ESSAYS IN INTERNATIONAL ECONOMICS

by

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This dissertation analyzes the sovereign default risk and trade of emerging countries. In the first two chapters, I investigate the default risk by laying particular emphasis on labor markets and income inequality. In Chapter 2, I propose a general equilibrium model with endogenous sovereign default. I assume that firms need to borrow from foreign lenders to finance a fraction of the cost of labor (also referred to as working capital conditions). In the model, the labor channel can amplify the effect of adverse TFP shocks on default risk. The main finding is that working capital conditions play a significant role in generating endogenous drops in output and labor in defaults. In addition, the model explains the main features of the business cycles observed in emerging markets, such as countercyclical spreads, procyclical labor, and high volatility of consumption relative to output, when calibrated to Argentine data.

In Chapter 3, I analyze income inequality as another channel that influences a country's risk of default. I extend a standard endogenous sovereign default model by including inequality shocks. The main finding is that inequality shocks can amplify the effect of output shocks on sovereign default risk. Moreover, the model generates reasonable business cycle characteristics of Argentina, and the consumption volatility of poor households relative to rich households is matched well with a data set on Mexico. Lastly, as a policy extension, I extend the model by introducing progressive income taxes and find that as the progressivity of the tax increases, the default risk decreases.

In Chapter 4, aside from the sovereign default risk, I examine export behavior of firms as another aspect of emerging countries. I use three empirical methods for a Chilean plant-level data set. I find evidence that firms with high productivity are more likely to become exporters and that firms enhance their productivity by participating in export markets. Furthermore, I show that the productivity growth of export firms is persistent even in a recession. Overall, my findings provide a better understanding of the effects of labor markets and income inequality on sovereign default risk and the productivity growth induced by exports.

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PREFACE

There are many people who helped me complete my research and my Ph.D. degree. I greatly appreciate my dissertation committee members: Daniele Coen-Pirani, Marla Ripoll, Sewon Hur, James Cassing, and Josephine E. Olson. I am sincerely grateful to my advisors, Daniele Coen-Pirani and Marla Ripoll, for their guidance, support, and most importantly patience. In addition, I am deeply indebted to my dissertation committee member, Sewon Hur. Without his support and guidance, I would not finish my graduate study and my dissertation. I owe a lot to my friend, Zeynep Kabukcuoglu. It was the most beautiful moment in my Ph.D. course to work with her. Lastly, I am grateful to my father Jaeryong Jeon and my mother Myoungsook Lee for their emotional support and care.

1.0 INTRODUCTION

This dissertation analyzes the sovereign default risk and trade of emerging countries. Emerging market economies are different from advanced countries in many dimensions. They are characterized by higher default risk, more volatile employment, and higher income inequality than more advanced economies. They also experience larger drops in GDP and employment during default episodes. In view of these facts, labor markets and income inequality may influence the government's incentives to borrow. I place a particular focus on the role of these two channels in generating the sovereign default risk. Along with these characteristics, emerging market economies have a significant share in the international trade. I investigate export behavior of individual firms and see if exports can explain productivity growth of firms in emerging countries. The connection between export decisions and firm productivity may shed light on the link between export-oriented growth and economic success.

In this dissertation, the first two chapters deal with the sovereign default risk and business cycles in emerging countries. Chapter 2 starts by investigating sovereign default risk through the labor channel, which has not received much attention in prior literature. Specifically, I explore the role labor markets play in the sovereign default risk among emerging economies. I propose a general equilibrium model with endogenous sovereign default. I assume that firms need to borrow from foreign lenders to finance a fraction of the cost of labor (also referred to as working capital conditions). In addition, I assume that interest rates on working capital loans are correlated with interest rate spreads on government bonds; which is a stylized fact observed in the data. Under this assumption, the labor channel can amplify the effect of adverse TFP shocks on sovereign default risk. Specifically, firms may demand less labor if the economy is hit by a low TFP shock. Consequently, equilibrium wages and output would decline. Since foreign lenders have perfect information about the state of the economy and the TFP shock process, they will demand higher premiums on government bonds. As borrowing becomes more expensive, firms face higher interest rates on working capital loans. As a result, firms cut back their production resulting in a higher default risk in the economy. This allows me to show that working capital conditions play a significant role in generating endogenous drops in output and labor in defaults. In addition, the model explains the main features of the business cycles observed in emerging markets. These include countercyclical spreads, procyclical labor, and high volatility of consumption relative to output, when the model is calibrated to Argentine data.

While Chapter 2 examines the link between the default risk and the labor market, Chapter 3 considers the effect of income inequality on sovereign default risk. Specifically, I investigate how changes in income inequality affect governments' decisions on borrowing and defaulting. First, I conduct an empirical analysis using data that cover the period 1994-2010 for 45 countries. I show that there exists a negative correlation between the Gini indices and countries' creditworthiness (measured by the credit ratings of the government bonds). Second, to study the role of income inequality in default risk, I extend a standard endogenous sovereign default model by introducing heterogeneous agents. In addition to output shocks, I adopt shocks to income distribution (also referred to as inequality shocks). The government makes decisions on borrowing and defaulting to maximize the life-time utility of households. In the model, inequality shocks amplify the effect of output shocks on the endowments of households, such that there is a larger dispersion in the marginal utilities of consumption across agents as well as across time when the economy receives low output and high inequality shocks. This induces the government to borrow more from foreign lenders, which increases spreads. However, as borrowing becomes more expensive, the government needs to impose higher taxes on households in order to roll over its debt. Increases in taxes generate an even larger dispersion in the marginal utilities of consumption between households. In this case, instead of issuing more debt, the government chooses to default on its debt to eliminate the tax burden on households. This allows me to show that inequality shocks can amplify the effect of output shocks on sovereign default risk. I calibrate the model to match a number of stylized facts for Argentina. Quantitatively, the results show that the default probability with output shocks alone is 0.5 percent. In contrast, with two shocks it is 3 percent. This highlights the role of inequality shocks as a channel that amplifies the default risk. Next, I calibrate the model to match the business cycle statistics of Mexico. A data set on Mexico is useful to test the performance of the model with heterogeneous agents since it provides information for the level of consumption across different income groups. The consumption volatility of poor households relative to rich households is matched well with the data. Lastly, as a policy extension, I extend the model to look at whether the progressive tax system has any effect on the default risk. I find that as the progressivity of the tax increases, the default risk decreases. In effect, the economy has lower mean spreads and can sustain larger levels of debt.

Aside from the sovereign default risk, more emerging countries tend to participate in export markets, so the volume of exports from emerging countries has grown over time. Chapter 4 looks into emerging economies from this aspect. I conduct an empirical study on Chilean plants during 1995-2007 to examine if firms with high productivity are more likely to become exporters and if firms in emerging countries improve their productivity by participating in export markets. To answer the first question (the self-selection hypothesis), I use the regression method and the stochastic dominance method. In particular, I regress the probability of becoming an exporter on the productivity and other control variables such as the size of firms and the ratio of skilled labor. In the stochastic dominance method, I compare the entire productivity distribution of export plants and non-exporting plants rather than marginal moment. Using these two methods, I find evidence that supports the self-selection hypothesis. For the second question (the learning-by-exporting hypothesis), I make use of the matched sampling technique and the regression method. In the matched sampling technique, I compare the productivity of export plants when they actually export and when they hypothetically do not export. Since export plants that hypothetically do not export are not observed in the data, I use a propensity score method to find non-exporting plants which have productivity similar with export plants in the beginning of the sample period. These non-exporting plants are the control group and the export plants are the treated group. By comparing productivity of the treated group and the control group, I show that the learning effect is persistent for two years after entry into export markets. In addition, plants who stay in export markets during the whole sample period enhance their productivity as well. In the regression method, I categorize plants as seven types. Among these types, I expect that entrants-stay and permanent exporters will gain the benefits from participating in export markets. I find that they enhance their productivity by participating in export markets compared to non-exporting plants. In addition to these two hypotheses, I consider a short recession in 1999 in Chile and see if it affects the productivity growth of export plants. Using the regression method, I find evidence that supports the learning effect even in a recession. Lastly, I examine the productivity growth within export plants. In particular, I compare the productivity growth of export plants between before and after entry into export markets using the regression method. The result indicates that there is no significant difference in productivity growth of export plants before and after entry.

Understanding emerging economies is crucial since their impact on the global economy is growing significantly. In light of this fact, this dissertation explores emerging economies from two different aspects. My main findings provide a better understanding of default risk in emerging economies and the productivity growth induced by exports. I explain how working capital conditions and income inequality incur high interest rates resulting in a high default risk in emerging markets. In addition, I provide evidence on the impact of export on the productivity growth of firms in emerging countries.

2.0 SOVEREIGN RISK AND PRIVATE CREDIT IN LABOR MARKETS[1]

2.1 INTRODUCTION

This paper examines sovereign default risk in emerging countries from a different viewpoint. Emerging economies experience countercyclical spreads over the business cycles, which are associated with default risk.[2] In addition, since emerging markets are also characterized by procyclical labor, labor will decrease after drops in output in default. In this paper, we focus on the labor market as a channel that amplifies the default risk in emerging economies. We propose a theory that links these observations such that financing of labor costs plays a critical role in generating countercyclical spreads, endogenous drops in output and procyclical labor. Particularly, we ask what is the role of labor markets in the default risk observed in emerging markets? Moreover, we address the question how they are related to the drops in output observed after sovereign defaults.

The stylized empirical facts for emerging economies are presented in Figure 2.1, focusing on a subset of countries that includes Argentina, Korea, Greece, and Spain. We look at the detrended real GDP and employment for these countries. Consistent with the findings in the literature shown by Li (2011), Neumeyer and Perri (2005) and Uribe and Yue (2006), we find that employment tends to move together with real GDP in these four emerging countries over the business cycle. In addition, in recession, the labor drops with the output drops in these economies.

[1]This research is a joint work with Zeynep Kabukcuoglu.

[2]Emerging economies tend to experience high risk of debt crisis and have incurred substantial losses in income from defaults in history. In Tomz and Wright (2013), they examined 251 defaults by 107 distinct entities and the most frequent defaulters are Ecuador, Mexico, Uruguay, and Venezuela. Similarly, Reinhart and Rogoff (2009) summarize that defaulting countries are mostly emerging economies since 1900 in history.

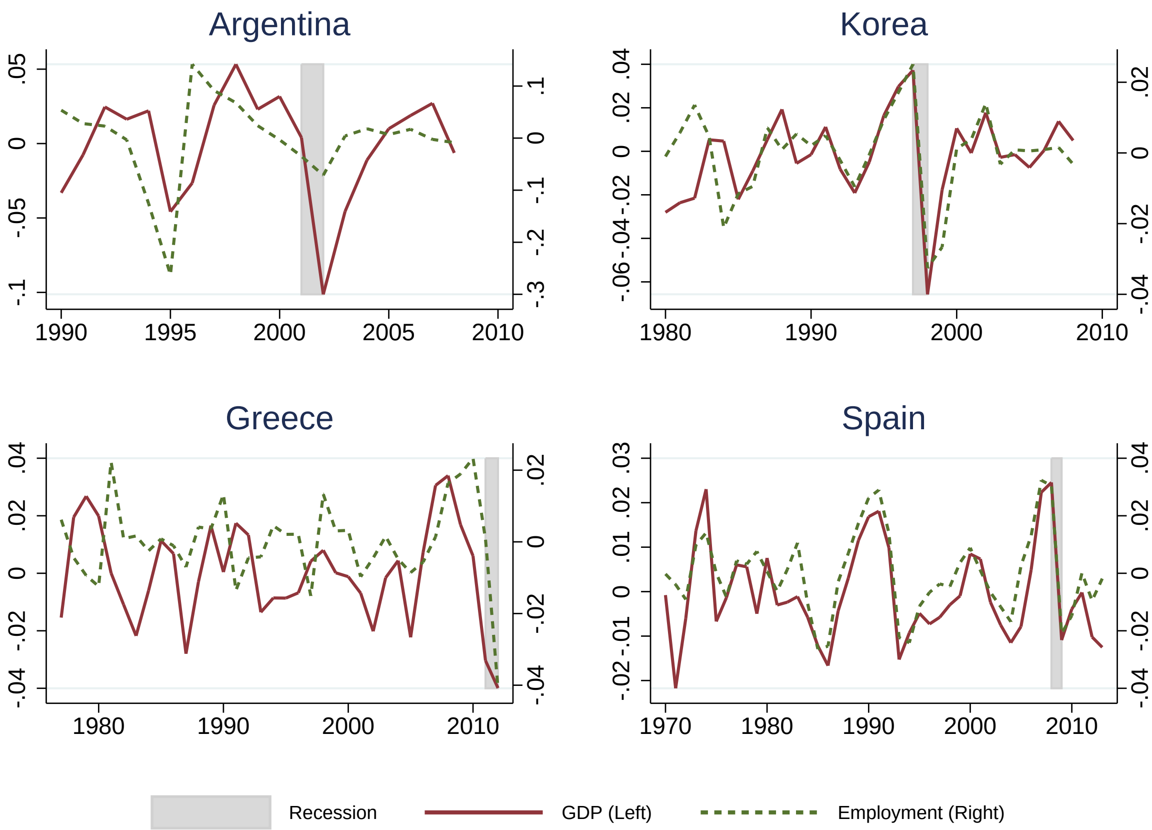


Figure 2.1: GDP and employment in emerging countries

In this paper, we examine these features of emerging markets using a stochastic general equilibrium model in a small open economy. The economy is subject to aggregate uncertainty about productivity. The problem of the representative households is standard in that they make consumption and labor decisions that optimize their life time utility subject to a budget constraint that entails wages, transfers from the government and profits from the firms. Similar to standard models with endogenous default, there is a benevolent government that can borrow from foreign lenders by issuing one-period, non-state contingent bonds, which are not enforceable and the government transfers the proceeds of debt operations to households. The government's incentives to borrow comes from the fact that the government tries to help households have smooth consumption across time, using these transfers. Foreign lenders extend loans to the government, taking into account the default risk. Endogenous default risk is associated with the government's default or repayment decisions and it depends on the level of bonds the government would like to issue and the size of the productivity shock. Default is more likely, if the economy is subject to low TFP shocks and has high levels of debt because they lead to an increase in the premium that the foreign lenders ask when they lend to the government. As foreign lenders ask for a higher premium, it becomes harder for the government to roll over its debt, so it has to incur large taxes on households to finance the existing debt. If this is the case, then default can become an optimal policy because it can help eliminate the tax burden and improve households utility. However, the government faces a trade-off. If the government chooses to default, the government is banned from the loan markets for a temporary period of time. This means government cannot issue bonds to help households have smooth consumption during this period.

To generate endogenous drops in output, we assume that firm's production requires the finance of working capital loans used to pay a fraction of the wage bill. Adopting the working capital condition from Neumeyer and Perri (2005) enables us to examine the role of labor in sovereign defaults. Firms maximize their profits by making labor decisions and taking into account the interest they need to pay on the working capital loans. They demand less labor as working capital loans become more expensive due to the increase in sovereign default risk. The drops in labor demand result in lower production. When the government decides to default on its debt, the firms can still borrow from foreign lenders, but at a high interest rate, even though the government cannot. In this sense, the high interest rate on working capital loans acts as a default penalty on firms. This assumption is consistent with the empirical findings in the literature. Arteta and Hale (2008) show that during sovereign debt crises, there is a significant decline in foreign credit to private firms. The paper suggests that the decrease in amount of credit available to private firms can be an important channel that generates large drops in output observed in defaults.

In addition, we assume that the debtor still has debt arrears following defaults. In a standard default model such as Arellano (2008), the defaulters start with zero debt when they again enter into the debt market. However, this assumption does not account for the debt restructuring in emerging countries. Benjamin and Wright (2009) show that the creditors lose 44 percent of their lending on average through the renegotiation process after the default. Partial default makes our model closer to the actual debt restructuring of the defaulters. It can play a role as another form of penalty on default because the debt arrears lower the future value of default, and therefore it affects the decision on default.

The model explains the main features of the business cycles observed in the emerging markets well, such as countercyclical spreads, countercyclical trade balance, and high consumption and output volatility, when calibrated to Argentine data. In addition, the model can generate reasonable drops in labor and output in defaults. We also obtain procyclical labor over the business cycles and the labor volatility is similar to Argentine economy. We obtain a procyclical labor supply because two things change when the economy is hit by an adverse TFP shock; first, the shock has a direct effect on production and it reduces output because productivity is lower and firms demand less labor, which is a standard result of an RBC model. Second, the shock has an indirect effect on production through the increase in endogenous default risk. Because the government is more likely to default, the interest rate on the working capital loan is also higher, which makes the production even more costly for the firms and it dampens firms' labor demand even more. Equilibrium wages also drop because they are inversely related with interest rates and positively related with the TFP shock. Because we assume that households have Greenwood-Hercovitz-Huffman (GHH) type of preferences, the substitution effect dominates the income effect and the households are willing to supply less labor.[3] Overall this generates even larger drops in output. When households' income drops so much due to the decreases in labor income, the government would like to borrow even more from foreign lenders, so that households can have smooth consumption. However, since the shocks are persistent, the foreign lenders adjust their expectations about the future state of the economy, such that they ask for an even higher premium on the government bonds. This generates a vicious cycle, in which output, labor, consumption and wages decrease further and it becomes harder for the government to roll over its debt. Consequently, the government may choose to default to eliminate the tax burden necessary to finance the existing debt, especially when the level of existing debt is already very high.

Our paper is related to the endogenous sovereign default literature that starts with the seminal paper of Eaton and Gersovitz (1981) and continues with Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2011), Chatterjee and Eyigungor (2012b) and Amador and Aguiar (2014), some of which were mentioned above.[4] These papers assume an exogenous output process and penalty in their models. Our paper is closely related to Mendoza and Yue (2011) in that they consider working capital conditions and endogenous sovereign default. They also combine the international business cycle model and the sovereign default model by considering the interaction between households, firms, government and foreign lenders, as we do in this paper. However, their work is different than ours in many dimensions. First, in their model the efficiency loss from sovereign default generates an endogenous output cost because firms should substitute imported inputs into other imported or domestic inputs, which are imperfect substitutes. However, in our model the default cost stems from the interest rate on working capital and the debt renegotiation. In addition, while their model adopts working capital conditions for imported intermediate goods, our model use working capital conditions for labor demand. Second, on the firms' side they assume that firms are excluded from the international debt market when the government

[3] The advantage of GHH preference specification is that it generates the right comovement between labor supply and production. GHH specification was introduced by Greenwood et al. (1988) and has been used in many papers with small open economy models, such as Mendoza (1991), Correia et al. (1995), Neumeyer and Perri (2005), and many others.

[4] Also see Panizza et al. (2009), Wright (2011) and Aguiar and Amador (2013) for good reviews of this literature.

decides to default. In our model, firms can still access to the international debt markets, but borrow at a high interest rate. In addition, our paper is related to papers on debt renegotiation and default such as D'Erasmo (2008), Bi (2008), Benjamin and Wright (2009), Yue (2010), and Pitchford and Wright (2011). Finally, our paper is related to the literature that studies the business cycle properties of labor market variables in emerging markets. Li (2011) explains countercyclical interest rates and procyclical wages in emerging economies by assuming exogenous default risk. As mentioned above we have endogenous default risk and working capital conditions in our model that generate fluctuations in labor together with productivity shocks.

The rest of the paper is organized as follows: Section 2 presents the model and defines the recursive equilibrium. Section 3 discusses the calibration, the quantitative analysis of the model and the simulation results. Section 4 concludes.

2.2 MODEL

In this section, we present a model economy in order to understand the role of labor supply on sovereign debt default. Basically, our model belongs to the class of models in the standard framework of Eaton and Gersovitz (1981), but richer in the sense that it has households, firms, foreign lenders and the government. We consider a discrete time, small open economy inhabited by representative households. Households choose optimal consumption and labor paths that maximize their lifetime utilities subject to the budget constraint. They receive transfers from the government, wages for supplying labor and profits from the ownership of the firms. Firms face stochastic TFP shocks and finance working capital before production takes place similar to Neumeyer and Perri (2005). There is a benevolent government that represents the preferences of households and has access to international markets. The government can issue one-period bonds to foreign lenders and distribute the proceeds of the debt payments to the households. The government can choose to default on its debt at any time, because contracts are not enforceable. The penalty for default is that the government is forced to stay in financial autarky for a period of time and the firms need to pay higher interest rates on their working capital. In addition, if the government gains access to the international bond markets, it needs to pay the debt arrears. That is, we allow for partial default.

