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## Essays On Asset Pricing, Debt Valuation, And Macroeconomics

#### Abstract

My dissertation consists of three chapters which examine topics at the intersection of financial markets and macroeconomics. Two of the sections relate to the valuation of U.S. Treasury and corporate debt while the third understands the role of banking frictions on equity markets.

More specifically, the first chapter asks the question, what is the role of monetary policy fluctuations for the macroeconomy and bond markets? To answer this question we design a novel asset-pricing framework which incorporates a time-varying Taylor rule for monetary policy, macroeconomic factors, and risk pricing restrictions from investor preferences. By estimating the model using U.S. term structure data, we find that monetary policy fluctuations significantly impact inflation uncertainty and bond risk exposures, but do not have a sizable effect on the first moments of macroeconomic variables. Monetary policy fluctuations contribute about 20% to the variation in bond risk premia.

Models with frictions in financial contracts have been shown to create persistence effects in macroeconomic fluctuations. These persistent risks can then generate large risk premia in asset markets. Accordingly, in the second chapter, we test the ability that a particular friction, Costly State Verification (CSV), has to generate empirically plausible risk exposures in equity markets, when household investors have recursive preferences and shocks occur in the growth rate of productivity. After embedding these mechanisms into a macroeconomic model with financial intermediation, we find that the CSV friction is negligible in realistically augmenting the equity risk premium. While the friction slows the speed of capital investment, its contribution to asset markets is insignificant.

The third chapter examines how firms manage debt maturity in the presence of investment opportunities. I document empirically that debt maturity tradeoffs play an important role in determining economic fluctuations and asset prices. I show at aggregate and firm levels that corporations lengthen their average maturity of debt when output and investment rates are larger. To explain these findings, I construct an economic model where firms simultaneously choose investment, short, and long-term debt. In equilibrium, long-term debt is more costly than short-term debt and is only used when investment opportunities present themselves in peaks of the business cycle.

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## ESSAYS ON ASSET PRICING, DEBT VALUATION, AND MACROECONOMICS

Ram S. Yamarthy

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Urban Jermann, Professor of Finance Nikolai Roussanov, Associate Professor of Finance Ivan Shaliastovich, Associate Professor of Finance, Wisconsin School of Business I dedicate this dissertation to my mom and dad, Lakshmi and Krishna. You have inspired my pursuit of knowledge and taught me the value of hard work. Without your love and support, this would not have been possible. Thank You.

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## ABSTRACT

## ESSAYS ON ASSET PRICING, DEBT VALUATION, AND MACROECONOMICS

Ram S. Yamarthy

João Gomes

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More specifically, the first chapter asks the question, what is the role of monetary policy fluctuations for the macroeconomy and bond markets? To answer this question we design a novel asset-pricing framework which incorporates a time-varying Taylor rule for monetary policy, macroeconomic factors, and risk pricing restrictions from investor preferences. By estimating the model using U.S. term structure data, we find that monetary policy fluctuations significantly impact inflation uncertainty and bond risk exposures, but do not have a sizable effect on the first moments of macroeconomic variables. Monetary policy fluctuations contribute about 20% to the variation in bond risk premia.

Models with frictions in financial contracts have been shown to create persistence effects in macroeconomic fluctuations. These persistent risks can then generate large risk premia in asset markets. Accordingly, in the second chapter, we test the ability that a particular friction, Costly State Verification (CSV), has to generate empirically plausible risk exposures in equity markets, when household investors have recursive preferences and shocks occur in the growth rate of productivity. After embedding these mechanisms into a macroeconomic model with financial intermediation, we find that the CSV friction is negligible in realistically augmenting the equity risk premium. While the friction slows the speed of capital

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## Chapter 1: Monetary Policy Risks in the Bond Markets

## (joint work with Ivan Shaliastovich)

## 1.1. Introduction

There is a significant evidence that monetary policy fluctuates over time. In certain periods the monetary authority reacts more strongly to fundamental concerns about real economic growth and inflation, thus affecting the dynamics and the risk exposures of the bond markets. Yet, there is no conclusive evidence on how these policy fluctuations impact the economy and asset prices. To assess the role of a time-varying monetary policy, we develop an economically motivated asset-pricing model which incorporates the link between the monetary policy fluctuations, aggregate macroeconomic variables, and nominal bond yields. We estimate the model using macroeconomic, forecast, and term structure data, and quantify the conditional implications of the monetary policy fluctuations above and beyond standard macro-finance risk channels. We find that while monetary policy fluctuations are not significantly related to the first moments of the macroeconomic variables, the inflation uncertainty and bond price exposures to economic risks significantly increase in aggressive relative to passive regimes. Consequently, through their direct impact on bond risk exposures and indirect effect on the quantity of inflation risk, fluctuations in monetary policy have a sizeable contribution to the time-variation in the levels of yields and bond risk premia.

Our asset-pricing framework features a novel recursive-utility based representation of the stochastic discount factor (SDF), the exogenous dynamics of the macroeconomic factors, and the time-varying Taylor rule for the interest rates. Specifically, the stochastic discount factor incorporates pricing conditions of the recursive-utility investor, but does not force an inter-temporal restriction between the short rate and the fundamental macroeconomic processes. This representation is an alternative to the decompositions in Bansal et al. (2013) and Campbell et al. (2012), and identifies long-run cash-flow, long-run interest rate news, and the uncertainty news as the key sources of risk for the investor. Our representation of the SDF is particularly convenient for our analysis. Similar to the reduced-form, no-arbitrage models of the term structure, our representation allows us to exogenously and flexibly model the dynamics of the short rates and the macroeconomic state variables. On the other hand, our stochastic discount factor is economically motivated, and the sources and the market prices of risks are disciplined to be consistent with recent economic term-structure models.

<sup>&</sup>lt;sup>1</sup>See Singleton (2006) for the review of the no-arbitrage term structure models.

To model the short rate, we assume a forward looking, time-varying Taylor rule in which the sensitivities of the short rate to expected real growth and expected inflation can vary across the monetary policy regimes. We further consider an exogenous specification of the dynamics of the real growth and inflation, which features persistent fluctuations in the conditional means and volatilities of the economic states. As in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005), we allow for inflation non-neutrality, so that expected inflation can have a negative impact on future real growth. This economic channel plays an important role to explain a positive bond risk premium and a positive slope of the nominal term structure. Following Bansal and Shaliastovich (2013), we also incorporate exogenous fluctuations in inflation volatility which drive the quantity of macroeconomic risks and bond risk premia. Novel in our paper, we allow the conditional expectations of future consumption and inflation as well as the inflation volatility to directly depend on the monetary policy regime. In this sense, we introduce a link between the time-varying monetary policy, the exposures of bond prices to economic risks, and the movements in the levels and volatilities of the underlying economic factors.

To estimate the model and assess the role of the monetary policy fluctuations, we utilize quarterly data on realized consumption and inflation, the survey expectations on real growth and inflation, and the data on bond yields for short to long maturities. The model-implied observation equations are nonlinear in the states, and all economic factors are latent. Similar to Schorfheide et al. (2013) and Song (2014), we rely on Bayesian MCMC methods to draw model parameters, and use particle filter to filter out latent states and evaluate the likelihood function.

We find that the estimation produces plausible parameter values and delivers a good fit to the observed macroeconomic and yield data. The expected consumption, expected inflation, and inflation volatility are very persistent, and expected inflation has a strong and negative feedback to future expected consumption. We further find substantial fluctuations in the monetary policy across the regimes. Interestingly, monetary policy fluctuations do not have a sizeable effect on the first moments of the macroeconomic variables: the expectations of future real growth and inflation are not significantly different across the regimes. On the other hand, inflation uncertainty significantly increases by about a quarter, and interest rates respond stronger to expected real growth and inflation risks in aggressive relative to passive regimes. Indeed, the median short rate loadings on expected real growth and expected inflation are equal to 0.7 and 1.7, respectively, in aggressive regimes, which are significantly larger than the estimated loadings of 0.5 and 0.9 in passive regimes.

These differences in inflation volatility and bond sensitivities have important implications for the dynamics of bond yields. We document that the loadings of bond yields and bond risk premia are magnified in aggressive relative to passive regimes. This results in elevated means and volatilities of bond yields and bond risk premia in aggressive regimes. Introducing time-variation in monetary policy increases variability of bond risk premia by about 20%. The time-varying bond exposures to expected inflation risk and the time-varying quantity of inflation risk due to monetary policy fluctuations both contribute about equally to a rise in bond risk premia. Interestingly, the variations in inflation and real growth sensitivities of interest rates have opposite effects on the bond risk premia. We further show that the bond risk premia can go negative when the inflation premium is relatively small. In the data, the model-implied bond risk premia turn negative post 2005 when the conditional inflation volatility is below the average, and the economy is in a passive regime.

Related Literature. There are several contributions of our approach to the existing literature. First, we rely on a novel representation of the stochastic discount factor which allows us to incorporate a flexible dynamics of a time-varying Taylor rule and macroeconomic factors, and yet impose economic pricing restrictions from the recursive preferences. Second, we consider the interaction between monetary policy fluctuations and movements in stochastic volatility, expected growth, and expected inflation. Finally, we estimate the model using the macroeconomic and asset price data, and perform a quantitative assessment of the importance of monetary policy fluctuations for the levels and volatilities of bond yields and bond risk premia.

Our paper is related to a large and growing macro-finance literature which studies the role of monetary policy for macroeconomic fundamentals and asset prices. In the context of general equilibrium models, Song (2014) considers a long-run risks type model to investigate a role of monetary policy and macroeconomic regimes for the dynamics of bond and equity prices, and specifically for the comovement between bond and equity returns. This work does not consider the link between monetary policy and macroeconomic uncertainties, which we find to be important for the movements in the bond risk premia. Campbell, Pflueger and Viceira (2014) use a New Keynesian habit formation model to study the variation in stock and bond correlation and movements in the bond risk premia across monetary policy regimes. The model is calibrated to target interest rate rules across data subsamples. In our work, persistent changes in monetary policy regimes which affect the dynamics of the macroeconomy and bond yields are taken into account when evaluating the Euler equation, and represent a priced source of risks for the investor. Within a DSGE framework, timevariation of the monetary policy is also considered in Andreasen (2012), Chib et al. (2010), and Palomino (2012), while constant coefficient Taylor rule are featured in general equilibrium models such as Rudebusch and Swanson (2012) and Gallmeyer et al. (2006). Kung (2015) embeds a constant coefficient Taylor rule in a production-based asset pricing model, and considers the monetary policy impact on the term structure of interest rates. Relative to this literature, our paper entertains an alternative and more flexible representation of the stochastic discount factor and macroeconomic factors, and further incorporates a link between inflation uncertainty and the monetary policy fluctuations.

In terms of the reduced-form term structure literature, Ang et al. (2011) highlight the importance of a time-varying monetary policy for the shape of the nominal term structure and the levels of the bond risk premia. They document substantial fluctuations in Fed's response to inflation, while the variations in policy stance to output gap shocks are much smaller. They do not entertain movements in macroeconomic volatilities. In our framework, we find that monetary policy coefficients to persistent growth and inflation risks are both quite volatile, and contribute to the fluctuations in the risk premia, alongside with movements in fundamental uncertainties. Bikbov and Chernov (2013) incorporate time-variation in monetary policy and stochastic volatilities of the fundamental shocks, and argue that interest rate data play an important role to identify movements in the underlying regimes. We follow their insight and use bond price data jointly with the macro and survey observations to identify the model parameters and states. Different from their paper, we take an economically-motivated, long-run risks pricing approach to the term structure, and allow for the link between monetary policy and economic uncertainty. Bekaert and Moreno (2010) and Ang et al. (2005) study the role of the monetary policy for the term structure using constant-coefficient Taylor rules and reduced-form specifications of the pricing kernel.

Our paper focuses on both the time-varying macroeconomic volatilities and regime shifts due to monetary policy as the main drivers of the bond risk premia. Hasseltoft (2012) and Bansal and Shaliastovich (2013) document the importance of the fluctuations in macroeconomic uncertainty for the time-variation in bond risk premia. In an alternative approach, Bekaert et al. (2009), Bekaert and Grenadier (2001) and Wachter (2006) use time-varying habits-formation models to study fluctuations in the bond risk premia. There is a large literature which incorporates discrete regimes changes into the term structure model specification. Bansal and Zhou (2002) implement regime shifting coefficients in the short rate of a one factor Cox-Ingersoll-Ross model. Beyond matching bond yield facts, they show that the regimes are related to business cycle movements. Dai et al. (2007) show that incorporating regime shift factors impact the dynamics of yields and increases time-variation in expected excess bond returns. Ang et al. (2008) use a regime-shift term structure model to study the implications for real rates and the inflation premium embedded in the cross-section of nominal bonds.

In terms of the earlier literature, Hamilton (1989) was the first to perform a Markov-

switching, regime shift model using purely macroeconomic data in a traditional VAR setting, finding that state parameters correspond to peaks and troughs in the business cycle. The seminal work of Sims and Zha (2006) extended the Markov-switching to a Bayesian framework with a structural VAR setup. The large conclusion of this work was that monetary policy shifts have been brief if at all existent. In fact the model that fits the best is one where there is stochastic volatility in the disturbances of fundamental variables. Related to that, Primiceri (2005) shows that while monetary policy significantly varies over time, it appears to have little effect on the real economy. Consistent with these works, we find little effect of monetary policy for the levels of real growth and inflation. However, we find a significant effect on the macroeconomic uncertainty, and the conditional dynamics of bond prices and bond risk premia.

Our paper is organized as follows. The next section discusses the economic model. In the following two sections, we provide an overview of our estimation method and discuss our results. The last section concludes.

## 1.2. Economic Model

## 1.2.1. Stochastic Discount Factor

We derive a convenient representation of the stochastic discount factor, similar to Bansal et al. (2013), which incorporates pricing conditions of the recursive-utility investor, but which does not force an inter-temporal restriction between the short rate and the fundamental macroeconomic processes. This approach allows us to specify a flexible link between short rates and macroeconomic factors, which can be identified directly in the data. At the same time, we maintain economic restrictions on the fundamental risk sources and their market prices of risks, useful for pricing long-term bonds.

As shown in Epstein and Zin (1989), under the recursive utility the log real stochastic discount factor (SDF) is given by,

$$m_{t+1}^r = \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{1.1}$$

where where  $\Delta c_{t+1}$  is the real consumption growth, and  $r_{c,t+1}$  is the return to the aggregate wealth portfolio. Parameter  $\gamma$  is a measure of a local risk aversion of the agent,  $\psi$  is the intertemporal elasticity of substitution, and  $\delta \in (0,1)$  is the subjective discount factor. For notational simplicity, parameter  $\theta$  is defined as  $\theta = \frac{1-\gamma}{1-\frac{1}{2}t}$ .

To obtain nominal SDF, we subtract inflation rate  $\pi_{t+1}$  from the real SDF:

$$m_{t+1} = m_{t+1}^r - \pi_{t+1}$$

$$= \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1} - \pi_{t+1}.$$
(1.2)

The nominal SDF is driven the consumption growth, inflation rate, and the unobserved return on the wealth portfolio.

Next we incorporate the budget constraint and the Euler equation for the short rates to characterize the expectations of the SDF,  $E_t m_{t+1}$ , and the innovation into the SDF,  $N_{m,t+1} = m_{t+1} - E_t m_{t+1}$ , in terms of the observed dynamics of the macroeconomic factors and the short rate.

The standard first-order condition implies that for any nominal return  $r_{t+1}$ , the Euler equation should hold:

$$E_t \left[ \exp(m_{t+1} + r_{t+1}) \right] = 1. \tag{1.3}$$

Using this condition for the one-period short-term nominal interest rate,  $r_{t+1} = i_t$ , we obtain that:

$$E_t m_{t+1} = -i_t - V_t. (1.4)$$

The last component  $V_t$  is the entropy of the SDF:

$$V_t = \log E_t(\exp(N_{m,t+1})). \tag{1.5}$$

Up to the third-order terms,  $V_t$  is driven by the variance of the SDF; indeed, in a conditionally Gaussian model,  $V_t$  is exactly equal to half of the conditional variance of the stochastic discount factor. This is why we refer to this component as capturing uncertainty or volatility risks.

Now let us relate the SDF innovation  $N_{m,t+1}$  to the fundamental economic shocks, consistent with the recursive utility specification in (1.2). Consider the news into the current and future expected stochastic discount factor,  $(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j m_{t+j+1}$ . Based on the recursive

utility formulation in (1.2), it is equal to,

$$(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j m_{t+j+1} = -\frac{\theta}{\psi} (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j \Delta c_{t+j+1}$$

$$+ (\theta - 1)(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j r_{c,t+j+1} - (E_{t+1} - E_t) \sum_{j=0} \kappa_1^j \pi_{t+j+1}.$$

$$(1.6)$$

Note that the return to the consumption claim  $r_{c,t+1}$  satisfies the budget constraint:

$$r_{c,t+1} = \log \frac{W_{t+1}}{W_t - C_t} \approx \kappa_0 + wc_{t+1} - \frac{1}{\kappa_1} wc_t + \triangle c_{t+1},$$
 (1.7)

where wc is the log wealth-consumption ratio and the parameter  $\kappa_1 \in (0, 1)$  corresponds to the log-linearization coefficient in the investor's budget constraint. Iterating this equation forward, we obtain that the cash-flow news, defined as the current and future expected shocks to consumption, should be equal to the current and future expected shocks to consumption return:

$$N_{CF,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j r_{c,t+j+1}.$$
 (1.8)

With that, the right-hand side of (1.6) simplifies to,

$$(E_{t+1} - E_t) \sum_{j=0} \kappa_1^j m_{t+j+1} = -\frac{\theta}{\psi} N_{CF,t+1} + (\theta - 1) N_{CF,t+1} - N_{\pi,t+1}$$

$$= -\gamma N_{CF,t+1} - N_{\pi,t+1},$$
(1.9)

where the long-run inflation news are defined as,

$$N_{\pi,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \pi_{t+j+1}.$$
 (1.10)

The news into the current and future expected stochastic discount factor, which is on the left-hand side of equation (1.9), incorporates current SDF shock  $N_{m,t+1}$  and shocks to future expectations of the SDF, which we can characterize using the representation in (1.4):

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j m_{t+j+1} = N_{m,t+1} - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j (i_{t+j} + V_{t+j})$$

$$= N_{m,t+1} - N_{i,t+1} - N_{V,t+1},$$
(1.11)

where the interest rate and volatility news are defined as follows:

$$N_{i,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j i_{t+j}, \quad N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j V_{t+j}.$$
(1.12)

Hence, we can represent the SDF shock as,

$$N_{m,t+1} = -\gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1}, \tag{1.13}$$

and the total SDF is given by:

$$m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1}. \tag{1.14}$$

That is, under the recursive utility framework, the agent effectively is concerned about the long-run real growth news  $N_{CF,t+1}$ , long-run risk free rate news (inflation-adjusted short rate news  $N_{i,t+1} - N_{\pi,t+1}$ ), and long-run uncertainty news  $N_{V,t+1}$ . The market price of the cash-flow risks is equal to the risk-aversion coefficient  $\gamma$ , while the market prices of both the interest rate and volatility shocks are negative 1: the marginal utility increases one-to-one with a rise in uncertainty or interest rates.

It is important to emphasize that the SDF representation above is common to all the recursive-utility based models. Indeed, to derive it we only used the Euler condition and the budget constraint, and did not make any assumptions about the dynamics of the underlying economy. In general equilibrium environments, the macroeconomic model assumptions are going to determine the decomposition of these underlying cash-flow, interest rate, and volatility risks into primitive economic shocks.

In our empirical approach, we rely on the SDF representation (1.14), instead of a more primitive specification in (1.2). This allows us to model short-term interest rates exogenously together with consumption and inflation processes, and yet maintain the pricing implications of the recursive utility SDF. Notably, the uncertainty term is still endogenous in our framework, as the volatility term  $V_t$  and the innovations  $N_{V,t+1}$  should be consistent with the entropy of the SDF in (1.5).

### 1.2.2. Economic Dynamics

In this section we specify the exogenous dynamics of consumption, inflation, and the short rates. This, together with the specification of the SDF and the Euler condition, allows us to solve for the prices of long-term nominal bonds.

We first specify a Markov chain to represent the time-variation in monetary policy regimes  $s_t$ . We assume N states with a transition matrix, T, given by:

$$T = \begin{pmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \pi_{N1} & \pi_{N2} & \dots & \pi_{NN} \end{pmatrix} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ T_1 & T_2 & \dots & T_N \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

where each  $\pi_{ji}$  indicates the probability of moving from state *i* to state *j*. Each column,  $T_i$ , is the vector of probabilities of moving from state *i* to all other states next period.

We next specify the exogenous dynamics of the macroeconomic state variables. Our assetpricing framework underscores the importance of long-run, persistent movements in consumption, inflation, and fundamental volatility. To capture these risks, we directly specify exogenous processes for consumption and inflation, which feature persistent fluctuations in their expectations and volatilities. In this sense, our approach is different from "New Keynesian" based models which feature alternative empirical specifications for output gap and a Phillips curve, such as those in Bikbov and Chernov (2013), Campbell et al. (2013), and Bekaert and Moreno (2010).

