess the joint probability for finding the same volatility signs as in the data combining the evidence across the horizons and across the consumption and dividend regressions. The table documents that across one- to five-year horizon consumption growth regressions, all bad volatility loadings turn out negative and all good volatility turn out positive in 3% of the cases. For dividend regressions, this number is 9%. Finally, combining the evidence from both the consumption and dividend predictability regression, the probabilities of finding the same signs in simulations as in the data are less than 1%. Thus, the simulation evidence clearly shows that the patterns in volatility loadings we find in the data cannot be simply attributed to the mechanics of construction of the realized variance estimators or the finite-sample considerations.

1.5.2. Empirical analysis

We consider various robustness checks concerning the construction of the realized variances in the data. For the first round of robustness checks, we maintain the industrial production growth data to measure the realized variances and modify the construction of exante good and bad uncertainties in several dimensions. First, to mitigate potential small-sample concerns with the realized variance estimators, we consider removing the conditional (rather than the unconditional) mean of industrial production growth in constructing good and bad realized variances. We do so by using the residuals based on fitting an AR(1) to industrial production growth. The summary of the key results for this specification is reported in Table 1.10. By and large, the findings are qualitatively and quantitatively similar to those reported in the benchmark specification. It is worth emphasizing that asymptotically, the conditional mean dynamics do not impact the properties of the realized variance. Our empirical results indicate the conditional mean dynamics also do not affect the realized variance in our finite sample.

Next, we consider changing the cutoff point for defining good and bad uncertainty. Instead of using the unconditional mean, now the good variance state is defined for the states in which industrial production is above its 75 percentile. Table 1.11 provides key results for this case and shows that the main findings for our benchmark specification are intact. Further, instead of taking the logs of the realized variances and exponentiating the fitted values, we run standard OLS regressions on the levels of the positive and negative realized variances and use directly the fitted values from these regressions as proxies for good and bad uncertainties, respectively. Alternatively, while in our benchmark approach we predict the realized variances over a three-year forecast window, for robustness, we also consider shorter and longer horizons, such as one- and five-year window specifications. We also expand the set of the predictive variables and include the term spread, defined as the difference between the 10-year and 3-month Treasury yield, to the benchmark set of predictors. We also experimented with removing some of the variables (e.g., default spread, price-dividend ratio, risk-free rate) from the benchmark set of predictive variables. Finally, we also consider using the cross-section of industry portfolios instead of size and book-to-market to identify the betas and market prices of risk. In the interest of space we do not report these additional tables but note that across all of these modifications of the benchmark specification, we confirm our key empirical results regarding: (i) the relation between good and bad uncertainties and the future macroeconomic growth rates, (ii) the relation between the two uncertainties and the aggregate asset prices, and (iii) the market prices and exposures to the three underlying risks.

For the second set of robustness checks, we consider monthly earnings data, instead of industrial production data, to construct realized variances. Table 1.12 shows a summary of the key macroeconomic and asset-pricing implications of the good and bad uncertainty using these alternative measures of volatility. The table shows that the earnings-based uncertainty measures deliver very similar implications to the industrial-production-based ones. Indeed, as shown in Panel A, with a single exception of R&D investment growth, all future macroeconomic growth rates increase following positive shocks to expected growth, positive shocks to good uncertainty, and negative shocks to bad uncertainty. As shown in Panel B of the table, the contemporaneous responses of aggregate asset prices to uncertainty based on earnings volatility measures are very similar to those based on industrial production measures of volatility. With the exception of the risk-free rate projection, this evidence again is consistent with interpreting the high expected growth, high good uncertainty, and low bad uncertainty as good states for asset valuations. This conclusion is confirmed in Panel C which documents that the market prices of expected consumption and good uncertainty risks are positive, and that of bad uncertainty is negative. As in the benchmark specification, the estimated equity exposures to these risk factors have the same sign as the market prices of risks, so the direct contribution of each macroeconomic risk to the equity risk premium is positive.

Using the estimated expected growth and uncertainty measures we verify whether the results are robust to the post-war sample. As shown in Table 1.13, for the majority of the considered projections, our benchmark conclusions for the relation of the macroeconomic volatilities to growth and asset prices are unchanged.

1.6. Conclusion

In this paper we present an economic framework and empirical measures for studying good and bad aggregate uncertainty. We define good and bad uncertainty as the variance associated with the respective positive and negative innovations of an underlying macroeconomic variable. We show that in the model and in the data, fluctuations in good and bad macroeconomic uncertainty have a significant and opposite impact on future growth and asset valuations.

We develop a version of the long-run risk model which features separate volatilities for good and bad consumption shocks, and feedback from volatilities to future growth. We show that the equity prices decline with bad uncertainty and rise with good uncertainty, provided there is a sufficiently large feedback from good uncertainty to future growth. Moreover, we show that the market price of risk and equity beta are both positive for good uncertainty, while they are both negative for bad uncertainty. This implies that both good and bad uncertainty risks contribute positively to the risk premia.

Empirically, we use the realized semivariance measures based on the industrial production data to construct good and bad uncertainties, and show the model implications are consistent with the data. Specifically, future economic growth, such as consumption, dividend, earnings, GDP, and investment, rise with good uncertainty, while they fall with bad uncertainty. Consistent with the model, equity prices and interest rates increase (decrease) with good (bad) uncertainty. Finally, using the cross-section of assets we estimate a positive market price of good uncertainty risk, and a negative one for bad uncertainty risk. In all, our theoretical and empirical evidence shows the importance of separate movements of good and bad uncertainty for economic growth and asset prices. We leave it for future work to provide explicit economic channels, linking good and bad uncertainty risks with technological aspects of production, investment, and financing opportunities.

Table 1.1: Data summary statistics

	Mean	Std. dev.	AR(1)
Panel A: Macro growth rates			
Consumption growth	1.84	2.16	0.50
GDP growth	2.04	12.91	0.41
Earnings growth	1.77	26.11	0.01
Market dividend growth	1.27	11.32	0.20
Capital investment growth	1.75	14.80	0.42
R&D investment growth	3.51	4.69	0.18
Panel B: Asset prices			
Market return	5.79	19.85	-0.01
Market price-dividend ratio	3.39	0.45	0.88
Real risk-free rate	0.34	2.55	0.73
Default spread	1.21	0.81	0.72
Panel C: Realized volatility			
RV_p	2.34	7.37	0.24
RV_n	2.27	5.68	0.29
RV	4.61	10.91	0.44

The table shows summary statistics for the macroeconomic variables (Panel A), aggregate asset prices (Panel B), and the realized variance measures (Panel C). Consumption, private GDP, as well as capital and R&D investment series are real and per capita. Dividends, earnings, stock prices, and returns are computed for a broad market portfolio. The real risk-free rate corresponds to a 3-month T-bill rate minus expected inflation. Default spread is the difference between the yields on BAA- and AAA-rated corporate bonds. The total realized variance, RV, is based on the sum of squared observations of demeaned monthly industrial production growth over one year, re-scaled to match the unconditional variance of consumption growth. The positive and negative realized semivariances, RV_p and RV_n, decompose the total realized variance into the components pertaining to only positive and negative movements in industrial production growth, respectively. All growth rates and returns are in percentages, and the realized variances are multiplied by 10,000. Data on R&D investment are annual from 1954 to 2008, and all the other data are annual from 1930 to 2012.

Table 1.2: Macroeconomic uncertainties and aggregate growth

	x	V_b	V_g	$Adj-R^2$	p-Value
Consumption growth:					
1Y Ahead	$1.98^{\diamond}$	$-64.76^{\dagger}$	12.97	0.51	0.25
	[4.98]	[−1.42]	[0.83]		
3Y Ahead	$1.07^{\diamond}$	−22.56	12.67	0.33	0.05
	[2.98]	[−0.68]	[1.21]		
5Y Ahead	$0.46^{\diamond}$	−2.20	6.81	0.18	< 0.01
	[2.93]	[−0.08]	[0.73]		
GDP growth:					
1Y Ahead	$4.87^{\diamond}$	$-733.08^{\dagger}$	$277.73^{\dagger}$	0.07	0.25
	[7.37]	[−1.62]	[1.44]		
3Y Ahead	2.53^{*}	$-410.36^{\dagger}$	$180.07^{\dagger}$	0.04	0.28
	[2.13]	[−1.29]	[1.44]		
5Y Ahead	$1.46^{\diamond}$	-142.27^{*}	$66.85^{\diamond}$	0.01	0.02
	[2.53]	[−1.72]	[2.51]		
Market dividend growth:					
1Y Ahead	$8.93^{\diamond}$	$-474.89^{\diamond}$	55.04	0.41	< 0.01
	[4.46]	[−2.41]	[0.84]		
3Y Ahead	$2.89^{\dagger}$	−107.83	60.23	0.08	0.16
	[1.45]	[−0.66]	[1.17]		
5Y Ahead	$1.22^{\dagger}$	-182.40^{*}	$79.83^{\diamond}$	0.04	0.01
	[1.52]	[−2.02]	[2.67]		
Earnings growth:					
1Y Ahead	$12.34^{\diamond}$	$-682.77^{\dagger}$	134.02	0.10	< 0.01
	[3.59]	[−1.29]	[0.66]		
3Y Ahead	0.78	60.55	21.86	−0.02	0.46
	[0.28]	[0.19]	[0.19]		
5Y Ahead	0.85	−155.41	98.54^{*}	0.01	< 0.01
	[0.78]	[−0.97]	[1.84]		

The table shows the predictability evidence from the projection of future macroeconomic growth rates on the current expected consumption growth x, good uncertainty V_g, and bad uncertainty V_b : $\frac{1}{h}\sum_{j=1}^{h}\Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + error$. The table reports the slope coefficients b_h, $t-$statistics, and the adjusted R^2s for the regression horizons of $h = 1, 3$, and 5 years for the corresponding aggregate series y. The $p-$values are computed for the Wald test for the joint significance of good and bad uncertainty, $H_0 : b_{gv} = b_{bv} = 0$. Standard errors are Newey-West adjusted. The notations †, ∗, and ⋄ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses $b_x > 0$, $b_{gv} > 0$, and $b_{bv} < 0$. The data are annual from 1930 to 2012.

Table 1.3: Macroeconomic uncertainties and investment

	x	V_b	V_g	$Adj-R^2$	p-Value
Gross private capital investment growth:					
1Y Ahead	$24.85^{\diamond}$	$-2309.41^{\diamond}$	$912.46^{\diamond}$	0.40	< 0.01
	[4.42]	[−2.85]	[3.41]		
3Y Ahead	$7.76^{\diamond}$	-891.16^{*}	$542.32^{\diamond}$	0.28	< 0.01
	[2.61]	[−2.18]	[3.60]		
5Y Ahead	$3.53^{\diamond}$	-399.32^{*}	$287.53^{\diamond}$	0.29	< 0.01
	[2.46]	[−2.17]	[4.31]		
Nonresidential capital investment growth:					
1Y Ahead	$13.81^{\diamond}$	-789.80^{*}	$226.19^{\dagger}$	0.45	0.07
	[6.74]	[−1.83]	[1.51]		
3Y Ahead	$5.72^{\diamond}$	-272.28	167.58^{*}	0.22	< 0.01
	[3.04]	[−1.26]	[2.11]		
5Y Ahead	$2.90^{\diamond}$	-124.54	93.97^{*}	0.18	0.01
	[3.44]	[−1.01]	[2.15]		
R&D investment growth:					
1Y Ahead	$4.45^{\diamond}$	$-822.83^{\diamond}$	571.37^{*}	0.28	0.05
	[4.05]	[−2.43]	[2.16]		
3Y Ahead	$1.53^{\diamond}$	$-980.22^{\diamond}$	$885.88^{\diamond}$	0.23	< 0.01
	[2.59]	[−2.59]	[4.76]		
5Y Ahead	$0.59^{\dagger}$	$-847.67^{\diamond}$	$775.23^{\diamond}$	0.24	< 0.01
	[1.59]	[−2.88]	[4.86]		
R&D stock growth:					
1Y Ahead	$1.13^{\diamond}$	$-983.80^{\diamond}$	308.73^{*}	0.55	< 0.01
	[3.83]	[−3.31]	[1.74]		
3Y Ahead	$1.05^{\diamond}$	$-950.27^{\diamond}$	$342.17^{\dagger}$	0.46	< 0.01
	[3.60]	[−2.86]	[1.57]		
5Y Ahead	$0.68^{\diamond}$	$-998.32^{\diamond}$	428.55^{*}	0.41	< 0.01
	[2.54]	[−2.86]	[1.81]		
Utility patents count growth:					
1Y Ahead	2.57^{*}	-209.98	13.11	0.11	0.11
	[1.72]	[−1.01]	[0.15]		
3Y Ahead	2.40^{*}	-158.15^{*}	18.55	0.13	0.02
	[1.88]	[−1.78]	[0.64]		
5Y Ahead	1.54^{*}	-159.60^{*}	26.64	0.14	< 0.01
	[1.96]	[−1.94]	[0.92]		

The table shows the predictability evidence from the projection of future investment growth rates on the current expected consumption growth x, good uncertainty V_g, and bad uncertainty V_b : $\frac{1}{h}\sum_{j=1}^{h}\Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + error$. The table reports the slope coefficients b_h, $t-$statistics, and the adjusted R^2s for the regression horizons of $h = 1, 3$, and 5 years for the corresponding investment series y. The $p-$values are computed for the Wald test for the joint significance of good and bad uncertainty, $H_0 : b_{gv} = b_{bv} = 0$. Standard errors are Newey-West adjusted. The notations †, ∗, and ⋄ indicate the significance of the coefficients at 10%, 5%, and 1%, levels respectively, against the economically motivated, alternative one-sided hypotheses $b_x > 0$, $b_{gv} > 0$, and $b_{bv} < 0$. R&D investment data are from 1954 to 2008, R&D stock data are from 1960 to 2007, and all the other data are annual from 1930 to 2012.

Table 1.4: Macroeconomic uncertainties and aggregate prices

Panel A: Level-based projection					
	x	V_b	V_g	$Adj - R^2$	p-Value
Market price-dividend ratio	8.82	$-2313.28^{\diamond}$	279.27	0.21	< 0.01
	[0.94]	[−2.67]	[0.93]		
Real risk-free rate	0.05	$-222.24^{\diamond}$	$80.50^{\diamond}$	0.21	< 0.01
	[0.08]	[−2.36]	[2.74]		
Default spread	-0.36^{*}	$50.54^{\diamond}$	−3.80	0.47	< 0.01
	[−1.81]	[2.99]	[−0.52]		
Panel B: First difference-based projection					
	Δx	ΔV_b	ΔV_g	$Adj - R^2$	p-Value
ΔMarket price-dividend ratio	$18.57^{\diamond}$	$-1353.26^{\diamond}$	$448.49^{\diamond}$	0.61	< 0.01
	[9.97]	[−4.21]	[3.11]		
ΔReal risk-free rate	0.01	-107.47^{*}	31.75	0.16	< 0.01
	[0.04]	[−1.65]	[1.19]		
ΔDefault spread	$-0.26^{\diamond}$	$40.46^{\diamond}$	-10.64^{*}	0.30	0.01
	[−2.61]	[2.84]	[−1.98]		

The table reports the evidence from the projections of the aggregate asset-price variables on the contemporaneous expected consumption growth x, and the good and bad uncertainty variables, V_g and V_b. Panel A shows the regression results based on the levels of the variables, and Panel B shows the output for the first differences. The table reports the slope coefficients, $t-$statistics, the adjusted R^2s, and the $p-$values for the Wald test for the joint significance of good and bad uncertainty, $H_0 : b_{gv} = b_{bv} = 0$. Standard errors are Newey-West adjusted. The notations †, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses $b_x > 0$, $b_{gv} > 0$, and $b_{bv} < 0$ for the market price-dividend ratio and the real risk-free rate, and $b_x < 0$, $b_{gv} < 0$, and $b_{bv} > 0$ for the default spread. The data are annual from 1930 to 2012.

Table 1.5: Macroeconomic uncertainties and equity returns

	x	V_b	V_g	$Adj-R^2$	p-Value
Panel A: Excess return projection					
1Y Ahead	-1.91	198.57	19.85	-0.01	0.05
	[-0.45]	[0.45]	[0.12]		
3Y Ahead	-1.41	82.02	69.42	0.09	< 0.01
	[-1.15]	[0.37]	[0.90]		
5Y Ahead	-2.04^{*}	$227.97^{\dagger}$	-30.34	0.08	0.14
	[-2.44]	[1.54]	[-0.74]		
Panel B: Return volatility projection					
1Y Ahead	$-1.94^{\dagger}$	24.39	49.10^{*}	0.34	< 0.01
	[-1.66]	[0.34]	[2.19]		
3Y Ahead	-1.39	64.42	3.35	0.16	0.02
	[-1.15]	[0.87]	[0.16]		
5Y Ahead	-0.63	34.51	10.75	0.15	< 0.01
	[-0.80]	[0.63]	[0.75]		

The table reports the evidence from the projections of future excess returns (Panel A) and realized variance of returns (Panel B) on expected consumption growth x, and the good and bad uncertainty variables, V_g and V_b. The table reports the slope coefficients, $t-$statistics, the adjusted R^2s, and the $p-$values for the Wald test for the joint significance of good and bad uncertainty, $H_0 : b_{gv} = b_{bv} = 0$. Standard errors are Newey-West adjusted. The notations $\dagger$, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated alternative hypotheses $b_x \neq 0$, $b_{gv} > 0$, and $b_{bv} > 0$. The data are annual from 1930 to 2012.

Table 1.6: Cross-sectional implications

Panel A: Market-prices of risk ($\Lambda/100$)

	λ_x	λ_{bv}	λ_{gv}
λ	$0.95^{\diamond}$	-66.06^{*}	38.58^{*}
	[4.37]	[−1.82]	[2.13]

Panel B: Exposures to risks ($\beta/100$)

	β_x	β_{bv}	β_{gv}		β_x	β_{bv}	β_{gv}
MKT	$25.08^{\diamond}$	$-1537.04^{\diamond}$	$613.89^{\diamond}$	S3BM4	$33.53^{\diamond}$	$-1591.13^{\diamond}$	$707.77^{\diamond}$
	[15.33]	[−4.87]	[3.95]		[13.47]	[−3.45]	[3.16]
S1BM1	$34.49^{\diamond}$	$-2888.64^{\diamond}$	$1016.18^{\diamond}$	S3BM5	$37.73^{\diamond}$	$-1493.40^{\diamond}$	$696.40^{\diamond}$
	[6.16]	[−4.24]	[2.99]		[13.10]	[−2.89]	[2.97]
S1BM2	$47.06^{\diamond}$	-1351.55^{*}	$939.72^{\diamond}$	S4BM1	$25.92^{\diamond}$	$-1809.04^{\diamond}$	$732.72^{\diamond}$
	[10.37]	[−1.72]	[2.72]		[9.56]	[−5.70]	[4.47]
S1BM3	$40.52^{\diamond}$	$-2058.56^{\diamond}$	$919.29^{\diamond}$	S4BM2	$28.82^{\diamond}$	$-1585.84^{\diamond}$	$757.34^{\diamond}$
	[14.79]	[−3.90]	[3.79]		[12.48]	[−3.89]	[4.10]
S1BM4	$42.95^{\diamond}$	$-1919.89^{\diamond}$	$1001.33^{\diamond}$	S4BM3	$32.46^{\diamond}$	$-1790.46^{\diamond}$	$802.42^{\diamond}$
	[12.81]	[−3.88]	[4.54]		[14.91]	[−4.30]	[3.97]
S1BM5	$46.40^{\diamond}$	$-1943.51^{\diamond}$	$1049.88^{\diamond}$	S4BM4	$35.95^{\diamond}$	$-2178.10^{\diamond}$	$975.61^{\diamond}$
	[14.18]	[−3.66]	[4.38]		[22.20]	[−8.36]	[7.03]
S2BM1	$35.86^{\diamond}$	-1351.02^{*}	615.07^{*}	S4BM5	$39.92^{\diamond}$	$-1871.59^{\diamond}$	$1085.74^{\diamond}$
	[10.85]	[−2.18]	[2.10]		[12.09]	[−3.91]	[4.38]
S2BM2	$37.06^{\diamond}$	-1225.38^{*}	$588.72^{\diamond}$	S5BM1	$23.11^{\diamond}$	$-1592.88^{\diamond}$	$563.17^{\diamond}$
	[15.11]	[−2.30]	[2.44]		[9.95]	[−4.26]	[3.01]
S2BM3	$37.04^{\diamond}$	$-1637.18^{\diamond}$	$775.61^{\diamond}$	S5BM2	$24.51^{\diamond}$	$-1684.39^{\diamond}$	$683.83^{\diamond}$
	[12.08]	[−2.56]	[2.79]		[14.23]	[−6.77]	[5.74]
S2BM4	$38.42^{\diamond}$	$-1672.86^{\diamond}$	$863.86^{\diamond}$	S5BM3	$27.41^{\diamond}$	$-1617.78^{\diamond}$	$733.52^{\diamond}$
	[15.77]	[−3.81]	[4.19]		[11.65]	[−6.94]	[6.90]
S2BM5	$36.69^{\diamond}$	$-2316.27^{\diamond}$	$1000.77^{\diamond}$	S5BM4	$31.47^{\diamond}$	$-1503.41^{\diamond}$	$734.06^{\diamond}$
	[13.68]	[−5.37]	[4.45]		[11.36]	[−4.17]	[4.26]
S3BM1	$33.92^{\diamond}$	$-1754.57^{\diamond}$	$812.16^{\diamond}$	S5BM5	$27.62^{\diamond}$	$-3842.95^{\diamond}$	$1570.43^{\diamond}$
	[12.46]	[−3.32]	[3.49]		[11.72]	[−15.80]	[15.55]
S3BM2	$31.04^{\diamond}$	$-1648.15^{\diamond}$	$634.32^{\diamond}$	DEF	$8.36^{\diamond}$	$-388.33^{\diamond}$	33.08
	[13.45]	[−3.87]	[3.32]		[6.60]	[−3.15]	[0.57]
S3BM3	$31.62^{\diamond}$	$-1542.86^{\diamond}$	$640.69^{\diamond}$	TERM	$1.80^{\dagger}$	$-227.44^{\diamond}$	89.18^{*}
	[17.24]	[−3.53]	[3.14]		[1.46]	[−2.46]	[1.91]

The table shows the estimates of the market prices of risks (Panel A) and asset exposures (Panel B) to expected growth, good uncertainty, and bad uncertainty risks. The cross-section includes the market (MKT), 25 portfolios sorted on size (S) and book-to-market (BM), and Credit (DEF) and Term premium (TERM) bond portfolios. The reported betas and the market prices of risks are divided by 100. *T*-statistics are in brackets, and are based on Newey-West standard errors from GMM estimation. The notations $\dagger$, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses that $\lambda_x, \beta_x, \lambda_{gv}$, and β_{gv} are positive, and λ_{bv} and β_{bv} are negative. Data are annual from 1930 to 2012.

Table 1.7: Risk premia decomposition

	Total		Model decomposition					
	Model	Data	$RP_{x,x}$	$RP_{bv,bv}$	$RP_{gv,gv}$	$RP_{x,bv}$	$RP_{x,gv}$	$RP_{bv,gv}$
MKT	8.15	7.17	10.27	9.92	12.98	1.12	−5.64	−20.51
S1BM1	11.04	5.48	14.12	18.65	21.49	1.81	−8.35	−36.68
S1BM2	16.15	10.78	19.27	8.72	19.87	1.58	−9.84	−23.46
S1BM3	13.35	13.93	16.59	13.29	19.44	1.67	−8.85	−28.79
S1BM4	14.55	16.43	17.59	12.39	21.17	1.68	−9.48	−28.80
S1BM5	15.74	18.68	19.00	12.55	22.20	1.77	−10.13	−29.65
S2BM1	11.52	8.47	14.68	8.72	13.01	1.32	−7.15	−19.05
S2BM2	11.91	11.97	15.17	7.91	12.45	1.30	−7.22	−17.70
S2BM3	12.21	13.71	15.17	10.57	16.40	1.44	−7.87	−23.50
S2BM4	12.94	14.59	15.73	10.80	18.27	1.49	−8.37	−24.98
S2BM5	12.27	15.90	15.03	14.95	21.16	1.67	−8.60	−31.94
S3BM1	11.31	9.40	13.89	11.33	17.17	1.41	−7.56	−24.93
S3BM2	9.85	11.39	12.71	10.64	13.41	1.30	−6.54	−21.67
S3BM3	10.17	12.09	12.95	9.96	13.55	1.28	−6.64	−20.92
S3BM4	10.95	13.28	13.73	10.27	14.97	1.34	−7.14	−22.22
S3BM5	12.25	14.73	15.45	9.64	14.72	1.41	−7.69	−21.29
S4BM1	8.58	8.54	10.61	11.68	15.49	1.24	−6.16	−24.28
S4BM2	9.78	9.18	11.80	10.24	16.01	1.23	−6.66	−22.85
S4BM3	10.80	11.28	13.29	11.56	16.97	1.39	−7.32	−25.08
S4BM4	12.10	12.46	14.72	14.06	20.63	1.60	−8.41	−30.50
S4BM5	14.07	12.91	16.35	12.08	22.96	1.59	−9.35	−29.57
S5BM1	7.30	7.07	9.46	10.28	11.91	1.10	−5.18	−20.27
S5BM2	8.10	7.17	10.04	10.87	14.46	1.16	−5.80	−22.63
S5BM3	9.23	8.27	11.22	10.44	15.51	1.21	−6.38	−22.78
S5BM4	10.55	8.54	12.89	9.70	15.52	1.26	−6.95	−21.88
S5BM5	10.23	11.03	11.31	24.81	33.21	1.98	−9.30	−51.77
DEF	2.15	1.98	3.42	2.51	0.70	0.33	−1.28	−3.53
TERM	0.64	1.82	0.74	1.47	1.89	0.12	−0.56	−3.01

The table shows the estimates of risk premia in the data and in the model, and the decomposition of the model risk premia into the compensations for expected growth, good uncertainty, bad uncertainty, and the covariance components.The cross-section includes the market (MKT), 25 portfolios sorted on size (S) and book-to-market (BM), and Credit (DEF) and Term premium (TERM) bond portfolios. Data are annual from 1930 to 2012.

Table 1.8: Simulation analysis of macroeconomic uncertainties and aggregate growth

	x				V_b				V_g				R^2			
	Pop	5%	Med	95%	Pop	5%	Med	95%	Pop	5%	Med	95%	Pop	5%	Med	95%
Benchmark model:																
Consumption:																
1Y Ahead	1.57	1.22	2.00	3.00	8.53	-10.58	53.56	144.74	-9.56	-147.41	-56.91	8.67	0.41	0.24	0.42	0.59
3Y Ahead	1.03	0.64	1.02	1.41	1.19	-35.10	1.57	39.67	-1.61	-38.30	-2.01	33.39	0.36	0.16	0.38	0.60
5Y Ahead	0.78	0.24	0.67	1.13	0.24	-45.65	-2.81	40.08	-0.53	-38.34	2.80	44.23	0.29	0.09	0.31	0.56
Dividend:																
1Y Ahead	3.95	-0.47	5.37	12.76	19.89	-480.68	106.35	788.62	-51.52	-1087.24	-282.10	254.73	0.09	0.01	0.14	0.33
3Y Ahead	2.57	-0.89	2.68	6.42	1.71	-357.82	-2.84	354.48	-12.72	-428.53	-52.35	277.72	0.09	-0.01	0.12	0.35
5Y Ahead	1.96	-1.21	1.75	4.82	-0.20	-304.46	-11.11	277.78	-6.72	-319.33	-21.01	257.18	0.08	-0.02	0.12	0.38
Straight OLS model:																
Consumption:																
1Y Ahead	3.15	1.78	2.60	3.71	132.97	35.13	115.83	241.12	-132.80	-240.56	-116.93	-37.91	0.46	0.26	0.45	0.62
3Y Ahead	1.00	1.00	1.00	1.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.36	0.15	0.37	0.59
5Y Ahead	0.58	0.14	0.57	0.95	-14.54	-56.22	-13.27	24.80	14.52	-23.91	13.59	55.94	0.30	0.08	0.30	0.56
Dividend:																
1Y Ahead	12.29	-2.05	7.76	18.49	660.74	-782.68	336.74	1518.28	-707.98	-1753.33	-545.05	528.96	0.13	0.01	0.16	0.36
3Y Ahead	3.96	-1.90	2.86	7.67	109.97	-488.25	9.05	503.43	-126.15	-575.38	-74.37	402.16	0.09	-0.01	0.11	0.34
5Y Ahead	2.33	-2.06	1.60	5.21	29.93	-406.45	-28.67	329.97	-39.86	-370.39	-8.94	355.17	0.08	-0.02	0.11	0.37
AR(1) Adjustment model:																
Consumption:																
1Y Ahead	1.57	1.22	2.00	2.99	8.88	-9.97	54.65	147.90	-9.83	-149.61	-58.09	8.32	0.41	0.23	0.42	0.59
3Y Ahead	1.02	0.64	1.01	1.41	1.12	-36.22	1.43	39.92	-1.50	-38.38	-1.80	34.87	0.36	0.16	0.38	0.60
5Y Ahead	0.78	0.24	0.67	1.13	0.12	-47.10	-3.13	40.17	-0.30	-38.21	3.09	45.67	0.29	0.09	0.31	0.56
Dividend:																
1Y Ahead	3.91	-0.48	5.35	12.71	19.92	-492.75	110.35	801.79	-50.16	-1098.71	-285.97	267.96	0.09	0.01	0.14	0.33
3Y Ahead	2.54	-0.88	2.67	6.42	1.95	-365.16	-3.93	360.27	-12.12	-431.43	-52.98	286.43	0.09	-0.01	0.12	0.35
5Y Ahead	1.92	-1.20	1.75	4.81	-0.35	-310.83	-12.29	282.25	-5.62	-322.12	-20.90	262.79	0.08	-0.02	0.12	0.38

The table shows the Monte-Carlo predictability evidence for the projection of future consumption and dividend growth rates on the current expected consumption growth x, good uncertainty V_g, and bad uncertainty V_b : $\frac{1}{h}\sum_{j=1}^{h}\Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + error$. The table reports the population and small-sample estimates (corresponding to $5\%, 50\%$, and 95% percentile of the distribution in simulations) of the slope coefficients and R^2s. The consumption, dividends, and asset prices are simulated on monthly frequency and aggregated to annual horizon under the long-run risks, single-volatility model configuration of Bansal, Kiku, and Yaron (2012). Realized positive and negative variances are constructed from the model-simulated demeaned monthly consumption growth rate over the year. The exante uncertainty measures correspond to the projections of the log realized variances on the set of predictors, such as realized positive and negative variances, consumption growth, market return, and the risk-free rate. Small-sample evidence is based on 100,000 simulations of 83 years of monthly data; the population estimates are based on a long simulation of 1,000,000 years of data.

Table 1.9: Model-implied significance of the volatility coefficients

	$Pr(b_{bv} < 0)$	$Pr(b_{gv} > 0)$	$Pr(b_{bv<0}$ & $b_{gv} > 0)$
	Benchmark model:		
Consumption growth:	0.05	0.04	0.03
Dividend growth:	0.24	0.12	0.09
Joint:	0.02	0.01	0.01
	Straight OLS model:		
Consumption growth:	0.004	0.004	0.002
Dividend growth:	0.21	0.13	0.11
Joint:	0.002	0.001	0.001
	AR(1) Adjustment model:		
Consumption growth:	0.05	0.04	0.03
Dividend growth:	0.24	0.12	0.09
Joint:	0.02	0.01	0.01

The table shows the Monte-Carlo predictability evidence for the projection of future consumption and dividend growth rates on the current expected consumption growth x, good uncertainty V_g, and bad uncertainty V_b : $\frac{1}{h}\sum_{j=1}^{h} \Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + error$. The table reports the fraction of samples in which bad (good) uncertainty loadings at 1-, 3-, and 5-year maturities are all negative (positive). The data are simulated on monthly frequency and aggregated to annual horizon under the long-run risks, single-volatility model configuration of Bansal, Kiku, and Yaron (2012). Realized positive and negative variances are constructed from the model-simulated demeaned monthly consumption growth rate over the year. The exante uncertainty measures correspond to the projections of the log realized variances on the set of predictors, such as realized positive and negative variances, consumption growth, market return, the market price-dividend ratio, and the risk-free rate. Small-sample evidence is based on 100,000 simulations of 83 years of monthly data.

Table 1.10: Conditionally demeaned industrial production-based uncertainties

	x	V_b	V_g	$Adj-R^2$
Panel A: Aggregate growth rate predictability				
Consumption growth	$2.11^{\diamond}$	-63.49	21.41	0.48
	[4.80]	[-1.03]	[0.79]	
GDP growth	$7.22^{\diamond}$	$-910.48^{\dagger}$	$460.51^{\dagger}$	0.13
	[2.69]	[-1.39]	[1.29]	
Market dividend growth	$7.26^{\diamond}$	76.37	-154.09	0.29
	[3.41]	[0.29]	[-1.18]	
Earnings growth	$13.05^{\diamond}$	-401.02	75.59	0.09
	[2.69]	[-0.67]	[0.26]	
Capital investment growth	$24.56^{\diamond}$	$-1574.59^{\dagger}$	885.52^{*}	0.32
	[3.65]	[-1.59]	[2.01]	
R&D investment growth	$4.11^{\diamond}$	-1046.42^{*}	594.90^{*}	0.22
	[4.92]	[-1.95]	[1.77]	
Panel B: Aggregate asset prices				
Level-based projections:				
Market price-dividend ratio	3.60	-588.21	-470.52	0.17
	[0.43]	[-0.47]	[-1.09]	
Real risk-free rate	-0.44	$-106.17^{\dagger}$	$39.29^{\dagger}$	0.07
	[-0.71]	[-1.45]	[1.53]	
Default spread	$-0.44^{\diamond}$	$63.02^{\diamond}$	-13.20	0.45
	[-2.43]	[2.66]	[-1.15]	
First difference-based projections:				
ΔMarket price-dividend ratio	$21.25^{\diamond}$	-706.94^{*}	$367.33^{\diamond}$	0.52
	[9.35]	[-2.23]	[3.22]	
ΔReal risk-free rate	$0.36^{\dagger}$	-111.12^{*}	$41.35^{\dagger}$	0.15
	[1.53]	[-2.17]	[1.57]	
ΔDefault spread	$-0.33^{\diamond}$	$33.99^{\diamond}$	-11.47^{*}	0.22
	[-2.83]	[2.52]	[-2.28]	
Panel C: Asset-pricing implications				
Prices of risk ($\Lambda/100$)	$1.04^{\diamond}$	-18.65	$36.98^{\dagger}$	
	[3.53]	[-0.52]	[1.58]	
Market exposures ($\beta/100$)	$30.92^{\diamond}$	$-1452.43^{\diamond}$	$811.16^{\diamond}$	
	[15.11]	[-3.82]	[6.33]	

The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly, conditionally demeaned, industrial-production data. The conditional mean is estimated based on an AR(1) model of industrial production growth. Panel A documents the slope coefficients, $t-$statistics, and the R^2 in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty V_g, and bad uncertainty V_b. Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations $\dagger$, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

Table 1.11: Industrial production-based uncertainties with shifted cutoff

	x	V_b	V_g	$Adj-R^2$
Panel A: Aggregate growth rate predictability				
Consumption growth	$2.01^{\diamond}$	$-80.25^{\dagger}$	16.91	0.52
	[5.14]	[−1.56]	[1.09]	
GDP growth	$4.48^{\diamond}$	$-680.39^{\dagger}$	$229.82^{\dagger}$	0.05
	[6.60]	[−1.58]	[1.50]	
Market dividend growth	$8.78^{\diamond}$	-459.88^{*}	38.09	0.40
	[4.37]	[−1.96]	[0.50]	
Earnings growth	$12.26^{\diamond}$	−724.71	129.06	0.10
	[3.58]	[−1.19]	[0.60]	
Capital investment growth	$24.08^{\diamond}$	-2319.51^{*}	$830.56^{\diamond}$	0.38
	[4.28]	[−2.31]	[2.65]	
R&D investment growth	$4.39^{\diamond}$	$-724.28^{\diamond}$	582.91^{*}	0.28
	[4.02]	[−2.73]	[2.31]	
Panel B: Aggregate asset prices				
Level-based projections:				
Market price-dividend ratio	10.46	$-3075.22^{\diamond}$	$493.02^{\dagger}$	0.24
	[1.16]	[−2.96]	[1.49]	
Real risk-free rate	−0.02	-226.10^{*}	$73.65^{\diamond}$	0.19
	[−0.03]	[−2.14]	[2.51]	
Default spread	$-0.41^{\diamond}$	$72.97^{\diamond}$	$-10.93^{\dagger}$	0.52
	[−2.42]	[4.30]	[−1.40]	
First difference-based projections:				
ΔMarket price-dividend ratio	$18.54^{\diamond}$	$-1588.31^{\diamond}$	$445.77^{\diamond}$	0.61
	[10.25]	[−4.72]	[3.58]	
ΔReal risk-free rate	−0.01	$-117.20^{\dagger}$	28.81	0.14
	[−0.06]	[−1.53]	[1.10]	
ΔDefault spread	$-0.28^{\diamond}$	$57.12^{\diamond}$	-14.03^{*}	0.39
	[−2.89]	[3.13]	[−2.21]	
Panel C: Asset-Pricing Implications				
Prices of risk ($\Lambda/100$)	$0.94^{\diamond}$	-76.20^{*}	37.39^{*}	
	[4.32]	[−1.73]	[2.06]	
Market exposures ($\beta/100$)	$24.39^{\diamond}$	$-1642.30^{\diamond}$	$560.40^{\diamond}$	
	[16.86]	[−5.18]	[4.19]	

The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly industrial-production data, computed under a shifted cutoff for the good and bad uncertainty observations. The expost positive (negative) semivariance is computed using observations above (below) the 75th percentile of industrial production growth. Panel A documents the slope coefficients, $t-$statistics, and the R^2 in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty V_g, and bad uncertainty V_b. Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations †, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

Table 1.12: Earnings-based uncertainties

	x	V_b	V_g	$Adj-R^2$
Panel A: Aggregate growth rate predictability				
Consumption growth	$1.86^{\diamond}$	-160.58^{*}	41.10^{*}	0.53
	[7.10]	[−2.00]	[1.97]	
GDP growth	4.86^{*}	$-371.75^{\diamond}$	$98.78^{\diamond}$	0.05
	[2.00]	[−2.73]	[2.84]	
Market dividend growth	$6.28^{\diamond}$	$-1448.37^{\diamond}$	$354.80^{\diamond}$	0.37
	[3.01]	[−3.99]	[3.85]	
Earnings growth	4.12	$-1319.52^{\dagger}$	166.00	0.21
	[0.67]	[−1.60]	[0.86]	
Capital investment growth	$18.98^{\diamond}$	$-3498.01^{\diamond}$	$901.57^{\diamond}$	0.43
	[5.40]	[−3.17]	[3.21]	
R&D investment growth	$2.63^{\diamond}$	$720.54^{\diamond}$	$-195.27^{\diamond}$	0.26
	[2.72]	[2.55]	[−2.69]	
Panel B: Aggregate Asset Prices				
Level-based projections:				
Market price-dividend ratio	2.38	$-5282.53^{\diamond}$	$1377.15^{\diamond}$	0.13
	[0.37]	[−2.59]	[2.66]	
Real risk-free rate	−0.40	51.03	−24.48	−0.01
	[−0.39]	[0.25]	[−0.44]	
Default spread	−0.08	76.82^{*}	−15.42	0.19
	[−0.51]	[1.68]	[−1.26]	
First difference-based projections:				
ΔMarket price-dividend ratio	$16.81^{\diamond}$	$-1688.33^{\diamond}$	$399.38^{\diamond}$	0.61
	[7.89]	[−4.93]	[4.48]	
ΔReal risk-free rate	−0.44	-74.01^{*}	12.36	0.05
	[−0.96]	[−2.02]	[1.24]	
ΔDefault spread	−0.09	$39.07^{\dagger}$	−6.31	0.44
	[−1.15]	[1.28]	[−0.81]	
Panel C: Asset-pricing implications				
Prices of risk ($\Lambda/100$)	0.46	$-238.78^{\diamond}$	$69.48^{\diamond}$	
	[1.24]	[−3.32]	[3.19]	
Market exposures ($\beta/100$)	$25.68^{\diamond}$	$-1502.40^{\diamond}$	$401.75^{\diamond}$	
	[13.54]	[−6.92]	[7.49]	

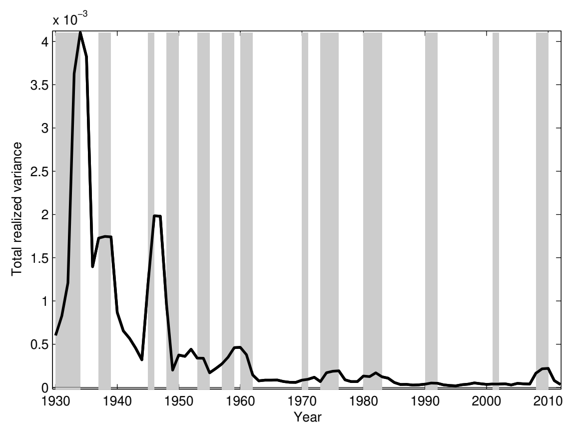
The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly corporate earnings data. Panel A documents the slope coefficients, $t-$statistics, and the R^2 in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty V_g, and bad uncertainty V_b. Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations †, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

Table 1.13: Benchmark uncertainties: post-war sample

	x	V_b	V_g	$Adj-R^2$
Panel A: Aggregate growth rate predictability				
Consumption growth	$1.43^{\diamond}$	$-275.72^{\diamond}$	$123.23^{\diamond}$	0.41
	[6.27]	[−3.57]	[2.88]	
GDP growth	$2.27^{\dagger}$	$-1127.75^{\diamond}$	1174.93^{*}	0.44
	[1.33]	[−2.41]	[2.09]	
Market dividend growth	2.41	−362.83	136.85	−0.01
	[1.26]	[−0.59]	[0.51]	
Earnings growth	11.39^{*}	−1941.22	666.54	0.02
	[1.78]	[−0.77]	[0.67]	
Capital investment growth	$8.33^{\diamond}$	$-2231.30^{\diamond}$	$1813.75^{\diamond}$	0.42
	[3.45]	[−4.51]	[3.88]	
Panel B: Aggregate asset prices				
Level-based projections:				
Market price-dividend ratio	−5.92	$-3987.38^{\dagger}$	−1011.95	0.34
	[−0.65]	[−1.59]	[−0.75]	
Real risk-free rate	$1.32^{\diamond}$	$-440.50^{\diamond}$	80.05	0.37
	[3.04]	[−2.35]	[1.12]	
Default spread	$-0.41^{\diamond}$	113.33^{*}	-49.41^{*}	0.33
	[−3.15]	[1.66]	[−2.11]	
First difference-based projections:				
ΔMarket price-dividend ratio	$20.93^{\diamond}$	$-2740.74^{\diamond}$	$665.55^{\diamond}$	0.61
	[12.62]	[−4.31]	[2.70]	
ΔReal risk-free rate	0.37	$-364.57^{\diamond}$	$174.82^{\diamond}$	0.46
	[1.09]	[−3.39]	[5.60]	
ΔDefault spread	$-0.40^{\diamond}$	$122.58^{\diamond}$	−4.44	0.61
	[−3.63]	[2.99]	[−0.77]	
Panel C: Asset-pricing implications				
Prices of risk ($\Lambda/100$)	$0.82^{\diamond}$	$-74.95^{\diamond}$	$38.15^{\diamond}$	
	[4.87]	[−2.81]	[3.43]	
Market exposures ($\beta/100$)	$28.84^{\diamond}$	$-2912.56^{\diamond}$	$938.32^{\diamond}$	
	[11.35]	[−4.74]	[2.57]	

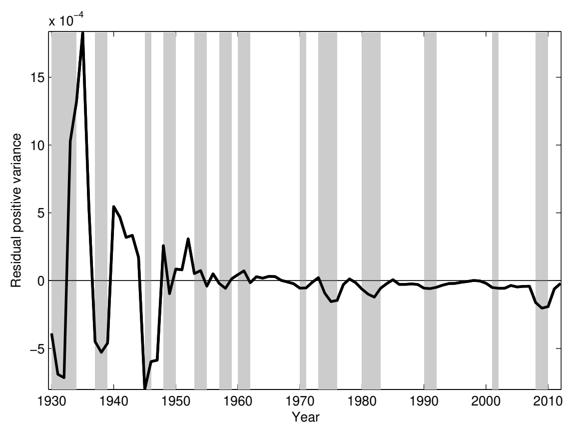
The table presents a summary of the macroeconomic and asset-price evidence using the benchmark uncertainty measures in the post-war period. Panel A documents the slope coefficients, $t-$statistics, and the R^2 in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty V_g, and bad uncertainty V_b. Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations $\dagger$, $*$, and $\diamond$ indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1947 to 2012.

Figure 1.1: Total realized variance



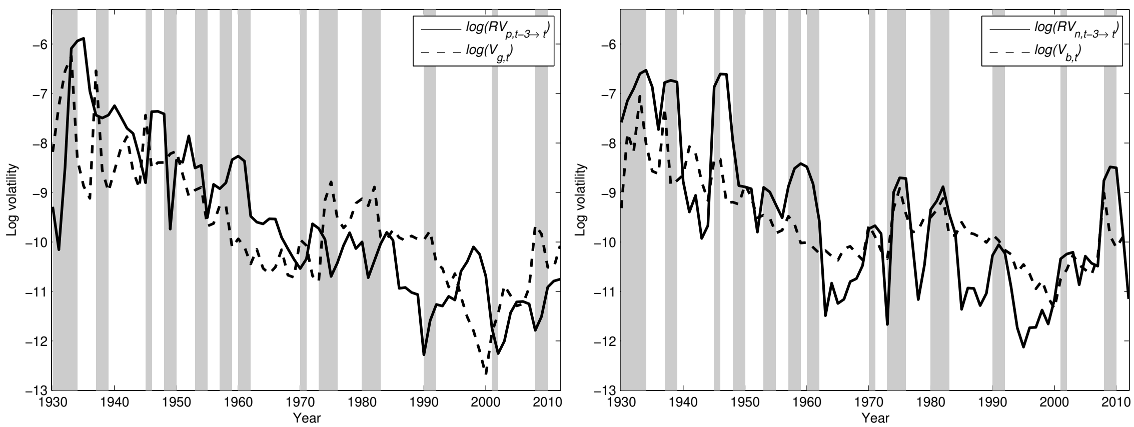
The figure shows the time series plot of the total realized variance smoothed over a 3-year window. The total realized variance is based on the sum of squared observations of demeaned monthly industrial production growth over 1-year, re-scaled to match the unconditional variance of consumption growth. The shaded areas represent National Bureau of Economic Research (NBER) recessions.

Figure 1.2: Residual positive variance



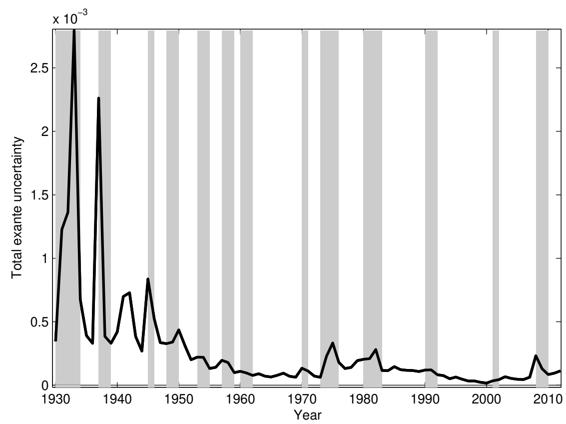
The figure shows the time series plot of the residual positive variance, smoothed over a 3-year window, which is orthogonal to the negative variance. The positive and negative realized semivariances decompose the total realized variance into the components pertaining only to positive and negative movements in industrial production growth, respectively. The residual positive variance is computed from the projection of the positive realized semivariance onto the negative one. The shaded areas represent NBER recessions.