2.2.1 Households

We assume that the households have GHH preferences which are used in open economy models by many international business cycle literatures. The GHH preferences are often adapted because they improve the performance of the model in terms of the business cycle statistics. In addition, these preferences remove the wealth effect on labor supply and the labor supply is determined independently of intertemporal considerations. The functional form of preference is:

$$u(c,l) = \frac{\left(c - \frac{l^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$$

where $\omega > 1$ and $\sigma > 0$.

The households have different budget constraints that depend on whether the government is in autarky or not. If the government decides to repay its debt, the household problem is given as:

$$\max_{c_t, l_t} E_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$
$$\text{subject to } c_t = w_t l_t + \pi_t + (B_t - q_t B_{t+1}).$$

If the government is in autarky, the budget constraint becomes $c_t = w_t l_t + \pi_t$. The optimal labor supply satisfies that

$$l_t^{\omega-1} = w_t. \tag{2.1}$$

2.2.2 Firms

Firms choose labor demand that maximizes their profits. Profits are equal to revenues net of the wage bill and interest payments on working capital loans. Firms have to borrow a certain fraction of the labor cost in order to complete production.

When the government decides to repay its debt, the interest rate on working capital, r_t, is equal to the interest rate on the government's debt.

$$\max_{l_t} z_t k^\alpha l_t^{1-\alpha} - w_t l_t - r_t \theta w_t l_t$$

where z_t is the TFP shock that is assumed to follow a Markov process with a transition function $f(z', z)$. The fraction of the labor cost that needs to be borrowed from foreign lenders at the interest rate, r_t, is denoted by θ.
When the government chooses to default, the firms' problem is:

$$\max_{l_t} z_t k^\alpha l_t^{1-\alpha} - w_t l_t - r_d \theta w_t l_t,$$

where r_d is the interest rate on working capital loans in default. It will be specified in detail in the government's problem.

In addition, we assume that r_d is an upper bound on the interest rate on working capital even when the government decides to repay its debt and the bond price is close to zero.

From the firm's problem, the wage should satisfy the following optimality condition obtained from the firm's problem:

$$w_t = \begin{cases} \frac{1-\alpha}{1+\theta r_t} z_t k^\alpha l_t^{-\alpha} & \text{(Repayment)} \\ \frac{1-\alpha}{1+\theta r_d} z_t k^\alpha l_t^{-\alpha} & \text{(Default).} \end{cases} \tag{2.2}$$

2.2.3 Government

The government of the economy can trade one period, non-state contingent bonds with foreign lenders that are risk free and competitive. Unlike standard default models, when the government defaults, the economy does not face direct output costs, but the government is in a temporary exclusion from borrowing in the debt markets. When the government gains access to the debt markets, it needs to pay a fraction of the debt, which is denoted by κ. In this sense, we allow for only partial default in our model. The government's goal is to maximize the households' expected lifetime utility, given as:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\right], \tag{2.3}$$

where β denotes the discount parameter and $\beta \in (0, 1)$.

The government makes two decisions in this model. The first one is whether to repay or default on its existing debt. Second, conditional on not defaulting, it chooses the amount of one-period bonds, B' to issue or buy. If the government chooses to buy bonds, the price it needs to pay is given as $q(B', z)$. The discount bonds, B', can take positive or negative values. If it is negative, it means that the government borrows $-q(B', z)B'$ amounts of period t goods and promises to pay B' units of goods in the next period, if it does not default. Similarly, if B' is positive, then it implies that the government saves $q(B', z)B'$ amounts of period t goods and it will receive B' units of goods in the next period. The bond price function $q(B', z)$ depends on the size of the bonds, B', and TFP shock, z. Government's incentive to default and the price functions are both endogenous.

The government's value of option is the maximum of value of default (v^d) or value of repayment (v^c) and it is given as:

$$V(B_t, z_t) = \max_{c,d} \left\{ v^c(B_t, z_t), v^d(B_t, z_t) \right\}.$$

The value of repayment is represented by

$$\begin{aligned} v^c(B_t, z_t) &= \max_{B_{t+1}} u(c_t, l_t) + \beta E_t \left[V(B_{t+1}, z_{t+1}) \right] \\ &\quad \text{subject to } c_t = z_t k^\alpha l_t^{1-\alpha} - r_t \theta w_t l_t + B_t - q_t(B_{t+1}, z_t) B_{t+1}. \end{aligned}$$

If the government chooses to repay its debt, the value function for this choice reflects the future options for default and staying in contract. The government chooses the optimal bond contract that maximizes the utility of the households and the discounted future value of option.

The value of default is given as:

$$\begin{aligned} v^d(B_t, z_t) &= u(c_t, l_t) + \beta E_t \left[(1-\phi) v^d(B_t, z_{t+1}) + \phi v^c(\kappa B_t, z_{t+1}) \right] \\ &\quad \text{where } c_t = z_t k^\alpha l_t^{1-\alpha} - r_d \theta w_t l_t \end{aligned}$$

The probability of having access to bond markets in the next period is denoted by ϕ. The value of default is equal to the utility of household plus the future expected discounted value that entails the value of default weighted by $1 - \phi$ and value of option in the next period weighted by ϕ. The value of option has κB_t as the state variable because the government enters into the international debt market with the debt arrears κB_t due to debt renegotiation process.

2.2.4 Foreign Lenders

Foreign creditors can perfectly monitor the state of the economy and have perfect information about the shock processes. They can borrow loans from international credit markets at a constant interest rate, $r^* > 0$, which is the risk free interest rate in this model. Taking the bond price function $q(B_{t+1}, z_t)$ as given, they choose loans B_{t+1} that maximize their expected profits π, given as:

$$\pi(B_{t+1}, z_t) = \begin{cases} q(B_{t+1}, z_t)B_{t+1} - \frac{B_{t+1}}{1+r^*} & \text{(if } B_{t+1} \geq 0) \\ \frac{1-\delta(B_{t+1},z_t)+\delta(B_{t+1},z_t)q^d}{1+r^*}B_{t+1} - q(B_{t+1}, z_t)B_{t+1} & \text{(if } B_{t+1} < 0), \end{cases} \tag{2.4}$$

where $\delta(B_{t+1}, z)$ is the probability of default and is determined endogenously. The expected bond price in default is denoted by q^d.

Because we assume that the market for new sovereign debt is completely competitive, the foreign investors' expected profit is zero in equilibrium. Hence, we have the bond price as following:

$$q_t(B_{t+1}, z_t) = \begin{cases} \frac{1}{1+r^*} & \text{(if } B_{t+1} \geq 0) \\ \frac{1-\delta(B_{t+1},z_t)+\delta(B_{t+1},z_t)q^d}{1+r^*} & \text{(if } B_{t+1} < 0), \end{cases} \tag{2.5}$$

where the expected bond price in default, q^d, is expressed as the following:[5]

$$q^d = \frac{\kappa\phi}{r^* + \phi}$$

That is, the bond price reflects both the default risk and the risk of debt restructuring.

[5] q^d is derived from the following process.

$$q^d = \frac{\kappa\phi}{1+r^*} + \frac{1-\phi}{1+r^*}\left[\frac{\kappa\phi}{1+r^*} + \cdots\right] = \frac{\kappa\phi}{1+r^*} + \frac{1-\phi}{1+r^*}q^d = \frac{\kappa\phi}{r^* + \phi}$$

Using the bond price function, the interest rate on working capital loans can be written as:

$$r_t(B_{t+1}, z_t) = \begin{cases} \frac{1}{q_t(B_{t+1}, z_t)} - 1 & (\text{if } r_t < r_d) \\ r_d & (\text{otherwise}). \end{cases} \tag{2.6}$$

When the government saves ($B_{t+1} > 0$) or does not default on it debt, the interest rate on working capital loans is a function of the bond price. However, if the government decides to default on its debt, then the interest rate is the maximum level in the economy, which is exogenously set in the model.

2.2.5 Recursive Equilibrium

We focus on a recursive equilibrium, where there is no enforcement. Based on the foreign creditors' problem, government's debt demand is met as long as the gross return on the bond equals to the risk free rate, $1 + r$.

We can characterize the government's default policy by default and repayment sets. Let $A(B)$ be the set of z for which repayment is optimal when assets are B, such that

$$A(B) = \left\{ z \in \mathbb{Z} : v^c(B, z) \geq v^d(B, z) \right\}, \tag{2.7}$$

and let $D(B) = \tilde{A}(B)$ be the set of z for which default is optimal for a level of assets B:

$$D(B) = \left\{ z \in \mathbb{Z} : v^c(B, z) < v^d(B, z) \right\}. \tag{2.8}$$

Also, let $s = \{B, z\}$ be the set of aggregate states for the economy.

Definition 1. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $c(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; (iii) the wage function $w(B', z)$; and (iv) the price function for bonds $q(B', z)$ such that:*

1. *Households' consumption $c(s)$ satisfies the resource constraints, taking the government policies as given.*

2. *The government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem, taking the bond price function $q(B', z)$ as given.*
3. *The optimal wage function $w(B', z)$ satisfies firms' optimization problem, taking the interest rate on working capital loans $r(B', z)$ as given.*
4. *Bonds prices $q(B', z)$ reflect the government's default probabilities and default probabilities satisfy creditors' expected zero profits.*
5. *Labor market clears.*

At the equilibrium, the bond price function should satisfy both the government's optimization problem and the expected zero profits in the lenders' problem, so that probability of default endogenously affects the bond price. Using the default sets, we can express the probability of default such that:

$$\delta(B', z) = \int_{D(B')} f(z', z) dz'.$$

2.3 QUANTITATIVE ANALYSIS

We solve the model numerically. In this section, we describe the estimation procedure for the shock processes. We calibrate the model to analyze the debt dynamics quantitatively, using Argentine data between 1990-2002.

2.3.1 Data

First, we begin with documenting the business cycle characteristics of the Argentine economy. For the business cycle statistics we use real output, consumption and trade balance data in quarterly, seasonally adjusted, real series covering the period 1993Q1 and 2001Q4 from the dataset in Arellano (2008).[6] We take logs of GDP and consumption series and detrend these series using an HP filter with a smoothing parameter of 1600. The trade balance data are reported as a fraction of GDP. We also borrow the spread data from Arellano (2008), which

[6] Arellano (2008) uses the data provided by the Ministry of Finance of Argentina.

Table 2.1: Business cycle statistics for Argentina from 1993Q1 to 2001Q4

	std(x)	corr(x, y)	corr(x, r^c)
Interest rates spread	3.08	−0.74	–
Trade balance	1.75	−0.58	0.70
Consumption	3.75	0.97	−0.68
Output	3.33	–	−0.74
Wage	4.18	0.49	−0.34
*Labor**	3.69	0.58	−0.85
Other Statistics in default in 2002			
Output drop	12.01		
Consumption drop	12.86		
Wage drop	13.88		
Labor drop	18.46		
Default probability	2.78		

* Quarterly labor data are available only between 1997Q1 and 2001Q4

are defined as the difference between the interest rate in Argentina and the yield of the five-year US Treasury bond. The interest rate series is EMBI for Argentina and start from 1983Q3. The quarterly wage series are available in International Financial Statistics (IFS) and Instituto Nacional de Estadistica y Censos (INDEC). We take logs and detrend the series using an HP filter with a smoothing parameter equal to 1600. For the labor data, we use the weekly hours of work from INDEC. However, these are only available starting from 1997. Hence, we use a short time series for labor.

Table 2.1 presents the business cycle statistics of all the data available up to the default episode that started in December 26, 2001. The first column shows the standard deviations up to the default episode. We find that consumption, wage, and labor are more volatile than output. The second and the third column present the correlations of each variable with the output and the interest rate spread, respectively. It has been shown that emerging market economies are characterized by countercyclical spread rates and net exports. Also, their consumption is highly correlated with output. We see similar empirical results for Argentina in the second column. In addition, we see that labor and wage are procyclical with output. The interest rate spread is negatively correlated with consumption and output, and positively correlated with trade balance. Wages and the labor are negatively correlated with the spread rate.

2.3.2 Calibration

The model is solved quantitatively. In the numerical solution, we define one period as a quarter. Our calibration strategy is largely based on Argentine data. Table 2.2 shows the calibrated parameter values.

The utility function represents GHH preferences as we explained in section 2.1. The risk aversion parameter, σ, is set to two, the risk-free interest rate is set to one percent, and the capital share to 0.32 percent, which are standard values in macroeconomics literature. The curvature parameter of labor in GHH preference is set to 1.455 which determines Frisch wage elasticity by $\frac{1}{\omega-1} = 2.2$. The debt recovery rate κ is set to 0.27 following Benjamin and Wright (2009). Benjamin and Wright (2009) estimate the recovery rate for all the default

Table 2.2: Parameters

Name	Parameters	Description
Risk-free interest rate	$r^* = 0.010$	Standard RBC value
Risk aversion	$\sigma = 2.000$	Standard RBC value
Capital share	$\alpha = 0.320$	Mendoza (1991)
Curvature parameter of labor supply	$\omega = 1.455$	Frisch wage elasticity=2.2
Debt recovery rate	$\kappa = 0.270$	Benjamin and Wright (2009)
Calibration	**Values**	**Target statistics**
Autocorrelation of TFP shocks	$\rho_z = 0.952$	GDP autocorrelation = 0.860
Standard deviation of TFP shocks	$\sigma_z = 0.017$	GDP std. deviation = 0.033
Discount factor	$\beta = 0.877$	Default probability = 2.78%
Interest rate on working capital in default	$r_d = 0.350$	Wage drop in default = 13.88%
Probability of reentry	$\phi = 0.150$	Trade Balance Volatility = 1.75
Fraction of working capital	$\theta = 0.145$	Output drop in default = 12.01%

episodes in recorded history. For Argentina's default in 2001, they estimate it as 27 percent.

For the TFP shock process, we assume that it follows an AR(1) process:

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t$$

with $\epsilon \sim N(0, \sigma_z^2)$. We use the quadrature method in Tauchen (1986) to construct a Markov approximation with 21 realizations. Data for labor is not available for 1993Q1 to 1996Q4, so we set ρ_z and σ_z to target the standard deviation and first-order autocorrelation of quarterly HP filtered GDP data of Argentina. We use seasonally-adjusted quarterly real GDP data from Arellano (2008) for the period 1980Q1 to 2005Q4. The standard deviation and autocorrelation of the cyclical component of GDP are 3.3 percent and 0.86, respectively. To match these targets, we set $\rho_z = 0.952$ and $\sigma_z = 0.017$.

The discount parameter β, the working capital interest rate in default r_d, the probability of reentry into international debt market, ϕ, and the fraction of working capital, θ, target default probability, wage drop in default, output drop in default, and trade balance volatility. We use SMM method to match these targets and the parameters are calibrated, such that $\beta = 0.877$, $r_d = 0.35$, $\phi = 0.150$, and $\theta = 0.145$.

2.4 SIMULATION RESULTS

In this section, we begin with the analysis of the benchmark model's results and we also elaborate on the intuition on the workings of the model. To solve the model numerically, we use the discrete state-space method. We discretize the asset space, making sure that the minimum and the maximum points on the grid do not bind when we compute the optimal debt decision. Our solution algorithm for the benchmark model is given in the appendix A.1.

Figure 2.2 shows the default risk and the bond price schedule generated by the model. As the model suggests the more the government borrows, the higher the default risk becomes. In addition, default risk increases as the economy is hit by low TFP shocks. Similar to the results presented in standard default literature, such as Arellano (2008) and Aguiar and