Specifically, similar to Bansal and Shaliastovich (2013), we specify an exogenous dynamics for consumption and inflation, which incorporates persistent movements in the conditional expectations and volatilities. The realized consumption and inflation are given by,

$$\Delta c_{t+1} = \mu_c(s_t) + x_{c,t} + \sigma_c^* \epsilon_{c,t+1}$$

$$\pi_{t+1} = \mu_{\pi}(s_t) + x_{\pi,t} + \sigma_{\pi}^* \epsilon_{\pi,t+1}.$$
(1.15)

Notably, we allow the components of the conditional means to depend on the monetary regime  $s_t$ , which allows us to identify the variation in expectations of future consumption and inflation related to monetary policy. Fundamentals are also driven by "non-policy" shocks such as movements in persistent expected components,  $x_{i,t}$ , as well as  $\epsilon_{i,t+1}$  which represent i.i.d. Gaussian short-run shocks.

The dynamics of the expected consumption and expected inflation states  $X_t = [x_{ct}, x_{\pi t}]'$  follows a VAR(1) process:

$$\begin{bmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{bmatrix} = \begin{bmatrix} \Pi_{cc} & \Pi_{c\pi} \\ \Pi_{\pi c} & \Pi_{\pi\pi} \end{bmatrix} \begin{bmatrix} x_{c,t} \\ x_{\pi,t} \end{bmatrix} + \Sigma_t \epsilon_{t+1}.$$
 (1.16)

This representation allows us to capture persistence of expected consumption and infla-

tion risks, as measured by the values of  $\Pi_{cc}$  and  $\Pi_{\pi\pi}$ . Further, this specification can also incorporate "inflation non-neutrality," that is, a negative response of future expected consumption to high expected inflation ( $\Pi_{c\pi} < 0$ ). As shown in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005), inflation non-neutrality is important to account for the bond market data. For simplicity, the persistence matrix  $\Pi$  is constant. The model can be extended to accommodate regime-dependent persistence coefficients.

In general, the volatilities of expected real growth and inflation states are given by:<sup>2</sup>

$$\Sigma_t = \begin{pmatrix} \sigma_{c,0} & 0 \\ 0 & \sigma_{\pi,t} \end{pmatrix} = \begin{pmatrix} \sqrt{\delta^c(s_t) + \tilde{\sigma}_{c,t}^2} & 0 \\ 0 & \sqrt{\delta^{\pi}(s_t) + \tilde{\sigma}_{\pi,t}^2} \end{pmatrix}$$

Each conditional variance depends on the monetary policy regime and is also driven by an orthogonal component  $\tilde{\sigma}_{i,t}$ . This component captures movements in macroeconomic volatilities which are independent from the monetary policy. Their dynamics is specified as follows:

$$\tilde{\sigma}_{ct}^2 = \tilde{\sigma}_{c,0}^2 + \varphi_c \tilde{\sigma}_{c,t-1}^2 + \omega_c \eta_{\sigma ct},$$

$$\tilde{\sigma}_{\pi t}^2 = \tilde{\sigma}_{\pi,0}^2 + \varphi_\pi \tilde{\sigma}_{\pi,t-1}^2 + \omega_\pi \eta_{\sigma \pi t}.$$
(1.17)

For simplicity, the exogenous volatilities are driven by Gaussian shocks. The specification can be extended to square root processes, as in Tauchen (2005), or positive Gamma shocks.

Notably, in our specification macroeconomic volatilities can be systematically different across the monetary policy regimes. This captures the link between the fluctuations in monetary policy and aggregate economic uncertainty. Many of the existing specifications which incorporate fluctuations in monetary policy and volatilities do not entertain comovements between the two (see e.g., Song (2014) and Bikbov and Chernov (2013)).

Finally, we specify the dynamics of the short rate. It follows a modified Taylor rule, in which the monetary authority reacts to expected growth and expected inflation, and the stance of the monetary policy can vary across the regimes. Specifically,

$$i_{t} = i_{0} + \alpha_{c}(s_{t}) (x_{ct} + \mu_{c}(s_{t})) + \alpha_{\pi}(s_{t}) (x_{\pi,t} + \mu_{\pi}(s_{t}))$$

$$= \underbrace{[i_{0} + \alpha_{c}(s_{t})\mu_{c}(s_{t}) + \alpha_{\pi}(s_{t})\mu_{\pi}(s_{t})]}_{\equiv \alpha_{0}(s_{t})} + \alpha_{c}(s_{t})x_{ct} + \alpha_{\pi}(s_{t})x_{\pi,t}.$$
(1.18)

The loadings  $\{\alpha_c(s_t), \alpha_{\pi}(s_t)\}$  are the key regime-dependent parameters of monetary policy. The interpretation of this Taylor rule is that the short rate loads stochastically on both

<sup>&</sup>lt;sup>2</sup>In empirical implementation, we focus on the time-variation in inflation volatility, and set consumption volatility to be constant.

expected growth and inflation. The justification for a "forward-looking" Taylor rule has been empirically founded and shown in Clarida, Gali and Gertler (2003). For parsimony, we abstract from other sources of variation in the interest rate rule, such as monetary policy shocks, dependence on lag rates, etc. While they can be easily introduced in our framework, we opt for a simpler specification to focus on the time-variation in the growth and inflation loadings of the Taylor rule and their relation to the macroeconomy and bond markets.

#### 1.2.3. Model Solution

In Appendix we show that the long-run cash-flow, inflation, interest rate, and volatility news can be expressed in terms of the underlying macroeconomic, interest rate, and regime-shift shocks. Specifically, the cash flow, inflation news and interest rates news are given by,

$$N_{CF,t+1} = F_{CF,0}(s_{t+1}, s_t) + F_{CF,\epsilon}(s_{t+1}, s_t)' \Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1}, \tag{1.19}$$

$$N_{\pi,t+1} = F_{\pi,0}(s_{t+1}, s_t) + F_{\pi,\epsilon}(s_{t+1}, s_t)' \Sigma_t \epsilon_{t+1} + \sigma_{\pi}^* \epsilon_{\pi,t+1}, \tag{1.20}$$

$$N_{I,t+1} = F_{I,0}(s_{t+1}, s_t) + F_{I,X}(s_{t+1}, s_t)' X_t + F_{I,\epsilon}(\dots)' \Sigma_t \epsilon_{t+1}, \tag{1.21}$$

where the functions F depend on the policy regimes and model parameters. Notably, the sensitivities of the long-term news to primitive economic shocks in general depend on current and future monetary policy regimes.

We can also show that the uncertainty term  $V_t$  is in general given by,

$$V_t(s_t, X_t, \tilde{\sigma}_{c,t}^2, \tilde{\sigma}_{\pi,t}^2) = V_0(s_t) + V_1(s_t)' X_t + V_{2c}(s_t) \tilde{\sigma}_{c,t}^2 + V_{2\pi}(s_t) \tilde{\sigma}_{\pi,t}^2, \tag{1.22}$$

so that the volatility news are given by,

$$N_{V,t+1} = F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)' X_t + F_{v,\sigma c}(s_{t+1}, s_t)' \tilde{\sigma}_{ct}^2 + F_{v,\sigma\pi}(s_{t+1}, s_t)' \tilde{\sigma}_{\pi t}^2 + F_{v,\epsilon}(\dots) \Sigma_t \epsilon_{t+1} + F_{v,nc}(\dots) \omega_c \eta_{\sigma c, t+1} + F_{v,n\pi}(\dots) \omega_{\pi} \eta_{\sigma\pi, t+1}.$$
(1.23)

The coefficients are determined as part of the model solution, and are given in the Appendix.

Combining all the components together, we can represent the SDF in terms of the underlying macroeconomic, interest rate, and regime shift shocks:

$$m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1}$$

$$= S_0 + S'_{1,X} X_t + S_{1,\sigma c} \tilde{\sigma}_{ct}^2 + S_{1,\sigma \pi} \tilde{\sigma}_{\pi t}^2 + S'_{2,\epsilon} \Sigma_t \epsilon_{t+1} + S_{2,\eta c} \omega_c \eta_{c,t+1} + S_{2,\eta \pi} \omega_{\pi} \eta_{\pi,t+1}$$

$$- \gamma \sigma_c^* \epsilon_{c,t+1} - \sigma_{\pi}^* \epsilon_{\pi,t+1}.$$
(1.24)

The SDF loadings and the market prices of risks depend on the primitive parameters of the model. In this sense, the pricing restrictions of the recursive utility provide economic discipline on the dynamics of our pricing kernel. Notably, because short rate loadings are time-varying, the SDF coefficients generally depend on monetary policy regimes. In a model with constant Taylor rule coefficients, the volatility of the SDF and the asset risk premia fluctuate only because the volatilities of expected growth and expected inflation are time-varying. In the model with time-varying monetary policy, the SDF volatility varies also due to movements in the Taylor rule coefficients.

### 1.2.4. Nominal Term Structure

In our model, log bond prices,  $p_t^n$ , and the bond yields  $y_t^n = -\frac{1}{n}p_t^n$  are (approximately) linear in the underlying expected growth, expected inflation, and volatility states, and the loadings vary across the regimes:

$$y_t^n = -\frac{1}{n}p_t^n = \mathcal{A}^n(s_t) + \mathcal{B}_X^{n'}(s_t)X_t + \mathcal{B}_{\sigma c}^n(s_t)\tilde{\sigma}_{ct}^2 + \mathcal{B}_{\sigma \pi}^n(s_t)\sigma_{\pi t}^2.$$
 (1.25)

For n=1 we uncover the underlying Taylor rule parameters:

$$\mathcal{A}^{1}(s_{t}) = \alpha_{0}(s_{t}),$$

$$\mathcal{B}^{1}_{X}(s_{t})' = \alpha(s_{t})',$$

$$\mathcal{B}^{1}_{\sigma c}(i) = 0,$$

$$\mathcal{B}^{1}_{\sigma \pi}(i) = 0.$$

We can further define one-period excess returns on n-maturity bond,

$$rx_{t\to t+1,n} = ny_{t,n} - (n-1)y_{t+1,n-1} - y_{t,1}. (1.26)$$

The expected excess return on bonds is approximately equal to,

$$E_{t}(rx_{t\to t+1,n}) + \frac{1}{2}Var_{t}(rx_{t+1,n}) \approx -Cov_{t}(m_{t+1}, rx_{t+1,n})$$

$$= Cons(s_{t}) + r_{\sigma c}(s_{t})\tilde{\sigma}_{ct}^{2} + r_{\sigma \pi}(s_{t})\tilde{\sigma}_{\pi t}^{2}.$$
(1.27)

The risk premia in our economy are time varying because there are exogenous fluctuations in stochastic volatilities, and because bond exposures fluctuate across monetary policy regimes. The second, monetary policy channel is absent in standard macroeconomic models of the term structure which entertain constant bond exposures and rely on time-variation in macroeconomic volatilities to generate movements in the risk premia (see e.g., Bansal and Shaliastovich (2013), Hasseltoft (2012)). In the next section we assess the importance of the monetary policy risks to explain the term structure dynamics, above and beyond traditional economic channels.

### 1.3. Model Estimation

## 1.3.1. Data Description

We use macroeconomic data on consumption and inflation, survey data on expected real growth and expected inflation, and asset-price data on bond yields to estimate the model. For our consumption measure we use log real growth rates of expenditures on non-durable goods and services from the Bureau of Economic Analysis (BEA). The inflation measure corresponds to the log growth in the GDP deflator. The empirical measures of the expectations are constructed from the cross-section of individual forecasts from the Survey of Professional Forecasts at the Philadelphia Fed. Specifically, the expected real growth corresponds to the cross-sectional average, after removing outliers, of four-quarters-ahead individual expectations of real GDP. Similarly, the expected inflation is given by the average of four-quarters-ahead expectations of inflation. The real growth and inflation expectation measures are adjusted to be mean zero, and are rescaled to predict next-quarter consumption and inflation, respectively, with a loading of one. The construction of these measures follows Bansal and Shaliastovich (2009). Finally, we use nominal zero-coupon bond yields of maturities one through five years, taken from the CRSP Fama-Bliss data files. We also utilize the nominal three-month rate from the Federal Reserve to proxy for the short rate. Based on the length of the survey data, our sample is quarterly, from 1969 through 2014.

Table 1 shows the summary statistics for our variables. In our sample, the average short rate is 5.2%. The term structure is upward sloping, so that the five-year rate reaches 6.4%. Bond volatilities decrease with maturity from 3.3% at short horizons to about 3% at five years. The yields are very persistent. As shown in the bottom panel of the Table, real growth and inflation expectations are very persistent as well. The AR(1) coefficients for the real growth and inflation forecasts are 0.87 and 0.98, respectively, and are much larger than the those for the realized consumption growth and inflation. Figure 1 shows the time series of the realized and expected consumption growth and inflation rate. As shown on the Figure, the expected states from the surveys capture low frequency movements in the realized macroeconomic variables.

### 1.3.2. Estimation Method

In our empirical analysis of the model, we focus on the stochastic volatility channel of the expected inflation, and set the volatility of the expected real growth to be constant.<sup>3</sup> To identify the volatility level parameters, we set the monetary policy component of the inflation volatility in state one to be zero; to identify the regimes, we impose that the short rate sensitivity to expected inflation is highest in regime 2. Finally, we set the log-linearization parameter  $\kappa_1$  to a typical value of .99 in the literature.

To estimate the model and write down the likelihood of the data, we represent the evolution of the observable macroeconomic, survey, and bond yield variables in a convenient state-space form:

(Measurement) 
$$y_{t+1}^{1:Ny} = \mathcal{A}^{1:Ny}(s_{t+1}) + \mathcal{B}_{X}^{1:Ny}(s_{t+1})X_{t+1} + \mathcal{B}_{\sigma\pi}^{1:Ny}(s_{t+1})\tilde{\sigma}_{\pi,t+1}^{2} + \Sigma_{u,y}u_{t+1,y},$$

$$\triangle c_{t+1} = \mu_{c}(s_{t}) + e'_{1}X_{t} + \sigma^{*}_{c}\epsilon_{c,t+1},$$

$$\pi_{t+1} = \mu_{\pi}(s_{t}) + e'_{2}X_{t} + \sigma^{*}_{\pi}\epsilon_{\pi,t+1},$$

$$x_{SPFcons,t+1} = \mu_{c}(s_{t+1}) - E\left[\mu_{c}(s_{t+1})\right] + x_{c,t+1} + \sigma_{u,xc}u_{t+1,xc},$$

$$x_{SPFinfl,t+1} = \mu_{\pi}(s_{t+1}) - E\left[\mu_{\pi}(s_{t+1})\right] + x_{\pi,t+1} + \sigma_{u,x\pi}u_{t+1,x\pi},$$
(Transition) 
$$X_{t+1} = \Pi X_{t} + \Sigma_{t}(\tilde{\sigma}_{\pi t}, s_{t})\epsilon_{t+1},$$

$$\tilde{\sigma}_{\pi t}^{2} = \tilde{\sigma}_{\pi,0}^{2} + \varphi_{\pi}\tilde{\sigma}_{\pi,t-1}^{2} + \omega_{\pi}\eta_{\sigma\pi,t},$$

$$s_{t} \sim \text{Markov Chain }(\mathbb{P}_{s}),$$

where Ny is the number of bond yields in the data. In the estimation we allow for Gaussian measurement errors on the observed yields and survey expectations, captured by  $u_{t+1,y}$  and  $u_{t+1,xc:x\pi}$ . For parsimony and to stabilize the chains, we fix the volatilities of the measurement errors to be equal to 20% of the unconditional volatilities of the factors. As we describe in the subsequent section, the ex-post measurement errors in the sample are much smaller than that.

The set of parameters, to be jointly estimated with the states, is denoted by  $\Theta$ , is given by:

$$\Theta = \{\Pi, \delta^{\alpha\pi}, \tilde{\sigma}_{c0}^2, \tilde{\sigma}_{\pi0}^2, \varphi_{\pi}, \omega_{\pi}, \sigma_{c}^*, \sigma_{\pi}^*, i_0, \gamma, \mu_c^{1:N}, \mu_{\pi}^{1:N}, \alpha_c^{1:N}, \alpha_{\pi}^{1:N}, \mathbb{P}_s\}.$$

The estimation problem is quite challenging due to the fact that the observation equations

<sup>&</sup>lt;sup>3</sup>Identification of real volatility is challenging in bond market data alone. In a related framework, Song (2014) incorporate equity market data, which are informative about movements in real uncertainty, to help estimate real volatility.

are nonlinear in the state variables, and the underlying expectation, volatility, and regime state variables are latent. Because of these considerations, we cannot use the typical Carter and Kohn (1994) methodology which utilizes smoothed Kalman filter moments to draw states. Instead, to estimate parameters and latent state variables we rely on a Bayesian MCMC procedure using particle filter methodology to evaluate the likelihood function. As in Andrieu et al. (2010) and Fernandez-Villaverde and Rubio-Ramirez (2007), we embed the particle filter based likelihood into a Random Walk Metropolis Hasting algorithm and sample parameters in this way. Schorfheide et al. (2013) and Song (2014) entertain similar approaches to estimate versions of the long-run risks model.

#### 1.4. Estimation Results

### 1.4.1. Parameter and State Estimates

Table 2 shows the moments of the prior and posterior distributions of the parameters. We chose fairly loose priors which cover a wide range of economically plausible parameters to maximize learning from the data. For example, a two-standard deviation band for the persistence of the expected inflation and expected consumption ranges from 0.5 to 1.0. The prior means for the scale parameters are set to typical values in the literature, and the prior standard deviations are quite large as well. Importantly, we are careful not to hardwire the fluctuations in monetary policy and their impact on macroeconomy and bond prices through the prior selection. That is, in our prior we assume that the monetary policy coefficients are the same across the regimes, and are equal to 1 for expected inflation and 0.5 for expected growth. Likewise, our prior distribution for the role of monetary policy on inflation volatility is symmetric and is centered at zero, and there is no difference in the conditional means of consumption and inflation across the regimes. Hence, we do not force any impact through the prior, and let the data determine the size and the direction of the monetary policy effects.

The table further shows the posterior parameter estimates in the data. The posterior median for the risk aversion coefficient is 13.6, which is smaller than the values entertained in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005). The expected consumption, expected inflation, and inflation volatility are very persistent: the median AR(1) coefficients are above 0.95. The expected inflation has a negative and non-neutral effect on future real growth:  $\Pi_{c\pi}$  is negative, consistent with findings in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005).

We further find that there are substantial fluctuations in the monetary policy in the data. The monetary policy regimes are quite persistent, with the probability of remaining in a passive regime of 0.955, and in the aggressive regime of 0.958. There is a significant difference in monetary policy across the regimes. Indeed, the median short rate loadings are equal to .75 and 1.68 on the expected growth and expected inflation, respectively, in aggressive regimes, which are larger than .54 and .94 in passive regimes. These differences are very significant statistically. Overall, our estimates for these regime coefficients corroborate the prior evidence for Taylor rule coefficients on inflation being above one; see e.g. Cochrane (2011), Gallmeyer et al. (2006), and Backus, Chernov and Zin (2013).

In terms of the impact of monetary policy on the macroeconomy, we find that the expected consumption and expected inflation are somewhat lower in aggressive regimes. This is consistent with the evidence in Bikbov and Chernov (2013) who show that future output and inflation tend to decrease following a monetary policy shock. However, in our estimation the difference in expectations is not statistically significant across the regimes, mirroring the findings in Primiceri (2005) that monetary policy appears to have little effect on the levels of economic dynamics. On the other hand, we find that inflation volatility is quite different across the regimes. The value of  $\delta^{\pi}$  is positive and significant statistically and economically: total inflation volatility rises by about a quarter in aggressive regimes.

Our filtered series for the latent expected growth, expected inflation, inflation volatility, and monetary policy regimes are provided in Figures 2-4. The estimated expectations are quite close to the data counterparts, and are generally in the 90% confidence set. Some of the noticeable deviations between the model and the data include post-2007 period, when model inflation expectations are systematically below the data. Notably, this is a period of a zero lower bound and unconventional monetary policy, which are outside a simple Taylor rule specification considered in this model.<sup>4</sup>

The exogenous component of inflation volatility is plotted in Figure 3. It is apparent that non-policy related volatility spikes up in the early to mid 1980's and gradually decreases over time. The inflation volatility is quite low in the recent period, which reflects low variability in inflation expectations in the data.

Finally, we provide model-implied estimates of the monetary regime in Figure 4. The figure suggests that a shift to an aggressive regime occurred in the late-70's / early-80's period, in accordance with the Volcker period. In mid 90's, there was a shift to a passive regime, consistent with the anecdotal evidence regarding the Greenspan loosening. These findings are consistent with the empirical evidence for the monetary policy regimes in Bikbov and Chernov (2013). In the crisis period our estimates suggest a passive regime, consistent with

<sup>&</sup>lt;sup>4</sup>Branger et al. (2015) discuss the impact of a zero lower bound on the inference of economic states and model-implied yields in a related framework.

the observed evidence of lower levels and volatilities of the bond yields and risk premia in this period.

## 1.4.2. Model Implications for Bond Prices

Figure 5 shows the time series of model-implied yields. The model matches the yields quite well in the sample: the average pricing errors range from 0.08% for 1-year yields to about 0.03% for 3-year yields, and a good fit is apparent from the Figure. As shown in Figure 6, the model generates an unconditional upward sloping term structure and a downward sloping volatility term structure. These patterns are consistent with the data.