Figure 1.3: Realized and predictive log volatilities



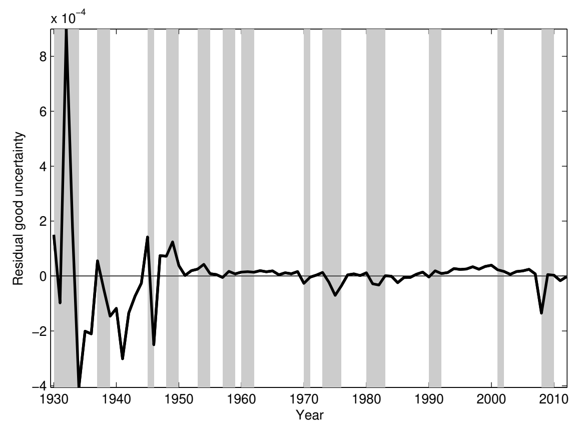
The figure shows the time series plots of the log positive (left panel) and negative (right panel) realized variances and their predictive values from the projection. The shaded areas represent NBER recessions. The benchmark predictive variables in the projection include positive and negative realized semivariances, consumption growth rate, the real-market return, the market price-dividend ratio, the real risk-free rate, and the default spread.

Figure 1.4: Total exante uncertainty



The figure shows the time series plot of the total exante uncertainty. The total exante uncertainty is constructed from the predictive regressions of future overall realized variance. The shaded areas represent NBER recessions.

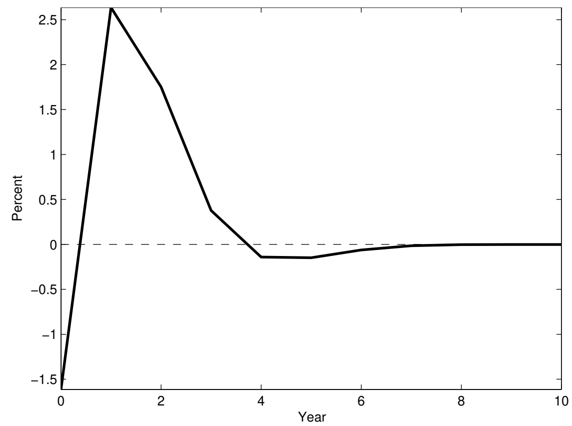
Figure 1.5: Residual good uncertainty



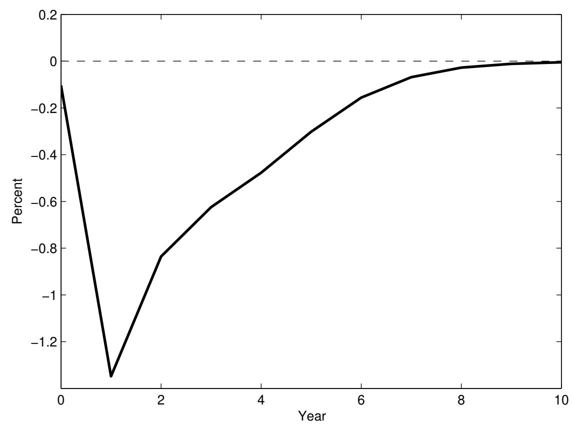
The figure shows the time series plot of the residual good uncertainty which is orthogonal to the bad uncertainty. The good and bad uncertainties are constructed from the predictive regressions of future realized positive and negative variances, respectively. The residual good uncertainty is computed from the projection of good uncertainty onto bad uncertainty. The shaded areas represent NBER recessions.

Figure 1.6: Impulse response of GDP to macro uncertainties

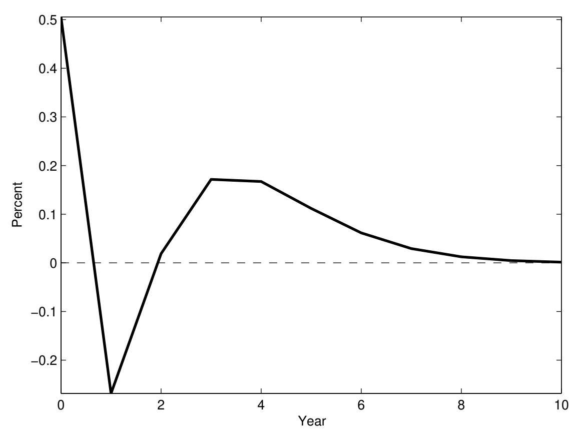
(a) GDP growth response to good uncertainty shock



(b) GDP growth response to bad uncertainty shock



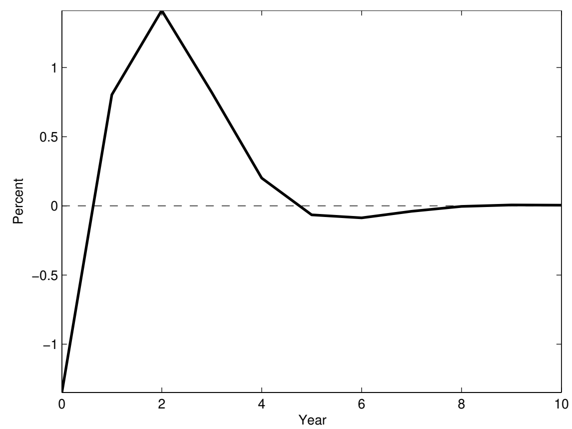
(c) GDP growth response to total uncertainty shock



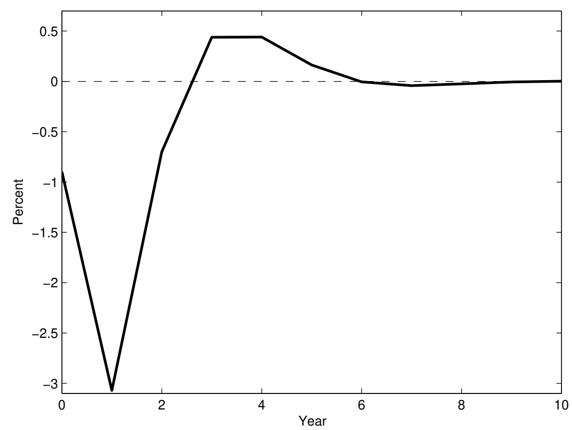
The figure shows impulse responses of private GDP growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and GDP growth rate. Data are annual from 1930 to 2012.

Figure 1.7: Impulse response of capital investment to macro uncertainties

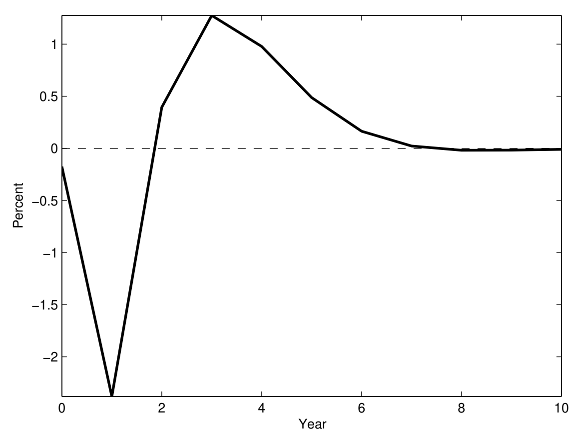
(a) Capital investment growth response to good uncertainty shock



(b) Capital investment growth response to bad uncertainty shock



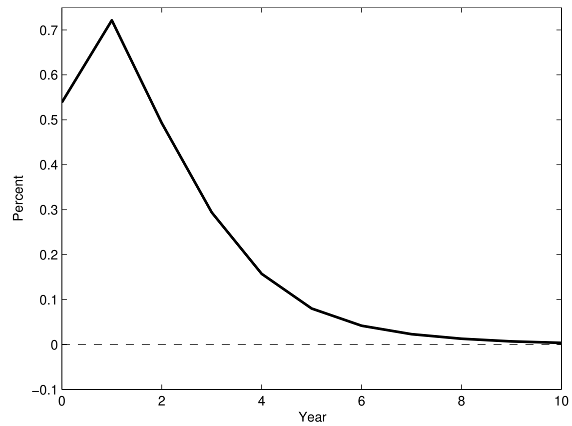
(c) Capital Investment growth response to overall uncertainty shock



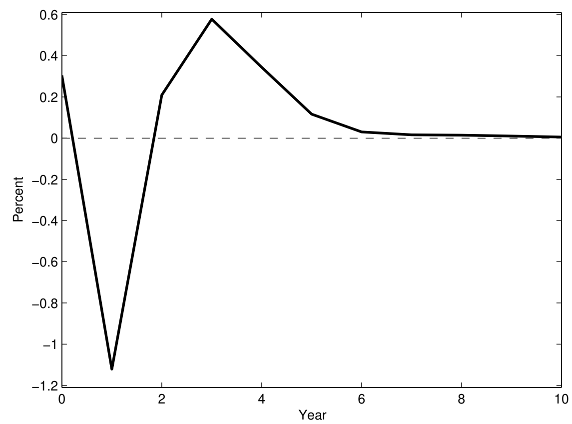
The figure shows impulse responses of capital investment growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and capital investment growth rate. Data are annual from 1930 to 2012.

Figure 1.8: Impulse response of R&D investment to macro uncertainties

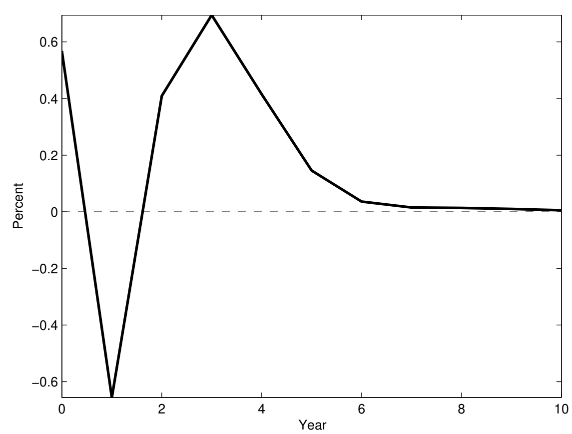
(a) R&D investment growth response to good uncertainty shock



(b) R&D investment growth response to bad uncertainty shock



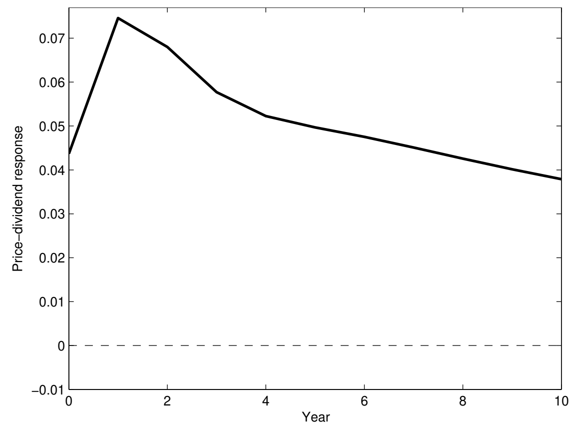
(c) R&D investment growth response to overall uncertainty shock



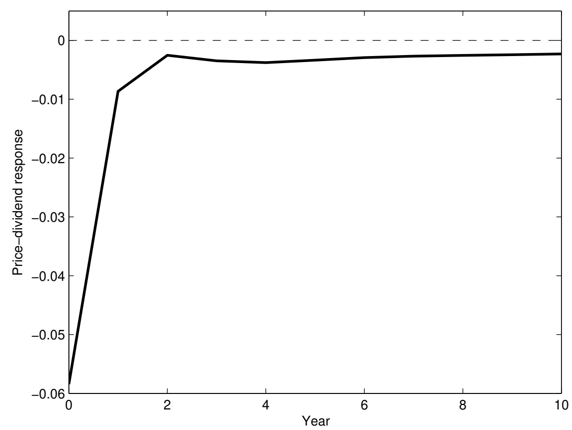
Impulse response of R&D investment to macro uncertainties. The figure shows impulse responses of R&D investment growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and R&D investment growth rate. Data are annual from 1954 to 2008.

Figure 1.9: Impulse response of price-dividend ratio to macro uncertainties

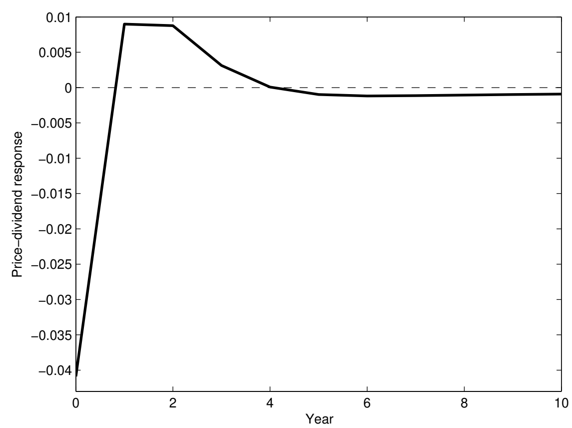
(a) Price-dividend ratio response to good uncertainty shock



(b) Price-dividend ratio response to bad uncertainty shock



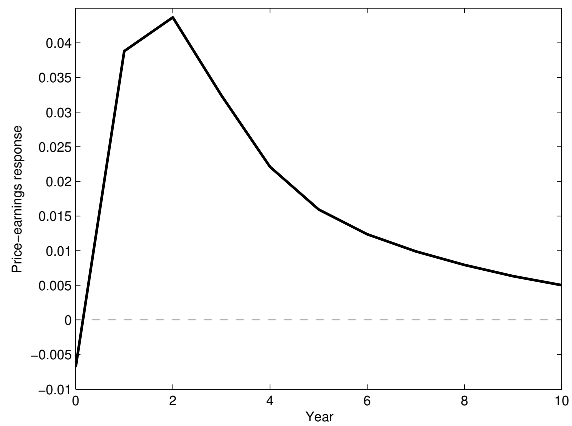
(c) Price-dividend ratio response to overall uncertainty shock



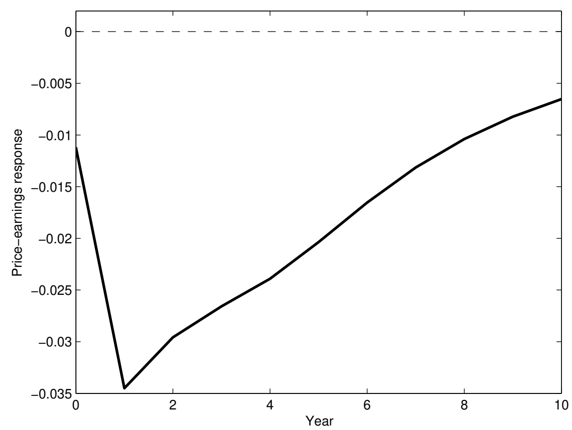
The figure shows impulse responses of the market price-dividend ratio to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and the market price-dividend ratio. Data are annual from 1930 to 2012.

Figure 1.10: Impulse response of price-earnings ratio to macro uncertainties

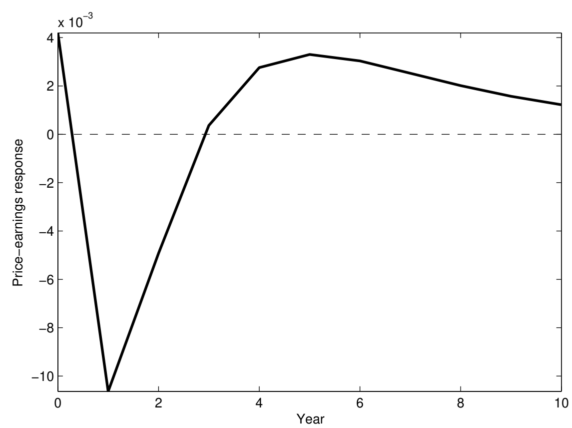
(a) Price-earnings ratio response to good uncertainty shock



(b) Price-earnings ratio response to bad uncertainty shock



(c) Price-earnings ratio response to overall uncertainty shock



Impulse response of price-earnings ratio to macro uncertainties. The figure shows impulse responses of the market price-earnings ratio to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and the market price-earnings ratio. Data are annual from 1930 to 2012.

CHAPTER 2 : A Tale of Two Volatilities: Sectoral Uncertainty, Growth, and Asset-Prices

2.1. Introduction

It is a common notion, especially among policymakers, that uncertainty played an important role in inhibiting economic recovery from the Great Recession. Consequently, there has been a growing research effort in macroeconomics and in finance to understand the implications of volatility shocks, yielding mixed evidence. In macroeconomic studies, it is debatable whether volatility, particularly in general equilibrium, lowers or increases investment. In asset-pricing, most studies argue that volatility drops asset-valuation ratios, while others claim it is a mechanism that boosts stock prices.[1] Corporate finance studies show that higher volatility increases the cost of capital, thus lowering investment and leverage.

In this study I show that it is possible to reconcile the mixed evidence about the implications of volatility by decomposing the source of uncertainty into sectoral origins. Specifically, I ask what is the impact of technological (TFP) volatility on asset-prices and aggregate cash-flows? I shed new light on this question, and find that the answer depends empirically and theoretically on the sector from which the volatility emanates. I split the economy into two super-sectors: the consumption sector and the investment sector. I study the pricing and the macroeconomic implications of sectoral innovations (first-moment sectoral TFP shocks), as well as sectoral volatility shocks (second-moment sectoral TFP shocks), of these two sectors.

I document a novel empirical regularity: the TFP-volatilities of the investment sector and the consumption sector have *opposite* impact on the real and financial economy. Contrary to the typical view of policymakers, TFP-volatility is not always contractionary empirically. The market's fear of uncertainty is well-justified when the productivity of the consumption sector is more uncertain. The TFP-volatility of the consumption sector depresses

[1] See a comprehensive discussion related to the implications of volatility shocks for economic growth and asset-prices in existing literature in Section 2.2.

stock prices and aggregate investment. By contrast, uncertainty about the productivity of investment-good producers boosts aggregate cash-flows, raises equity valuations, and lowers credit spreads. Moreover, investment TFP-volatility helps explain return spreads based on momentum and Tobin's Q, beyond the ability of first-moment sectoral TFP innovations.

I explain the empirical findings using a quantitative general-equilibrium production-based model. The model features two-sectors, consumption and investment, whose production is subject to sectoral TFP shocks with time-varying volatility. While a standard perfect-competition model fails to fully explain the data, I show that a model that features monopolistic power for firms and sticky prices, as well as early resolution of uncertainty under Epstein and Zin (1989) and Weil (1989) preferences, is capable of explaining the differential impact of sectoral volatilities on real and financial variables.

The implications of this study contribute to several disciplines. On the macroeconomic front, this paper shows that higher uncertainty should not be suppressed if it stems from investment firms. On the asset-pricing front, my work highlights that sectoral volatility shocks, in particular in the investment sector, can go beyond first-moment innovations in explaining return spreads. On the corporate-finance front, I demonstrate that sectoral volatilities lead to differential impact on credit spreads, and on firms' incentive to take leverage.

A starting-point of my study is that uncertainty takes many different forms, and therefore, can lead to the mixed findings in the literature regarding its effect on economic growth and valuations. Focusing on the consumption versus the investment sector's TFP-volatility, stems from a voluminous macro-finance literature which divides the economy to these two classifications. This literature stresses the importance of innovations to the level of investment TFP (*first-moment* shocks) for the business-cycle, the equity premium, and certain return spreads.[2] To the best of my knowledge, my work is the first to examine the differen-

[2]More precisely, these works discuss investment-specific technological shocks, or IST. IST shocks refer to the log-difference between investment and consumption TFP level innovations. With some contrast, in my work I examine the total Solow residual in both sectors. For symmetry, I use the terms investment-TFP and

tial role of consumption and investment TFP-volatility (*second-moment* shocks) for prices and growth.

The focus on the TFP-volatility of the two sectors can be motivated economically. In a reduced form manner, higher investment TFP-volatility could be thought of as a bundle of R&D growth options, which raises uncertainty. Some of these options would turn out to be bad, but some in the right tail would be successful. Future exercising of successful options could be manifested in improved productivity and welfare. For example, uncertainty about the productivity of a firm like "Delta Airlines", classified as consumption-producing firm (service producer), can be quite different than uncertainty about "Pratt & Whitney" productivity (a large aircraft engine producer), classified under the investment sector.[3] Perhaps, creative R&D work done at "Pratt & Whitney", which is a source for higher uncertainty, would generate the next technological advancement (e.g. fuel efficient engine), from which "Delta Airlines" could also benefit? Interestingly, I find results along this intuition.

Empirically, using measures of sectoral innovations and volatility shocks, I document four novel stylized facts:[4] (1) While consumption-sector's TFP-volatility is associated with lower investment, output, and consumption, investment-sector's TFP-volatility is associated with boosting these quantities; (2) Investment TFP-volatility risk has a positive market-price, and consumption TFP-volatility has a negative market-price. The sectoral volatilities also affect the default spread in opposite directions: investment TFP-volatility lowers it; (3) By and large, equities are exposed in a similar fashion to the sectoral volatilities. Investment TFP-volatility increases firms' stock-prices (positive exposures, or positive "betas"), while firms' beta to consumption TFP-volatility is negative; (4) I show that investment TFP-volatility is important for the market risk-premium, and for explaining the momentum spread.

consumption-TFP innovations. Both terms in my paper refer to *sectoral* Hicks-neutral technology shocks.

[3] I follow here the classification suggested by Gomes et al. (2009), of SIC codes into industries.

[4] I measure the TFP-volatility of the consumption and investment sectors via the predictable component of sectoral TFP realized variances. For more details, see discussion in Section 2.3.2.

Why does investment TFP-volatility impact differ from consumption TFP-volatility? Using a quantitative DSGE theory, my work explains the impact of sectoral volatilities on aggregate cash-flows and aggregate valuations. The model features a consumption sector, and an investment sector, and builds on Smets and Wouters (2007), Liu et al. (2012), and Garlappi and Song (2013b). The output of the consumption sector is a final-good used for consumption only, and it is subject to a consumption TFP shock. The output of the investment sector is the economy's aggregate investment, and it is subject to an investment TFP shock. It flows to both consumption firms, and investment firms that wish to invest.

Given the economy's structure, a consumption TFP innovation is a multiplicative shock that only rescales consumption, and thus has a transitory impact. By contrast, an investment TFP innovation affects multi-period stock of aggregate capital, and thus has a prolonged impact. As a result, consumption TFP-volatility resembles pure short-run capital risk, while investment TFP-volatility resembles more long-run income risk. As discussed below, this implies that the strength of the motive to save (invest) in order to hedge against higher uncertainty differs between the two volatilities.

When TFP-volatility of the investment sector rises, it implies that in future periods the probability of having sub-optimal amount of investment goods rises. In this case, the household has a strong incentive to invest more, and consume less, due to "precautionary saving". Investing more ensures higher aggregate capital in the future. Capital can be used for both consumption and investment production. Hence, it acts as a buffer of savings. If a bad investment TFP shock is realized, the buffer can be used to smooth consumption.

By contrast, I show that under early resolution of uncertainty preferences, more consumption TFP-volatility makes the household more impatient. This triggers more consumption, and less investment. Intuitively, under early resolution of uncertainty case, the agent dislikes uncertainty. To minimize her exposure to volatility build-up in the future, she shifts her consumption profile as much as possible to the present, which implies lower investment.

The former discussion demonstrates that consistently with the data, a model with perfect competition leads investment expenditures to rise (drop) with investment (consumption) TFP-volatility. However, because consumption and investment are substitutes, with perfectly competitive firms, a sectoral TFP-volatility shock would cause consumption and aggregate investment to counterfactually diverge.[5] The model therefore features time-varying markups, which builds upon monopolistic competition and sticky prices.[6] Time-varying markups make consumption and aggregate investment comove in response to sectoral volatility shocks, consistently with the data.[7] Consequently, sticky prices play an important role in explaining macroeconomic facts and volatility risk premia.

As is common in production models, aggregate investment and stock prices comove. Consequently, the two sectoral TFP-volatilities have opposite impact on stock prices. In particular, since higher investment TFP-volatility increases investment, it increases the demand for capital goods, and also their relative price. As a result, the value of firms' capital rises, and stock prices appreciate. This implies, as in the data, a positive beta to investment TFP-volatility. The opposite logic applies to consumption TFP-volatility, and implies negative betas to consumption TFP-volatility, consistently with the data.

The behavior of the market-prices of risk is derived from consumption dynamics and preferences. Consumption TFP-volatility depresses aggregate consumption, and generates a more volatile consumption profile. Both effects, under early resolution of uncertainty, increase the marginal utility of the investor, and imply a negative market-price of risk. Investment TFP-volatility increases consumption's volatility on one hand. On the other hand, this volatility has a prolonged effect on the economy through capital build-up. This capital build-up leads to a rise in long-run consumption. Quantitatively, the latter channel can

[5]This divergence implies that consumption TFP-volatility would counterfactually boosts consumption, not only contemporaneously but also in the future. Counterfactual consumption behavior could also adversely affect the market-price of consumption TFP-volatility risk.

[6]Markups in the model are countercyclical: They increase with consumption TFP-volatility. As higher consumption TFP-volatility has a contractionary impact, this is consistent with some empirical evidence suggesting that markups are countercyclical (see e.g. Barsky et al., 1994; and Chevalier and Scharfstein, 1996).

[7]See related discussion in Basu and Bundick (2012), and Fernández-Villaverde et al. (2015).

dominate the first, implying a positive market-price of risk for investment TFP-volatility, as in the data.

The rest of this paper is organized as follows. In Section 2.2, I review related literature. Section 2.3 documents the novel empirical facts regarding sectoral TFP volatilities. In Section 2.4, I present the general-equilibrium model, and discuss its intuition in Section 2.5. Section 2.6 presents the quantitative results. Section 2.7 provides concluding remarks.

2.2. Related Literature

My paper relates to three main strands of literature. *First, my study is related to the growing literature discussing the implications of volatility shocks for macroeconomic growth, and asset-prices.* I contribute to this line of works by documenting and rationalizing novel channels, through which fundamental volatilities can interact both positively and negatively with macro-aggregates and prices. Volatility in this work refers to the time-series conditional volatility of shocks, to an economic variable of interest (in my case, TFP).

Empirically, the typical relation between volatility and the macroeconomy is *negative.* This negative link is documented in the early work of Ramey and Ramey (1995), Martin and Rogers (2000), and more recently by Engel and Rangel (2008), Bloom (2009), and Baker and Bloom (2013). Fewer empirical works, document a positive impact of volatility, such as Kormendi and Meguire (1985) on output, and Stein and Stone (2013) on R&D expenditures.[8]

From a theoretical perspective, there is an on-going debate regarding the impact of volatility on investment. *On one hand,* some studies highlight a negative impact on investment. The works of McDonald and Siegel (1986), Dixit and Pindyck (1994), and recently Bloom (2009), use real-option effects to explain why volatility suppresses investment and hiring. The work of Fernandez-Villaverde et al. (2011) discusses uncertainty in an open-economy, showing

[8]Related, the work of Imbs (2007) shows that on average, within-industry volatility of value-added is non-negatively (or weakly positively) related to the same industry's growth. Yet, average within-sector volatility across industries, relates negatively to aggregate growth. Differently from my work, Imbs does not identify which sectors' volatility interact positively or negatively with *aggregate* growth, or why.

that volatility lowers domestic investment. Other works argue that volatility increases the cost of capital, or credit spreads, making investment more costly (see e.g. Christiano et al., 2014; Arellano et al., 2012; and Gilchrist et al., 2014). Basu and Bundick (2012) and Fernández-Villaverde et al. (2015) rely on nominal rigidities to show that both consumption and investment drop in response to volatility shocks. *On the other hand,* other studies rely on economic forces which yield a positive link between volatility and investment, including precautionary savings, time-to-build, and investment irreversibility.[9]

Most asset-pricing studies argue for a *negative* effect of volatility on financial variables. Focusing first on the impact of aggregate-fundamental's volatility, Bansal et al. (2005b) show that higher aggregate volatility depresses asset-valuation ratios. Related, Drechsler and Yaron (2011), and Shaliastovich (2015), show that higher aggregate volatility increases risk premia. Bansal et al. (2014) find that the market-price of aggregate volatility risk is negative. In the context of real options, Ai and Kiku (2012) argue that higher aggregate volatility may decrease the value of growth options, as the volatility is priced, and affects discount rates.

Other works, argue also for a *negative* impact, yet of *different* facets of volatility. Croce et al. (2012), and Pastor and Veronesi (2012), demonstrate the negative impact of policy uncertainty on prices. In the context of learning, Van Nieuwerburgh and Veldkamp (2006), show that slower learning and higher belief uncertainty in bad times, generates slow recoveries and countercyclical movements in asset prices.[10]

Some financial studies argue for a more *positive* link between volatility and valuations. Campbell et al. (2012) analyze aggregate volatility in an extended version of the intertem-

[9] see e.g. Abel and Eberly, 1996; Bar-Ilan and Strange, 1996; Gilchrist and Williams, 2005; Jones et al., 2005; Malkhozov, 2014; and Kung and Schmid, 2014. Related, Johnson (2007) highlights that higher uncertainty, accompanied with technological revolutions, encourages investment as a mean of optimal learning. For an excellent survey of uncertainty impact on macroeconomic quantities, the reader may also refer to Bloom (2014).

[10] Related, Fajgelbaum et al. (2015) also show that higher belief uncertainty discourages investment. Herskovic et al. (2015) show that the common component of idiosyncratic volatility among firms raises the households marginal utility, and is negatively priced. Krishnan et al. (2009), show that correlation risk carries a significant negative price of risk.

poral capital asset-pricing model (ICAPM), and find that in a recent-sample, equities have positive exposure to volatility. Pastor and Veronesi (2009b) show that stock prices of firms rise as a result of higher uncertainty during times of technological revolutions.[11]

Different frameworks exhibit a more *ambivalent* link between volatility and returns. Segal et al. (2015) show that the positive and negative semivariances of industrial-production have opposite impact on stock and bond prices. Patton and Sheppard (2015) show that negative semivariances of returns leads to higher future return volatility.[12]

The second strand of literature related to my paper, discusses the role of investment TFP innovations for the business cycle and asset prices. Yet, the focus of this literature so far evolved around first-moment TFP innovations, as opposed to second-moment TFP shocks, which are at the focus of the current work. A long strand of macroeconomic works stress the importance of investment technology innovations for business-cycle fluctuations.[13]

In the context of asset-pricing, the works of Christiano and Fisher (2003), Papanikolaou (2011), and Garlappi and Song (2013a) among others, highlight the ability of investment specific technological shocks (IST) to explain the equity premium puzzle. Nonetheless, while Papanikolaou (2011) finds that IST shocks carry a negative beta and a negative market-price of risk, Garlappi and Song (2013a) and Li et al. (2013) find that these shocks carry a positive beta and a positive market-price. Importantly, in this work I do not examine IST shocks, but rather focus on the total TFP of the investment sector (in comparison to the consumption sector's TFP). I document a negative beta to investment (first-moment) TFP innovations. I document that the sign of the market-price of risk of investment first-moment TFP innovations, is not a strictly robust feature of the quarterly data. In my benchmark

[11] Other related papers include Johnson and Lee (2014), which highlight that the the common component of firm-specific cash-flow volatility increases equity valuation ratios, especially for levered equity claims. In the context of executive compensation, Cohen et al. (2000) argue that since executive options increase in stock's volatility, they provide incentives for managers to take actions that increase firm risk, thus pursuing more projects.

[12] Other related papers include Feunou et al., 2013; Bekaert and Engstrom, 2009; Bekaert et al., 2015; Colacito et al., 2013; McQuade, 2014; and Feunou et al., 2015.

[13] see e.g. Greenwood et al., 1997; Greenwood et al., 2000; Fisher, 2006; Jaimovich and Rebelo, 2009; Justiniano et al., 2010; and Basu et al., 2006.

analysis, I find a positive market-price for investment first-moment TFP innovations, though this market-price turns negative in some of the robustness checks. More relevant, the market-prices of sectoral TFP-*volatility* shocks are robust features of the quarterly data.

Investment specific innovations are shown to be helpful in explaining certain return spreads: Value spread (see Papanikolaou, 2011), spreads based on past-investment, market-betas and idiosyncratic volatility (see Kogan and Papanikolaou, 2014; and Kogan and Papanikolaou, 2013), and commodity-based spreads (see Yang, 2013). Li (2014) argues that investment specific innovations can explain the momentum spread, though Garlappi and Song (2013a) find that the magnitude of this spread captured by these shocks is low, in particular at quarterly frequency. My work documents that investment TFP-*volatility* shocks, are capable of explaining a significant fraction of the momentum spread at quarterly frequency.

The last voluminous literature that my paper is more broadly related to, are production/ investment based asset-pricing papers. These works, study the role of (neutral) technological innovations for the joint dynamics of asset-prices and macroeconomic quantities.[14] For example, Liu et al. (2009), find that conditional expectations of stock returns are positively correlated with expectations of investment returns. Belo et al. (2014), provide an investment-based model to explain why firms with high hiring rates earn lower returns, while Jones and Tuzel (2013) offer an investment-based framework to explain why firms with higher inventory growth earn lower returns, relying on adjustment costs channels. Lastly, Gârleanu et al. (2012) study "displacement risk", that is, that innovation works to the advantage of new generations of innovators at the expense of older generations, helping to rationalize the value premium.[15]

[14]For a survey of this comprehensive literature, the reader may also refer to Kogan and Papanikolaou (2012).

[15]Other works discussing asset-pricing moments in a general-equilibrium production models include Jermann, 2010; Berk et al., 1999; Tallarini, 2000; Boldrin et al., 2001; Gomes et al., 2003; Carlson et al., 2004; Zhang, 2005; Croce, 2014; Kaltenbrunner and Lochstoer, 2010; Gomes and Schmid, 2010; Favilukis and Lin, 2013; Eisfeldt and Papanikolaou, 2013; Lustig et al., 2011; Lin, 2012; and Ai et al., 2013, to name a few.

2.3. The Facts

In this section I empirically examine the implications of sectoral first-moment TFP innovations and volatility shocks. Sections 2.3.1 and 2.3.2, describe the data and the methodology used to construct first- and second- moment sectoral TFP shocks empirically. In Section 2.3.3, I analyze the effects of sectoral shocks, and in particular volatility shocks, on aggregate macro quantities such as output, consumption, and investment. In Section 2.3.4, I examine the implications of sectoral shocks for cross-sectional risk-premia. I further highlight the asset-pricing role of sectoral TFP-volatility shocks, above and beyond sectoral first-moment TFP shocks, in Section 2.3.5. In the robustness section, Section 2.3.7, I show that the key results are maintained for alternative methods of extracting TFP-volatility shocks from the data.

2.3.1. Data

In my benchmark analysis I use quarterly data from 1947-Q1 to 2014-Q4. Consumption and output data come from the Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) tables. Consumption corresponds to the real per capita expenditures on non-durable goods and services and output is real and per capita gross domestic product. Capital investment data are from the NIPA tables; Data on average weekly hours worked, and average hourly earnings, of production and nonsupervisory employees in good-producing industries are from BLS. Quarterly sales, capital-expenditures, and net-earnings for publicly traded firms are taken from Compustat. All nominal time-series are adjusted for inflation using Consumer-Price Index from BEA. Data on price deflators of non-durables and services, and on equipment and software goods, are from NIPA tables as well. Total-factor productivity data, are taken from the Federal Reserve Bank of San-Fransisco. I elaborate more on the TFP data used in section 2.3.2.

Aggregate asset-prices data include 3-month Treasury bill rate, the stock price and dividend on the broad market portfolio from the Center for Research in Security Prices (CRSP). I adjust the nominal short-term rate by the expected inflation to obtain a proxy for the real risk-free rate. Additionally, I collect data on equity portfolios sorted on key characteristics, such as size, book-to-market ratio, momentum, operating profitability and idiosyncratic return volatility from the Fama-French Data Library. To measure the default spread, I use the difference between BAA and AAA corporate yields, obtained from the Federal Reserve Bank of St. Louis.

2.3.2. Construction of Sectoral Shocks

I obtain quarterly aggregate and sectoral TFP data (Solow residual) from Fernald (2012). In computing the TFP, labor includes an adjustment for "quality" or composition. Capital services are also adjusted for changes in composition over time. I further obtain capacity-utilization adjusted TFP data from Basu et al. (2006). Using the relative prices of investment-goods, the aggregate TFP series is decomposed into separate sectoral TFP series, for the (non-structures, non-residential) "investment" sector, and "consumption" sector. "Consumption" in this context means everything that is not in the investment sector, i.e., everything other than private business equipment (e.g. non-durables and services).[16]

The use of the relative price of investment goods to obtain investment TFP innovations was originally proposed by Greenwood et al. (1997). It can be shown that if producers in both sectors have equal factor shares of capital and labor, pay the same factor prices (i.e., wages

[16]See Fernald (2012) for details. To be specific, the log-growth in aggregate TFP is defined as:

$$\Delta TFP_t = \Delta Y_t - \alpha_t \Delta K_t - (1-\alpha_t)(\Delta \text{hours}_t + \Delta \text{labor-productivity}_t)$$

where ΔY is the log-growth in gross value-added, ΔK is the log-growth in perpetual inventory stocks (calculated from disaggregated quarterly NIPA investment data), and α is capital's share of output. Let $\Delta \tilde{P}_{i,t}$ be the log-growth in the *relative* price of investment (equipment):

$$\Delta \tilde{P}_{i,t} = log(P_i/P_c)_t - log(P_i/P_c)_{t-1},$$

where P_i is the price deflator of investment-goods, and P_c is the price deflator of non-equipment goods and services. Let $w_{i,t}$ be equipment share of business output. Then consumption TFP log-growth ΔC-TFP, and investment TFP log-growth ΔI-TFP are computed by solving:

$$\begin{aligned}\Delta TFP_t &= w_{i,t}\Delta \text{I-TFP}_t + (1-w_{i,t})\Delta \text{C-TFP}_t \\ \Delta \tilde{P}_{i,t} &= \Delta \text{C-TFP}_t - \Delta \text{I-TFP}_t.\end{aligned}$$

and capital rents), have similar markups, and capital flows freely between the two-sectors intra-temporally, then changes in relative TFP of both sectors equal changes in the relative price of investment.

In my benchmark case, I use the sectoral TFP time-series proposed by Fernald (2012). The sectoral TFP data of Fernald (2012) account for the time-varying output share of the investment-sector, and capture the overall TFP in each of the sector. As such, these data correspond well with my general-equilibrium setup, in which sectors' sizes are also time-varying. Yet, in section 2.3.7, I demonstrate that the empirical results are robust to other proxies as well.[17]

As is common in the investment literature, the log-growth in consumption TFP and investment TFP are the respective sectoral *first-moment* innovations.[18] I denote these innovations as ΔC-TFP and ΔI-TFP, respectively, where C is a short for consumption, and I is a short for investment.

To obtain second-moment (volatility) TFP shocks I follow four steps. First, I filter the sectoral TFP growth rates using an AR(k) filter, where k is chosen by Akaike Information Criterion. I do so, in order to remove any potential conditional mean from the time series, and obtain sectoral TFP residuals, denoted $\{\varepsilon_{j,t}\}, \quad j \in \{C, I\}$.

Second, I construct sectoral realized variances $RV_j, \quad j \in \{C, I\}$, from the sectoral TFP residuals, over a window of W quarters:

$$RV_{j,t-W+1\rightarrow t} = \Sigma_{k=t-W+1}^{t} \varepsilon_{j,k}^2 \tag{2.1}$$

[17]In particular, the results are robust when sectoral TFPs are adjusted for capacity-utilization, as in Basu et al. (2006). Furthermore, it is very common in the investment literature to use only the relative investment price deflator as a proxy for investment-*specific* shocks (see e.g. Greenwood et al. (1997), Fisher (2006), and Garlappi and Song (2013a)). The results are robust to the use of the relative-price of investment deflator proxy instead. Other proxies considered are described in section 2.3.7.

[18]The construction of first-moment TFP innovations via log growth is identical to the empirical construction of TFP innovations in the works of Garlappi and Song (2013a) and of Kogan and Papanikolaou (2014). It is also consistent with the fact that in the model, the sectoral TFPs are random walks. However, the results are robust to filtering the sectoral TFP growth series using an $AR(k)$ filer, and using the residuals as the first-moment TFP innovations.

These realized variances capture ex-post (or backward-looking) volatility in each sector. Third, to make the volatilities forward-looking, in-line with the model, I project future sectoral log-realized variances on a set of predictors, denoted by Γ_t:

$$\log(RV_{j,t+1\to t+W}) = b_0 + b'\Gamma_t + error \tag{2.2}$$

The exponentiated fitted value of these projections are the sectoral ex-ante TFP-volatilities ($V_j = exp(\hat{b}_0 + \hat{b}'\Gamma), \quad j \in \{C, I\}$). The log transformation ensures that the ex-ante volatility measures remain strictly positive, in a similar fashion to Segal et al. (2015).

Lastly, I use the logarithm first-difference of the sectoral ex-ante TFP-volatility series, as the sectoral TFP-volatility shocks. I consider this step as a reduced-form way to obtain a proxy of second-moment shocks, that is both in-line with the construction of the first-moment innovations, and does not require further filtering. Taking the first-difference of the volatility series, also reduces their auto-correlation, and thus, alleviates potential Stambaugh (1986) biases in predictive projections. However, the results are still robust when the total ex-ante volatilities are used as well in the various projections, instead of their first-difference.

In the benchmark case, I set $k = 3$, and $W = 8$ quarters. Motivated by the general-equilibrium setup, the benchmark predictors I use, Γ_t, are the four variables which from a production perspective, are sufficient describe the economy's evolution: consumption and investment TFP growth, and the two sectoral realized variances. However, the results are robust to exclusion or inclusion of other predictors. Following these steps, I obtain four shocks: ΔC-TFP and ΔI-TFP, capturing (first-moment) sectoral TFP innovations, and ΔC-TFP-VOL and ΔI-TFP-VOL capturing second-moment sectoral TFP shocks. With these four shocks, I obtain a set of novel empirical facts, as illustrated in sections 2.3.3 - 2.3.5.

As the TFP of the consumption and the investment sectors comove, their volatilities are also correlated. To emphasize the distinction between the two sectoral volatilities, Figure

2.1 shows the component of investment TFP-volatility which is orthogonal to consumption TFP-volatility. The orthogonal component is obtained from the projection of investment TFP-volatility on consumption TFP-volatility. The residual investment TFP-volatility is procyclical. Specifically, we can see a decrease in the residual investment TFP-volatility during the Great Recession. On the other hand, the residual volatility rises during the high-tech revolution of mid to late 1990s.

2.3.3. Sectoral Shocks and The Macroeconomy

In this section, using the empirical proxies for sectoral volatility shocks, I document the first stylized fact, related to the interaction of sectoral volatilities and the macroeconomy.

Fact I: *Investment-sector's TFP-volatility predicts positively both the* growth rates *and the* business-cycle *component of key macroeconomic variables: consumption, output, investment, and labor; Consumption-sector's TFP-volatility predicts these quantities negatively.*

I first project contemporaneous and future cumulative macroeconomic growth rates, for horizon h quarters, on the current proxies for sectoral shocks: first-moment sectoral TFP innovations of the two-sectors, and second-moment TFP shocks of the two-sectors. In other words, I run the following regressions:

$$\begin{cases} \Delta y_t = \beta_0 + \beta_0' X_t + error & \text{if } h = 0 \\ \frac{1}{h}\sum_{j=1}^{h} \Delta y_{t+j} = \beta_{0,h} + \beta_h' X_t + error & \text{if } h > 1. \end{cases} \tag{2.3}$$

where $X_t = [\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t]$. The variable y is a macroeconomic log-variable of interest, and the forecast horizon h varies between $h \in \{0, 1, 4, 12, 20\}$ quarters. Table 2.1 shows the slope coefficients, along with the adjusted R^2 of the regressions, for aggregate cash-flow (macroeconomic) growth variables – consumption, GDP, corporate sales and earnings. Table 2.2 shows the evidence for inputs growth measures – capital inputs: non-residential capital investment, corporate capital expenditures, and the relative-price of investment, as well as labor inputs: average hours worked and wages.

It is evident from these two tables that across the various macroeconomic growth rates and across all the horizons, the slope coefficient on consumption TFP innovation is always positive and almost always significant (with the exception of an insignificant negative slope for sales growth contemporaneously). This evidence is consistent with the notion that higher productivity is associated with higher growth, and increased economic activity.

With some contrast, investment TFP innovation's loadings are positive contemporaneously (and also in shorter predictive horizons), but turn negative for medium and long-run predictive projections. Investment innovations also have strictly negative loadings in aggregate prices projections: wages and the relative-price of investment. As shown in Panel A of Table 2.2, investment TFP innovations increase capital investment growth contemporaneously. This finding is broadly consistent with the empirical evidence of Kogan and Papanikolaou (2014) and Kogan and Papanikolaou (2013), that investment-specific shocks (measured via the relative price of investment, or via investment-minus-consumption portfolio returns), also raise firms' investment-rates contemporaneously. Yet, some of the negative loadings on investment TFP innovations, in particular for consumption and GDP, are consistent with recent empirical evidence of Basu et al. (2006) and Liu et al. (2012), that investment technology shocks can be contractionary.

Consumption TFP-volatility carries always a negative slope coefficient. It is statistically significant mostly in shorter horizons of zero quarters up to one year. This is the typical negative interaction of volatility and growth, documented by Bloom (2009) and others. By sharp contrast, investment TFP-volatility has always a positive correlation with contemporaneous and future growth. This positive interaction is also statistically significant at horizons of one-year, and three-years ahead. However, in the case of capital investment, the positive loading on investment TFP-volatility is also significant contemporaneously.

It is worth noting that the adjusted R^2s for the contemporaneous projections of GDP and capital investment are quite substantial. For GDP growth the adjusted R^2 is close to 50%, and for capital investment it is 36%. Generally, the R^2s decline with the forecast horizon.

The positive interaction of investment TFP-volatility is not limited to growth rates. In Table 2.3, I repeat the same projections of the former Tables, but now the dependent variable is the business-cycle component of an economic variable of interest y, averaged over the predictive horizon h. The business-cycle component is obtained via filtering the level data using a one-sided HP-filter, with a smoothing parameter of 1600. Averaging the business-cycle component is made to reduce the amount of noise, and extract the "long-term" business-cycle component of the variables of interest. For brevity, I consider in Table 2.3 a subset of macroeconomic variables, including consumption, GDP, capital investment, hours, and the relative-price of investment.

The Table conveys a similar message to the growth-rate evidence. Consumption TFP-volatility shocks predict negatively, while investment TFP-volatility shocks predict positively, the cyclical component of macroeconomic variables. The significance of the volatilities generally drops with the predictive horizon. For some variables, such as hours-worked, the significance of the volatility loadings is stronger in the business-cycle evidence, than in the growth-rate evidence.[19]

Though the projections in Tables 2.1 - 2.3 are multivariate, and account for the correlations between the factors, I further illustrate the impact of TFP-volatility shocks via impulse-responses, shown in Figure 2.2. The impulse-response functions are computed from a first-order vector autoregression (VAR(1)) that includes investment TFP-volatility, consumption TFP-volatility, investment TFP innovation, consumption TFP innovation, and the *detrended* macroeconomic variable of interest. The detrended macroeconomic variable is

[19] From the fact that the sectoral TFP volatilities impact both growth rates and cyclical components similarly, one may learn that the impact of TFP-volatilities on macroeconomic variables is not only persistent, but even tends to amplify some period after the volatility shock hits. This pattern is theoretically consistent with the existence of adjustment costs, that prevent firms from fully responding to the volatility shocks upon impact.

also standardized. I plot one-standard deviation Cholesky TFP-volatility shock responses, to detrended consumption, output and capital investment.

Figure 2.2 illustrates again the expansionary pattern for investment TFP-volatility, and the contractionary pattern for consumption TFP-volatility. Panels (a), (b) and (c), demonstrate that a one-standard deviation of consumption TFP-volatility shock, drops the cyclical component of consumption, investment and output by 0.13, 0.16 and 0.24 standard deviations, respectively, one-quarter after the shock hits. The negative impact persists up to ten quarters ahead. In particular for investment, the negative response is persistent up to 20 quarter ahead.

By contrast, Panels (d), (e), and (f) show that one standard-deviation shocks to investment TFP-volatility increase one-quarter ahead detrended consumption, investment and output by 0.04, 0.12, and 0.13 standard deviations, and the positive impact persists up to 20 quarters onwards. Economically, the negative impact of consumption TFP-volatility is somewhat larger than the positive impact of investment TFP-volatility.