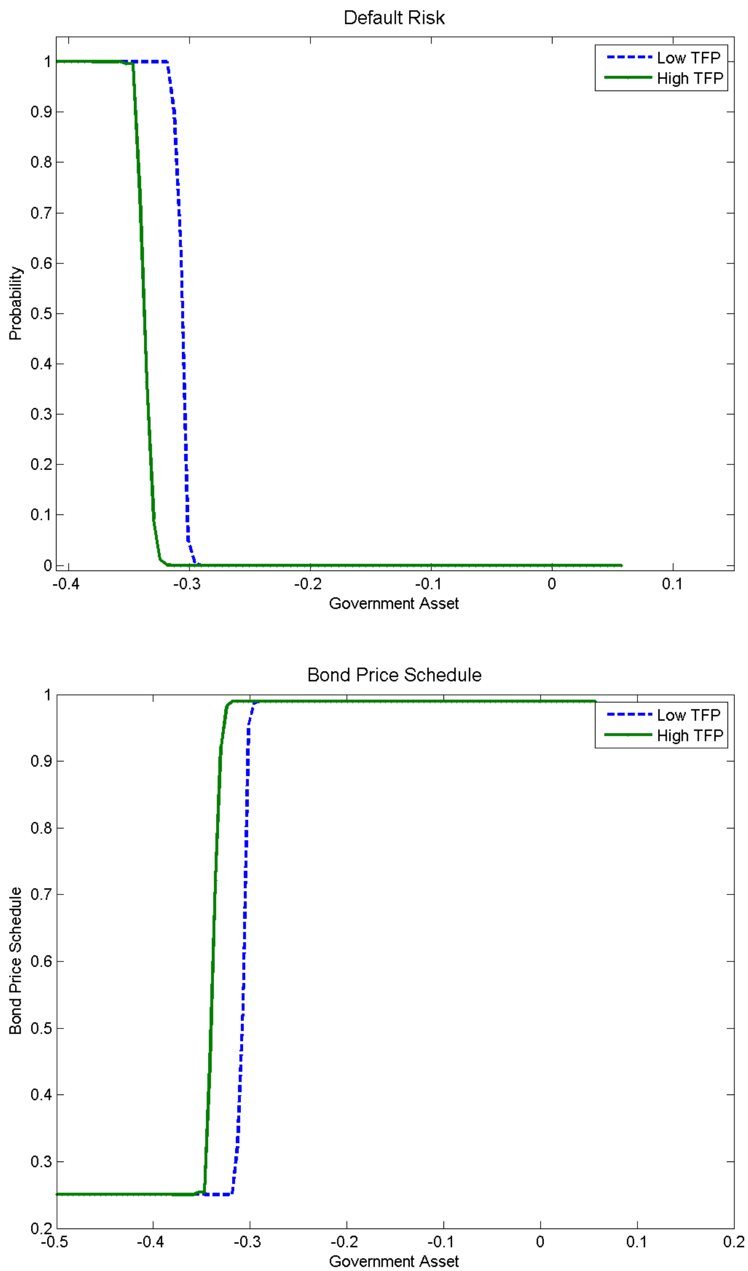


Figure 2.2: Default risk and bond prices

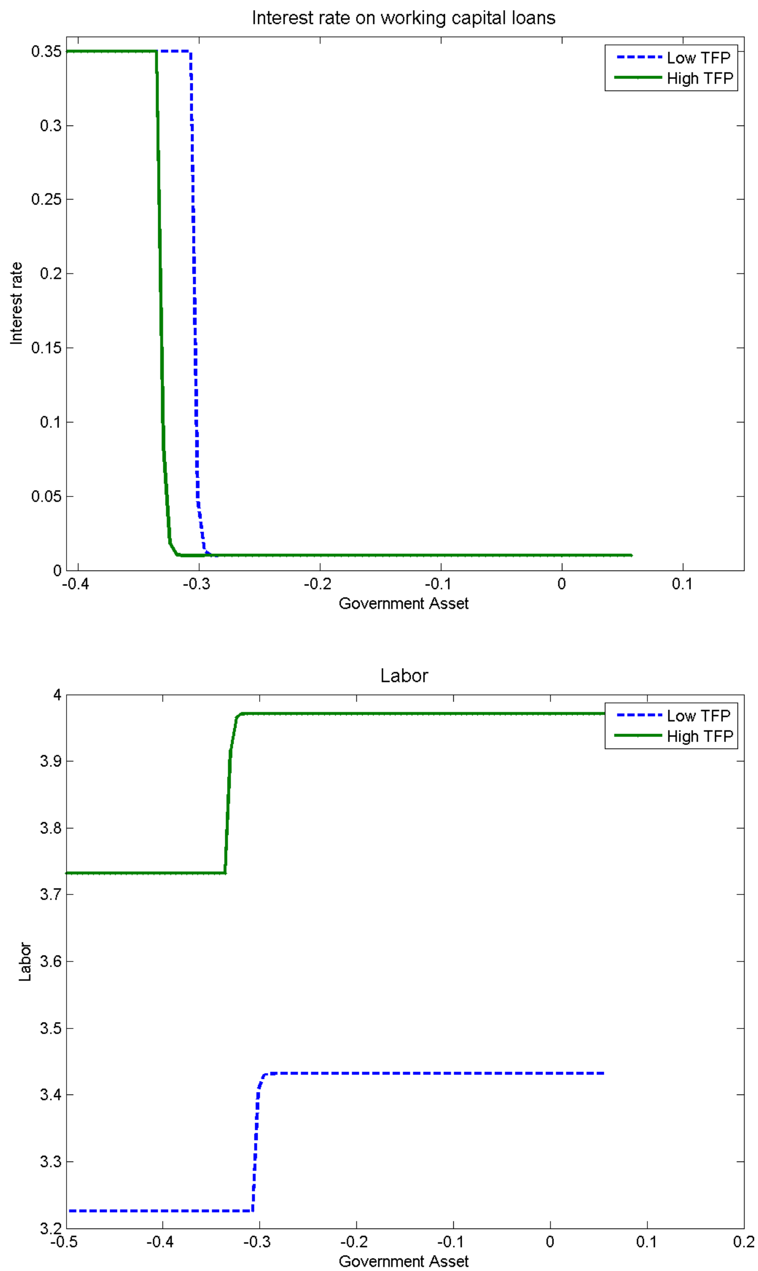


Figure 2.3: Interest rate on working capital and labor supply

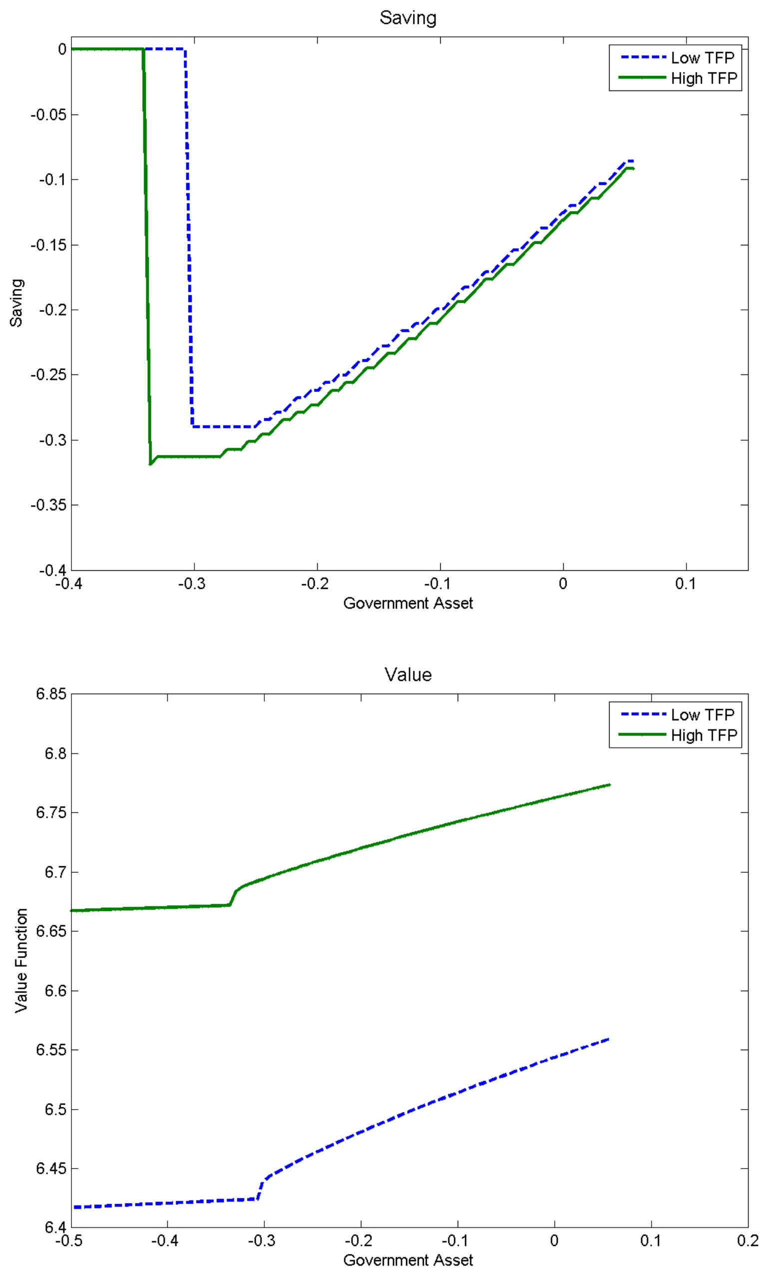


Figure 2.4: Savings and value functions

Gopinath (2006), we observe that the bond price is an increasing function of the assets, such that high levels of debt entails a low bond price. The bond price schedule is determined by not only the default risk, but also the risk of debt restructuring and the expected bond price in default q_d, which is constant regardless of the size of the TFP shock. Even though the government is not able to borrow in default, its debt arrears are evaluated at the bond price in default, q_d. In addition, the bond price is an increasing function in TFP shock. That is, the economy with high TFP shock pays less interest on its debt than the one subject to low TFP shock.

The first panel in Figure 2.3 shows the interest rates on working capital loans generated by the model. The interest rate is calculated using (2.6). Unlike the standard sovereign default models, the interest rate on working capital has an upper bound of r_d and it is the level that the firms need to pay for borrowing working capital, when the sovereign declares default. The interest rate on working capital is a decreasing function in government assets and TFP shocks. Firms in the economy with high TFP shock and low debt pay less interest on working capital compared to those in the economy with low TFP shock and high debt. On the second panel in Figure 2.3, we show that the labor supply increases as the government assets increase and the state of the world gets better. Intuitively, wages increase in expansions, so households are willing to supply more labor. Also, firms face lower interest rates on working capital loans, which reduces labor costs, therefore in equilibrium they demand more labor during expansions.

The first panel in Figure 2.4 shows the saving policy function conditional on not defaulting, which is similar with the standard default model. Our results show that the government borrows more in expansions and is less likely to default in good states of the world. This result is consistent with countercyclical interest rates, since it becomes more costly to borrow in bad states of the world. The second panel of Figure 2.4 is the value of option as a function of assets. The small kink shows the level of assets above which repayment is optimal.

Next, we move on to the business cycle statistics generated by the benchmark model and we evaluate the performance of the model with Argentine data. The simulation results for the benchmark model are presented in Table 2.3. The benchmark model generates a default probability of 0.03 percent, which is relatively smaller than the data (3 percent). In our

Table 2.3: Statistical moments in the benchmark model and in the data

Statistics	(1) Data	(2) Model	(3) Model ($\theta = 0$)
Average debt/GDP ratio (%)	33.20	30.60	-
Average bond spreads (%)	6.60	0.02	0.00
Std. dev. of bond spreads (%)	3.08	0.03	7.36
Std. dev. of labor (%)	3.69	5.96	-
Std. dev. of consumption / Std. dev. of GDP	1.13	1.01	0.97
Std. dev. of labor / Std. dev. of GDP	0.86	0.69	0.68
Correlations with GDP			
Bond spreads	-0.74	-0.25	-
Trade balance	-0.58	-0.22	0.99
Labor	0.58	1.00	1.00
Correlations with bond spreads			
Trade balance	0.70	0.02	-
Labor	-0.85	-0.25	-
Other statistics in default			
Output drop (%)	12.01	20.12	-
Consumption drop (%)	12.86	23.03	-
Wage drop (%)	13.88	7.79	-
Labor drop (%)	18.46	17.12	-
Default probability (%)	2.78	0.02	-

model, we don't have ad-hoc default penalty as other literatures on sovereign default. Even without this type of output penalty, the simulation results from our model are fairly similar to the business cycle statistics in Argentina. The model also generates large drops in output and wage during default episodes as in the data. The model can also generate high volatility in labor supply.

In terms of correlations with output, consumption shows a positive correlation and the interest rate spread shows a negative correlation consistent with the data. Moreover, there is a negative correlation between output and trade balance. The reason is that households can consume more than their income from wages and profits during expansions, because the government can borrow easily. On the other hand, when there is a recession, borrowing is constrained, therefore the consumption is less than the income from wages and profits of the firms. This generates a countercyclical trade balance over the business cycle. Correlations with interest rate show consistent results with the data.

Our model also performs well in terms of generating a procyclical labor supply. We obtain a procyclical labor supply because two things change when the economy is hit by an adverse TFP shock; first, the shock has a direct effect on production and it reduces output because productivity is lower and firms demand less labor, which is a standard result of an RBC model. Second, the shock has an indirect effect on production through the increase in endogenous default risk. Because the government is more likely to default, the interest rate on the working capital loan is also higher, which makes production even more costly for the firms and it dampens firms' labor demand even more. Equilibrium wages also drop because they are inversely related to interest rates and positively related to the TFP shock. Because of the GHH preferences, the substitution effect dominates the income effect and the households are wiling to supply less labor. Overall this generates even larger drops in output. When households' income drops so much due to the decreases in labor income, the government would like to borrow even more from foreign lenders so that households can have smooth consumption. However, since the shocks are persistent, the foreign lenders adjust their expectations about the future state of the economy, such that they ask for an even higher premium on the government bonds. This generates a vicious cycle, in which output, labor, consumption and wages decrease further and it becomes harder for the government to roll over its debt. Thus, the government may choose to default to eliminate the tax burden necessary to finance the existing debt, especially when the level of existing debt is already very high.

To look at the role of the working capital condition, we set θ equal to zero. We find that this model generates no default. In addition, consumption becomes less volatile than output and the trade balance becomes procyclical. The results show that the financing of working capital plays an important role in generating the default risk.

2.5 CONCLUSION

This paper studies the relationship between endogenous default risk and labor decisions using a stochastic general equilibrium model in a small open economy. With the assumptions on working capital loans and the debt renegotiation in default, our model performs well in matching the business cycle characteristics of the Argentine economy. We obtain countercyclical interest rates and procyclical labor. An increase in default risk yields a lower bond price and it implies a high interest rate on working capital loans. As the cost of production increases, firm's labor demand decreases. Since equilibrium wages also drop and the substitution effect dominates the income effect, the households are willing to supply less labor. In equilibrium we find that both production and labor are lower, when the economy is hit by an adverse TFP shock. The reduction in labor income and output induces the government to want to borrow more from foreign lenders; however, the lenders ask for higher premiums due to endogenous default risk. This makes borrowing even harder for the government and the government may choose to default to eliminate the tax burden necessary to finance the existing debt.

Sovereign defaults are related to a number of factors domestically as well as internationally. Unlike the earlier papers in the literature, in this paper we focus on the labor supply and demand decisions as channels that induce default. We show that working capital conditions can be a channel for amplifying the effects of adverse TFP shocks on labor supply and demand decisions and it can increase the default risk. We see this paper as a first step to look into the how labor supply and demand decisions are linked with sovereign default risk. However, we believe that one might get even stronger amplification effects by introducing other labor market frictions, such as matching frictions similar to the ones used in Mortensen-Pissarides type of models. We leave this for future study.

3.0 INCOME INEQUALITY AND SOVEREIGN DEFAULT[1]

3.1 INTRODUCTION

Emerging markets tend to experience high rates of sovereign default and they are also characterized by high income inequality that changes over time. In sovereign default literature, the line of research following Eaton and Gersovitz (1981) has focused mainly on the effects of output shocks on the government's borrowing and default decisions. In this paper, we study the role of income inequality in government's borrowing and default decisions. Does higher income inequality increase the probability of default? Furthermore, how do changes in inequality compare to changes in output in explaining the variation in default probability?

Figure 3.1 presents empirical evidence that shows the relationship between income inequality and the sovereign bond ratings. We estimate the partial regressions, controlling for external debt-to-GDP ratio, income per capita, country and time fixed effects, using a panel dataset that covers 23 emerging economies from 1994-2012.[2] The figure shows the scatter plots and the negative relationship between income inequality measured by the Gini index and the creditworthiness of countries' bond ratings, which proxy the default risk.[3] The results suggest that there is a positive correlation between income inequality and default risk.

In order to explore the relationship between endogenous default risk and income inequality, we consider a stochastic general equilibrium model following an approach similar to that

[1]This research is a joint work with Zeynep Kabukcuoglu.

[2]Following the empirical papers that study the determinants of a country's default risk, we also use ratings grade data and total external debt scaled by GDP in this analysis. See Cantor and Packer (1996), Reinhart (2002) and Reinhart et al. (2003).

[3]Reinhart et al. (2003) show that sovereign bond ratings are good predictors of default.

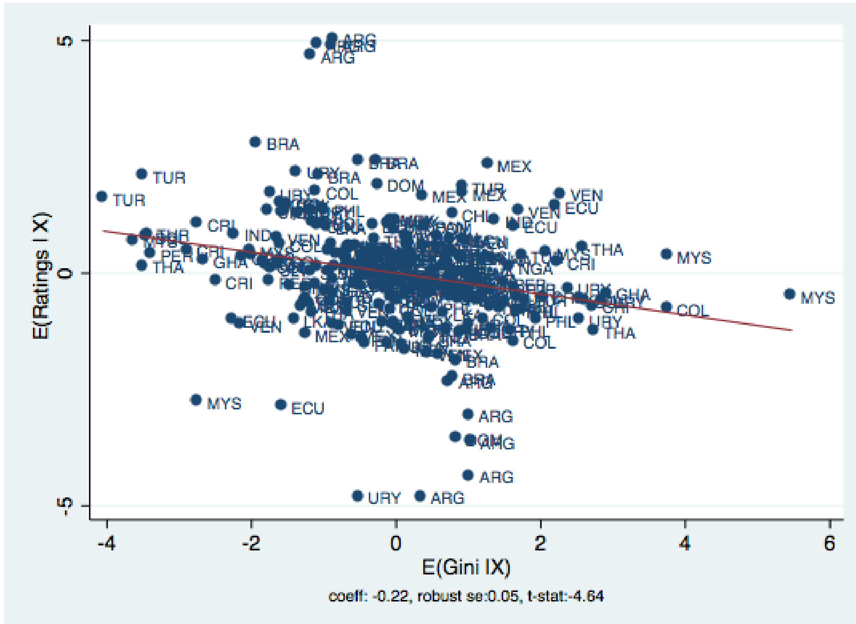


Figure 3.1: Partial Regression Plots $E(ratings|X)$ vs $E(Gini|X)$

of Eaton and Gersovitz (1981). We model a small open economy with two types of households. In addition to output shocks that affect the average level of endowment, we introduce shocks that affect its distribution, which we call inequality shocks.[4] The economy is subject to aggregate uncertainty about future endowments, and households cannot completely insure against the shocks. The output and inequality shocks have different effects on the endowments; an adverse output shock lowers the endowments of both types, but an adverse inequality shock raises the endowment of the rich households and reduces the endowment of the poor households, increasing the dispersion between the endowments. There is also a benevolent government that represents the preferences of the households and can issue non-state contingent, one-period bond contracts to borrow from risk-neutral foreign lenders, retaining the option to default at any time. We assume that default entails exogenous drops in output and that the economy goes into autarky temporarily. The government internal-

[4]Even though our model treats the changes in income inequality as exogenous, these shocks can be motivated by the fact that idiosyncratic labor earnings risk exhibits countercyclical volatility, as shown by Storesletten et al. (2004).

izes how its borrowing decisions affect the default risk, as well as the price of bonds, which determines the interest rates.

In our model, the government would like to borrow on behalf of households for two reasons. First, the government uses bond contracts and rebates the proceeds of debt operations equally across households to help them smooth consumption. Second, the equilibrium interest rate is lower than the discount rate of the government, so the government would like to shift future consumption to today by borrowing. The level of existing debt and the size of the shocks are crucial for government's borrowing decision. As the debt accumulates, it becomes harder to roll over because the benefits of borrowing diminish. Defaults are particularly more attractive in recessions, in high inequality states and when there is high debt accumulation because foreign lenders offer bond contracts that have higher interest rates in those states, which creates a borrowing constraint for the government. The government's goal is to maximize household's expected lifetime utilities, so it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. Default can reduce the gap in the marginal utilities of consumption between the two types of households because the burden of debt payment can be eliminated. Consequently, in our model, default can serve as a redistribution mechanism that improves households' welfare. The main finding of this paper is that inequality shocks can increase the default risk significantly. The key intuition for this result is that when the economy is subject to both adverse inequality and output shocks, the marginal utility of consumption of the poor increases significantly relative to the marginal utility of consumption of the rich. This generates a large tax burden, particularly on poor households, and the government chooses to default more often to wipe out the debt burden.

When we consider the role of each shock, we find that default risk is slightly higher when there are output shocks than inequality shocks. This is because the implied default penalties are different in the two models. In the case of output shocks, the default penalty is higher in good states of the world and smaller otherwise. So with smaller penalty and tighter borrowing constraints in bad states, we observe a larger default rate. On the other hand, in case of inequality shocks the default penalty is constant across all states because aggregate output is constant. This generates a lower probability of default in this model.

However, each shock alone can generate only about one sixth of the probability of default observed when there are both shocks in the model. Thus, we show that the joint effect of these shocks helps the model generate a default probability consistent with the data. The reason behind this result is the VAR(1) process estimated from the Argentine data. Based on the estimates of the structural parameters, we find that high inequality at time $t-1$ leads to lower output at time t. Also, the estimates of the covariances of the shocks are negative, which implies that there is more likely to be an adverse output shock together with an adverse inequality shock. These characteristics play an important role in lenders' and the government's expectations about the future state of the economy. An adverse inequality shock not only amplifies the effect of a low output shock today, but also creates a deep-seated pessimism that the recession will be more severe in the future. As a result, foreign lenders ask for a higher premium even for smaller levels of debt. This increases the borrowing constraints for the government, and default becomes the optimal decision.

The model is calibrated using Argentine data between 1990-2002, and we simulate the model to generate the business cycle statistics. Our model's results regarding the default probabilities can be compared to the results in Aguiar and Gopinath (2006).[5] Similar to Aguiar and Gopinath (2006), the default probability when the economy is hit by an output shock is quite low, only 0.52 percent. Aguiar and Gopinath (2006) also use shocks to the trend of output and generate a default probability of around two percent. On the other hand, the inequality shocks generate a default probability of 0.32 percent. Using shocks to both output and inequality, our model can match countercyclical interest rates, high volatility of consumption and output, and a positive correlation between the trade balance and interest rate spreads. In addition to that, inequality is countercyclical with output and positively correlated with interest rate spreads.

We also calibrate the model to match the business cycle statistics of Mexico. We find that the model can explain the business cycle statistics observed in Mexico well. In addition, it can also generate high consumption volatility of the poor households relative to the rich households. We find that the ratio of the volatilities is close to its data counterpart. Incom-

[5] Aguiar and Gopinath (2006) assume a representative agent model; yet their default penalty structure and calibration strategy are similar to ours.

plete asset markets together with output and inequality shocks are key for this result. In the model, income inequality shocks amplify the effect of output shocks particularly on the poor households' endowment. Since there are no other assets that the households can use to insure against these shocks, poor households have higher volatility of consumption than the rich households. This is an important contribution of the paper that has not been shown by the existing papers in the literature before.

As a policy implication, we extend the model by introducing progressive income taxes and analyze the effect of these taxes on the debt levels and the default probability. When it is costly to borrow for the government, i.e. the proceeds of the debt operations are negative, the government finances the existing debt by issuing progressive income taxes. We adopt the progressive tax regime that Heathcote et al. (2014) present. However, when it is cheap to borrow, the government does not tax households, it simply distributes the transfers across households. We show that as the progressivity of the tax increases, the probability of default decreases. The tax system helps eliminate the effect of inequality shocks in the model and reduces the dispersion in the marginal utilities of consumption between households. Therefore, we obtain larger debt in the simulated economies.