We next consider the conditional dynamics of bond prices implied by the model. In Figure 7 we report standardized bond loadings on the expected growth, expected inflation and inflation volatility. The Figure shows that bond yields increase at times of high expected real growth. This captures a standard inter-temporal trade-off effect: at times of high expected real growth agents do not want to save, so bond prices fall and yields increase. Because we are looking at the nominal bonds which pay nominal dollars, their prices fall at times of higher anticipated inflation, so bond yields also increase with expected inflation. Finally, while short rates do not respond to inflation volatility, long term yields increase at times of high volatility of inflation. This reflects a positive risk premium component which is embedded in long term yields, and which increases at time of high inflation volatility.

Interestingly, all the bond loadings are uniformly larger in aggressive relative to passive regimes. Hence, a higher sensitivity of short-term bonds to expected consumption and expected inflation risks in aggressive regime, embedded in the Taylor rule, persists across all the bond maturities. As bonds are riskier in aggressive regimes, the average levels and volatilities of bond yields are higher in aggressive relative to passive regimes, as shown in Figure 6.

### 1.4.3. Model Implications for Bond Premia

In the benchmark model, the market price of the expected growth risk is positive, while the market prices of risks are negative for expected inflation and volatility risks. Indeed, high marginal utility states are those associated with low expected real growth, high expected inflation, or high inflation volatility. As bond yield loadings are all positive to these risks, it implies that the bond exposure to expected real growth contributes negatively to bond risk premia, while bond exposures to inflation risks contribute positively to the bond risk premia. Table 3 shows the average bond risk premium in the model, and its decomposition into the underlying economic sources of risk. Quantitatively, the expected inflation risk premium is quite large, so the average bond risk premia are positive.

One of the key model parameters which determines the magnitude of the inflation premium, and thus the level of the risk premia and slope of the nominal term structure, is the inflation non-neutrality coefficient  $\Pi_{c\pi}$ . When this parameter is negative, as in the benchmark model, high expected inflation is bad news for future real growth. The inflation non-neutrality implies that investors are significantly concerned about expected inflation news. Long-term bonds which are quite sensitive to expected inflation are thus quite risky, and require a positive inflation premium. In the middle panel of Table 3 we show the risk premia implications when the inflation non-neutrality parameter is set to zero. In this case, expected inflation risk premium is virtually zero, the bond risk premia are negative, and the entire term structure is downward sloping.

We plot the in-sample risk premia in Figure 8. Consistent with the above discussion, the bond risk premia are positive on average, and the term-structure of bond risk premia is upward-sloping, so that long-term bonds are riskier and have higher expected excess returns than short-term bonds. The risk premia fluctuate over the sample, and can even go negative, as in in the post 2000 sample when the volatility of expected inflation is quite below its average, and the economy is in a passive regime.

We next quantify the contribution of the monetary policy risks to the levels of the bond risk premia. Specifically, we set all the regime-dependent coefficients to be equal to their unconditional means, based on the median set of parameters. This includes the regime-shifting Taylor rule coefficients, the policy component of expected inflation volatility, as well as drift components in the fundamental consumption and inflation processes. As displayed in the last panel, we find that risk premia decrease, with larger absolute differences at the long end of the curve. However, the impact of the time-variation in monetary policy to the levels of the risk premia is quite modest, about 10-15 basis points.

On the other hand, we find that monetary policy fluctuations contribute significantly to the time-variation in bond risk compensation. Similar to the regime dependent structures in Bansal and Zhou (2002) and Dai et al. (2007), the time-variation in monetary policy coefficients creates nonlinearities in yields via regime dependent bond loadings that affect the fluctuations in the risk premia. We can think of this as a time-varying risk exposure channel, which is different from a time-varying quantity of risk generated through the conditional volatility present in the inflation expectations. Both of these channels help generate risk premia variability.

To examine the quantitative impact of monetary policy on risk premia fluctuations, in Table 4 we present the volatilities of risk premia under different model specifications, in sample and population. First, we consider a case where all the regime-shifting parameters are set to

the constant unconditional averages, and only the non-policy portion of inflation volatility is present. Next we consider the case where we add the time-varying short rate sensitivity related to inflation,  $\alpha_{\pi}$ . Subsequently, in the third line, we allow for time-variation in the policy portion of inflation volatility,  $\delta^{\pi}$ . The fourth line represents the case with variation in the growth-related Taylor rule coefficient, while the baseline configuration additionally allows for regime-dependent movements in expected growth and inflation. We find that only allowing for exogenous inflation volatility generates about 80% of the risk premia variation in population, and about 70% to 75% of the variation in sample. Incorporating movements in the inflation-related Taylor rule coefficient increases the variance of the risk premia by about 15\% of the total risk premia variance. Interestingly, movements in the policy portion of inflation volatility then contribute an additional 15%. Allowing for the movements in the short rate sensitivity to growth brings down the risk premia variability back to value in the benchmark model, and incorporating movements in expected consumption and inflation does not materially alter the volatility of the risk premia. In total this suggests that the effects of monetary policy on risk premia variability is substantial – about 20% of baseline in total.

The impact of monetary policy on bond risk premia is quite substantial in the sample as well, as shown in Figure 9. Here the solid line is the case with only exogenous inflation volatility, while the dashed and circled lines represent cases with movements in inflation-related policy variables and the baseline parameters. The central takeaway is that adding monetary policy fluctuations increases the volatility of the risk premia. Relative to the model with constant monetary policy, the benchmark bond risk premia rise in the 1980s, and fall below zero, at a greater degree in the recent period.

In our model, the risk premia volatility is higher when the short rate sensitivity to growth is constant. To help interpret this result, consider the bond risk premia decomposition:

$$rp_t^n = Cons(s_t) + r_{\sigma c}(s_t)\tilde{\sigma}_{c0}^2 + r_{\sigma \pi}(s_t)\tilde{\sigma}_{\pi t}^2.$$

The first component captures the risk premia due to the inflation volatility and regime shifts risks. The second component captures the risk premia due to the expected consumption risks. This is constant within the regime because the amount of expected consumption risks, the real growth volatility, is assumed to be constant. Finally, the last component contains the compensation for the expected inflation risks. It is driven by the monetary policy fluctuations, and the movements in exogenous inflation volatility. As we showed in Table 3, the risk premium portion coming from the expected real growth risks is negative while that from expected inflation risk is positive:  $r_{\sigma c} < 0, r_{\sigma \pi} > 0$ . Further, both coefficients become larger, in absolute value, in aggressive regimes, as bond riskiness increases. Hence,

in the benchmark model movements in expected growth risk premia offset the movements in expected inflation risk premia. When short-rate sensitivity to real growth is constant, the expected real growth component of the bond premia is also constant, and thus the volatility of the risk premia increases.

The above decomposition also shows that bond risk premia can turn negative at times when expected inflation compensation is relatively low. This is more likely to happen in passive regimes, at times of low inflation volatility, and for long-maturity bonds, as shown in Figure 10. This is why the in-sample bond risk premia turn negative post 2000 when inflation volatility is low, and the economy is in the passive regime.

## 1.5. Conclusion

We estimate a novel, structurally motivated asset-pricing framework to assess the role of monetary policy fluctuations for the macroeconomy and bond markets. We find substantial fluctuations in the monetary policy in the data. Interestingly, monetary policy fluctuations do not seem to have a sizeable effect for the first moments of the macroeconomic variables: the expectations of real growth and inflation are not significantly different across the regimes. On the other hand, inflation uncertainty significantly increases, and interest rates respond stronger to economic risks in aggressive relative to passive regimes. The monetary policy fluctuations help increase persistent variations in the bond risk premia, and the policy fluctuations in regard to real growth and inflation concerns have offsetting effects on the level and volatility of the bond premia.

Our empirical findings for the impact of monetary policy on the expectations and volatilities of the macroeconomic variables have important implications for the conduct of monetary policy and the understanding of the transmission of the monetary policy risks. We leave the study of the economic mechanisms which can explain our evidence for future research. Further, in our paper we focus on a "conventional" monetary policy represented by changing coefficients in the Taylor rule. Other policy issues, such as a zero lower bound and unconventional policy channels, can play an important role to explain the recent evidence, and are also left for future research.

## 1.6. Appendix: Analytical Model Solution

In this section we present the details of the model solution.

## 1.6.1. Long-Run Cash Flow and Inflation News

Recall that the cash flow news is solved through

$$N_{CF,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1}$$

$$= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j \mu_c(s_{t+j}) + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j e_1' X_{t+j} + \sigma_c^* \epsilon_{c,t+1}.$$
(1.28)

We start by calculating the first portion of the news related to  $\mu_c(...)$ . Note that,

$$(E_{t+1} - E_t)\mu_c(s_{t+1}) = \mu_c(k) - \sum_{j=1}^{N} \pi_{ji}\mu_c(j) = \mu_c(k) - T_i'\mu_c = (e_k' - T_i')\mu_c,$$

$$(E_{t+1} - E_t)\mu_c(s_{t+2}) = T_k'\mu_c - \sum_{\tilde{j}} \pi_{\tilde{j}\tilde{j}}\sum_j \pi_{j\tilde{j}}\mu_c(j) = T_k'\mu_c - T_i'T'\mu_c,$$

$$\dots$$
(1.29)

 $(E_{t+1} - E_t)\mu_c(s_{t+j}) = \left[T'_k(T')^{j-2} - T'_i(T')^{j-1}\right]\mu_c \text{ for } j > 1.$ 

Summing over j, we obtain that,

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \mu_c(s_{t+j}) = \left\{ \kappa_1 \left( e_k' - T_i' \right) + \kappa_1^2 \left( T_k' - T_i' T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} \mu_c.$$
 (1.30)

On the other hand, the revisions in the expectations of the continuous factors are given by,

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j e_1' X_{t+j} = \kappa_1 e_1' (I - \kappa_1 \Pi)^{-1} \Sigma_t \epsilon_{t+1}.$$
 (1.31)

Hence, the long-run cash flow news is given by,

$$N_{CF,t+1} = \left\{ \kappa_1 \left( e'_k - T'_i \right) + \kappa_1^2 \left( T'_k - T'_i T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} \mu_c$$

$$+ \kappa_1 e'_1 \left( I - \kappa_1 \Pi \right)^{-1} \Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1}$$

$$= F_{CF,0}(s_{t+1}, s_t) + F_{CF,\epsilon}(\dots)' \Sigma_t \epsilon_{t+1} + \sigma_c^* \epsilon_{c,t+1}.$$
(1.32)

In an analogous way we can show that the long-run inflation news is:

$$N_{\pi,t+1} = \left\{ \kappa_1 \left( e'_k - T'_i \right) + \kappa_1^2 \left( T'_k - T'_i T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} \mu_{\pi}$$

$$+ \kappa_1 e'_2 \left( I - \kappa_1 \Pi \right)^{-1} \Sigma_t \epsilon_{t+1} + \sigma_{\pi}^* \epsilon_{\pi,t+1}$$

$$= F_{\pi,0}(s_{t+1}, s_t) + F_{\pi,\epsilon}(\dots)' \Sigma_t \epsilon_{t+1} + \sigma_{\pi}^* \epsilon_{\pi,t+1}.$$
(1.33)

## 1.6.2. Long-Run Nominal Interest Rate News

The interest rate news is specified by:

$$N_{I,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \left[ \alpha_0(s_{t+j}) + \alpha(s_{t+j})' x_{t+j} \right]. \tag{1.34}$$

Similar to the cash flow news, the first part is equal to,

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \alpha_0(s_{t+j}) = \left\{ \kappa_1 \left( e_k' - T_i' \right) + \kappa_1^2 \left( T_k' - T_i' T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} \alpha_0.$$
 (1.35)

To compute the second component, note that:

$$(E_{t+1} - E_t)\alpha(s_{t+1})'x_{t+1} = \alpha'_k x_{t+1} - \sum_{j=1}^N \pi_{ji}\alpha(j)'(\Pi X_t)$$
  
=  $\alpha'_k (\Pi X_t + \Sigma_t \epsilon_{t+1}) - T'_i \alpha(\Pi X_t) = (\alpha'_k - T'_i \alpha)\Pi X_t + \alpha'_k \Sigma_t \epsilon_{t+1},$   
(1.36)

where  $\alpha$  indicates the stacked matrix of  $\alpha(j)'$  of all states. For the t+2 state,

$$(E_{t+1} - E_t)\alpha(s_{t+2})'x_{t+2} = T_k'\alpha\Pi X_{t+1} - \sum_j \pi_{ji} \sum_{\tilde{j}} \pi_{\tilde{j}j}\alpha(\tilde{j})' (\Pi^2 X_t)$$

$$= T_k'\alpha\Pi X_{t+1} - T_i'T'\alpha\Pi^2 X_t$$

$$= (T_k'\alpha\Pi - T_i'T'\alpha\Pi) \Pi X_t + T_k'\alpha\Pi \Sigma_t \epsilon_{t+1}.$$

$$(1.37)$$

More generally for  $j \geq 2$ ,

$$(E_{t+1} - E_t)\alpha(s_{t+j})'x_{t+j} = \left[T_k' - T_i'T'\right] \left[ (T')^{j-1} \alpha \Pi^j \right] X_t + T_k'(T')^{j-2} \alpha \Pi^{j-1} \Sigma_t \epsilon_t (1.38)$$

Denote the infinite sums,  $\{S_{0,I}^1, S_{0,I}^2\}$  which can be solved through the Ricatti equations,

$$S_{0,I}^{1} = \sum_{j=2}^{\infty} (T')^{j-1} \alpha \Pi^{j} \kappa_{1}^{j} = \kappa_{1}^{2} T' \alpha \Pi^{2} + \kappa_{1} T' S_{0,I}^{1} \Pi, \qquad (1.39)$$

$$S_{0,I}^2 = (T')^{-1} S_{0,I}^1(\Pi)^{-1}. (1.40)$$

After summing across j we obtain that,

$$N_{I,t+1} = \left\{ \kappa_1 \left( e_k' - T_i' \right) + \kappa_1^2 \left( T_k' - T_i' T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} \alpha_0$$

$$+ \left\{ \kappa_1 \left[ \alpha_k' \Pi - T_i' \alpha \Pi \right] + \left[ T_k' - T_i' T' \right] S_{0,I}^1 \right\} X_t + \left\{ \kappa_1 \alpha_k' + T_k' S_{0,I}^2 \right\} \Sigma_t \epsilon_{t+1}$$

$$= F_{I,0}(s_{t+1}, s_t) + F_{I,X}(s_{t+1}, s_t)' X_t + F_{I,\epsilon}(\dots)' \Sigma_t \epsilon_{t+1}.$$
(1.41)

## 1.6.3. Long-Run Volatility News

We guess and verify that the uncertainty term  $V_t$  satisfies,

$$V_t(s_t, X_t, \tilde{\sigma}_{c,t}^2, \tilde{\sigma}_{\pi,t}^2) = V_0(s_t) + V_1(s_t)' X_t + V_{2c}(s_t) \tilde{\sigma}_{c,t}^2 + V_{2\pi}(s_t) \tilde{\sigma}_{\pi,t}^2, \tag{1.42}$$

where  $\{V_0, V_1, V_{2c}, V_{2\pi}\}$  are the volatility loadings that are determined endogenously.

The long-run volatility news is given by,

$$N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j V_{t+j}. \tag{1.43}$$

Similar to cash-flow and interest rate news, we can solve for the components of the volatility news as follows. The first term is given by,

$$(E_{t+1} - E_t) \sum_{i=1}^{\infty} \kappa_1^j V_0(s_{t+j}) = \left\{ \kappa_1 \left( e_k' - T_i' \right) + \kappa_1^2 \left( T_k' - T_i' T' \right) \left( I - \kappa_1 T' \right)^{-1} \right\} V_0(1.44)$$

where  $V_0$  denotes the stacked matrix of  $V_0(j)'$ . The second portion of the volatility news is given by:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_1(s_{t+j})' X_{t+j} = \left\{ \kappa_1 \left[ V_{1,k}' \Pi - T_i' V_1 \Pi \right] + \left[ T_k' - T_i' T' \right] S_{0,V}^1 \right\} X_t + \left\{ \kappa_1 V_{1,k}' + T_k' S_{0,V}^2 \right\} \Sigma_t \epsilon_{t+1},$$
(1.45)

where the volatility loadings  $\{S^1_{0,V}, S^2_{0,V}\}$  are given by,

$$S_{0,V}^{1} = \kappa_{1}^{2} T' V_{1} \Pi^{2} + \kappa_{1} T' S_{0,V}^{1} \Pi, \qquad (1.46)$$

$$S_{0,V}^2 = (T')^{-1} S_{0,V}^1(\Pi)^{-1}. (1.47)$$

The final two portions of the volatility news can be derived in an analogous way. They are

equal to,

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2c}(s_{t+j}) \tilde{\sigma}_{c,t+j}^2 = \left\{ \kappa_1 \left[ V_{2c}(k) \varphi_c - T_i' V_{2c} \varphi_c \right] + \left[ T_k' - T_i' T' \right] S_{0,v2c}^1 \right\} \tilde{\sigma}_{c,t}^2$$

$$+ \left\{ \kappa_1 V_{2c}(k) + T_k' S_{0,v2c}^2 \right\} \omega_c \eta_{\sigma c,t+1},$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2\pi}(s_{t+j}) \tilde{\sigma}_{\pi,t+j}^2 = \left\{ \kappa_1 \left[ V_{2\pi}(k) \varphi_\pi - T_i' V_{2\pi} \varphi_\pi \right] + \left[ T_k' - T_i' T' \right] S_{0,v2\pi}^1 \right\} \tilde{\sigma}_{\pi,t}^2$$

$$+ \left\{ \kappa_1 V_{2\pi}(k) + T_k' S_{0,v2\pi}^2 \right\} \omega_\pi \eta_{\sigma\pi,t+1}.$$

$$(1.48)$$

For  $i = c, \pi$  the vector terms satisfy:

$$S_{0,v2i}^{1} = \kappa_{1}^{2} T' V_{1} \varphi_{i}^{2} + \kappa_{1} T' S_{0,v2i}^{1} \varphi_{i}, \tag{1.49}$$

$$S_{0,v2i}^2 = (T')^{-1} S_{0,v2i}^1(\varphi_i)^{-1}. (1.50)$$

Hence, the volatility news is given by,

$$N_{V,t+1} = F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)' X_t + F_{v,\sigma c}(s_{t+1}, s_t)' \tilde{\sigma}_{ct}^2 + F_{v,\sigma\pi}(s_{t+1}, s_t)' \tilde{\sigma}_{\pi t}^2$$
(1.51)  
+ 
$$F_{v,\epsilon}(\ldots) \Sigma_t \epsilon_{t+1} + F_{v,\eta c}(\ldots) \omega_c \eta_{\sigma c, t+1} + F_{v,\eta\pi}(\ldots) \omega_{\pi} \eta_{\sigma \pi, t+1},$$
(1.52)

where the loadings satisfy:

$$F_{v0} = \left\{ \kappa_{1} \left( e'_{k} - T'_{i} \right) + \kappa_{1}^{2} \left( T'_{k} - T'_{i} T' \right) \left( I - \kappa_{1} T' \right)^{-1} \right\} V_{0},$$

$$F'_{v,X} = \left\{ \kappa_{1} \left[ V'_{1,k} \Pi - T'_{i} V_{1} \Pi \right] + \left[ T'_{k} - T'_{i} T' \right] S_{0,V}^{1} \right\},$$

$$F_{v,\sigma c} = \left\{ \kappa_{1} \left[ V_{2c}(k) \varphi_{c} - T'_{i} V_{2c} \varphi_{c} \right] + \left[ T'_{k} - T'_{i} T' \right] S_{0,v2c}^{1} \right\}$$

$$F_{v,\sigma\pi} = \left\{ \kappa_{1} \left[ V_{2\pi}(k) \varphi_{\pi} - T'_{i} V_{2\pi} \varphi_{\pi} \right] + \left[ T'_{k} - T'_{i} T' \right] S_{0,v2\pi}^{1} \right\},$$

$$F'_{v,\epsilon} = \left\{ \kappa_{1} V'_{1k} + T'_{k} S_{0v}^{2} \right\},$$

$$F_{v,\eta c} = \left\{ \kappa_{1} V_{2c}(k) + T'_{k} S_{0,v2c}^{2} \right\},$$

$$F_{v,\eta \pi} = \left\{ \kappa_{1} V_{2\pi}(k) + T'_{k} S_{0,v2\pi}^{2} \right\}.$$

$$(1.53)$$

Now we solve for coefficients on the volatility factor. From the definition of the uncertainty component  $V_t$  it follow that,

$$V_t = \log E_t \exp\left(N_{m,t+1}\right). \tag{1.54}$$

The SDF shock be related to the primitive macroeconomic and regimes shocks,

$$N_{m,t+1} = -\gamma N_{CF,t+1} + N_{I,t+1} - N_{\pi,t+1} + N_{V,t+1}$$

$$= M_0 + M'_{1,X} X_t + M_{1,\sigma c} \tilde{\sigma}_{ct}^2 + M_{1,\sigma \pi} \tilde{\sigma}_{\pi t}^2 + M'_{2,\epsilon} \Sigma_t \epsilon_{t+1} + M_{2,\eta c} \omega_c \eta_{c,t+1} + M_{2,\eta \pi} \omega_{\pi} \eta_{\pi,t+1}$$

$$- \gamma \sigma_c^* \epsilon_{c,t+1} - \sigma_{\pi}^* \epsilon_{\pi,t+1}, \qquad (1.55)$$

where each loading in general depends on current and future monetary policy regime:

$$M_{0}(s_{t}, s_{t+1}) = -\gamma F_{CF,0} + F_{I,0} - F_{\pi,0} + F_{v,0},$$

$$M_{1,X}(s_{t}, s_{t+1})' = F'_{I,X} + F'_{v,X},$$

$$M_{1,\sigma c}(s_{t}, s_{t+1}) = F_{v,\sigma c},$$

$$M_{1,\sigma\pi}(s_{t}, s_{t+1}) = F_{v,\sigma\pi},$$

$$M_{2,\epsilon}(s_{t}, s_{t+1})' = -\gamma F'_{CF,\epsilon} + F'_{I,\epsilon} - F'_{\pi,\epsilon} + F'_{v,\epsilon},$$

$$M_{2,\eta c}(s_{t}, s_{t+1}) = F_{v,\eta c},$$

$$M_{2,\eta\pi}(s_{t}, s_{t+1}) = F_{v,\eta\pi}.$$

$$(1.56)$$

Conditioning on next-period regime, one can show that,

$$V_{t} = \log E_{t} \exp (N_{m,t+1})$$

$$= \log E_{t} \exp \left(\tilde{M}_{0} + \tilde{M}'_{1,X} X_{t} + \tilde{M}_{1,\sigma c} \tilde{\sigma}_{ct}^{2} + \tilde{M}_{1,\sigma \pi} \tilde{\sigma}_{\pi t}^{2}\right),$$
(1.57)

where

$$\tilde{M}_{0} = M_{0} + \frac{1}{2} \left[ \left( M_{2,\epsilon}^{1} \right)^{2} \delta^{\alpha c} \alpha_{c}(s_{t}) + \left( M_{2,\epsilon}^{2} \right)^{2} \delta^{\alpha \pi} \alpha_{\pi}(s_{t}) + M_{2,\eta c}^{2} \omega_{c}^{2} + M_{2,\eta \pi}^{2} \omega_{\pi}^{2} \right] + \frac{1}{2} \gamma^{2} \left( \sigma_{c}^{*} \right)^{2} + \frac{1}{2} \left( \sigma_{\pi}^{*} \right)^{2}, 
\tilde{M}_{1}' = M_{1}', 
\tilde{M}_{1,\sigma c} = M_{1,\sigma c} + \frac{1}{2} \delta^{\sigma c} \left( M_{2,\epsilon}^{1} \right)^{2}, 
\tilde{M}_{1,\sigma \pi} = M_{1,\sigma \pi} + \frac{1}{2} \delta^{\sigma \pi} \left( M_{2,\epsilon}^{2} \right)^{2}.$$
(1.58)

To integrate out next-period regimes, similar to Bansal and Zhou (2002) and Song (2014), we use the approximation,  $\exp(y) \approx 1 + y$ , which holds for small enough y. It follows that,

$$V_{t} = T'_{i}\tilde{M}_{0}(i) + \left(T'_{i}\tilde{M}_{1,X}(i)\right)X_{t} + T'_{i}\tilde{M}_{1,\sigma c}(i)\tilde{\sigma}_{ct}^{2} + T'_{i}\tilde{M}_{1,\sigma\pi}(i)\tilde{\sigma}_{\pi t}^{2},$$
(1.59)

where  $\tilde{M}_0(i)$  is the stacked vector of  $\tilde{M}_0(i,:)$ , and  $\tilde{M}_{1,X}(i)$  is the stacked matrix of  $\tilde{M}_{1,X}(i,k)'$ .