2.3.4. Sectoral Shocks and The Cross-Section of Returns

In this section I show the implications of sectoral first-moment TFP innovations and TFP-volatility shocks for the cross-section of stock returns. To the extent that sectoral volatilities interact with aggregate consumption growth in an opposite way, it may suggest that the marginal utility of the household is affected differently by sectoral volatilities. In-line with this conjecture, my empirical analysis yields the second stylized fact:

Fact II: *Consumption TFP-volatility has a negative market price of risk, while the market price of investment TFP-volatility is positive. Hence, the high-risk states for the investors are associated with low investment uncertainty, and high consumption uncertainty.*

Generally, a portfolio risk premium is given by the product of the market prices of fundamental risks $\Lambda = (\lambda_{\text{C-TFP}}, \lambda_{\text{I-TFP}}, \lambda_{\text{C-TFP-VOL}}, \lambda_{\text{I-TFP-VOL}})$, the variance-covariance matrix of the risk-factors, denoted by Ω, which captures the quantity of risk, and the exposure of the portfolio to the underlying macroeconomic risk β_i:

$$E[R_{i,t+1} - R_{f,t}] = \Lambda' \Omega \beta_i. \tag{2.4}$$

Given a cross-section of returns, and the risk-factors' shocks, I can estimate the equity exposures and the market prices of risks using a standard Fama and MacBeth (1973) procedure, described below.

First, I obtain the return betas by running a multivariate regression of each portfolio returns on the sectoral shocks[20]:

$$\begin{aligned} r_{i,t} \quad = \quad & \text{const} + \beta_{i,C-TFP} \Delta\text{C-TFP}_t \\ & + \beta_{i,I-TFP} \Delta\text{I-TFP}_t \\ & + \beta_{i,C-TFP-VOL} \Delta\text{C-TFP-VOL}_t \\ & + \beta_{i,I-TFP-VOL} \Delta\text{I-TFP-VOL}_t \\ & + \text{ error}. \end{aligned} \tag{2.5}$$

The slope coefficients in the above projection, represent the portfolio's exposures to sectoral TFP innovation risks and sectoral TFP-volatility risks. Next, I obtain factor risk premia $\tilde{\Lambda}$ by running a cross-sectional regression of average excess returns on the estimated betas:

$$\begin{aligned} \overline{R_i^e} = {} & \tilde{\lambda}_{\text{C-TFP}} \beta_{i,\text{C-TFP}} + \tilde{\lambda}_{\text{I-TFP}} \beta_{i,\text{I-TFP}} + \tilde{\lambda}_{\text{C-TFP-VOL}} \beta_{i,\text{C-TFP-VOL}} + \tilde{\lambda}_{\text{I-TFP-VOL}} \beta_{i,\text{I-TFP-VOL}} \\ & + \text{ error}. \end{aligned} \tag{2.6}$$

[20] I obtain similar results when I use excess returns, or first-difference of the returns as a dependent variable, as a reduced-form return innovation.

I impose a zero-beta restriction in the estimation and thus run the regression without an intercept. The implied factor risk premia, $\tilde{\Lambda} = (\tilde{\lambda}_{\text{C-TFP}}, \tilde{\lambda}_{\text{I-TFP}}, \tilde{\lambda}_{\text{C-TFP-VOL}}, \tilde{\lambda}_{\text{I-TFP-VOL}})$, encompass both the vector of the underlying prices of risks Λ, and the quantity of risks Ω :

$$\tilde{\Lambda} = \Omega\Lambda.$$

To compute the underlying prices of risk Λ, I pre-multiply the factor risk premia $\tilde{\Lambda}$ by the inverse of the quantity of risk matrix Ω.To obtain standard errors, I embed the two-state procedure into Generalized Method of Moments (GMM), which allows to capture statistical uncertainty in estimating jointly asset exposures and market-prices of risk.

In the benchmark implementation, the menu of cross-sectional assets includes the market return, the cross-section of ten portfolios sorted on size, ten portfolios sorted on book-to-market, and ten portfolios sorted on momentum. Panel A of Table 2.4 shows the market prices of risks estimates along with their t-statistics.

Panel A documents that consumption sector's TFP first-moment innovations have a positive and significant market price. This is in-line with several works (e.g. Garlappi and Song (2013a)).

The market-price of risk of investment TFP first-moment innovations is positive yet not statistically significant. Importantly, this finding is not at odds with Papanikolaou (2011) and Kogan and Papanikolaou (2013), who find that IST shocks are negatively priced. This is because investment TFP positively shares a common component with consumption TFP, which has a positive market price as well, while an IST shock is the log-difference between investment and consumption TFPs.

I find that the market-price of risk of investment TFP innovations, is not a strictly robust feature of the data – at least not at quarterly frequency. Though the benchmark analysis yields a positive market-price for investment innovations, this market-price turns negative in some of the robustness checks. For example, when ten industry portfolios are added to the cross-section, this market price turns negative, yet with a very low t-statistic. In my model, I choose to adopt the view that investment TFP innovations are positively priced. In *most* of the robustness checks this market price is positive. A positive sign is also consistent with an intertemporal elasticity of substitution greater than one, which is important for explaining the impact of sectoral volatilities on investment. More importantly, the market-prices of sectoral TFP-volatility shocks, as I discuss next, are robust features of the data.

Panel A also shows that the market price of investment TFP-volatility is positive, while the market-price of consumption TFP-volatility is negative. Both market prices are statistically significant. This is consistent with the effect of sectoral volatilities on the evolution of aggregate cash-flows (and consumption in particular).

The next stylized fact is evident from Panel B of Table 2.4:

Fact III: *For most equities, the risk exposures (betas) to consumption TFP-volatility are negative, and the risk exposures to investment TFP-volatility are positive.*

All assets have a positive exposure to consumption TFP first-moment innovations, and a negative exposure to investment TFP first-moment innovations. In addition, all equities except for portfolios comprised of very small stocks, are exposed in a similar fashion to sectoral TFP-volatility shocks. By and large, consumption TFP-volatility lowers equity valuations (negative betas), while investment TFP-volatility raises equity valuations (positive betas). Table 2.4 reports exposures without t-statistics to save space. In Table 2.5, I report industry (sectoral) portfolios' exposures to the sectoral shocks, along with t-statistics, obtained from running projection (2.5). Sorting stocks into industry portfolios is based on Gomes et al. (2009) SIC classifications for sectors.

Similarly to Panel B of Table 2.4, Table 2.5 shows that all sectors' exposures to sectoral shocks share the same pattern described earlier. In particular, the non-durables, services, and investment portfolios have positive exposure to investment TFP-volatility, and a negative one to consumption TFP-volatility. Except for two-cases in the Table, all betas are statistically significant.

2.3.5. The Pricing Role of Sectoral Volatilities

Section 2.3.4 demonstrates that the sectoral TFP-volatility shocks are priced in the cross-section of returns. Consequently, a production-based stochastic discount factor that excludes the volatility shocks is misspecified. Yet, is this misspecification also *economically* important for matching asset-pricing moments? In this section I argue that sectoral volatility shocks, and in particular in the investment sector, contribute positively and significantly to the equity premium, and can also explain a significant variation of the momentum spread, and of investment-based spreads.

To highlight the importance of the TFP-volatility shocks, I compare two factor model specifications. In the first specification, I include four risk factors: consumption and investment (first-moment) TFP innovations, and consumption and investment TFP-volatility shocks. The second model specification excludes the sectoral volatilities, and only includes two risk factors: the first-moment TFP innovations of the two-sectors. I tabulate a summary of the asset-pricing implications for the two models – in Table 2.6, for the four-factor model, and in Table 2.7, for the two-factor model.

Panel A of Tables 2.6 and 2.7 reports the *adjusted* R^2 of the second-stage projection in the Fama-Macbeth procedure (i.e., mean excess returns on cross-sectional risk exposures, as in equation (2.6)), performed separately for each of the models. The cross-sectional assets in each case are identical to those used in Section 2.3.4, and include ten portfolios sorted on size, ten portfolios sorted on book-to-market, and ten portfolios sorted on momentum. The fit of the four-factor model is significantly better than the two-factor specification. The adjusted R^2 rises from about 50% with only sectoral first-moment innovations, to 70% when volatilities are included.[21]

Furthermore, panel B of Tables 2.6 and 2.7 reports the factor model-implied quantile based

[21] The Akaike Information Critertion of the second-stage projection also rises from the two- to four- factor specification.

quarterly return spreads, of several cross-sections, against their data counterpart. The dimensions tabulated include size, book-to-market, momentum, lagged firm value to capital value (Tobin's Q), operating profitability, and idiosyncratic return volatility spreads.[22]

The fit of the four-factor model is significantly improved along the momentum, Q, operating profitability, and idiosyncratic volatility dimensions, in comparison to the two-factor specification. In the data, the quarterly momentum spread amounts to 2.65%. When volatilities are included, the factor model-implied spread amounts to 0.83%. While this is only 30% of the data-spread's magnitude, in the model without volatilities, the model-implied spread bears the wrong sign (that is, low momentum portfolio earns a higher return than high momentum portfolio), and amounts to -0.50%. Similarly, model-implied spreads based on operating profitability and idiosyncratic volatility bear the opposite sign compared to the empirical counterparts when the sectoral volatilities are excluded, but become close to the empirical estimates once the volatility shocks are included. The model-implied quarterly Q-spread is 1.28% when volatilities are included, but only 0.41% without volatilities, while the data-spread is 0.98%.

The literature on investment-specific shocks documents that IST shocks are helpful in explaining the Value spread (see Papanikolaou, 2011), and commodity-based spreads (see Yang, 2013). Li (2014) argues that IST innovations can explain the momentum spreads at annual frequency. Yet, the ability of IST innovations to explain the momentum spread is disputed in the literature. Garlappi and Song (2013a) find that the fraction of the momentum spread, captured by these innovations is low at *quarterly* frequency. Although I do not explicitly consider IST shocks, but rather the total investment TFP innovations, Table 2.7 is broadly consistent with the notion that investment first-moment innovations alone are not enough to explain the momentum spread. By sharp contrast, I find that the ability of investment TFP-*volatility* shocks to explain the momentum spread is large and

[22] Tobin's Q is measured as Market-to-Book ratio as in Hennessy et al. (2007). Operating profitability is measured via operating profits divided by book equity. Idiosyncratic volatility is measured via the variance of the residuals from the Fama-French three-factor model over 60 days.

economically significant.

Panel B of Table 2.6 shows the decomposition of the model-implied momentum spread to the contribution of each risk factor.[23] The momentum spread, emanating from investment TFP-volatility risk channel, is 2.43% compared to 2.65% in the data (90% of the momentum spread's magnitude).

In Panel C of Tables 2.6 and 2.7, I tabulate the model-implied market excess return, along with its decomposition to the risk-premia contributions coming from the different risk factors. The model-implied quarterly market excess return, when volatilities are included, is 1.63%, strikingly close to the empirical counterpart of 1.64%. For comparison, the model-implied market excess return in a model without volatilities is 1.39%. Panel C of Table 2.6 shows that most of the market risk premium stems from consumption TFP innovation risk, and investment TFP-*volatility* risk.

Tables 2.6 and 2.7 thus lead to the following stylized fact:

Fact IV: *Investment TFP-volatility shocks are important for the market risk premium, and explaining the magnitude of the momentum spread.*

Lastly, I examine the differential impact of sectoral TFP volatilities on the default spread in Panel A of Table 2.8. I project contemporaneous and future cumulative log growth rates of the default spread, on the current proxies of sectoral shocks, as specified in projection (2.3). Interestingly, I find that while consumption TFP-volatility raises the spread, investment TFP-volatility significantly *lowers* it, in predictive horizons of up to three years ahead. This evidence may suggest that investment TFP-volatility lowers the cost of capital for firms, thus spurring investment, consistently with the evidence of Section 2.3.3. The differential impact of the volatilities on the economy-wide default spread seems to be translated into an opposite incentive of firms to issue debt. Panel B of Table 2.8 shows that consumption

[23]The contribution of a risk factor to a model-implied return includes the risk-premium from the factor's own quantity of risk, as well as one-half of the risk-premium from the covariance terms between the risk factor and other shocks in the model.

TFP-volatility drops total debt growth, whereas investment TFP-volatility raises it.

2.3.6. Volatility Feedback to Technological Growth

Section 2.3.3 shows that the sectoral TFP volatilities have a significant impact on the growth of aggregate cash-flows. In this section I examine whether the sectoral volatilities also affect the evolution of production technology itself, positively or negatively. I project one-quarter ahead consumption- and investment- TFP growth rates on the current level of the four factors: two sectoral first-moment TFP innovations, and two sectoral TFP-volatilities. The results are reported in Table 2.9.

Table 2.9 shows that investment and consumption TFP growth rates depend significantly (and positively) only on their own lagged value. Beyond that, there is a positive and significant feedback between investment TFP-volatility today to one-quarter ahead consumption TFP growth. I denote this feature the 'volatility feedback'. This is the only significant interaction between second-moment shocks to first moment TFP innovations predictively.

Although not micro-founded, one can think of this volatility feedback as delayed culmination of growth options in the investment sector. In a reduced form manner, higher investment TFP-volatility could be thought of as a bundle of R&D growth options, which raises uncertainty. Some of these options would turn out to be bad, but some in the right tail would be successful. Because higher volatility also causes a delay in exercising growth options, the positive impact of the successful growth options would not be seen immediately today. But in the future (one quarter), these successful growth options are exercised. This could be manifested as improved productivity in the final good sector one quarter ahead.

The economic significance of this finding will be clarified in the model section. The empirical feedback of investment TFP-volatility to future consumption productivity would be used to quantitatively explain the positive market-price of risk of investment TFP-volatility.

2.3.7. *Robustness*

I consider various robustness checks regarding the construction of the ex-ante sectoral volatilities in the data. First, I consider different predictors for predicting future realized variances, as in projection (2.2). I add to the benchmark predictors additional variables such as the risk-free rate and the market-price dividend ratio. The summary of the key results are shown in Table 2.12. In unreported results, I consider different sets of predictors as well: including the default-spread as an additional predictor, or including *only* the lagged sectoral realized-variances as predictors. In all cases, the results are broadly unchanged. I also consider a different window for the realized variances construction, as in equation (2.1). In Table 2.13, I tabulate a summary of the results when the window is expanded to three years. The results are also largely robust when the window is shortened to just four quarters.

Next, I consider the usage of the total ex-ante volatilities as risk-factors in the various projections, as opposed to their first-difference (referred to in this work as their reduced-form shocks). The results are reported in Table 2.10. Similarly, I also replace the ex-ante volatilities by their realized-variance counterparts (i.e., backward-looking volatilities) in Table 2.14. In both cases, by and large, the findings are qualitatively similar to those reported in the benchmark specification.

I also consider the usage of a different proxy for sectoral volatilities. Specifically, I split the universe of Compustat firms into consumption and investment sectors, according to the classifications of Gomes et al. (2009). I then consider the dispersion of sales growth for consumption firms, versus the dispersion of sales for investment firms, as proxies for the two-sectors' technological volatilities. The summary results are reported in Table 2.11. Notably, dispersion differs conceptually from time-series conditional volatility of aggregate shocks. Yet, I obtain qualitatively the same results as with the benchmark proxies. Sales dispersion of consumption firms generates a contractionary impact, while sales dispersion of investment firms an expansionary one.

Lastly, I consider other modifications: (1) Filtering the sectoral TFP growth rates using an $AR(k)$ filter, and using the residuals as first-moment TFP innovations; (2) Using capacity-utilization adjusted TFPs, as in Basu et al. (2006), for sectoral productivity shocks; (3) Using the relative-price of investment goods as an investment *specific* technology shock, against a neutral TFP; (4) Constructing the sectoral volatilities by estimating GARCH(1,1) processes; (5) Accounting for estimation errors in the ex-ante sectoral volatilities, by collapsing projection (2.2) and projection (2.3) into a single GMM system. In the interest of space, I do not report these additional tables but note that across all of these modifications, I broadly confirm the key empirical results qualitatively.

2.4. The Model

Why is consumption TFP-volatility contractionary for macroeconomic quantities and prices, while investment TFP-volatility expansionary? I rationalize the findings using a quantitative framework. This section describes the general-equilibrium model. The model is quite rich, and I provide intuition regarding the role of the various model ingredients in Section 2.5.[24]

An overview of the economy follows below. Figure 2.3 provides a schematic illustration of the model players and their interactions.[25] The economy is populated by a continuum of identical households, deriving felicity from an Epstein and Zin (1989) and Weil (1989) utility over a stream of consumption-goods and leisure. The household supplies labor to two good-producing sectors: a "consumption" sector and an "investment" sector.

In each sector, there is a mass of intermediate good producers, who produce *differen-*

[24]Only a subset of the model assumptions are needed to rationalize the impact of volatility shocks on investment behavior. Namely, even without monopolistic competition and nominal rigidities, a two-sector of perfect competition is sufficient to explain volatilities' impact on investment, as I illustrate in section 2.5. Other model ingredients are placed to generate *comovement* of consumption and investment in response to volatility shocks (see Basu and Bundick (2012)), and to *quantitatively* amplify the impact of volatility shocks on real and financial quantities.

[25]The economy structure builds on the two-sector production economies of Papanikolaou (2011), Liu et al. (2012), and Garlappi and Song (2013b), but also features Epstein and Zin (1989) utility and stochastic volatility in the productivity of both sectors.

tiated products: either differentiated-intermediate consumption goods or differentiated-intermediate investment goods. The intermediate good producers produce their output using a Cobb-Douglas production function over capital and labor, which is subject to sectoral TFPs, that also feature stochastic volatilities. The intermediate good producers face monopolistic competition in the product markets. They pick their nominal product price, but face adjustment costs in doing so.

In each sector, a representative aggregator converts the intermediate goods to a final composite good. The consumption-sector's aggregator sells the final consumption-good to the household for consumption. The investment aggregator produces final investment goods (capital), and sells them back to the intermediate-good producers in both sectors, who buy these goods when they wish to invest. In the economy a monetary policy authority also operates, and sets the nominal interest rate according to a Taylor (1993) rule. This Taylor rule, along with the pricing kernel of the household, endogenously pins down inflation. Next, I describe in more detail each model ingredient.

2.4.1. Aggregation

The aggregator in the consumption (investment) sector produces composite or "final" consumption (investment) goods, denoted $Y_{c,t}$ ($Y_{i,t}$). $Y_{c,t}$ will be used for consumption by the household, while $Y_{i,t}$ will be equal to aggregate investment in the economy. Production of the composite consumption (investment) good requires a continuum of differentiated intermediate goods as inputs, denoted by $\{y_{c,t}(n)\}_{\{n\in[0,1]\}}$ ($\{y_{i,t}(n)\}_{\{n\in[0,1]\}}$). The aggregation technology in both sectors is symmetric, so I describe it below jointly.

The production of the final composite $Y_{j,t}$, in sector $j \in \{c, i\}$, converts the intermediate goods of sector j into a final-good using a constant elasticity of substitution (CES) technology:

$$Y_{j,t} = \left[\int_0^1 (y_{j,t}(n))^{\frac{\mu_j - 1}{\mu_j}} dn \right]^{\frac{\mu_j}{\mu_j - 1}}, \quad j \in \{c, i\}. \tag{2.7}$$

The parameter $\mu_j, \quad j \in \{c, i\}$, measures the degree of substitutability among the intermediate goods. Perfect competition among the intermediate good producers implies $\mu_j \to \infty$. Under finite μ_j, the intermediate goods in sector j are not perfect substitutes, and thus each intermediate good producer possesses some monopolistic power.

Each intermediate good producer of variety n in sector j sells its intermediate good to the aggregator at a nominal price $p_{j,t}(n)$. Each final good producer (aggregator) in sector j, sells its composite output $Y_{j,t}$ at nominal price $P_{j,t}$. The aggregator in each sector $j \in \{c, i\}$ faces perfectly competitive market, thus solving:

$$\max_{\{y_{j,t}(n)\}} P_{j,t} Y_{j,t} - \int_0^1 p_{j,t}(n) y_{j,t}(n) dn, \quad j \in \{c, i\}, \tag{2.8}$$

where $Y_{j,t}$ is given by (2.7), and the prices are taken as given. The first-order condition of (2.8) yields the demand for differentiated intermediate good of type n in sector j:

$$y_{j,t}(n) = \left[\frac{p_{j,t}(n)}{P_{j,t}}\right]^{-\mu_j} Y_{j,t}, \quad j \in \{c, i\}. \tag{2.9}$$

As the market for final goods is perfectly competitive, the final-good producing firm (aggregator) in sector j earns zero profits in equilibrium. This condition, along with equations (2.8) and (2.9), yields the aggregate price index in sector j, given by:

$$P_{j,t} = \left[\int_0^1 (p_{j,t}(n))^{1-\mu_j} dn\right]^{\frac{1}{1-\mu_j}}, \quad j \in \{c, i\}. \tag{2.10}$$

2.4.2. Intermediate Good Production

2.4.2.1. Sectoral Intermediate-Good Producers

This section describes the production and price-setting decisions of intermediate goods. To save space, and since the description of production in the consumption sector and investment sector is symmetric, I describe them jointly.

Intermediate goods in sector $j \in \{c, i\}$ are differentiated, and each variety is denoted by $n \in [0, 1]$. Each intermediate-good producer n in sector j rents labor $n_{j,t}(n)$ from the household, and owns capital stock $k_{j,t}(n)$. The intermediate-good producer n in sector j produces an intermediate good $y_{j,t}(n)$, using a constant returns-to-scale Cobb-Douglas production function over capital and labor, and subject to sectoral TFP shocks $Z_{j,t}$:

$$y_{j,t}(n) = Z_{j,t} k_{j,t}(n)^{\alpha_j} n_{j,t}(n)^{1-\alpha_j}, \quad j \in \{c, i\}, \tag{2.11}$$

where α_j is the capital share of output of intermediaries in sector j, and $Z_{j,t}, \quad j \in \{c, i\}$, are the sectoral TFPs. Each intermediate good producer who wishes to invest an amount $i_{j,t}(n)k_{j,t}(n)$, where $i_{j,t}(n)$ is the investment-rate, must purchase $\Phi_k(i_{j,t}(n))k_{j,t}(n)$ units of capital goods, under an equilibrium price of investment goods $P_{i,t}$. Following Papanikolaou (2011) and Garlappi and Song (2013b), the convex adjustment cost function $\Phi_k(i)$ is given by:

$$\Phi_k(i) = \frac{1}{\phi}(1+i)^{\phi} - \frac{1}{\phi}. \tag{2.12}$$

The parameter ϕ captures the degree of adjustment cost. When $\phi = 1$ there are no adjustment costs. When $\phi = 2$, adjustment costs are quadratic. Capital of each producer of type n in sector j, depreciates at rate δ, and evolves according to:

$$k_{j,t+1}(n) = (1 - \delta + i_{j,t}(n))k_{j,t}(n). \tag{2.13}$$

Intermediate good producers in both sectors are price takers in the input market, and monopolistic competitors in the product market. They face a quadratic costs of changing their nominal output price $p_{j,t}(n)$ each period, similarly to Rotemberg (1982), given by:

$$\Phi_{P,j}(p_{j,t}(n), p_{j,t-1}(n)) = \frac{\phi_{P,j}}{2}\left[\frac{p_{j,t}(n)}{\Pi_j p_{j,t-1}(n)} - 1\right]^2 p_{j,t-1}(n)Y_{j,t}, \quad j \in \{c, i\}, \tag{2.14}$$

where $Y_{j,t}$ is the final composite good in sector j, Π_j is the steady-state inflation in the j sector, and $\phi_{P,j}$ governs the degree of nominal rigidity in sector j. The assumption of Rotemberg (1982), as opposed to Calvo (1983) pricing, implies that I can model the intermediate good production in each sector as a single representative intermediate goods-producing firm. In all, the period nominal dividend of intermediate good producer of type n in sector j, $d^{\$}_{j,t}(n)$, in terms on nominal consumption goods, is given by:

$$d^{\$}_{j,t}(n) = p_{j,t}(n)y_{j,t}(n) - W_t n_{j,t}(n) - P_{i,t}\Phi_k(i_{j,t}(n))k_{j,t}(n) - \Phi_{P,j}(p_{j,t}(n), p_{j,t-1}(n)), \quad j \in \{c, i\}. \tag{2.15}$$

Each intermediate good producer n, chooses optimal hiring, investment, and nominal output price, to maximize the firm's market value, taking as given nominal wages W_t, the nominal price of investment goods $P_{i,t}$, the demand for differentiated intermediate good n in sector j given by (2.9), and the nominal stochastic discount factor of the household $M^{\$}_{t,t+1}$. Specifically, the intermediate good-producers maximize:

$$V^{\$}_{j,t}(n) = \max_{\{n_{j,s}(n), k_{j,s}(n), p_{j,s}(n)\}} \quad E_t \Sigma_{s=t}^{\infty} M^{\$}_{t,t+s} d^{\$}_{j,t+s}(n), \tag{2.16}$$

subject to (2.13), (2.15), and the demand constraint:

$$\left[\frac{p_{j,t}(n)}{P_{j,t}}\right]^{-\mu_j} Y_{j,t} \leq Z_{j,t} k_{j,t}(n)^{\alpha_j} n_{j,t}(n)^{1-\alpha_j}, \quad j \in \{c, i\}. \tag{2.17}$$

Notice that $V^{\$}_{j,t}(n), \quad j \in \{i, c\}$, is measured in nominal consumption units. Define the real firm value $V_{j,t}(n)$, and real dividend $d_{j,t}(n)$ (in terms of real consumption goods), for firm n in sector j, by:

$$V_{j,t}(n) = V^{\$}_{j,t}(n)/P_{c,t}; \quad d_{j,t}(n) = d^{\$}_{j,t}(n)/P_{c,t}. \tag{2.18}$$

Lastly, define the real *growth* rate in aggregate investment expenditures (in terms of real consumption goods) as $\Delta I_t = \frac{(P_{i,t}/P_{c,t})Y_{i,t}}{(P_{i,t-1}/P_{c,t-1})Y_{i,t-1}}$, and the *growth* rate in the *relative* price of investment goods by $\Delta P_{i,t} = \frac{P_{i,t}/P_{c,t}}{P_{i,t-1}/P_{c,t-1}}$.

2.4.2.2. Productivity Shocks

The production in the investment sector is subject to a sectoral TFP shock, denoted $Z_{i,t}$, and similarly, the production in the consumption sector is subject to a sectoral TFP shock denoted $Z_{c,t}$. The sectoral TFP growth rates are characterized as follows:

$$\frac{Z_{i,t}}{Z_{i,t-1}} = \mu_{z,i} + \tilde{\varepsilon}_{i,t}, \tag{2.19}$$

$$\frac{Z_{c,t}}{Z_{c,t-1}} = \mu_{z,c} + \tilde{\varepsilon}_{c,t}, \tag{2.20}$$

where $\tilde{\varepsilon}_{i,t} = \sigma_{zi,t-1}\varepsilon_{i,t}$, and $\tilde{\varepsilon}_{c,t} = \tau(\sigma^2_{zi,t-1} - \sigma^2_{zi,0}) + \sigma_{zc,t-1}\varepsilon_{c,t}$. The shocks $\varepsilon_{i,t}$ and $\varepsilon_{c,t}$ are orthogonal, and are i.i.d. standard Normal.[26] Driven by the empirical findings of Section 2.3.6, equation (2.20) shows that I incorporate a positive volatility feedback from investment-technology volatility, $\sigma^2_{zi,t-1}$ to one-period-ahead consumption TFP growth, which is governed by the parameter $\tau > 0$. The processes $\sigma_{zc,t}$ and $\sigma_{zi,t}$ capture time-variation in the volatility of sectoral growth shocks. They follow independent AR(1) processes:

$$\sigma^2_{zi,t} = (1 - \rho_{\sigma,zi})\sigma^2_{zi,0} + \rho_{\sigma,zi}\sigma^2_{zi,t-1} + \sigma_{w,i}\varepsilon_{\sigma,i,t}, \tag{2.21}$$

$$\sigma^2_{zc,t} = (1 - \rho_{\sigma,zc})\sigma^2_{zc,0} + \rho_{\sigma,zc}\sigma^2_{zc,t-1} + \sigma_{w,c}\varepsilon_{\sigma,c,t}, \tag{2.22}$$

where the volatility shocks $\varepsilon_{\sigma,i,t}$ and $\varepsilon_{\sigma,c,t}$ are i.i.d. over time and are standard Normal.

2.4.3. Household

The economy is populated by a mass of identical households, or alternatively, by a one representative household. The representative household supplies total labor N_t, which flows to the consumption and investment sectors. It derives utility from an Epstein and Zin (1989) and Weil (1989) utility over a stream of consumption-goods C_t and disutility from labor

[26] Notice that I do not exponentiate the right-hand side of the sectoral growth rates in equations (2.19), and (2.20). Thus, TFP growth rates are normal, instead of log-normal. The motivation for this modeling choice is to exclude any hard-wired Jensen effect that can mechanically yield an impact of volatility on the *mean* growth rate. Moreover, the parameters μ_{zc} and μ_{zi} will be set to values above one, while the shocks are small, ensuring the growth rate is never negative in any population simulation. However, exponentiating the growth rates to ensure positivity does not change the qualitative or quantitative results of this work.

N_t:

$$U_t = \left\{ (1-\beta)\left[C_t(1-\xi N_t^{\eta})\right]^{1-1/\psi} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{2.23}$$

where β is the time discount-rate, γ is the relative risk aversion, ψ is the intertemporal elasticity of substitution (IES), ξ is the amount of disutility from labor, and η is the sensitivity of disutility to working hours. When $\gamma = \frac{1}{\psi}$, the utility becomes time-separable power utility. When $\gamma > (<)\frac{1}{\psi}$ the household has preferences exhibiting early (late) resolution of uncertainty. The preferences nest a class of multiplicative preferences over consumption and labor, as discussed in King et al. (1988).

The household derives income from labor, as well as from the dividends of well-diversified portfolio of intermediate consumption and investment good producers. She chooses the labor supply and consumption to maximize her lifetime utility, subject to the following budget constraint:[27]

$$\max_{\{C_s, N_s\}} \quad U_t, \quad \text{s.t. } P_{c,t}C_t = W_t N_t + \int_0^1 d_{c,t}^{\$}(n)dn + \int_0^1 d_{i,t}^{\$}(n)dn, \tag{2.24}$$

where $P_{c,t}$ is the nominal price of final consumption goods, and W_t is the nominal market wage.

From the consumer problem, I can obtain the nominal SDF used to discount the nominal dividend of intermediate-good producing firms in *both* sectors:

$$M_{t+1}^{\$} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left(\frac{1-\xi N_{t+1}^{\eta}}{1-\xi N_t^{\eta}} \right)^{1-1/\psi} \left(\frac{U_{t+1}}{\left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{1/\psi - \gamma} \frac{P_{c,t}}{P_{c,t+1}}. \tag{2.25}$$

[27] This is a simplified budget constraint. I implicitly imposed the market-clearing condition that the nominal bond holding of the household is zero every period ($B_t = B_{t+1} = 0$), and the household is the owner of all shares for all firms ($\omega_{j,t}(n) = \omega_{j,t+1}(n) = 1, \quad j \in \{i, c\}, n \in [0,1]$, where $\omega_{j,t}(n)$ is the fraction of firm n in sector j held by the household).

2.4.4. Monetary Authority

The economy is cashless. The monetary authority sets the nominal log-interest rate $r_t^{\$}$ according to a Taylor (1993) rule. Thus, $r_t^{\$}$ evolves as follows:

$$r_t^{\$} = \rho_r r_{t-1}^{\$} + (1 - \rho_r)(r_{ss}^{\$} + \rho_\pi(\pi_t - \pi_{ss}) + \rho_y(\Delta y_t - \Delta y_{ss})) \tag{2.26}$$

where π_t is log inflation (in the consumption sector) defined as $\pi_t = \log\left(\frac{P_{c,t}}{P_{c,t-1}}\right)$, and where Δy_t is log-growth of real total output, $\Delta y_t = \log\left(\frac{Y_{c,t}+P_{i,t}/P_{c,t}Y_{i,t}}{Y_{c,t-1}+P_{i,t-1}/P_{c,t-1}Y_{i,t-1}}\right)$. $r_{ss}^{\$}$, π_{ss}, and Δy_{ss} are the steady-state log-levels of nominal interest rate, inflation, and output growth.

2.4.5. Equilibrium

In equilibrium, (nominal) wage W_t, price of investment goods $P_{i,t}$, and consumption-sector inflation π_t, are set to clear all markets:

- Labor market clearing:

$$\int_0^1 n_{c,t}(n)dn + \int_0^1 n_{i,t}(n)dn = N_t. \tag{2.27}$$

- Consumption-good market clearing:

$$C_t = Y_{c,t}. \tag{2.28}$$

- Investment-good market clearing:

$$\int_0^1 \Phi_k(i_{c,t}(n))K_{c,t}(n)dn + \int_0^1 \Phi_k(i_{i,t}(n))K_{i,t}(n)dn = Y_{i,t}. \tag{2.29}$$

- Zero net supply of nominal bonds:

$$\frac{1}{R_t^{\$}} = E_t[M_{t+1}^{\$}] \tag{2.30}$$

An equilibrium consists of prices and allocations such that (i) taking prices and wage as given, each household's allocation solves (2.24); taking aggregate prices and wage as given, firm's allocations in each sector $j \in \{c, i\}$ solve (2.16); (iii) labor, consumption-good, investment-good and bond markets clear.

I am looking for a symmetric equilibrium, in which all intermediate good firms, in both sectors, choose the same price $P_{j,t}(n) = P_{j,t}$, employ the same amount of labor $n_{j,t}(n) = n_{j,t}$, and choose to hold the same amount of capital $k_{j,t}(n) = k_{j,t}$.

2.5. Model Intuition

To understand the model intuition, in this section I shut down certain channels, to highlight the core economic forces of the model. As I illustrate below, even in a stripped-down perfect-competition model, I am able to rationalize the impact of volatility shocks on investment, and the risk-exposures of firms to the sectoral shocks. Yet, the simplified model described below generates divergence of investment and consumption in response to volatility shocks, and hence, cannot rationalize the impact of volatility shocks on consumption. This could also result in counterfactual market-prices of volatility risks. The layers of monopolistic competition and nominal rigidities address this matter.

To facilitate the discussion, assume a two-sector economy under perfect competition, inelastic labor supply, and without adjustment costs.[28] Under these assumptions, I can collapse

[28]Specifically, I assume that (1) There is no disutility from labor $\xi = 0$, so labor supply is inelastic; (2) $\mu_j \to \infty, \quad j \in \{c, i\}$, implying perfect competition in both sectors; (3) Assume $\tau = 0$, that is, no volatility feedback to future TFP growth; (4) $\phi = 1$, so there are no capital adjustment costs ; (5) The capital share of output is the same in both sectors $\alpha_c = \alpha_i = \alpha$.

the model of Section 2.4 to a representative agent problem, as follows:

$$V_t(\Gamma_t) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \left\{ (1-\beta) C_t^{1-1/\psi} + \beta \left(E_t V_{t+1}(\Gamma_{t+1})^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{2.31}$$

s.t.

$$C_t = z_{ct} k_{ct}^{\alpha} n_{ct}^{1-\alpha} \tag{2.32}$$

$$Y_{i,t} = z_{it} k_{it}^{\alpha} n_{it}^{1-\alpha} \tag{2.33}$$

$$k_{c,t+1} = (1-\delta) k_{ct} + I_{ct} \tag{2.34}$$

$$k_{i,t+1} = (1-\delta) k_{it} + I_{it} \tag{2.35}$$

$$I_{ct} + I_{it} = Y_{i,t} \tag{2.36}$$

$$n_{ct} + n_{it} = 1, \tag{2.37}$$

where $\Gamma_t = [k_{ct}, k_{it}, z_{ct}, z_{it}]$, and $\frac{z_{jt}}{z_{j,t-1}} = \sigma_{zj,t-1} \varepsilon_{j,t}, \quad j \in \{c, i\}$.[29] In appendix A.2.1, I show that the solution to program (2.31) is equal to the solution of the maximization program (2.38), given by:

$$\tilde{V}_t(k_{ct}, k_{it}, z_{it}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \left\{ (1-\beta) \left(k_{ct}^{\alpha} n_{ct}^{1-\alpha} \right)^{1-1/\psi} + \beta \underbrace{\left(E_t \left(\frac{z_{ct+1}}{z_{ct}} \right)^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}}}_{\tilde{\beta}_t} \left(E_t \tilde{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1})^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{2.38}$$

s.t.

$$(2.33), (2.34), (2.35), (2.36), \text{ and } (2.37),$$

$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t} \varepsilon_{zc,t+1},$$

$$\frac{z_{it+1}}{z_{it}} = \mu_{zi} + \sigma_{zi,t} \varepsilon_{zi,t+1}.$$

[29] In program (2.31) the sectoral volatilities, $\sigma_{zc,t}$ and $\sigma_{zi,t}$, are also carried as state variables. For brevity of notation, I omitted them from the vector of state variables.

The equivalence of programs (2.31) and (2.38) relies on the fact that the detrended value function of the social planner is homogeneous of degree one in consumption TFP growth. Homogeneity of degree one in consumption TFP growth stems from the fact that z_{ct} is random walk, and from the fact that an Epstein-Zin utility is a homogeneous of degree one function.

2.5.1. Sectoral Volatilities and Investment Implications

To understand the impact of consumption TFP-volatility on investment, it is constructive to realize that higher consumption TFP-volatility, $\sigma_{zc,t}$, increases the social planner's effective impatience, under the case of early resolution of uncertainty. To see this, notice first that in maximization program (2.38), the ex-ante expectation of consumption TFP growth (that is, the expression $\tilde{\beta}_t = \beta \left(E_t \left(\frac{z_{ct+1}}{z_{ct}} \right)^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}}$) acts like a time "preference shock" that changes in the effective time-discount rate of the planner.

When $\gamma > 1$, $(\cdot)^{(1-\gamma)}$ is a convex function. With more consumption TFP-volatility (higher σ_{zc}), $E_t \left(\frac{z_{ct+1}}{z_{ct}} \right)^{1-\gamma}$ increases by Jensen's inequality. When the agent has *early resolution of uncertainty* preferences, $\psi > 1$, and the expression $\frac{1-1/\psi}{1-\gamma}$ is negative. Thus, higher σ_{zc} translates into a *lower* effective discount factor $\tilde{\beta}$. In other words, the representative agent puts lower weight on the continuation value. This implies a more impatient agent. Moreover, consumption TFP-volatility only affects impatience, as the growth of z_c appears nowhere else in the program, except for its *ex-ante* impact on $\tilde{\beta}_t$.

As a result of greater impatience, when σ_{zc} rises, the agent decides to shift her consumption profile to the present.[30] To implement such policy, the agent would shift labor to the consumption sector, to increase consumption today; she would also increase investment in the consumption sector, to ensure higher consumption in near-future. Consequently, investment sector's labor drops ($n_{i,t} \downarrow$), and investment sector's investment drops. Since

[30] An alternative intuitive argument for this claim, is that under early resolution of uncertainty case the agent dislikes uncertainty. To minimize her exposure to volatility build-up in the future, and void capital loss, she prefers shifting her consumption profile as much as possible to the present.

capital in the investment sector is predetermined, but investment's labor drops, *higher consumption TFP-volatility lowers aggregate investment, in-line with the empirical findings.*

Notice, that the impact of σ_{zc} on the agent's patience depends on the preference parameters. When $\gamma = 0, 1$, there is no Jensen effect, and so consumption TFP-volatility would not impact $\tilde{\beta}_t$. If $\psi < 1$ (late resolution of uncertainty preferences), consumption technology volatility would boost investment, as the agent becomes more patient (higher $\tilde{\beta}_t$).[31]

The program (2.38) shows that consumption TFP *ex-ante* expectations change the effective discount rate. Beyond that, under the specification for z_c growth, consumption TFP shocks have no other effect ex-post except for "rescaling" the *flow* of consumption (see equation (2.32)). This is a transitory (short-run) impact.[32] By contrast, investment innovations z_i affect multi-period *stock* of aggregate capital dynamics, which flows to *both* sectors. Investment innovations, consequently, have a long-run and persistent impact. As a result, when these shocks become more volatile they induce a strong *precautionary saving* motive.[33]

When TFP-volatility of the investment sector rises (higher σ_{zi}), it implies that in the future the probability of having sub-optimal amount of investment-goods rises. This would inhibit the ability to smooth consumption, as aggregate investment goods flow to both sectors, much like total output in a one-sector economy. The household has a strong incentive to invest more in the investment sector, and consume less, by shifting labor to the investment sector ($n_{i,t} \uparrow$). Implementing such a policy, ensures higher aggregate capital in the future. Capital can be used for both consumption and investment production. Hence, it acts as a

[31] The dependence of investment's response to consumption TFP-volatility on the value of IES is consistent with the works of Levhari and Srinivasan (1969), Sandmo (1970), and Obstfeld (1994). In a one-sector context, these studies analyze the impact of higher volatility of multiplicative shocks, which only affect the riskiness of capital (similarly to consumption TFP). These models share the prediction that for high values of IES, the substitution effect dominates, and higher volatility induces less investment.

[32] The notion that consumption TFP has a short-run impact, that is, rescales consumption traces back to Kimball (1994).

[33] A necessary condition for precautionary saving is Decreasing Absolute Risk Aversion (see e.g. Leland, 1968 ;Kimball and Weil, 2009), satisfied by Epstein and Zin (1989) utility. Quantitatively, I find that the motive to hedge against low consumption states, in response to higher investment TFP-volatility, prevails the substitution effect for both high and low IES values. This is consistent with the study of Jones et al. (2005), who show that in a one-sector economy, and under most realistic calibrations, higher volatility raises savings and growth in equilibrium.

buffer of savings. If a bad investment TFP shock is realized, the buffer of capital can be used to smooth consumption. Higher investment partially hedges the investment TFP-volatility shock. Consequently, *higher investment TFP-volatility increases aggregate investment, in-line with the empirical findings.*

2.5.2. Sectoral Volatilities and Pricing Implications

The illustrated logic shows that investment TFP-volatility, σ_{zi}, increases the demand for investment goods, while consumption TFP-volatility, σ_{zc}, lowers the demand for investment goods (when IES is greater than one). As a result, the relative price of new investment goods (in the decentralized economy) intuitively increases when σ_{zi} rises, but drops when σ_{zc} rises. To see this more formally, let $q_{c,t}$ be the Lagrange multiplier of constraint (2.34), let $q_{i,t}$ be the Lagrange multiplier of constraint (2.35), and let $P_{i,t}$ be the Lagrange multiplier of constraint (2.33).[34] From first-order conditions of program (2.31), and in particular, from equating the marginal productivity of labor in both sectors, one obtains:

$$P_{i,t} = q_{i,t} = q_{c,t}, \tag{2.39}$$

$$P_{i,t} = \frac{z_{c,t}}{z_{i,t}} \left(\frac{k_{c,t}}{k_{i,t}} \frac{n_{i,t}}{n_{c,t}} \right)^{\alpha}. \tag{2.40}$$

Since higher consumption TFP-volatility σ_{zc} lowers $n_{i,t}$ and raises $n_{c,t}$, the price of new investment goods $P_{i,t}$ (measured here in real consumption units) must fall by equation (2.40). Likewise, higher investment TFP-volatility σ_{zi} increases $n_{i,t}$ and lowers $n_{c,t}$, causing the price $P_{i,t}$ to rise.

Generally, the marginal value of assets in place (i.e., Tobin's Q: q_c or q_i), should equal the marginal cost of new capital (P_i), times the marginal adjustment cost (installation cost). In the absence of adjustment costs, we obtain equation (2.39), which implies that the price of installed capital is equal in the consumption and investment sectors. Thus, consumption TFP-volatility lowers $q_{c,t}$ and $q_{i,t}$, and the opposite happens in response to investment

[34] I normalize all multipliers by the marginal utility from consumption at time t, to parallel the multipliers with prices of the decentralized economy

TFP-volatility.

Firms in the model exhibit constant returns to scale in capital and labor. By a standard argument, Tobin's Q is a sufficient statistic for the (ex-dividend) firm values. Higher σ_{zi} increases $P_{i,t}$ and so increases both $q_{i,t}$ and $q_{c,t}$. This implies that higher investment TFP-volatility increases firms' values in both sectors, and by definition, $\beta_{\text{j,I-TFP-VOL}} > 0, \quad j \in \{c, i\}$. The exact opposite logic applies to consumption TFP-volatility, and implies $\beta_{\text{j,C-TFP-VOL}} < 0, \quad j \in \{c, i\}$. *The risk-exposure patterns with respect to sectoral volatility shocks, are consistent with the data.*

2.5.3. Sectoral Volatilities and The Role of Nominal Rigidities

The intuition of sections 2.5.1 and 2.5.2 demonstrates that in the perfect-competition model, investment expenditures rise in response to investment TFP-volatility, and drop in response to consumption TFP-volatility, in-line with the data. Yet, in the simplified setup, consumption and aggregate investment diverge in response to volatility shocks. As a result, consumption TFP-volatility counterfactually boosts consumption, not only contemporaneously but also in the future.[35] Counterfactual consumption behavior also induces a counterfactual impact on the market-price of consumption TFP-volatility risk.

The full version of the model features time-varying markups, that rely on monopolistic competition in the two sectors, along with sticky prices. As suggested in Basu and Bundick (2012) and Fernández-Villaverde et al. (2015), these model features make consumption and aggregate investment expenditures to commove with respect to sectoral volatility shocks. Specifically, when sticky prices are added (in particular, to consumption producing firms), consumption and investment expenditures *both* decrease in response to consumption TFP-volatility. The intuition is described below.

Section 2.5.1 shows that higher consumption TFP-volatility makes the agent more impatient. This increases the demand for consumption goods, and causes the agent to desire to

[35]In the data, consumption TFP-volatility drops both consumption and investment. See Figure 2.2.

supply more labor to the consumption sector. As a consequence, wages and the price of investment drop.[36] Thus, the marginal cost of producing consumption goods declines. When monopolistic competition is added, along with nominal price rigidity, higher consumption TFP-volatility causes the markups of consumption producing firms to rise, due to a drop in their marginal production costs.[37]

Higher markups of consumption producing firms lower the demand of these firms for labor at any given level of wages. This is because higher markups are equivalent to a higher degree of monopolistic power, which has a rationing impact on the quantity produced, and involves less utilization of labor.[38] Differently put, facing higher markups the consumption good producers would have optimally liked to reduce their prices, in order to drop markups, and increase their capacity. However, due to the nominal price rigidities, the consumption producing firms are limited in doing so. Since these firms cannot expand their capacity by lowering their output price, they demand less labor.

If the decline in labor demand from consumption producing firms (due to higher markups) is sufficiently strong, higher consumption TFP-volatility would cause these firms to hire less. Hence, *consumption drops upon a positive consumption TFP-volatility shock, which is consistent with the data.*

As labor flows out of the consumption sector, and into the investment sector, production of investment goods ($Y_{i,t}$) rises. The increased supply of investment goods (rise in $Y_{i,t}$), along with the reduced demand for these products from the household (due to higher effective impatience), causes their relative price P_i to decline even further.

[36] Wages move in the simplified model in an opposite direction to consumption labor. The price of investment drops due to decreased demand for capital.

[37] With Rotemberg pricing, gross markup equals the inverse of the real marginal costs.

[38] The demand curve for labor from consumption producing firms is given by:

$$W_t = (1-\alpha)\frac{1}{\theta_{c,t}} k_{ct}^{\alpha} n_{ct}^{-\alpha},$$

where W_t is aggregate wage, and $\theta_{c,t}$ is the markup in the consumption sector. Thus, a higher markup $\theta_{c,t}$, shifts the labor demand curve of consumption producing firms downwards. As a result, consumption TFP-volatility shock makes consumption producing firms to demand less labor.

If the decline in the relative price of investment P_i is strong enough, investment *expenditures*, defined as $I_t = P_{i,t}Y_{i,t}$, would drop in response to consumption TFP-volatility shock. This would happen simultaneously with a decline in consumption, as seen in the data.

2.6. Quantitative Model Results

2.6.1. Calibration

Table 2.15 shows the parameter choices of the model in the Benchmark case. The model is calibrated at quarterly frequency. There are three main parameter groups.