This paper relates to the recent quantitative models that explore emerging markets' business cycles and sovereign debt. We contribute to the sovereign default literature by incorporating the role of income inequality as an additional source of default risk. The endogenous sovereign default literature starts with the seminal paper of Eaton and Gersovitz (1981) and continues with Aguiar and Gopinath (2006), Arellano (2008), Martin and Ventura (2010), Yue (2010), Pitchford and Wright (2011), Chatterjee and Eyigungor (2012a), Amador and Aguiar (2014) and Gennaioli et al. (2014), some of which were mentioned above.[6] Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012b) consider long maturity bonds in a representative agent framework. Cuadra and Sapriza (2008) and Hatchondo et al. (2009) study the role of political uncertainties in sovereign default risk. Martin and Ventura (2010) and Broner et al. (2008) show that well-functioning secondary markets can eliminate the default risk. All these pa-

[6] Also see Panizza et al. (2009), Wright (2011) and Aguiar and Amador (2013) for good reviews of this literature.

pers use representative agent models and focus on the role of output shocks. Our paper is also closely related to D'Erasmo and Mendoza (2012) and D'Erasmo and Mendoza (2013), the main focus of which is the relationship between wealth inequality and default using a heterogeneous agent framework. D'Erasmo and Mendoza (2012) have endogenous wealth heterogeneity that comes from idiosyncratic income shocks; however, the amount of bonds is determined by a fiscal reaction function and does not come from the maximization of household utility. As mentioned above, in our model, the government optimally chooses the level of next-period bonds taking into account the welfare of the households. Furthermore, we show that income inequality shocks tend to have a systematic relationship with output shocks, so we incorporate this dimension into our model to generate inequality. D'Erasmo and Mendoza (2013) study the distributional effects of sovereign debt default in a two-period, closed economy model, assuming an exogenous initial wealth distribution. In their closed economy setup, they study optimal debt and default decisions on domestic debt. However, in our model, we focus on borrowing and default on external debt in a small open economy. In this sense, our paper is complementary to these two papers. Cuadra et al. (2010) study fiscal policy and default risk using a representative agent model, in which tax on consumption is endogenously determined and the revenues are used to finance public goods. In our paper, we assume progressive taxes on income.

Our paper is also related to the immense empirical literature that studies the determinants of sovereign default. Cantor and Packer (1996) show that income, external debt and economic development are significant determinants of credit risk. Reinhart et al. (2003) show that a country's past behavior about meeting its debt obligations can be a good predictor of its ability to pay future debt, pointing out the importance of financial institutions. Hatchondo et al. (2007) argue that countries are more likely to default during periods with low resources, high borrowing costs and changes in political circumstances, and González-Rozada and Yeyati (2008) examine the role of global factors, such as liquidity, risk appetite and contagion, in explaining the emerging market spreads.

The rest of the paper is organized as follows: We provide a more formal analysis of the empirical results, showing the relationship between income inequality and credit scores in Section 2. We then present the model and define the recursive equilibrium in Section 3.

We discuss the calibration, the quantitative analysis of the model and the simulation results with counter factual experiments in Section 4. Section 5 presents the business cycle statistics obtained for Mexico and discusses the differences in consumption volatilities between rich and poor households. Section 6 shows the effects of income taxes. Section 7 concludes.

3.2 EMPIRICAL MOTIVATION

In this section, we provide empirical results that support the relationship between income inequality and default risk. We use credit ratings dataset as a measure of default risk. Reinhart (2002) shows that credit ratings can predict defaults well.[7] First, we show that income inequality is positively correlated with the creditworthiness of sovereign bonds. Next, we provide evidence on the fact that income inequality is countercyclical over the business cycle.

3.2.1 Income Inequality and Credit Ratings

Reinhart et al. (2003) show that there is a strong relationship between external debt and credit ratings. In order to present some empirical evidence for the effect of inequality on the creditworthiness of sovereign bonds, we follow an approach similar to that in Reinhart et al. (2003). We use the following specification to estimate the effect of inequality on credit scores:

$$\begin{aligned} \textit{Credit Score}_{i,t} &= \alpha_0 + \alpha_1 \mathrm{Gini}_{i,t-1} + \alpha_2 \textit{Debt-to-GDP}_{i,t-1} \\ &+ \alpha_3 \textit{GDP per capita}_{i,t-1} + u_i + z_t + \textit{error}_{i,t} \end{aligned} \tag{3.1}$$

To measure the creditworthiness of sovereign bonds, we use the Fitch credit ratings data for long-term bonds that are issued under foreign currency. This dataset covers a period between 1994 and 2012. For income inequality, we use the Gini indices provided by the Standardized World Income Inequality Database (SWIID) (Solt, 2009). This is an unbalanced panel

[7] They show that this relationship is robust using various credit-score datasets such as Institutional Investor ratings, Standard and Poor's and Moody's.

dataset that has information on inequality for 153 countries covering 1960 to 2012. Debt-to-GDP ratio is the external debt-to-GDP ratio from the Reinhart-Rogoff series that extends until 2010. Most of this dataset comes from IMF's Standard Data Dissemination Service, and it is defined as the outstanding amount of those actual current liabilities that require payments of principal and/or interest that residents of an economy owe to non-residents (Statistics, 2003). The GDP per capita series is from the World Bank, and we take its log for this estimation. The Net foreign assets-to-GDP (NFA/GDP) data used in Table 3.1 are from the External Wealth of Nations Mark II database by Lane and Milesi-Ferretti (2007). This series includes net foreign assets (NFA) using FDI or equity assets and liabilities estimated using different methodologies. NFA is defined as the sum of the net debt position, the net equity position and the net FDI position in Lane and Milesi-Ferretti (2001). In order to perform a regression using the credit ratings, we assign a numerical value similar to that in Cantor and Packer (1996) and Reinhart (2002). Table B1 shows the conversion of the ratings to scores in Appendix B.1.

We expect to obtain a negative coefficient on Gini and debt-to-GDP ratio and a positive coefficient on GDP per capita. This implies that higher inequality in country i at time $t-1$ reduces the credit score in the next period. The credit score of a country shows how risky that country's bond is, and higher inequality increases the riskiness, which is reflected by a lower credit score.

Table 3.1 shows the summary statistics for the variables used for the regression sample, which covers the period 1994-2010 and contains 45 countries. A couple of differences stand out when we compare observations of emerging markets and advanced economies. First, emerging markets have low ratings even though their debt-to-GDP ratios are not very high. When the ratio of net foreign assets to GDP is considered, emerging markets are, on average, more indebted than advanced countries. Second, they also have higher income inequality and lower GDP per capita than advanced economies have.

We estimate equation 3.1 using year (z_t) and country (u_i) fixed effects. We are interested in analyzing the effect of inequality that varies over time; therefore, country fixed effects will control for time-invariant characteristics unique to a country. In the first specification, we find that an increase in a country's external debt-to-GDP ratio is associated with lower

Table 3.1: Country ratings, debt, income inequality, GDP per capita and net foreign assets

Country	Average Fitch Rating	Average external debt/GDP	Average Inequality	Average GDP per capita	Average NFA/GDP
		Emerging market economies			
Argentina	CCC-/CCC	72.49	45.19	4,483	-21.82
Bolivia	CCC+/B-	55.64	50.16	1,086	-64.71
Brazil	B+/BB-	28.82	50.22	4,739	-32.47
Bulgaria	BB-/BB	85.21	28.38	3,613	-53.71
Chile	BBB+/A-	47.71	49.34	7,131	-30.29
China	BBB+/A-	12.99	48.27	1,573	8.65
Colombia	BB/BB+	30.64	50.38	3,386	-25.00
Costa Rica	BB-	29.18	44.08	4,680	-21.82
Dominican Rep.	CCC+	26.71	45.80	3,928	-35.65
Ecuador	CCC-/CCC	49.55	50.02	3,005	-49.82
Egypt, Arab Rep.	BB/BB+	33.13	35.43	1,220	-11.76
El Salvador	BB-/BB	40.51	45.47	2,698	-38.53
Ghana	B-/B	75.86	40.01	504	-53.53
India	BB/BB+	19.45	49.57	762	-19.71
Indonesia	B/B+	65.47	55.45	1,283	-60.76
Korea, Rep.	BBB+/A-	31.30	31.60	16,643	-12.94
Malaysia	BBB-/BBB	46.50	47.61	5,296	-21.18
Mexico	BB/BB+	30.22	47.03	7,586	-34.94
Nigeria	B+	18.33	42.46	920	-57.76
Panama	BB/BB+	56.91	49.64	4,747	-77.94
Peru	BB-/BB	40.70	50.92	3,038	-41.59
Philippines	BB-/BB	62.05	50.79	1,195	-42.29
Romania	BB-	38.69	30.06	4,447	-28.47
Russia	BB	43.34	30.31	4,928	1.24
Sri Lanka	CCC+/B-	44.52	41.16	1388	-45.24
Thailand	BB+/BBB-	46.69	52.70	2,623	-39.59
Turkey	B/B+	43.44	45.29	6,584	-35.29
Uruguay	B+/BB-	43.91	43.18	5,447	-11.53
Venezuela	B/B+	35.65	41.59	5,500	5.71
		Advanced economies			
Australia	AA-/AA	59.96	31.32	30,7901	-54.47
Canada	AA/AA+	69.65	30.18	30,870	-18.47
Denmark	AA/AA+	96.08	22.40	43,164	-12.53
Finland	AA-/AA	72.71	22.35	29,175	-46.24
Greece	BBB/BBB+	93.64	33.64	19,689	-53.94
Hungary	BBB-/BBB	85.08	28.36	9,881	-81.76
Italy	A+/AA-	84.03	33.60	29,355	-17.53
Japan	AA-/AA	33.66	28.38	34,743	33.94
New Zealand	AA-/AA	84.00	32.38	27,242	-84.35
Norway	AA+	44.65	23.41	57,064	38.59
Poland	BB/BB+	41.43	30.01	6,960	-39.41
Portugal	A-/A	138.62	35.05	17,497	-59.29
Singapore	AA/AA+	154.52	42.92	25,595	1.78
Spain	AA/AA+	90.52	32.88	23,920	-47.76
Sweden	AA-	92.11	23.39	34,421	-25.41
United States	AA+	60.68	36.80	41,165	-14.76

List of countries used in the panel regression. Time period covers 1994-2010. Data sources from left to right: Fitch, Rainhart-Rogoff series, SWIID, the World Bank and the External Wealth of Nations Mark II database.

creditworthiness in the next period. This is a standard result in the literature, as well. In the second specification, we introduce GDP per capita in log terms, and we find that an increase in income is associated with an increase in country's creditworthiness. Finally, the last specification shows the relationship between income inequality and credit ratings. We find that an increase in Gini index is negatively associated with the creditworthiness in the next period. The estimate is significant at ten percent and robust to country and time fixed effects. In order to get an economic interpretation of the estimates, we do a simple back-of-the-envelope calculation based on the third specification. The median score in the sample is 13, which corresponds to BB+. We estimate the third specification separately for each country. Then, we increase each variable by its one standard deviation and compare their effects on the score for each country. We find that, on average, a one standard deviation increase in external debt reduces the credit score by 0.97 and a one standard deviation increase in log GDP increases the credit score by only 0.01 point. On the other hand, a one standard deviation increase in Gini reduces the credit score by 0.21 point. The largest effect comes from the external debt-to-GDP, but the change in the Gini index also has a substantial effect.

The regression results are also robust to the use of the Gini series obtained from The World Bank. The regression sample constructed using this series has 40 countries and covers the period 1994-2009. This sample has similar statistics for the median and standard deviation of debt and GDP per capita data. The median inequality in this sample is 40.8, and it has a larger standard deviation of 5.57. The estimation is again done with the inclusion of country and time fixed effects. The results are reported in Table B2 in the Appendix B.1. Using the same specification, we find that the estimate for inequality is larger and significant at five percent. However, external debt-to-GDP loses its significance when GDP per capita and the Gini index are included in the specification.

3.2.2 Income Inequality over the Business Cycle

In order to support our theory that income inequality plays a role in default decisions, we also need to determine whether there is countercyclical inequality over the business cycle.

Table 3.2: Panel regressions explaining creditworthiness with debt ratios, GDP per capita and inequality

	Dependent Variable: Score of country i in year t.		
Independent Variable	(1)	(2)	(3)
External debt-to-GDP at $t-1$	-0.0221^{***}	-0.0146^{***}	-0.0122^{**}
	(0.0057)	(0.0050)	(0.0048)
GDP per capita at $t-1$	–	9.5976^{***}	10.0130^{***}
		(2.5606)	(2.5013)
Gini at $t-1$			-0.0698^{*}
			(0.0360)
Year fixed effects	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes
No of countries	45	45	45
N	568	568	568

Sample period is between 1994 and 2010. The dependent variable is the credit score of country i in year t. Estimation is by robust standard errors. Standard errors are reported in parentheses. Per capita GDP is in logs. (***,**,* represent significance level at 1%, 5% and 10%, respectively.)

Using household-level data from several countries, Krueger et al. (2010) show that during recessions, earnings inequality increases.[8] We perform a similar exercise using our country-level data. We use the countries that have continuous series for Gini and GDP, leaving us with 77 countries. We compute the correlation between detrended GDP and inequality and find that, on average, inequality is countercyclical over the business cycle, with a mean correlation equal to -0.02. This result is robust to using the Gini series from The World Bank, as well. In this sample, there are only 46 countries and the mean correlation is equal to -0.03. Both results support the idea inequality is, on average, countercyclical over the business cycle in our sample.

[8] They have several inequality measures, such as Gini coefficient, variance of logs, 50/10 and 90/50 percentile ratios, and the countries they study are Canada, Germany, Italy, Mexico, Russia, Spain, Sweden and the USA.

3.3 MODEL

In this section, we present a model economy in order to structurally analyze the role of inequality in sovereign debt default. Our model is similar to the model presented by Arellano (2008) and belongs to the class of models in the standard framework of Eaton and Gersovitz (1981). We consider a discrete time, small open economy inhabited by heterogeneous agents that are hand-to-mouth and differ in the stochastic endowments they receive. The endowment is subject to aggregate output and inequality shocks that cannot be completely insured against. There is a benevolent government that represents the preferences of households and has access to international markets. The government can issue one-period bonds to foreign lenders and rebate the proceeds of the debt operations to the households. The government can choose to default fully on its debt at any time, because contracts are not enforceable. The penalty for default is that the economy is forced into financial autarky for a period of time, and there is an exogenous drop in output. Now, we move on to the details of the model.

3.3.1 Households

There are two types of infinitely-lived households indexed by $i = 1, 2$, and their preferences over consumption of the good, c_t, is assumed to be

$$u(c_t^i) = \frac{c_t^{i,1-\sigma}}{1-\sigma} \tag{3.2}$$

where σ is the constant relative risk-aversion parameter, and $\sigma > 0$ and $\sigma \neq 1$. The type 1 household receives a stochastic stream of a tradable good, $\frac{(1+\gamma)y}{2}$, and type 2 receives $\frac{(1-\gamma)y}{2}$, where y and γ denote output and inequality, respectively. The output y and the inequality γ follow a Markov process with a transition function $f(y', \gamma' | y, \gamma)$. Households also receive an equal amount of transfer from (or pay taxes on goods to) the benevolent government in a lump sum fashion. Households live hand-to-mouth, which means they do not make any individual saving or borrowing decisions.

3.3.2 Government

The government of the economy can trade one-period, non-state contingent bonds with foreign lenders that are risk neutral and competitive. As in a standard default model, when the government defaults, the economy faces two types of exogenous default penalties: direct output costs and a temporary exclusion from borrowing in the debt markets. The government's goal is to maximize social utility, which is the expected discounted sum of lifetime utilities of both types with equal weights given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_t^1) + u(c_t^2) \Big], \tag{3.3}$$

where β denotes the discount parameter and $\beta \in (0, 1)$. The government makes two decisions in this model. First, it decides whether to repay or default on its existing debt. Second, conditional on not defaulting, it chooses the amount of one-period bonds, B', to issue or buy. If the government chooses to buy bonds, the price it needs to pay is given as $q(B', y, \gamma)$. The discount bonds, B', can take a positive or negative value. If it is negative, this means that the government borrows $-q(B', y, \gamma)B'$ amounts of period t goods and promises to pay B' units of goods in the next period, if it does not default. Similarly, if B' is positive, then this implies that the government saves $q(B', y, \gamma)B'$ amounts of period t goods and will receive B' units of goods in the next period. The bond price function $q(B', y, \gamma)$ depends on the size of the bonds, B', income shock, y, and inequality shock, q. Government internalizes how its borrowing decisions affect the default risk and the price of the bond.

When the government chooses to repay its debt, the resource constraint for household 1 is

$$c^1 = \frac{(1+\gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}, \tag{3.4}$$

and the resource constraint for household 2 is

$$c^2 = \frac{(1-\gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}. \tag{3.5}$$

The economy faces three types of uncertainty that cannot be insured away with non-state-contingent bonds. The first one is the dispersion in incomes induced by shocks to γ. The second one is the output shock y that affects the aggregate output in the economy. Finally, the third one is the endogenous default risk. The goal of the government is to maximize the expected utilities of households, and it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. One government policy is to choose optimal B' that satisfies its goal, and the level of existing debt and the size of the shocks are crucial for this decision. As debt accumulates, it becomes harder to roll it over because of increases in default risk.

When the government chooses to default, consumption of the types are:

$$c^1 = \frac{1+\gamma}{2} y^d \tag{3.6}$$

and

$$c^2 = \frac{(1-\gamma)}{2} y^d, \tag{3.7}$$

where y^d is the level of output in default and $y^d = y - \kappa(y)$. The penalty is a function of the output such that $\kappa(y) = \min\{y, \max\{0, d_0 y + d_1 y^2\}\}$. The default penalty is higher, if default happens in the good states of the world. This default penalty structure has been used in many papers in the literature, such as Chatterjee and Eyigungor (2012b).

3.3.3 Foreign Creditors

Foreign creditors can perfectly monitor the state of the economy and have perfect information about the shock processes. They can borrow loans from international credit markets at a constant interest rate $r > 0$, which is the risk-free interest rate for this model. Taking the bond price function $q(B', y', \gamma)$ as given, they choose loans B' that maximize their expected profits ϕ, given as

$$\phi = q(B', y, \gamma)B' - \frac{1 - \delta(B', y, \gamma)}{1+r} B', \tag{3.8}$$

where $\delta(B', y, \gamma)$ is the probability of default and it is determined endogenously.

$$q(B', y, \gamma) = \begin{cases} \frac{1}{1+r} & B' \geq 0 \\ \frac{1-\delta(B',y,\gamma)}{1+r} & B' < 0. \end{cases}$$

The price function depends on the sign of B'. It is never optimal to default when the government saves ($B' \geq 0$), so in that case, the price is a constant function of the risk-free interest rate. On the other hand, if the government borrows ($B' < 0$), then the price reflects the default probability. This implies that as the default probability increases, the price of the bond falls.

3.3.4 Timing

The timing in the model is as follows.

1. The government starts with initial assets B.
2. The output shock y and the inequality shock γ are realized.
3. The government decides whether to repay its debt obligations or default.
   a. If the government decides to repay, then taking as given the bond price schedule $q(B', y, \gamma)$, the government chooses B' subject to the resource constraint. Then creditors, taking $q(B', y, \gamma)$ as given, choose B'. Finally, households consume c^1 and c^2 with respect to their types.
   b. If the government chooses to default, then the economy is in financial autarky and remains in autarky in the next period with probability θ. Households simply consume their endowments.