To guarantee an internally-consistent solution to the model, we equate the volatility specification in (1.42) to the equation above. This implies that:

$$V_{0}(i) = T'_{i}\tilde{M}_{0}(i),$$

$$V_{1X}(i) = T'_{i}\tilde{M}_{1,X}(i),$$

$$V_{2c}(i) = T'_{i}\tilde{M}_{1,\sigma c}(i),$$

$$V_{2\pi}(i) = T'_{i}\tilde{M}_{1,\sigma\pi}(i).$$
(1.60)

for all i. This system is exactly identified, and allows us to endogenously relate volatility news to the primitive shocks and model parameters.

#### 1.6.4. Nominal Bond Prices

The solution to the stochastic discount factor satisfies,

$$m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1}$$

$$= S_0 + S'_{1,X} X_t + S_{1,\sigma c} \tilde{\sigma}_{ct}^2 + S_{1,\sigma \pi} \tilde{\sigma}_{\pi t}^2 + S'_{2,\epsilon} \Sigma_t \epsilon_{t+1} + S_{2,\eta c} \omega_c \eta_{c,t+1} + S_{2,\eta \pi} \omega_{\pi} \eta_{\pi,t+1}$$

$$- \gamma \sigma_c^* \epsilon_{c,t+1} - \sigma_{\pi}^* \epsilon_{\pi,t+1}, \qquad (1.61)$$

where each of the coefficients are given by,

$$S_{0}(s_{t}, s_{t+1}) = M_{0} - \alpha_{0} - V_{0},$$

$$S_{1,X}(s_{t}, s_{t+1})' = M'_{1,X} - \alpha' - V'_{1},$$

$$S_{1,\sigma c}(s_{t}, s_{t+1}) = M_{1,\sigma c} - V_{2c},$$

$$S_{1,\sigma \pi}(s_{t}, s_{t+1}) = M_{1,\sigma \pi} - V_{2\pi},$$

$$S_{2,\epsilon}(s_{t}, s_{t+1})' = M'_{2,\epsilon},$$

$$S_{2,\eta c}(s_{t}, s_{t+1}) = M_{2,\eta \pi},$$

$$S_{2,\eta \pi}(s_{t}, s_{t+1}) = M_{2,\eta \pi}.$$

$$(1.62)$$

In the model, log bond prices,  $p_t^n$ , are linear in states, and the bond loadings vary with the monetary policy regime:

$$p_t^n = \tilde{\mathcal{A}}^n(s_t) + \tilde{\mathcal{B}}_X^{n'}(s_t)X_t + \tilde{\mathcal{B}}_{\sigma c}^n(s_t)\tilde{\sigma}_{ct}^2 + \tilde{\mathcal{B}}_{\sigma \pi}^n(s_t)\tilde{\sigma}_{\pi t}^2, \tag{1.63}$$

where for the short term bond n = 1 the loadings satisfy

$$\tilde{\mathcal{A}}^{n}(s_{t}) = -\alpha_{0}(s_{t}), 
\tilde{\mathcal{B}}^{n}_{X}(i)' = -\alpha(s_{t})', 
\tilde{\mathcal{B}}^{n}_{\sigma c}(s_{t}) = 0, 
\tilde{\mathcal{B}}^{n}_{\sigma \pi}(s_{t}) = 0.$$
(1.64)

Using similar approach as before to solve for the bond prices, we obtain that the bond

loadings for longer maturities satisfy,

$$\tilde{\mathcal{A}}^{n}(i) = \frac{1}{2} \left( \gamma^{2} (\sigma_{c}^{*})^{2} + (\sigma_{\pi}^{*})^{2} \right), 
+ \sum_{k} \pi_{ki} \left\{ \tilde{S}_{0} + \frac{1}{2} \left[ \left( \tilde{S}_{2,\epsilon}^{1} \right)^{2} \delta^{\alpha c} \alpha_{c}(s_{t}) + \left( \tilde{S}_{2,\epsilon}^{2} \right)^{2} \delta^{\alpha \pi} \alpha_{\pi}(s_{t}) + \tilde{S}_{2,\eta c}^{2} \omega_{c}^{2} + \tilde{S}_{2,\eta \pi}^{2} \omega_{\pi}^{2} \right] \right\}, 
\tilde{\mathcal{B}}_{X}^{n'}(i) = \sum_{k} \pi_{ki} \tilde{S}_{1,X}', 
\tilde{\mathcal{B}}_{\sigma c}^{n'}(i) = \sum_{k} \pi_{ki} \left\{ \tilde{S}_{1,\sigma c} + \frac{1}{2} \delta^{\sigma c} \left( \tilde{S}_{2,\epsilon}^{1} \right)^{2} \right\}, 
\tilde{\mathcal{B}}_{\sigma \pi}^{n'}(i) = \sum_{k} \pi_{ki} \left\{ \tilde{S}_{1,\sigma c} + \frac{1}{2} \delta^{\sigma c} \left( \tilde{S}_{2,\epsilon}^{1} \right)^{2} \right\}.$$
(1.65)

Table 1: Summary Statistics

Bond Yields:							
	3M	1Y	2Y	3Y	4Y	5Y	
Mean (%) Std. Dev. (%) AR(1)	3.27	3.33		3.18		6.39 2.99 .969	

## Macro and Survey Data:

Mean (%) 1.81 3.64 0.00 0.00 Std. Dev. (%) 1.74 2.41 .957 2.13		$\triangle c$	$\pi$	$x_c$	$x_{\pi}$
AR(1) .513 .892 .865 .983	\ /	1.74	2.41	.957	2.13

The table shows summary statistics for bond yields, consumption growth, inflation, and expectations of real growth and inflation constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4.

Table 2: Prior and Posterior Distributions

			Prior		Posterio			
	Distr.	Mean	Std. Dev.	5%	50%	95%		
	Transition Parameters:							
$\Pi_{cc}$	$N^T$	.9	.2	.956	.976	.991		
$\Pi_{\pi c}$	$N^T$	0	_	-	0	-		
$\Pi_{c\pi}$	$N^T$	0	.2	040	027	015		
$\Pi_{\pi\pi}$	$N^T$	.9	.2	.950	.966	.977		
	Non-	Policy V	olatility Para	meters:				
$\frac{\tilde{\sigma}_{c0}^2}{1-\varphi_c} \times 10^5$	IG	.01	.02	.011	.018	.041		
$\frac{\tilde{\sigma}_{\pi 0}^2}{1-\omega_{\pi}} \times 10^5$	IG	.02	.01	.009	.016	.024		
$\varphi_{\pi}$	$N^T$	0.95	0.1	.962	.977	.992		
$\omega_{\pi} \times 10^6$	IG	0.15	0.2	.182	.184	.189		
$\sigma_c^* \times 10$	IG	.05	.02	.030	.038	.051		
$\sigma_{\pi}^* \times 10$	IG	.05	.02	.025	.033	.045		
Regime-Shifting Coefficients:								
$\mu_c(s_1)$	N	.0045	.001	.0024	.0042	.0074		
$\mu_c(s_2)$	N	.0045	.001	.0009	.0033	.0068		
$\mu_{\pi}(s_1)$	N	.0091	.001	.0085	.0102	.0121		
$\mu_{\pi}(s_2)$	N	.0091	.001	.0065	.0081	.0101		
$\delta^{\pi}(s_1) \times 10^5$	_	0.00	-	-	0.00	-		
$\delta^{\pi}(s_2) \times 10^5$	N	0.00	0.1	.005	.006	.007		
$\alpha_c(s_1)$	N	0.5	1	.287	.535	.652		
$\alpha_c(s_2)$	N	0.5	1	.541	.748	.918		
$\alpha_{\pi}(s_1)$	N	1	1	.892	.936	.949		
$\alpha_{\pi}(s_2)$	N	1	1	1.56	1.68	1.88		
$\pi_{11}$	$N^T$	.9	.2	.941	.955	.970		
$\pi_{22}$	$N^T$	.9	.2	.942	.958	.986		
Other Parameters:								
$\kappa_1$	_	.995	-	-	.995	-		
$\gamma$	G	10	5	12.92	13.58	14.25		
$i_0$	N	0.00	.01	004	0008	0003		

The table summarizes the prior and posterior distributions for the model parameters. G refers to Gamma distribution, N to Normal distribution,  $N^T$  is truncated (at zero and/or one) Normal distribution, and IG is Inverse-Gamma. Dashed line indicates that the parameter value is fixed.

Table 3: Risk Premia Levels

Baseline Parameters:							
	n = 6M	1Y	3Y	5Y			
Exp Growth Shocks (%)	205	601	-2.02	-3.19			
Exp Inflation Shocks (%)	.324	.920	2.74	3.91			
Other Shocks (%)	0.00	0.03	.430	1.09			
Total (%)	.119	.348	1.16	1.81			
$In flation\ Neutrality:$							
	n = 6M	1Y	3Y	5Y			
Exp Growth Shocks (%)	205	601	-2.02	-3.19			
Exp Inflation Shocks (%)	.009	.022	.026	009			
Other Shocks (%)	0.00	0.00	0.00	0.00			
Total (%)	196	579	-1.99	-3.20			
Constant Monetary Policy:							
	n = 6M	1Y	3Y	5Y			
Exp Growth Shocks (%)	205	601	-2.02	-3.19			
Exp Inflation Shocks (%)	.310	.884	2.66	3.81			
Other Shocks (%)	0.00	0.03	.429	1.09			
Total (%)	.105	.311	1.07	1.71			

The table shows the decomposition of the average bond risk premia into the risk contributions due to expected growth shocks, expected inflation shocks, and the remaining shocks (inflation volatility and the regime shifts). The Baseline case refers to the benchmark estimation of the model. For the Constant Monetary Policy case the regime-dependent coefficients are fixed at their unconditional averages. For the Inflation Neutrality case the feedback between expected consumption and expected inflation is set to zero. All statistics are in annual terms and in percentages, and are computed at the median parameter draw.

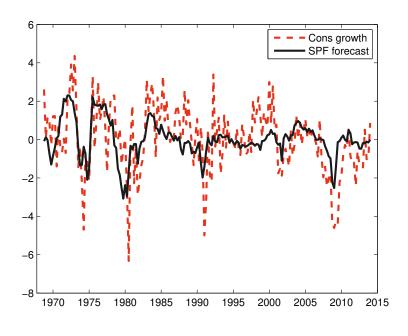
Table 4: Risk Premia Volatilities

$In ext{-}Sample:$							
% of Baseline	n = 6M	1Y	3Y	5Y			
Movements in $\tilde{\sigma}_{\pi t}$	68.8	69.7	72.7	74.5			
Movements in $\{\tilde{\sigma}_{\pi t}, \alpha_{\pi t}\}$	95.1	94.8	93.7	92.9			
Movements in $\{\tilde{\sigma}_{\pi t}, \delta_t^{\pi}, \alpha_{\pi t}\}$	109.5	109.2	108.4	107.7			
Movements in $\{\tilde{\sigma}_{\pi t}, \delta_t^{\pi}, \alpha_{\pi t}, \alpha_{ct}\}$	100.0	100.0	100.0	100.0			
Baseline (%)	.231	.649	1.87	2.62			
Population:							
% of Baseline	n = 6M	1Y	3Y	5Y			
Movements in $\tilde{\sigma}_{\pi t}$	79.9	80.8	83.4	85.0			
Movements in $\{\tilde{\sigma}_{\pi t}, \alpha_{\pi t}\}$	94.9	94.6	93.8	93.3			
Movements in $\{\tilde{\sigma}_{\pi t}, \delta_t^{\pi}, \alpha_{\pi t}\}$	110.6	110.2	108.8	107.8			
Movements in $\{\tilde{\sigma}_{\pi t}, \delta_t^{\pi}, \alpha_{\pi t}, \alpha_{ct}\}$	100.0	100.0	100.0	100.0			
Baseline (%)	.178	.503	1.47	2.06			

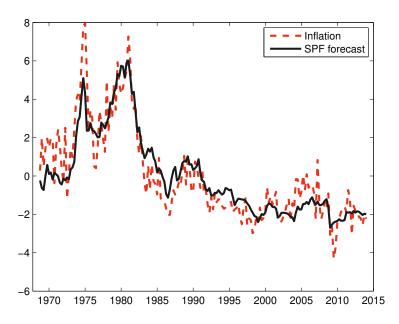
This table shows the in-sample and population risk premia volatilities in the restricted models which incorporate time-varying exogenous inflation volatility  $(\tilde{\sigma}_{\pi})$ , monetary portion of inflation volatility  $(\delta^{\pi})$ , Taylor rule coefficients on inflation and consumption  $(\alpha_{\pi}, \alpha_{c})$ . The baseline model additionally includes movements in the consumption and inflation drifts  $(\mu_{c}, \mu_{\pi})$ . The bottom line represents the annualized volatility, in percentage terms. The top lines represent volatilities as a percentage of the baseline value.

Figure 1: Realizations and Survey Expectations of Macroeconomic States

Real Growth:



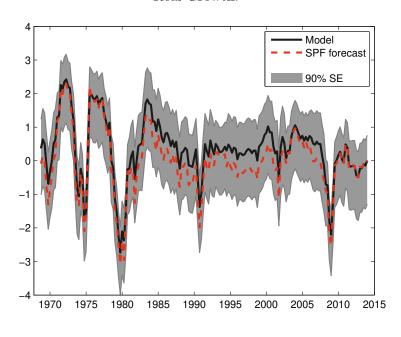
## Inflation:



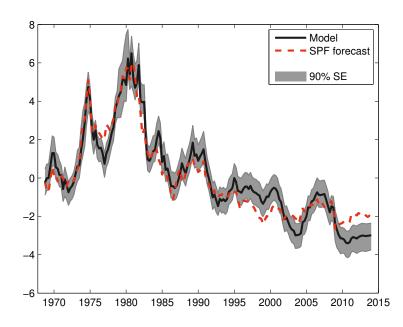
The top panel of the figure shows the realized (dashed line) and expected (solid line) consumption growth. The bottom panel shows the realized and expected inflation. Real growth and inflation expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4. The variables are demeaned, and are reported at the annual basis in percentage terms.

Figure 2: Estimated Macroeconomic States

Real Growth:

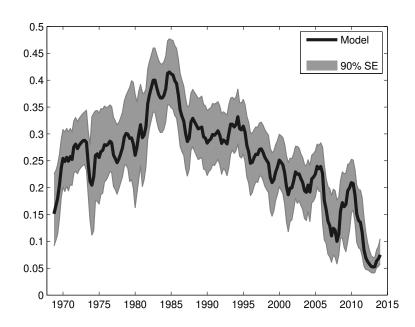


Inflation:



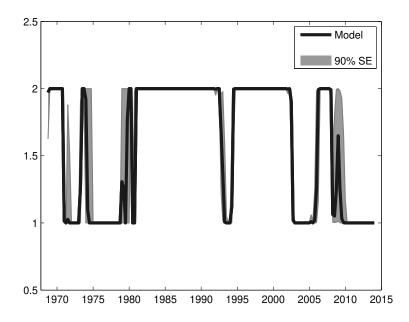
The top panel of the figure shows the expected real growth in the survey data (dashed line), and the estimated posterior median from the model (solid line). The bottom panel shows the expected inflation in the survey data and in the model. Grey region represents posterior (5%, 95%) credible sets. Data expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4. The variables are reported at the annual basis in percentage terms.

Figure 3: Estimated Inflation Volatility



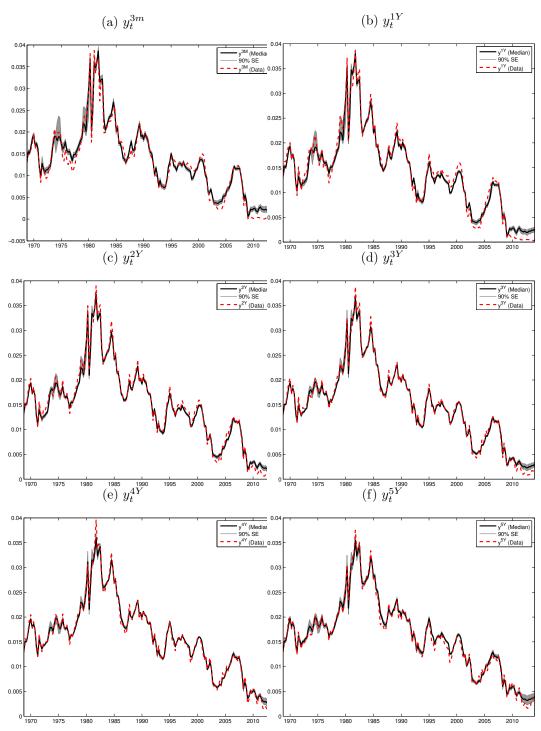
The figure shows the estimated posterior median of the exogenous component of inflation volatility. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4. The variables are reported at the annual basis in percentage terms.

Figure 4: Estimated Monetary Policy Regime



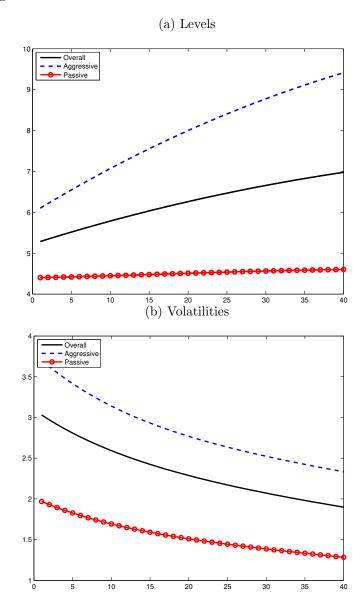
The figure shows the estimated posterior median of the monetary policy regime. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.

Figure 5: Estimated and Observed Yields



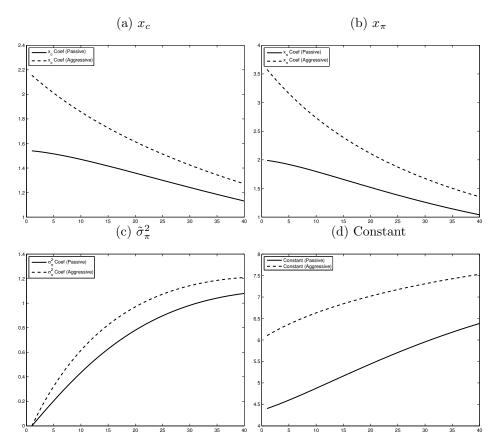
The figure shows the nominal bond yields in the data (red line), and the estimated posterior median from the model. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.