Production and technologies parameters. I set $\alpha_i = \alpha_c = 0.33$, so that the labor share in each sector is about 2/3. The quarterly depreciation rate is 0.015, which implies annual depreciation of 6%. Similarly to Papanikolaou (2011), the capital adjustment cost parameter is $\phi = 1.2$. I set the growth rates of the sectoral TFPs, μ_{zc} and μ_{zi}, to values that are consistent with the empirical estimates of Fernald (2012) and Basu et al. (2006), and such that the steady state growth rate of per-capita consumption is about 2%. The ratio between the log of μ_{zi} and μ_{zc} is about 2, which is consistnet also with the estimates obtained by Liu et al. (2011). This ratio also matches the model-implied growth rate for the relative price of investment to the data. The unconditional volatilities of the sectoral TFP shocks $\sigma_{zc,0}$ and $\sigma_{zi,0}$, are also close to the empirical estimates of Fernald (2012) and of Justiniano et al. (2010). They are set to match the volatility of consumption growth and investemnt growth. The ratio of $\sigma_{zi,0}$ to $\sigma_{zc,0}$ is 2, which is in-line with the calibration of Garlappi and Song (2013b). The peristence of the stochastic volatility in both sectors ρ_σ is set to 0.95, which is higher than Basu and Bundick (2012), but smaller than the estimate of Bansal and Shaliastovich (2013). The standard deviation of the volatility shock in each sector is set such that the ratio between the standard deviation of the sectoral volatility process to its unconditional mean is similar to the empirical estimate. The feedback from investment TFP-volatility to one quarter ahead consumption TFP growth is $\tau = 1.5$, which falls in the 90%-confidence interval of its empirical estimate.

Preference parameters. The time discount factor is $\beta = 0.997$, close to the value set in both Liu et al. (2012) and Garlappi and Song (2013b), and allows to closely match the value of the real risk-free rate. The relative risk aversion γ is set to 25. Though this number is quite high, it is consistent with and even smaller than some estimates at quarterly frequency (see e.g. Bansal and Shaliastovich, 2013; Van Binsbergen et al., 2012; and Rudebusch and Swanson, 2012). The intertemporal elasticity of substitution is set to 1.7, consistently with Bansal et al. (2012) and Bansal and Shaliastovich (2013). The sensitivity of disutility to working hours η is set to 1.4, consistently with Jaimovich and Rebelo (2009). The degree of disutility to working hours ξ is chosen such that in the deterministic steady state, the household works roughly 20% of their time.

Nominal rigidities and monetary policy parameters. Monetary policy parameters are consistent with Basu and Bundick (2012) and are standard in the literature. I set $\rho_r = 0.5$, $\rho_\pi = 1.5$, and $\rho_\pi = 0.5$. The nominal risk-free steady-state is set such that the deterministic steady-state inflation rate is 0.005 per quarter, or 2% per annum. I choose market power parameters of $\mu_c = \mu_i = 4$, which implies on average a 25% markup for firms in both sectors, and is identical to the market power set in the work of Garlappi and Song (2013b). Lastly, the nominal adjustment cost parameter is set to $\phi_C = 250$, and contributes to matching the volatility of the relative price of investment. This value is slightly higher, but of a similar magnitude to the parameter used in Basu and Bundick (2012) of 160.

I solve the model numerically via third-order perturbations method around the stochastic steady state, and using the above benchmark calibration. A characterization of the equilibrium conditions is specified in Appendix A.2.2.

2.6.2. Macroeconomic Moment Implications

I simulate the model at quarterly frequency and time-aggregate the model-implied time-series to form annual observations. The mean, standard deviation, and auto-correlation moments of annual real (log) consumption growth, investment expenditure growth, output growth, and the growth in the relative price of investment, are reported in Table 2.16, along with their empirical counterparts. Almost all data moments fall inside the model-implied 90%-confidence intervals.

Specifically, consumption growth mean is about 2% in the model and in the data. In the model, the standard deviation of consumption growth is about 2.2%, while the standard deviation of output growth is 3%. These estimates are slightly higher than the data counterparts of 1.52% and 2.53%, respectively, for the sample of 1947-2014. Yet, the model-implied standard deviations are consistent with the long-run sample (1930-2014) data volatilities. The auto-correlation of consumption growth is 0.54 in the model, versus 0.49 in the data. The standard deviation and auto-correlation of investment expenditure growth are 6.6% and 0.30 in the model, closely related to a standard deviation of 6.75% and autocorrelation of 0.18 in the data. The mean growth rate of the relative price of investment is -0.97% in the data, while it is -0.95% in the model. The model-implied volatility of the relative investment price growth is 3.48%, closely matching its empirical counterpart of 3.62%.

2.6.3. Sectoral Shocks and Macroeconomic Implications

In this Section I analyze the impact of sectoral first-moment and volatility TFP shocks on macroeconomic quantities in the model. I document in the benchmark case a positive impact of investment TFP-volatility on macro aggregates, and a negative impact of consumption TFP-volatility on macro aggregates, consistently with the data.

I plot impulse-responses from sectoral shocks to key macroeconomic variables.[39] The

[39] The impulse responses are computed by Monte-Carlo simulations. In each simulation $i \in \{1, 2, .., S\}$, I simulate the economy for 140 periods. Denote the simulated path of simulation i from period 100 onward by $\{p_i\}$. I then simulate the economy again, using the same shocks as were drawn before, but in period 100, I increase shock j by one standard deviation. Let the second simulated path from period 100 onward be $\{p'_i\}$. The impulse-responses of shock j are given by the matrix $\frac{1}{S}\Sigma_{s=1}^{S}(p'_i - p_i)$. I pick $S = 10,000$ simulations for the impulse-response computations.

Similar results are obtained by computing the impulse-responses using Vector Auto-regression of order one, as in the empirical section. In unreported results, I construct first- and second- moment TFP shocks from simulated model sample, in an identical fashion to the empirical construction. I then project detrended model variables on these shocks. Consistently, I obtain negative loadings on consumption TFP-volatility, and positive loadings on investment TFP-volatility. The quantitative magnitude of the model-implied loadings is similar to the data for detrended output and investment projections.

impulse-responses are computed for three separate model calibrations: (1) the benchmark case; (2) an identical calibration to the benchmark case, but in which there is no volatility feedback from investment TFP-volatility to future consumption TFP (i.e., $\tau = 0$); (3) an identical calibration to the benchmark case, but in which there is no volatility feedback, *and* no monopolistic competition or nominal rigidities (i.e., perfect competition).[40] Specifications (2) and (3) allow to highlight the role of volatility feedback and nominal rigidities in the model.

Since sectoral volatilities are the main focus of this work, I first analyze the implications of sectoral TFP-volatility shocks, $\varepsilon_{\sigma,c}$ and $\varepsilon_{\sigma,i}$. Figure 2.4 shows model-implied impulse responses from consumption TFP-volatility and investment TFP-volatility shocks, to aggregate consumption (Panels (a) and (d)), aggregate investment expenditures (Panels (b) and (e)), and aggregate output (Panels (c) and (f)). All variables are real and detrended using the model's stochastic trend. Each impulse-response is in units of percent change from the stochastic steady-state. Observing first Panels (b) and (e), one can see a negative impact of consumption TFP-volatility on investment, both at the time of the shock and up to 40 quarters ahead, and a positive impact of investment TFP-volatility on investment, that persists 40 quarters ahead as well, for all three model calibrations. This pattern aligns with the empirical findings. Though the magnitude and shape of the graphs may change somewhat between the specifications, the plots demonstrate that neither a volatility feedback, nor time-varying markups, are crucial to explain the impact of volatilities on investment. As discussed in section 2.5.1, it fundamentally stems from precautionary saving motive induced by investment TFP-volatility, and from higher effective impatience induced by consumption TFP-volatility.

Panel (a) of Figure 2.4 shows that in the benchmark case, consumption TFP-volatility lowers consumption, contemporaneously and predictively, in-line with the data. The neg-

[40] Case 3 is almost identical quantitatively to the case in which there is monopolistic competition but no sticky prices, that is, constant markups. To save space, I do not report the results of the constant-markups case.

ative impact on consumption is a result of higher markups, which rise when consumption TFP-volatility rises. As discussed in Section 2.5.3, higher markups lower the demand of consumption good producers for labor at any given wage, causing them to hire less, and the production of the consumption sector falls. By contrast, and consistently with the data, Panel (d) shows that investment TFP-volatility generates in the benchmark case mostly a positive impact on consumption (a large overshoot), a few periods after the shock.[41] This is a consequence of prolonged capital build-up, which occurs upon the impact of this volatility shock. The build-up of capital translates into higher consumption in the future. Panel (a) shows that under perfect competition, consumption TFP-volatility shock increases consumption upon impact, and the response remains positive 20 quarters ahead. This feature is counterfactual to the empirical evidence. As explained in Section 2.5.3, in a perfect competition model consumption and aggregate investment diverge in response to volatility shocks. The layer of sticky prices, featured in the benchmark model, allows to flip the sign of consumption's response to consumption TFP-volatility, making it negative, consistently with the data.

Panels (c) and (f) show that output's response is strictly negative to consumption TFP-volatility, and strictly positive to investment TFP-volatility, for all three model configurations. This pattern is consistent with the empirical impulse-responses. Adding sticky prices *amplifies* in absolute value the magnitude of output's impulse-responses. This is a result of the fact that sticky prices cause consumption and investment expenditures, that comprise total output, to comove, instead of offsetting each other.

In figure 2.5, I plot impulse-responses from sectoral TFP-volatility shocks to hours, to detreded real wages, and the relative price of investment. In general, in the benchmark case, the Figure shows that investment TFP-volatility boosts these variables, while consumption TFP-volatility depresses these quantities. These volatility impacts are consistent with the

[41] Panels (a) and (d) also show that the impulse-responses to consumption in a model without volatility feedback, closely track the benchmark plots. Thus, the volatility feedback is not responsible for qualitatively generating these results.

data. In addition, all sub-plots illustrate that the volatility feedback is not qualitatively material for these macro responses. Observing Panels (c) and (f) of Figure 2.5, qualitatively, the price of investment-goods drops with consumption TFP-volatility, due to lower demand for investment goods (as the household is more impatience), and rises with investment TFP-volatility, due to higher demand for these goods (as the household desires to save more). Nominal rigidities amplify the magnitude of sectoral volatilities impulse-responses to investment-price. This feature arises as the price of investment-goods is inversely related in equilibrium to the markup of the consumption sector. This markup rises with consumption TFP-volatility, and drops with investment TFP-volatility.[42] A similar pattern arises for wages.

In all, figures 2.4 and 2.5 illustrate the ability of the benchmark model to rationalize the impact of sectoral TFP-volatilities on macro aggregates. The volatility feedback channel is not a (qualitative) driving force behind the macro results. The volatility feedback only plays a role in rationalizing the behavior of market-prices of investment volatility risk, as discussed in Section 2.6.4. Nominal rigidities can help reverse the shape of consumption's responses to volatility shocks, and quantitatively amplify other responses.

In figure 2.6, I plot the impulse-responses of sectoral first-moment TFP innovations to detrended consumption, investment expenditures and output. Panels (a)-(c) show that consumption TFP impact on these quantities is positive, at the time that the shock hits, but revert to zero shortly afterwards (or immediately afterwards, in the case of perfect-competition). Consumption TFP raises consumption by definition, and raises investment expenditures to the extent that it rescales positively the relative price of investment. Yet, all responses are short-lived. By contrast, investment TFP impact is very persistent on all three variables.[43] Panels (d)-(f) show that investment TFP raises investment, as the

[42] Differently put, with sticky prices the supply of investment-goods increases in response to consumption TFP-volatility, while the demand for these goods drops by higher impatience. The increased supply amplifies the depreciation in investment price, compared to the perfect-competition case.

[43] As highlighted in Section 2.5.1, the persistent (long-run) nature of investment TFP innovations implies that when their volatility rises, it induces a strong desire to hedge against low consumption states (precautionary savings).

investment sector becomes more productive, and in absence of labor frictions, labor flows into the investment sector. As a result, investment TFP drops consumption, as resources are allocated to the investment sector.[44] Output's response to an investment TFP innovation is mixed: positive contemporaneously, but negative predictively, as is also the case in the data.

Next, I examine the role of IES in the model. In figure 2.7, I plot the impulse-responses of sectoral TFP-volatility shocks to consumption, aggregate investment and output, for two cases: (1) the benchmark calibration (IES = 1.7); (2) A calibration that is identical to the benchmark case, but in which there is no monopolistic competition or volatility feedback, and in which IES is calibrated to 0.8. When the IES is less than one, the impact of either consumption TFP-volatility or investment TFP-volatility on the macro quantities is qualitatively the same. The reason is that when the IES is less than one, higher consumption TFP-volatility acts as a preference shock that increases the household *patience* (see Section 2.5.1). As a result, with more consumption TFP-volatility, the household desires to invest more. Similarly, upon a positive shock to investment TFP-volatility, the household also desires to invest more due to a strong precautionary saving motive. By sharp contrast, allowing the IES to be greater than one allows to obtain a *differential* volatility impact: positive for investment TFP-volatility, and negative for consumption TFP-volatility, consistently with the data.

2.6.4. Sectoral Shocks and Asset-Pricing Implications

In this Section I analyze the impact of sectoral TFP first-moment and second-moment shocks on asset-pricing quantities in the model. I show that the benchmark model is able to rationalize the signs of the market prices of risk, and the signs of cross-sectional exposures to the different sources of risk.

The model-implied log-returns for consumption firms', and investment firms', are defined

[44] In longer horizons, investment TFP generates an overshoot in consumption, as a result of a build-up in the amount of capital.

as:

$$r_{c,t+1} = \log\left(\frac{V_{c,t+1}}{V_{c,t} - d_{c,t}}\right); \quad r_{i,t+1} = \log\left(\frac{V_{i,t+1}}{V_{i,t} - d_{i,t}}\right), \tag{2.41}$$

where $V_{j,t}$ is the cum-dividend real market firm values, defined in equation (2.18), for $j \in \{c, i\}$. At each time t, the aggregate market value is the sum of the market values for consumption and investment firms, $V_{m,t} = V_{c,t} + V_{i,t}$. The market log-return is given by:

$$r_{m,t+1} = \log\left(\frac{V_{m,t+1}}{V_{m,t} - d_{c,t} - d_{i,t}}\right). \tag{2.42}$$

In Table 2.17, I show the mean, standard deviation, and auto-correlation moments of annualized equity premium and real risk free rate, along with their empirical counterparts.[45] For the most part, the data moments fall inside the model-implied 90%-confidence intervals. In the model, the annualized (levered) equity premium is 6.6%, while it is 6.20% in the data for the period of 1947-2014.[46] The volatility of the equity premium is smaller compared to the data. This is an artifact of the relatively high value of risk aversion, coupled with the absense of investment efficiency shocks in the model.[47] The real risk free rate in the model is 1.37%, while the data counterpart is slightly below 1%. The volatility and autocorrelation of the risk-free rate closely match their empirical counterparts.

Allowing for market-prices and betas to (potentially) time-vary, and using a log-linear approximation for the log-SDF and log-returns, the innovation to the real log-SDF ($m_{t,t+1}$),

[45] Following Papanikolaou (2011), I multiply the model-implied market excess return by a factor of 5/3, to account for the fact the firms in the model are unlevered.

[46] A significant contribution to the equity premium stems from the volatility risks-premia, and in particular investment TFP-volatility risk-premium. This is a result of the fact that the volatilities are persistent processes, and the preferences are Epstein and Zin (1989). This resembles long-run volatility risk-premia in a Long-Run Risks model (see Bansal and Yaron, 2004).

[47] As demonstrated in Papanikolaou (2011), a model that does not include shocks to the efficiency of capital goods, in addition to investment TFP shocks, tends to generate too little quantity of risk in asset returns. I refrain from including such efficiency shocks in my model, in order to keep the number of shocks in the model the same as in the empirical section. This fasilitates a comparison between the model-implied signs of betas and market-prices of each shock against the data.

and real log-return of asset $k \in \{c, i, m\}$, $(r_{k,t+1})$, are given by:

$$
\begin{aligned}
m_{t,t+1} - E_t m_{t,t+1} = & -\lambda_{zc,t}\sigma_{zc,t}\varepsilon_{c,t+1} - \lambda_{zi,t}\sigma_{zi,t}\varepsilon_{i,t+1} - \lambda_{\sigma,zc,t}\sigma_{w,c}\varepsilon_{\sigma,c,t+1} \\
& - \lambda_{\sigma,zi,t}\sigma_{w,i}\varepsilon_{\sigma,i,t+1}; \qquad (2.43) \\
r_{k,t+1} - E_t r_{k,t+1} = & \beta_{k,zc,t}\sigma_{zc,t}\varepsilon_{c,t+1} + \beta_{k,zi,t}\sigma_{zi,t}\varepsilon_{i,t+1} + \beta_{k,\sigma,zc,t}\sigma_{w,c}\varepsilon_{\sigma,c,t+1} \\
& + \beta_{k,\sigma,zi,t}\sigma_{w,i}\varepsilon_{\sigma,i,t+1}, \qquad (2.44)
\end{aligned}
$$

where $\lambda_{\mathbf{t}} = [\lambda_{zc,t}, \lambda_{zi,t}, \lambda_{\sigma,zc,t}, \lambda_{\sigma,zi,t}]'$ is the vector of market-prices of risk, and $\beta_{\mathbf{k,t}} = [\beta_{k,zc,t}, \beta_{k,zi,t}, \beta_{k,\sigma,zc,t}, \beta_{k,\sigma,zi,t}]'$ is the vector of risk-exposures of asset k, to consumption TFP, investment TFP, consumption TFP-volatility and investment TFP-volatility risks, respectively.

Consider a projection of long-sample simulated paths of log-SDF and log-returns, on long-sample paths of simulated shocks in the model:

$$
m_{t,t+1} = m_0 + \tilde{\lambda}_{zc}\varepsilon_{c,t+1} + \tilde{\lambda}_{zi}\varepsilon_{i,t+1} + \tilde{\lambda}_{\sigma,zc}\varepsilon_{\sigma,c,t+1} + \tilde{\lambda}_{\sigma,zi}\varepsilon_{\sigma,i,t+1} + error; \qquad (2.45)
$$

$$
r_{k,t+1} = r_{k,0} + \tilde{\beta}_{k,zc}\varepsilon_{c,t+1} + \tilde{\beta}_{k,zi}\varepsilon_{i,t+1} + \tilde{\beta}_{k,\sigma,zc}\varepsilon_{\sigma,c,t+1} + \tilde{\beta}_{k,\sigma,zi}\varepsilon_{\sigma,i,t+1} + error. \qquad (2.46)
$$

From identities (2.43) and (2.44), I define the model-implied *average* market-prices of risk, as the negative of the factor loadings of projection (2.45), dividend by the *average* quantity of risks that corresponds to each shock, as in the data. Similarly, I define the *average* exposures of asset k, as the factor loadings of projection (2.46), dividend by the *average* quantity of risks that corresponds to each shock, as in the data:

$$
\lambda = \left[-\frac{1}{\sigma_{zc,0}}\tilde{\lambda}_{zc},\ -\frac{1}{\sigma_{zi,0}}\tilde{\lambda}_{zi},\ -\frac{1}{\sigma_{w,c}}\tilde{\lambda}_{\sigma,zc},\ -\frac{1}{\sigma_{w,i}}\tilde{\lambda}_{\sigma,zi}\right]', \qquad (2.47)
$$

$$
\beta_k = \left[\frac{1}{\sigma_{zc,0}}\tilde{\beta}_{k,zc},\ \frac{1}{\sigma_{zi,0}}\tilde{\beta}_{k,zi},\ \frac{1}{\sigma_{w,c}}\tilde{\beta}_{k,\sigma,zc},\ \frac{1}{\sigma_{w,i}}\tilde{\beta}_{k,\sigma,zi}\right]'. \qquad (2.48)
$$

I simulate population paths of the log-SDF and log-returns, and project them onto the shocks paths', to obtain the market-prices of risk and exposures, as defined in (2.45) and (2.46). Importantly, in both projections, the R^2 is close to 99%. This indicates that the model-implied log-SDF and log-returns are almost *log-linear*, as specified in identities (2.43) and (2.44). Thus, I ignore any higher-order, non-linear SDF specifications. The model-implied market-prices and exposues are reported in Table 2.18. The Table shows the results for two model calibrations: (1) The benchmark case, in Panel A; (2) An identical calibration to the benchmark case, but without a volatility feedback ($\tau = 0$), and under perfect competition ($\mu_j \to \infty, \quad j \in \{c, i\}$), in Panel B.

2.6.4.1. Risk Exposures Implications

Panel A of Table 2.18 shows the risk exposures (betas) in the benchmark model. The risk exposures of the market, of consumption firms, and of investment firms to the sectoral shocks, are all consistent with the empirical findings. Namely, all assets have a positive exposure to consumption TFP, and investment TFP-volatility, and a negative exposure to investment TFP, and consumption TFP-volatility. For volatility risks, the exposures are also of roughly similar magnitude as their empirical counterparts, as can be seen in Table 2.5. Panel B of Table 2.18 shows the risk exposures in a simplified framework, in which firms are perfectly competitive. The signs of the risk exposures are unaltered.

The intuition behind the *signs* of the volatility exposures is explained in Section 2.5.2. For completeness, I briefly repeat it here. Since firms in the model exhibit constant returns to scale, the sign of an exposure is determined primarily by the impact of the volatility shock on the firm's Tobin's Q. In the model, Tobin's Q of firms, and the aggregate price of investment goods are positively related (they are identical in the absence of adjustment costs). Consumption TFP-volatility causes the household to be more impatient. This lowers the demand for investment goods, causing their price to drop, and consequently, depreciates the value of installed capital of firms. A reduction in the firms' value implies a negative exposure to consumption TFP-volatility ($\beta_{j,\text{C-TFP-VOL}} < 0, \quad j \in \{c, i, m\}$). By contrast, investment TFP-volatility raises the incentive of the household to save. In turn, it increases the demand for investment goods, appreciates the value of the price of capital, and raises firms' value. Thus, firms are positively exposed to investment TFP-volatility ($\beta_{j,\text{I-TFP-VOL}} > 0, \quad j \in \{c, i, m\}$).

The signs of exposures to first-moment TFP innovations are also rationalized through their impact on the relative price of investment. The relative price of investment drops with higher investment TFP, as a positive investment TFP innovation increases the supply of investment goods, and drops their price. Alternatively, a positive investment TFP innovation implies that it is cheaper to produce and replace assets-in-place and so their marginal value falls. A decline in the price of capital implies a negative impact on firms' valuations, and a negative exposure to investment TFP ($\beta_{j,\text{I-TFP}} < 0, \quad j \in \{c, i, m\}$). A positive consumption TFP innovation increases the productivity of consumption firms, causing an increase in the demand for new capital goods, and increases their price. As a result, the marginal value of firms' installed capital appreciates, suggesting a positive exposure to consumption TFP ($\beta_{j,\text{C-TFP}} > 0, \quad j \in \{c, i, m\}$). Panel B shows that neither monopolistic competition, nor a volatility feedback, are necessary to rationalize the signs of the empirical betas.

2.6.4.2. Market-Prices of Risk Implications

Panel A of Table 2.18 shows that the benchmark model is capable of explaining the signs of the empirical market prices of risk: positive market-price for consumption TFP, investment TFP, and investment TFP-volatility, and a negative market-price for consumption TFP-volatility. The magnitudes of the market prices are of roughly similar magnitude as their empirical counterparts reported in Table 2.4.

The real SDF in the economy is given by:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi} \left(\frac{1-\xi N_{t+1}^{\eta}}{1-\xi N_t^{\eta}}\right)^{1-1/\psi} \left(\frac{U_{t+1}}{\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1/\psi-\gamma}. \tag{2.49}$$

Expression (2.49) shows that under early resolution of uncertainty ($\gamma > \frac{1}{\psi}, \psi > 1$), the SDF, $M_{t-1,t}$, falls under three scenarios: (i) Consumption C_t rises; (ii) The continuation utility (which includes today's consumption as well) U_t rises; (iii) Labor N_t rises. Quantitatively, channels (i) and (ii) dominate fluctuations in channel (iii). Consequently, I analyze below the impact of sectoral shocks on the SDF through their immediate impact on consumption, and their impact on the continuation utility.

Upon a positive consumption TFP innovation, consumption increases by definition. The continuation utility also rises due to the positive impact on today's consumption. Both channels operate to drop the SDF, and thus, yield a positive market price for consumption TFP innovations in the benchmark model, consistently with the data.

When investment TFP rises, in absence of labor frictions, labor flows to the investment sector, as it is becomes more productive. As a result, the immediate impact on consumption is negative. If preferences excluded the impact of the continuation utility (i.e., power utility), this would imply a negative market price for investment TFP innovations. However, since labor and capital are shifted to the investment sector, the economy builds-up more capital goods. This is translated into a large consumption overshoot in the future, and to an increase in the continuation utility. In addition, a positive investment TFP innovation triggers more working hours.[48] The rise in the continuation utility (along with the rise in total working hours) is sufficiently strong to compensate for the immediate decline in consumption. Consequently, investment TFP innovations drop the SDF, and are priced positively in the benchmark model. This is in-line with the results of the empirical analysis. Panel B of Table 2.18 shows the market-prices of risk in the simplified model, in which firms are perfectly competitive, and that excludes the volatility feedback. The signs of the market-prices of consumption TFP and investment TFP are still positive.

The market-price of risk of consumption TFP-volatility is negative, both in the benchmark

[48]Under King et al. (1988) preferences, total hours moves in an opposite direction to consumption-sector's hours. Since investment TFP increases labor in the investment sector, total hours worked also rises.

model (Panel A), and in a perfect-competition model (Panel B). A negative market-price is consistent with the data. When consumption TFP-volatility rises, in the case of monopolistic competition and nominal rigidities, consumption drops both contemporaneously and predictively (see explanation in Sections 2.5.3 and 2.6.3). In addition, future consumption profile becomes more volatile. Under early resolution of uncertainty, the agent dislikes a rise in consumption's volatility, and the continuation utility drops. Both effects generate an increase in the SDF, and yield a negative market price for consumption TFP-volatility.[49]

By contrast, investment TFP-volatility has two opposite impacts on the SDF: (a) Higher investment TFP-volatility drops immediate consumption, and generates a more volatile consumption profile in the future (investment TFP-volatility shocks are *capital-embodied* shocks, that affect the volatility of capital allocations in the consumption sector). This lowers the continuation utility; (b) Higher investment TFP-volatility increases future consumption, due to capital build-up in the present (see Panel D of Figure 2.4). Higher future consumption can operate to raise the continuation utility. Under a reasonable calibration for aggregate macroeconomic moments, and in the absence of a volatility feedback, I find that channel (a) dominates. As a result, the market-price of risk of investment TFP-volatility in Panel B is counterfactually negative.

Once the empirically-borne volatility feedback is added to the model ($\tau > 0$), Panel A of Table 2.18 shows that in the benchmark model the market-price of investment TFP-volatility turns positive. This is in-line with the empirical findings, and implies that investment TFP-volatility is welfare improving. Intuitively, a positive feedback from investment TFP-volatility to future consumption TFP, strengthens quantitatively channel (b) above. When channel (b) dominates, investment TFP-volatility is positively priced.

Economically, in the benchmark model investment TFP-volatility has a prolonged multi-

[49]Without sticky prices, consumption TFP-volatility raises consumption today and in the near future. Under CRRA preferences this implies a counterfactual positive market-price of risk. Under Epstein and Zin (1989) preferences, the market-price is negative through the impact of higher uncertainty on the continuation utility. Yet, it is not as negative as in the benchmark case (with sticky prices).

period effect on the economy, through capital build-up. The capital build-up consequently leads to an overshoot in future consumption, and to improved welfare in the economy. This capital build-up happens because of two reasons. The first is that when investment TFP-volatility rises, it induces precautionary savings. The second reason for capital build-up is the volatility feedback. In a reduced form manner, this feedback could be interpreted as slightly delayed culmination of successful growth options (see discussion in Section 2.3.6).

2.6.5. Monte-Carlo Experiment: Using the Model to Rule-Out Mechanical Empirical Results

A general concern regarding the empirical results presented in Section 2.3 can be that the results are mechanically driven by the methodology in which the volatilities are constructed, as discussed in Section 2.3.2. Specifically, if the conditional mean of the TFP growth rates is not fully removed from the time-series, then the constructed realized variances are contaminated by the impact of first-moment shocks.

To try to alleviate such a concern, I solve the model presented in section 2.4, yet with two modifications: (1) No volatility feedback from investment TFP-volatility to future consumption TFP growth; (2) No stochastic-volatility: the conditional volatilities of sectoral TFP growth rates are constant, set at their unconditional values.

I simulate the economy, and construct from the simulated data first- and second- moment sectoral TFP shocks, in an exact fashion to the empirical construction. I then repeat the various data projections, as outlined in sections 2.3.3 - 2.3.4. I perform the projections using a a small sample of 272 quarters (same length as data observations), and in a population sample (half-million observations). The results for the volatilities' loadings are reported in Table 2.19. Under the Null conjecture of this model, one should not find a positive (negative) feedback from investment (consumption) TFP-volatility to future growth.

The Table shows that in finite-samples, the sectoral volatility loadings are indeed insignificant for the macroeconomic projections. In the population sample, the Table shows that in almost all cases, the slope coefficients on consumption TFP-volatility are positive, while the slope coefficients on investment TFP-volatility are negative. This is the *opposite* of what I find in the data. Moreover, for the market portfolio, the betas for both consumption TFP-volatility and investment TFP-volatility are negative, while in the data, investment TFP-volatility exposure is positive.

2.7. Conclusion

In this paper I empirically document a novel empirical puzzle: consumption-sector's technological volatility and investment-sector's technological volatility oppositely impact economic growth, aggregate asset-prices, and the cross-section of returns. I further develop a general-equilibrium two-sector model, that explains the opposite roles of the sectoral volatilities, and also studies the implications of sectoral first-moment technological innovations.

On the macroeconomic front, the paper sheds new light on the on-going debate regarding the impact of volatility shocks on investment. I find that consumption TFP-volatility inhibits investment, consumption, output and wages. Investment TFP-volatility, on the other hand, stimulates investment and output. It also raises welfare inside the model. Thus, economic policies that are designed to curb uncertainty, may not yield a desired result if the volatility stems from the investment sector. The positive impact of investment TFP-volatility on investment is explained via precautionary-saving channel in equilibrium. The contractionary impact of consumption TFP-volatility on investment hinges on the preferences of the agent. Under *early* resolution of uncertainty, the agent hedges against consumption sector's volatility by shifting her consumption profile to the present, and investing less. In fact, higher consumption TFP-volatility is equivalent to a demand shock (or a time-preference shock), that makes the agent more impatient, thus discouraging investment.

On the asset-pricing front, I find that a production SDF that excludes the sectoral volatilities is misspecified. The misspecification is important, as first-moment sectoral innovations are not able to fully explain certain return spreads (e.g. momentum), while TFP-volatility shocks improve the factor-model's fit to the data. The sectoral volatility risks have market-prices of risk of opposite signs. Moreover, sectoral volatilities have an opposite impact on stock prices. Investment TFP-volatility increases equity valuations, empirically and in the model, as it increases the demand for capital-goods, appreciating the marginal value of installed capital. Consumption TFP-volatility lowers equity valuations, for the opposite reason. From a corporate finance perspective, I document that investment-sector's TFP-volatility lowers the default spread, while consumption TFP-volatility raises it. This differential impact can affect firms' incentive to take leverage oppositely.

In all, the theoretical and empirical evidence show the importance of separate movements in sectoral TFP-volatilities for economic growth and asset prices, beyond first-moment innovations. Future research can explicitly model debt in a two-sector model, to explore the sectoral volatility implications for leverage taking and defaults in equilibrium. Another research direction, which I currently explore, is to endogenize the heterogeneity of risk exposures to investment TFP-volatility in the cross-section, in relation to the momentum spread.

Table 2.1: Sectoral Shocks and Aggregate Cash-Flow (Macroeconomic) Growth

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	$Adj-R^2$
Consumption growth:					
0Q Ahead	0.27 [2.96]	0.08 [0.96]	-0.01 [-1.14]	0.02 [1.45]	0.085
1Q Ahead	0.27 [3.29]	0.05 [0.62]	-0.04 [-3.78]	0.06 [4.68]	0.067
4Q Ahead	0.28 [3.35]	-0.14 [-2.10]	-0.02 [-2.72]	0.03 [2.49]	0.089
12Q Ahead	0.21 [2.79]	-0.14 [-2.72]	-0.02 [-2.23]	0.02 [2.06]	0.086
20Q Ahead	0.17 [1.77]	-0.11 [-1.64]	-0.01 [-1.71]	0.02 [1.67]	0.086
GDP growth:					
0Q Ahead	0.72 [7.66]	0.31 [2.67]	-0.01 [-0.48]	0.02 [1.09]	0.485
1Q Ahead	0.50 [4.05]	0.16 [1.49]	-0.09 [-4.09]	0.12 [4.45]	0.148
4Q Ahead	0.48 [2.88]	-0.21 [-1.62]	-0.04 [-2.21]	0.04 [2.06]	0.109
12Q Ahead	0.27 [2.32]	-0.18 [-2.17]	-0.02 [-1.99]	0.02 [1.74]	0.071
20Q Ahead	0.18 [1.53]	-0.14 [-1.62]	-0.01 [-1.36]	0.01 [1.22]	0.057
Sales growth:					
0Q Ahead	-0.45 [-0.46]	1.88 [2.57]	-0.13 [-1.47]	0.11 [1.36]	0.004
1Q Ahead	0.27 [0.40]	0.66 [1.08]	-0.14 [-1.63]	0.15 [1.37]	0.014
4Q Ahead	1.21 [2.76]	-0.21 [-0.59]	-0.14 [-3.29]	0.17 [3.32]	0.110
12Q Ahead	0.95 [2.37]	-0.37 [-1.43]	-0.09 [-2.23]	0.10 [2.28]	0.116
20Q Ahead	0.80 [1.97]	-0.44 [-1.66]	-0.06 [-1.67]	0.07 [1.64]	0.115
Net earnings growth:					
0Q Ahead	3.58 [1.45]	1.98 [1.22]	-0.42 [-1.72]	0.59 [1.90]	0.061
1Q Ahead	4.02 [1.67]	1.06 [0.85]	-0.58 [-1.93]	0.69 [2.28]	0.062
4Q Ahead	3.19 [1.56]	-0.62 [-0.40]	-0.26 [-1.59]	0.32 [1.66]	0.028
12Q Ahead	1.77 [1.24]	-1.00 [-0.84]	-0.10 [-0.93]	0.12 [0.89]	0.004
20Q Ahead	0.70 [1.11]	-0.45 [-0.73]	-0.03 [-0.66]	0.04 [0.91]	0.000

The Table shows the evidence from the projection of contemporaneous and future aggregate cash-flow growth rates on the current sectoral shocks: consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. The predictive projection ($h > 1$) is: $\frac{1}{h}\sum_{j=1}^{h}\Delta y_{t+j} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error$. The contemporaneous projection ($h = 0$) is the same, but the dependent variable is Δy_t. The Table reports the slope coefficients β_h, $t-$statistics, and the adjusted R^2s for the contemporaneous projection ($h = 0$), and the predictive horizons of $h = 1, 4, 12$ and 20 quarters, for the corresponding aggregate growth series Δy. Standard errors are Newey-West adjusted. The data on consumption and GDP are quarterly from 1947Q1-2014Q4. Data on sales and earnings are from 1964Q1-2014Q4.

Table 2.2: Sectoral Shocks and Aggregate Inputs Growth

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	$Adj-R^2$
Panel A: Aggregate growth of investment measures					
Capital investment growth:					
0Q Ahead	1.23 [4.03]	1.03 [3.70]	-0.12 [-3.17]	0.19 [3.91]	0.362
1Q Ahead	1.10 [3.53]	0.66 [2.50]	-0.15 [-3.21]	0.21 [4.09]	0.223
4Q Ahead	1.09 [2.41]	-0.28 [-0.85]	-0.09 [-1.93]	0.11 [1.93]	0.109
12Q Ahead	0.63 [1.61]	-0.41 [-1.37]	-0.05 [-1.45]	0.05 [1.33]	0.052
20Q Ahead	0.41 [1.46]	-0.36 [-1.48]	-0.03 [-1.25]	0.03 [1.10]	0.055
Capital expenditures growth:					
0Q Ahead	0.10 [0.03]	4.66 [1.88]	-1.49 [-2.37]	1.30 [1.64]	0.087
1Q Ahead	-3.93 [-1.24]	5.05 [1.96]	-1.15 [-2.00]	1.09 [1.52]	0.077
4Q Ahead	0.73 [0.56]	0.78 [0.77]	-0.20 [-1.88]	0.24 [1.84]	0.078
12Q Ahead	0.75 [1.29]	0.17 [0.25]	-0.10 [-2.59]	0.13 [2.80]	0.050
20Q Ahead	0.08 [0.15]	0.20 [0.35]	-0.03 [-0.54]	0.03 [0.63]	-0.012
Relative investment-price growth:					
0Q Ahead	0.78 [4.50]	-0.93 [-5.04]	-0.01 [-1.01]	0.01 [0.71]	0.152
1Q Ahead	0.61 [4.09]	-0.63 [-3.76]	-0.04 [-1.45]	0.04 [1.39]	0.059
4Q Ahead	0.36 [1.97]	-0.43 [-3.24]	-0.02 [-1.00]	0.01 [0.74]	0.111
12Q Ahead	0.34 [1.92]	-0.36 [-2.61]	-0.02 [-1.23]	0.02 [1.02]	0.105
20Q Ahead	0.38 [2.19]	-0.35 [-2.40]	-0.03 [-1.96]	0.03 [1.93]	0.149
Panel B: Aggregate growth of labor measures					
Hours growth:					
0Q Ahead	0.33 [2.57]	0.22 [2.05]	-0.02 [-1.67]	0.04 [1.15]	0.066
1Q Ahead	0.20 [1.64]	0.08 [0.68]	-0.03 [-1.39]	0.02 [1.93]	0.049
4Q Ahead	0.12 [1.06]	-0.16 [-1.75]	-0.00 [-0.31]	-0.00 [-0.19]	0.037
12Q Ahead	0.02 [0.45]	-0.07 [-1.67]	0.00 [0.51]	-0.00 [-1.14]	0.045
20Q Ahead	0.03 [0.54]	-0.09 [-1.50]	0.00 [0.52]	-0.01 [-0.99]	0.031
Wage growth:					
0Q Ahead	0.51 [4.46]	-0.21 [-1.92]	-0.00 [-0.17]	0.01 [0.70]	0.057
1Q Ahead	0.35 [2.90]	-0.10 [-0.91]	-0.05 [-2.62]	0.06 [2.69]	0.020
4Q Ahead	0.31 [3.37]	-0.23 [-2.71]	-0.03 [-2.65]	0.03 [2.69]	0.060
12Q Ahead	0.26 [2.67]	-0.21 [-2.64]	-0.02 [-2.00]	0.02 [1.92]	0.074
20Q Ahead	0.24 [1.93]	-0.16 [-1.58]	-0.02 [-1.92]	0.02 [2.03]	0.078

The Table shows the evidence from the projection of contemporaneous and future aggregate investment growth rate measures (Panel A), and labor growth rate measures (Panel B) on the current sectoral shocks: consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. The predictive projection ($h > 1$) is: $\frac{1}{h}\sum_{j=1}^{h}\Delta y_{t+j} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error$. The contemporaneous projection ($h = 0$) is the same, but the dependent variable is Δy_t. The Table reports the slope coefficients β_h, $t-$statistics, and the adjusted R^2s for the contemporaneous projection ($h = 0$), and the predictive horizons of $h = 1, 4, 12$ and 20 quarters, for the corresponding aggregate growth series Δy. Standard errors are Newey-West adjusted. The data are quarterly from 1947Q1-2014Q4.

Table 2.3: Sectoral Shocks and Detrended Macroeconomic Variables

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	$Adj - R^2$
Detrended consumption:					
0Q Ahead	0.21 [0.33]	1.33 [2.05]	-0.08 [-1.19]	0.14 [1.62]	0.060
1Q Ahead	0.51 [0.70]	1.39 [2.15]	-0.16 [-2.22]	0.22 [1.47]	0.098
4Q Ahead	1.09 [1.26]	0.59 [0.94]	-0.17 [-1.97]	0.21 [1.98]	0.093
12Q Ahead	1.08 [1.13]	-0.25 [-0.37]	-0.11 [-1.39]	0.13 [1.34]	0.034
20Q Ahead	0.53 [0.62]	-0.42 [-0.63]	-0.04 [-0.67]	0.04 [0.56]	-0.001
Detrended GDP:					
0Q Ahead	0.79 [1.38]	1.41 [2.20]	-0.14 [-2.55]	0.22 [3.27]	0.160
1Q Ahead	1.14 [2.03]	1.38 [2.51]	-0.23 [-3.52]	0.32 [4.06]	0.214
4Q Ahead	1.56 [2.30]	0.54 [1.10]	-0.20 [-2.81]	0.26 [2.94]	0.177
12Q Ahead	0.98 [1.41]	-0.27 [-0.53]	-0.09 [-1.52]	0.10 [1.44]	0.038
20Q Ahead	0.22 [0.43]	-0.25 [-0.63]	-0.01 [-0.28]	0.00 [0.11]	-0.009
Detrended capital investment:					
0Q Ahead	0.33 [0.13]	4.12 [1.80]	-0.29 [-1.35]	0.48 [1.89]	0.063
1Q Ahead	1.92 [0.77]	4.03 [1.84]	-0.53 [-2.26]	0.76 [2.70]	0.111
4Q Ahead	4.21 [1.54]	1.65 [0.82]	-0.62 [-2.35]	0.80 [2.55]	0.113
12Q Ahead	4.37 [1.64]	-1.89 [-0.95]	-0.41 [-1.88]	0.47 [1.84]	0.051
20Q Ahead	2.04 [1.37]	-2.17 [-1.54]	-0.10 [-1.03]	0.08 [0.78]	0.036
Detrended relative-price of investment:					
0Q Ahead	1.25 [2.00]	-1.11 [-2.36]	-0.09 [-1.50]	0.08 [1.28]	0.112
1Q Ahead	1.11 [1.69]	-1.15 [-2.12]	-0.05 [-0.92]	0.04 [0.75]	0.102
4Q Ahead	0.98 [1.54]	-1.05 [-1.89]	-0.04 [-0.79]	0.03 [0.64]	0.087
12Q Ahead	0.59 [1.08]	-0.79 [-1.51]	-0.00 [-0.04]	0.01 [0.24]	0.057
20Q Ahead	-0.37 [-1.19]	0.10 [0.31]	0.05 [2.09]	-0.06 [-2.14]	0.023
Detrended hours:					
0Q Ahead	0.24 [1.11]	0.41 [1.99]	-0.04 [-2.01]	0.07 [2.90]	0.132
1Q Ahead	0.27 [1.33]	0.43 [2.50]	-0.05 [-2.39]	0.07 [2.64]	0.182
4Q Ahead	0.31 [1.67]	0.30 [1.96]	-0.05 [-2.60]	0.06 [2.81]	0.174
12Q Ahead	0.24 [1.15]	0.11 [0.74]	-0.03 [-1.21]	0.03 [1.20]	0.082
20Q Ahead	0.04 [0.22]	0.03 [0.21]	0.00 [0.01]	-0.00 [-0.06]	-0.003

The Table shows the results from the projection of contemporaneous and future business-cycle component of selected macroeconomic variables, averaged over h periods, on the current sectoral shocks: consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. The predictive projection ($h > 1$) is: $\frac{1}{h}\sum_{j=1}^{h} y_{t+j}^{\text{cycle}} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error$. The contemporaneous projection ($h = 0$) is the same, but the dependent variable is $\Delta y_t^{\text{cycle}}$. The cyclical component y^{cycle} of a variable y is obtained from one-sided HP-filtering the trending level-series of y with a smoothing parameter of 1600. The Table reports the slope coefficients β_h, $t-$statistics, and the adjusted R^2s for the contemporaneous projection ($h = 0$), and the predictive horizons of $h = 1, 4, 12$ and 20 quarters, for the corresponding business-cycle variable y^{cycle}. Standard errors are Newey-West adjusted. The data are quarterly from 1947Q1-2014Q4.

Table 2.4: Sectoral Shocks and the Cross-section of Returns

Panel A: Market-prices of risk (Λ)				
	$\lambda_{\text{C-TFP}}$	$\lambda_{\text{I-TFP}}$	$\lambda_{\text{C-TFP-VOL}}$	$\lambda_{\text{I-TFP-VOL}}$
λ	2.39	1.36	-0.43	0.70
	[2.30]	[1.37]	[-3.49]	[4.75]

Panel B: Exposures to risks (β)				
	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$
Market	2.80	-0.94	-0.06	0.08
bm1	3.04	-0.78	-0.01	0.03
bm2	2.48	-1.06	-0.04	0.04
bm3	2.35	-1.23	-0.04	0.05
bm4	2.43	-0.85	-0.13	0.16
bm5	2.60	-1.17	-0.04	0.03
bm6	2.88	-0.90	-0.13	0.16
bm7	2.42	-0.75	-0.07	0.09
bm8	3.27	-0.58	-0.19	0.25
bm9	2.76	-0.53	-0.07	0.10
bm10	3.13	0.08	-0.07	0.09
mom1	5.41	-2.44	-0.06	0.04
mom2	4.07	-1.58	0.01	-0.01
mom3	3.19	-1.39	-0.12	0.12
mom4	2.73	-0.77	-0.03	0.05
mom5	2.78	-0.97	-0.11	0.16
mom6	2.51	-0.95	-0.09	0.11
mom7	2.25	-0.94	-0.14	0.18
mom8	2.43	-0.83	-0.07	0.09
mom9	2.71	-0.76	-0.07	0.11
mom10	3.15	-0.64	-0.04	0.16
size1	3.58	-0.31	0.08	-0.07
size2	3.37	-0.83	0.08	-0.08
size3	3.05	-0.70	0.07	-0.06
size4	3.39	-1.06	0.01	0.00
size5	3.06	-1.01	0.05	-0.06
size6	2.96	-1.10	0.01	0.00
size7	2.87	-0.97	-0.04	0.04
size8	2.52	-0.85	-0.01	0.02
size9	2.42	-0.75	-0.08	0.09
size10	2.78	-0.90	-0.08	0.11

The Table shows the estimates of the market-prices of risks (Panel A) and the exposures (Panel B) to consumption TFP, C-TFP, investment TFP, I-TFP, consumption TFP-volatility, C-TFP-VOL, and investment TFP-volatility, I-TFP-VOL, risks for the cross-section of equity returns. The cross-section includes the market, ten portfolios sorted on book-to-market (bm), ten portfolios sorted on momentum (mom), and ten portfolios sorted on size (size). The reported market prices of risks are divided by 10. *T*-statistics are in brackets, and are based on Newey-West standard errors from GMM estimation. For brevity, the significance of exposures is omitted. The data are quarterly from 1947Q1-2014Q4.

Table 2.5: Sectoral (Industry) Exposures to Sectoral Shocks

Sector	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$
All	2.80 [9.72]	-0.94 [-3.62]	-0.06 [-1.69]	0.08 [1.86]
Services	2.17 [4.24]	-0.98 [-2.18]	-0.13 [-2.95]	0.17 [3.34]
Nondurables	2.33 [5.58]	-1.18 [-3.56]	-0.05 [-0.75]	0.05 [1.67]
Durables	4.56 [6.42]	-1.50 [-3.05]	-0.25 [-2.49]	0.28 [2.13]
Investment	4.07 [8.85]	-1.01 [-2.13]	-0.02 [-0.32]	0.04 [1.35]

The Table shows the exposures of sectoral (industry) portfolios, to consumption TFP, C-TFP, investment TFP, I-TFP, consumption TFP-volatility, C-TFP-VOL, and investment TFP-volatility, I-TFP-VOL risks. Each portfolio is comprised of value-weighted returns from CRSP. Sorting firms into industry portfolios is made each June, based on Gomes et al. (2009) SIC classifications for sectors. T-statistics are in brackets, and are Newey-West adjusted. The data are quarterly from 1947Q1-2014Q4.