3.3.5 Recursive Equilibrium

We focus on a recursive equilibrium, in which there is no enforcement. Based on the foreign creditors' problem, government's debt demand is met as long as the gross return on the bond equals $(1 + r)$. Given loan size B', inequality state γ and income state y, the bond price is

$$q(B', y, \gamma) = \frac{1 - \delta(B', y, \gamma)}{1 + r}. \tag{3.9}$$

The value function for the government that has the option to default or pay its debt is given as $v^o(B, y, \gamma)$. Government chooses the option that maximizes the welfare of agents. The default option will be optimal only if the government has debt. The value of default is denoted by the function $v^d(y, \gamma)$, and the value of repayment is denoted by $v^c(B, y, \gamma)$.

$$v^o(B, y, \gamma) = \max_{c,d}\{v^c(B, y, \gamma), v^d(y, \gamma)\}. \tag{3.10}$$

The value of default is expressed by

$$\begin{aligned} v^d(y, \gamma) &= u\left(\frac{(1+\gamma)y^{def}}{2}\right) + u\left(\frac{(1-\gamma)y^{def}}{2}\right) \\ &+ \beta \int_{\gamma'} [\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] f(y', \gamma'|y, \gamma) d(\gamma', y'). \end{aligned} \tag{3.11}$$

Under default, individuals only consume their income. The government can gain access to debt markets with probability θ, and the economy stays in autarky with probability $1 - \theta$. The transition probabilities are given by the joint density function, f. Similarly, the value of staying in contract is

$$\begin{aligned} v^c(B, y, \gamma) &= \max_{B'} u\left(\frac{(1+\gamma)y - q(B', y, \gamma)B' + B}{2}\right) + u\left(\frac{(1-\gamma)y - q(B', y, \gamma)B' + B}{2}\right) \\ &+ \beta \int_{y', \gamma'} v^o(B', y', \gamma') f(y', \gamma'|y, \gamma) d(\gamma', y'). \end{aligned} \tag{3.12}$$

If the government chooses to repay its debt, the value function for this choice reflects the future options for default and staying in contract. The government chooses the optimal bond contract that maximizes the sum of utilities of the households and expected discounted future value of option.

We can characterize the government's default policy by default and repayment sets. Let $A(B)$ be the set of y and γ for which repayment is optimal when assets are B, such that

$$A(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) \geq v^d(y, \gamma)\right\}, \tag{3.13}$$

and let $D(B) = \tilde{A}(B)$ be the set of y, γ for which default is optimal for a level of assets B:

$$D(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) < v^d(y, \gamma)\right\}. \tag{3.14}$$

Proposition 1. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, if default is optimal for B^2 then default will be optimal for B^1, and the probability of default at equilibrium satisfies $\delta(B^1, y, \gamma) > \delta(B^2, y, \gamma)$.*

Proof. See Appendix B.2.1. □

This proposition formally states a feature of the model that Eaton and Gersovitz (1981) also have. It shows that in equilibrium default sets expand and the probability of default increases as the level of debt in a country increases. The following proposition states that equilibrium bond price decreases as the level of debt increases.

Proposition 2. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, equilibrium bond price satisfies $q(B^1, y, \gamma) \leq q(B^2, y, \gamma)$.*

Proof. See Appendix B.2.2. □

Now we define the recursive equilibrium for this economy. Let $s = \{B, y, \gamma\}$ be the set of aggregate states for the economy.

Definition 2. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumptions $c^1(s)$, $c^2(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B', y, \gamma)$ such that:*

1. *Agents' consumption $c^1(s)$ and $c^2(s)$ satisfy the resource constraints, taking the government policies as given.*
2. *The government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem, taking the bond price function $q(B', y, \gamma)$ as given.*
3. *Bonds prices $q(B', y, \gamma)$ reflect the government's default probabilities and default probabilities satisfy creditors' expected zero profits.*

In equilibrium, the bond price function should satisfy both the government's optimization problem and the expected zero profits in the lenders' problem. As mentioned, the probability of default endogenously affects the bond price. Using the default sets, we can express the probability of default such that:

$$\delta(B', y, \gamma) = \int_{D(B')} f(y', \gamma' | y, \gamma) d(y', \gamma'). \tag{3.15}$$

When default sets are empty, default is never optimal at the asset level B', so the probability of default equals zero, independent of the realized shock. When $D(B') = (\mathbb{Y}, \Gamma)$, government always chooses to default for all shock levels. Default sets are shrinking in assets.

3.4 QUANTITATIVE ANALYSIS AND SIMULATION

3.4.1 Quantitative Analysis

In this section, we describe the estimation procedure for the shock processes and then explain the calibration strategy. We use the model to analyze the debt dynamics in Argentina between 1990-2002, quantitatively. Focusing on an Argentine default episode enables us to compare our results with the ones in the existing literature.

3.4.1.1 Calibration and Functional Forms We solve the model assuming that both output and inequality shocks are in play. We call this the benchmark model. In the benchmark model, output and inequality shocks are modeled as a VAR process. Next, in order to quantify the role of each shock and to assess the importance of the shocks in matching the high volatilities and particularly high default rates observed in emerging economies, we solve the model subject to only one shock at a time. Model II has only output shocks, and we assume that output follows an AR(1) process. Model III has only inequality shocks and, again, the inequality shock is modeled as an AR(1) process.

In the benchmark model, we assume that the VAR process for log output and inequality is as follows:

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix},$$

where

$$
\begin{aligned}
\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} &= \left[\boldsymbol{I} - \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \right] \begin{bmatrix} \mu_y \\ \mu_\gamma \end{bmatrix} \\
\boldsymbol{\varepsilon} &= \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix} \\
E\left[\boldsymbol{\varepsilon}\right] = \mathbf{0} \quad &\text{and} \quad Var\left[\boldsymbol{\varepsilon}\right] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}.
\end{aligned}
$$

The estimated values are derived from Argentina's GDP and income inequality data between 1991Q1 and 2005Q2. We use real output in quarterly, seasonally adjusted, real series and covering the period 1993Q1 to 2001Q4 from the dataset in Arellano (2008).[9] We take logs of GDP and detrend these series using an HP filter. The data pertaining to inequality are constructed using the distribution of income series in World Development Indicators provided by the World Bank. We choose the same period as for GDP. In order to construct the inequality measure, we compute the total income share of the upper 50th percentile and lower 50th percentile. Then, we take the difference of the income shares and divide it by two, which gives us the dispersion from the mean income. Since only annual data are available, we adopt the Boots-Feibes-Lisman method to disaggregate the annual data into quarterly data. Both output and inequality shocks are then discretized into a 21-state Markov chain, using Tauchen (1986).

The discount factor β, and default penalty parameters d_0 and d_1 are jointly calibrated to target a default probability of 3 percent, debt-to-GDP ratio of 5.53 percent and mean spread of 6.23. We set the probability of reentry to 0.25, which implies it takes a year to gain access to bond markets.[10]

[9] Arellano (2008) uses the data provided by the Ministry of Finance of Argentina.

[10] The calibrated value of β and the value of θ are close to the values used in the default literature. For instance, Yue (2010) assumes that $\beta = 0.72$, and Aguiar and Gopinath (2006) assume that $\beta = 0.925$. The value of parameter θ implies that, on average, autarky takes four quarters, assuming that the distribution of default lengths is exponential (Tomz and Wright (2007) and Pitchford and Wright (2011)). Dias et al. (2007) empirically show that it takes 5.7 years, on average, for countries to regain partial access to international capital markets and Gelos et al. (2011) document that average exclusion from the international markets declined to two years in the 1990s; however, endogenous sovereign default models with exogenous entry to the debt markets calibrate the parameter θ around 0.25. (Arellano (2008) chooses 0.282 and Aguiar and Gopinath (2006) choose 0.10).

Table 3.3: A priori parameters for Model I

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.95 & -0.38 \\ 0.00 & 0.95 \end{bmatrix}$	Argentina's GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0003 & -0.0001 \\ -0.0001 & 0.0001 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.01 \end{bmatrix}$	

Table 3.3 shows the parameters that we use for the benchmark model's calibration. We set the risk-free interest rate to 1.7 percent to match the US five-year Treasury bond quarterly yield. The risk-aversion parameter σ is set to 2, as it is standard in the macro literature. We also report the estimates of the parameters in the stochastic shock process. Note that the correlation of the output at t and the inequality at $t-1$, $\rho_{y\gamma}$, is negative. This means that high inequality generates low output in the next period. Similarly, since $\rho_{\gamma y}$ is equal to zero, the output in the previous quarter does not affect the inequality in the current period. This relationship between inequality and output is not unique to Argentina. We find that other frequently defaulting economies, such as Brazil, Costa Rica, Dominican Republic, Ecuador and Uruguay, also have similar results in terms of the signs of the estimates. These results are reported in Table B3 in the Appendix B.1.

For Model II, we remove the stochastic inequality shocks by setting the level of inequality to the mean inequality up to the default episode. This corresponds to setting γ equal to 0.66. The stochastic process for output is assumed to be a log-normal AR(1) process such that

$$log(y_t) = \rho_y log(y_{t-1}) + \epsilon_{yt}, \tag{3.16}$$

where $E[\epsilon_{yt}] = 0$ and $E[\epsilon_{yt}^2] = \sigma_y^2$, which are estimated from Argentina's GDP. We again discretize the output process into a 21-state Markov chain using the Tauchen method. We

Table 3.4: Calibrated parameters for Model I

Name	Parameters	Calibrated Parameter	Target	Value Target
Discount rate	β	0.925	Default probability	3 percent
Default penalty	d_0	-0.691	Debt service-to-GDP	5.45 percent
	d_1	0.095	Mean spread	6.23

keep all else the same as in the benchmark model. Table 3.5 presents the parameters for the second model.

Similarly, we need to estimate the stochastic inequality process for Model III. We estimate the following AR(1) process:

$$\gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \gamma_{t-1} + \epsilon_{\gamma t}, \tag{3.17}$$

where $E[\epsilon_{\gamma t}] = \mu_\gamma$ and $Var(\epsilon_{\gamma t}) = \sigma_\gamma^2$, which are estimated from Argentina's inequality data. As with Model III, we discretize the inequality process into a 21-state Markov chain using the Tauchen method. We keep all else the same as in benchmark model. The parameters for the third model are presented in Table 3.6.

3.4.2 Model Solution

In this section, we begin with the analysis of the benchmark model's results and then elaborate on the intuition behind the workings of the model. Our solution algorithm is given in the Appendix B.3.

In our model, the benevolent government has two policy decisions to make: whether to repay the existing debt or default; and how much to borrow or save using one-period bonds. The government borrows to help households have smooth consumption and to shift future consumption to today because the equilibrium interest rate is lower than government's discount rate. The level of optimal debt depends on the current assets and the state of the world. Since lenders have full information about the state of the world and contracts are

Table 3.5: A priori parameters for Model II

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.925$	
Default penalty	$d_0 = -0.691$	
	$d_1 = 0.095$	
Probability of reentry	$\theta = 0.25$	
Inequality	$\gamma = 0.66$	Mean income inequality in Argentina
Stochastic structure	$\rho_y = 0.9351$	Argentina's GDP
	$\sigma_y = 0.0190$	

Table 3.6: A priori parameters for Model III

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.925$	
Default penalty	$d_0 = -0.691$	
	$d_1 = 0.095$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\rho_\gamma = 0.9851$	Argentina's Inequality
	$\sigma_\gamma = 0.0037$	
	$\mu_\gamma = 0.38$	

not state-dependent, borrowing constraints can bind for the government, particularly in bad states of the world, such as high inequality and low output. Therefore, we observe that bond prices depend on the level of assets and the types of shocks that the economy is subject to.

In the model, since the endowment is shared unequally among households, even in the absence of the shocks, the poor agents' marginal utility of consumption is higher than that of rich agents. An adverse output shock increases both agents' marginal utility of consumption, but an adverse inequality shock raises the marginal utility of the poor and reduces the marginal utility of the rich, increasing the dispersion between the marginal utilities of consumption. Defaults are more likely when there are adverse shocks and high levels of debt because the lenders offer bond contracts that have higher interest rates in these states. This makes the government borrowing-constrained and imposes large taxes on households in order to finance the debt. An adverse inequality shock exacerbates the burden of the tax, particularly on the poor, because it increases the poor's marginal utility of consumption disproportionately. In this case, the government can choose to default and use default as a redistribution mechanism. This policy improves welfare because, by eliminating the tax burden, the government can alleviate the dispersion.

First, we analyze our results related to policy functions and value functions in the benchmark model. We report the results based on four different combinations of output and inequality shocks. A low (high) shock is one standard deviation below (above) its mean for each type of shock. The level of assets is denoted as a fraction of GDP. Then, we look at the business cycle statistics that the model generates.

Figure 3.2 shows the bond price schedule and the interest rate generated by the model. On the x-axis we have assets as a fraction of output. Similar to the results presented in the standard default literature, such as Arellano (2008) and Aguiar and Gopinath (2006), we observe that bond prices are an increasing function of assets, such that high levels of debt entail a low bond price and a high interest rate. Fixing the level of inequality shocks, we observe that it is easier to borrow during expansions than during recessions. However, the results also show that the effect of a high output shock can be dominated by the effect of a high inequality shock. In other words, an economy that is subject to both high output and high inequality shocks can have a bond price that is lower than that when there are low

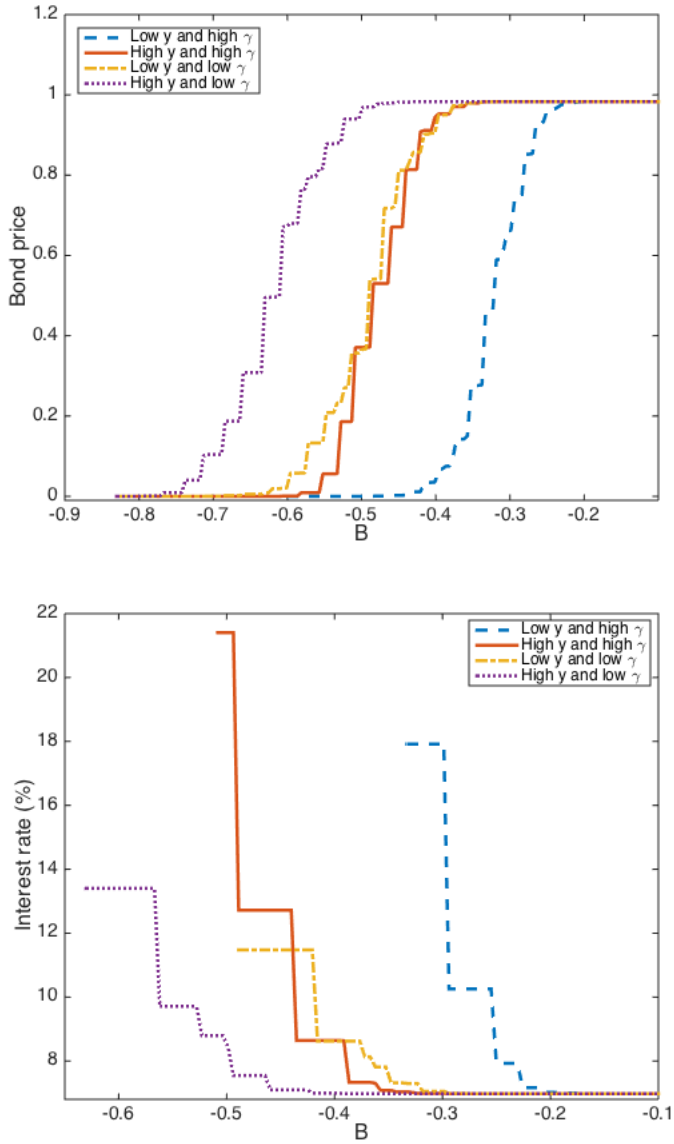


Figure 3.2: Bond prices and interest rate (Model I)

output and low inequality shocks.

The lower panel in Figure 3.2 shows the annual equilibrium interest rates generated by the model. The interest rate is calculated as $1/q(B', y, \gamma) - 1$. Inequality shocks generate another source of risk that is reflected in interest rates. The highest level of borrowing is possible when there is high output and low inequality in the economy. Government borrowing is subject to higher interest rates, even for small amounts of debt that are above the level of default in high-inequality and or low-output states.

The top panel in Figure 3.3 shows the saving policy function conditional on not defaulting. Our results show that the government borrows more in expansions and when there is low inequality. This result is consistent with the countercyclical interest rates, since it becomes more costly to borrow in bad states of the world. The bottom panel of Figure 3.3 is the value function for the option to default or repay as a function of assets. Again, inequality plays a significant role in the default decision. The flat regions of the value function show the range of debt for which default is optimal. The value functions show that the highest debt can be supported, when there is high output and low inequality in the economy.

3.4.3 Business Cycle Results

3.4.3.1 Data First, we document the business cycle characteristics of the Argentine economy. For the business cycle statistics, we use real output, consumption and trade balance data in quarterly, seasonally adjusted, real series for the period 1993Q1 and 2001Q4 from the dataset in Arellano (2008).[11] We take logs of GDP and consumption series and apply a linear trend on these series following Arellano (2008).[12] The trade balance data are a fraction of GDP. We also borrow Arellano (2008)'s spread data, which are defined as the difference between the interest rate in Argentina and the yield of the five-year U.S. treasury bond. The interest rate series is EMBI for Argentina and starts from 1983Q3. For the mean and standard deviation of the spread we use the period between 1993Q1 and 2001Q1. The inequality series is the one we constructed to generate a shock process, as explained in the

[11] Arellano (2008) uses the data provided by the Ministry of Finance of Argentina.

[12] Analysis using HP filtered series (with smoothing parameter 1600) also produces similar results for correlations.

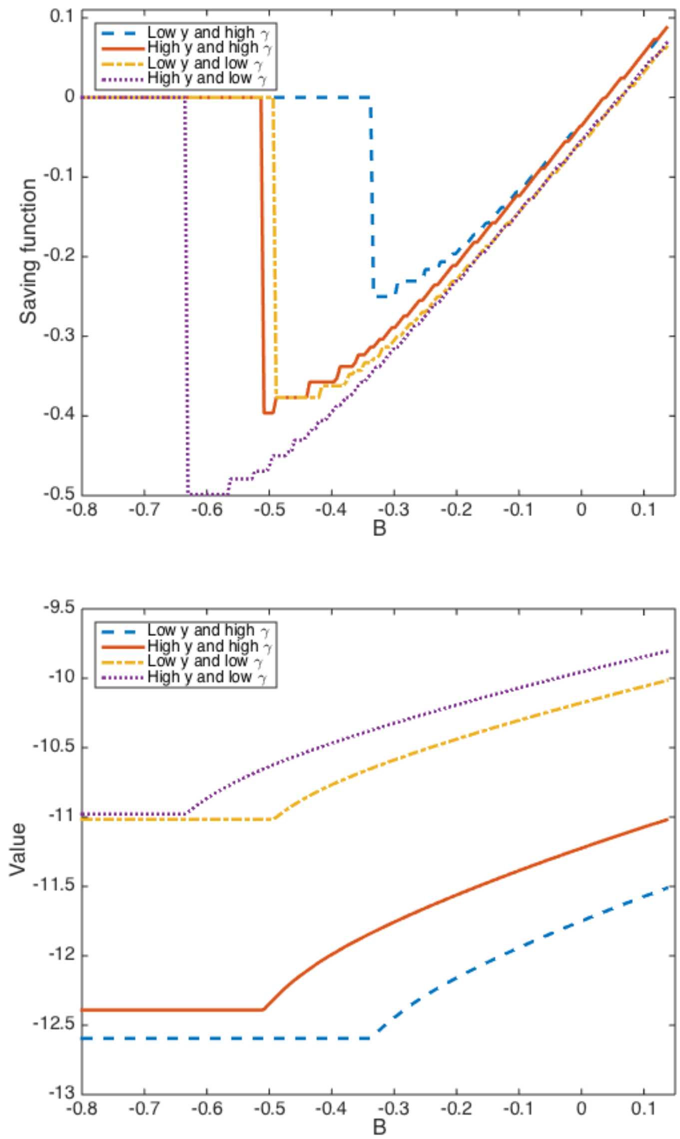


Figure 3.3: Savings and value functions (Model I)

Table 3.7: Business cycle statistics for Argentina

	Default episode x: Q1-2002	std(x)	corr(x, y)	corr(x, r^c)
Interest rate spread (%)	28.60	2.77	-0.88	
Trade balance (% of GDP)	9.90	1.75	-0.64	0.70
Consumption (% deviation from trend)	-16.01	8.59	0.98	-0.89
Output (% deviation from trend)	-14.21	7.78		-0.88
Inequality (% deviation from mean ineq.)	8.6	1.71	-0.23	0.55

previous section.