Figure 6: Unconditional Levels and Volatilities of Yields



The figure shows model-implied unconditional levels and volatilities of bond yields across monetary policy regimes.





The figure shows the model-implied bond loadings on the expected real growth, expected inflation and inflation volatility, and the unconditional bond yields in aggressive and passive regimes. Bond loadings are standardized to capture a one standard deviation movement in each factor, and are computed at the median parameter draw.

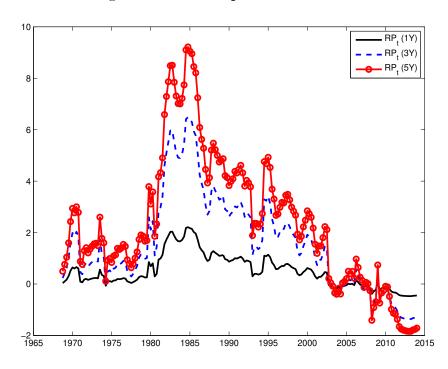


Figure 8: Model-Implied Risk Premia

The figure shows the estimated bond risk premia in the sample. We display the one-quarter risk premia for one-, three-, and five-year to maturity bonds. All model-implied values are computed at median parameter values and states.

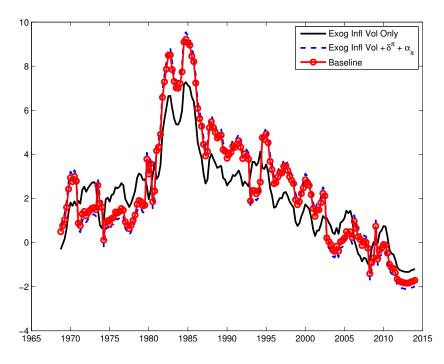


Figure 9: Risk Premia Decompositions

This figure displays the in-sample time series of the five year bond risk premia in the restricted model which incorporates only time-varying exogenous inflation volatility  $(\tilde{\sigma}_{\pi})$ ; and the model which also adds monetary portion of inflation volatility  $(\delta^{\pi})$  and Taylor rule coefficients on inflation  $\delta_{\pi}$ . The baseline model additionally includes movements in the Taylor rule coefficients to real growth, and monetary-policy components of the consumption and inflation drifts  $(\mu_c, \mu_{\pi})$ . The economic states correspond to the benchmark estimate of the model.

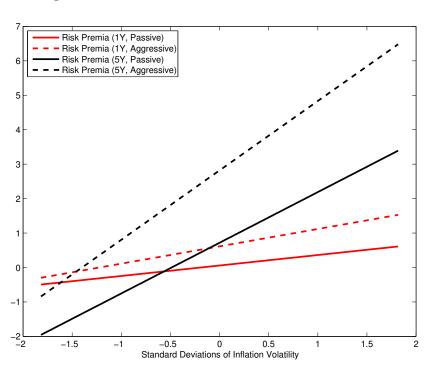


Figure 10: Conditional Risk Premia in the Model

The figure shows the model-implied risk premia for one- and five-year bonds with respect to standardized movements of inflation volatility. The solid lines are the risk premia that result in passive regimes while the dashed ones result in aggressive regimes.

# Chapter 2: The Asset Pricing Implications of Contracting Frictions

## (joint work with João Gomes and Amir Yaron)

#### 2.1. Introduction

The success of Epstein and Zin (1989) preferences in endowment-based economies has led to an offspring in research that asks whether recursive preferences can generate plausible macroeconomic and financial dynamics in production-based frameworks. In a parallel line of work, financial frictions have been shown to create an amplification mechanism that increases the quantity of risk associated with total factor productivity shocks. In this paper we ask what quantitative role do contracting frictions play in amplifying the risk exposures of financial assets. Furthermore, how do agency frictions compare to other popular mechanisms in the literature, such as recursive preferences and adjustment costs to capital?

We examine these questions through the tractable framework of Carlstrom and Fuerst (1997). There are four agents in the model - (1) households who make consumption and savings decisions under Epstein-Zin preferences, (2) intermediaries who borrow household savings and lend them out, (3) risk-neutral entrepreneurs who borrow from intermediaries and invest into a capital generating technology, and (4) final goods producers who rent capital and labor, set marginal prices, and close the economy. There is moral hazard between the intermediaries and entrepreneurs, regarding the productivity of the capital-generating technology, which leads to a Costly State Verification (CSV) contract that allows for a role of monitoring costs. While the contract is decided in a static fashion, it is a function of the aggregate state and hence plays a dynamic role.

Households own capital in this economy and we calculate asset returns from their perspective. More precisely, this is the rate of return from purchasing a unit of capital today, collecting dividends, and reselling the undepreciated portion to the market tomorrow. Under a baseline calibration that has neglibile bankruptcy costs and implements quadratic costs of capital adjustment we are able to capture many key macro-financial moments. These include the volatility of output growth, the relative volatility of consumption growth, the correlation of consumption and investment growth, and the level of the risk free rate. Additionally we receive an equity risk premium of about three percent (3%) and a Sharpe ratio of .4, values that get close to empirical estimates. Finally, we are also able to receive a reasonable leverage ratio for the entrepreneurs, of about thirty percent (30%), and

countercyclical credit risk premia.

Having fitted the data reasonably well with a model that only implements direct adjustments costs to capital, we find that bankruptcy frictions have a minor effect on the aggregate market return. Increasing monitoring costs from zero to twenty percent raises the levered excess return on market capital by only eighty basis points. While the contracting friction acts as an implicit adjustment cost, distorting the price of capital away from one and decreasing investment volatility, it is a weak one. Instead, a standard convex adjustment cost to capital accounts for 250 basis points. Equivalently stated, the overall volatility of the cost of capital is less sensitive to a movement in the agency friction than it is to a change in the physical capital adjustment.

Another weakness of the contracting friction is that it drives a procyclicality of the credit spread. As the monitoring cost increases, entrepreneur leverage in the model becomes more procyclical, due to the fact that lending becomes highly restricted when aggregate states are adverse. This increased leverage in good states of the world results in a procyclical default rate. The compensation for the entrepreneur's default risk must pay off in the very states where default risk and leverage are high. Hence the credit spreads are procyclical in a model with costly state verification. These results are parallel to those in Gomes et al. (2003), keeping in mind that our model includes mechanisms (recursive preferences, shocks to growth rates) that are traditionally known to get closer to asset pricing data.

The original intuition of this class of models is that costly state verification increases the autocorrelation of economic aggregates. This is what the literature determines to be the "persistence effects" of financing frictions and is the result of hump-shaped impulse response functions. We show that financial frictions do not have such an effect in our model. In the baseline model, one with no physical adjustment costs, and one featuring a lower separation between risk aversion and the intertemporal elasticity of substitution, we show that the autocorrelation of both output and consumption are largely in-sensitive to financial frictions. Put in another way, when we calibrate a financial frictions model to handle asset pricing data as well as movements in macroeconomic aggregates, the persistence effects of contracting frictions are not present. This stands in direct contrast to the key results in Carlstrom and Fuerst (1997).

Another major result is regarding the role of Epstein-Zin preferences. We examine values of the intertemporal elasticity of substitution while fixing risk aversion. We find that there is a clear, monotonically decreasing risk premium when we shift IES from two to below one. As our model setup is dependent on shocks to growth rates of productivity as opposed to levels of productivity the IES plays a strong role in influencing the persistence of investment

flows. As a result, the procycality of dividend payments are diminished when IES decreases. This result is very similar to those in Croce (2014) and Favilukis and Lin (2013).

Related Literature. Our work is focused on measuring the asset market impact of contracting frictions from intermediation, in a production economy setting. In order to make a quantitative statement however, we need to properly calibrate our model to match key features of the financial and macroeconomic time series. This is not an easy task. As discussed in Jermann (1998) and much of the production-based asset pricing literature, households can smooth consumption by managing investment. Consumption growth becomes less volatile across the business cycle and assets decay in their level of risk premia. To get around this smoothness of consumption, one can input adjustment costs to capital to help slow the rate of investment and increase consumption volatility. This is also the spirit behind planning on investment in advance (Christiano et al. (2001)) and rigidities in wage adjustment (Favilukis and Lin (2015)). In our setup we will also input adjustment costs to capital and compare it to an indirect investment friction: the intermediation link between entrepreneurs and households.

Starting from the Long Run Risks literature (Bansal and Yaron (2004), Bansal and Shaliastovich (2013)), the use of Epstein and Zin (1989) preferences has helped explain asset pricing dynamics, by increasing risk exposures on asset valuations. From a production economy standpoint, Tallarini Jr. (2000) and Kaltenbrunner and Lochstoer (2010) have shown the benefits of utilizing recursive preferences to better match macro-financial data. In our model, we will show that positive sizeable equity premia are only generated when we have a large EIS parameter, close to two.

Using productivity shocks that are more long-lived naturally amplifies the quantities of risk that assets bear. This is the motivation behind using persistent shocks to the growth rates of total factor productivity. As shown in Croce (2014), these shocks are not only advantageous in terms of matching asset pricing properties in a model-based setting but are also empirically justified when we examine growth rates of Cobb Douglas residuals in the data. Further, Favilukis and Lin (2013) discuss the desirable implications that shocks to growth rates have with respect to capturing autocorrelation and heteroskedasticy of investment rates in a neoclassical model. If we would like to micro-found these production-based long run risks, one could examine the properties of an endogenous growth model, as in Kung and Schmid (2015). In this model, per-period investment into a patent sector generates endogenous, upwards movement in a persistent productivity process. The collective patent formation gives rise to desirable asset pricing results.

Our model focuses on the quantitative implications arising from financial frictions, which were first motivated as mechanisms to generate endogenous magnification in aggregate fluctuations. In particular, we can think of Kiyotaki and Moore (1997), Bernanke et al. (1999), and Carlstrom and Fuerst (1997) as primary examples of models in which firms' financing constraints create distortions in capital markets. In Kiyotaki and Moore, firms are forced to pledge collateral for borrowing, which creates distortions across borrowers of different types. In Carlstrom and Fuerst, hereafter CF, firms (entrepreneurs) can only borrow on the basis of their ex-ante net worth, which factors into a costly state verification agreement with intermediaries. As our model builds upon CF we also will use the net worth channel to create magnified cyclical variations. And as business cycle variations are magnified in quantities, we expect assets with positive exposures (betas) to be subject to higher risk premia. We will show in our paper that when we calibrate our model to more saliently capture macroeconomic and financial time series data (through different household preferences, shocks, and adjustment costs), many of these traits that amplify financial frictions are no longer robust. At a business cycle frequency, it is more so the case that direct, physical costs to adjustment are more advantageous relative to the financial friction.

If we interpret the net worth channel as an active financial constraint for entrepreneurs, this paper also connects to the literature that discusses the relationship between firm financing constraints and asset pricing. Gomes et al. (2006) quantitatively examines whether financial constraints are significantly priced in asset returns, through the Euler condition of a firm that dynamically chooses capital each period. Similarly, Whited and Wu (2006) construct a model-based financial constraints index that predicts returns significantly, relative to other predictive factors in the literature. Livdan et al. (2009) takes a similar approach but uses model-based Lagrange multipliers on firm constraints in order to predict returns.

Our article can best be thought of as a combination of Gomes et al. (2003), hereafter GYZ, and Croce (2014). While GYZ studies the asset pricing implications of the Carlstrom and Fuerst model, it does not allow for recursive preferences and also does not employ shocks to growth rates of productivity as in Croce and Favilukis and Lin. We find that both of these mechanisms are crucial to better capture macro-financial dynamics. Under the scope of a much more plausible environment, we then make statements regarding the financial friction.

The roadmap for our paper is as follows. In the next section we document the general

<sup>&</sup>lt;sup>1</sup>In Gomes et al. (2003), the average annual equity premium in a model with agency costs was .02% on an annual basis. In our model, an analogous environment generates about 3.30%, a much more realistic amount relative to the data.

equilibrium model that we will use to study the role of financial frictions in asset prices. In the following section we document the quantitative details of the model calibration, including parameter choice and the data used. In the fourth section, we document the main results regarding the role of contracting frictions. In the fifth and final section, we conclude.

#### 2.2. Model

The model consists of four agents: (1) Households that exhibit recursive preferences (2) Intermediaries whose role wil largely be to propagate a costly state verification contract (3) Entrepreneurs who are capital-producing and make investment decisions under the previously stated friction (4) and Final Goods Producers that convert capital goods into consumption goods for the household and entrepreneur. We will describe each agent's role individually. For more details, Appendices 2.6 and 2.7 discuss the timing of the model and derivations of first order conditions.

#### 2.2.1. Households

Identical households are infinitely lived with Epstein-Zin (EZ) preferences over consumption and labor at wage  $w_t$ . They also own and rent capital,  $k_{t+1}$ , to the final goods producer at rate  $r_t$ . The decision problem can be written in the following manner:

$$U_{t} = \max_{\{c_{t}, l_{t}, k_{t+1}\}} \left( (1 - \beta) \left( V(c_{t}, l_{t}) \right)^{\frac{1 - \gamma}{\theta}} + \beta \left( E_{t} \left[ U_{t+1}^{1 - \gamma} \right] \right)^{\frac{1}{\theta}} \right)^{\frac{\theta}{1 - \gamma}}$$

s.t. 
$$c_t + q_t k_{t+1} + q_t \Phi_a(k_t, k_{t+1}) k_t = w_t l_t + r_t k_t + q_t (1 - \delta) k_t$$

where  $V(c_t, l_t)$  represents the *intra*temporal utility from consumption and labor choice. The price of capital is  $q_t$  and  $\delta$  is its depreciation rate. The EZ parameters are standard with  $\gamma$  representing risk aversion,  $\psi$  representing IES, and  $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ . Additionally, there are adjustment costs that directly enter the household's problem, as provided by  $\Phi_a \times k$ . Because the previous expression is in terms of capital units, it is multiplied by the price of capital to maintain consumption units.

In this model the household lends to an intermediary that provides an instantaneous, intraperiod return of one. Hence the household's problem does not need to take into account the "zero profit" intermediary. Similarly, the final goods producer has constant returns to scale and will provide zero profits to the households. Therefore the above budget constraint does not to take into account these two elements.

We can solve this problem for first order and envelope conditions. We receive an equation

governing the trade-off of labor and consumption and the usual Euler equation:

$$w_{t}V_{c}(c_{t}, l_{t}) = -V_{l}(c_{t}, l_{t})$$

$$E_{t}\left[M_{t+1}R_{t+1}^{k}\right] = 1$$
s.t. 
$$M_{t+1} = \beta \frac{U_{t+1}^{\frac{1}{\psi}-\gamma}}{\left(E_{t}\left[U_{t+1}^{1-\gamma}\right]\right)^{1-\frac{1}{\theta}}} \left(\frac{V_{t+1}}{V_{t}}\right)^{-\frac{1}{\psi}} \left(\frac{V_{c,t+1}}{V_{c,t}}\right)$$

$$R_{t+1}^{k} = \frac{r_{t+1} + q_{t+1}(1-\delta) - q_{t+1}\left(\Phi_{a,t+1} + \frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}}k_{t+1}\right)}{q_{t}\left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}}k_{t}\right)}$$

where  $\Phi_{a,t}$  are adjustment costs at time t, evaluated at  $(k_t, k_{t+1})$ .

We will interpret the bottom quantity as the return on capital in the model. Put in other words, it is the return from purchasing a unit at time t, selling back the depreciated unit at time t+1 and collecting the appropriate dividend from the final goods producer. With adjustment costs, additional terms reflect the opportunity cost of investment. Note that when adjustment costs are set to zero, the expression boils down to:

$$R_{t+1}^{k,noadj} = \frac{r_{t+1} + (1 - \delta)q_{t+1}}{q_t}$$

and the dividend becomes the rental rate on capital, r.

#### 2.2.2. Contract between Intermediary and Entrepreneurs

Intermediaries take household savings and lend to entrepreneurs at rate  $r_t^l$ . In turn entrepreneurs invest these funds along with their own net worth, a total investment of  $i_t$ , into a linear technology of capital production that is subject to idiosyncratic productivity shocks. Let the TFP shocks be represented by  $\omega_t$ , which are IID random variables such that  $\mathbb{E}(\omega_t) = 1$ . Additionally,  $\omega$  takes on a CDF  $\Phi(\cdot)$  and PDF  $\phi(\cdot)$  over non-negative support. This will suggest that the capital generated will be  $\omega_t i_t$ . In consumption units it will be  $q_t \omega_t i_t$ .

To finance his investment the entrepreneur needs to borrow an external debt from intermediaries, over his current net worth, represented by  $i_t - n_t$ . This will be at an endogenously determined interest rate. As  $\omega$  is an ex-ante unknown object to the bank, the bank will contract on the future realization of these shocks. In particular it will be the case that

when the realization of  $\omega$  is too low, the enterpreneur will not be able to repay the loan.<sup>2</sup> Accordingly, he will give up all of his realized investment,  $\omega_t i_t$ . Hence, default only occurs when:

$$\omega_t i_t < \bar{\omega}_t i_t \equiv (1 + r_t^l)(i_t - n_t)$$

where  $\bar{\omega}_t$  is determined prior to the realization of the investment shock.

As we will discuss, the payoffs and participation constraint are linear. This will allow us to formulate an optimal debt contract in the form of Townsend (1979) and Gale and Hellwig (1985), where banks pay a fractional fee,  $\mu$ , to audit or monitor entrepreneurs when default occurs. Based on the previous cutoff value, the bank's expected income of a loan of size  $i_t - n_t$  will be given by:

$$q_t i_t g(\bar{w}_t) \equiv q_t \left[ \int_0^{\bar{\omega}_t} \omega_t i_t d(\Phi(\omega_t)) - \Phi(\bar{\omega}_t) \mu i_t + (1 - \Phi(\bar{\omega}_t))(1 + r_t^l)(i_t - n_t) \right]$$

$$= q_t i_t \underbrace{\left[ \int_0^{\bar{\omega}_t} \omega_t d(\Phi(\omega_t)) - \Phi(\bar{\omega}_t) \mu + (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t \right]}_{g(\bar{\omega}_t)}$$

Similarly, the enterpreneur's expected income with the given loan size will be:

$$q_t i_t f(\bar{w}_t) \equiv q_t \left[ \int_{\bar{\omega}_t}^{\infty} (\omega_t i_t - (1 + r_t^l)(i_t - n_t)) d(\Phi(\omega_t)) \right]$$
$$= q_t i_t \underbrace{\left[ \int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t) d(\Phi(\omega_t)) \right]}_{f(\bar{\omega}_t)}$$

We maximize the expected income of the entrepreneur given that the lender is (at-least) returned his original loan amount. This results in the following contracting problem:

$$\operatorname{Max}_{\{\bar{\omega}_t, r_t^l, i_t\}} q_t i_t f(\bar{\omega}_t)$$
s.t. 
$$q_t i_t g(\bar{\omega}_t) \ge i_t - n_t$$

Notice that given  $i_t$  and  $n_t$  there is a one to one mapping from  $\bar{\omega}_t$  to  $r_t^l$  – this comes from the definition of  $\bar{\omega}_t$ . Hence we will simply solve for  $i_t$  and  $\bar{\omega}_t$ . It can be shown that these

<sup>&</sup>lt;sup>2</sup>More specifically, we use a debt contract in which partial disclosure is optimal. When the entrepreneur defaults it will be incentive compatible to disclose the severity of the shock. Likewise, the lender's participation constraint will hold under non-disclosure, which will only occur when the entrepreneur does not default.

values will satisfy the first order conditions:

$$q_t f(\bar{\omega}_t) = \frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)} (q_t g(\bar{\omega}_t) - 1)$$

$$i_t = \frac{n_t}{1 - q_t g(\bar{\omega}_t)}$$
s.t. 
$$f'(\bar{\omega}_t) = -(1 - \Phi(\bar{\omega}_t))$$

$$g'(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t) - \mu \phi(\bar{\omega}_t)$$

To solve for the contract terms, notice first  $\bar{\omega}_t = \bar{\omega}(q_t)$  to satisfy the first equation. For the given level of  $\bar{\omega}_t$ ,  $q_t$ , and  $n_t$ , we can then solve for  $i_t = i(q_t, n_t)$  from the second equation and also pin down  $r_t^l$ . For additional details see Appendix 2.7.

From this contract it is clear that the intermediary's participation constraint is binding; in other words, he will make zero profits. Hence the household does not receive any additional funds through deposits. In terms of timing, the contract will determine what  $i_t$  is before the entrepreneur makes his consumption / saving decisions.

#### 2.2.3. Entrepreneur

There is a continuum of entrepreneurs that make consumption and capital decisions each period, following the determination of contract conditions and realization of technology shocks. For entrepreneurs that default on loan terms, they choose zero amounts of consumption and next period capital, while solvent entrepreneurs optimize. Due to the linear structure of the budget constraint, the risk neutral preferences of the entrepreneur, and the fact that entrepreneurs will receive the continuum of IID technology shocks we can aggregate the dynamic decision problem to solve for the entrepreneurs' average policies. For additional details see Appendix 2.6.