Table 2.6: Summary of Pricing Statistics from a Four-Factor Model

Panel A: Adjusted R^2 of Fama-Macbeth second-stage projection						
Adj-R^2		0.73				

Panel B: Cross-sectional spreads						
Spread	Data	Model	$Spr_{\text{C-TFP}}$	$Spr_{\text{I-TFP}}$	$Spr_{\text{C-TFP-VOL}}$	$Spr_{\text{I-TFP-VOL}}$
MOM	-2.65	-0.83	1.15	-0.95	1.39	-2.43
BM	-1.00	-0.69	-0.08	-0.32	1.31	-1.60
SIZE	0.46	0.27	0.50	0.22	5.25	-5.70
Q	0.98	1.28	0.12	0.35	-0.70	1.52
OP	-0.79	-1.03	0.27	-0.20	-0.20	-0.90
RVAR	1.55	0.88	-2.84	1.18	-8.18	10.72

Panel C: Market risk premium decomposition						
	Data	Model	$Prm_{\text{C-TFP}}$	$Prm_{\text{I-TFP}}$	$Prm_{\text{C-TFP-VOL}}$	$Prm_{\text{I-TFP-VOL}}$
Market Premium	1.64	1.63	1.88	-0.67	-1.91	2.34

The Table shows summary asset-pricing results of a four-factor model: consumption TFP, C-TFP, investment TFP, I-TFP, consumption TFP-volatility, C-TFP-VOL, and investment TFP-volatility, I-TFP-VOL, risk factors. Panel A reports the adjusted R^2 of the second-stage regression (mean-excess returns projected on risk-exposures) from a Fama-Macbeth procedure, using a cross-section of ten book-to-market sorted portfolios, ten momentum sorted portfolios, ten size sorted portfolios, and the market portfolio. Panel B reports data and model counterpart quarterly spreads of quantile sorted portfolios, along the momentum dimension (MOM), book-to-market dimension (BM), size dimension (SIZE), Tobin's Q dimension (Q), operating profitability dimension (OP), and residual (idiosyncratic) variance of return (RVAR) dimension. The operating profitability is measured via operating profits divided by book equity value. Residual variance refers to the variance of the residuals from the Fama-French three-factor model using 60 days of lagged returns. Each spread is computed by subtracting the return of portfolios 5 (the portfolio of stocks with the highest characteristic), from the return of portfolio 1 (the portfolio of stocks with the lowest characteristic). Panel C reports the market risk-premium in the data versus the model. Panels B and C also show the decomposition of the model-implied spreads (*Spr*), and model-implied risk premia (*Prm*), into the compensations for the four risk factors. The data for OP and RVAR sorted portfolios are from 1964Q1-2014Q4. All other data are quarterly from 1947Q1-2014Q4.

Table 2.7: Summary of Pricing Statistics from a Two-Factor Model

Panel A: Adjusted R^2 of Fama-Macbeth second-stage projection				
Adj-R^2		0.52		

Panel B: Cross-sectional spreads				
Spread	Data	Model	$Spr_{\text{C-TFP}}$	$Spr_{\text{I-TFP}}$
MOM	-2.65	0.50	1.87	-1.37
BM	-1.00	-0.55	0.05	-0.59
SIZE	0.46	1.29	1.36	-0.07
Q	0.98	0.41	0.07	0.34
OP	-0.79	0.51	0.43	0.08
RVOL	1.55	-3.12	-5.26	2.14

Panel C: Market risk premium decomposition				
	Data	Model	$Prm_{\text{C-TFP}}$	$Prm_{\text{I-TFP}}$
Market Premium	1.64	1.39	2.47	-1.08

The Table shows summary asset-pricing results of a two-factor model: consumption TFP, C-TFP, and investment TFP, I-TFP. Panel A reports the adjusted R^2 of the second-stage regression (mean-excess returns projected on risk-exposures) from a Fama-Macbeth procedure, using a cross-section of ten book-to-market sorted portfolios, ten momentum sorted portfolios, ten size sorted portfolios, and the market portfolio. Panel B reports data and model counterpart quarterly spreads of quantile sorted portfolios, along the momentum dimension (MOM), book-to-market dimension (BM), size dimension (SIZE), Tobin's Q dimension (Q), operating profitability dimension (OP), and residual (idiosyncratic) variance of return (RVAR) dimension. The operating profitability is measured via operating profits divided by book equity value. Residual variance refers to the variance of the residuals from the Fama-French three-factor model using 60 days of lagged returns. Each spread is computed by subtracting the return of portfolios 5 (the portfolio of stocks with the highest characteristic), from the return of portfolio 1 (the portfolio of stocks with the lowest characteristic). Panel C reports the market risk-premium in the data versus the model. Panels B and C also show the decomposition of the model-implied spreads (*Spr*), and model-implied risk premia (*Prm*), into the compensations for the four risk factors. The data for OP and RVAR sorted portfolios are from 1964Q1-2014Q4. All other data are quarterly from 1947Q1-2014Q4.

Table 2.8: Sectoral Volatilities and Debt Measures

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	$Adj-R^2$
Panel A: Sectoral volatilities and the default Spread					
0Q Ahead	-3.32 [-1.52]	-1.57 [-0.69]	0.57 [1.89]	-0.60 [-1.71]	0.065
1Q Ahead	-3.81 [-2.15]	-1.06 [-0.62]	0.48 [1.97]	-0.58 [-1.97]	0.017
4Q Ahead	-3.96 [-1.96]	2.58 [1.61]	0.26 [1.23]	-0.28 [-1.12]	0.035
12Q Ahead	-2.71 [-2.00]	2.74 [2.23]	0.13 [1.11]	-0.12 [-0.90]	0.055
20Q Ahead	-0.26 [-0.62]	0.92 [2.20]	-0.02 [-0.57]	0.05 [1.10]	0.048
Panel B: Sectoral volatilities and real debt growth					
0Q Ahead	4.70 [1.39]	-2.70 [-1.19]	-1.13 [-2.26]	1.28 [2.00]	0.011
1Q Ahead	7.61 [2.22]	-4.32 [-1.68]	-0.36 [-0.55]	0.55 [0.71]	0.015
4Q Ahead	3.82 [2.02]	-2.27 [-1.53]	-0.24 [-1.62]	0.28 [1.63]	0.048
12Q Ahead	1.87 [1.58]	-1.63 [-1.50]	-0.10 [-0.97]	0.11 [0.94]	0.020
20Q Ahead	1.54 [1.26]	-1.12 [-1.19]	-0.10 [-1.01]	0.11 [1.01]	0.017

The Table shows the results of projecting contemporaneous and future default-spread growth rates (Panel A) and real-debt growth rates (Panel B) on the current sectoral shocks: consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. The predictive projection ($h > 1$) is: $\frac{1}{h}\sum_{j=1}^{h} \Delta y_{t+j} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error$. The contemporaneous projection ($h = 0$) is the same, but the dependent variable is Δy_t. The default-spread is computed as the difference between the yield of BAA and AAA rated corporate bonds. Total debt for publicly traded firms is computed as debt in current liabilities (*dlcq*) plus long term debt (*dlttq*). The Table reports the slope coefficients β_h, $t-$statistics, and the adjusted R^2s for the contemporaneous projection ($h = 0$), and for the predictive horizons of 1 up to 20 quarters. Standard errors are Newey-West adjusted. Data on the default spread are quarterly from 1947Q1-2014Q4. Data on real debt growth span from 1966Q1-2014Q4.

Table 2.9: Sectoral Volatility Feedback to Future Technological Growth

$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	$Adj-R^2$
1Q Ahead C-TFP:				
0.46 [3.69]	-0.23 [-1.77]	-0.78 [-1.06]	0.82 [2.20]	0.047
1Q Ahead I-TFP:				
-0.10 [-0.67]	0.38 [3.32]	-0.32 [-0.67]	0.42 [0.88]	0.056

The Table shows the volatility feedback evidence from projections of one-quarter ahead sectoral TFP growth rates, on the current sectoral shocks: consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL: $\Delta j - TFP_{t+1} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error, \quad j \in \{C, I\}$. The Table reports the slope coefficients β_h, $t-$statistics, and the adjusted R^2s. Standard errors are Newey-West adjusted. The data are quarterly from 1947Q1-2014Q4.

Table 2.10: Summary Results Based on Total Ex-Ante Volatilities as Factors

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	R^2
Panel A: Macroeconomic growth rate predictability					
Consumption growth	0.32 [0.66]	-0.24 [-0.51]	-21.89 [-1.29]	19.95 [3.22]	0.256
GDP growth	1.00 [1.58]	-0.91 [-1.45]	-49.75 [-2.11]	30.88 [3.43]	0.214
Capital investment growth	3.09 [3.42]	-2.80 [-3.16]	-136.13 [-3.76]	75.92 [4.37]	0.181
Capex growth	4.56 [1.35]	-3.85 [-1.06]	-256.49 [-1.31]	158.59 [0.92]	0.096
Relative price growth	-0.28 [-0.55]	0.30 [0.61]	-12.95 [-0.77]	30.88 [3.56]	0.357
Wage growth	-0.11 [-0.24]	0.15 [0.34]	-7.45 [-0.43]	16.21 [1.64]	0.183
Hours growth	0.27 [1.51]	-0.28 [-1.56]	-12.83 [-1.93]	6.97 [2.14]	0.061
Panel B: Macroeconomic business-cycle predictability					
Detrended consumption	3.58 [1.47]	-2.76 [-1.16]	-218.44 [-2.10]	159.15 [2.02]	0.148
Detrended GDP	5.00 [2.75]	-4.21 [-2.28]	-264.34 [-3.40]	171.11 [2.95]	0.225
Detrended capital investment	24.40 [4.01]	-21.82 [-3.72]	-969.05 [-4.20]	457.18 [3.66]	0.129
Detrended capex	14.10 [0.94]	-9.58 [-0.58]	-933.08 [-1.14]	761.00 [1.13]	0.119
Detrended relative price	-1.88 [-1.77]	1.71 [1.66]	57.65 [1.22]	-13.31 [-0.58]	0.032
Detrended wage	1.39 [1.43]	-1.46 [-1.45]	-57.48 [-1.56]	20.54 [1.08]	0.005
Detrended hours	0.32 [0.59]	-0.24 [-0.42]	-26.29 [-1.38]	24.00 [2.55]	0.115
Panel C: Asset-pricing implications					
Market prices of risk	13.72 [4.72]	-12.50 [-4.23]	-720.05 [-6.65]	435.18 [6.70]	
Market betas	4.60 [3.53]	-2.90 [-2.61]	-88.92 [-1.93]	38.20 [2.51]	

The Table presents the summary of the macroeconomic and asset-pricing implications of sectoral factors, using the (total) ex-ante sectoral TFP volatilities as risk-factors, as opposed to their shocks (first differences) as in the benchmark case. Panel A documents the slope coefficients, $t-$statistics and the R^2 in the projections of 12-quarters ahead macroeconomic growth rates on consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility, C-TFP-VOL, and investment TFP-volatility, I-TFP-VOL. Panel B shows the evidence from projecting 12-quarters ahead average business-cycle component of macroeconomic variables on the sectoral innovations and ex-ante volatilities. Panel C shows the estimates of the market-prices of risks and the market return exposures to the four risk factors, constructed and reported as in Table 2.4. The data are quarterly from 1947Q1-2014Q4.

Table 2.11: Summary Results Based on Sale-Dispersion as Sectoral Volatility Factors

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-DISP}}$	$\beta_{\text{I-DISP}}$	R^2
Panel A: Macroeconomic growth rate predictability					
Consumption growth	0.09 [1.49]	-0.04 [-0.74]	-0.04 [-1.21]	0.01 [1.63]	0.014
GDP growth	0.16 [1.89]	-0.09 [-1.10]	-0.06 [-1.43]	0.01 [1.75]	0.031
Capital investment growth	0.72 [2.05]	-0.44 [-1.43]	-0.08 [-0.76]	0.03 [1.70]	0.063
Capex growth	0.37 [0.96]	0.13 [0.29]	-0.31 [-2.25]	0.14 [2.44]	0.015
Relative price growth	0.33 [2.57]	-0.53 [-3.41]	0.10 [1.17]	0.01 [0.10]	0.122
Wage growth	0.03 [0.34]	-0.02 [-0.29]	0.01 [0.39]	0.01 [1.68]	-0.019
Hours growth	0.03 [0.97]	-0.08 [-2.27]	-0.02 [-1.99]	-0.01 [-0.35]	0.096
Panel B: Macroeconomic business-cycle predictability					
Detrended consumption	0.43 [0.89]	0.08 [0.20]	-0.24 [-1.40]	0.02 [1.33]	0.030
Detrended GDP	0.61 [1.48]	-0.10 [-0.28]	-0.33 [-1.76]	0.02 [1.38]	0.044
Detrended capital investment	3.67 [2.24]	-1.44 [-1.04]	-0.74 [-1.26]	0.17 [1.80]	0.072
Detrended capex	6.51 [2.11]	-0.62 [-0.18]	-0.65 [-0.64]	0.58 [1.85]	0.074
Detrended relative price	0.94 [2.24]	-0.95 [-2.40]	-0.16 [-1.47]	0.06 [1.31]	0.126
Detrended wage	0.03 [0.14]	-0.12 [-0.60]	-0.13 [-1.76]	0.01 [1.37]	0.007
Detrended hours	0.13 [1.01]	-0.06 [-0.52]	-0.06 [-1.28]	0.00 [1.07]	0.005
Panel C: Asset-pricing implications					
Market prices of Risk	0.84 [1.04]	0.38 [0.45]	-0.20 [-10.81]	0.12 [6.70]	
Market betas	3.85 [1.79]	-0.89 [-0.79]	-0.02 [-1.12]	0.06 [1.60]	

The Table presents the summary of the macroeconomic and asset-pricing implications of sectoral factors, using an alternative measure of sectoral volatilities: sales growth dispersion in the consumption sector as a substitute for consumption TFP-volatility, and sales growth dispersion in the investment sector as a substitute for investment TFP-volatility. Panel A documents the slope coefficients, $t-$statistics and the R^2 in the projections of 12-quarters ahead macroeconomic growth rates on consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption sales dispersion, C-DISP, and investment sales dispersion, I-DISP. Panel B shows the evidence from projecting 12-quarters ahead average business-cycle component of macroeconomic variables on the sectoral innovations and sale dispersions. The slope coefficients on I-TFP-VOL and C-TFP-VOL are multiplied by 10. Panel C shows the estimates of the market-prices of risks and the market return exposures to the four risk factors, constructed and reported as in Table 2.4. The data are quarterly from 1964Q1-2014Q4.

Table 2.12: Summary Results Based on Different Predictors of Future Volatility

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	R^2
Panel A: Macroeconomic growth rate predictability					
Consumption growth	0.18 [2.70]	-0.13 [-2.55]	-0.01 [-2.17]	0.01 [1.78]	0.070
GDP growth	0.23 [2.21]	-0.17 [-2.05]	-0.01 [-2.42]	0.01 [1.85]	0.060
Capital investment growth	0.52 [1.51]	-0.38 [-1.28]	-0.02 [-1.60]	0.01 [1.17]	0.043
Capex growth	0.33 [0.60]	0.25 [0.37]	-0.01 [-0.30]	0.00 [0.11]	0.029
Relative price growth	0.35 [2.29]	-0.37 [-2.75]	-0.01 [-2.29]	0.01 [2.44]	0.111
Wage growth	0.23 [2.65]	-0.20 [-2.59]	-0.01 [-1.87]	0.01 [1.80]	0.067
Hours growth	0.03 [0.63]	-0.07 [-1.64]	-0.00 [-1.92]	0.00 [0.69]	0.043
Panel B: Macroeconomic business-cycle predictability					
Detrended consumption	0.71 [0.85]	-0.14 [-0.22]	-0.02 [-0.72]	0.01 [1.64]	0.022
Detrended GDP	0.72 [1.17]	-0.19 [-0.38]	-0.02 [-0.89]	0.02 [0.75]	0.028
Detrended capital investment	3.33 [1.41]	-1.62 [-0.82]	-0.10 [-1.29]	0.09 [1.15]	0.035
Detrended capex	3.50 [1.93]	-0.77 [-0.31]	-0.09 [-0.77]	0.12 [1.37]	0.045
Detrended relative price	0.65 [1.28]	-0.80 [-1.54]	-0.01 [-0.75]	0.01 [1.62]	0.055
Detrended wage	0.00 [0.00]	-0.11 [-0.39]	-0.01 [-0.63]	-0.01 [-0.21]	-0.004
Detrended hours	0.01 [0.08]	0.05 [0.32]	-0.00 [-0.04]	-0.00 [-0.49]	0.002
Panel C: Asset-pricing implications					
Market prices of risk	7.21 [0.70]	6.59 [8.20]	-0.26 [-4.07]	0.21 [3.38]	
Market betas	0.00 [1.10]	0.05 [0.19]	-0.31 [-4.26]	0.08 [2.84]	

The Table presents the summary of the macroeconomic and asset-pricing implications of sectoral shocks, using alternative construction of ex-ante TFP volatilities, in which the set of predictive variables Γ_t includes the benchmark predictors, as well as the risk-free rate and the market price-dividend ratio. Panel A documents the slope coefficients, $t-$statistics and the R^2 in the projections of 12-quarters ahead macroeconomic growth rates on consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. Panel B shows the evidence from projecting 12-quarters ahead average business-cycle component of macroeconomic variables on the sectoral innovations and volatility shocks. Panel C shows the estimates of the market-prices of risks and the market return exposures to the four risk factors, constructed and reported as in Table 2.4. The data are quarterly from 1947Q1-2014Q4.

Table 2.13: Summary Results Based on Different Window in Construction of Realized Variances

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-VOL}}$	$\beta_{\text{I-TFP-VOL}}$	R^2
Panel A: Macroeconomic growth rate predictability					
Consumption growth	0.21 [2.86]	-0.14 [-2.60]	-0.03 [-2.07]	0.03 [1.96]	0.082
GDP Growth	0.27 [2.44]	-0.16 [-1.96]	-0.04 [-2.03]	0.05 [1.89]	0.070
Capital investment growth	0.64 [1.66]	-0.43 [-1.34]	-0.09 [-1.49]	0.10 [1.40]	0.054
Capex growth	0.71 [1.24]	0.21 [0.32]	-0.24 [-2.45]	0.30 [2.38]	0.054
Relative price growth	0.33 [1.87]	-0.36 [-2.42]	-0.02 [-0.85]	0.02 [0.70]	0.096
Wage growth	0.25 [2.60]	-0.19 [-2.32]	-0.03 [-1.86]	0.03 [1.81]	0.063
Hours growth	0.04 [0.84]	-0.06 [-1.56]	-0.00 [-0.33]	0.00 [0.05]	0.023
Panel B: Macroeconomic business-cycle predictability					
Detrended consumption	0.89 [0.93]	-0.14 [-0.20]	-0.15 [-1.91]	0.18 [1.89]	0.025
Detrended GDP	0.84 [1.13]	-0.15 [-0.27]	-0.14 [-1.04]	0.17 [1.01]	0.029
Detrended capital investment	4.44 [1.51]	-2.09 [-0.91]	-0.65 [-1.35]	0.75 [1.30]	0.043
Detrended capex	5.79 [1.88]	0.14 [0.04]	-1.53 [-2.13]	1.95 [2.11]	0.111
Detrended relative price	0.08 [0.20]	-0.32 [-0.74]	0.04 [0.77]	-0.06 [-0.84]	0.016
Detrended wage	0.15 [0.38]	-0.11 [-0.32]	-0.01 [-0.09]	0.00 [0.01]	-0.011
Detrended hours	0.22 [1.07]	0.14 [0.84]	-0.03 [-0.71]	0.04 [0.70]	0.085
Panel C: Asset-pricing implications					
Market prices of risk	4.81 [3.25]	-1.41 [-1.03]	-1.58 [-5.05]	2.12 [5.43]	
Market betas	2.75 [11.12]	-1.11 [-3.24]	-0.04 [-1.51]	0.04 [1.85]	

The Table presents the summary of the macroeconomic and asset-pricing implications of sectoral shocks, using alternative construction of ex-ante volatilities, in which the sectoral TFP realized variances are computed over a window of 12 quarters, as opposed to 8 quarter in the benchmark case. Panel A documents the slope coefficients, $t-$statistics and the R^2 in the projections of 12-quarters ahead macroeconomic growth rates on consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. Panel B shows the evidence from projecting 12-quarters ahead average business-cycle component of macroeconomic variables on the sectoral innovations and volatility shocks. Panel C shows the estimates of the market-prices of risks and the market return exposures to the four risk factors, constructed and reported as in Table 2.4. The data are quarterly from 1947Q1-2014Q4.

Table 2.14: Summary Results Based on Realized Volatilities as Factors

Offset	$\beta_{\text{C-TFP}}$	$\beta_{\text{I-TFP}}$	$\beta_{\text{C-TFP-RV}}$	$\beta_{\text{I-TFP-RV}}$	R^2
Panel A: Macroeconomic growth rate predictability					
Consumption growth	0.12 [2.21]	0.02 [0.87]	-2.80 [-1.89]	1.61 [2.29]	0.117
GDP growth	0.13 [1.43]	0.03 [0.96]	-1.98 [-1.48]	2.64 [2.64]	0.142
Capital investment growth	0.36 [1.39]	0.07 [0.72]	-0.18 [-0.01]	2.84 [0.77]	0.051
Capex growth	-0.32 [-0.72]	0.37 [1.26]	-27.84 [-0.58]	34.75 [4.14]	0.193
Relative price growth	0.06 [0.57]	-0.07 [-1.78]	23.66 [3.80]	0.41 [0.48]	0.366
Wage growth	0.08 [1.24]	-0.00 [-0.11]	-6.99 [-1.24]	1.56 [1.34]	0.196
Hours growth	0.00 [0.08]	-0.02 [-2.07]	-1.65 [-0.76]	1.13 [2.59]	0.095
Panel B: Macroeconomic business-cycle predictability					
Detrended consumption	0.52 [0.90]	0.47 [2.04]	-79.06 [-1.74]	22.89 [2.21]	0.130
Detrended GDP	0.50 [1.06]	0.41 [2.24]	-51.19 [-1.23]	16.31 [1.65]	0.100
Detrended capital investment	2.77 [1.58]	1.04 [1.58]	-66.28 [-0.55]	18.36 [0.69]	0.037
Detrended capex	1.42 [0.65]	2.43 [1.39]	-101.58 [-0.38]	104.61 [2.24]	0.126
Detrended relative price	-0.02 [-0.08]	-0.20 [-1.41]	34.64 [1.22]	-4.94 [-0.96]	0.062
Detrended wage	0.21 [0.59]	0.03 [0.23]	-29.77 [-1.28]	5.81 [1.57]	0.031
Detrended hours	0.03 [0.27]	0.08 [1.48]	-4.65 [-0.46]	1.50 [0.54]	0.005
Panel C: Asset-pricing implications					
Market prices of risk	-0.11 [-0.20]	1.29 [2.34]	-37.44 [-4.12]	304.14 [5.19]	
Market betas	2.25 [5.48]	-0.76 [-2.89]	-12.13 [-1.62]	15.33 [4.28]	

The Table presents the summary of the macroeconomic and asset-pricing implications of sectoral factors, using the realized variances of sectoral TFP growth rates as the volatility risk-factors. Panel A documents the slope coefficients, $t-$statistics and the R^2 in the projections of 12-quarters ahead macroeconomic growth rates on consumption TFP innovation, ΔC-TFP, investment TFP innovation, ΔI-TFP, consumption TFP realized variance, C-TFP-RV, and investment TFP realized variance, I-TFP-RV. Panel B shows the evidence from projecting 12-quarters ahead average business-cycle component of macroeconomic variables on the sectoral innovations and realized variances. Panel C shows the estimates of the market-prices of risks and the market return exposures to the four risk factors, constructed and reported as in Table 2.4. The data are quarterly from 1947Q1-2014Q4.

Table 2.15: Calibration of the Benchmark Model

Symbol	Value	Parameter
γ	25	Relative risk aversion
ψ	1.7	Intertemporal Elasticity of Substitution
β	0.997	Time discount factor
ξ	3	Disutility from labor
η	1.4	Sensitivity of disutility to working hours
$\alpha_c = \alpha_i$	0.33	Share of capital in output
δ	0.015	depreciation rate
μ_{zc}	1.0024	Drift of consumption sector TFP
μ_{zi}	1.0050	Drift of investment sector TFP
$\sigma_{zc,0}$	0.01	Unconditional volatility of consumption TFP shock
$\sigma_{zi,0}$	0.02	Unconditional volatility of investment TFP shock
ρ_σ	0.95	Persistence of volatilities
μ_c	4	Markup of 25% in the consumption sector
μ_i	4	Markup of 25% in the investment sector
ϕ_P	250	Nominal price rigidity (Rotemberg)
π_{ss}	0.005	Steady state inflation
ρ_π	1.5	Weight on inflation gap in Taylor rule
ρ_y	0.5	Weight on output gap in Taylor rule
τ	1.5	Feedback from investment TFP-volatility to future consumption TFP

The Table presents parameter choice of the model parameters in the Benchmark case.

Table 2.16: Model-Implied Macroeconomic Moments against Data Counterparts

	Model (Annualized)			Data (1947-2014)		
	Mean	Std.dev.	Ac(1)	Mean	Std.dev.	Ac(1)
ΔC	1.92 [0.99,2.84]	2.17 [1.70,2.67]	0.54 [0.33,0.70]	1.92	1.52	0.49
ΔY	1.93 [0.98,2.81]	3.01 [2.49,3.52]	0.43 [0.23,0.59]	1.98	2.53	0.18
ΔI	1.88 [0.89,2.99]	6.64 [5.54,7.90]	0.30 [0.10,0.48]	1.67	6.75	0.18
ΔP_I	-0.95 [-2.08,0.24]	3.48 [2.89,4.08]	0.30 [0.07,0.47]	-0.97	3.62	0.45

The Table presents model-implied mean, standard deviation, and auto-correlation for key macroeconomic growth rates, against their empirical counterparts. The macroeconomic growth rates reported include (log-real growth rates of) consumption growth ΔC, output growth ΔY, investment-expenditures growth ΔI, and relative-price of investment growth ΔP_I. The model-implied macroeconomic moments are computed from simulated data. I simulate the model at a quarterly frequency and then time-aggregate the data to annual observations. I report median moments along with the 5% and 95% percentiles, across 10,000 simulations, each with a length of 272 quarters, similarly to the length of the data time-series. The data moments are computed using annual data from 1947-2014.

Table 2.17: Model-Implied Pricing Moments against Data Counterparts

	Model (Annualized)			Data (1947-2014)		
	Mean	Std.dev.	Ac(1)	Mean	Std.dev.	Ac(1)
R_m^e	6.64 [6.16,7.20]	8.01 [7.06,9.03]	-0.00 [-0.19,0.14]	6.20	17.63	-0.03
R_f	1.37 [0.75,2.02]	2.27 [2.02,2.59]	0.79 [0.69,0.87]	0.89	2.24	0.73

The Table presents model-implied mean, standard deviation, and auto-correlation for the real market excess return, R_m^e, and the real risk-free rate, R_f, against their empirical counterparts. In the model, the market excess return is levered-up using a factor of 5/3. The model-implied macroeconomic moments are computed from simulated data. I simulate the model at a quarterly frequency and then time-aggregate the data to annual observations. I report median moments along with the 5% and 95% percentiles, across 10,000 simulations, each with a length of 272 quarters, similarly to the length of the data time-series. The data moments are computed using annual data from 1947-2014.

Table 2.18: Model-Implied Market-Prices of Risk and Risk Exposures

	C-TFP	I-TFP	C-TFP-VOL	I-TFP-VOL
Panel A: Benchmark				
Market prices of risk	2.452	0.974	-0.194	0.611
Market betas	0.595	-0.016	-0.030	0.064
C-Sector betas	0.597	-0.061	-0.029	0.061
I-Sector betas	0.587	-0.001	-0.031	0.074
Panel B: No monopolistic competition and no volatility feedback ($\tau = 0$)				
Market prices of risk	2.499	0.969	-0.140	-0.127
Market betas	1.000	-0.638	-0.012	0.097
C-Sector betas	1.000	-0.697	-0.010	0.088
I-Sector betas	1.000	-0.516	-0.016	0.117

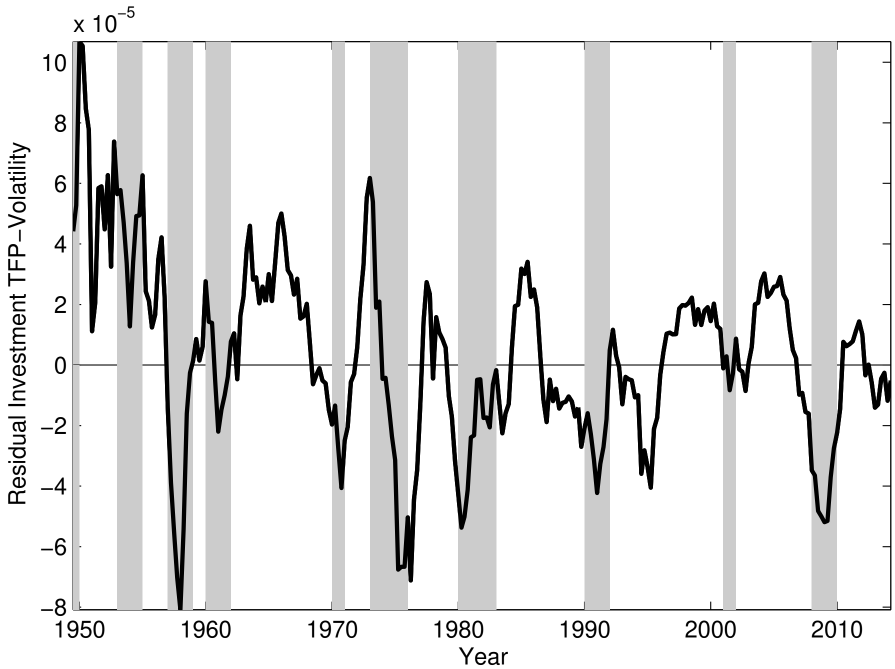
The Table presents model-implied market-prices of risk (λ) and risk exposures (β) to consumption TFP innovation risk (C-TFP shock $\varepsilon_{c,t}$), investment TFP innovation risk (I-TFP shock $\varepsilon_{i,t}$), consumption TFP-volatility risk (C-TFP-VOL shock $\varepsilon_{\sigma,c,t}$) and investment TFP-volatility risk (I-TFP-VOL shock $\varepsilon_{\sigma,i,t}$). The exposures (betas) to the risk factors are reported for consumption firms (V_c), investment firms (V_i), and the market ($V_m = V_c + V_i$). Panel A reports model implied market-prices and betas for the benchmark model. Panel B shows the results for a model with no volatility feedback ($\tau = 0$) and no monopolistic competition. The reported market prices of risks are divided by 10. The construction of market-prices of risk and betas is described in section 2.6.4.

Table 2.19: Simulation Analysis of Sectoral Volatilities in a Model of Constant Volatility

	C-TFP-VOL				I-TFP-VOL				R^2			
Offset	Pop	Med	5%	95%	Pop	Med	5%	95%	Pop	Med	5%	95%
					No Stochastic Volatility and $\tau = 0$:							
Consumption growth:												
0Q	0.64	0.42	[0.17,	0.83]	-0.55	-0.36	[-0.82,	0.01]	0.12	0.17	[0.02,	0.39]
1Q	0.57	0.38	[0.12,	0.76]	-0.47	-0.30	[-0.74,	0.07]	0.19	0.19	[0.04,	0.37]
12Q	0.42	0.15	[-0.05,	0.61]	-0.31	-0.10	[-0.52,	0.17]	0.12	0.06	[-0.08,	0.28]
Output growth:												
0Q	0.42	0.30	[0.16,	0.53]	-0.36	-0.25	[-0.50,	-0.03]	0.33	0.48	[0.21,	0.71]
1Q	0.35	0.27	[0.06,	0.50]	-0.28	-0.21	[-0.49,	0.06]	0.25	0.22	[0.09,	0.37]
12Q	0.20	0.09	[-0.01,	0.30]	-0.15	-0.06	[-0.25,	0.07]	0.15	0.11	[-0.04,	0.33]
Investment growth:												
0Q	0.07	0.12	[-1.31,	1.58]	-0.17	-0.09	[-0.54,	0.23]	0.20	0.36	[0.08,	0.72]
1Q	0.05	0.06	[-1.77,	1.98]	-0.19	-0.08	[-0.64,	0.37]	0.03	-0.02	[-0.08,	0.10]
12Q	-0.03	-0.02	[-1.47,	1.45]	-0.21	-0.07	[-0.60,	0.22]	0.05	-0.01	[-0.10,	0.18]
Market betas:												
	-0.02	-0.00	[-0.21,	0.19]	-0.03	-0.02	[-0.08,	0.03]	0.98	0.99	[0.98,	0.99]

The Table shows the Monte-Carlo evidence of various projections, in a model without stochastic volatility and no volatility feedback ($\tau = 0$). For an economic variable of interest y, the table reports the model-implied loadings on the sectoral volatilities shocks, in the projection of contemporaneous and future growth rates of y, on the current sectoral shocks: consumption and investment TFP innovations, consumption TFP-volatility shock, ΔC-TFP-VOL, and investment TFP-volatility shock, ΔI-TFP-VOL. The predictive projection ($h > 1$) is: $\frac{1}{h}\sum_{j=1}^{h} \Delta y_{t+j} = \beta_0 + \beta_h'[\Delta\text{C-TFP}_t, \Delta\text{I-TFP}_t, \Delta\text{C-TFP-VOL}_t, \Delta\text{I-TFP-VOL}_t] + error$. The variables of interest y include consumption, output, and investment growth rates. The predictive horizons are $h = 0, 1, 12$ quarters. Market betas are based on projection of contemporaneous market returns on the contemporaneous sectoral shocks.The Table reports the population and small-sample estimates (corresponding to $5\%, 50\%$ and 95% percentile of the distribution in simulations) of the slope coefficients and R^2s. Consumption, investment, output and market-returns are simulated at quarterly frequency under a model that is identical to the benchmark model, but in which the volatility of shocks are fixed at their mean value. Sectoral shocks are then computed from the simulated time-series of TFP growth rates, in an identical fashion to the empirical benchmark construction, as described in Section 2.3.2. Small-sample evidence is based on 10,000 simulations of 272 observations of quarterly data; the population estimates are based on a long simulation of 500,000 quarters of data.

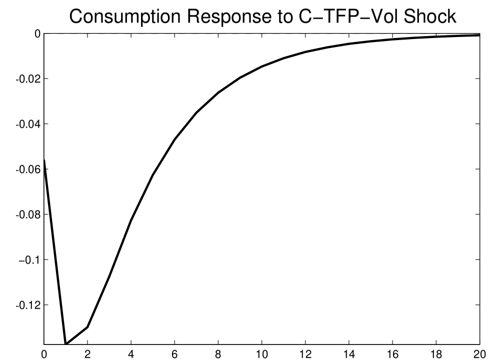
Figure 2.1: Residual Investment TFP-Volatility



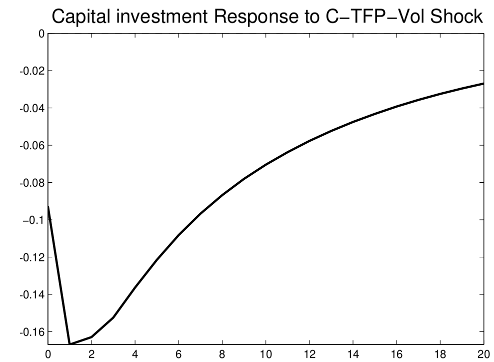
The figure shows the time series plot of the residual investment TFP-volatility which is orthogonal to consumption TFP-volatility. The sectoral TFP-volatilities are constructed from the predictive regressions of future sectoral TFP realized variances. The residual investment TFP-volatility is computed from the projection of investment TFP-volatility onto consumption TFP-volatility. The shaded areas represent NBER recessions.

Figure 2.2: Data Impulse Response of Detrended Consumption, Output and Investment to Sectoral Volatilities

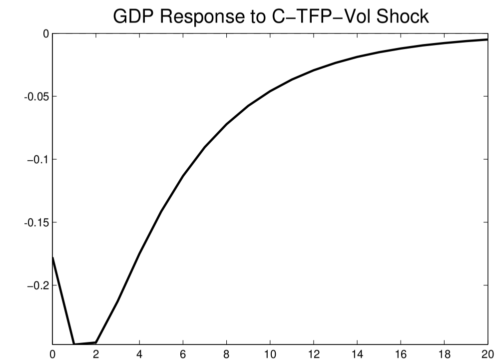
(a) Consumption response to consumption TFP-volatility shock



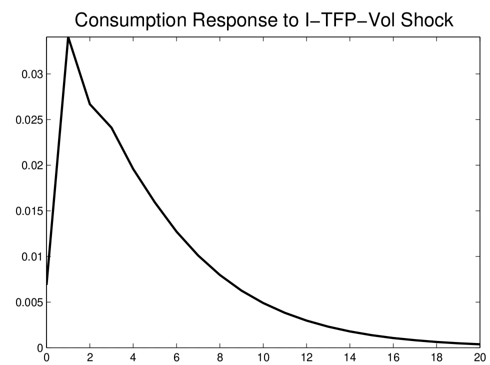
(b) Capital investment response to consumption TFP-volatility shock



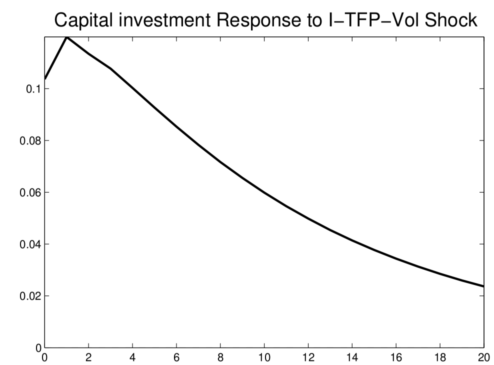
(c) Output response to consumption TFP-volatility shock



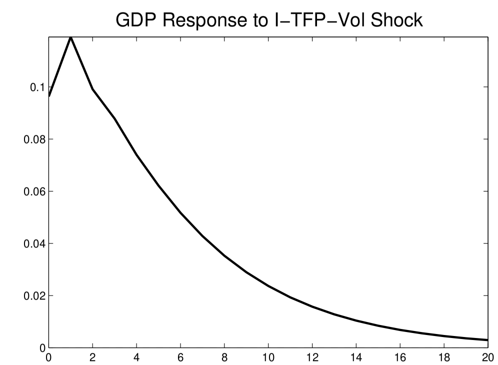
(d) Consumption response to investment TFP-volatility shock



(e) Capital investment response to investment TFP-volatility shock

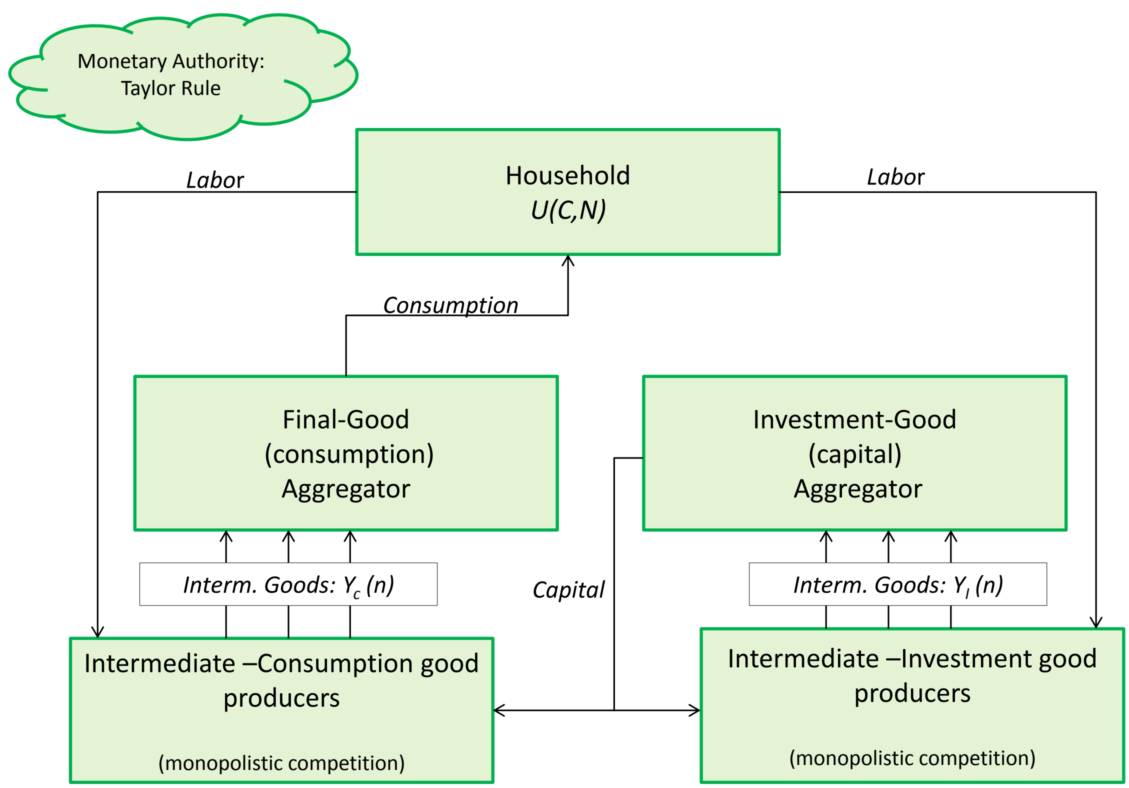


(f) Output response to investment TFP-volatility shock



The Figure shows impulse responses of the cyclical component of consumption, GDP, and capital investment to one-standard deviation shocks of consumption TFP-volatility (C-TFP-VOL) and investment TFP-volatility (I-TFP-VOL). The impulse responses are computed from a VAR(1) which includes sectoral volatilities, sector first-moment innovations, and the economic variable of interest. The cyclical component of each macroeconomic variable is obtained using a one-sided HP-filter. Each cyclical component is standardized. The horizontal axis represents quarters. The vertical axis represents response in standard deviation units of the cyclical component. The data are quarterly from 1947Q1-2014Q4.

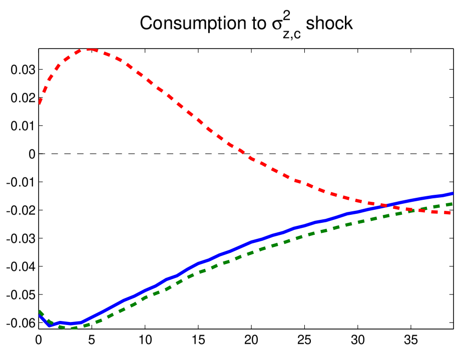
Figure 2.3: Model Scheme



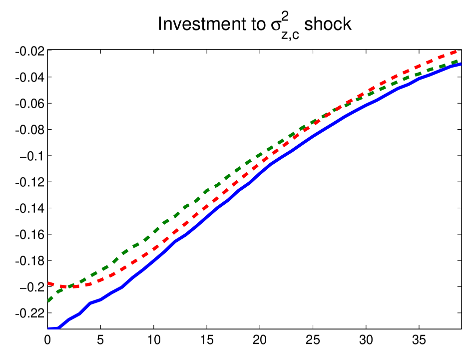
The figure outlines the structure of the benchmark two-sector economy.

Figure 2.4: Model Impulse Response of Detrended Consumption, Output and Investment to Sectoral Volatilities

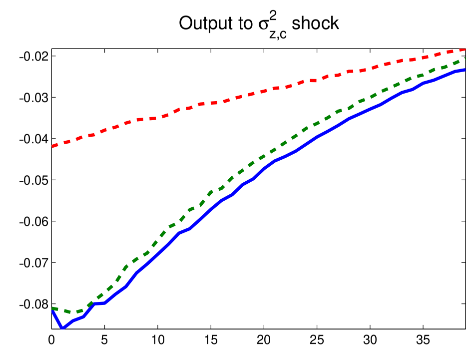
(a) Consumption response to consumption TFP-volatility shock



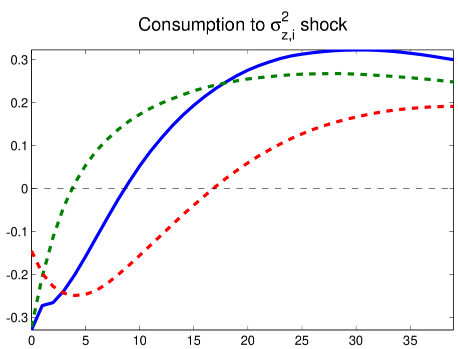
(b) Investment response to consumption TFP-volatility shock



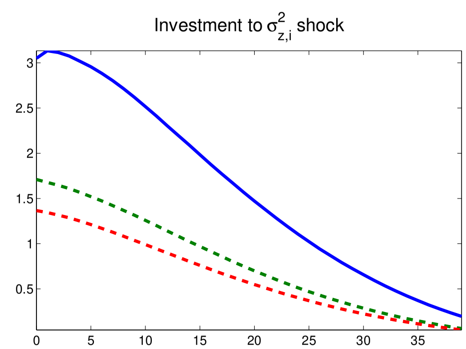
(c) Output response to consumption TFP-volatility shock



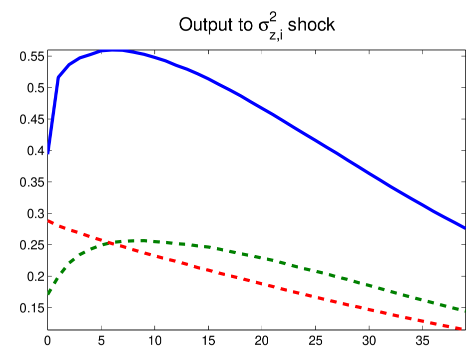
(d) Consumption response to investment TFP-volatility shock



(e) Investment response to investment TFP-volatility shock



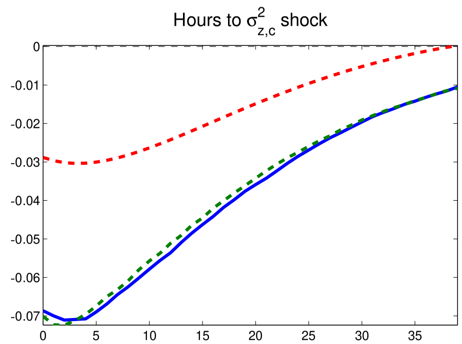
(f) Output response to investment TFP-volatility shock



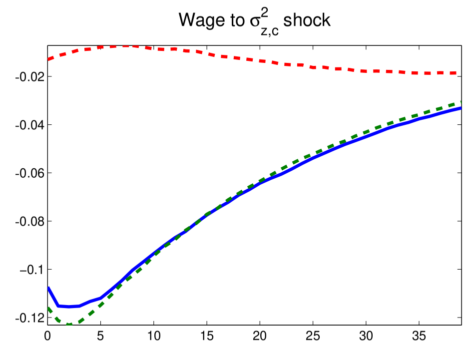
The Figure shows impulse responses of model-detrended real consumption, investment expenditures, and output to one-standard deviation shocks of consumption TFP-volatility (σ^2_{zc}) and investment TFP-volatility (σ^2_{zi}). The impulse responses are computed using simulated model-data. The solid blue-line shows impulse-responses from the benchmark model. The dashed green line shows impulse responses from a model with an identical calibration to the benchmark model, but without a feedback from investment TFP-volatility to future consumption TFP growth ($\tau = 0$). The dashed red line shows impulse responses from a model without volatility feedback ($\tau = 0$), and without monopolistic competition or nominal rigidities ($\mu_j \to \infty \quad j \in \{c, i\}, \quad \phi_P = 0$). The horizontal axis represents quarters. The vertical axis represents percent deviations from the steady-state.