Table 3.7 presents the business cycle statistics of all the data available up to the default episode that started on December 26, 2001. Consumption and output in the first column show the deviations from the trend, and the other values are in levels in the first quarter of 2002. Relative to the average inequality in the series, in the default episode, inequality increased by 8.6 percent. The second column shows the standard deviations up to the default episode. We find that consumption is more volatile than output. The third and the fourth columns present the correlations of each variable with the output and the interest rate spread, respectively. It has been shown that emerging market economies are characterized by countercyclical spread rates and net exports. Also, their consumption is highly correlated with output. We see similar empirical results for Argentina in column 3. In addition, we show that inequality is countercyclical with output, so the economy has high inequality during recessions. The interest rate spread is negatively correlated with consumption and output, and positively correlated with trade balance. The data show that inequality is positively correlated with the spread, which implies that inequality increases during times of risky borrowing.

3.4.3.2 Simulation Results Next, we move on to the business cycle statistics generated by the benchmark model and evaluate the performance of the model with Argentine data. The upper panel of Table 3.8 presents the simulation results for the benchmark model, which generates a default probability of 2.80, debt-to-GDP ratio of 5.53 percent and mean spread of 4.90. High volatility of interest rates is a consequence of high default probability. We observe a large increase in the spread during default episodes, which is close to the data. In Argentina, in the couple of months following the default, quarterly spreads reached to 5,000-6,000 basis points. The model also generates large drops in consumption and output during default episodes. Inequality increases by 9.09 percent relative to its mean, which is also close to the increase observed during the default episode (8.60 percent). The model can also generate high volatility in consumption and output. The volatility of inequality is slightly lower than the value observed in the data.

Table 3.8: Simulation results for the benchmark model

	Default episodes	std(x)	corr(x,y)	$corr(x, r^c)$
Model I: Shocks to output and inequality				
Interest rate spread (%)	59.82	9.94	−0.20	-
Trade balance (% of GDP)	−0.01	0.91	−0.12	0.29
Total Consumption (% deviation from trend)	−7.19	5.82	0.99	−0.25
Output (% deviation from trend)	−7.29	5.63	–	−0.20
Inequality (% deviation from mean ineq.)	7.45	0.70	−0.28	0.16
Other Statistics				
Mean debt (percent output)	5.53	Mean spread		4.90
Default probability	2.80			

In terms of correlations with output, the simulations can generate a positive correlation with consumption and a negative correlation with the interest rate spread.[13] We also obtain a negative correlation between output and trade balance. The reason is that when there are only output shocks, households can consume more than the level of the output during expansions because the government can borrow easily. On the other hand, when there is a recession, borrowing is constrained; therefore, the consumption is less than the output. This generates a countercyclical trade balance over the business cycle. We see a positive correlation between the spread and the trade balance. Since the spread reflects the risk due to both inequality and output shocks, it is more correlated with the bad states of the

[13]See Neumeyer and Perri (2005), Uribe and Yue (2006) and Aguiar and Gopinath (2007) for the role of countercyclical interest rates in emerging markets.

world, in which the government is more likely to face borrowing constraints and experience large trade balances. As we expected, inequality is negatively correlated with output and positively correlated with the spread. Table 3.9 shows our model's performance relative to Arellano (2008). The benchmark model does quantitatively a similar job with Arellano (2008) in terms of matching the data.

Table 3.9: Simulation results for the benchmark model: Comparison with Arellano (2008)

	Data	Benchmark Model	Arellano (2008)
Volatilities			
$\sigma(c)/\sigma(y)$	1.09	1.04	1.10
$\sigma(tb)/\sigma(y)$	0.17	0.16	0.26
Correlations			
corr(y,spread)	−0.88	−0.20	−0.29
corr(y,c)	0.98	0.98	0.97
corr(y,tb)	−0.64	−0.12	−0.25
corr(y,inequality)	−0.23	−0.28	–
corr(spread,c)	−0.89	−0.25	−0.36
corr(spread,tb)	0.70	0.29	0.43
corr(spread,inequality)	0.55	0.16	–
Other Statistics			
Mean Debt (percent output)	5.53	5.41	5.95
Mean Spread	6.23	4.90	3.54
Default Probability	3.00	2.80	3.00

We solve and simulate Model II and Model III, in order to assess the role of output shocks and inequality shocks in default risk. The simulation results for Model II and Model III are given in Table 3.10. We find that the default probability is around 0.52 percent when there are output shocks and 0.32 percent when there are inequality shocks. We obtain a probability of default when the economy is subject to output shocks that is slightly higher than the model with inequality shocks because the default penalties are different in two models. In the case of output shocks, the default penalty increases in good states of the world and decreases in bad states of the world; thus with smaller penalty and tighter borrowing constraints in bad states, we observe a larger default rate. On the other hand, in model III the default penalty is constant across all states because aggregate output is constant. This generates a lower probability of default in model III.

We also find that the default risk in both Model II and Model III is lower than that in the benchmark model. This is strong evidence that shows that the amplification effect comes from the underlying joint shock process. The reason behind this result is the VAR(1) process that we systematically estimated from the Argentine data. Based on the estimated process, it is more likely to have adverse output and inequality shocks together. Moreover, high inequality at time $t-1$ leads to lower output at time t. These characteristics play an important role in altering the expectations of foreign lenders and the government about the future state of the economy. An adverse inequality shock not only amplifies the effect of an adverse output shock today, but also generates pessimism that the recession with increasing inequality may be long-lasting.[14] As a result, foreign lenders ask for a higher premium, even for smaller levels of debt. This increases the borrowing constraints on the government, and default becomes an optimal decision.

Table 3.10: Simulation results for Model II and Model III

	Model II		Model III	
	Default episodes	std(x)	Default episodes	std(x)
Interest rate spread (%)	9.70	1.46	1.78	0.68
Trade balance	−0.03	2.15	−0.02	0.98
Total Consumption	−2.78	5.65	−2.92	0.99
Output	−8.00	4.46	−7.99	0.00
Inequality	0.00	0.00	2.63	1.15
Other Statistics				
Mean debt (percent output)	52.76		48.46	
Default probability	0.52		0.32	
Mean Spread	0.63		0.44	

[14] In order to disentangle the effect of inequality on output in the next period, when we generate the Markov process, we assume that $\rho_{y\gamma} = 0$ and $\rho_{\gamma y} = 0$. Under this specification, we find that the probability of default falls to 1.96 percent. This result shows that two thirds of the default risk comes from the fact that the covariances of the shocks are negative.

Table 3.11: A priori parameters for Mexico

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.90 & -0.17 \\ 0.02 & 0.94 \end{bmatrix}$	Mexico GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0001 & 0.0002 \\ 0.0002 & 0.00005 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.02 \end{bmatrix}$	

3.5 ALTERNATIVE CALIBRATION

We obtain the main results regarding the effects of inequality shocks using Argentine data. In this section, we calibrate the model for Mexico. Our goal in this exercise is to see whether the model can match the business cycle statistics of Mexico and whether the model can also explain differences in consumption volatilities across income groups that we observe in the data. Since we do not have the consumption distribution data for Argentina, we focus on Mexican economy for this exercise. Like Argentina, Mexico experienced several default episodes. We focus on the crises in the last century when we compute the default probability. According to Reinhart and Rogoff (2011), Mexico experienced external default or restructuring in 1914, 1928 and 1982 and it was near default in 1994. Depending on whether we include the last incidence, we get a default probability between 3-4 percent; therefore we choose 3.5 percent as the default rate.[15]

We repeat the steps for Argentina when we estimate the shock processes for Mexico. Table 3.11 shows a-priori parameters that we used in the simulations in order to obtain the

[15] If we count the number of default or restructuring episodes starting from the country's year of independence, then we consider the period between 1828 and 2015 for Mexico. There are in total 9 crises episodes, which lead to a higher default rate around 4-5 percent. The data on external debt crisis are from (Reinhart and Rogoff, 2011). An external debt crisis is defined as the failure to meet the principal or interest payment on the due date by Reinhart and Rogoff (2011). The episodes also include instances where the principal or interest payment is rescheduled at less favorable terms than the original contract.

Table 3.12: Calibrated parameters for Mexico

Name	Parameters	Calibrated Parameter	Target	Value Target
Discount rate	β	0.90	Default probability	3.5 percent
Default penalty	d_0	-1.37	Debt service-to-GDP	4.5 percent
	d_1	0.15	Mean spread	4.2

business cycle statistics for Mexico. We use the same values for the risk-free interest rate and the risk aversion parameter as in the previous sections. The stochastic shock processes come from the VAR estimations based on Mexico's GDP and income distribution data. The data cover the period between 1995-2012. We find the estimate for $\rho_{y\gamma}$ is negative and it implies that inequality in the previous period reduces the output in the current period. However the covariances of the errors are not negative, which implies that Mexico is more likely to receive a low inequality shock together with a low output shock.

We follow the same calibration strategy. Table 3.12 shows the calibrated parameter values. We jointly calibrate the discount rate (β) and the output cost in autarky parameters (d_0 and d_1), in order to match the default probability of 3.5 percent, debt service-to-GDP ratio of 4.5 percent and the mean spread 4.2 in Mexico.[16] We compute the business cycle statistics using quarterly seasonally adjusted real GDP, real consumption and trade balance data from FRED, Federal Reserve Bank of St. Louis. We detrend the consumption and output series and we focus on the period between 1993q1 and 2012q4.

The simulation results are given in Table 3.13. In terms of matching the targets, the model does well except that it generates higher debt service-to-GDP ratio than we observe in the data. We see that the model can match the main business cycle characteristics for Mexico, such as spreads that are countercyclical, consumption that is procyclical over the business cycle and consumption that is highly correlated with output. We get high volatility

[16] We borrow debt service-to-GDP statistic from Cuadra et al. (2010). Debt service to GDP data cover the years from 1980 to 2007 and the spread covers the period from 2000q1 to 2012q4. We compute the spread as the difference between the interest rates on government securities and treasury bills of Mexico and the U.S., both data are retrieved from FRED database provided by Federal Reserve Bank of St. Louis.

of consumption relative to output. Also, the trade balance is positively correlated with the spread; however, it is acyclical with output.

Table 3.13: Business cycle statistics for Mexico

Mexico	Simulation	Data
corr(spread, y)	-0.57	-0.52
corr(spread, tb)	0.44	0.68
corr(spread, tc)	-0.72	-0.53
corr(tb, y)	0.01	-0.87
corr(tc, y)	0.86	0.97
std(tc)/std(y)	1.19	1.09
std(c_{poor})/std(c_{rich})	1.05	1.10
Targets		
Default probability	3.78%	3.5%
Debt service-to-GDP	9.9%	4.5%
Mean spread	4.44	4.20

Note: Total consumption and trade balance are denoted by tc and tb, respectively. The consumption volatilities of the rich and poor are yearly, the rest of the statistics are at quarterly frequency.

We can also compute consumption volatilities for the rich and poor households using this model. We use Mexico Household Income and Expenditure Survey data between 1992 and 2008. We compute the consumption of the upper and lower 50 percentile of the households in order to make the statistics comparable with the model. In the data, we find that consumption volatility of the poor household is slightly higher than the rich household's and the ratio of volatilities is 1.09. Since survey data set is annual, using the simulated results and aggregating the data we convert the consumption of the poor and rich households to annual frequency. In our model, consumption volatility of the poor households is also higher than that of the rich and the ratio is close to its data counterpart. Incomplete asset markets together with income shocks are key for this result. In our model income inequality shocks amplify the effect of output shocks particularly on the poor households' endowment. Since there are no other assets that the households can use to insure against these shocks, poor households have higher volatility of consumption than rich households.

3.6 PROGRESSIVE INCOME TAXES

In the previous sections, we assume that government distributes the proceeds of the debt payments equally between the households. As mentioned above, these proceeds can function as taxes when they are negative and they can function as transfers, otherwise. Since these payments are lump sum, the burden (benefit) of taxes (transfers) relative to endowment is quite different across the households. Particularly, the burden of lump sum taxes is on the poor. Therefore, this brings up the question: How would the probability of default change in an economy if the government could use progressive income taxes to finance the debt when it is costly to borrow?

We impose the following tax regime:

$$T(y^i) = \begin{cases} 0 & B - qB' \geq 0, \\ y^i - \lambda (y^i)^{1-\tau} & B - qB' < 0. \end{cases}$$

As τ increases the tax function becomes more progressive, and when $\tau = 1$, both types of households consume equally. The parameter λ is called the shift parameter and determines the average tax rate. If $B - qB'$ is positive, the government only distributes the proceeds of the debt operations across households as transfers similar to the benchmark model. If $B - qB'$ is negative, then the government uses the revenues from the taxes to finance the debt. The budget constraint of the government for the latter case is given as:

$$T(y^1) + T(y^2) + B - qB' = 0. \tag{3.18}$$

One can solve for λ using the budget constraint of the government:

$$
\begin{aligned}
y^1 - \lambda(y^1)^{1-\tau} + y^2 - \lambda(y^2)^{1-\tau} + B - qB' &= 0 \\
y - \lambda[(y^1)^{1-\tau} + (y^2)^{1-\tau}] + B - qB' &= 0 \\
\lambda[(y^1)^{1-\tau} + (y^2)^{1-\tau}] &= y + B - qB' \\
\lambda &= \frac{y + B - qB'}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}.
\end{aligned}
$$

The disposable incomes are denoted by $\tilde{y}^i$ for each type of household i. When $B - qB' < 0$, we get:

$$
\begin{aligned}
\tilde{y}^1 &= \lambda(y^1)^{1-\tau}, \\
&= \frac{(y + B - qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \\
\tilde{y}^2 &= \lambda(y^2)^{1-\tau}, \\
&= \frac{(y + B - qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}.
\end{aligned}
$$

We can write the budget constraints of the households if the government does not choose to default as:

$$
c^1 = \begin{cases} y^1 + \frac{B-qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}
$$

$$
c^2 = \begin{cases} y^2 + \frac{B-qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}
$$

If the government chooses to default, we assume that the progressive taxes are in effect. The budget constraints during autarky are:

$$c^1 = \frac{y^d (y^{d,1})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}},$$
$$c^2 = \frac{y^d (y^{d,2})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}}.$$

Table 3.14: Effect of τ_1 on default probability and debt

	$\tau = 0$	$\tau = 0.1$	$\tau = 0.20$	$\tau = 0.3$	$\tau = 0.4$
Mean debt (% output)	26.32	27.44	28.02	28.65	28.98
Mean spread (%)	5.43	4.78	4.60	4.50	4.28
Probability of default (%)	2.92	2.32	1.85	1.68	1.46

We recalibrate the model in order to match 3 percent default probability and mean spread of 6.3, when $\tau = 0$.[17] We simulate the model for five different values of $\tau \in \{0, 0.1, 0.2, 0.3, 0.4\}$ and analyze how the progressivity of the tax system affects the probability of default and debt-to-output ratio. Table 3.14 shows the results. The model with $\tau = 0$ has mean debt of 26.32 percent. As τ increases, we obtain higher mean debt. Moving from $\tau = 0$ to $\tau = 0.4$, the probability of default decreases from 2.92 percent to 1.68 percent. The reason is that taxes reduce the dispersion in marginal utilities of consumption between households, by taxing the rich more than the poor. As the dispersion gets smaller, the government has less incentive to default. Therefore, foreign lenders lend higher levels of debt to the government and the mean spread declines monotonically.

[17] The calibrated parameters are $\beta = 0.895$, $d_0 = -0.56$ and we fix $d_1 = 0.095$.

3.7 CONCLUSION

This paper studies the role of increasing income inequality in sovereign borrowing and default decisions using a stochastic general equilibrium model in a small open economy with endogenous default risk. To motivate the idea, we analyze the nexus among the Gini index, sovereign bond ratings and GDP per capita using a panel data set. The results show that high inequality lowers the creditworthiness of long-term government bonds significantly. The paper also shows that the inequality is countercyclical over the business cycle for the average country. Next, using a model that belongs to the class of models of Eaton and Gersovitz (1981) and extending it to allow for heterogeneous agents and shocks to the distribution of income, the paper shows analytically that inequality shocks can generate a high probability of default when the markets are incomplete. Using Argentine data, the model predicts a default probability of 2.8 percent and also matches the business cycle characteristics observed in the data, such as the high volatility of consumption and output, the counter-cyclical interest rates, and positive correlations between the trade balance and interest rates, as well as, inequality and interest rates. Our model's contribution is to highlight the redistributive effects of default as a policy that improves the welfare of the households. The model can also explain the differences in consumption volatilities across different income groups, which has not been shown by the earlier papers in the literature. As a policy extension, we show that progressive income taxes can reduce the default risk and increase the debt-to-output ratio.

Rising income inequality is a general problem that many countries have experienced. Therefore, it is important to understand how inequality induces economic crises and sovereign defaults that last several years and cause large losses. Even though our paper provides a first step toward analyzing the role of income inequality, we abstract from the determinants of income inequality and model it as an exogenous shock. We think that it is also important to study what drives high income inequality and how it affects agents' welfare and a government's decision to default. We leave these issues for future study.

4.0 EXPORT AND PRODUCTIVITY GROWTH: EVIDENCE FROM CHILEAN PLANTS

4.1 INTRODUCTION

Export-oriented growth has been an important component of the economic success of a number of countries. There have been numerous empirical studies on firm's performance and entry into export markets. These studies have generally found that more productive firms have a higher tendency to enter into export markets and less productive firms have a higher tendency to exit from international markets, which is called the self-selection hypothesis. Research has also suggested that export firms may improve their productivity more than non-exporting firms, which is called the learning-by-exporting hypothesis. While self-selection is a common finding in a number of empirical studies, conclusive evidence for the learning-by-exporting hypothesis is not always found.

To test the hypotheses, three empirical methods are used: regression method, matched sampling technique and the stochastic dominance method. In the regression method, self-selection is tested by whether a firm's export decision is affected by productivity of the firm in the previous year. The learning-by-exporting hypothesis is tested by whether productivity of export firms increases more than non-exporting firms. Bernard and B. Jensen (1999) propose a method to test the hypotheses. While their results support the former hypothesis, they fail to confirm the latter hypothesis. Following Bernard and B. Jensen (1999), a number of other studies commonly confirm the self-selection hypothesis using firm-level data from various countries and periods. However, the learning-by-exporting hypothesis is only detected in some specific countries and times.[1] The matched sampling technique fo-

[1]See Alvarez and López (2005), Clerides et al. (1998), Damijan et al. (2004), and Fernandes and Isgut

cuses on the learning-by-exporting hypothesis. After setting up non-exporting firms as a control group, one tests how significantly different productivity of export firms and that of the control group are. A number of previous studies rely on data from a specific country and period.[2] The stochastic dominance method makes use of the Kolmogorov-Smirnov test to see whether the productivity distribution of export firms dominates that of non-exporting firms. If the hypothesis on the equality of distribution is rejected and the one on the difference favorable to export firms' distribution over that of non-exporting firms is accepted, it is said that export firms self-select when entering into international markets. To examine the learning-by-exporting hypothesis, the distributions of productivity growth of export firms and non-exporting firms are tested.[3]

To test the self-selection hypothesis and the learning-by-exporting hypothesis with the three methods, I use Chilean plant-level data for 1995-2007. Alvarez and López (2005) examined self-selection and learning effects with Chilean plants during 1990-1996. They used a regression method with categorized types of plants and concluded that both effects are detected from the data. While Chile's economy showed stable growth during 1990-1996 in terms of GDP, its behavior changed after 1997. Specifically, Chile's economy suffered from a short-term recession in 1999 and it affected the behavior of plants because more exits and fewer entries in export markets happened around 1999 than before. Therefore, I conjecture that Chilean plants which enter into export markets after a recession will show higher productivity growth than those that entered before a recession. Besides the learning-by-exporting hypothesis, I examine the productivity growth of plants before and after entry into export markets.

I confirm that the self-selection hypothesis and the learning-by-exporting hypothesis are detected from Chilean plants even with different empirical methods. In particular, the self-selection hypothesis is confirmed by the regression method and the stochastic dominance method. The regression method and the matched sampling technique provide us with the evidence of the learning-by-exporting hypothesis.

(2005).

[2]See Arnold and Hussinger (2005), Kostevc (2005), De Loecker (2007), Girma et al. (2003), Girma et al. (2004a), Greenaway and Kneller (2007), Greenaway and Kneller (2008), and Yasar and Rejesus (2005).