We maximize average lifetime discounted consumption. The entrepreneur's decision problem is the following:

$$\max_{\substack{\{c_t^e, k_{t+1}^e\}}} E \sum_{t=0}^{\infty} (\beta \gamma^e)^t c_t^e$$
s.t. 
$$c_t^e + q_t k_{t+1}^e = q_t i_t f(\bar{\omega}_t)$$

$$n_t = w_t^e + r_t k_t^e + q_t (1 - \delta) k_t^e$$

The first constraint represents the entrepreneur's budget balance, where  $c_t^e$  is his consumption choice and  $k_{t+1}^e$  is his capital choice. For an investment of  $i_t$  (which is predetermined by the point in time of this decision) the aggregate entrepreneur receives  $q_t i_t f(\bar{\omega}_t)$  as a re-

turn in the same period, which represents the aggregation of each individual entrepreneur's contract proceeds. – The second constraint represents the components of net worth: wages  $w_t^e$ , rent on capital (at same rate as household)  $r_t k_t^e$ , and the current value of undepreciated capital,  $q_t(1-\delta)k_t^e$ . These are all observable at the time of the entrepreneur solving the problem.

Using the definition of  $f(\bar{\omega}_t)$  from above we can solve the previous problem and derive the following Euler equation:

$$E_{t} \left[ \beta \gamma^{e} \left\{ \frac{\left( \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right) (q_{t+1} (1 - \delta) + r_{t+1})}{q_{t}} \right\} \right] = 1$$

In this equation, there is an additional wedge that serves as additional compensation for taking on investment. This is given by:

$$\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})}$$

The principal reason for this wedge is that it captures further risk that the entrepreneur bears. While he gets the standard gross capital return, he also needs to be compensated for the additional risk that comes with investment – additional leverage and hence more default risk, all else equal. One can think of this wedge as the compensation for credit risk. In the empirical evaluation of the model, we will treat it similar to a credit spread.

#### 2.2.4. Final Goods Producer

The final goods firm exhibit constant returns to scale (CRS) with labor-augmenting shocks inside the production function. This will be given by:

$$Y_t = F(K_t, Z_t L_t, Z_t L_t^e)$$

where  $K_t$  denotes aggregate capital,  $L_t$  aggregate household labor, and  $L_t^e$  aggregate entrepreneur labor.  $Z_t$  represents a (symmetric) technology shock.

The share of entrepreneurs in the economy is  $\eta$ , while that of households is  $1 - \eta$ , which suggest  $K_t = (1 - \eta)k_t + \eta k_t^e$ . For labor market clearing,  $L_t = (1 - \eta)l_t$  and  $L_t^e = \eta l_t^e = \eta$ , where the last statement follows from labor not entering the entrepreneur's utility. Due to

<sup>&</sup>lt;sup>3</sup>One can also think of this wedge as the additional compensation for financial constraintedness. In a firm problem where leverage or equity is bounded, the dynamic Lagrange multiplier provides a wedge for additional return compensation. This is the focus in Livdan et al. (2009).

the CRS, zero-profit nature of the final goods producers we will have:

$$r_{t} = F_{K}(K_{t}, Z_{t}L_{t}, Z_{t}L_{t}^{e})$$

$$w_{t} = F_{L}(K_{t}, Z_{t}L_{t}, Z_{t}L_{t}^{e})$$

$$w_{t}^{e} = F_{L^{e}}(K_{t}, Z_{t}L_{t}, Z_{t}L_{t}^{e})$$

at the equilibrium quantities.

## 2.2.5. Market Clearing

Beyond the labor markets clearing we will require that supply and demand match in the capital markets and goods markets. That is to say:

(Capital Markets) 
$$K_{t+1} = (1 - \delta)K_t + \underbrace{\eta i_t (1 - \mu \Phi(\bar{\omega}_t))}_{\text{Investment across all entrepreneurs}}$$
(Goods Markets) 
$$Y_t = F(K_t, Z_t L_t, Z_t L_t^e) = (1 - \eta)c_t + \eta c_t^e + \eta i_t$$

Notice the correction for the surplus loss in the capital markets clearing. As investment is chosen before productivity shocks are realized, there are deadweight losses to be bourne from the monitoring costs that are triggered, for  $\Phi(\bar{\omega}_t)$  worth of defaults.

Since all variables are aggregate quantities for a measure one, we scale households and entrepreneurs by  $\eta$  and  $1-\eta$  for market clearing. Going forward we will denote aggregate consumption and investment as:

$$I_t = \eta i_t (1 - \mu \Phi (\bar{\omega}_t))$$

$$C_t = \eta c_t^e + (1 - \eta) c_t$$

In measurement of aggregate quantities we compute growth rates of these variables.

#### 2.2.6. Measurement of Quantities and Returns

We will be interested in examining log growth rates of macroeconomic variable X given by  $\log\left(\frac{X_{t+1}}{X_t}\right)$  where  $X_t \in \{Y_t, C_t, I_t\}$ . The risk free rate we will be targeting will be given by:

$$\exp\left(r_{t+1}^f\right) = \frac{1}{E_t\left[M_{t+1}\right]}$$

We will focus on one capital return in the model – that of the household, denoted here by  $r^k$ :

$$\exp\left(r_{t+1}^{k}\right) = \frac{r_{t+1} + q_{t+1}(1-\delta) - q_{t+1}\left(\Phi_{a,t+1} + \frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}}k_{t+1}\right)}{q_{t}\left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}}k_{t}\right)}$$

$$\exp\left(r_{t+1}^{e}\right) = \frac{\left(\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})}\right)(q_{t+1}(1-\delta) + r_{t+1})}{q_{t}}$$

The last value specified,  $r^e$ , denotes a return-type object from the entrepreneur's perspective of owning and collecting on capital proceeds. For completeness, we also report this in the final tables but will not utilize this measure in our calibration procedure.

It is well founded in the data that excess returns in equities are levered and some volatility in the excess returns is due to idiosyncratic noise. To take our model's equity excess return to the data we finally define a levered, excess equity return given by:

$$R_{ex,t+1}^{lev} = 2 \times \left(r_{t+1}^k - r_{t+1}^f\right) + \sigma_{lev}\epsilon_{lev,t+1}$$

The leverage parameter here, two, comes from literature. It can be originally traced back to work by Rauh and Sufi (2011) as well as Garca-Feijo and Jorgensen (2010).

To understand the credit spread dynamics in the model we will use the difference between the returns of the entrepreneur and household,  $r^e - r^k$ . While the household has adjustment costs, it will be the case that much of the variation in this difference will be governed by the additional default premium – the wedge that was discussed earlier.

## 2.3. Baseline Calibration and Data

In this section we specify the processes governing productivity shocks, household utility, and adjustment costs. We will also go over the baseline calibration which corresponds to a model with solely convex costs of adjustment to capital and roughly zero costs of bankruptcy (equivalently stated, a low value of  $\mu$ ). The goal in later parts of the paper will be to make adjustments to this baseline and observe marginal behavior of the model.

#### 2.3.1. Productivity Shocks

As demonstrated in Croce (2014), the use of productivity shocks to *growth rates* of total factor productivity generates dynamics that are both empirically plausible as well as quantitatively advantageous for general equilibrium models. Explained in Favilukis and Lin

(2013), these modifications generate persistent investment flows that will result in a larger procyclicality of dividend flows and hence larger risk premia. In models where shocks are to the *level* of TFP, we are not able to generate such features.

We model log growth rates in productivity as having a small, persistent component:

$$\Delta z_{t+1} = g_z + x_t + \varphi_z \epsilon_{z,t+1}$$
$$x_t = \rho_x x_{t-1} + \varphi_x \epsilon_{x,t+1}$$

The setup of TFP shocks here allows us to interpret movements in  $\epsilon_{z,t+1}$  as "short run risks" and those in  $x_t$  as "long run risks." We follow from the literature and impose the ratio of  $\frac{\varphi_x}{\varphi_z}$  to be 10% while the persistence of  $x_t$  will be  $\rho_x = .96$  which is consistent with the estimate in Croce. Similarly we will choose  $g_z$  to be roughly .5%, which allows us to match average growth rates of the US economy in steady state. All that is left to calibrate is  $\varphi_z$  which we do so to approximately match the volatility of US GDP.

In addition to the productivity shocks of the TFP, we also need to calibrate the shocks related to the entrepreneur's capital investment technology. We assume that  $\omega$  follows a log-normal distribution with zero mean. That is to say that:

$$\log \omega_t \sim \mathcal{N}\left(-.5\sigma_\omega^2, \sigma_\omega^2\right)$$

Because of the log-normal distribution, we can compute as a function of the normal distribution, the share of surplus going to entrepreneurs and banks in closed form.<sup>4</sup>

#### 2.3.2. Intratemporal Utility and Adjustment Costs

Given the generality of the model we will need to take a stand on the forms of household's intratemporal utility function,  $V_t$ . For the purposes of this exposition we will shut down the household's labor supply and make it inelastic. That is to say:

$$V(c_t, l_t) = c_t$$

and hence  $l_t$  is equal to one at all states. Including labor in the model will more than likely make the quantitative performance weaker. This is due to the fact that households will have another hedging instrument to smooth over consumption flows, which will reduce volatility of the SDF and decrease excess equity returns. In principle we could add other modeling

<sup>&</sup>lt;sup>4</sup>We have tested an alternative form of entrepreneur shocks. When implementing a uniform shock distribution centered around a unitary mean, results do not materially change (investment volatility increases while asset pricing conclusions are unchanged).

devices, such as a time to plan assumption for labor (Christiano et al. (2001)) or rigidity in wages (Favilukis and Lin (2015)), but we keep the model simple in order to focus on the effects of contracting frictions.

Additionally we will set the final goods production function to be Cobb-Douglas with labor-augmenting TFP shocks, which is to say:

$$Y_t = K_t^{\alpha_k} (Z_t L_t)^{\alpha_l} (Z_t L_t^e)^{\alpha_e}$$

$$Z_t = \exp(z_t)$$

where  $\alpha_k + \alpha_l + \alpha_e = 1$  and the transmission of  $z_t$  is as given in the previous subsection. Finally we set the household adjustment costs to take a convex, quadratic form given by:

$$\Phi_t = \Phi(k_t, k_{t+1}) = \frac{\phi_k}{2} \left( \frac{k_{t+1} - (1 - \delta)k_t}{k_t} - \delta \right)^2$$

The use of this form of adjustment costs is certainly different than the irreversibility form used in Jermann (1998). As a result, investment by each individual is not limited to be be positive in equilibrium. Furthermore, it might be the case that our results are not directly at a parallel of Kaltenbrunner and Lochstoer (2011) and Croce (2014). Nonetheless we find that the qualitative attributes of our model are very reasonable. For example, increasing  $\phi_k$  decreases investment volatility and increases the market return.

#### 2.3.3. Baseline Calibration

In Table 5 we list the baseline calibration model where bankruptcy costs are set at very low levels. This is given through  $\mu=.5\%$ . Regarding other parameters of the model, we set the capital share of the final goods producer  $(\alpha_k)$  to be .30 while setting the household labor share to be .6999 and the remainder to the entrepreneur share. This calibration follows directly from Carlstrom and Fuerst (1997) and Gomes, Yaron and Zhang (2003). Also similar to literature are the depreciation rate of capital  $(\delta=.02)$  and the share of entrepreneurs in the economy  $(\eta=.10)$ . We also fix the household's risk aversion to a reasonable level  $(\gamma=15)$  throughout our experiments. This value is in between those calibrated in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013). We also fix the intertemporal elasticity of substitution at 2.5 in the baseline. This level of IES is necessary to merit reasonable consumption and dividend dynamics. We will show this more explicitly in our counterfactuals.

There are a few parameters that we have left to more carefully calibrate; the first one being the subjective time discount factor of the household,  $\beta$ . Naturally we would expect

 $\beta$  to influence the level of the risk free rate negatively. Different from the mechanism in Kaltenbrunner and Lochstoer (2011) however we do not get a sole identification of the time discount rate through the risk free rate; rather the volatility of investment is also highly dependent on  $\beta$ , negatively. As  $\beta$  increases, the penchant to smooth consumption decreases which increases the volatility share of consumption. For a fixed output volatility and correlation of consumption and investment growth, this means that the volatility of investment will drop. Altogether, this strong tradeoff makes it difficult to hit both moments simultaneously and hence we choose a  $\beta$  that gets the risk free rate at a reasonable expected level; in our calibration this amounts to an annual risk free rate of 72 basis points.

As the baseline model is selected to operate with purely adjustment costs we modify the value of  $\phi$ , which is the value of the multiplier on the household's quadratic adjustment costs. What we find is that this parameter has a strong bearing on whether consumption and investment growth correlate in a positive manner. In particular, as  $\phi$  increases so too does the value of  $\rho(\Delta c, \Delta i)$ . We set  $\phi = 10$  to receive a correlation of about .67 which is very close to the annual data counterpart.

The amount of volatility on the entrepreneur's shock has a direct impact on the equilibrium leverage that is taken by entrepreneurs. We set  $\sigma_w$  to correspond to the roughly thirty percent market leverage that is found in the Compustat universe of firms. Finally we have left one parameter to set which is the volatility parameter on the idiosyncratic noise in the levered excess equity return. We set this quarterly number to 3.25% to match the monthly leverage volatility found in Croce (2014). This helps us match the Sharpe ratio of the excess, levered equity return.

#### 2.3.4. Data

The macroeconomic data we use as a calibration comparison come largely from the National Income Product Accounts (NIPA). Annual consumption from 1929 through 2008 is constructed as the sum of real, per capita, nondurables and services consumption. Similarly investment is taken to be the sum of real, per-capita, private residential and non-residential fixed investment.<sup>5</sup> To construct a comparable output series in the context of the model we sum the constructed consumption and investment series.

The annual return series for the realized excess equity returns is given through the excess market return on Ken French's website. We also take the nominal one month risk free rate on his website and subtract inflation constructed from the GDP deflator index to receive a measure for the real risk free rate. Credit spread data is received by taking the difference

<sup>&</sup>lt;sup>5</sup>We do not subtract software processing equipment as the time series for this does not go back to 1929. Even if we are to extrapolate the series going backwards, investment statistics do not drastically change.

between BAA and AAA corporate bond indices from Moody's.

#### 2.4. Results

## 2.4.1. The Fit of the Baseline Model

Based on the parameters discussed in the previous section we examine the model's implied behavior through a long sample simulation of 50,000 quarters (the model is calibrated at a quarterly frequency). Following the simulation, we time aggregate the data by four quarters to receive annual data. All reported statistics are hence in annual units. Table 6 provides the baseline fit of the model alongside standard error bounds. Quantities that are bolded indicate those that are used to calibrate the model (this is explained in detail in the previous section). We find that we are able to do a reasonable job in matching the relative volatility of consumption (.64 in the data versus .84 in the model), the volatility of output (3.36 in the data versus .3.74 in the model), and the correlation of consumption and investment growth (.68 in the data versus .718 in the model). As discussed earlier we choose  $\beta$  to hit the expected level of the risk free rate and as a result the model warrants an equivalent risk free rate of 72 basis points.

While we reasonably succeed on our desired calibration metrics, we perform differentially with respect to other ones. In particular, the relative volatility of investment is too low in the model. This boils down to facing a tradeoff in matching the macroeconomic moment (investment volatility) versus targeting the asset pricing moment (market equity returns). As the asset pricing moment is more relevant for our study, we compromise on the other metric. As a result, we are able to capture the excess equity return reasonably well with an equity premium of about three percent (2.51 %) and a Sharpe ratio that is well within error bounds (.356). To better understand the mechanism through which we capture the equity premium, we provide a more granular image of the returns in Table 8. In the column titled  $\mu = .5\%$  we outline the results of the baseline model. As the unlevered market return is only about 1.25% with a rather high Sharpe ratio of .87, leverage provides a much more reasonable final result. It is interesting to note that the "return-like" entrepreneur object,  $r^e$ , provides just under 4.3% of excess returns. Finally, the model is able to deliver the procyclicality of the market excess return and the countercylicality of the model-based credit spread. We will discuss this last result in length shortly.

## 2.4.2. Behavior of the Model

To provide a basic understanding of the model mechanism, we examine impulse response functions with respect to a long-run shock, a one-standard deviation shock to  $\epsilon_x$ , in Figure 11. As the model is non-linear and history-dependent, we compute a number of model

simulations and take the average deviation from steady state values to compute the impulse response function. In the top-right panel, we see that a shock to the long-run component of TFP growth results in a positive and persistent response to output growth. As the shock to TFP growth is persistent via the autocorrelation of x, this has long-lasting effects for output growth. This is compounded by the fact that the amount of capital increases in the economy. In the bottom left panel, we see that investment immediately shoots up, albeit less persistently. This happens in conjunction with an upwards movement in the price of capital, q, which is hence procyclical as well. As output growth does not respond following the impact of the shock, and investment does in a positive way, consumption growth decreases at time 0. While this seems odd in conjunction with a positive TFP shock, we see that consumption growth immediately rebounds to a positive level in period one and after.

#### 2.4.3. Adjustment versus Bankruptcy Costs

We start by examining the model results with respect to a perturbation of the household's adjustment costs, as given in Table 7. Broadly speaking, as we slow down the movement of capital, this should lead to a lower volatility in investment, dividends that are more procyclical, and larger equity premia. When adjustment costs are shut down (first column), the relative share of investment volatility is much higher and levered returns collapse to .30% on an annual basis. The second column is our baseline model, and already represents a roughly eight-fold increase in premia, to 2.51%. In the final column we shift adjustment costs to a, perhaps, unreasonable level. This brings the equity premium even higher, to about 4.5%. The adjustment cost mechanism works similar to how we would expect it to.

Next, we explore the effects of bankruptcy costs. In Table 8 we incrementally shift the monitoring costs from the baseline, one half of a percentage point, upwards to ten and twenty percent. We find that there is a very marginal effect on this dimension, amounting to only about eighty basis points on the final levered market return. This underscores the fact that the financing friction acts as a micro-founded adjustment cost to investment. The relative volatility of investment decreases roughly 25% in total as we shift the monitoring costs to the highest level. If we compare this tradeoff to those related to direct adjustment costs to capital, we find that, at least as a modeling device, physical adjustment costs are much more quantitatively appealing.

We discuss the impact of the bankruptcy costs on the behavior of the model, in Figure 12. We look at the change in the price of capital, output growth, and investment growth under different model configurations with respect to agency costs (given by  $\mu = .005, .10, .20$ ). In the previous paragraph, we discussed the fact that the financing friction is an adjustment

cost to investment. In this sense, increasing the value of  $\mu$  adds to the volatility of q and this is prominently displayed in the first graph. We see that the responses of q increase monotonically at time 0, as we adjust  $\mu$  upwards. As expected losses given default are higher to the intermediary, when monitoring costs increase, it will be the case that the amount of lending by banks will be less sensitive to the cycle. Put equivalently, the total amount of investment from the entrepreneurs will be less sensitive to business cycle shocks, or simply less volatile. This message is conveyed in the bottom left most panel. We see that indeed as monitoring costs increase, investment growth reacts less sensitively. This is an illustration of the fact that the monitoring friction acts as an adjustment cost.

## 2.4.4. Monitoring Costs, Leverage, and the Cyclicality of Returns

In Table 9 we examine how returns covary with the business cycle. In the first row we display the correlation between consumption growth and the leveraged excess market return. The second row is the same, except using output growth. Similar to data, our baseline calibration  $(\mu = .5\%)$ , delivers a procyclical, realized market return. This is largely due to the fact that when the economy is in a boom period, the price of capital (q) goes up, which means that the resale value on a unit of capital is procyclical. Additionally the rental rate on capital is procyclical in the model. Together these facts generate a correlation of roughly 25 - 30%. One can also see that increasing monitoring costs does not significantly change these results.

When we compute the same statistics regarding the effective credit spread in the model,  $r^e - r^k$ , we find that the baseline calibration delivers a countercyclical default premium, with a correlation of roughly -92%. As discussed earlier, we interpret this as a credit spread due to the fact that the difference is driven by the additional default risk being priced into the entrepreneur's return.<sup>6</sup> Put differently,

$$r^e - r^k \approx \frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})}$$

The intuition for this result is the following. From the model simulations we can see that leverage is countercyclical in the baseline model. This is given in the fifth and sixth lines. This implies that not only does equilibrium lending increase in bad times, but also too does default, as shocks are IID and increasing leverage is correlated with increasing default. As leverage is used to generate more capital, and broadly own more capital, the return has to be sufficient to warrant the default risk that is taken on by entrepreneurs. In particular, there must be higher return in bad times to compensate for the excess default, in those very

<sup>&</sup>lt;sup>6</sup>If we compute a variance decomposition of  $r^e - r^k$ , we can show that much of the variation is driven by the wedge in the entrepreneur's return.

times. This then provides the intuition behind the baseline result.

Using this very same intuition we can gain more insight as to why agency costs push the model towards delivering counterfactual results. As the monitoring costs increase, not only does the average amount of leverage decrease, but also leverage becomes procyclical. This is due to the fact that lending becomes highly restricted when aggregate states are low, or correspondingly when the price of capital is low. As a result of more equilibrium leverage in good times, and hence more default, then entrepreneurs must be compensated with additional returns to capital in good times. Put equivalently, the price of default risk is higher in good times. In line with this thought process, the table shows that increasing the monitoring cost raises the procyclicality of leverage and hence the credit spread. Moreover, even when we incorporate the agency cost friction into a setting with recursive preferences and adjustment costs, we are not able to overturn the results of Gomes et al. (2003). As a graphical confirmation of this logic, in Figure 13 we display the response of the credit spread with respect to changes in  $\mu$ . For a negligible amount of bankruptcy costs, the impulse is negative. When we increase the costs to higher levels, the movement becomes more pronounced in the positive direction, in accordance with higher rates of procyclicality.