Figure 2.5: Model Impulse Response of Hours, Detrended Wages and Investment-Price to Sectoral Volatilities

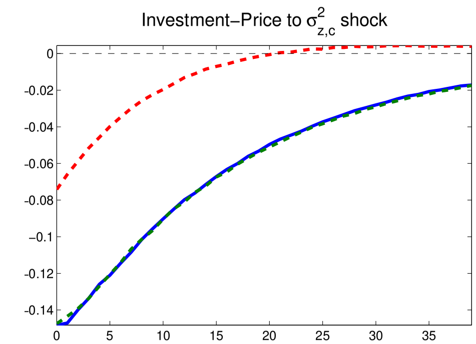
(a) Hours response to consumption TFP-volatility shock



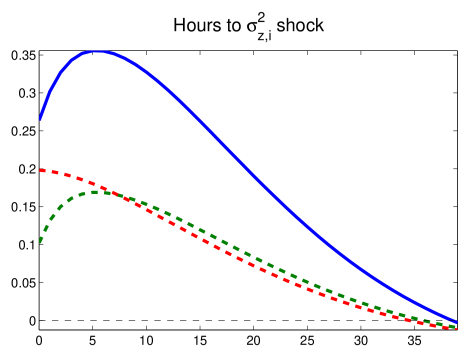
(b) Wage response to consumption TFP-volatility shock



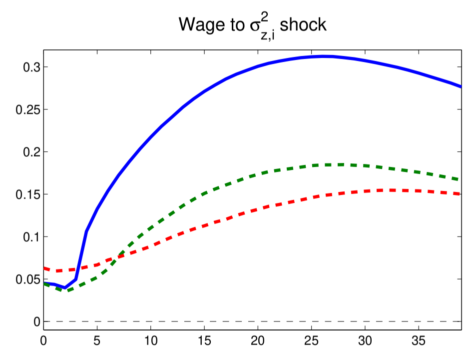
(c) Investment-price response to consumption TFP-volatility shock



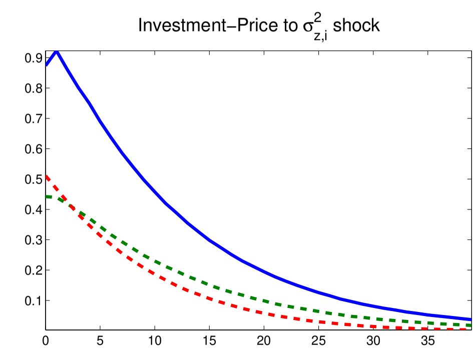
(d) Hours response to investment TFP-volatility shock



(e) Wage response to investment TFP-volatility shock



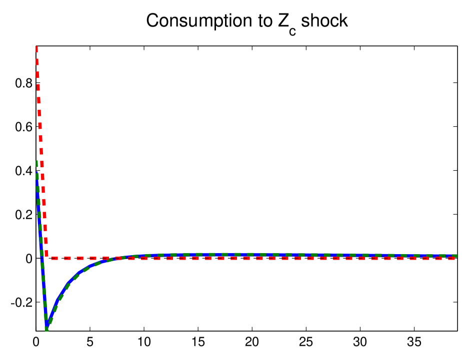
(f) Investment-price response to investment TFP-volatility shock



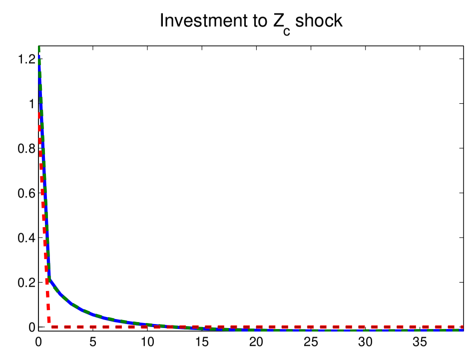
The Figure shows impulse responses of model-implied hours, detrended real wages, and real relative price of investment to one-standard deviation shocks of consumption TFP-volatility (σ^2_{zc}) and investment TFP-volatility (σ^2_{zi}). The impulse responses are computed using simulated model-data. The solid blue-line shows impulse-responses from the benchmark model. The dashed green line shows impulse responses from a model with an identical calibration to the benchmark model, but without a feedback from investment TFP-volatility to future consumption TFP growth ($\tau = 0$). The dashed red line shows impulse responses from a model without volatility feedback ($\tau = 0$), and without monopolistic competition or nominal rigidities ($\mu_j \to \infty \quad j \in \{c, i\}, \quad \phi_P = 0$). The vertical axis represents percent deviations from the steady-state.

Figure 2.6: Model Impulse Response of Detrended Consumption, Output and Investment to Sectoral Innovations

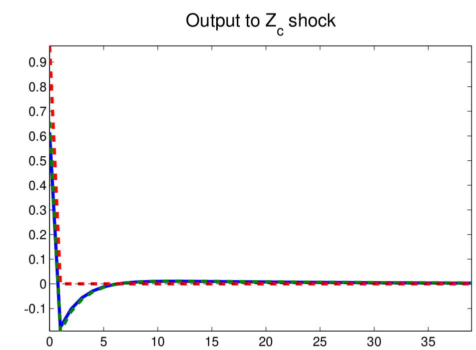
(a) Consumption response to consumption TFP innovation



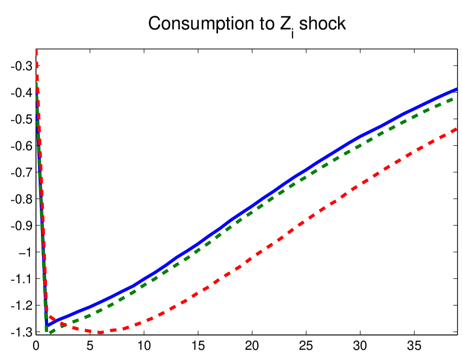
(b) Investment response to consumption TFP innovation



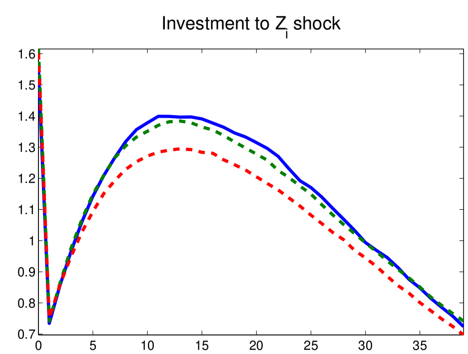
(c) Output response to consumption TFP innovation



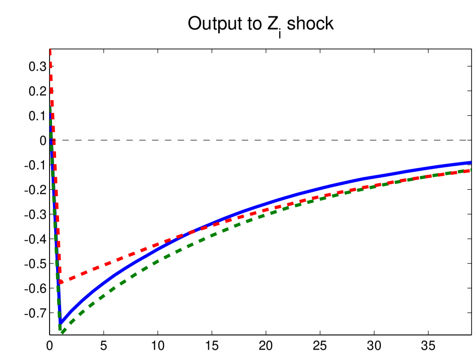
(d) Consumption response to investment TFP innovation



(e) Investment response to investment TFP innovation



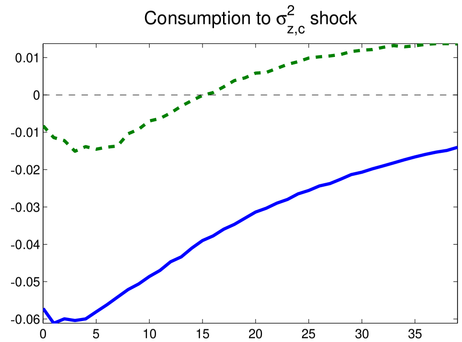
(f) Output response to investment TFP innovation



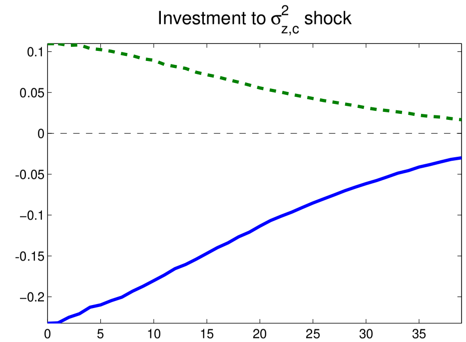
The Figure shows impulse responses of model-detrended consumption, investment expenditures, and output to one-standard deviation shocks of consumption TFP growth ($Z_{c,t}/Z_{c,t-1}$) and investment TFP growth ($Z_{i,t}/Z_{i,t-1}$). The impulse responses are computed using simulated model-data. The solid blue-line shows impulse-responses from the benchmark model. The dashed green line shows impulse responses from a model with an identical calibration to the benchmark model, but without a feedback from investment TFP-volatility to future consumption TFP growth ($\tau = 0$). The dashed red line shows impulse responses from a model without volatility feedback ($\tau = 0$), and without monopolistic competition or nominal rigidities ($\mu_j \to \infty \quad j \in \{c, i\}, \quad \phi_P = 0$). The vertical axis represents percent deviations from the steady-state.

Figure 2.7: Model Impulse Responses to Sectoral Volatilities: The Role of IES

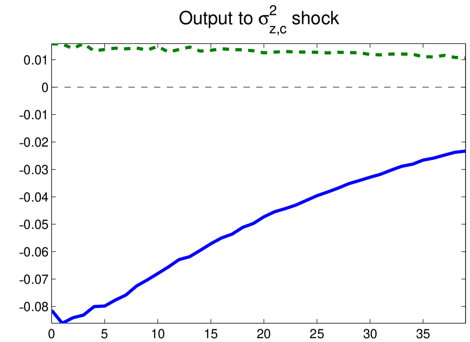
(a) Consumption response to consumption TFP-volatility shock



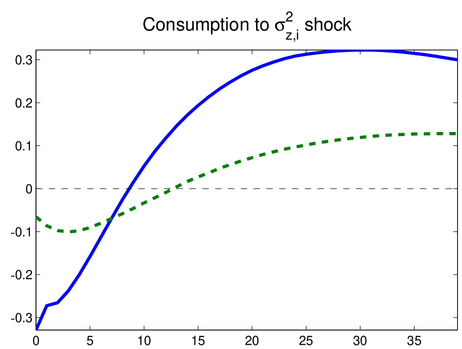
(b) Investment response to consumption TFP-volatility shock



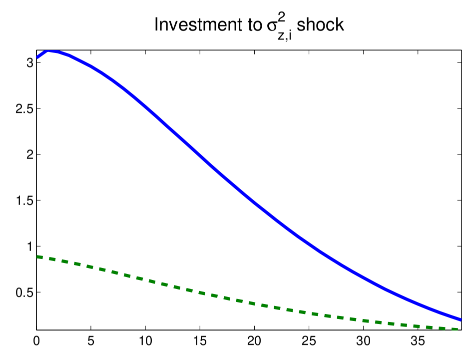
(c) Output response to consumption TFP-volatility shock



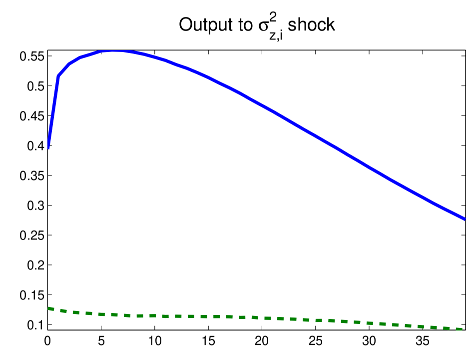
(d) Consumption response to investment TFP-volatility shock



(e) Investment response to investment TFP-volatility shock



(f) Output response to investment TFP-volatility shock



The Figure shows impulse responses of model-detrended consumption, investment expenditures, and output to one-standard deviation shocks of consumption TFP-volatility (σ^2_{zc}) and investment TFP-volatility (σ^2_{zi}). The impulse responses are computed using simulated model-data. The solid blue-line shows impulse-responses from the benchmark model. The dashed green line shows impulse responses from a model with an identical calibration to the benchmark model, but without a feedback from investment TFP-volatility to future consumption TFP growth ($\tau = 0$), no nominal rigidities ($\phi_P = 0$), and the IES (ψ) is set to 0.8. The horizontal axis represents quarters. The vertical axis represents percent deviations from the steady-state.

CHAPTER 3 : From Private-Belief Formation to Aggregate-Vol Oscillation

3.1. Introduction

What are the quantitative implications for aggregate and cross sectional volatility fluctuations, that are induced by Bayesian learning and informational asymmetry? This study provides a micro-founded model that endogenously generates time-varying volatility for aggregate growth rates, via imperfect information channels, while generating realistic unconditional real business-cycle moments. Macroeconomic volatility in this paper is built in a bottom-up approach. The model suggests that endogenous oscillations in the correlation between firms' policies are an important source for aggregate volatility fluctuations. In recessionary periods, correlations rise as a result of stronger reliance on public information. Higher between-firm correlation is translated into higher aggregate volatility. Both learning *and* asymmetric information are crucial to generate economically significant fluctuations in aggregate volatility. Empirically, correlations do rise at bad times, in spite of an increase in cross-sectional variation of real variables (dispersion), since the average between-firm covariance rises in recessions more than dispersion does.

The importance of this study lies in the growing body of literature in macroeconomics and finance, which stresses the pivotal role of higher volatility in hindering economic recovery, growth, and asset-prices. Specifically, consider the following stylized facts regarding the time-varying behavior of volatility, and its implications:

Fact (I): Aggregate and cross-sectional volatilities are stochastic:

a. The conditional volatility of real aggregate macroeconomic variables, such as output and investment growth, rises in economic downturns. Quarterly GDP growth has about 35% more conditional volatility in NBER recessions (Bloom (2014)); Consumption growth's volatility increases by 30% in bad times (This paper).

b. The cross-sectional dispersion of real quantities produced by firms is countercyclical. Firms' output growth, and employment growth, are negatively correlated with detrended GDP (see e.g. Bachmann and Bayer (2013)).

c. The average correlation between firm-level real-variables (e.g. output, investment), increases in economic slowdowns (This paper). This fact complements the more established notion that the average correlation amongst stock-returns significantly increases in recessions (see e.g. Moskowitz (2003), Krishnan et al. (2009)).

Fact (II): An increase in the volatility of macroeconomic fundamentals has an adverse effect on the real and financial economy; in particular -

a. Volatility reduces investment and economic activity due to a real-option effect (see e.g. Dixit and Pindyck (1994); Bloom (2009)), or due to a rise in the cost of capital (see e.g. Christiano et al. (2014); Arellano et al. (2012); and Gilchrist et al. (2014)).

b. Volatility lowers asset valuations, raises risk premia and increases return volatility (see e.g. Bansal et al. (2005b); Drechsler and Yaron (2011); Bansal et al. (2014)).

The studies that examine the impacts of macro-volatility on the economy, that is, explore fact (II) above, treat the shocks to volatility, detailed in fact (I), as exogenous and independent of other fundamentals. Differently put, the evolution of stochastic volatility is traditionally modeled using an exogenous process. Yet, as illustrated in this study, a perfect-information neo-classical growth model without exogenous stochastic volatility, generates only a negligible increase in the conditional volatility during downturns. This raises a gap, to uncover the economic forces that lead the volatility of aggregate fundamentals to fluctuate and rise in recessions.

This study proposes a theory for the endogenous emergence of stochastic macro volatility, in an environment of only homoscedastic first-moment shocks. It is thus aimed at *quantitatively* explaining facts (I.a) - (I.c), while generating realistic business-cycle unconditional moments. By doing so, the work helps to bridge the gap between the econometric findings of fact (I), and the macroeconomic and financial literature of fact (II). I demonstrate that learning is quantitatively important to understand the dynamics of volatility and correlations over the business-cycle[1].

The model presented in this paper relies on five main ingredients: (1) Existence of a mass of atomistic firms; (2) The aggregate TFP shock is latent, but can be fully recovered with a continuum of signals; (3) Firms use Bayesian learning to update their belief about the current TFP level, and by doing so they rely on both public and private information; (4) Higher economic activity at the firm level, helps the firm to learn about the unobserved TFP, by endowing it with more signals; and (5) It takes a lag of one period to publish any macroeconomic quantity, including any public information about the aggregate TFP.

In a nutshell, the economic narrative of the paper is as follows. Each period, the firm receives two signals regarding the aggregate latent TFP shock. The first is the firm's own privately observed output, and the second is the *lagged* aggregate TFP, which serves as a public signal. On one hand, firms produce using capital and labor. The productivity of each hour of labor is subject to an unobserved, homoscedastic, and idiosyncratic labor efficiency shock, that captures the effect of time-varying tiredness, motivation, and focus on human capital. Consequentially, every hired hour of labor provides an idiosyncratic signal, with fixed precision on the aggregate state. Thus, firms that hire more labor, have a better ratio of signal to noise. In bad times, firms choose to reduce their rented working-hours, due to decreased profitability. Reducing the amount of labor drops the precision of the firm's private idiosyncratic signal (output) in recessionary periods. On the other

[1]Notably, more than one explanation is plausible for the observed behavior of volatility. The economic forces described in this work should be viewed as a significant source of macro volatility fluctuations, among possible others.

hand, as all firms are atomistic, a central-household that observes all firms' outputs can become perfectly informed about the underlying TFP by the end of the period. Publishing this recovered aggregate TFP *in a lag*, is equivalent to a signal on today's TFP with fixed precision, as all shocks are homoscedastic. As a result, in bad times, firms place more weight on public information, and less on their idiosyncratic information, when constructing their beliefs on the current state of the economy. This generates a greater comovement in the beliefs of firms in economic slowdowns, and hence, a larger comovement in their investment and labor policies. The higher degree of correlation amongst firms in bad times increases the volatility of aggregate quantities, such as output and consumption.

When calibrated at quarterly frequency to match the unconditional moments of consumption and output growth rates, the learning model amplifies the oscillations in the conditional volatility of aggregate growth rates, while a no-learning model, similar to the neo-classical firm problem, produces only minuscule changes in the conditional volatility. In the learning environment, consumption growth's volatility rises in bad periods (when TFP growth is low) by 29% in the model versus 32% in the data, while it falls in good times (when TFP growth is high) by 20% and 14% in the model and in the data, respectively. For comparison, in the no-learning environment, the volatility of aggregate consumption fluctuates by merely 3% in bad times. Similar results are obtained for other macro growth rates. Capital's growth volatility increases in bad times by 57% and by 56%, in the model and in the data.

I further establish that the movements in the conditional volatility of aggregates, capture shifts in the average conditional covariation between firms, as all firm-specific volatility is effectively diversified away at the aggregate level. While this claim holds exactly in the model in which firms are atomistic, I show that it roughly holds in the data as well, in spite of the fact that empirically some firms are non-atomistic (see e.g. Gabaix (2011)).

Since aggregate volatility amounts to the average covariation between firms, I can then decompose aggregate volatility. The fluctuation in *aggregate* volatility between bad and normal periods is equal to the fluctuation in *firm-level* volatility multiplied by the fluctuation in the average between-firm *correlation*. In the no-learning model, the small fluctuations in the conditional volatility of aggregates, are shown to be driven by small changes in the conditional firm-level volatility, while the average correlation between firms is approximately constant. By contrast, in the learning model, the fluctuations in the conditional volatility of aggregates, are largely due to shifts in the conditional correlation between firms, that rises in bad times. The average correlation between firms' outputs increases by 32% in bad times, and drops by 29% in good times. For all aggregate growth variables, about 80% to 90% of the increase in the conditional aggregate volatility in bad times is attributed to an increase in the conditional correlations.

The endogenous fluctuations in the average correlation in the learning model stem from two main model ingredients: (1) Bayesian leaning, with time varying gains, and (2) Informational asymmetry. To show this, I shut down each of those channels separately. Namely, I solve a modified learning model in which the labor noise shocks are aggregate rather than idiosyncratic, thus eliminating informational asymmetries between firms. In addition, I solve an alternative model in which the weights that firms place on the public and private signals are fixed, and do not vary with the actual gain (non-Bayesian learning). Both of the modified models lack the ability to produce significant volatility fluctuations. Thus, a rise in (belief) uncertainty in bad times, as some earlier works feature, is not sufficient to produce enough *realized* volatility at the aggregate level.

Lastly, I show that the cross-sectional variation in the model, or dispersion, is also countercyclical, as is also the case empirically. The correlation of output growth dispersion with TFP is negative in the model and in the data. This is a result of a rise not only in aggregate volatility, but also in firm-level volatility in bad times, in the model. Ostensibly, an increase in dispersion seems to contradict a rise in the expected correlation between firms. I reconcile the two by showing that if the average between-firm covariation, increases in magnitude more than dispersion does in downturns, the average correlation increases as well. In the data, the rise in the average covariation in bad times ranges between 19% to 55%, while dispersion rises by less.

The rest of the paper is organized as follows. Section 2 offers a discussion of related literature. In section 3, I provide the economic model. Section 4 presents the data and the econometric methodology used to construct the conditional volatilities and correlations. In Section 5, I report the model calibration and its implications for unconditional business-cycle moments. Section 6 discusses the main results of this paper: the implications of learning for the fluctuations in the aggregate conditional volatility, both in the model and empirically, and its decomposition into firm-level volatility and average correlation. The section also presents the implications of the learning model for cross-sectional volatility, and establishes the robustness of the results. Section 7 provides concluding remarks.

3.2. Related Literature

This study relates to several strands of literature. The closest studies that my work is relates to, are theoretical macroeconomic models that aim at explaining why objects like *uncertainty*, *volatility* and *dispersion* vary over time, both at the firm-level, and at the macro level. A growing number of recent papers attempt to endogenize *uncertainty*, mainly in centralized economies, over the business cycle (that is, why the volatility of agents' *beliefs* over the state of the economy increases in recessions). In Van Nieuwerburgh and Veldkamp (2006), procyclical learning about productivity generates countercyclicality in firm-level uncertainty that may relate to countercyclical movements in asset prices.[2] Fajgelbaum et al. (2015) also endogenize uncertainty level, and link it to economic activity via learning: higher uncertainty about the fundamental discourages investment, which in turn results in fewer signals about the fundamental, thus keeping uncertainty levels high, which discourages investment further. Similarly, Orlik and Veldkamp (2013) show that a Bayesian forecaster who revises model parameters in real-time, experiences countercyclical uncertainty shocks,

[2]Related work to Van Nieuwerburgh and Veldkamp (2006) includes Ordoñez (2013). Ordoñez (2013) argues that the speed of boom and busts depends on the financial system of the country. In his work however, beliefs are only public and the state of the economy is the volatility of productivity. Thus, volatility is exogenously stochastic, while this work features homoscedastic volatility.

even if the underlying process is homoscedastic. This occurs as the agent is more confident in predicting the future when growth is normal, while sudden "unfamiliar" events in recessions make it harder for the forecaster to make predictions. *A key difference* between this paper and the former works, is that I focus on *volatility*, or in other words, the time-varying predictable variation of *realized* quantities, while the former discuss *uncertainty* levels, that is, the forecasting error squared is time-varying. The former works do not predict that firms' actual policies are necessarily becoming more volatile, or that the correlation between firms' policies is fluctuating.

Other recent works endogenize *firm-level volatility*, or *dispersion*, in good versus bad periods. Bachmann and Moscarini (2011) show that downturns offer the opportunity for firms to drastically alter their pricing policy, or to "experiment", allowing them to better learn their firm-specific demand function. This experimentation, mainly performed to decide whether to exit the market, is the driver of *cross-sectional dispersion* in the prices of firms. In Decker et al. (2013), first moment TFP shocks enable firms to expand to more markets and expose firms to an increased number of market-specific shocks, which reduces firm-level volatility by diversification. Related, Tian (2015) also endogenizes productivity dispersion over the business-cycle. These works focus on *(micro) cross-sectional volatility.* They do not explicitly examine whether this micro volatility feeds into higher aggregate quantities. In contrast, this work propagates the notion that an important source of aggregate volatility is not merely an increase in individual firms' idiosyncratic volatility, but rather an increase in the correlation between individual firms' policies.

Some related papers explicitly discuss aggregate volatility, which is the main focus of this work. One contributor to aggregate volatility may come from Governments and Central Banks. Pastor and Veronesi (2012) argue that policy becomes more volatile during recessions because policy makers wish to experiment. In economic downturns, politicians are drawn to experiment as they attempt to boost growth. While this explanation directly feeds to macro volatility, it differs from the bottom-up approach of the decentralized economy, taken in this paper. Gabaix (2011) shows that idiosyncratic firm-level fluctuations can explain a significant portion of aggregate shocks, when some firms are non-atomistic or "granular". His study however, focuses on unconditional aggregate volatility, not on its cyclical behavior. Herskovic et al. (2015) endogenize firm level volatility (dispersion) using a different framework than mine: consumer-supplier business networks, with some implications for aggregate volatility. More related, Ilut et al. (2013) show that when hiring decisions respond more to bad signals, due to ambiguity about the level of noise, both aggregate conditional volatility and dispersion of labor growth are countercyclical. A similar idea is used in the context of stock return correlations, in the works of Ribeiro and Veronesi (2002), and Ozsoy (2013). As opposed to my quantitative study, that embeds learning in a real business cycle environment, the work of Ilut et al. (2013) is mostly qualitative. My work also employs a different learning mechanism. The papers of Thesmar and Thoenig (2004), Comin and Philippon (2006), and Comin and Mulani (2006), also target aggregate volatility by trying to explain the so-called "great moderation" in the volatility of aggregate returns and output (see Stock and Watson (2003)). However, these works target the ostensible trend in aggregate volatility, while they do not generate fluctuations of aggregate volatility over the business cycle.

The second body of works related to this paper are studies discussing the social value of public information, starting with the influential work of Morris and Shin (2002). The work of Amador and Weill (2012), shows that increasing public information slows down learning in the long run, and may reduce welfare. While aggregate volatility fluctuates in their model, their stylized framework exhibits a hump shape for volatility over time, that converges to zero in the long run, and does not explain why volatility increases in recessionary periods.

Related, Angeletos and La'O (2013) show that even without aggregate TFP shocks, sunspot public shocks that purely affect agents' belief about the state of the economy, without altering the underlying technology or preferences, termed "sentiments", create aggregate fluctuations. While their framework highlights that public information can serve as an important source of aggregate fluctuations, it produces fixed volatility for aggregate output. A closer work of Angeletos et al. (2016) demonstrates, in a comparative static manner, that more precise public information reduces dispersion, but can increase the volatility of aggregate output. In contrast, in my work the precisions of the signals is time-varying, allowing to obtain stochastic volatility. In addition, my work is quantitative in nature, and targets objects that are absent from the former works, such as investment rate and capital growth. My work complements these works in that my focal point is different. I harness the use of time-varying weights on public and private information to obtain aggregate volatility that varies over the business cycle.

The third branch of studies my paper is related to, are econometric papers that document that macro volatility, micro volatility (or cross-sectional volatility), and also correlations, rise in recessions. Bloom (2014) documents that industrial production growth, based on GARCH models, has about 35% more conditional volatility in recessions. In the context of stock returns, Bloom (2014) and Bekaert et al. (2013), report that the VIX level is countercyclical, and increases by 58% in recessions. Other meaures of macro uncertainty also increase in bad times. Jurado et al. (2013) use monthly economic series in a system of forecasting equations and look at the implied forecasting errors. They find a sharp increase in recessionary periods, and in particular, in the Great Recession. The works of Higson et al. (2002), Jorgensen et al. (2012), Kehrig (2011), Bloom et al. (2012), and Bachmann and Bayer (2013), provide extensive evidence that cross-sectional variance, or dispersion, is also highly countercyclical, for various economic outcomes including output growth, sales growth, employment growth, earnings growth, and Solow residuals. Investment-rate dispersion however, seems to be procyclical, as pointed by Bachmann and Bayer (2014). Moskowitz (2003), in the context of stock returns, uses a multivariate-GARCH approach to show that conditional correlations exhibit significant time variation, increase during recessions, and were extremely large during the 1987 stock market crash. Similarly, Krishnan et al. (2009) use average realized correlations of stock returns, and show that it significantly rises in recessions. My work contributes to these findings by empirically showing that the correlation of *fundamentals*, such as investment-rate and output growth, increases in recessionary periods, and explains fluctuations in aggregate volatility of fundamentals.

The last strand of papers my work relates to are macroeconomics and asset-pricing works that stress the importance of aggregate volatility in explaining business-cycle fluctuations, economic growth and risk premia. Bloom (2009) shows that increased volatility, measured via VIX, leads to an immediate drop in output and investment growth rates as firms delay their investment decisions. The work of Fernandez-Villaverde et al. (2011) discusses uncertainty in an open-economy context, showing that higher volatility lowers domestic investment. Other works argue that higher volatility increases the cost of capital, or credit spreads, hence makes investment more costly (see e.g. Christiano et al. (2014); Arellano et al. (2012); and Gilchrist et al. (2014)). Basu and Bundick (2012) rely on nominal rigidities to show that both consumption and investment can drop in response to volatility shocks. Other works rely on alternative economic forces which can yield a positive relationship between volatility and investment. These channels include precautionary savings, time-to-build, or investment irreversibility (see e.g. Abel and Eberly (1996); Bar-Ilan and Strange (1996); Gilchrist and Williams (2005); Jones et al. (2005); Malkhozov (2014); and Kung and Schmid (2014)). Importantly, these papers treat volatility shocks as exogenous, while in this paper I treat volatility as an endogenous object.

3.3. Model

This section describes the theoretical framework that generates stochastic aggregate volatility in a homoscedastic world. The economy is comprised of a mass of firms, indexed by $i \in [0,1]$, and one representative household, who owns all firms and consume their dividends. Below I describe the problem faced by firms, the household, and a definition of an equilibrium in this setup.

3.3.1. *Aggregate Productivity*

Aggregate productivity, denoted by G_t, evolves as geometric random walk with time varying drift. Specifically, $G_{t+1} = G_t \cdot g_t$, where

$$g_t = (1 - \rho_g)g_0 + \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t},$$

and where $\varepsilon_{g,t} \sim N(0,1)$[3]. Notice that the conditional volatility of aggregate productivity growth is constant. Further, note that g_t is the gross-growth rate of productivity, and I assume that the mean growth rate $g_0 > 1$ is sufficiently large, in comparison to the volatility of the shock σ_g such that g_t is always positive.

It is assumed that aggregate productivity is a latent variable. This is also the case in the real world: total factor productivity is unobserved, but can be recovered by observing real aggregate macroeconomic growth rates. Both firms and the household learn about the current and past levels of productivity from publicly and privately observed signals. All information regarding the productivity shock is obtained from real (noisy) economic outcomes. As explained later, all agents become perfectly informed about any lagged level of aggregate productivity, but there is uncertainty regarding the current period's productivity growth g_t.

3.3.2. *Firms*

Each firm is operated by a manager. The firm operates on an island. As a result, all aggregate quantities, including aggregate productivity level, become observable to the firm in a lag of one period. This assumption parallels to the real world, in the sense that aggregate quantities are usually published in some lag. Specifically, at the beginning of every period t, the manager of the firm gets an input from its owner (the household): last period's aggregate productivity growth g_{t-1} [4]. This assumption is consistent with the availability

[3]Notice that G_t is predetermined.

[4]Alternatively, the firm's input is the aggregate productivity *level* of period $t-1$. This is an equivalent assumption, as the productivity level of time $t-2$ is already in the information set of the firm at time t.

of data in reality: the San-Fransisco Federal Reserve Bank, for instance, publishes a TFP time-series, in a lag of one-quarter[5]. In return, the firm ships back to the owner its current period dividend after producing and investing.

Firms produce output using capital and labor. Firm i has a stock of capital $k_{i,t}$, and rented labor inputs (measured in time-units, or hours) $l_{i,t}$.

Capital evolves according to:

$$k_{i,t+1} = (1-\delta)k_{i,t} + \Lambda(\frac{I_{i,t}}{k_{i,t}}),$$

where δ is the depreciation rate, and $I_{i,t}$ is the investment level at period t. The capital adjustment cost function Λ is specified as in Jermann (1998): $\Lambda(\frac{I_{i,t}}{k_{i,t}}) = \frac{\alpha_1}{1-\frac{1}{\zeta}}(\frac{I_{i,t}}{k_{i,t}})^{1-\frac{1}{\zeta}} + \alpha_2$. The parameter ζ represents the elasticity of the investment rate with $\zeta \to \infty$ representing infinitely costly adjustments. The parameters α_1 and α_2 are set such that there are no adjustment costs in the deterministic steady state.

Labor to be used in period t is rented in the period $t-1$ for a wage w_t per unit of time (hour). The wage exogenously grows at the same rate as aggregate productivity, and is given by $w_t = w \cdot G_{t-1}$ [6]. Adjusting the labor force, requires a non-pecuniary adjustment cost, and is given by $\Phi_L(l_{i,t}, l_{i,t+1}) = G_t \cdot \frac{\kappa}{2} \cdot (l_{t+1} - l_t)^2$. These adjustment costs capture, in a reduced form manner, the costs induced by the friction of search. In the absence of consumption smoothing in a risk-neutral setting, this adjustment cost is vital to make labor growth, and hence output and consumption growths, sufficiently persistent.

Firms also face two idiosyncratic shocks. First, firms revenue is affected by an observed

Dividing aggregate productivity level of time $t-1$ with that of time $t-2$ yields g_{t-1}.

[5] The quarterly TFP data relies on Basu et al. (2006) and Fernald (2012).

[6] As labor is hired in period $t-1$, I specify a wage that incorporates only time $t-1$ information. The reason that labor is pre-hired in my setting is that otherwise, one could potentially learn with certainty the current level of productivity growth g_t simply by observing the current labor wage. By making labor predetermined, the current wage reflects merely g_{t-1}, which is already known to the firm at time t.

demand shock $z_{i,t}$, that evolves according to an AR(1) process:

$$z_{i,t} = (1 - \rho_z)z_0 + \rho_z z_t + \sigma_z \varepsilon_{i,z,t},$$

where the innovation is conditionally homoscedastic, and $\varepsilon_{i,z,t} \sim N(0, 1)$. The second idiosyncratic shock, $\varepsilon_{i,l,t}$, is a shock to the efficiency of labor. It is assumed to be a latent i.i.d. shock across time and across firms, with $\varepsilon_{i,l,t} \sim N(0, 1)$. This conditionally homoscedastic shock captures disturbances to the efficiency of the labor force, that are unobserved to the firm, such as time-varying levels of focus, tiredness and motivation that may affect a human resource.

The production technology of firm i at time t is therefore given by:

$$y_{i,t} = G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha} (g_t l_{i,t} + \sigma_l \varepsilon_{i,l,t})^{\nu-\alpha}, \tag{3.1}$$

where $\nu \in (0, 1)$ is the the total returns to scale. This specification is similar to that used in Van Nieuwerburgh and Veldkamp (2006), but augmented to support labor and growth. It is a reduced-form production function that captures a very basic notion: bigger firms who acquire more labor, and have a higher economic activity, have a higher loading on the aggregate TFP growth g_t, and enjoy a preferable signal to noise ratio, as illustrated next. This assumption can be motivated explicitly by breaking the total labor time stock $l_{i,t}$ into operating time-units (hours), each of which provides another signal on the aggregate TFP shock. Below I outline briefly a microfounded explanation for the emergence of such a production function.

Suppose that each firm operates by hiring its labor force to work for $l_{i,t}$ hours. For motivational purposes think of $l_{i,t}$ as discrete. The productivity of the labor force, per hours $\ell \in [1..l_{i,t}]$, varies. As mentioned earlier, this assumption captures the effect of time-varying tiredness, or motivation. Specifically, in every hour ℓ, the labor force productivity, in labor efficiency units, is $g_t + \sigma_l \eta_{i,\ell,t}$, where g_t is the aggregate shock of the labor augmenting technology, and $\eta_{i,\ell,t}$ is an idiosyncratic efficiency shock, independent over firms and hours, and distributed $N(0,1)$. All $\eta_{i,\ell,t}$ shocks are latent, and so is g_t.

By integrating over all hours, the firm's total labor input, in efficiency units can be written as: $g_t l_{i,t} + \sigma_l \sqrt{l_{i,t}} \varepsilon_{i,l,t}$, where $\varepsilon_{i,l,t} \sim N(0,1)$. To make sure that all shocks are explicitly homoscedastic, I choose to solve a version of the model in which the labor efficiency is simply $g_t l_{i,t} + \sigma_l \varepsilon_{i,l,t}$, as specified in equation (3.1)[7].

It is assumed that neither the current aggregate productivity growth g_t, nor the additive idiosyncratic productivity shock to labor $\varepsilon_{i,l,t}$ is observed by the firm. Firms learn about the state of the economy, that is on g_t, by receiving two types of signals. The first signal, is the firm's own privately observed idiosyncratic output. Rewriting the firm's output as a signal on g_t, one obtains:

$$s_{i,t} = \frac{1}{l_{i,t}} \left(\frac{y_{i,t}}{G_t^{1-\alpha} z_t k_{i,t}^{\alpha}} \right)^{\left(\frac{1}{\nu-\alpha}\right)} = g_t + \frac{\sigma_l}{l_{i,t}} \varepsilon_{i,l,t}. \tag{3.2}$$

Thus, the precision of the private signal is $l_{i,t}^2 \sigma_l^{-2}$, which is time varying and increases with the amount of labor the firm rents. This assumption makes some intuitive sense: bigger firms have better access to information due to more operating branches, and access to different segments of the market. The firm's output can be written in terms of the observed signal: $y_{i,t} = G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha} (s_{i,t} l_{i,t})^{\nu-\alpha}$.

The second signal, which is analyzed next, is the publicly observed level of lagged aggregate productivity growth g_{t-1}. It is assumed that as the firm lives on an island, the only information its manager observes is what is shipped by its owner. In other words, each firm can observe aggregate productivity, sent from its owner to the island, and all other aggregate quantities or prices without restriction, in a lag of one period.

[7] Solving a version of the model in which $\varepsilon_{i,l,t}$ is pre-multiplied by $\sigma_l \sqrt{l_{i,t}}$ yields quantitatively very similar results.

With a mass of firms, the assumption that aggregate consumption growth, or output growth are fully observed by the firm in a one-period lag, is equivalent to assuming that g_{t-1} is observed in a lag with certainty. The intuition is that at the aggregate level, all idiosyncratic shocks are diversified, thus fully revealing aggregate productivity growth. Aggregate consumption growth at time t, for instance, would be a function of g_t and the distribution of capital and labor. Assuming that the distribution of capital becomes known to the manager in a lag, once the distribution of resources is fixed, consumption is monotonically increasing with g_t. Hence, one can find a one-to-one mapping between aggregate consumption growth level, and g_t, conditioning on the distribution of capital and labor. The conclusion is therefore that observing aggregate real quantities in a lag does not provide any further information on today's g_t, beyond observing g_{t-1} directly, which is sent to the firm at the beginning of period t.

By equation (3.3.1), g_{t-1} can be perceived as a public signal on g_t with fixed precision, where the mean of the signal is $(1-\rho_g)g_0 + \rho g_{t-1}$ and the precision is σ_g^{-2}, by the assumption of homoscedastic shocks. As firms cannot obtain any information on the current level of g_t that is not contained in g_{t-1}, this public signal determines the *common* prior for all firms on g_t, at the beginning of the period.

At the beginning of each period, the firm first produces using its capital and labor stocks that are predetermined in the last period. Then, using the public signal g_{t-1}, and using its own private idiosyncratic signal (its output, or alternatively $s_{i,t}$), it forms a posterior belief on what today's level of g_t is. Using this belief, the firm picks its level of next period capital $k_{i,t+1}$, that is, the firm chooses its investment level, and also hires its next period labor force, $l_{i,t+1}$.

The private and the public signals the firm obtains can be collapsed into one posterior belief, that weights the private and the public information with their respective relative precisions. By Bayes rule, the weight the firm will put on the private signal $s_{i,t}$, and on the public signal, are given by:

$$w_{private,i,t} = \frac{l_{i,t}^2 \sigma_l^{-2}}{l_{i,t}^2 \sigma_l^{-2} + \sigma_g^{-2}}; \quad w_{public,i,t} = 1 - w_{private,i,t}. \tag{3.3}$$

In bad times, when aggregate TFP growth is smaller, $l_{i,t}$ is on average smaller as firms optimally choose to scale down, invest less, and hire less labor. Consequentially, expression (3.3) demonstrates that the firm puts more weight on the public information in recessions, and less on its own idiosyncratic signal. Thus, posterior beliefs are becoming more correlated among firms in recessions, triggering a higher correlation between the policies of firms, and contributing to a higher aggregate volatility.

The manager is trying to maximize the firm's value, given his own public and private information. The information set of the manager at the beginning of the period t, right after producing, is given by: $k_{i,t}$, the firm's capital, $l_{i,t}$, the firm's labor, $s_{i,t}$, the productivity signal obtained from the firm's private output, g_{t-1}, the public signal, and lagged level of aggregate productivity G_{t-1}. Given this information set, the manager solves the following maximization problem:

$$
\begin{aligned}
V_{i,t}(k_{i,t}, l_{i,t}, G_{t-1}, g_{t-1}, s_{i,t}, z_{i,t}) \quad = \quad & max_{k_{i,t+1}, l_{i,t+1}} \quad G_t^{1-\alpha} z_{i,t} k_{i,t}^{\alpha} (s_{i,t} l_{i,t})^{\nu-\alpha} - w_t l_{i,t} - I_{i,t} \\
& \qquad -\Phi_L(l_{i,t}, l_{i,t+1}) \\
& \qquad +\beta E_t[V_{i,t+1}(k_{i,t+1}, l_{i,t+1}, G_t, \hat{g}_t, s_{i,t+1}, z_{i,t+1})] \\
& \text{s.t.} \\
& k_{i,t+1} = (1-\delta)k_{i,t} + \Lambda(\frac{I_t}{k_t}) \\
& \Lambda(i) = \frac{\alpha_1}{1-\frac{1}{\zeta}}(i)^{1-\frac{1}{\zeta}} + \alpha_2 \\
& \Phi_L(l_{i,t}, l_{i,t+1}) = G_t \frac{\kappa}{2}(l_{i,t+1} - l_{i,t})^2 \\
& w_t = G_{t-1} w \\
& V_{i,g,t} = [\frac{1}{\sigma_g^2} + \frac{l_{i,t}^2}{\sigma_l^2}]^{-1} \\
& \mu_{i,g,t} = V_{i,g,t}[\frac{1}{\sigma_g^2}((1-\rho_g)g_0 + \rho_g g_{t-1}) + \frac{l_{i,t}^2}{\sigma_l^2}(s_{i,t})] \\
& \hat{g}_t = \mu_{i,g,t} + \sqrt{V_{i,g,t}}\varepsilon_{i,\mu,t}; \quad G_t = G_{t-1} g_{t-1} \\
& s_{i,t+1} = [(1-\rho_g)g_0 + \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}] + \frac{\sigma_l}{l_{i,t+1}} \varepsilon_{l,t+1} \\
& \varepsilon_{i,\mu,t} = (g_t - \mu_{i,g,t})/V_{i,g,t} \sim N(0,1), \qquad (3.4)
\end{aligned}
$$

where $\mu_{i,g,t}$ and $V_{i,g,t}$ are the posterior mean and variance (uncertainty) of the belief on g_t. $\hat{g}_t$, the stochastic belief on g_t, is defined as $\hat{g}_t = \mu_{i,g,t} + (g_t - \mu_{i,g,t}) = \mu_{i,g,t} + \sqrt{V_{i,g,t}}\varepsilon_{\mu,i,t}$, where $\varepsilon_{\mu,i,t} \sim N(0,1)$. When computing the continuation value, the manager uses his belief $\hat{g}_t$ to project the evolution of all variables that are contingent on g_t, including future aggregate and private signals.

3.3.3. Household

There is one infinitely lived representative household in the economy, that holds all firms, and exerts utility from a consumption stream of C_t. It is assumed that the household is risk neutral. The time discount rate of the household is β. The household derives income from dividend payments from its diversified portfolio of corporate stocks.

After firms produce, and ship back their dividend to the household, the representative household gets to observe the output of all firms, comprising together a mass of signals $\{s_{i,t}|i \in [0,1]\}$ on g_t with finite precisions. As a result, the household becomes perfectly informed about aggregate productivity growth by the end of period t, and consequentially, she sends the recovered g_t to the managers at the beginning of period $t+1$. In other words, fully learning the value of g_t by the household occurs at the end of the period. This assumption captures the notion that collecting the data from a mass of individual firms, and analyzing it to extract productivity growth requires some time and effort. This assumption also ensures that the household information regarding the fundamentals, at the beginning of period t when prices are set, is not better than that of the managers who operate on the islands. The information set of the household, at the beginning of period t is therefore any aggregate real quantity shipped back from the firms, including aggregate output, capital, and labor growth rates, and lagged aggregate productivity.

3.3.4. Equilibrium

An equilibrium is comprised of capital and labor policies for each firm $i \in [0,1]$, $k^*_{i,t+1}$ and $l^*_{i,t+1}$, and firm valuations $V_{i,t}$, such that:

1. Given the information set of the manager, the policies k^*_{t+1} and l^*_{t+1} solve the firm problem in (3.4).

2. Markets clear: aggregate consumption satisfies, $C_t = \int_{i\in[0,1]} y_{i,t} - I^*_{i,t}$.

3. The valuation of a firm i is given by $V_{i,t}$.

3.4. Data and Volatility Measures

3.4.1. Data

I collect both annual and quarterly data on real macroeconomic aggregate growth rates, from 1946 to 2013. Annual time-series are used for calibration purposes, while the higher frequency quarterly time-series are used for the construction of aggregate volatility measures. While some aggregate time-series span longer into pre-war era, I use only postwar data to ensure that all aggregate time-series correspond to the same time span, given the availability of the data. Consumption and output data come from the Bureau of Economic Analysis (BEA) NIPA tables. Consumption corresponds to the real per capita expenditures on non-durable goods and services and output is real per capita gross domestic product. Quarterly time-series are seasonally adjusted. Data on capital and investment are taken from the Flow of Funds for all private non-financial corporate businesses. Capital corresponds to total assets, and investment corresponds to total capital expenditures[8]. CPI data are taken from the Federal Reserve Bank of St. Louis. The real per-capita growth rate of capital, is computed by dividing capital by the mid-point population estimate from NIPA tables, and subtracting inflation obtained from CPI data. Annual and quarterly Data on Average Weekly Hours of Production per worker are taken from the Bureau of Labor Statistics (BLS). Data on TFP growth are obtained from the San-Fransisco Federal Reserve Bank. All aggregate growth time-series, including investment to capital ratio, are in log form.

To obtain cross-sectional data, for the purposes of constructing cross-sectional volatility and between-firm correlation measures, I use quarterly Compustat data. To construct a cross-sectional menu of assets, I group Compustat firms into industry portfolios. I choose to work with industry portfolios, instead of firm-specific data, as this reduces the amount of noise and measurement error in each individual asset time-series, and mitigates biases that may result from entry and exit of firms. Notice further, that there are no shifts of individual firms between portfolios over time. Industry portfolios are formed using the SIC code definitions as in Fama-French Data Library, for 38 industry portfolios. I exclude financial and utility industry firms from the sample, and hence, left with 31 industry portfolios. I use sales, capital expenditures, and total assets as proxies for firms' output, investment and capital.

[8]Though there are other suitable variables to measure investment, the use of capital expenditures allows better comparison to Compustat data in which capex is also available.

Industry levels of output, investment, and capital are therefore defined as the sum of the total sales, capex, and assets levels, for all firms within the industry at time t. Industry sales, total assets, and capital expenditures time-series begin at 1966-Q1, 1975-Q1, and 1985-Q1, respectively. Prior to these starting dates, some portfolios, or all, have missing observations. All industry time-series end at 2013-Q4. As data are quarterly, they exhibits strong seasonality. I remove seasonality from industry level time-series, by using X-12-ARIMA filter at the quarterly frequency. The real growth rates of the seasonally adjusted time-series are then computed by subtracting the quarterly inflation rate.

3.4.2. Measurement of Aggregate and Cross-Sectional Conditional Volatilities

To measure the conditional volatility of an aggregate time-series, in the data or in the model, we first need to specify the information set of the econometrician at time t. To ensure the construction of the conditional volatility in the data is identical to the construction procedure within the model, I assume that the information available to the econometrician is the same as the information set available to the household at time t. Differently put, our household, who collects the data from individual firms, and publishes aggregate quantities, is the econometrician.