[3]See Delgado et al. (2002).

In the next section, I present empirical methods with related literatures. In section 3, I discuss the characteristics of the data. Section 4 and 5 contain the results of self-selection and learning effects, respectively. Section 6 concludes.

4.2 LITERATURE REVIEW AND EMPIRICAL METHODS

In empirical studies on productivity and export behavior, three analysis methods are primarily used. One representative method is the regression method proposed by Bernard et al. (1995). Another one is the matched sampling technique which was recently used in a number of empirical studies. The last one is a non-parametric method with the concept of stochastic dominance devised by Delgado et al. (2002). In the following sections, I review some studies of exports and productivity using different methods and their results for the self-selection and the learning-by-exporting hypotheses.

4.2.1 Regression Method

The regression approach follows Bernard and B. Jensen (1999). For self-selection, they consider the following linear probability model with fixed effects.

$$X_{it} = \alpha + \beta\, Control_{it-1} + \gamma\, X_{it-1} + \varepsilon_{it}$$

where X_{it} is the export decision of firm i at time t, $Control_{it-1}$ is a vector of firm i's characteristics including firm's productivity.

To avoid the biasedness and inconsistency of estimates, they make use of the following linear probability models in first differences, using as instruments, X_{it-2}, X_{it-3}, $Control_{it-2}$, $Control_{it-3}$.

$$\triangle X_{it} = \beta\, \triangle Control_{it-1} + \gamma\, \triangle X_{it-1} + \triangle\varepsilon_{it}$$

Recent studies in Table 4.1 make use of the probit model for the self-selection as below.

$$Pr(X_{it} = 1|X_{it-1} = 0) = \Phi(\beta\, PROD_{it-1} + \delta' Control_{it-1} + \gamma_s + \gamma_t + \epsilon_{it})$$

where $PROD_{it-1}$ is firm i's productivity at time $t-1$, γ_s and γ_t are the dummies for sector and time, respectively. We can predict the probability of exporting in the current period for a firm that was not in export markets in the previous period based on its characteristics and productivity in the previous period.

For the learning effect, they use the following regression model with dummy variables of some specific firm types.

$$PROD_{iT} - PROD_{i0} = \alpha + \beta_1 Start_{iT} + \beta_2 Both_{iT} + \beta_3 Stop_{iT} + \gamma Control_{i0} + \epsilon_{iT}$$

where $Control_{i0}$ is a vector of firm i's characteristics at the initial time, and the dummies for export status are defined as follows:

$$\begin{aligned} Start_{iT} &= 1 \quad if \quad (X_{i0} = 0)\, and\, (X_{iT} = 1) \\ Both_{iT} &= 1 \quad if \quad (X_{i0} = 1)\, and\, (X_{iT} = 1) \\ Stop_{iT} &= 1 \quad if \quad (X_{i0} = 1)\, and\, (X_{iT} = 0) \end{aligned}$$

In the regression, the productivity difference between the initial and the final period is affected by a firm's exporting behavior in the initial and the final period. If I have a significant β_1 in estimation, I can say that export firms improve their productivity more than non-export firms.

Table 4.1 present studies using the regression method with their findings. Alvarez and López (2005) detect both effects using Chilean plant-level data. Bernard and B. Jensen (1999) discover self-selection but not the learning effect with the U.S. firm-level data for 1984-1992. Clerides et al. (1998) consider Colombia, Mexico and Morocco and find scant evidence for the learning effect. Interestingly, they set a dynamic model of export decisions and compare the simulation results with the data results. Damijan et al. (2004) consider the heterogeneity in export markets as well as firms. They conclude that different destinations affect the productivity improvement after entry into export markets. Exporters to advanced countries benefit more in terms of productivity than exporters to less advanced countries.

Table 4.1: Empirical studies using the regression method

Study	Country and period	Findings	
		Self-selection	Learning-by-exporting
Alvarez and López (2005)	Chile (1990-1996)	Yes	Yes
Bernard and B. Jensen (1999)	U.S. (1984-1992)	Yes	No
Clerides et al. (1998)	Colombia (1981-1991), Mexico (1986-1990), Morocco (1984-1991)	Yes	No
Damijan et al. (2004)	Slovenia (1994-2002)	Yes	Yes only when serving advanced countries
Fernandes and Isgut (2005)	Colombia (1981-1991)	Yes	Yes

Fernandes and Isgut (2005) focus on young firms and export experience rather than on export participation using Arrow (1962)'s characterization of learning-by-doing. The learning effect is the main interest of their study. They use the matched sampling approach as well as regression and the general method of moment. With Colombian plant-level data, they conclude that young plants in export markets show learning-by-exporting but not young non-exporting plants or old exporting plants.

4.2.2 Matched Sampling Technique

The matched sampling technique has recently been used in many studies on the learning-by-exporting hypothesis. The main idea of the method is to compare the differences of the same exporting firms when they actually export and when they hypothetically do not export.

$$E[PROD^1_{it+s}|X_i = 1] - E[PROD^0_{it+s}|X_i = 1]$$

where $PROD^1_{it+s}$ is productivity of export firm i at time $t+s$ and $PROD^0_{it+s}$ is productivity of non-exporting firm i at time $t+s$. Since the latter term is not observed in the data, I use a propensity score method to find a non-exporting firm which has productivity in the initial time similar with a export firm. Specifically, I adopt the following probit model to give each firm its own score based on its characteristics in the previous period.

$$P(X_{it} = 1) = \Phi(h(Control_{it-1}, PROD_{it-1}, \cdots))$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. From the probit model above, each firm has a probability of entering into export markets. The export firms are the treated group and the non-exporting firms with a probability as much as the export firms are the control group.

With the control group as matched samples, they derive the following estimator of the learning-by-exporting effect.

$$\beta^s_{LBE} = \frac{1}{N_s} \sum_i \{PROD^1_{is} - \frac{1}{n_i} \sum_{j \in C(i)} PROD^0_{js}\}$$

where $s \in \{0, 1, 2, \cdots, S\}$ denotes the years after entering into export markets, N_s is the number of firms who entered the markets at time s, $PROD^1$ and $PROD^0$ are the estimated productivity of the treated and the control group, and $C(i)$ is the set of the control firms matched to firm i based on propensity score. The weight is n_i which is the size of $C(i)$.

Table 4.2 presents some studies with the matched sampling technique for different countries and periods. De Loecker (2007), Girma et al. (2003), Girma et al. (2004a) and Wagner (2002) focus only on the learning effect and present conclusive evidence on it. Greenaway et al. (2005) do not confirm the learning effect as well as the self-selection hypothesis due to the extremely high openness of Swedish economy. Greenaway and Kneller (2007) argue that post-entry productivity growth of new export firms is faster than non-export firms and also that the magnitude of these effects is affected by industry characteristics. Greenaway and Kneller (2008) not only present unambiguous evidence on self-selection and learning-by-exporting, but also show that regional and industry agglomerations positively influence the international market participation.

Table 4.2: Empirical studies using the matched sampling approach

Study	Country and period	Findings	
		Self-selection	Learning-by-exporting
Arnold and Hussinger (2005)	Germany (1992-2000)	Yes	No
De Loecker (2007)	Slovenia (1994-2000)	-	Yes
Fernandes and Isgut (2005)	Colombia (1981-1991)	Yes	Yes
Girma et al. (2003)	UK (1991-1997)	-	Yes
Girma et al. (2004a)	UK (1988-1999)	Yes	Yes
Greenaway et al. (2005)	Sweden (1980-1997)	No	No
Greenaway and Kneller (2007)	UK (1989-1998)	Yes	Yes
Greenaway and Kneller (2008)	UK (1989-2002)	Yes	Yes
Kostevc (2005)	Slovenia (1994-2002)	Yes	No
Wagner (2002)	Germany (1978-1989)	-	Yes

4.2.3 Stochastic Dominance Method

The non-parametric method with the concept of stochastic dominance was proposed by Delgado et al. (2002). This method examines the whole productivity distribution of exporters and non-exporters rather than marginal moments. Thus, it gives us information on whether productivity of the entire set of export firms is better than that of the entire set of non-exporters. Denote $F(\cdot)$ and $G(\cdot)$ the cumulative distribution functions of exporters and non-exporters, respectively. Then, they test the following hypotheses:

(i) Two-sided test

$$H_0:\ F(z)-G(z)=0\ for\,all\,z\in\mathbb{R}\ \ vs.\ \ H_1:\ F(z)-G(z)\neq 0\ for\,some\,z\in\mathbb{R}$$

(ii) One-sided test

$$H_0:\ F(z)-G(z)\leq 0\ for\,all\,z\in\mathbb{R}\ \ vs.\ \ H_1:\ F(z)-G(z)>0\ for\,some\,z\in\mathbb{R}$$

If the null hypothesis for the two-sided test is rejected, it means that the productivity distribution of exporters cannot be the same as that of non-exporters. If the one-sided test cannot be rejected after rejecting the hypothesis of the equality of the productivity distributions between exporters and non-exporters, I can say that the productivity distribution of exporters stochastically dominates that of non-exporters. For these two-sided and one-sided tests, the following Kolmogorov-Smirnov test statistics are used, respectively.

$$\delta_N=\sqrt{\frac{nm}{N}}\max_{1\leq i\leq N}|T_N(Z_i)|\ \ and\ \ \eta_N=\sqrt{\frac{nm}{N}}\max_{1\leq i\leq N}\{T_N(Z_i)\}$$

where $T_N=F_n(Z_i)-G_m(Z_i)$, $N=n+m$, n and m are the number of exporters and non-exporters, $F_n(\cdot)$ and $G_m(\cdot)$ are the empirical distribution functions of $F(\cdot)$ and $G(\cdot)$. The limiting distributions of both test statistics, δ_N and η_N, are known under independence. For more details see the footnote 1 in Delgado et al. (2002).

In Table 4.3, I review studies using stochastic dominance method for their empirical analysis. Delgado et al. (2002) examine self-selection and learning-by-exporting, and present supporting evidence of self-selection. However, the learning effect is supported only by young export firms. In addition, a number of studies pay attention to foreign direct investment

Table 4.3: Empirical studies using the stochastic dominance method

Study	Country and period	Findings
Arnold and Hussinger (2010)	Germany (1996-2002)	Exporters' productivity distribution dominates that of non-exporters
Delgado et al. (2002)	Spain (1991-1996)	Self-selection for all export firms and learning-by-exporting for young export firms
Girma et al. (2004b)	Ireland (2000)	Domestic multinationals have highest productivity distribution than domestic non-exporters and domestic exporters

(FDI) as well as export with this method. Girma et al. (2004b) and Girma et al. (2005) consider three types of firms: domestic non-exporters, domestic exporters and domestic multinationals which invest through FDI. They compare the productivity measures and cumulative distribution of productivity for those three types of firms. Interestingly, domestic multinationals have the highest level of productivity and distribution. Wagner (2006) also considers FDI as a way for a firm to participate in international markets and derives the similar result as Girma et al. (2004b) and Girma et al. (2005).

4.3 DATA

I use the Chilean plant-level data from 1995 to 2007 based on the Annual National Industrial Survey (ENIA) carried out by the National Institute of Statistics of Chile. This survey is done annually for about 5,000 plants and so during the analysis period, I have approximately 70,000 observations. Table 4.4 shows the periods and export experience of plants in the data. There are 10,909 plants that appear in the data for at least one year. Among them, 34.3 percent appear in the panel for less than three years and 29.0 percent stay in the panel for more than 10 years. In addition, 15.9 percent of them stayed in the data for 13 years. 74.8

percent of all plants are not in the export market; 11.6 percent of the plants participate in export markets less than three years and 2.1 percent stay in international markets for 13 years.

In Table 4.5, I show the average characteristics of plants in the data. On average, among the 5,378 plants exporters are 20.1 percent and non-exporters are 79.9 percent. When it comes to the ownership of plants, plants with national private ownership are the majority which account for 92.7 percent of all plants. Small plants are 70.6 percent of all plants; medium and large ones are 18.6 percent and 10.8 percent, respectively. The distribution of export plants is different from that of non-export plants. While small plants are the majority of non-exporters, medium-sized and large plants are the majority of exporters.

Table 4.6 presents the annual rate of plants' exit from and entry into export markets. While the exit rate has kept stable at around 19 percent between 1996 and 1998, it started to increase in 1999, 2000, and 2001. The entry rate dropped significantly in 1998 and 1999, and jumped back in 2000. It seems that plants behaved differently before and after the short recession in 1999. Before the recession, plants were stable in terms of entry and exit rate, but around the recession, exit rates increased and entry rates dropped a lot. The drop in GDP growth rate in 1998 and the short-term recession in 1999 may have caused Chilean plants to exit from and enter into export markets more often between 1999 and 2001.

To analyze self-selection and learning-by-exporting, I divide plant types into more categories. I have seven types of plants based on their export behaviors following Alvarez and López (2005). Permanent exporters stayed in export markets for 13 years. Non-exporters are the opposite of permanent exporters. Quitters, entrants-stay, entrants-exit, switchers and participants are introduced instead of entrants and exiters. Quitters started to export before 1995 and ended exporting before 2007. Entrants-stay entered into export markets during the period and stayed in the markets at the end of the period. Entrants-exit also entered during the period but exited before 2007. Switchers changed their export behaviors more than once during the period. Lastly, participants are plants that appeared in export markets only once in the period of analysis.

In Figure 4.1, I present the rank of each type of plants based on value added per worker. Permanent exporters have the highest productivity during the periods except in 1998 and

Table 4.4: Plants in the panel and years of exporting

No. of years	Plants in the panel		Years of exporting	
	Obs.	(%)	Obs.	(%)
13	1,731	15.9	227	2.1
12	547	5.0	99	0.9
11	384	3.5	95	0.9
10	498	4.6	99	0.9
9	450	4.1	91	0.8
8	551	5.1	103	0.9
7	525	4.8	121	1.1
6	678	6.2	172	1.6
5	861	7.9	198	1.8
4	932	8.5	276	2.5
3	1,216	11.1	288	2.6
2	1,245	11.4	424	3.9
1	1,291	11.8	561	5.1
0	-	-	8,155	74.8
Total	10,909	100.0	10,909	100.0

Table 4.5: Descriptive statistics (Average, 1995-2007)

	Number		Exporters		Non-exporters	
A. Total Number of Plants	5,378	(100.0)	1,082	(20.1)	4,297	(79.9)
B. Ownership						
- National Private	4,988	(92.7)	867	(80.1)	4,121	(95.9)
- Foreign Private	172	(3.2)	108	(10.0)	64	(1.5)
- Mixed	150	(2.8)	90	(8.3)	60	(1.4)
- State	69	(1.3)	17	(1.6)	52	(1.2)
C. Size						
- Small (10-49 workers)	3,795	(70.6)	328	(30.3)	3,467	(80.7)
- Medium (50-149 workers)	1,002	(18.6)	372	(34.4)	630	(14.7)
- Large (¿= 150 workers)	582	(10.8)	382	(35.3)	200	(4.7)

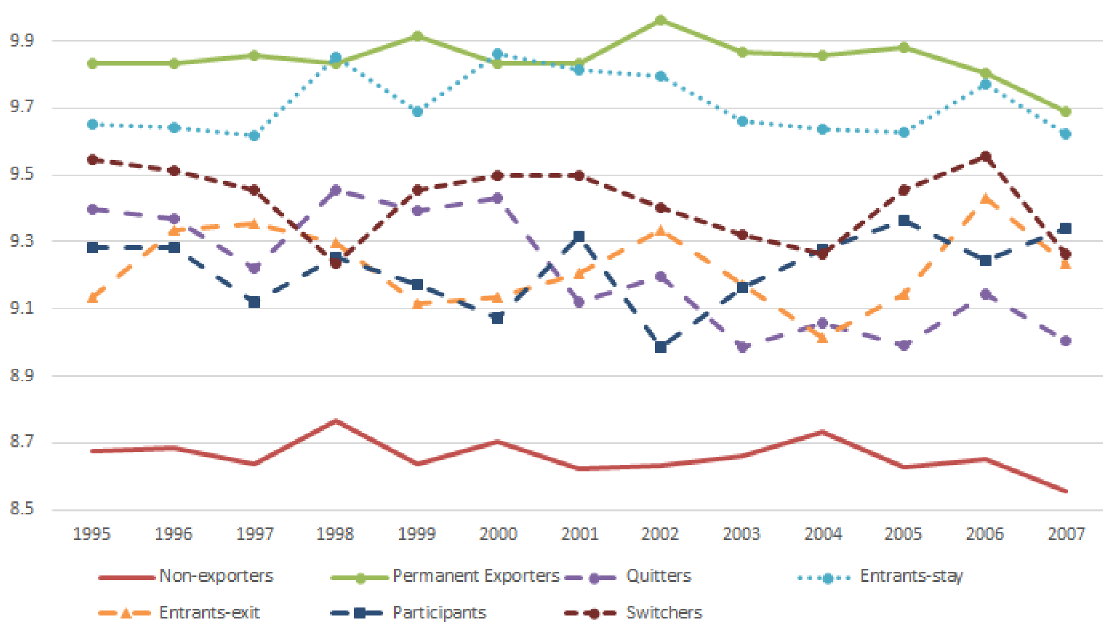
Table 4.6: Plant's exit and entry for export markets

Year	Exit	Exporters in previous year	Exit rate (%)	Entry	Exporters in current year	Entry rate (%)
1996	214	1,170	18.3	248	1,204	20.6
1997	225	1,204	18.7	198	1,177	26.8
1998	220	1,177	18.7	160	1,117	14.3
1999	222	1,117	19.9	130	1,025	12.7
2000	234	1,025	22.8	175	966	18.1
2001	208	966	21.5	197	955	20.6
2002	175	955	18.3	245	1,025	23.9
2003	179	1,025	17.5	217	1,063	20.4
2004	200	1,063	18.8	252	1,115	22.6
2005	229	1,115	20.5	237	1,123	21.1
2006	191	1,123	17.0	155	1,087	14.3
2007	203	1,087	18.7	152	1,036	14.7

Table 4.7: Plant's type (Average, 1995-2007)

Type	Observations		Employment	Value added per worker
	Number	(%)		
Non-exporters	3,682	69.7	460	8.7
Permanent exporters	212	4.0	3,139	9.8
Quitters	289	5.5	1,953	9.2
Entrants-stay	499	9.4	2,092	9.7
Entrants-exit	198	3.7	1,266	9.2
Switchers	256	4.8	1,583	9.4
Participants	198	2.8	866	9.2
Total	5,285	100.0	-	-

Figure 4.1: Value added per worker of plants by type and year



2000. The second highest type is entrants-stay and they showed higher productivity in 1998 and 2001 than permanent exporters. Non-exporters are the least productive type among them. Other types of plants show fluctuations in their productivity during the overall periods.

Table 4.7 shows the average number of observations, employment, and value added per worker of each type. Even though permanent exporters account for only 4.0 percent of all plants, they have the highest productivity and the biggest employment as well. Entrants-stay are in the second rank in employment and productivity. In addition, non-exporters are the majority of all plants but show the least productivity and employment.

4.4 SELF-SELECTION

As a productivity measure, the logarithm of value added per worker is used in this paper. I use the following probit model to confirm that productive plants in Chile select into export markets during 1995-2007.

$$Pr(X_{it} = 1 | X_{it-1} = 0) = \Phi(\beta\, PROD_{it-1} + \delta' Control_{it-1} + \gamma_s + \gamma_t + \epsilon_{it})$$

where X_{it} is a dummy variable for export of a plant i at time t, $PROD_{it-1}$ is productivity of plant i at time $t-1$, $Control_{it-1}$ is plant i's characteristics at time $t-1$, and γ_s and γ_t are dummy variables for sector and year. From this model, I can detect how much plant i's characteristics at time $t-1$ contribute to plants i's export decision at time t. Following Alvarez and López (2005), I use dummies for the size of a plant, the ratio of skilled labor to total labor as plant i's characteristics as shown in table 8.

Table 4.8 presents the result from the probit model. Compared to other characteristics such as size and the ratio of skilled labor, the coefficient on productivity has a smaller but significant effect on the probability of export decision. One standard deviation increase in productivity raises the probability of becoming exporters by 0.35 percent. It is clear from the results of the probit model that the more productive plants tend to enter into export markets.