## 2.4.5. Persistence of Output Growth

As discussed in Carlstrom and Fuerst, the inclusion of financial frictions creates a hump-shaped response to the level of output, and hence an endogenous persistence to the growth rate of output. We test whether we can make similar conclusions in our model, given that we calibrate to a wider array of macroeconomic and financial data using different modeling devices.

As displayed in the top right corner of Figure 12, we see that output growth is barely modified by the inclusion of monitoring costs. In particular we find that the shocks are indistinguishable across calibrations, but more relevantly, the autocorrelation is roughly equivalent. This can be deduced from the responses of output being on top one another. These sort of effects can be more precisely seen if we directly compute autocorrelations of macro aggregates, as in Table 11. In the top panel, we provide the first-order autocorrelation, of annualized data generated from the baseline model. The bottom two panels examine similar statistics shutting off various model elements, including adjustment costs to capital, as well as a lower Epstein-Zin friction. What we find is that, starkly, the autocorrelation of output growth is relatively unchanged in all three cases, when we increase the bankruptcy friction. In line with previous literature, the investment growth autocorrelation certainly increases as well, likely due to the contracting friction dampening the volatility of investment.

What explains the result with respect to output growth? While the new model frictions we incorporate (adjustment costs and recursive preferences) slightly change the *levels* of output growth autocorrelation, they do not affect the *in-sensitivity of autocorrelation* to the monitoring costs. This suggests that the use of shocks to growth rates of productivity, as opposed to levels of TFP as in previous literature, plays a substantial role in mitigating the persistence effects of financial frictions. It can be explained in the following way. Output growth is fundamentally a function of capital growth and TFP growth. As we embed TFP growth with a persistent component, by definition, this gives rise to the results we see.

## 2.4.6. The Role of Recursive Preferences

In Table 10 we examine the behavior of the model when shifting the intertemporal elasticity of substitution from  $\psi=2.5$  to values below 1. The first column represents the baseline version of the model. What we find is that the unlevered excess return on household capital decreases to -1.2%. A large amount of the shift downwards is due to the increase in the risk free rate. As  $\psi$  decreases agents are much more less willing to trade off consumption between periods. This will decrease the need for the hedging asset and the risk free rate increases in equilibrium as a result. Moreover, as  $\psi$  moves towards  $\frac{1}{\gamma}$ , the model returns to one with time-separable preferences, and wipes away any existing risk premia.

#### 2.4.7. A Comparison to He and Krishnamurthy

He and Krishnamurthy (2013), hereafter HK, discuss the importance of intermediation for asset markets. In a set up with households, intermediaties, and riskless and risky asset markets, they are able to show that intermediation constraintedness and countercyclical leverage are key drivers for reaching large levels of risk premia. As our model provides a differing opinion on the quantitative relevance of intermediation, it is natural to ask how can we reconcile our differences.

There are key modeling contrasts between our setup and HK. One difference is that dividends with respect to the risky asset are calibrated to be an exogenous cash flow, as opposed to the endogenous dividends in our model. As discussed earlier, endogenous dividends pose a significant challenge in terms of matching both macro quantities and financial prices in the data. A more substantial element in HK is that the intermediary is the marginal agent in pricing risky assets. This means that asset prices are determined through the covariation of the specialist's pricing kernel and the risky asset return. In our work, the household is still the marginal agent as we use the recursive preference based Euler equation to evaluate equity returns. If the focus is on understanding the effect of financial frictions on broader asset markets, then it might be more appropriate to utilize our setup. This is due to the rea-

son that equity markets are operated to a great extent with household capital (via mutual and pension funds, money market accounts), and resultingly household preferences seem to be a central concern. Additionally, our analysis with respect to macroeconomic quantities requires individuals that are critical participants. It would be tough to argue that the high wealth clients that use proprietary trading firms to access risky assets in He and Krishnamurthy (2013), can be interpreted as representative of the larger macroeconomy.

HK also argues that the existence of an occasionally binding equity constraint is crucial to drive risk premia upwards. When the wealth of banks drop too low, intermediaries become equity capital constrained, and exceedingly leveraged. In order to compensate investors for the bank's risky positions, the required excess return on the risky asset increases dramatically. Similarly in our model, capital-producing firms face constraints on their investment. However, the net worth constraint dictates how much debt, as opposed to equity, the entrepreneurs can take on. More generally however, we argue that the optimal calibration that captures key features of the data, is one that has a negligible amount of agency costs. Put otherwise, the most realistic economy is one in which investment is not constrained by financial frictions.

#### 2.5. Conclusion

We revisit the quantitative role of financial frictions and discuss its impact on financial markets. In particular, the use of Epstein and Zin preferences and shocks to growth rates of productivity make a large impact on how we plausibly interpret the effects of costly state verification. After fitting a model jointly to business cycle and financial data, without monitoring costs associated with the entrepreneur's contract, we then activate the friction and examine its effects. For a very mild increase in equity risk premia, our setup sugests counterfactual results that are associated with the intermediation contract: a large decrease in investment volatility and a procylicality of credit spreads. In total, we are able to suggest that volatility of the price of capital due to physical adjustment costs, as opposed to contracting frictions, allows for more quantitatively satisfying results.

- 2.6. Appendix: Model Timing and Related Issues
- 2.6.1. Model Timing (from start of t to end of t)
  - (I) Aggregate shock is realized,  $Z_t$ .
  - (II) Households / entrepreneurs supply previously chosen capital  $(k_t, k_t^e)$  and labor  $(l_t, l_t^e)$  to Final Goods Producer. Output  $(Y_t)$  is created and rental payments (capital and labor income at now determined prices) are given to households and entrepreneurs. The budget constraint of the household will read:

$$c_t + \underbrace{q_t \left( k_{t+1} - (1 - \delta) k_t + \Phi_{a,t} k_t \right)}_{\text{Goes to Intermediary Intraperiod}} = \underbrace{w_t l_t + r_t k_t}_{\text{Realized Income}}$$

(III) The intermediary collects the non-consumption portion of realized income and uses it to lend to the entrepeneur. Each entrepreneur's loan will be  $i_t - n_t$  and the amount of loan will equate:

$$q_t (k_{t+1} - (1 - \delta)k_t + \Phi_{a,t}k_t) = (i_t - n_t) = q_t i_t g(\bar{\omega}_t)$$

At this point, the entrepreneur's net worth will be given by:

$$n_t = w_t^e + (r_t + q_t(1 - \delta)) k_t^e$$

Note here that  $k_t^e$  represents an average capital for the entire set of entrepreneurs, taking into account those that default last period. This allows us to not keep track of each individual entrepreneur.

(IV) Entrepreneurs enter into a contract with intermediaries and decide terms such that they solve:

$$\operatorname{Max}_{\{\bar{\omega}_t, r_t^l, i_t\}} q_t i_t f(\bar{\omega}_t)$$
s.t. 
$$q_t i_t g(\bar{\omega}_t) \ge i_t - n_t$$

This pins down the lending rate, cutoff threshold, and investment level. All entrepreneurs (total measure one) now invest the level  $i_t$ .

(V) The idiosyncratic shock is realized and if  $\omega_t^j > \bar{\omega}_t$  then the *j*th individual will do the following: (a) pay back loan to the bank at  $q_t\bar{\omega}i_t$ , (b) choose a consumption level,  $c_t^{e,j}$  and (c) choose next period capital,  $k_{t+1}^{e,j}$ .

However, if  $w_t^j < \bar{\omega}_t$ , then the jth entrepeneur will set  $c_t^{e,j} = 0$  and  $k_{t+1}^{e,j} = 0$ . In this case also, the bank will pay a monitoring cost  $\mu$  which will count as a loss in capital goods.

(VI) The bank will take both loans that are paid back fully (in solvent entrepreneurs' case) and also claim remaining assets minus monitoring costs (in non-solvent cases) and pay back the household at a short term borrowing rate of 1. This will reenter the household's budget constraint as:

$$q_t (k_{t+1} - (1 - \delta)k_t + \Phi_{a,t}k_t) = q_t i_t g(\bar{\omega}_t)$$

(VII) The entrepreneurs that are living (realized shocks greater than  $\bar{\omega}_t$ ) now choose a consumption level. Instead of keeping track of individual consumption and capital plans, we solve for the average consumption and capital levels accounting for income that would go to the entire measure of entrepreneurs. This can be seen from the budget constraint:

$$c_t^e + q_t k_{t+1}^e = \underbrace{q_t i_t f(\bar{\omega}_t)}_{\text{Total income across measure}}$$

Hence  $\{c_t^e, k_{t+1}^e\}$  are average quantities across  $\eta$  entrepeneurs, some of which have defaulted on their loans.

2.6.2. Average Entrepreneur's Decision Problem

Entrepreneur j's problem, regardless of default, is given by:

$$\max_{\{c_t^{e,j}, k_{t+1}^{e,j}\}} E \sum_{t=0}^{\infty} (\beta \gamma^e)^t c_t^{e,j}$$
s.t. 
$$c_t^{e,j} + q_t k_{t+1}^{e,j} = q_t i_t \max\{0, \omega_t^j - \bar{\omega}_t\}$$

Investment is not entrepreneur dependent as it is decided before the shock is realized. We know at each t,  $\Phi(\bar{\omega}_t)$  will default, with consumption and next period capital equal to zero, so *average* policies will be given by:

$$c_t^e = \int_{\omega_t^j > \bar{\omega_t}} c_t^{e,j} d\Phi\left(\omega_t^j\right) \qquad k_{t+1}^e = \int_{\omega_t^j > \bar{\omega_t}} k_{t+1}^{e,j} d\Phi\left(\omega_t^j\right)$$

and the overall income (RHS of the budget constraint) will be given by:

$$q_t i_t \int_{\omega_t^j > \bar{\omega}_t} (\omega_t - \bar{\omega}_t) d\Phi \left(\omega_t^j\right) = q_t i_t f\left(\bar{\omega}_t\right)$$

using the definition from earlier. To get the average utility we can utilize the risk neutrality to receive the time t utility to be:

$$(\beta \gamma^e)^t \Phi\left(\bar{\omega}_t\right)(0) + (\beta \gamma^e)^t \int_{\omega_t^j > \bar{\omega}_t} c_t^{e,j} d\Phi\left(\omega_t^j\right) = (\beta \gamma^e)^t c_t^e$$

# 2.6.3. Accounting for Market Clearing

Note that the market clearing is given by:

(Capital Markets) 
$$K_{t+1} = (1 - \delta)K_t + \underbrace{\eta i_t (1 - \mu \Phi(\bar{\omega}_t))}_{\text{Investment across all entrepreneurs}}$$
  
(Goods Markets)  $Y_t = F(K_t, Z_t L_t, Z_t L_t^e) = (1 - \eta)c_t + \eta c_t^e + \eta i_t$ 

The reason we take into account the actual distribution, via  $\Phi$ , in the first equation is because investment is made at the per-entrepeneur level, before default is realized. In the second equation,  $c_t^e$  is accounted for as an average, taking into account the entire distribution of possibilities. Because it is an average for the unit measure of entrepreneurs, we also multiply it by  $\eta$ .

# 2.7. Appendix: Analytical Derivations

#### 2.7.1. Household Problem

The decision problem of the Epstein-Zin household can be written as:

$$U_{t} = \max_{\{c_{t}, l_{t}, k_{t+1}, s_{t+1}\}} \left( (1 - \beta) \left( V(c_{t}, l_{t}) \right)^{\frac{1-\gamma}{\theta}} + \beta \left( E_{t} \left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}}$$

s.t. 
$$c_t + q_t k_{t+1} + q_t \Phi_a(k_t, k_{t+1}) k_t = w_t l_t + r_t k_t + q_t (1 - \delta) k_t$$

where  $V(c_t, l_t)$  represents the *intra*temporal utility from consumption and labor choice. The price of capital is  $q_t$  and  $\delta$  is its depreciation rate. The EZ parameters are standard with  $\gamma$  representing risk aversion,  $\psi$  representing IES, and  $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ . We proceed to solve the household problem with the following Lagrangian:

$$\mathcal{L} = U_t + \lambda_t \left( w_t l_t + r_t k_t + q_t (1 - \delta) k_t - c_t - q_t k_{t+1} - q_t \Phi_a \left( k_t, k_{t+1} \right) k_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial c_t} : \qquad \frac{\theta}{1 - \gamma} \left( U_t^{\frac{1 - \gamma}{\theta}} \right)^{\frac{\theta}{1 - \gamma} - 1} (1 - \beta) (1 - \frac{1}{\psi}) \left( V_t \right)^{-\frac{1}{\psi}} V_{c.t} - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial l_t} : \qquad \frac{\theta}{1 - \gamma} \left( U_t^{\frac{1 - \gamma}{\theta}} \right)^{\frac{\theta}{1 - \gamma} - 1} (1 - \beta) (1 - \frac{1}{\psi}) \left( V_t \right)^{-\frac{1}{\psi}} V_{l.t} + \lambda_t w_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : \qquad \frac{\theta}{1 - \gamma} \left( U_t^{\frac{1 - \gamma}{\theta}} \right)^{\frac{\theta}{1 - \gamma} - 1} \beta \left( \frac{1}{\theta} \right) \left( E_t \left[ U_{t+1}^{1 - \gamma} \right] \right)^{\frac{1}{\theta} - 1} E_t \left[ (1 - \gamma) U_{t+1}^{-\gamma} \frac{\partial U_{t+1}}{\partial k_{t+1}} \right] - \lambda_t \left( q_t \left[ 1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t \right] \right) = 0$$

$$\text{Invelope} : \qquad \frac{\partial U_t}{\partial k_t} = \frac{\partial \mathcal{L}}{\partial k_t} = \lambda_t \left( r_t + q_t (1 - \delta) - q_t \left[ \Phi_{a,t} + \frac{\partial \Phi_{a,t}}{\partial k_t} k_t \right] \right)$$

If we bring  $\lambda$  terms to the right hand side of the first two equations, and divide the first equation by the second we will receive the following relationship governing consumption-labor tradeoffs:

$$w_t V_c(c_t, l_t) = -V_l(c_t, l_t)$$

To receive the Euler condition, we start by dividing the third FOC by the first one and receive:

$$q_t \left( 1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t \right) = E_t \left[ \frac{\beta}{1 - \beta} \frac{U_{t+1}^{-\gamma}}{\left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta} - 1}} \frac{V_t^{\frac{1}{\psi}}}{V_{ct}} \frac{\partial U_{t+1}}{\partial k_{t+1}} \right]$$

If we substitute the Envelope condition into  $\frac{\partial U_{t+1}}{k_{t+1}}$  we receive:

s.t. 
$$M_{t+1} = \beta \frac{U_{t+1}^{\frac{1}{\psi} - \gamma}}{\left(E_t \left[U_{t+1}^{1 - \gamma}\right]\right)^{1 - \frac{1}{\theta}}} \left(\frac{V_{t+1}}{V_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{c,t+1}}{V_{c,t}}\right)$$

$$R_{t+1}^k = \frac{r_{t+1} + q_{t+1}(1 - \delta) - q_{t+1} \left(\Phi_{a,t+1} + \frac{\partial \Phi_{a,t+1}}{\partial k_{t+1}} k_{t+1}\right)}{q_t \left(1 + \frac{\partial \Phi_{a,t}}{\partial k_{t+1}} k_t\right)}$$

which matches the result from the main text.

# 2.7.2. Intermediary Contract

The contract between intermediaries and entrpreneurs is a static, one period agreement, that maximizes the entrepreneur's portion of the surplus while ensuring that the participation constraint of the intermediary holds.

$$\operatorname{Max}_{\{\bar{\omega}_t, i_t\}} q_t i_t f(\bar{\omega}_t)$$
s.t. 
$$q_t i_t g(\bar{\omega}_t) \ge i_t - n_t$$

where  $n_t$  and  $q_t$  are assumed to be known at the time of the contract.

As the lender is assumed to be risk neutral we know that his participation constraint will bind, which will result in:

$$i_t = \frac{n_t}{1 - q_t g(\bar{\omega}_t)} = i(\bar{\omega}_t)$$

Notice that solving for i is interchangeable with  $\omega_t$  as this is a one-to-one mapping (based on properties of f). Hence the maximization problem has  $\omega_t$  as a sole control variable. The first order condition of the original maximand is:

$$\frac{\partial}{\partial \bar{\omega}_{t}} (q_{t} i_{t} f(\bar{\omega}_{t})) = q_{t} i_{t} f'(\bar{\omega}_{t}) + q_{t} f(\bar{\omega}_{t}) i'(\bar{\omega}_{t}) = 0$$

$$\iff = i_{t} f'(\bar{\omega}_{t}) + f(\bar{\omega}_{t}) i_{t} \frac{q_{t} g'(\bar{\omega}_{t})}{1 - q_{t} g(\bar{\omega}_{t})} = 0$$

Canceling  $i_t$  and rearranging terms we receive the condition provided in the main text.

### 2.7.3. Entrepreneur Problem

The Lagrangian of the entrepreneur's problem can be written as:

$$\mathcal{L}^{e} = \sum_{t=0}^{\infty} \sum_{s^{t}} \pi \left( s^{t} \right) \left( \beta \gamma^{e} \right)^{t} \left[ c_{t}^{e} + \mu_{t} \left( q_{t} i_{t} f_{t} - c_{t}^{e} - q_{t} k_{t+1}^{e} - q_{t} \Phi_{a,t}^{e} k_{t}^{e} \right) \right]$$

where the entrepreneur's choice variables are simply  $\{c_t^e, k_{t+1}^e\}$  as investment,  $i_t$  is predetermined due to the ex-ante contract.  $\mu_t$  denotes the Lagrange multiplier on the budget constraint. The first order conditions will be:

$$\frac{\partial \mathcal{L}^e}{\partial c_t^e} : \qquad \pi \left( s^t \right) \left( \beta \gamma^e \right)^t \left[ 1 - \mu_t \right] = 0$$

$$\iff \mu_t = 1$$

$$\begin{split} \frac{\partial \mathcal{L}^e}{\partial k_{t+1}^e} : & \quad \pi \left( s^t \right) (\beta \gamma^e)^t \left[ \mu_t \left( -q_t \left( 1 + \frac{\partial \Phi_{a,t}^e}{\partial k_{t+1}^e} k_t^e \right) \right) \right] \\ & \quad + \sum_{s^{t+1}} \pi(s^{t+1}) \left( \beta \gamma^e \right)^{t+1} \left[ \mu_{t+1} \left( q_{t+1} f(\bar{\omega}_{t+1}) \frac{\partial i_{t+1}}{\partial k_{t+1}^e} - q_{t+1} \left( \Phi_{a,t+1}^e + \frac{\partial \Phi_{a,t+1}^e}{\partial k_{t+1}^e k_{t+1}^e} \right) \right) \right] = 0 \\ \text{s.t.} & \quad \frac{\partial i_{t+1}}{\partial k_{t+1}} = \frac{\partial n_{t+1} / \partial k_{t+1}}{1 - q_{t+1} q_{t+1}} = \frac{r_{t+1} + (1 - \delta) q_{t+1}}{1 - q_{t+1} q_{t+1}} \end{split}$$

In the second condition, we treat  $\bar{\omega}$  as a price which explains why there are no partials of  $f(\bar{\omega})'$  or  $g(\bar{\omega})'$  with respect to  $k^{e'}$ . Also we use the definition of net worth as earlier introduced in the text:

$$n_t = w_t^e + r_t k_t^e + q_t (1 - \delta) k_t^e$$

If we rewrite the latter first order condition and plug in  $\mu$ , we will receive the result from the text:

$$E_{t} \left[ \beta \gamma^{e} \left\{ \frac{\left( \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right) \left( q_{t+1} (1 - \delta) + r_{t+1} \right) - q_{t+1} \left( \frac{\partial \Phi_{a,t+1}^{e}}{\partial k_{t+1}^{e}} k_{t+1}^{e} + \Phi_{a,t+1}^{e} \right)}{q_{t} \left( 1 + \frac{\partial \Phi_{a,t}^{e}}{\partial k_{t+1}^{e}} k_{t}^{e} \right)} \right\} \right] = 1$$

# 2.8. Appendix: Data Sources

### 2.8.1. Consumption

We construct real, per capita consumption  $(C_t)$  by summing real measures of nondurables and services consumption. Nominal consumption data for these items come from National Income and Product Accounts (NIPA) table 1.1.5. To get corresponding price deflators for real quantities, we use those in table 1.1.9. Per-capita measures utilize population data in NIPA table 2.1.

#### 2.8.2. Investment

To measure investment  $(I_t)$  we use fixed investment, from NIPA table 1.1.5. We adjust this by its price deflator in table 1.1.9 and the population series from earlier.

### 2.8.3. Output

Output for model moments is provided by the sum of the constructed consumption and investment. As there is no government or trade in the model these are the only series of quantitative relevance for us.

#### 2.8.4. Inflation and Financial Market Data

The annual, nominal data on risk free rates and market returns come from Ken French's website. We compute the market excess return as the annualized difference between the two. To calculate the real risk free rate, we adjust the risk free rate by the growth rate of the GDP deflator index from NIPA table 1.1.4.

To obtain statistics regarding credit spreads, we use Moody's data series on seasoned BBB and AAA corporate yields from the St. Louis FRED.