Let ΔX_t be the *log* growth of some aggregate time-series ($\Delta X_t = log(\frac{X_t}{X_{t-1}})$). To compute the conditional volatility $V_t(\Delta X_{t+1})$, I follow two steps. First, I remove the conditional *mean* of the time-series by projecting future ΔX_{t+1} on a set of time t predictors $\mathbf{Z}_t$:

$$\Delta X_{t+1} = b_0 + b_x' \mathbf{Z}_t + \varepsilon_{x,t+1}, \tag{3.5}$$

where $\varepsilon_{x,t+1}$ captures the conditionally demeaned, or innovation time-series of ΔX_t. Second, I project future squared innovations on their own lag, and the same set of time t predictors $\mathbf{Z}_t$:

$$\varepsilon^2_{x,t+1} = \nu_0 + \nu_x' [\varepsilon^2_{x,t}, \quad \mathbf{Z}_t] + error, \tag{3.6}$$

and take the *fitted value* of the projection above as the ex-ante conditional volatility of

ΔX_{t+1}, that is, $V_t(\Delta X_{t+1}) = \nu_0 + \nu_x'[\varepsilon_{x,t}^2, \quad \mathbf{Z}_t]$. In the benchmark implementation of the above procedure, both in the model and in the data, the set of the benchmark predictors Z_t includes real aggregate log output growth ΔY_t, real aggregate log capital growth ΔK_t, real aggregate log labor growth ΔL_t, real aggregate log investment to capital ratio I/K_t, and the log lagged productivity growth rate. This information set is equivalent to all aggregate variables that are observed by the household at the beginning of period t. Although this information set is log-linear in the underlying state variables, I find that it maximizes the Akaike Information Criterion of projection (3.5), and the results are robust to the inclusion of higher order powers of the underlying aggregate state variables.

Similarly, let $\Delta x_{i,t}$ be the *log* growth of some single-firm (indexed by $i \in [1, ..., N]$) time-series (or alternatively, some single-industry portfolio i time-series in the data), where N is the number of individual assets in the sample. To measure the conditional volatility of a one-firm i time-series, I follow a similar procedure. At the firm stage, I remove the conditional mean of the one-firm time-series by projecting future one-firm growth rates on their own lag and the set of predictors $\mathbf{Z}_t$:

$$\Delta x_{i,t+1} = b_{i,0} + b_{i,x}'[\Delta x_{i,t}, \quad \mathbf{Z}_t] + \varepsilon_{i,x,t+1}. \tag{3.7}$$

Next, the ex-ante conditional one-firm i volatility $V_t^{one-firm}(x_{i,t+1})$ is the fitted value of the predictive projection:

$$\varepsilon_{i,x,t+1}^2 = \nu_{i,0} + \nu_{i,x}'[\varepsilon_{i,x,t}^2, \quad \mathbf{Z}_t] + error. \tag{3.8}$$

Measuring the conditional covariation between two firms' time-series, $\Delta x_{i,t}$ and $\Delta x_{j,t}$ ($i, j \in [1, .., N]$), involves a two-stage procedure, consistently with the conditional volatility measurements. First, the conditional mean is removed from $\Delta x_{i,t}$ and $\Delta x_{j,t}$, by applying the projection (3.7) twice: once for firm i, and once for firm j. The first stage provides two demeaned (innovation) time-series $\varepsilon_{i,x,t}$ and $\varepsilon_{j,x,t}$. Second, I project the interaction of future firm i and firm j shocks, $\varepsilon_{i,x,t+1}\varepsilon_{j,x,t+1}$, on its own lagged value, and the set of predictors $\mathbf{Z}_t$:

$$\varepsilon_{i,x,t+1}\varepsilon_{j,x,t+1} = c_0 + c_x'[\varepsilon_{i,x,t}\varepsilon_{j,x,t}, \quad \mathbf{Z}_t] + error, \tag{3.9}$$

The ex-ante conditional covariation, is obtained from the fitted value of the above projection: $COV_t(\Delta x_{i,t+1}, \Delta x_{j,t+1}) = c_0 + c_x'[\varepsilon_{i,x,t}\varepsilon_{j,x,t}, \quad \mathbf{Z}_t]$.

Lastly, the dispersion of a growth variable Δx, at time t, is directly computed as the *cross-sectional* variance of $\{\Delta x_{i,t}|i = 1..N\}$, that is: $DISP_t(\Delta x_t) = V_n(\Delta x_{i,t})$. The residual (or ex-post) dispersion of a growth variable is defined as the cross-sectional variance of the innovations $\{\varepsilon_{i,x,t}|i = 1..N\}$ at time t, or: $DISP_t(\varepsilon_{x,t}) = V_n(\varepsilon_{i,x,t})$.

3.5. Calibration and Unconditional Moments

3.5.1. Parameter Choice

Table 3.1 reports the parameters that I use for the benchmark calibration of the model, under risk neutrality. The model is calibrated at a quarterly frequency. Some choices of the production parameters are dictated by standard choices in macroeconomics. I set the degree of returns to scale to $\eta = 0.9$ consistent with Basu and Fernald (1997) and Gomes et al. (2009). The elasticity of capital input is $\alpha = 0.22$, generating a capital share of output $\frac{\alpha}{\nu}$ of approximately 25%, and a share of labor of 75%. I select a depreciation rate of capital to be a conservatively standard value of $\delta = 2\%$, or an effective rate of 8.2% at an annual frequency, consistently with the annual depreciation rate of capital in the data. This depreciation rate yields an annual investement-to-capital ratio of about 10%, which is comparable with the data.

The key parameters that affect the learning ability are the standard deviations of aggregate productivity and noise shocks. The standard deviation of aggregate productivity shock determines the amount of prior uncertainty a manager has regarding today's level of productivity growth g_t. I set the standard deviation of aggregate productivity shock at a relatively high value for quarterly frequency, of $\sigma_g = 0.02$. Calibrating this parameter at lower, more conservative value, reduces the ability of the model to amplify conditional volatilities via learning, as the prior uncertainty becomes too small to provoke a significant impact.

The standard deviation of aggregate productivity is given by $\sqrt{\sigma_g^2/(1-\rho_g^2)}$, where ρ_g is the autocorrelation parameter of aggregate productivity. To ensure this standard deviation is not too high (in the presence of high σ_g), I then need to pick a relatively small value for ρ_g. I set $\rho_g = 0.5$, which then implies a standard deviation of 2.3% for aggregate productivity. While this standard deviation is still high, setting ρ_g at significantly lower values then implies uncrealistically low autocorrelations for real growth rates in the model.

Since the autocorrelation parameter is now relatively low, while the standard deviation of aggregate productivity is large, I smooth consumption and output growth using the adjustment costs parameters. I set the adjustment cost parameter of capital to $\zeta = 1.2$, comparably with $\zeta = 0.8$ in Kung and Schmid (2014). The adjustment cost of labor is set to $\kappa = 7$. These adjustment costs facilitate targeting the standard deviation of output growth, and the autocorrelation of consumption and output growth rates. Notably, the adjustment costs for labor are quite large. I introduce this adjustment cost, to target the autocorrelation of consumption. It is crucial in the absence of consumption smoothing in a risk neutral setup.

The standard deviation of the labor efficiency (noise) shock σ_l, governs the posterior uncertainty a manager has regarding today's level of productivity growth g_t. Consequentially, this parameter governs the amplification of the conditional volatility of real quantities in bad times. Naturally, a choice of noise close to zero yields no amplification at all, as we are back in a perfect information case. I pick $\sigma_l = 0.265$, to target the increase in consumption's conditional volatility in bad times.

I set the (gross) growth of aggregate productivity g_0 to 1.005, ensuring that annual consumption growth is approximately 2%. In a risk neutral setup, the discount rate parameter

β must satisfy $\beta g_0 < 1$, to ensure that the detrended value function is a contraction. I therefore pick a value of $\beta = 0.994$. This implies an annual real risk free rate of slightly above 2%.

As wages are exogenously specified, I set the (detrended) wage for labor as a numéraire, with $w = 1$. Lastly, the idiosyncratic demand shock parameters $\sigma_z = 0.01$ and $\rho_z = 0.9$ are set to approximately match the correlation between output and investment-rate dispersions with the business cycle (that is, with TFP growth).

3.5.2. Model Numerical Solution and Implications for Unconditional Aggregates

I solve the model using a second order perturbation method, as in Judd (1998)[9]. To solve the model, I detrend the growing model variables by the lagged value of the stochastic productivity trend. Details regarding detrending the firm problem, are provided in the Appendix. I simulate the model at the quarterly frequency for 100,000 quarters, after truncation to remove dependence on initial values. I simulate a cross section of 10,000 firms, to ensure that all idiosyncratic shocks are diversified at the aggregate level. Aggregate model-implied level time-series, of capital, labor, output, consumption and investment, are obtained by averaging the respective firm-level quantities over all firms.

To facilitate the comparison between the benchmark model (with Bayesian learning) and the data, I also solve a version of the model without any learning. This no-learning model specification is identical to the learning model. Namely, the production function including the labor efficiency shock, the evolution of capital, and the adjustment costs are the same, except for the fact that the firm *knows* every period the true value of g_t (zero prior and posterior uncertainty). The calibration used for the no-learning model is identical to that used for the benchmark learning model, and is specified in Table 3.1.

I report the model-implied unconditional moments of aggregate consumption, output, labor, and capital log-growth rates and the log aggregate investment-to-capital rate, versus their

[9]Solving the model using third-order perturbation method yields similar results.

empirical counterparts in Table 3.2. The simulated quarterly model-implied time-series is time-aggregated to form annual observations, to be compared with the annual data.

For the most part, the moments implied from the model with learning, are close to or match their empirical estimates. The growth rates of aggregate consumption, capital, and output are all roughly 2% in the model and in the data. Log investment-rate is slightly higher in the model than in the data (-2.31 in the model versus -2.91 in the data). The model-implied volatilities align generally well with the data. The volatility of output growth is about 3% in the model and in the data. The volatilities of labor growth and investment-rate and close to their empirical counterparts. Consumption growth has excess volatility in the model (3.1% and 1.4% in the model and data, respectively). However, in the long same of 1930-2012, consumption growth's volatility is 2.2%, and the upper-bound of its volatility 90%-confidence interval of 2.6%, which is much closer to the model. Capital growth is less volatile in the model than the data, due to the effect of adjustment costs, that compensate for the lack of consumption smoothing.

The learning model implied autocorrelations of consumption, output and labor growth fall into the data 90%-confidence intervals. Labor growth is much more persistent in the model at the annual frequency, yet at the quarterly frequency this problem vanishes. In the model, the quarterly auto-correlation of labor is 0.11, and in the data the quarterly auto-correlation of labor growth is 0.23 with a confidence interval of [0.048, 0.419]. Likewise, capital growth is overly persistent in the model. However, the upper-bound of the 90%-confidence interval for quarterly capital growth autocorrelation is 0.75, which is closer to the model quarterly autocorrelation of 0.94. In all, the model is capable of producing reasonable unconditional aggregate moments, in-light of the absense of risk-aversion.

While I do not target any moment implied by the no-learning model (this model bears the same calibration as the learning model for comparative reasons), the no-learning model produces similar moments to the learning model. The volatilities in the no-learning model are slightly higher. This makes intuitive sense: in the no-learning model, all firms share the same belief on the state of the economy, or aggregate TFP growth. As "beliefs" in the no-learning model are perfectly correlated, this increases the correlation between firms policies, in-comparison to the learning model in which beliefs are heterogeneous. As a result of a higher unconditional correlation between firms, aggregate volatilities are higher.

3.6. Results

This section illustrates the implications of the learning model for aggregate and cross-sectional volatilities, in a risk-neutral environment. In section 3.6.1, I show how the learning model is capable of amplifying fluctuations in the conditional volatility of aggregate growth rates, while the no-learning model, produces minute changes in the conditional volatility, which is the main result of this study. Sections 3.6.2 and 3.6.3 are dedicated to decompose the aggregate volatility movements into firm-level volatility and cross-sectional correlation fluctuations. I demonstrate the importance of the endogenous, time-varying correlation channel to produce endogenous shifts in aggregate volatility. Next, section 3.6.4 provides evidence that it is the combination of Bayesian learning, along with asymmetric information, that is responsible for the countercyclical correlation between the growth rates of firms, in-line with the model's economic narrative. Section 3.6.5 explains how non-linearities in the measurement of volatility, can produce small fluctuations in the measured conditional volatility under a homoscedastic environment. This section illustrates that the no-learning model is isomorphic to a constant conditional volatility world. Section 3.6.6 demonstrates that dispersion in the learning model is by large countercyclical, in spite of an increase in the conditional correlations in bad times, and reconciles the two. Finally, section 3.6.7 deals with the robustness of the results.

3.6.1. Implications of Learning for Aggregate Conditional Volatility

The learning model is capable of generating fluctuations in the conditional volatility of aggregates, that are much larger than those produced by a no-learning model, and are also comparably close to the magnitude of fluctuations observed in the data. Table 3.3 demonstrates this claim. The table shows by how much the conditional volatility of macroeconomic variables of interest, increases or decreases, in bad times compared to normal periods. Likewise, the table shows the fluctuations in the conditional volatility in good times compared to normal ones. Bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The table presents the volatility fluctuations induced by quarterly data from the learning benchmark model, as well as from a no-learning model, and empirical estimates of the fluctuations in the data.

In the data, the conditional volatility of real macroeconomic variables is clearly countercyclical. For all variables, including output and consumption growth, the volatility is higher (lower) in bad (good) times, in comparison to normal times. For all variables, except for the investment rate, the rise (drop) in the conditional volatility in bad (good) times is significantly above (below) zero, as can be seen from the confidence intervals. The magnitude of the positive (and significant) fluctuations in volatility in bad times ranges from an increase of 30% to 56%. Specifically, the estimated increase in output's (GDP) conditional volatility in bad times is about 30%. This figure aligns well with Bloom (2014), who finds that quarterly GDP and industrial production growth, has about 35% more conditional volatility in NBER recessions.

In the learning model, almost all of the oscillations in the conditional volatility in good and bad times for the variables of interest, fall into the empirical 90% confidence intervals. For some variables the fluctuations induced by the model are very close to the data point-estimates. For example, capital's growth volatility rises in bad times by 57% and 56% in the model and in the data, while it drops in good times by 41% and 47% in the model and the data. Consumption growth's volatility increases in bad times by 29% in the model versus 32% in the data, and it falls in good times by 20% and 14% in the model and the data, respectively. For the investment rate, the model tends to overstate the fluctuations in volatility, compared to the data. In the learning model, the magnitude of the positive fluctuations in volatility in bad times ranges from an increase of 29% to 58% [10].

By contrast, the no-learning model-implied volatility oscillations are muted, not only in comparison to the learning model, but also in comparison to the data. The positive fluctuations in volatility during bad times range from an increase of 1.8% to 4.3%, outside the data confidence intervals. Similarly, the fluctuations in good times are mixed in sign, and range from -1.5% to 0.76%.

Two questions arise. First, and most importantly, what triggers the large volatility fluctuations in the learning model? This questions is addressed in the following sections 3.6.2 - 3.6.4. Second, why are the volatility fluctuations in the no-learning model very small, and yet, non-zero? I provide an answer in section 3.6.5.

3.6.2. Implications of Learning for Average Between-Firm Conditional Covariation

The fluctuations in the conditional volatility of aggregates in the model, reported in Table 3.3, capture movements in the average conditional covariation between firms.

To see this, notice that if X is an aggregate variable, x_i is a firm level (single-firm indexed by i) variable, and N is the number of firms in the cross-section, then:

$$\begin{aligned} V_t(X_{t+1}) &= V_t(\frac{1}{N}\sum_{i=1}^{N} x_{i,t+1}) \\ &= \frac{1}{N^2}\left(\sum_{i=1}^{N} V_t(x_{i,t+1}) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} COV_t(x_{i,t+1}, x_{j,t+1}).\right) \end{aligned} \tag{3.10}$$

Denote the average conditional one-firm volatility as $\overline{V_t^{one-firm}}(x_{i,t+1}) = \overline{\sigma^2_{ii,t}}$ (all firms are ex-ante identical), and the average conditional covariation as $\overline{COV_t}(x_{i,t+1}, x_{j,t+1}) = \overline{\sigma_{ij,t}}$.

[10] The learning model is also capable of generating fluctuations in the conditional volatility of aggregate labor growth. For example, the conditional volatility of aggregate labor growth rises by 63% in the model, and by 75% in the data.

Then, the expression in (3.10) can be written as:

$$V_t(X_{t+1}) = \frac{1}{N^2}\left(N\overline{\sigma_{ii,t}}^2 + N(N-1)\overline{\sigma_{ij,t}}\right). \tag{3.11}$$

With a mass of atomistic firms, $N \to \infty$ and $V_t(X_{t+1}) \to \overline{\sigma_{ij,t}}$ [11]. That is, the aggregate volatility equals the average between-firm covariation. This claim therefore implies that the aggregate volatility oscillations in Table 3.3, are driven by fluctuations in the conditional covariation.

While this claim is straightforward algebraically, I provide direct evidence that this claim holds in the model. I construct a measure of the changes in the (average) conditional pairwise covariation between firms in the model (and data). The methodology of constructing the pairwise covariation is described in section 3.4.2.

Table 3.4 shows by how much the conditional between-firm pairwise covariation of variables of interest, increases or decreases, in bad times, and in good times, compared to normal periods. As in the previous section, bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The table presents the covariation fluctuations from the learning model, and empirical counterparts.

In the model, the changes in the covariation as reported in Table 3.4, coincide with the fluctuations in aggregate volatility reported in Table 3.3. All oscillations are identical, up to the units digit. Notice that the fluctuations in Table 3.3 are based on aggregate time-series only, while fluctuations in Table 3.4 are computed using firm-level data only. This

[11] Equation (3.10) is an approximation when the aggregate variable X is a growth rate, not a level. The exact decomposition for growth rates is as follows:

$$V_t(\Delta X_{t+1}) \quad = \quad V_t(\sum_{i=1}^{N} w_{i,t}\Delta x_{i,t+1}),$$

where $w_{i,t} = \frac{x_{i,t}}{\sum_{j=1}^{N} x_{j,t}}$. Hence, the aggregate volatility of a growth rate converges to the average "value-weighted" covariation between-firms. When firms are atomistic, this equals approximately to the "equal-weighted" covariation between-firms.

exercise demonstrates that the methodology used in this study to measure the unobserved ex-ante conditional volatility and covariation satisfy equation 3.11. In unreported results, I verify that the conditional aggregate volatility fluctuations in the no-learning model, are also identical to the oscillations in covariations.

In the data, the fluctuations in the conditional covariations are countercyclical: covariation rises in bad times, and drops in good times. Perhaps surprisingly, the fluctuations in the empirical pairwise covariations, are very close in magnitude to the fluctuations in the empirical aggregate volatility. For example, the conditional volatility of aggregate output in the data rises by 29.8% in bad times, while the increase in the average covariation between firms' outputs in those periods is 28.9%. Similarly, the empirical conditional volatility of aggregate capital growth, and the conditional covariation of capital growth rates, rise by 56.2% and 55.8%, respectively. Given that in the data some firms are non-atomistic, as illustrated in Gabaix (2011), the similarity of the figures is non-trivial.

What causes the conditional covariation to rise in bad times, and drop in good times? The next section provides an answer.

3.6.3. Aggregate Volatility Decomposition: Implications for Average Conditional Correlations

Sections 3.6.1 and 3.6.2 show that the conditional aggregate volatility is countercyclical in the learning model, due to an increase in the conditional covariation between firms in bad times. In this section, the aggregate volatility (or average covariation) is decomposed into firm-level volatility and average between-firm correlation. This decomposition yields that:

A. In the model without learning, the fluctuations in the conditional volatility of aggregates (or alternatively, in the conditional between-firm covariation), are purely due to small changes in the conditional one-firm volatility. The average correlation between firms is fixed.

B. In the model with learning, the fluctuations in the conditional volatility of aggregates (or alternatively, in the conditional between-firm covariation), are largely due to shifts in the conditional correlation between firms, that rises in bad times.

Let x_i be a firm-level variable. Denote, as before, the average conditional one-firm volatility as $\overline{V_t^{one-firm}(x_{i,t+1})}$, and the average conditional correlation between firms as $\overline{CORR_t(x_{i,t+1}, x_{j,t+1})}$. Using equation (3.11), the volatility of the aggregate variable X can be decomposed as:

$$V_t^{agg}(X_{t+1}) \approx \overline{COV_t(x_{i,t+1}, x_{j,t+1})} = \overline{V_t^{one-firm}(x_{i,t+1})} \cdot \overline{CORR_t(x_{i,t+1}, x_{j,t+1})}.$$

As a consequence, the oscillation in aggregate volatility between bad and normal times is equal to the fluctuation in firm level volatility multiplied by the fluctuation in the average between-firm correlation, between bad and normal times:

$$\frac{\overline{V_t}(\cdot|\text{Bad})}{\overline{V_t}(\cdot|\text{Normal})} \approx \frac{\overline{V_t^{one-firm}}(\cdot|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot|\text{Normal})} \cdot \frac{\overline{CORR_t}(\cdot|\text{Bad})}{\overline{CORR_t}(\cdot|\text{Normal})}. \tag{3.12}$$

A similar decomposition can be made for good versus normal period oscillations. Thus, if the fluctuations in the aggregate volatility are *very close* to those in the one-firm volatility, there are no fluctuations in the conditional correlation. However, if the fluctuations in aggregate volatility *differ* from the one-firm volatility movements, this indicates shifts in the conditional correlation between firms.

Tables 3.5 and 3.6 respectively show by how much the average one-firm conditional volatility, and the average between-firm correlation of variables of interest, fluctuate in bad times and in good times compared to normal periods. As before, bad, normal and good times refer to periods is which the aggregate TFP growth is between its 0-25th, 25-75-th, and 75-100th percentiles, respectively. The tables present oscillations induced by model-implied quarterly data from the learning benchmark model, and from a no-learning model.

Comparing the figures of Table 3.3 and Table 3.5 in the no-learning case, reveals that the fluctuations in the aggregate and one-firm volatility are small and roughly the same. As a result, the fluctuations in the average conditional between-firm correlations are minuscule, as illustrated in Table 3.6.

In contrast, comparing Tables 3.3, 3.5 and 3.6 in the learning case, demonstrates that the one-firm volatility fluctuations are *amplified* by a counter-cyclical movement in the conditional correlation between firms. The conditional correlation between firms' outputs rises by 32% in bad times, and drops by 29% in good times. For investment rate, the conditional correlation increases (drops) by 27.5% (28.8%) in bad (good) times. In fact, for output growth, about 90% of the increase in the conditional aggregate volatility in bad times is attributed to an increase in the conditional correlations. For capital growth and the investment rate, the oscillation in the conditional correlation explains about 80% of the contemporaneous increase in the aggregate volatility. These numbers are comparable to the findings of Veldkamp and Wolfers (2007), who decompose (unconditional) aggregate volatility into sector specific volatility, and comovement of sectors, and attribute about 80% of aggregate volatility to the comovement term.

3.6.4. The Role of Bayesian Learning and Asymmetric Information for Correlation Fluctuations

The fluctuations in the conditional correlations between firms, that drive the conditional aggregate volatility in the learning model, are a result of the Bayesian learning and Asymmetric information: in the bad states, firms put more weight on public (common) information, and less on private (idiosyncratic) information. An increase in the correlation between the posterior belief of firms, triggers policies that comove more, and making aggregate growth rates more volatile. The Tables in this section provide evidence in support of these claims.

First, I solve a modified learning model, having the same calibration as the benchmark learning model, but in which the (noise) shocks to labor efficiency, $\varepsilon_{i,l,t}$ are *aggregate* shocks.

In other words, the shocks $\varepsilon_{i,l,t}$ are i.i.d over time, but the same over all firms, and hence can be denoted by omitting the i index as $\varepsilon_{l,t}$. Now, privately observed signals $s_{i,t}$, obtained from firms' output, all have the same ex-port bias (per unit of labor), driven by the aggregate shock $\varepsilon_{l,t}$. Thus, in this model, both the lagged value of productivity growth g_{t-1}, *and* the signal $s_{i,t}$ obtained from firms' output, are driven by public-common information shocks. Importantly, there is still learning: the ex-ante and ex-post uncertainties about g_t are positive, and the weights on the private and public signals are still time-varying with the amount of rented labor. Yet, as no signal is idiosyncratic, shifts in the weights placed on the public and the private signals should not trigger significant changes in the average correlation between firms' posterior (or policies), as there are no effective informational asymmetries.

The results of the no-informational asymmetries model, for aggregate volatility and average correlation fluctuations in bad and good times compared to normal periods, are shown in Table 3.7. As conjectured, the correlation fluctuations are all close to zero. As a result, the fluctuations in aggregate volatility are small, and all range between 0.3% to 0.6% in absolute value. Notice that the speed of learning in this model is procyclical, in a similar fashion to the model of Van Nieuwerburgh and Veldkamp (2006), yet without asymmetric information, the model is not capable of producing significant fluctuations in *volatility.*

Second, suppose the learning model is altered such that there are both public signals (g_{t-1}) and private signals ($s_{i,t}$, driven by idiosyncratic shocks), but learning is not Bayesian. That is, I fix the gains (the weights) on the public and private signals at their steady state values. The posterior mean $\mu_{i,g,t}$ and variance $V_{i,g,t}$ on g_t satisfy:

$$\begin{aligned}\mu_{i,g,t} &= \overline{w}_{public}((1-\rho_g)g_0+\rho_g g_{t-1})+\overline{w}_{private}\left(s_{i,t}\right),\\ V_{i,g,t} &= \left\{\sigma_g^{-2}+l_{ss}^2\sigma_l^{-2}\right\}^{-1},\end{aligned}$$

where:

$$\overline{w}_{public} \;=\; \frac{\sigma_g^{-2}}{\sigma_g^{-2} + l_{ss}^2 \sigma_l^{-2}}, \quad \overline{w}_{private} = 1 - \overline{w}_{public},$$

and where l_{ss} is the steady-state level of labor (ex-ante, it is identical for all firms). In this model, there is still learning (posterior uncertainty is positive), and there is still asymmetric information, hence belief heterogeneity. However, since the weight on public common information is fixed, in bad times firms *do not* place, by construction, more weight on common information. Consequentially, the correlation between firms should not fluctuate. Table 3.8 demonstrates that this is indeed the case. The correlation oscillations between good and bad times versus normal periods are minuscule, and thus, aggregate volatility fluctuations are small. The fluctuations in the conditional aggregate volatility are quite close to the no-learning case, as reported in Table 3.3. For instance, aggregate output volatility increases in bad times by 4.4% and 4.3% in the Non-Bayesian learning and No-learning models, respectively, while the volatility drops by 1.5% and 2.1% in good times in these two models, respectively.

Importantly, the oscillations in aggregate volatility in the No-Information Asymmetry model or in the Non-Bayesian learning models, should not coincide precisely with the no-learning model results: in both cases there is still some posterior uncertainty that can deviate the results from the exact full-information case. These two alternated learning model illustrate the importance of two separate model ingredients: (1) Bayesian leaning, with time varying gains, and (2) Informational asymmetry.

Next, I solve the benchmark learning model (with Bayesian learning and Asymmetric information), but calibrated with different standard deviation for the noise labor efficiency shock (changing σ_l). All other model parameters are calibrated as in the benchmark calibration outlined in Table 3.1. Panel A of Table 3.9 presents the results for four noise levels: $\sigma_l \in \{0.3, 0.265 \text{ ``benchmark-level''}, 0.2, 0\}$. Intuitively, the less noise (smaller σ_l), the closer the model is to the no-learning case, and the amplification effect on aggregate volatility induced by average correlation fluctuations become smaller. Aggregate volatility and average correlation fluctuations, in good and bad times, monotonically decrease in absolute value with the noise level. In the case where $\sigma_l = 0$, the private signal is perfectly revealing of the fundamental. As a consequence, the results for the learning and no-learning models coincide, despite different model first-order-conditions in the two cases, as shown in Panel B of Table 3.9.

3.6.5. Volatility Fluctuations in the No-Learning Model: Falsification Tests

Table 3.3 shows that the oscillations in the conditional volatility of aggregates between good and bad states, in the no-learning model are very small, yet non-zero. This section demonstrates that the small changes in the aggregate conditional volatility in the no-learning model, are a result of some non-linearities in the econometric construction of the conditional volatility, mainly the usage log-growth rates, and the usage of squared residuals in realized-volatility construction. The conclusion is that the no-learning model results *do not differ* from results that one would expect to find in a homoscedastic world.

It is hard to isolate a single source of non-linearity that generates small fluctuations in the aggregate volatility in a no-learning environment. To deal with this issue, I use a "falsification" test. I verify that constant conditional volatility processes, having the same unconditional moments as model-implied aggregate variables, yield the same minuscule fluctuations in the conditional volatility, when using the *econometric* methodology for the construction of volatility, as described in section 3.4.2.

Specifically, let $log(X_t)$ be some log aggregate time-series induced from the model. I calibrate a process $\tilde{X}_t$ of the form:

$$\tilde{X}_t = (1-\rho_x)x_0 + \rho_x \tilde{X}_{t-1} + \beta_g(g_{t-1} - g_0) + \beta_{g,2}(g_{t-1} - g_0)^2 + \sigma_x \varepsilon_{g,t} + \sigma_{x,2}(\varepsilon_{g,t}^2 - 1),$$

where g_{t-1} is the lagged value of productivity growth, and $\varepsilon_{g,t}$ are the shocks to productivity used in the model simulation. The process $\tilde{X}_t$ is calibrated such that the process has the same mean, same standard deviation, same skewness, same correlation with TFP growth, and same correlation with TFP growth squared, as the original model-implied exponentiated (level) time-series X_t process[12]. Then, I construct the conditional volatility fluctuations for $log(\tilde{X}_t)$ time-series, under the econometric methodology of section 3.4.2, while including the lag of $log(\tilde{X})$ as a predictor.

Notice that: (1) the process $\tilde{X}_t$ has, by construction, constant conditional volatility; (2) the shocks to $\tilde{X}$ depend on the shocks to the aggregate TFP only, and are taken from the model simulation (the only aggregate shock in the model is ε_g); (3) the process can depend on the state g in a linear, and non-linear fashion.

The results for aggregate volatility fluctuations, against the no-learning model, are reported in Panel A of Table 3.10. Two main features arise. First, the fluctuations in the volatility for the no-learning model time-series and for the matched homoscedastic processes are quite similar. Second, the fluctuations reported for the matched homoscedastic processes are small yet non-zero. Non-zero results can arise as I apply the log function on the Gaussian process $\tilde{X}$, and as the residuals of $log(\tilde{X})$ are squared in the volatility construction. Both of these are non-linear operations, that introduce some small skewness, which is manifested in small volatility movements. In unreported results, I notice that when I do not use log-growth rates, or use absolute residuals (as opposed to squared residuals) in the volatility construction, the oscillations in the conditional volatility are even smaller.

Next, I repeat the same "falsification" test for the learning model. For each log aggregate time-series $log(X_t)$ given by the learning model, I calibrate a matched process $\tilde{X}_t$, that has the same unconditional moments as X_t, but constant conditional volatility. Panel B of Table 3.10 shows that in this case, the fluctuations in the volatility of $log(\tilde{X}_t)$ are tiny in comparison to the learning model-implied volatility changes. This fact provides further evidence that the learning model results are not spurious.

[12] If $log(X_t)$ is a log aggregate growth time-series, X_t is a gross-growth time-series.

Finally, I consider another source of non-linearity that stems from within the model: the decreasing returns to scale technology ($\nu < 1$). Intuitively, a higher ν implies a closer to linear production function, which then attenuates the non-linearity and the fluctuations in the conditional volatility, both in a learning and a no-learning environments. In unreported results, I find that increasing (decreasing) ν reduces (amplifies) the reported conditional volatility movements.

3.6.6. Implications for Cross-Sectional Dispersion

The volatility implications discussed in the previous sections referred to the conditional volatility, which is the *predictable* variation of future shocks. Another concept of volatility is the *cross-sectional* variance, or dispersion. This section shows that dispersion in the model is counter-cyclical, in spite of an increase aggregate volatility and between-firm correlations. While ostensibly, an increase in dispersion seems to contradict a rise in expected correlation between firms, I reconcile the two in the data. When the average between-firm covariation increases more than dispersion does, correlations increase too, as is also the case empirically.

It is a well-known established fact, that the dispersion of real economic outcomes, such as output growth and earnings growth, is countercyclical (see among others Bachmann and Bayer (2013), Bloom et al. (2012), and Bachmann and Bayer (2014)). There are a few exceptions in the data. Recently, the work of Bachmann and Bayer (2014) showed that the dispersion of investment rate is procyclical.

I construct a time-series of dispersion for log output and capital growth, and investment-rate in the model using the methodology detailed in section 3.4.2. To measure the amount of cyclicality of dispersions, I correlate each dispersion time-series with the business-cycle, namely, productivity growth. A negative correlation indicated counter-cyclical dispersion. The results for the learning and for the no-learning models, along with empirical counterparts are reported in table 3.11.

Empirically, output growth and investment-rate dispersions exhibit a small amount of countercyclicality (their dispersion correlates with productivity by -0.03 and -0.05, respectively). Capital growth dispersion is procyclical, as this is consistent with the finding of Bachmann and Bayer (2014).

In the learning model, all real growth rates exhibit almost the same amount of slight countercyclicality. The correlation of investment-rate dispersion with TFP growth is -0.04 in the model and in the data. Output growth's dispersion is slightly more countercyclical in the model than in the data.

The evidence presented for the countercyclicality of dispersion, coincides with the fluctuations in the one-firm volatility in the model. Dispersion, as discussed below, is a measure of the idiosyncratic one-firm volatility (in the limit, and under certain assumptions, the two are the same). Since one-firm volatility in the learning model rises in bad times, and drops in good times, (see Table 3.5) it explains the model-implied dispersion behavior.

The claim that the between-firm correlations rise in bad times, seems to be, at first glance, at odds with a simultaneous increase in cross-sectional dispersion. The two can be reconciled.

Let $\{x_{i,t}\}_i$ be a cross-section of some variable x time-series. Denote by $\{\varepsilon_{i,x,t}\}_i$ the cross-section of demeaned times-series of x, or equivalently, the shocks to x_i. In the model, the average predictable correlation of $\{\varepsilon_{i,x,t+1}\}_i$ at time t increases in bad periods. Does this contradict a greater cross-sectional dispersion?

To put some structure into the answer, suppose further that one can find some factor structure for the demeaned (shocks) time-series. In other words, assume:

$$\varepsilon_{i,x,t} = \beta_i F_t + e_{i,t},$$

where $e_{i,t}$ and $e_{j,t}$ are independent for $i \neq j$. Assume that $VAR_t(e_{i,t+1}) = \sigma^2_{e,t} \quad \forall i$, and that $VAR_t(F_{t+1}) = \sigma^2_{F,t}$. Here, for simplicity, I assume a single common-factor, F_t, in explaining the residuals of x.

One can write the conditional correlation between the innovations of firms i and j, as follows:

$$\begin{aligned} CORR_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) &= \frac{\beta_i \beta_j VAR_t(F_{t+1})}{\sqrt{\beta_i^2 VAR_t(F_{t+1}) + VAR_t(e_{i,t+1})}\sqrt{\beta_j^2 VAR_t(F_{t+1}) + VAR_t(e_{j,t+1})}} \\ &= \frac{1}{\sqrt{1+\frac{\sigma^2_{e,t}}{\beta_i^2\sigma^2_{F,t}}}} \frac{1}{\sqrt{1+\frac{\sigma^2_{e,t}}{\beta_j^2\sigma^2_{F,t}}}} \end{aligned}$$

As an approximation (or by ignoring β heterogeneity, and denoting the average β as $\overline{\beta}$), the average pairwise correlation can be expressed as:

$$\begin{aligned} \overline{CORR}_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) &= \frac{1}{1+\frac{\sigma^2_{e,t}}{\overline{\beta}^2\sigma^2_{F,t}}}, \\ &= \frac{1}{1+\frac{\sigma^2_{e,t}}{\overline{COV}_t(\varepsilon_{i,t+1},\varepsilon_{j,t+1})}}. \end{aligned}$$

Using a result from Garcia et al. (2011), the dispersion of the residuals $DISP(\varepsilon_{x,t}) = VAR_n(\varepsilon_{i,x,t})$, is a consistent measure of the idiosyncratic volatility $\sigma^2_{e,t} = VAR_t(e_{i,t+1})$, assuming that a factor structure that satisfies the standard arbitrage pricing-theory (APT) assumptions exists. The dispersion provides a consistent measure of idiosyncratic volatility, *without* a need to know the actual underlying factor structure.

Suppose that dispersion rises in recessions. The above decomposition reveals that if the average pairwise between-firm covariation, $\overline{COV_t}$, which equals to the variation of the underlying factors, $(\overline{\beta}^2\sigma^2_{F,t})$, increases more than residual-dispersion, $(DISP(\varepsilon_{x,t}) = \sigma^2_{e,t})$ does in bad times, the average correlation increases too in bad times.

The last claim holds in the data, as shown in Table 3.12. The rise in average covariation in bad times, ranges between 19% to 55%, while dispersion rises by no more than 26%. For all variables, dispersion's increase is always less than the point estimate increase of covariation.

Moreover, dispersion even drops in bad times for some variables.

3.6.7. Robustness

The main result of this study is the ability of the learning model to yield oscillations in the conditional aggregate volatility that are comparably close to the magnitude of volatility fluctuations in the data, and are of a much larger scale than those induced from a no-learning model. I show in this section that this main result is robust to altering some of the benchmark implementation choices.

Throughout the previous sections, I defined the business cycle (namely, good, normal, and bad times) using the TFP growth variable percentiles. I consider other economic outcomes (that vary procyclically) for the business-cycle definition. Table 3.13 reports the aggregate volatility fluctuations, for the learning model, the no-learning model, and empirically, when bad, normal and good times are defined as the 0-25th, 25-75th, and 75-100th percentiles of *aggregate output growth*. Both in the learning model and empirically, the implied fluctuations in the conditional volatility are quite close to the benchmark result of Table 3.3. The magnitude of the positive (negative) fluctuations in the conditional volatility in bad (good) times tend to be larger (smaller) in absolute value when output growth is used to define the cycle, compared to TFP growth. For the no-learning model, the volatility fluctuations are still far less pronounced in comparison to the learning model.

Another modification to the definition of good and bad periods are the percentile breakpoints. In Table 3.14, I still define the business-cycle using TFP growth, but I use more extreme definitions for good and bad times. Good (bad) times, are periods in which TFP growth is between its 90-100th (0-10th) percentiles. Normal times, are periods in which TFP lies between the 10-90th percentiles. The Table shows that, as expected, this modification amplifies, in absolute terms, the changes in the conditional volatility both in the model and in the data. The no-learning model results are largely unchanged. Most of the learning model-implied volatility oscillations still fall into the empirical 90%-confidence intervals.

Lastly, the results are also robust to the predictors used to demean aggregate and firm-level times series, and obtain the ex-ante volatility time-series. In the benchmark specification, I use a log-linear set of predictors $\mathbf{Z}$. I show in Table 3.15, that when the squared-values of the variables in $\mathbf{Z}$ are also added as additional predictors, the model-implied results are almost identical. In the data, adding the non-linear predictors tend to increase the volatility fluctuations in bad times. Empirically, it also causes volatility to increase by a small amount in good times (though volatility in general is still counter-cyclical, not U-shaped). In unreported results, I verify that no-single predictor in the set of predictors $\mathbf{Z}$ is responsible for producing the observed behavior, by dropping a different single predictor each time, and confirming that there are no significant changes in the results.

3.7. Conclusion

The volatility of aggregate fundamentals, such as output and consumption growth, is time-varying and increases in recessions. Recent work in macroeconomics and finance has shown that this volatility is important for recession duration and asset-pricing: it inhibits investment and recovery, and depresses assets' valuation-ratios. Traditional models that examine the impact of volatility on the real and financial economy treat aggregate volatility as an exogenous object. This paper attempts to fill the gap in our understanding of macroeconomic volatility, by proposing a theory of how aggregate volatility arises endogenously in a decentralized economy. The theory suggests that the correlation structure between firms is an important source of macro volatility. When firms do not observe the state of the economy, they learn about it from public information, whose precision is constant over time, and from private idiosyncratic information - their own output, that become more noisy in recessions as firms scale down and produce less information. Consequentially, the correlation in the beliefs of firms about the state of the economy rises in recessions, as firms scale down and put more weight on public information. As a result, the policies of firms become more correlated, contributing to a rise in aggregate volatility.

The study produces some important *quantitative* results. In the learning model, the conditional volatility of aggregate output rises when TFP growth is low (bad times) by 43%, and it drops when TFP growth is high (good times) by 32%. These numbers fall into the 90%-confidence intervals for volatility fluctuations in the data. Likewise, aggregate consumption's volatility increases by 30% in bad times, in the model and also empirically. The main economic force behind these fluctuations are endogenous shifts in the average between-firm correlations. The average correlation between firms' outputs and investment-rates rises (drops) by about 30%, in absolute value, in bad (good) times. Consquentially, about 80% of the increase in the conditional aggregate volatility of total output growth, and other macro-quantities, during slowdowns is attributed to an increase in the conditional correlations. Without Bayesian learning, or when all information is symmetric between firms, the oscillations in the correlations over time are minute, and are therefore translated to very small fluctuations in the conditional volatility of aggregates.

Table 3.1: Benchmark Calibration

Parameter	Symbol	Value
Depreciation rate of capital	δ	2%
Discount factor	β	0.994
Aggregate Productivity:		
Growth rate	g_0	1.005
Autocorrelation of aggregate productivity	ρ_g	0.5
Standard deviation of shock	σ_g	0.02
Idiosyncratic Demand Shock:		
Mean of shock	z_0	1
Autocorrelation of idiosyncratic component	ρ_g	0.9
Standard deviation of shock	σ_z	0.01
Production:		
Returns to scale	ν	0.9
Elasticity of capital input	α	0.22
Adjustment cost for capital	ζ	1.2
Adjustment cost for labor	κ	7
(Detrended) wage	w	1
Standard deviation of labor efficiency shock	σ_l	0.265

The Table presents the benchmark calibration of the learning model, at the quarterly frequency.

Table 3.2: Unconditional Aggregate Annual Moments

	Model with Learning			Model without Learning			Data				
	Mean	Std. Dev.	AR(1)	Mean	Std. Dev.	AR(1)	Mean	Std. Dev.	AR(1)		
ΔC	0.019	0.031	0.384	0.019	0.044	0.305	0.019	0.014^{a}	0.341	[0.07,	0.62]
ΔK	0.019	0.013	0.950	0.019	0.014	0.952	0.021	0.036	0.369^{b}	[0.18,	0.56]
ΔY	0.019	0.035	0.315	0.019	0.044	0.286	0.017	0.030^{c}	0.234	[-0.02,	0.49]
I/K	-2.315	0.135	0.936	-2.313	0.138	0.941	-2.916	0.150	0.646	[0.35,	0.94]
ΔL	-0.000	0.024	0.416	-0.000	0.015	0.819	0.000	0.018	-0.296^{d}	[-0.45,	-0.15]

The Table shows model-implied moments and data counterparts, for log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, log aggregate labor growth ΔL, and log aggregate investment rate I/K. Columns two to four present the mean, standard deviation and autocorrelation implied from the benchmark model with learning. The next three columns show similar moments implied from an identical model, with full information and no-learning, and under the same calibration. All model moments are based on quarterly simulated data that is time-aggregated to annual observations. The left-most columns present the empirical counterparts. The numbers in brackets present the 90%-confidence intervals of the data AR(1) coefficients. The data figures are estimates using annual observations from 1946-2013. All growth rates in the data are real and per-capita. Data on capital and investment correspond to total assets and capital expenditures from the Flow of Funds for all private non-financial firms. Data on output correspond to GDP. Data on labor correspond to Average Weekly Hours of Production per Worker from BLS.

Notes:

(a) In a long sample of 1930-2013, the standard deviation is 0.022.

(b) The quarterly auto-correlation of capital growth in the data is 0.651 with a confidence interval of [0.550,0.753]. In the model, the quarterly auto-correlation is 0.945.

(c) In a long sample of 1930-2013, the standard deviation is 0.049.

(d) The quarterly auto-correlation of labor growth in the data is 0.234 with a confidence interval of [0.048,0.419]. In the model, the quarterly auto-correlation is 0.107.

Table 3.3: Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle

	Model with Learning		Model without Learning		Data					
	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V}_t(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$		
ΔC	29.00%	-20.23%	2.91%	-0.47%	32.51%	[16.53%,	48.50%]	-13.74%	[-24.35%,	-3.14%]
ΔY	42.73%	-32.91%	4.32%	-1.49%	29.88%	[15.99%,	43.77%]	-12.87%	[-23.36%,	-2.39%]
ΔK	57.88%	-41.53%	1.79%	0.76%	56.22%	[28.66%,	83.77%]	-47.83%	[-60.80%,	-34.87%]
I/K	58.69%	-42.41%	3.44%	-1.10%	4.24%	[-9.01%,	17.48%]	-7.35%	[-19.18%,	4.48%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b_x'[\mathbf{Z}_t] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, V_t is the fitted value of the following projection: $\varepsilon^2_{x,t+1} = const + \nu_x'[\varepsilon^2_{x,t}, \mathbf{Z}_t] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90%-confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 3.4: Fluctuations in the Average-Conditional (Pairwise) Covariation between Firms with the Business-Cycle

	Model with Learning		Data	
	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$
ΔY	43.82%	-34.05%	28.96% [-2.98%, 60.90%]	-28.44% [-53.03%, -3.85%]
ΔK	58.53%	-41.88%	55.85% [28.41%, 83.29%]	-36.42% [-53.71%, -19.13%]
I/K	59.39%	-42.81%	19.55% [-2.17%, 41.28%]	-7.64% [-28.22%, 12.94%]

The Table presents by how much, in percentages, the conditional between-firms average covariation of variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{COV}_t(\cdot|Period)$ refers to the between-firm pairwise conditional covariation at time t, $\{COV_t(\cdot_i,\cdot_j)\}_{i,j}$, of a firm-level variable specified in the left-most column, averaged between all firm tuples (i,j) in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The average *pairwise* conditional covariation of a variable at time t, is constructed using three steps. First, the conditional mean is removed for each firm time-series separately by projecting $x_{i,t+1} = const + b'[x_{i,t}, \mathbf{Z}_t] + \varepsilon_{i,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, the ex-ante covariation $COV_t(x_{i,t+1}, x_{j,t+1})$ between firm i and firm j is computed as the fitted value of the following projection: $\varepsilon_{i,t+1}\varepsilon_{j,t+1} = const + c'[\varepsilon_{i,t}\varepsilon_{j,t}, \mathbf{Z}_t] + error$. Third, the average ex-ante covariation at time t is the average of $\{COV_t(x_{i,t+1}, x_{j,t+1})\}_{i,j}$ for all tuples (firms i, firm j) in the cross-sectional sample. Columns two and three present the fluctuations of average pairwise covariation in bad and good periods, induced by quarterly simulated data from the learning model. The two left-most columns present the empirical counterparts, along with 90% confidence intervals in brackets. The empirical estimates are based on the average covariation between quarterly data of 31 assets (industry portfolios). Likewise, the model results are based on a sub sample of 31 firms. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time-series end at 2013-Q4.

Table 3.5: Fluctuations in the Average-Conditional Volatility of One-Firm with the Business-Cycle

	Model with Learning		Model without Learning	
	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Good})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Bad})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t^{one-firm}}(\cdot\|\text{Good})}{\overline{V_t^{one-firm}}(\cdot\|\text{Normal})} - 1$
ΔY	8.09%	-4.96%	3.13%	-1.52%
ΔK	23.22%	-17.82%	1.79%	0.61%
I/K	24.46%	-19.09%	3.44%	-1.24%

The Table shows by how much, in percentages, the average conditional volatility of firm-level variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{V^{one-firm}}_t(\cdot|Period)$ refers to the conditional one-firm volatility of a firm-level variable specified in the left-most column, $\{V_t^{one-firm}(\cdot)\}_i$, averaged between all firms i in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t^{one-firm}(x_{i,t+1})$ of some one-firm variable x_i at time t is constructed using two projections. First, the conditional mean is removed by projecting $x_{i,t+1} = const + b'_{i,x}[x_{i,t}\mathbf{Z}_t] + \varepsilon_{i,x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, $V_t^{one-firm}(x_{i,t+1})$ is the fitted value of the following projection: $\varepsilon^2_{i,x,t+1} = const + \nu'_{i,x}[\varepsilon^2_{i,x,t}, \mathbf{Z}_t] + error$. The average one-firm volatility $V_{i,t}^{one-firm}$ at time t is the average of all $\left\{V_t^{one-firm}(x_{i,t+1})\right\}_i$ for firms i in the cross-sectional sample. Columns two and three (four and five) present the fluctuations of one-firm volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. I use a sub-sample of 31 firms in the model for the construction of the table, for consistency with the empirical analysis.