Table 4.8: Probability of beginning to export

Independent variable	(1)
$PROD_{t-1}$	0.026***
	(0.007)
$Medium_{t-1}$	0.480***
	(0.026)
$Large_{t-1}$	0.740***
	(0.037)
$log(\frac{Skilled\,labor}{Employment})_{t-1}$	-0.023
	(0.035)
Constant	-2.304
	(5.187)
Sector Fixed Effect	Yes
Year Fixed Effect	Yes
N	51,384

Table 4.9: The difference in productivity distribution between exporters and non-exporters

Year	Number of observations		Equality of distribution		Differences favorable to exporters	
	Exporters	Non-exporters	Estimate	p-value	Estimate	p-value
1995	1,120	4,260	0.350	0.000	0.001	1.000
1996	1,158	4,573	0.351	0.000	0.002	0.995
1997	1,134	4,387	0.360	0.000	0.006	0.930
1998	1,073	4,249	0.344	0.000	0.009	0.856
1999	982	4,217	0.373	0.000	0.008	0.913
2000	926	4,113	0.366	0.000	0.003	0.988
2001	921	4,055	0.387	0.000	0.005	0.970
2002	991	4,316	0.359	0.000	0.001	0.998
2003	1,030	4,235	0.337	0.000	0.002	0.993
2004	1,080	4,403	0.350	0.000	0.014	0.700
2005	1,093	4,306	0.377	0.000	0.006	0.938
2006	1,061	4,097	0.377	0.000	0.006	0.948
2007	1,012	3,910	0.372	0.000	0.005	0.959

Table 4.9 shows the results from the stochastic dominance method. In this method, I compare the productivity distribution of exporters and non-exporters every year. Since the hypothesis for the two-sided test is rejected, it cannot be said that the productivity distribution of exporters is the same as that of non-exporters. In addition, the hypothesis for the one-sided test is not rejected as shown in Table 4.9. Therefore, the productivity distribution of exporters stochastically dominates that of non-exporters every year. From the results, it is confirmed that the more productive plants show a higher tendency to enter into export markets. Therefore, the clear evidence of the self-selection hypothesis is provided using the regression method and the stochastic dominance method.

From the probit model and the stochastic dominance method, I confirm that the more productive plants in Chile tend to participate in export markets.

4.5 LEARNING-BY-EXPORTING

4.5.1 Productivity Growth Between Exporters and Non-exporters

I consider two concepts for the learning-by-exporting hypothesis. The first concept is commonly accepted by the literatures.[4] The learning-by-exporting hypothesis implies that plants that select into export markets raise their productivity more than those that stay in domestic market. That is, it focuses on the difference in productivity growth between exporters and non-exporters. The regression method and the matched sampling technique are used to confirm the first concept.

For the first concept of the learning effect, I examine the productivity growth for different types of plants in the following regression.

$$PROD_{iT} - PROD_{i0} = \alpha + \beta' TYPE_i + \delta' Control_{i0} + \gamma_s + \varepsilon_{iT} \tag{4.1}$$

where $PROD_{iT}$ and $PROD_{i0}$ are plant i's productivity at the final year and the initial year in the data, $TYPE_i$ is a dummy vector for plant i's types(permanent exporters, entrants-stay, quitters, entrants-exit, switchers and participants), $Control_{i0}$ is plant i's characteristics

[4] See Aw et al. (2000).

at the initial year, and γ_s is a dummy variable for sector. Since I do not consider the dummy variable for non-exporters, the coefficient β is interpreted based on non-exporters.

In addition, I examine the annual productivity growth of each type of plants with the following econometric model.

$$g_i = \alpha + \beta' TYPE_i + \delta' Control_i + \gamma_s + \varepsilon_i \tag{4.2}$$

where g_i is the average annual growth rate of productivity. I expect that permanent exporters and entrants-stay will satisfy higher productivity growth compared to other types of plants.

Table 4.10 shows the results of the regression for learning-by-exporting hypothesis. Column (1) - (4) are the result for Equation (1). In column (1), entrants-stay, entrants-exit, and participants show significant effect on the productivity growth over the period. Quitter takes negative effect on the productivity growth. Coefficients for entrants-stay, quitter, and participants are robust even with other control variables as shown in column (2). Among them, I am interested in entrants-stay because they may show different results according to when they entered into export market. Hence, I divide entrants-stay into early entrants-stay and late entrants-stay. Early entrants-stay enter into export market between 1996 and 1999 which is before the recession. Late entrants-stay enter between 2000 and 2006 which is after the recession. As I see in column (3), both types of entrants-stay show positive effect on the productivity growth. However, late entrants-stay is more robust with other control variables in column (4). That is, early entrants-stay were hit by the recession and their productivity in the final year did not grow significantly compared to the one in the initial year. However, late entrants-stay are those that survived the domestic recession and are ready to start to export. That's why they improve their productivity significantly.

Column (5) and (6) in Table 4.10 show the result for Equation (2). Permanent exporters show significant faster productivity growth compared to other types of plants. It is robust with other control variables in the regression. In terms of increases in productivity level, entrants-stay show significant productivity growth. However, in terms of average annual productivity growth, permanent exporters are those that learn by exporting faster than other types of plants.

Table 4.11 provides the result from the matched sampling technique. I estimate the

Table 4.10: The learning-by-exporting from the regression method

Dependant variable:	Productivity difference				Productivity growth rate	
Independent variable	(1)	(2)	(3)	(4)	(5)	(6)
Entrants-stay	0.336***	0.297***	-	-	0.003	0.001
	(0.117)	(0.124)	-	-	(0.010)	(0.010)
Early entrants-stay	-	-	0.389***	0.355	-	-
	-	-	(0.233)	(0.236)	-	-
Late entrants-stay	-	-	0.322***	0.281**	-	-
	-	-	(0.130)	(0.136)	-	-
Permanent	0.238	0.162	0.238	0.162	0.027**	0.023*
	(0.148)	(0.162)	(0.162)	(0.162)	(0.012)	(0.013)
Quitter	-0.280***	-0.332**	-0.280***	-0.332**	-0.020	-0.022*
	(0.150)	(0.157)	(0.258)	(0.157)	(0.012)	(0.013)
Entrants-exit	0.301***	0.253	0.301***	0.253	0.002	0.001
	(0.162)	(0.164)	(0.000)	(0.164)	(0.013)	(0.013)
Participants	0.410***	0.400*	0.410***	0.400*	-0.007	-0.007
	(0.207)	(0.206)	(0.000)	(0.206)	(0.017)	(0.017)
Switcher	-0.129	-0.226	-0.129	-0.226	-0.009	-0.011
	(0.150)	(0.155)	(0.000)	(0.155)	(0.012)	(0.013)
Medium	-	0.034	-	0.033	-	0.002
	-	(0.091)	-	(0.091)	-	(0.007)
Large	-	0.162	-	0.163	-	0.007
	-	(0.124)	-	(0.124)	-	(0.010)
$log(\frac{Skilled\,labor}{Employment})$	-	0.026	-	0.026	-	0.004
	-	(0.044)	-	(0.044)	-	(0.004)
Constant	-0.167	-0.128	-0.166	-0.127	-0.026	-0.021
	(0.258)	(0.262)	(0.258)	(0.263)	(0.022)	(0.022)
Sector Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
N	3,980	3,939	3,980	3,939	3,980	3,980

Table 4.11: The learning-by-exporting using the matched sampling technique

	Productivity		Annual growth rate	
s	Estimate	Standard Error	Estimate	Standard Error
0	0.260***	0.077	0.015	0.019
1	0.307**	0.131	0.018	0.017
2	0.169	0.126	0.035	0.029
3	0.098	0.131	-0.002	0.029
4	0.114	0.118	0.014	0.033
5	0.071	0.144	-0.027	0.018
6	0.310**	0.153	-0.005	0.025
7	0.054	0.157	-0.017	0.022
8	0.012	0.148	-0.001	0.030
9	0.183	0.166	-0.041	0.024
10	-0.130	0.130	0.029	0.025
11	0.378**	0.180	0.035*	0.018

difference in productivity and the annual productivity growth rate between the treated and the controls. That is, I estimate β^s_{LBE} after s years of entering into international markets in Section 2.2. For the productivity level, one year after entering into export market, plants show productivity growth. However, the effect of entry on productivity growth disappears after the second year. In addition, permanent exporters show higher productivity than plants which had the similar level of productivity in the beginning. It is also applied to the productivity growth. Permanent exporters show faster productivity growth than those that started their business domestically with a similar level of productivity.

4.5.2 Productivity Growth of Exporters

The second concept of the learning-by-exporting hypothesis considers the productivity growth of exporters before and after entry into export markets. I compare the productivity growth of entrants-stay to confirm the second concept because entrants-stay serve both of domestic market and export markets during the sample period. If the previous export status helps their productivity growth faster, I can argue that exporting experience contributes to the faster productivity growth of export plants. To see the second concept, I use the following regression model.

$$g_t = \alpha + \beta EXP_{t-1} + \delta' Control_{t-1} + \varepsilon_t$$

where g_t is the annual growth rate of productivity at time t, EXP_{t-1} denotes export status of a plant at $t-1$. In addition, I repeat the same regression for early entrants-stay and late entrants-stay because the short recession can affect the productivity growth of entrants-stay.

Table 4.12 provides the results of the regression above. Column (1) shows the result for the whole entrants-stay and I see that the previous export status does not contribute to the faster productivity growth of export plants. Hence, the evidence for the learning effect for exporters is not found.[5] In column (2), I regress the productivity growth of early entrants-stay on the previous export status and control variables. Similar to Column (1), the previous export status does not have a significant coefficient for the productivity growth of

[5] Bernard and B. Jensen (1999) have consistent results using US firm-level data during 1984-1992.

Table 4.12: Learning effect of entrants-stay

	Entrants-stay	Early entrants-stay	Late entrants-stay
	(1)	(2)	(3)
EXP_{t-1}	0.006	-0.043	0.008
	(0.012)	(0.063)	(0.012)
$Medium_{t-1}$	0.037***	0.057	0.030**
	(0.013)	(0.037)	(0.014)
$Large_{t-1}$	0.041***	0.075*	0.027*
	(0.014)	(0.039)	(0.015)
$log(\frac{Skilled\,labor}{Employment})_{t-1}$	0.004	0.041**	-0.002
	(0.007)	(0.020)	(0.008)
Constant	-0.106**	0.055	-0.115**
	(0.051)	(0.197)	(0.052)
Sector Fixed Effect	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes
N	4,863	921	3,942

early entrants-stay. Column (3) presents the result for late entrants-stay and their previous export status does not help their productivity grow faster. However, while early entrants-stay have a negative coefficient on the previous export status, late entrants-stay have a positive coefficient. That is, the previous export status contributes to the faster productivity growth of late entrants-stay even though it is not significant. From column (2) and (3), one can see that the short recession generates different effects on productivity growth of entrants-stay.

The first concept of the learning effect is detected in Chilean plants during 1995-2007. In the probit model, entrants-stay enjoy higher productivity growth than non-exporters and permanent exporters show faster productivity growth than non-exporters. In matched sampling technique, I compare differences in productivity between exporters and non-exporters with similar initial productivities. The productivity growth rate of exporters is faster than non-exporters, but the learning effect lasts only for one year of entry. Lastly, the productivity of entrants-stay does not grow significantly faster after entry into export markets. Apparently, export plants experience higher productivity growth than non-exporting plants but the productivity of export plants does not continue to grow faster after participating in export markets.

4.6 CONCLUSION

In this paper, I examine the self-selection and the learning-by-exporting hypothesis with a recent plant-level data set from Chile using three different empirical methods. The plant-level dataset from Chile covers from 1995 to 2007 and Chile suffered from a short recession in 1999. Hence, I study two hypotheses considering the recession in Chile.

Three empirical methods are used to derive consistent results on firm performance and exporting behavior. From the methodological point of view, a majority of previous studies focus on regression method to see the self-selection and the learning effects. The matched sampling technique is used for the learning effect. Research using the stochastic dominance method pays more attention to the self-selection. When it comes to the learning effect using the stochastic dominance method, Delgado et al. (2002) successfully provide evidence of the learning effect.

I find evidence that supports the self-selection from the regression method and the stochastic dominance method. The more productive plants self-select into international markets. Moreover, the learning-by-exporting hypothesis is supported by the regression method and the matched sampling technique. After entry into export markets, productivity of entrants-stay grows more than that of non-exporting plants. While entrants-stay who entered after the recession show significantly higher productivity growth compared to non-exporters, entrants-stay who entered before the recession do not show significant higher productivity growth. I also confirm that permanent exporters during the sample period show faster productivity growth than non-exporters. Lastly, I confirm that export experience does not contribute to productivity growth of export plants. However, I see that the previous export status has a positive coefficient for productivity growth of early entrants-stay and a negative coefficient for that of late entrants-stay. Even though coefficients are not statistically significant, while export plants that entered after a recession show faster productivity growth, export plants that entered before a recession show slower productivity growth.

APPENDIX A

CHAPTER 2

A.1 SOLUTION ALGORITHM

1. Start with a discretized state space for government bonds on a grid.
2. Calculate the government's value function in permanent autarky v^{aut}. In permanent autarky, the optimal wage, labor, and consumption are computed for each state of TFP shock.
3. Start with a guess for the value of the government, the bond price schedule, the wage function, and the interest rate on working capital such that $V_0 = v^{aut}$, $q_0(B, z) = \frac{1}{1+r^*}$, $w_0(B, z) = 1$, and $r_0(B, z) = r^*$.
4. Derive the optimal labor supply (2.1) from the households' problem using the initial wage obtained in Step 3.
5. Given the value of the government, the bond price schedule, the wage function, and the interest rate on working capital, we solve the optimal policy function for government's bond decisions, repayment sets, and default sets via value function iteration. For each iteration of the value function, we compute the value of default, which is endogenous because it depends on the value of contract at $B = 0$.
6. Using default sets and repayment sets obtained in Step 5, we update the bond price schedule $q_1(B, z)$ via (2.5) and compare it to the previous bond price schedule $q_0(B, z)$. Using $q_1(B, z)$, we update the interest rate on working capital $r_1(B, z)$ following (2.6). Using the updated $r_1(B, z)$, we update the wage function $w_1(B, z)$ following (2.2) and

the labor supply. In each iteration of the value function, we check whether the value function, the wage schedule, and the bond price schedule converge, simultaneously. If any of three fails to converge, we go back to Step 5.

APPENDIX B

CHAPTER 3

B.1 TABLES

Table B1: Fitch credit rating conversion table

Fitch Rating	Score
AAA	23
AA+	22
AA	21
AA-	20
A+	19
A	18
A-	17
BBB+	16
BBB	15
BBB-	14
BB+	13
BB	12
BB-	11
B+	10
B	9
B-	8
CCC+	7
CCC	6
CCC-	5
CC	4
C	3
DDD	2
D	1
RD	1

Table B2: Panel regressions explaining creditworthiness with debt ratios and inequality using Gini data from the World Bank

Independent Variable	(1)	(2)	(3)
External debt-to-GDP at $t-1$	-0.0214^{**}	-0.0068	-0.0060
	(0.0102)	(0.0067)	(0.0061)
GDP per capita at $t-1$	–	15.3950^{***}	14.7987^{***}
		(2.4341)	(2.3667)
Gini at $t-1$			-0.1762^{**}
			(0.0687)
Year fixed effects	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes
No of countries	40	40	40
N	364	364	364

Sample period is 1994-2009. The dependent variable is the credit rating of country i in year t. Estimation is by robust standard errors. Standard errors are reported in parentheses. Per capita GDP is in logs. (***,**,* represent the significance level at 1%, 5% and 10%, respectively.)

Table B3: VAR estimations for different countries

Country	ρ_{yy}	$\rho_{y\gamma}$	$\rho_{\gamma y}$	$\rho_{\gamma\gamma}$	σ_y^2	$\sigma_{y\gamma}$	$\sigma_{\gamma y}$	σ_γ^2
Brazil	0.34	-0.25	0.09	0.64	5.6x10^{-4}	5x10^{-5}	5x10^{-5}	8x10^{-5}
Colombia	0.44	0.09	-0.15	0.33	1.4x10^{-4}	2x10^{-5}	2x10^{-5}	8x10^{-5}
Costa Rica	0.33	-0.07	0.05	0.74	4.5x10^{-5}	-1x10^{-5}	-1x10^{-5}	9x10^{-5}
Dominican Republic	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Ecuador	0.01	-0.33	0.20	0.82	1.3x10^{-3}	-1.8x10^{-4}	-1.8x10^{-4}	2.3x10^{-4}
Paraguay	-0.74	0.24	-0.05	0.73	4x10^{-4}	-4x10^{-5}	-4x10^{-5}	7x10^{-5}
Uruguay	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Argentina	0.28	-0.56	0.05	0.79	1.2x10^{-3}	-2x10^{-4}	-2x10^{-4}	1.3x10^{-4}

In this VAR analysis, we assume that log output and the inequality follow a VAR(1) process such that

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

where

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

$$E[\boldsymbol{\varepsilon}] = \mathbf{0} \quad \text{and} \quad Var[\boldsymbol{\varepsilon}] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}$$

B.2 PROOFS OF PROPOSITIONS

B.2.1 Proof of Proposition 1

The proof is similar to Arellano (2008).

First we show that value of repayment is increasing i asset holdings. For all $\{y, \gamma\} \in D(B^2)$,

$$\begin{aligned} \frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} &> \frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2}, \\ \frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} &> \frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2}. \end{aligned}$$

So,

$$\begin{aligned} u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) &+ u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq \\ u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) &+ u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma'). \end{aligned}$$

Therefore, for all $\{y, \gamma\} \in D(B^2)$,

$$\begin{aligned} & u\left(\frac{y(1-\gamma)}{2}\right) + u\left(\frac{y(1+\gamma)}{2}\right) + \beta E[\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] > \\ & u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq \\ & u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma'). \end{aligned}$$

Hence, any pair of $\{y, \gamma\}$ that is in $D(B^2)$, we have $\{y, \gamma\} \in D(B^1)$.
Let $d(B, y', \gamma')$ denote the equilibrium default decision rule. Default probability satisfies

$$\delta(B, y', \gamma') = \int d(B, y', \gamma') f((y', \gamma'), y, \gamma) d(y', \gamma')$$

Since any $\{y, \gamma\} \in D(B^2)$, we have $D(B^2) \subseteq D(B^1)$, if $d(B^2, y', \gamma') = 1$, then $d(B^1, y', \gamma') = 1$. Hence, $\delta(B^1, y, \gamma) \geq \delta(B^2, y, \gamma)$. □

B.2.2 Proof of Proposition 2

The bond price is defined as $q(B', y, \gamma) = \frac{1-\delta(B',y,\gamma)}{1+r}$. Using Proposition 1, we have $B^1 < B^2 \leq 0$ and $\delta(B^2, y, \gamma) < \delta(B^1, y, \gamma)$. Hence, we get $q(B^2, y, \gamma) > q(B^1, y, \gamma)$. $\square$

B.3 SOLUTION ALGORITHM

To solve the model numerically, we use the discrete state-space method. We discretize the asset space using a finite set of grid points, making sure that the minimum and the maximum points on the grid do not bind when we compute the optimal debt decision. Our solution algorithm for the benchmark model is the following:[1]

1. Guess that the initial price is the reciprocal of the risk-free interest rate, and the initial value function is equal to the autarky value.
2. Given a price $q(B', y, \gamma)$ and $v^o(B, y, \gamma)$, solve for the optimal policy functions and update the value of option given as equation (3.10) by comparing $v^c(B, y, \gamma)$ and $v^d(y, \gamma)$.
3. Given the price function, compute the default probabilities.
4. Update the price function using equation (3.9).
5. We simultaneously check whether the initial guesses for price and the value of option are close enough to their updated values. If not, we update the initial values and iterate steps 2-4 until both bond price and the value of option functions converge.

[1]We use the same algorithm to solve the models with a single type of shock. For instance, for Model II, the price function is denoted as $q(B', y)$, and value of option for default or repayment is denoted as $v^o(B, y)$.

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