Table 3.6: Fluctuations in the Average-Conditional Correlation between Firms with the Business-Cycle

	Model with Learning		Model without Learning	
	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})} - 1$
ΔY	32.04%	-29.41%	0.95%	0.11%
ΔK	28.13%	-28.86%	0.02%	0.10%
I/K	27.50%	-28.82%	0.03%	0.09%

The Table presents by how much, in percentages, the conditional between-firm average correlation of variables increases (decreases) in bad (good) times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad times refer to periods is which the TFP growth is between its 0-25-th percentiles. Normal times refer to periods is which the TFP growth is between its 25-75-th percentiles. Good times refer to periods is which the TFP growth is between its 75-100-th percentiles. The notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm pairwise correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-section, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The fluctuations in correlations are derived using the decomposition: $\frac{\overline{V_t}(\cdot|\text{Period})}{\overline{V_t}(\cdot|\text{Normal})} = \frac{\overline{V_t^{one-firm}}(\cdot|\text{Period})}{\overline{V_t^{one-firm}}(\cdot|\text{Normal})} \cdot \frac{\overline{CORR_t}(\cdot|\text{Period})}{\overline{CORR_t}(\cdot|\text{Normal})}$, where $Period \in \{Bad, Good\}$, where V_t denotes the conditional aggregate volatility, and $\overline{V^{one-firm}}_t$ denotes the conditional one-firm volatility, averaged over all firms i in the cross-section. The construction of the aggregate and one-firm volatility fluctuations are detailed in Tables 3.3 and 3.5.

Table 3.7: Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: A Model with No Informational Asymmetries

	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Bad})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t^{agg}}(\cdot\|\text{Good})}{\overline{V_t^{agg}}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Bad})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{CORR_t}(\cdot\|\text{Good})}{\overline{CORR_t}(\cdot\|\text{Normal})}-1$
ΔY	1.73%	3.66%	0.27%	0.66%
ΔK	1.26%	0.35%	-0.04%	0.04%
I/K	2.01%	-0.82%	-0.04%	0.03%

The Table presents a summary of results implied from a learning model, having the same calibration as the benchmark learning model, but in which the (noise) shocks to labor efficiency are *aggregate* shocks, making all information symmetric. The left (right) panel shows by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm (pairwise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The construction of aggregate volatility and average correlation fluctuations are the same as detailed in Tables 3.3 and Table 3.6.

Table 3.8: Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: A Model with Fixed Weights on Private and Public Signals (Non-Bayesian Learning)

	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\vert\text{Bad})}{\overline{V_t^{agg}}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\vert\text{Good})}{\overline{V_t^{agg}}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\vert\text{Bad})}{\overline{CORR_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\vert\text{Good})}{\overline{CORR_t}(\cdot\vert\text{Normal})} - 1$
ΔY	4.45%	-2.18%	1.07%	-0.53%
ΔK	0.62%	1.60%	0.54%	0.77%
I/K	2.18%	-0.54%	0.00%	0.93%

The Table presents a summary of results implied from a learning model, having the same calibration as the benchmark learning model, but in which weights on the public signal, and on the private signal in the posterior updating equation are fixed to their steady state values, making leaning non-Bayesian. The left (right) panel show by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR}_t(\cdot|Period)$ refers to the between-firm (pairwise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The construction of aggregate volatility and average correlation fluctuations are the same as detailed in Tables 3.3 and Table 3.6.

Table 3.9: Fluctuations in the Conditional Volatility of Aggregates and in Average-Conditional Correlation with the Business-Cycle: Comparison Between Different Noise Levels

	Panel A: Results for A Learning Model - Different Noise Levels			
	$\sigma_l = 0.3$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Bad})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Good})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Bad})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Good})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$
ΔY	46.25%	-37.80%	37.14%	-35.26%
ΔK	90.41%	-76.05%	36.74%	-64.09%
I/K	91.83%	-77.22%	36.46%	-65.17%
	$\sigma_l = 0.2$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Bad})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Good})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Bad})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Good})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$
ΔY	12.08%	-3.74%	8.33%	-2.86%
ΔK	39.65%	-10.82%	24.02%	-4.23%
I/K	40.01%	-12.95%	22.80%	-4.89%
	$\sigma_l = 0$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Bad})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Good})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Bad})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Good})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$
ΔY	4.13%	-1.22%	1.61%	0.51%
ΔK	1.78%	0.78%	0.02%	0.10%
I/K	3.43%	-1.09%	0.04%	0.08%
	Panel B: Results for A No-Learning Model - No-Noise Model			
	$\sigma_l = 0$			
	Aggregate Volatility Fluctuations		Average Correlation Fluctuations	
	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Bad})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{V_t^{agg}}(\cdot\mid\text{Good})}{\overline{V_t^{agg}}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Bad})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$	$\frac{\overline{CORR_t}(\cdot\mid\text{Good})}{\overline{CORR_t}(\cdot\mid\text{Normal})} - 1$
ΔY	4.13%	-1.22%	1.61%	0.51%
ΔK	1.78%	0.78%	0.02%	0.10%
I/K	3.43%	-1.09%	0.04%	0.08%

The sub-panels in Panel A present a summary of results implied from learning models, identical to the benchmark learning model, but calibrated with different standard deviation for the noise labor efficiency shock. All other parameters in the model are calibrated as in the benchmark calibration outlined in Table 3.1. Panel B shows a summary of results implied from a no-learning, but calibrated with *zero* standard deviation for the labor efficiency shock. In each sub-panel the two-left (right) most columns show by how much, in percentages, the conditional volatility of aggregate variables (the average correlation between-firms) fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, normal, and good times refer to periods is which the TFP growth is between its 0-25-th, 25-75th, and 75-100th percentiles, respectively. The notation $\overline{V^{agg}}_t(\cdot|Period)$ refers to the average conditional aggregate volatility $V_t^{agg}(\cdot)$ of an *aggregate* variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Likewise, the notation $\overline{CORR_t}(\cdot|Period)$ refers to the between-firm (pair-wise) correlation at time t of a firm-level variable specified in the left-most column, $\{CORR_t(\cdot_i, \cdot_j)\}_{i,j}$, averaged between all firm tuples (i, j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$.

Table 3.10: Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Comparison Between Model-Implied Data and Matched Data from Simulated Constant Conditional Volatility Processes

	Panel A: Falsification-test for the No-Learning Model			
	No-Learning Model		Simulation of Constant Conditional Volatility Process	
	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$
ΔC	2.91%	-0.47%	4.01%	-1.45%
ΔY	4.32%	-1.49%	3.89%	-1.33%
ΔK	1.79%	0.76%	1.73%	0.83%
I/K	3.44%	-1.10%	1.49%	1.18%
	Panel B: Falsification-test for the Learning Model			
	Learning Model		Simulation of Constant Conditional Volatility Process	
	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Bad})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\vert\text{Good})}{\overline{V_t}(\cdot\vert\text{Normal})} - 1$
ΔC	29.00%	-20.23%	2.06%	1.16%
ΔY	42.73%	-32.91%	2.09%	0.82%
ΔK	57.88%	-41.53%	0.58%	0.68%
I/K	58.69%	-42.41%	0.69%	0.49%

The Table shows by how much, in percentages, the conditional volatility of aggregate variables fluctuates in bad and good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods in which the TFP growth is between its 0-25-th, 25-75th and 75-100th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the average conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. Columns two and three present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the no-learning model, in Panel A, and from the learning model, in Panel B. Each row in columns four and five presents the volatility fluctuations, induced by a simulated constant conditional volatility process, having the same unconditional moments as the model-implied time-series, for the aggregate variable specified in the left-most column. In Panel B (Panel A), the simulated processes imitate aggregate time-series from the (no-) learning model. Specifically, for each model-implied aggregate log variable $log(X)$, denoted in the left-most column, I calibrate a process $\tilde{X}_t$ of the form: $\tilde{X}_t = (1-\rho_x)x_0 + \rho_x \tilde{X}_{t-1} + \beta_g(g_{t-1} - g_0) + \beta_{g,2}(g_{t-1} - g_0)^2 + \sigma_x \varepsilon_{g,t} + \sigma_{x,2}(\varepsilon_{g,t}^2 - 1)$ such that the process $\tilde{X}_t$ has the same mean, standard deviation, skewness, and same correlations with TFP growth and TFP growth squared, as the original (exponentiated) model-implied X_t process. The volatility fluctuations are then reported for the $log(\tilde{X}_t)$ process. The measurement of the conditional volatility for both model-implied and matched homoscedastic processes, is the same, and detailed in Table 3.3.

Table 3.11: Correlation between Dispersion and the Business-Cycle

	Model with Learning	Model without Learning	Data	
	$CORR(DISP_t(\cdot), g_t)$	$CORR(DISP_t(\cdot), g_t)$	$CORR(DISP_t(\cdot), g_t)$	
ΔY	-0.0635	-0.0945	-0.0303	[-0.1232, 0.0625]
I/K	-0.0447	0.0316	-0.0499	[-0.2302, 0.1304]
ΔK	-0.0125	0.0823	0.1275	[-0.0849, 0.3399]

The Table shows the correlation between the dispersion of variables in the cross-section and the business-cycle, measured via TFP growth g_t. The correlation between the dispersion and TFP growth is reported for firms' output growth ΔY, capital growth ΔK, and investment-to-capital ratio I/K. For some real (growth) variable x, the dispersion at time t is computed as the cross-sectional variance of $\{x_{i,t}\}_i$ for all firms i in the cross-sectional sample, or $VAR_n(x_{i,t})$. The notation $DISP_t(\cdot)$ refers to the cross-sectional dispersion of a variable specified in the left-most column. The second (third) column reports the results using the (no-) learning model-implied data. The fourth column shows empirical counterparts, along with a 90% confidence interval in brackets. The empirical estimates are based on a sample of 31 assets (industry portfolios). Likewise, the model results are based on a sub-sample of 31 firms. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time series end at 2013-Q4.

Table 3.12: Empirical Fluctuations in the Average-Conditional Between-Firm Covariation, and in Residual Dispersion, with the Business-Cycle

	Average-Conditional Covariation						Residual Dispersion					
	$\frac{\overline{COV_t}(\cdot\|\text{Bad})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{COV_t}(\cdot\|\text{Good})}{\overline{COV_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{DISP_t}(\cdot\|\text{Bad})}{\overline{DISP_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{DISP_t}(\cdot\|\text{Good})}{\overline{DISP_t}(\cdot\|\text{Normal})}-1$		
ΔY	28.96%	[-2.98%,	60.90%]	-28.44%	[-53.03%,	-3.85%]	-21.41%	[-52.55%,	9.74%]	-41.48%	[-65.55%,	-17.42%]
I/K	19.55%	[-2.17%,	41.28%]	-7.64%	[-28.22%,	12.94%]	18.95%	[-35.68%,	73.57%]	31.37%	[-42.15%,	104.90%]
ΔK	55.85%	[28.41%,	83.29%]	-36.42%	[-53.71%,	-19.13%]	26.26%	[-80.42%,	132.95%]	71.61%	[-45.61%,	188.84%]

The Table shows empirically by how much, in percentages, the conditional between-firm average covariation of variables, and the cross-sectional residual-dispersion, fluctuates in bad and good times, in comparison to normal periods, for the following variables: log output growth ΔY, log capital growth ΔK, and log investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods in which the TFP growth is between its 0-25-th, 25-75th and 75-100th percentiles, respectively.

The notation $\overline{COV}_t(\cdot|Period)$ refers to the between-firm pairwise conditional covariation at time t of a firm-level variable specified in the left-most column, $\{COV_t(\cdot_i,\cdot_j)\}_{i,j}$, averaged between all firm tuples (i,j) in the cross-sectional sample, and then time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$. The notation $\overline{DISP}_t(\cdot|Period)$ refers to the cross-sectional residual-dispersion at time t, of a variable specified in the left-most column, that is time-averaged over all times t belonging to a certain $Period \in \{Bad, Good, Normal\}$.

The average *pairwise* conditional covariation of a variable at time t, is constructed using three steps. First, the conditional mean is removed for each firm time-series separately by projecting $x_{i,t+1} = const + b'[x_{i,t}, \mathbf{Z}_t] + \varepsilon_{i,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, the ex-ante covariation $COV_t(x_{i,t+1}, x_{j,t+1})$ between firm i and firm j is computed as the fitted value of the following projection: $\varepsilon_{i,t+1}\varepsilon_{j,t+1} = const + c'[\varepsilon_{i,t}\varepsilon_{j,t}, \mathbf{Z}_t] + error$. Third, the average ex-ante covariation at time t is the average of $\{COV_t(x_{i,t+1}, x_{j,t+1})\}_{i,j}$ for all tuples (firms i, firm j) in the cross-sectional sample. The residual-dispersion of a variable is defined as the cross-sectional variance of the shocks to x_t, $\{\varepsilon_{i,t}|i = 1..N\}$, at time t, or $DISP_t(\varepsilon_t) = V_n(\varepsilon_{i,t})$.

Columns two and three (four and five) present the fluctuations of average pairwise covariation (residual-dispersion), computed by quarterly empirical observations. The brackets show the 90%-confidence intervals. The empirical estimates are based on data of 31 industry portfolios. Data on output (sales) start at 1966-Q1, on capital (assets) start at 1975-Q1, and on investment (capex) at 1985-Q1. All time series end at 2013-Q4.

Table 3.13: Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Defining the Cycle using Output Growth

	Model with Learning		Model without Learning		Data					
	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$		
ΔC	37.99%	-14.06%	2.39%	-1.89%	48.82%	[31.63%,	66.01%]	-8.14	[-18.59%,	2.32%]
ΔY	58.35%	-22.30%	5.51%	-4.95%	37.75%	[23.12%,	52.38%]	-8.79	[-19.09%,	1.52%]
ΔK	82.83%	-28.39%	-0.12%	0.60%	61.75%	[36.43%,	87.06%]	-65.25	[-73.87%,	-56.64%]
I/K	81.92%	-27.86%	1.92%	-1.28%	20.42%	[4.16%,	36.68%]	-0.32	[-12.28%,	11.64%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which aggregate output growth (ΔY) is between its 0-25-th, 50-75-th, and 75-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. Columns two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90%-confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 3.14: Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: Defining the Cycle using Alternative Percentiles

	Model with Learning		Model without Learning		Data					
	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})}-1$			$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})}-1$		
ΔC	44.02%	-22.93%	4.28%	-0.15%	69.58%	[52.06%,	87.10%]	-21.02	[-34.26%,	-7.79%]
ΔY	64.70%	-38.72%	6.38%	-1.20%	58.65%	[43.10%,	74.19%]	-22.74	[-36.39%,	-9.10%]
ΔK	85.34%	-47.25%	2.67%	1.58%	98.88%	[46.45%,	151.32%]	-67.73	[-86.78%,	-48.68%]
I/K	86.23%	-48.70%	4.96%	-1.02%	18.59%	[0.22%,	36.95%]	-5.07	[-19.17%,	9.03%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which TFP growth is between its 0-10-th, 10-90-th, and 90-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b_x'[\mathbf{Z}_t] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors. Second, V_t is the fitted value of the following projection: $\varepsilon^2_{x,t+1} = const + \nu_x'[\varepsilon^2_{x,t}, \mathbf{Z}_t] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90% confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

Table 3.15: Fluctuations in the Conditional Volatility of Aggregates with the Business-Cycle: using Nonlinear Predictors

	Model with Learning		Model without Learning		Data			
	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$	$\frac{\overline{V_t}(\cdot\|\text{Bad})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$		$\frac{\overline{V_t}(\cdot\|\text{Good})}{\overline{V_t}(\cdot\|\text{Normal})} - 1$	
ΔC	16.55%	-14.96%	3.39%	0.26%	42.50%	[23.94%, 61.06%]	1.90	[-8.96%, 12.77%]
ΔY	24.60%	-25.46%	4.95%	-1.21%	35.63%	[12.41%, 58.86%]	4.38	[-9.30%, 18.07%]
ΔK	52.05%	-32.06%	2.11%	1.74%	92.68%	[51.36%, 134.01%]	-8.30	[-22.77%, 6.16%]
I/K	48.50%	-33.08%	3.97%	-0.40%	7.45%	[-0.48%, 15.37%]	10.06	[2.77%, 17.34%]

The Table shows by how much, in percentages, the conditional volatility of macroeconomic variables fluctuates in bad and in good times, in comparison to normal periods, for the following variables: log aggregate consumption growth ΔC, log aggregate output growth ΔY, log aggregate capital growth ΔK, and log aggregate investment-to-capital ratio I/K. Bad, Normal and Good times refer to periods is which TFP growth is between its 0-25-th, 50-75-th, and 75-100-th percentiles, respectively. The notation $\overline{V}_t(\cdot|Period)$ refers to the conditional volatility $V_t(\cdot)$ of a variable specified in the left-most column, averaged over all observation belonging to a certain $Period \in \{Bad, Good, Normal\}$. The conditional volatility $V_t(X_{t+1})$ of some aggregate variable X at time t is constructed using two projections. First, the conditional mean is removed by projecting $X_{t+1} = const + b_x'[\mathbf{Z}_t, \mathbf{Z}_t^2] + \varepsilon_{x,t+1}$, where $\mathbf{Z}$ is the set of benchmark predictors, and $\mathbf{Z}_t^2$ is the set of benchmark predictors squared. Second, V_t is the fitted value of the following projection: $\varepsilon_{x,t+1}^2 = const + \nu_x'[\varepsilon_{x,t}^2, \mathbf{Z}_t, \mathbf{Z}_t^2] + error$. Column two and three (four and five) present the fluctuations of volatility in bad and good periods, induced by quarterly simulated data from the (no-) learning model. The last two columns present the empirical counterparts, along with 90% confidence intervals in brackets. Quarterly data is available from 1952Q1-2013Q4 for all aggregate variables.

APPENDIX

A.1. Appendix for Chapter 1

A.1.1. Realized variance asymptotics

Consider a jump-diffusion process y_t:

$$y_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s + J_t, \tag{A.1}$$

where μ_s is a locally bounded predictable drift process, σ_s is a strictly positive càdlàg volatility process, J_t is a finite activity jump process, and μ_s, σ_s, and J_t are adapted to some common filtration $\mathcal{F}_t$.

The realized semivarainces are defined as follows:

$$\begin{aligned} RV_{p,t+1} &= \textstyle\sum_{i=1}^N \mathbb{I}(\Delta y_{t+\frac{i}{N}} \geq 0)\Delta y^2_{t+\frac{i}{N}}, \\ RV_{n,t+1} &= \textstyle\sum_{i=1}^N \mathbb{I}(\Delta y_{t+\frac{i}{N}} < 0)\Delta y^2_{t+\frac{i}{N}}. \end{aligned}$$

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) derive the behaviour of this statistic under in-fill asymptotics, and in particular, they show that:

$$\begin{aligned} RV_{p,t+1} &\xrightarrow{p} \frac{1}{2}\int_t^{t+1} \sigma_s^2 ds + \sum_{t \leq s \leq t+1} \mathbb{I}(\Delta J_s \geq 0)\Delta J_s^2, \\ RV_{n,t+1} &\xrightarrow{p} \frac{1}{2}\int_t^{t+1} \sigma_s^2 ds + \sum_{t \leq s \leq t+1} \mathbb{I}(\Delta J_s < 0)\Delta J_s^2. \end{aligned}$$

Intuitively, the positive (negative) semivariances are informative about positive (negative) squared jumps.

While the above theory considers jump-diffusion processes, Diop et al. (2013) provide asymptotic convergence results and Central Limit theorems for the quadratic variations of pure jump Itô semimartingales.

A.1.2. Benchmark model solution

In case when $\epsilon_{i,t+1},\ i = \{g, b\}$, follows a compensated compound Poisson distribution with time-varying intensity l_t, its log moment-generating function is given by,

$$\log E_t e^{u\epsilon_{i,t+1}} = l_t(\alpha(u) - u\alpha'(0) - 1), \tag{A.2}$$

where $\alpha(u) = E_t e^{uJ_{i,t+1}}$ denotes the moment-generating function of the underlying positive jumps. The conditional variance of the compound Poisson is given by $V_{i,t} = Var_t\epsilon_{i,t+1} = l_t\alpha''(0)$, which implies that its log moment-generating function is linear in its variance,

$$\log E_t e^{u\epsilon_{i,t+1}} = V_t f(u), \quad \text{for} \quad f(u) = \frac{\alpha(u) - u\alpha'(0) - 1}{\alpha''(0)}. \tag{A.3}$$

Because the underlying Poisson jumps are positive, ($J_{i,t+1} > 0$), $f(u)$ is positive, convex, and asymmetric, so that $f(u) > f(-u)$ for $u > 0$.[1]

In case when $\epsilon_{i,t+1},\ i = \{g, b\}$, follows a demeaned Gamma distribution with a unit scale and time-varying shape parameter V_t, Bekaert and Engstrom (2009) show that its log moment-generating function satisfies

$$\log E_t e^{u\epsilon_{i,t+1}} = V_t f(u), \tag{A.4}$$

for $f(u) = -(\log(1-u) + u)$. The function $f(u)$ is positive, convex, and asymmetric, so that $f(u) > f(-u)$ for $u > 0$.

The solution of the model relies on a standard log-linearization of returns,

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}. \tag{A.5}$$

[1]To prove asymmetry, note that because jump distribution is positively skewed, $f'''(u) > 0$. This implies that $f(u) - f(-u)$ is increasing in u, so that $f(u) - f(-u) > 0$ for $u > 0$.

In equilibrium, the price-consumption ratio is linear in the expected growth and uncertainty factors, as shown by Eq. (1.10). The log-linearization parameter κ_1 satisfies the equation,

$$\begin{aligned}\log\,\kappa_1 &= \log\,\delta + (1-\frac{1}{\psi})\mu_c + A_{gv}(1-\kappa_1\nu_g)V_{g0} \\ &+ A_{bv}(1-\kappa_1\nu_b)V_{b0} \\ &+ \theta\kappa_1^2\left[\frac{1}{2}A_{gv}^2\sigma_{gw}^2 + \frac{1}{2}A_{bv}^2\sigma_{bw}^2 + \alpha A_{gv}A_{bv}\sigma_{gw}\sigma_{bw}\right]. \qquad (A.6)\end{aligned}$$

The real stochastic discount factor is equal to:

$$\begin{aligned}m_{t+1} &= m_0 + m_x x_t + m_{gv}V_{gt} + m_{bv}V_{bt} \\ &- \lambda_x\sigma_x(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) - \lambda_{gv}\sigma_{gw}w_{g,t+1} \\ &- \lambda_{bv}\sigma_{bw}w_{b,t+1}, \qquad (A.7)\end{aligned}$$

where the market prices of risk are specified in Eqs. (1.14)–(1.16), and the loadings on the state variables are given by,

$$\begin{aligned}m_x &= -\gamma + (1-\theta)(1-\kappa_1\rho)A_x = -\frac{1}{\psi}, \\ m_{gv} &= (1-\theta)(A_{gv}(1-\kappa_1\nu_g) - \kappa_1 A_x\tau_g) \\ &= \frac{1-\theta}{\theta}f(\theta((1-\frac{1}{\psi})\sigma_c + \kappa_1 A_x\sigma_x)), \\ m_{bv} &= (1-\theta)(A_{bv}(1-\kappa_1\nu_b) + \kappa_1 A_x\tau_b) \\ &= \frac{1-\theta}{\theta}f(\theta(-(1-\frac{1}{\psi})\sigma_c - \kappa_1 A_x\sigma_x)).\end{aligned}$$

Note that we can alternatively rewrite the stochastic discount factor in terms of $V_{gt} + V_{bt}$

and $V_{gt} - V_{bt}$, which capture the total variance and skewness of consumption dynamics:

$$\begin{aligned} m_{t+1} &= m_0 + m_x x_t + \frac{m_{gv} + m_{bv}}{2}(V_{gt} + V_{bt}) \\ &+ \frac{m_{gv} - m_{bv}}{2}(V_{gt} - V_{bt}) \\ &- \lambda_x \sigma_x(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ &- \frac{\lambda_{gv} + \lambda_{bv}}{2}(\sigma_{gw} w_{g,t+1} + \sigma_{bw} w_{b,t+1}) \\ &- \frac{\lambda_{gv} - \lambda_{bv}}{2}(\sigma_{gw} w_{g,t+1} - \sigma_{bw} w_{b,t+1}). \end{aligned} \tag{A.8}$$

The last two shocks are equal to the innovations into the total variance and skewness of consumption. As $\lambda_{gv} > 0$ and $\lambda_{bv} < 0$, the market price of risk of skewness is positive: agents dislike the states with low (negative) skewness.

The bond loadings satisfy the recursive equations:

$$B_{x,n} = \rho B_{x,n-1} - m_x, \tag{A.9}$$

$$\begin{aligned} B_{gv,n} &= \nu_g B_{gv,n-1} - m_{gv} \\ &\quad - f(-\sigma_x(\lambda_x + B_{x,n-1})) + \tau_g B_{x,n-1}, \end{aligned} \tag{A.10}$$

$$\begin{aligned} B_{bv,n} &= \nu_b B_{bv,n-1} - m_{bv} \\ &\quad - f(\sigma_x(\lambda_x + B_{x,n-1})) - \tau_b B_{x,n-1}, \end{aligned} \tag{A.11}$$

for $B_{x,0} = B_{gv,0} = B_{bv,0} = 0$.

Similarly, the return of the dividend-paying asset can be expressed by:

$$r_{d,t+1} \approx \kappa_{0,d} + \kappa_{1,d} pd_{t+1} - pd_t + \Delta d_{t+1}, \tag{A.12}$$

where $\kappa_{0,d}$ and $\kappa_{1,d}$ are the log-linearization parameters, and $\kappa_{1,d}$ satisfies:

$$\begin{aligned}\log\kappa_{1,d} &= m_0 + \mu_d + H_{gv}(1-\kappa_{1,d}\nu_g)V_{g0} \\ &+ H_{bv}(1-\kappa_{1,d}\nu_b)V_{b0} \\ &+ \kappa_{1,d}^2\left[\frac{1}{2}H_{gv}^2\sigma_{gw}^2 + \frac{1}{2}H_{bv}^2\sigma_{bw}^2 + \alpha H_{gv}H_{bv}\sigma_{gw}\sigma_{bw}\right]. \end{aligned} \tag{A.13}$$

The return dynamics can be expressed in the following way:

$$\begin{aligned} r_{d,t+1} &= E_t[r_{d,t+1}] + \beta_x\sigma_x(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ &+ \beta_{gv}\sigma_{gw}w_{g,t+1} + \beta_{bv}\sigma_{bw}w_{b,t+1} + \sigma_d u_{d,t+1}, \end{aligned} \tag{A.14}$$

where the equity betas are given by,

$$\beta_x = \kappa_{1,d}H_x,\ \beta_{gv} = \kappa_{1,d}H_{gv},\ \text{and}\ \ \beta_{bv} = \kappa_{1,d}H_{bv}. \tag{A.15}$$

H_x, H_{gv}, and H_{bv} are the equilibrium loadings of the price-dividend ratio on predictable consumption growth, good uncertainty, and bad uncertainty, respectively, and are given by:

$$H_x = \frac{\phi_x + m_x}{1-\kappa_{1,d}\rho}, \tag{A.16}$$

$$H_{gv} = \frac{f(\kappa_{1,d}H_x\sigma_x - \lambda_x\sigma_x) + \kappa_{1,d}H_x\tau_g + m_{gv}}{1-\kappa_{1,d}\nu_g}, \tag{A.17}$$

$$H_{bv} = \frac{f(-\kappa_{1,d}H_x\sigma_x + \lambda_x\sigma_x) - \kappa_{1,d}H_x\tau_b + m_{bv}}{1-\kappa_{1,d}\nu_b}. \tag{A.18}$$

Note that we can alternatively rewrite the return dynamics in terms of $V_{gt}+V_{bt}$ and $V_{gt}-V_{bt}$, which capture the total variance and skewness of consumption dynamics:

$$
\begin{aligned}
r_{d,t+1} &= E_t[r_{d,t+1}] + \beta_x \sigma_x (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\
&+ \frac{\beta_{gv} + \beta_{bv}}{2} \sigma_{gw} (w_{g,t+1} + w_{b,t+1}) \\
&+ \frac{\beta_{gv} - \beta_{bv}}{2} \sigma_{bw} (w_{g,t+1} - w_{b,t+1}) + \sigma_d u_{d,t+1}.
\end{aligned} \tag{A.19}
$$

As $\beta_{gv} > 0$ and $\beta_{bv} < 0$, equity exposure to skewness risk is positive: equities fall at times of low (negative) skewness.

It follows that the conditional variance of returns is time varying and driven by good and bad uncertainties:

$$
Var_t r_{d,t+1} = \beta_{gv}^2 \sigma_{gw}^2 + \beta_{bv}^2 \sigma_{bw}^2 + \sigma_d^2 + \beta_x^2 \sigma_x^2 (V_{gt} + V_{bt}). \tag{A.20}
$$

In particular, the variance of returns increases at times of high good or bad uncertainty.

In levels, the equity risk premium satisfies,

$$
\begin{aligned}
E_t R_{d,t+1} - R_{f,t} &\approx log \quad E_r e^{r_{d,t+1} - r_{f,t}} \\
&= \left[f(-\lambda_x \sigma_x) - f((\beta_x - \lambda_x)\sigma_x) + f(\beta_x \sigma_x) \right] V_{gt} \\
&+ \left[f(\lambda_x \sigma_x) - f((\lambda_x - \beta_x)\sigma_x) + f(-\beta_x \sigma_x) \right] V_{bt} \\
&+ \beta_{gv} \lambda_{gv} \sigma_{gw}^2 + \beta_{gv} \lambda_{bv} \sigma_{bw}^2 \\
&+ \alpha \sigma_{bw} \sigma_{gw} (\beta_{gv} \lambda_{bv} + \beta_{bv} \lambda_{gv}).
\end{aligned} \tag{A.21}
$$

Under standard parameter configuration, the equity premium loadings on good and bad volatility are positive. Indeed, notice that these loadings can be written as, $f(a) + f(b) - f(a+b)$. As $\lambda_x > 0$ and $\beta_x > 0$, a and b have opposite signs. Without loss of generality, let $a > 0$ and $b < 0$. Suppose $a + b > 0$. Then, as $f(u)$ is positive and increasing for $u > 0$, $a > a + b > 0 \Rightarrow f(a) + f(b) > f(a) > f(a+b)$. Alternatively, suppose $a + b < 0$. Then, as

$f(u)$ is positive and decreasing for $u < 0$, $0 > a + b > b \Rightarrow f(a) + f(b) > f(b) > f(a + b)$. In both cases, $f(a) + f(b) - f(a + b) > 0$.

A.1.3. Long-run risks model specification

In a standard long-run risks model, consumption dynamics satisfies

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \tag{A.22}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \tag{A.23}$$

$$\sigma_{t+1}^2 = \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, \tag{A.24}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{d,t+1}, \tag{A.25}$$

where ρ governs the persistence of expected consumption growth x_t, and ν determines the persistence of the conditional aggregate volatility σ_t^2. η_t is a short-run consumption shock, ϵ_t is the shock to the expected consumption growth, and w_{t+1} is the shock to the conditional volatility of consumption growth; for parsimony, these three shocks are assumed to be independent and identically distributed (i.i.d.) Normal. The parameter configuration for consumption and dividend dynamics used in our model simulation is identical to Bansal, Kiku, and Yaron (2012), and is given in Table A.1.

Table A.1: Long-Run Risk Model calibration

Preferences	δ	γ	ψ	
	0.9987	10	2	
Consumption	μ	ρ	φ_e	
	0.0015	0.975	0.038	
Volatility	σ_c	ν	σ_w	
	0.0072	0.999	2.8e-06	
Dividend	μ_d	ϕ	φ_d	π
	0.0015	2.5	5.96	2.6

The table shows the calibrated parameters of the long-run risks model at monthly frequency. The parameter δ is the subjective discount factor, γ is the coefficient of relative risk aversion, and ψ is the intertemporal elasticity of substitution. μ is the unconditional expectation of consumption growth, ρ captures the persistence of expected consumption, and ϕ_e governs the scale of expected consumption shocks. The parameters σ_c, ν, and σ_w represent the level, persistence, and the standard deviation of volatility shocks, respectively. μ_d is the unconditional growth rate of dividends, ϕ captures the exposure of dividends to expected consumption shocks, and π reflects the exposure of dividends to realized consumption shocks. The parameter ϕ_d governs the volatility of the idiosyncratic dividend shock.

A.2. Appendix for Chapter 2

A.2.1. Consumption TFP as a Preference Shock

In this appendix I show that the maximization programs (2.31) and (2.38) are equivalent. For notational ease, I denote the budget constraint of program (2.31), with the exclusion of consumption production (that is, equations (2.33)-(2.37)), as $\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$.

Define $\hat{C}_t = \frac{C_t}{z_{ct-1}}$ and $\hat{V}_t = \frac{V_t}{z_{ct-1}}$. It is straightforward to show using first-order condition equivalence, that the solution to the program (2.31) solves the partially *detrended* value-function given by:

$$\hat{V}_t(k_{ct}, k_{it}, z_{it}, \frac{z_{ct}}{z_{ct-1}}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \left\{ (1-\beta)\hat{C}_t^{1-1/\psi} + \beta \left(\frac{z_{ct}}{z_{ct-1}} \right)^{1-\frac{1}{\psi}} \left(E_t \hat{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1}, \frac{z_{ct+1}}{z_{ct}})^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{A.26}$$

s.t.

$$\hat{C}_t = \frac{z_{ct}}{z_{ct-1}} k_{ct}^{\alpha} n_{ct}^{1-\alpha}$$

$$\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$$

$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1},$$

and where $\mathbb{B}(k_{it}, k_{ct}, z_{it})$ is the same budget constraint as of program (2.31).

The detrended value function $\hat{V}$ of program (A.26) is homogeneous of degree one in $\frac{z_{ct}}{z_{ct-1}}$.

To see this, plug the expression for $\hat{C}_t$ in the objective function to obtain:

$$\hat{V}_t(k_{ct}, k_{it}, z_{it}, \frac{z_{ct}}{z_{ct-1}}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \Bigg\{ (1-\beta)\left(\frac{z_{ct}}{z_{ct-1}} k_{ct}^{\alpha} n_{ct}^{1-\alpha}\right)^{1-1/\psi}$$
$$+\beta\left(\frac{z_{ct}}{z_{ct-1}}\right)^{1-\frac{1}{\psi}}\left(E_t \hat{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1}, \frac{z_{ct+1}}{z_{ct}})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}} \Bigg\}^{\frac{1}{1-1/\psi}}$$

s.t.

$$\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$$
$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1}.$$

Notice, that $\frac{z_{ct}}{z_{ct-1}}^{1-1/\psi}$ multiplies both terms inside the maximand $\{\cdot\}^{\frac{1}{1-1/\psi}}$ expression. Thus, one can re-write the program as follows:

$$\hat{V}_t(k_{ct}, k_{it}, z_{it}, \frac{z_{ct}}{z_{ct-1}}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \left(\frac{z_{ct}}{z_{ct-1}}\right) \Bigg\{ (1-\beta)\left(k_{ct}^{\alpha} n_{ct}^{1-\alpha}\right)^{1-1/\psi}$$
$$+\beta\left(E_t \hat{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1}, \frac{z_{ct+1}}{z_{ct}})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}} \Bigg\}^{\frac{1}{1-1/\psi}} \tag{A.27}$$

s.t.

$$\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$$
$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1}$$

For any scalar $\lambda > 0$, specification (A.27) permits the following identity:

$$
\begin{aligned}
\hat{V}_t(k_{ct}, k_{it}, z_{it}, \lambda\frac{z_{ct}}{z_{ct-1}}) = &\max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \lambda\left(\frac{z_{ct}}{z_{ct-1}}\right)\Bigg\{(1-\beta)\left(k_{ct}^{\alpha}n_{ct}^{1-\alpha}\right)^{1-1/\psi} \\
&+\beta\left(E_t\hat{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1}, \frac{z_{ct+1}}{z_{ct}})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\Bigg\}^{\frac{1}{1-1/\psi}} \\
&\text{s.t.} \\
&\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it}) \\
&\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1} \\
=&\lambda \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \left(\frac{z_{ct}}{z_{ct-1}}\right)\Bigg\{(1-\beta)\left(\hat{k}_{ct}^{\alpha}n_{ct}^{1-\alpha}\right)^{1-1/\psi} \\
&+\beta\left(E_t\hat{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1}, \frac{z_{ct+1}}{z_{ct}})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\Bigg\}^{\frac{1}{1-1/\psi}} \\
&\text{s.t.} \\
&\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it}) \\
&\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1} \\
=&\lambda\hat{V}_t(k_{ct}, k_{it}, z_{it}, \frac{z_{ct}}{z_{ct-1}}).
\end{aligned} \tag{A.28}
$$

The second equality of equation (A.28) stems from the fact that λ is only a multiplicative constant that rescales the objective function, but does not affect the budget constraints, or the continuation value's state variables (as the growth in z_{ct} is independent over time). The third equality establishes homogeneity of degree one in consumption TFP growth. As a corollary, it is possible to write:

$$
\hat{V}_t(k_{ct}, k_{it}, z_{it}, \frac{z_{ct}}{z_{ct-1}}) = \left(\frac{z_{ct}}{z_{ct-1}}\right)\tilde{V}_t(k_{ct}, k_{it}, z_{it}). \tag{A.29}
$$

Lastly, the ex-ante expectation of $\frac{z_{ct+1}}{z_{ct}}$ behaves like a preference shock in the problem (A.27). To see this, divide both hands of (A.27) by $\frac{z_{ct}}{z_{ct-1}}$, and use the corollary (A.29), to obtain:

$$\tilde{V}_t(k_{ct}, k_{it}, z_{it}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \Bigg\{(1-\beta)\left(k_{ct}^{\alpha} n_{ct}^{1-\alpha}\right)^{1-1/\psi}$$

$$+\beta\left(E_t\left(\frac{z_{ct+1}}{z_{ct}}\right)^{1-\gamma}\tilde{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\Bigg\}^{\frac{1}{1-1/\psi}} \tag{A.30}$$

$$\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$$

$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1}$$

As $\tilde{V}_{t+1}$ is independent of $\frac{z_{ct+1}}{z_{ct}}$, we can separate the expectation in the objective function of (A.30) to obtain:

$$\tilde{V}_t(k_{ct}, k_{it}, z_{it}) = \max_{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}} \Bigg\{(1-\beta)\left(k_{ct}^{\alpha} n_{ct}^{1-\alpha}\right)^{1-1/\psi}$$

$$+\beta\underbrace{\left(E_t\left(\frac{z_{ct+1}}{z_{ct}}\right)^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}}_{\tilde{\beta}_t}\left(E_t\tilde{V}_{t+1}(k_{ct+1}, k_{it+1}, z_{it+1})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\Bigg\}^{\frac{1}{1-1/\psi}}$$

$$\{I_{i,t}, I_{c,t}, n_{c,t}, n_{i,t}\} \in \mathbb{B}(k_{it}, k_{ct}, z_{it})$$

$$\frac{z_{ct+1}}{z_{ct}} = \mu_{zc} + \sigma_{zc,t}\varepsilon_{zc,t+1}.$$

This program is identical to that specified in (2.38). Thus, the solution of program (2.38), is identical to the solution of (A.26), which is equal to the solution of (2.31). When z_{ct} is a random walk, the expression $\tilde{\beta}_t$ behaves like a preference shock, that depends only on the conditional volatility $\sigma_{zc,t}$.

A.2.2. Characterization of Model's Solution

A.2.2.1. Equilibrium Conditions

This section describes the equilibrium first-order conditions of the model described in section 2.4. The first-order condition of firm $n \in [0,1]$ in sector $j \in \{c, i\}$:

$$0 = q_{j,t} - P_{it}\Phi'_k(i_{j,t}(n)) \tag{A.31}$$

$$0 = W_t n_{j,t}(n) - (1-\alpha_j)\theta_{j,t} Z_{j,t} k_{j,t}(n)^{\alpha_j} n_{j,t}(n)^{1-\alpha_j} \tag{A.32}$$

$$\begin{aligned} 0 = -q_{j,t} + E_t \Big[M^{\$}_{t+1} \{ & -P_{i,t+1}\Phi_k(i_{j,t+1}) + q_{j,t+1}(1-\delta+i_{j,t+1}(n)) \\ & +\theta_{j,t+1} Z_{j,t+1} \alpha_j k_{j,t+1}(n)^{\alpha_j - 1} n_{j,t+1}(n)^{1-\alpha_j} \} \Big] \end{aligned} \tag{A.33}$$

$$\begin{aligned} 0 = &(1-\mu_j)\left[\frac{p_{j,t}(n)}{P_{j,t}}\right]^{-\mu_j} + \theta_{j,t}\mu_j \left[\frac{p_{j,t}(n)}{P_{j,t}}\right]^{-\mu_j - 1} \frac{1}{P_{j,t}} - \phi_P \left[\frac{p_{j,t}(n)}{\Pi_j p_{j,t-1}(n)} - 1\right] \frac{1}{\Pi_j} \\ & + \phi_P E_t \left[M^{\$}_{t+1} \left(\frac{Y_{j,t+1}}{Y_{j,t}}\right) \left\{ \left[\frac{p_{j,t+1}(n)}{\Pi_j p_{j,t}(n)} - 1\right] \frac{p_{j,t+1}(n)}{\Pi_j p_{j,t}(n)} - \frac{1}{2}\left[\frac{p_{j,t+1}(n)}{\Pi_j p_{j,t}(n)} - 1\right]^2 \right\} \right] \end{aligned} \tag{A.34}$$

$$0 = k_{j,t+1}(n) - (1-\delta+i_{j,t}(n))k_{j,t}(n) \tag{A.35}$$

$$0 = y_{j,t}(n) - Z_{j,t} k_{j,t}(n)^{\alpha_j} n_{j,t}(n)^{1-\alpha_j} \tag{A.36}$$

where $q_{j,t}$ be the price of a marginal unit of installed capital in sector j (the Lagrange multiplier of constraint (2.13)), and $\theta_{j,t}$ is the marginal cost of producing an additional unit of intermediate good in sector $j \in \{c, i\}$ (the Lagrange multiplier of constraint (2.17)).
The first-order condition of the household:

$$0 = \frac{W_t}{P_{c,t}} - \frac{C_t}{1-\xi N_t^{\eta}} \xi \eta N_t^{\eta-1} \tag{A.37}$$

The nominal SDF, nominal interest rate, as well as the household utility, are given in equations (2.25), (2.26) and (2.23), respectively. The last equilibrium conditions include four market clearing conditions (labor, investment-goods, consumption-goods, and bond market) specified in equations (2.27), (2.28), (2.29), and (2.30), respectively. We are looking for a symmetric equilibrium in which $P_{j,t}(n) = P_{j,t}$, $n_{j,t}(n) = n_{j,t}$, and $k_{j,t}(n) = k_{j,t}$, for all $n \in [0,1]$ and $j \in \{c,i\}$. Thus, the above equations can be rewritten in terms of only aggregate quantities. There are 20 endogenous variables: C_t, N_t, $Y_{c,t}$, $Y_{i,t}$, $N_{c,t}$, $N_{i,t}$, $K_{c,t}$, $K_{i,t}$, $i_{c,t}$, $i_{i,t}$, $q_{c,t}$, $q_{i,t}$, $\theta_{c,t}$, $\theta_{i,t}$, $P_{i,t}$, $P_{c,t}$, W_t, $R_t^{\$}$, U_t, $M_t^{\$}$. In turn, there are 20 equations: 13 equations for household's and firms' first-order conditions (in both sectors), 4 market clearing conditions, and 3 definitions of SDF, utility and Taylor-rule). Other quantities, such as the real SDF, and firm-valuations, are derived from the endogenous decision variables (see e.g. equation (2.16)).

A.2.2.2. Detrended Problem

Covariance-stationary first-order conditions can be achieved by rescaling the non-stationary variables of the problem as follows:

- Divide $k_{c,t}$, $k_{i,t}$, $Y_{i,t}$ by $Z_{i,t-1}^{\frac{1}{1-\alpha_i}}$.
- Divide C_t, $Y_{c,t}$, U_t by $Z_{c,t-1}Z_{i,t-1}^{\frac{\alpha_c}{1-\alpha_i}}$.
- Divide W_t by $P_{c,t}Z_{c,t-1}Z_{i,t-1}^{\frac{\alpha_c}{1-\alpha_i}}$.
- Divide $\theta_{c,t}$ by $P_{c,t}$.
- Divide $\theta_{i,t}$, $q_{i,t}$, $q_{c,t}$, $P_{i,t}$ by $P_{c,t}Z_{c,t-1}Z_{i,t-1}^{\frac{\alpha_c-1}{1-\alpha_i}}$.

After plugging the rescaled variables in the first-order equations, the equilibrium conditions can be written using stationary variables (in particular, using the rescaled variables, and using the *growth* rates of $Z_{c,t}, Z_{i,t}$ and of $P_{c,t}$).

A.3. Appendix for Chapter 3

To detrend the firm problem in 3.4, I divide the following quantities by the lagged trend level:

$$\tilde{k}_{i,t} = \frac{k_{i,t}}{G_{t-1}}; \quad \tilde{I}_{i,t} = \frac{I_{i,t}}{G_{t-1}}; \quad \tilde{\Phi}_{i,t} = \frac{\Phi_{i,t}}{G_{t-1}}; \quad \tilde{V}_t = \frac{V_t}{G_{t-1}}.$$

This allows re-writing the firm problem in a stationary form, as follows:

$$\begin{aligned}
\tilde{V}(\tilde{k}_{i,t}, l_{i,t}, g_{t-1}, s_{i,t}, z_{i,t}) \quad = \quad & max_{\tilde{k}_{t+1}, l_{t+1}} \quad g_{t-1}^{1-\alpha} z_{i,t} \tilde{k}_{i,t}^{\alpha} (s_{i,t} l_{i,t})^{\nu-\alpha} - w \cdot l_{i,t} - \tilde{I}_{i,t} \\
& \qquad\qquad -\tilde{\Phi}_L(l_{i,t}, l_{i,t+1}) \\
& \qquad\qquad +\beta g_{t-1} E[\tilde{V}(k_{i,t+1}, l_{i,t+1}, G_t, \hat{g}_t, s_{i,t+1}, z_{i,t+1})] \\
& g_{t-1} \tilde{k}_{i,t+1} = (1-\delta)\tilde{k}_{i,t} + \Lambda(\frac{\tilde{I}_t}{\tilde{k}_t}) \\
& \Lambda(i) = \frac{\alpha_1}{1-\frac{1}{\zeta}} (i)^{1-\frac{1}{\zeta}} + \alpha_2 \\
& \tilde{\Phi}_L(l_{i,t}, l_{i,t+1}) = g_{t-1} \frac{\kappa}{2} (l_{i,t+1} - l_{i,t})^2 \\
& V_{i,g,t} = [\frac{1}{\sigma_g^2} + \frac{l_{i,t}^2}{\sigma_l^2}]^{-1} \\
& \mu_{i,g,t} = V_{i,g,t} [\frac{1}{\sigma_g^2}((1-\rho_g) g_0 + \rho_g g_{t-1}) + \frac{l_{i,t}^2}{\sigma_l^2} (s_{i,t})] \\
& \hat{g}_t = \mu_{i,g,t} + \sqrt{V_{i,g,t}} \varepsilon_{i,\mu,t} \\
& s_{i,t+1} = [(1-\rho_g) g_0 + \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}] + \frac{\sigma_l}{l_{i,t+1}} \varepsilon_{l,t+1} \\
& \varepsilon_{i,\mu,t} = (g_t - \mu_{i,g,t})/V_{i,g,t} \sim N(0,1)
\end{aligned} \tag{A.38}$$

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