Table 5: Calibration Parameters for Baseline Model

	37.1	
Parameter	Value	Selection Criteria
eta	.991	Approximately match $E(r^f)$
$rac{\zeta}{\delta}$	1	Labor Supply Shut Down
$\delta$	.02	
$lpha_k$	.3000	
$lpha_l$	.6999	
$lpha_e$	.0001	
$\eta$	.1	
$\sigma_{\omega}$	.75	Set to receive leverage ratio of roughly 30%
$\gamma_e$	_	
$\mu$	.5%	Key parameter to control amount of bankruptcy costs
$\gamma$	15	
$\psi$	2.5	
$g_z$	.45%	Approximately match $E(\triangle y)$
$arphi_z$	2%	Approximately match $\sigma(\triangle y)$
$ ho_x$	.96	Croce(2014)
$arphi_x$	$1 \times \varphi_z$	Capture appropriate LRR portion of volatility
$\phi$	10	
$\sigma_{lev}$	.0325	Croce (2014)

This table provides the baseline calibration for the model, at a quarterly frequency. All parameters with z and x subscripts refer to parameters that calibrate the growth rate process for TFP. The wedge in the discount rates of the entrepreneur and household is given through  $\gamma_e$  while that of the household is  $\beta$ . The level of monitoring costs for the intermediary is provided through  $\mu$ .

Table 6: Baseline Model Fit

		T
Statistic	Data (Annual, 1929 - 2008)	Baseline Model
Macro (Annual):		
$\sigma(\triangle c)/\sigma(\triangle y)$	$.641\;(.578,.664)$	.840
$\sigma(\triangle i)/\sigma(\triangle y)$	4.46 (3.19, 5.19)	2.07
$\sigma(\triangle y)(\%)$	$3.36 \ (1.90,  4.35)$	3.74
$ ho(\triangle c, \triangle i)$	.678 (.436, .889)	.718
Financial (Annual):		
$E(r^f)(\%)$	.722 (690, 2.16)	.72
$\sigma(r^f)(\%)$	3.78 (2.02, 4.78)	.93
$E(R_{ex,t+1}^{lev})(\%)$	5.04 (3.07, 7.03)	2.51
$\sigma(R_{ex.t+1}^{lev})(\%)$	20.3 (16.4, 23.6)	7.05
$Sharpe(R_{ex,t+1}^{lev})$	.248 (.137, .390)	.356
$\rho(\triangle c, R_{ex}^{lev})$	.224 (.008, .333)	.294
$\rho(\triangle y, R_{ex}^{lev})$	.209 (099, .360)	.358
$\rho(\triangle c, r^e - r^k)$	503 (637,244)	922
$\rho(\triangle y, r^e - r^k)$	539 (685,345)	969
	, ,	

This table provides the results of the baseline calibration. The in-sample moment estimates are constructed from annual data (1929-2008). Numbers in parentheses provide a bootstrapped standard error distribution at the (5%, 95%) levels. Details for data construction are provided in the Appendix. The baseline calibration results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 7: Role of Household Adjustment Costs

	1 0	/ 10	/ 00
Statistic	$\phi = 0$	$\phi = 10$	$\phi = 20$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.680	.840	923
$\sigma(\triangle i)/\sigma(\triangle y)$	2.69	2.07	1.91
$\sigma(\triangle y)(\%)$	3.83	3.74	3.70
$\rho(\triangle c, \triangle i)$	.52	.72	.68
Financial (Annual):			
$E(r^f)(\%)$	.43	.72	.72
$\sigma(r^f)(\%)$	.90	.93	.96
$E(r^k - r^f)(\%)$	.15	1.25	2.26
$\sigma(r^k - r^f)(\%)$	.30	1.44	2.21
$Sharpe(r^k - r^f)$	.492	.870	1.02
$E(r^e - r^f)(\%)$	4.45	4.31	4.46
$\sigma(r^e - r^f)(\%)$	1.08	1.01	1.01
$Sharpe(r^e - r^f)$	4.10	4.28	4.43
P ( )	1.10		1.10
$E(R_{ex,t+1}^{lev})(\%)$	.30	2.51	4.52
$\sigma(R_{ex,t+1}^{lev})(\%)$	6.53	7.05	7.77
$Sharpe(R_{ex,t+1}^{lev})$	.046	.356	.582

This table provides the results of perturbing the level of adjustment costs, that enter the household's budget constraint. The results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 8: Role of Bankruptcy Costs

Statistic	$\mu = .5\%$	$\mu = 10\%$	$\mu = 20\%$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.840	.861	.881
$\sigma(\triangle i)/\sigma(\triangle y)$	2.07	1.65	1.60
$\sigma(\triangle y)(\%)$	3.74	3.71	3.70
$\rho(\triangle c, \triangle i)$	.72	.78	.78
Financial (Annual):			
$E(r^f)(\%)$	.72	.49	.42
$\sigma(r^f)(\%)$	.93	.93	.94
$E(r^k - r^f)(\%)$	1.25	1.56	1.64
$\sigma(r^k - r^f)(\%)$	1.44	1.75	1.83
$Sharpe(r^k - r^f)$	.870	.892	.896
$E(r^e - r^f)(\%)$	4.31	14.8	26.1
$\sigma(r^e-r^f)(\%)$	1.01	2.94	3.63
$Sharpe(r^e - r^f)$	4.28	5.04	7.17
$E(R_{ex.t+1}^{lev})(\%)$	2.51	3.12	3.29
$\sigma(R_{ex,t+1}^{lev})(\%)$	7.05	7.31	7.39
$Sharpe(R_{ex,t+1}^{lev})$	.356	.427	.445
pc(rex,t+1)	.000	.741	OFF.

This table provides the results of perturbing the level of monitoring costs, where we fix the adjustment cost parameter,  $\phi=10$ . The results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 9: The Cyclicality of Returns – Current Model

Statistic	Data (Annual, 1929 - 2008)	$\mu = .5\%$	$\mu = 10\%$	$\mu = 20\%$
$\rho(\triangle c, R_{ex}^{lev}) \\ \rho(\triangle y, R_{ex}^{lev})$	.224 (.008, .333)	.243	.279	.303
	.209 (099, .360)	.299	.349	.362
$ \rho(\triangle c, r^e - r^k) \\ \rho(\triangle y, r^e - r^k) $	503 (637,244)	922	035	.199
	539 (685,345)	969	.162	.367
$\rho(\triangle c, \log(i_t/n_t))$ $\rho(\triangle y, \log(i_t/n_t))$	_	496	.212	.305
	_	323	.430	.499
$E\left[rac{i_t-n_t}{k_t^e} ight] \ E\left[rac{i_t-n_t}{i_t} ight]$		.549 .335	.147	.145

This table provides the results of perturbing the level of monitoring costs, where  $\rho$  represents contemporaneous correlation statistics. The in-sample moment estimates are constructed from annual data (1929 – 2008). Numbers in parentheses provide a bootstrapped standard error distribution at the (5%, 95%) levels. Details for data construction are provided in the Appendix. Model results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above.

Table 10: Role of IES

Statistic	$\psi = 2.5$	$\psi = 1.5$	$\psi = .8$
Macro (Annual):			
$\sigma(\triangle c)/\sigma(\triangle y)$	.840	.843	.954
$\sigma(\triangle i)/\sigma(\triangle y)$	2.07	1.94	2.13
$\sigma(\triangle y)(\%)$	3.74	3.69	3.61
$\rho(\triangle c, \triangle i)$	.72	.86	.58
Financial (Annual):			
$E(r^f)(\%)$	.72	2.10	3.88
$\sigma(r^f)(\%)$	.93	1.39	2.19
$E(r_k - r_f)(\%)$	1.25	.50	49
$\sigma(r_k - r_f)(\%)$	1.44	1.28	1.57
$Sharpe(r_k - r_f)$	.870	.394	310
$E(r_e - r_f)(\%)$ $\sigma(r_e - r_f)(\%)$ $Sharpe(r_e - r_f)$	4.31 1.01 4.28	3.44 1.44 2.39	2.75 2.25 1.22
$E(R_{ex}^{lev})(\%)$	2.51	1.01	97
$\sigma(R_{ex}^{lev})(\%)$	7.05	6.94	7.20
$Sharpe(R_{ex}^{lev})$	.356	.146	135

This table provides the results of perturbing the IES parameter that enters the household preferences, where we also fix the household adjustment costs parameter at  $\phi=10$ . The calibration results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above. All statistics refer to quantities that are defined in the main portion of the text.

Table 11: Persistence of Macroeconomic Variables

Statistic	Data (Annual, 1929 - 2008)	u = 5%	u = 10%	$\mu = 20\%$		
	Data (11111dai, 1929 2000)	$\mu = .070$	$\mu - 1070$	$\mu = 2070$		
	$Baseline\ Mo$	odel:				
$AR1(\triangle c)$	.480 (.222, .562)	.515	.490	.449		
$AR1(\triangle i)$	.435 (.200, .580)	018	.280	.337		
$AR1(\triangle y)$	.444 (.125, .589)	.406	.408	.408		
		•	•	'		
	No Adjustment Costs ( $\phi = 0$ ):					
		1				
$AR1(\triangle c)$	$.480 \; (.222,  .562)$	.795	.516	.513		
$AR1(\triangle i)$	$.435 \; (.200,  .580)$	009	.298	.321		
$AR1(\triangle y)$	.444 (.125, .589)	.432	.437	.438		
Lower Epstein-Zin Friction ( $\psi = .8$ ):						
4 D1 / A \	400 ( 000 - 700)		1 070	055		
$AR1(\triangle c)$	.480 (.222, .562)	.335	.378	.355		
$AR1(\triangle i)$	.435 (.200, .580)	061	.203	.273		
$AR1(\triangle y)$	$.444 \ (.125, .589)$	.373	.374	.374		

This table provides the first order autocorrelation statistics of macroeconomic growth variables. The in-sample moment estimates are constructed from annual data (1929-2008). Numbers in parentheses provide a bootstrapped standard error distribution at the (5%, 95%) levels. Details for data construction are provided in the Appendix. Model results are generated through a long sample simulation (population) of 50000 quarters. After a burn-in of 5000 quarters, we annualize all moments, in log terms and report figures above.

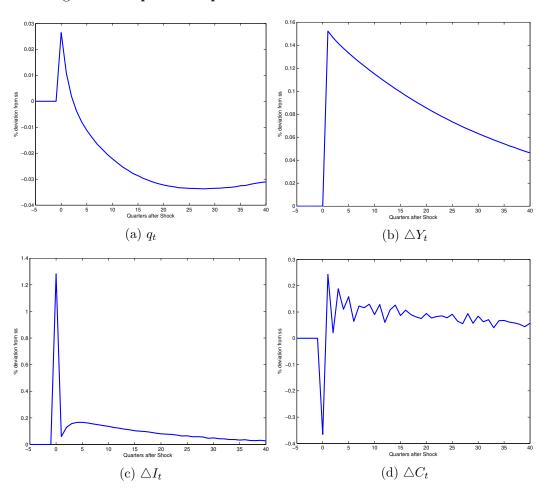


Figure 11: Impulse Response Functions – Baseline Calibration

Impulse response functions are given here with respect to a one standard deviation shock to innovations of the long run component of TFP growth,  $\epsilon_x$ . Parameters are fixed at baseline values. All units are given in quarterly values. To compute impulses, we simulate 200 economies under two sets of shocks, one of which includes an additional long run growth impulse. We then take the average, across all economies, of the deviation of responses for both sets of shocks.

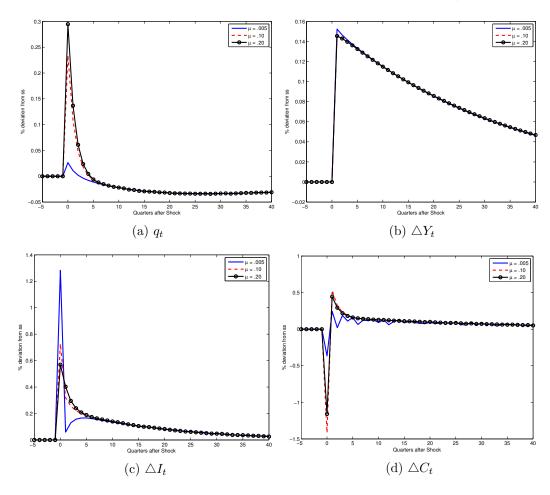
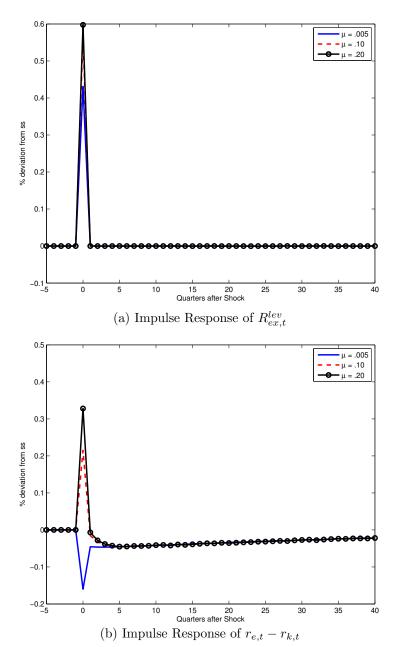


Figure 12: Impulse Response Functions – Shifting  $\mu$ 

Impulse response functions are given here with respect to a one standard deviation shock to innovations of the long run component of TFP growth,  $\epsilon_x$ . The solid, dashed, and lines marked with a circle represent model responses under different values of  $\mu$  (.005, .10, .20). All units are given in quarterly values. To compute impulses, we simulate 200 economies under two sets of shocks, one of which includes an additional long run growth impulse. We then take the average, across all economies, of the deviation of responses for both sets of shocks.

Figure 13: Impulse Response Functions – Financial Market Variables



Impulse response functions are given here with respect to a one standard deviation shock to innovations of the long run component of TFP growth,  $\epsilon_x$ . The solid, dashed, and lines marked with a circle represent model responses under different values of  $\mu$  (.005, .10, .20). All units are given in quarterly values. To compute impulses, we simulate 200 economies under two sets of shocks, one of which includes an additional long run growth impulse. We then take the average, across all economies, of the deviation of responses for both sets of shocks.

# Chapter 3: Corporate Debt Maturity and the Real Economy

# 3.1. Introduction

The financial crisis of the late 2000's placed debt maturity concerns at the forefront of the economic policy debate. As firms with relatively more short-term debt were exposed to rollover and liquidity crises, new questions arose as to how long-term debt could affect firm and economic stability. While some academic work tackle these issues, they do not provide a link between debt maturity and investment behavior. My work provides a comprehensive, quantitative framework that connects debt maturity choice, corporate bond yields, and the endogenous assets of the firm's balance sheet.

Empirically, I document several novel facts that discuss the positive link between business cycles and the long-term debt share. At the aggregate level, I use US Federal Reserve Financial Accounts data to identify the portion of total non-financial, corporate liabilities that are considered long-term. This ratio is significantly correlated with GDP and aggregate investment growth, with predictive power up to six quarters in the future. Using Compustat data at the firm level as well, I show that when firms shift their long-term debt ratio to a longer average maturity, profitability and investment rates are higher. These results are robust to controlling for a variety of macroeconomic and financial factors.

In order to understand these phenomena, I design a dynamic, heterogeneous firm, capital structure model in which corporations optimally issue equity and debt of short and long maturities. Using external financing and cash flows from production, firms finance investment into profit-generating capital. I compute and calibrate the model to target cross-sectional and aggregate data related to investment, leverage, default, and credit spreads.

The model generates a pro-cyclical long-term debt ratio through an endogenously generated, time-varying pecking order of capital market securities. The framework also implies that stable firms, which are more capitalized and have a larger portion of long-term debt, matter more for the real economy. Despite higher quantities of leverage and long-term debt, their average credit spreads are lower. Altogether, endogenous investment plays a crucial role in

<sup>&</sup>lt;sup>1</sup>It is well-documented that the over abundance of short-term liabilities on corporate balance sheets helped cause runs in commercial paper and repurchase agreement markets during the financial crisis (see eg. BNP Paribas, Bear Stearns, Lehman Brothers, General Electric). In non-crisis events as well, survey evidence suggest that CFO's take on long-term debt to "reduce risk of having to borrow in 'bad times'" (see Graham and Harvey, 2002; Servaes and Tufano, 2006).

<sup>&</sup>lt;sup>2</sup>My focus on the endogenous asset choice of firms separates my paper from Chen et al. (2013) and He and Milbradt (2016), where cash flows are taken to be exogeneous. Ivashina and Scharfstein (2010) and Campello et al. (2011) discuss the impact of the financial crisis event on investment, but do not discuss the explicit role of long-term debt in a larger context.

driving leverage and debt maturity choice, default dynamics, and credit spreads.

The conditions of Miller and Modigliani (1959) would suggest that firms are indifferent to debt of varying maturities, under a pari passu treatment of securities. To break this indifference, the model splits the proportion of distress costs that are held by short and long-term debt. Each period, firms have access to debt issuance in a short-term, collateralized debt contract which prices in relatively less default risk. Meanwhile the longer term debt market inherits a greater burden of default risk and associated distress costs.

In a higher aggregate state of the model economy, firms optimally choose to invest more than they generate in profits. In order to do so, they need to acquire external financing. Due to a tax advantage of debt, and the fact that short-term debt is the least costly form of financing, via the collateral constraint, corporations seek to first finance their need for cash using short-term debt issuance.

For additional financing, the firm has the choice of issuing long-term debt or equity. In positive economic environments, it is desirable for corporations to take on more long-term debt as its effective cost is lower; this is due to a lower probability of default that results in lower expected distress costs. Put differently, in good states of the world, yields on long-term debt compare favorably to issuance costs in equity.

In poor aggregate states, when the marginal gains from investing are lower, the firm doesn't require as much external financing. However, if some firms do seek to externally finance more than is available through short-term markets, the marginally higher credit spreads and effective costs of longer term debt make it an unattractive option. In this case, firms would rather obtain external financing using equity than long-term debt.

As my model involves a heterogeneous firm setup, I also provide implications for the cross section. Sorting firms on distance to default, I find that firms that are closer to exiting have higher credit spreads, less capital, less leverage, and less long-term debt. The italicized statements are counter-intuitive; we would expect leverage to have a negative relationship with firm stability. These results directly imply that in the model's equilibrium cross-section, capital is a much larger driver of firm default. Furthermore, these findings suggest that economic stability is positively linked to a higher long-term debt ratio.

To underscore the importance of endogenous investment, I show that default events are largely precipitated by a joint drop in productivity levels and capital. During a sequence of negative productivity shocks in an economic recession, eventually-defaulting firms disinvest in response to a decreasing marginal product of capital. However, due to the burden of long-term debt on their balance sheet, firms eventually choose to exit. Firms do have the

option to purchase back debt, but for defaulting firms, this option becomes limited as they have reduced funds from production and constraints on borrowing from short-term debt markets.

In comparison to the classical leverage and default framework of Leland and Toft (1996), there are three major differences: (i) via investment, firms shift the level of capital resulting in an endogenous dividend stream, (ii) the firm continually rebalances the book values of short and long-term debt (a setup with "non-commitment"), (iii) when using external financing, the firm chooses between additional equity issuance and debt issuance of two different maturity types.

As documented in Chatterjee and Eyigungor (2012), models that feature long-term debt with fair pricing have difficulties with numerical convergence. Due to the discrete nature of firm default and the dependence on future policy functions, the long-term debt pricing function fluctuates greatly, which leads to non-convergence in value as well. In order to remedy this problem, I extend the computational method cited in the above paper, to allow for a simultaneous choice of both capital and debt policies. The additional choice variables complicate the problem substantially.<sup>3</sup> I introduce zero-mean IID noise with a low amount of variance into the dividend payments. By calculating default breakpoints perfectly as a function of these noise variables and taking expectations over default and non-default regions of the noise, I am able to smooth out the discrete jumps and improve convergence of the model.

Having described the key results, I now provide a roadmap for the rest of the paper. I conclude this section by providing a literature review. In the following section, I discuss the procyclicality of the long-term debt share. The third section is dedicated to discussing the model I use to study these issues, via dynamic issuance of multiple debt maturities. Following this, I delve into the quantitative implications arising from the model, including the key mechanisms and cross-sectional implications. In the final section I conclude.

**Previous Literature.** This paper relates to many strands of literature regarding corporate credit spreads, capital structure, and the macroeconomy. I discuss my work in the context of each area and provide differences. The overarching theme is that I connect capital structure with multiple debt maturities, endogenous investment and output, and asset prices in a dynamic structural model.

<sup>&</sup>lt;sup>3</sup>In the cited paper, consumer income is taken to be an exogenous process, while in my paper the firm's income is chosen endogenously, as investment is a choice variable. My extension compares very similarly to the methodology used in Gordon and Guerron-Quintana (2016). The key difference is that I also have short-term debt (which I account for through a collateral constraint).

As my model provides endogenous prices for short and long-term debt, this paper connects to the literature regarding structural models of credit. Merton (1974), Leland (1994), and Leland and Toft (1996) serve as historical benchmarks in this area. In the large majority of this work, firms are risk neutral with cash flows expressed as exogenous Gaussian diffusion processes. Using the known statistical distribution of firm value given the current state, alongside optimal choice for debt and default boundaries, we can compute closed form expressions for bond prices. However, as discussed in Huang and Huang (2012), the large conclusion of this literature is that structural credit models undershoot credit risk premia when matching default rates. This is partially due to the fact that model-based state prices (Arrow-Debreu prices) are not volatile or countercyclical enough. In order to correct for these problems, Bhamra et al. (2010) and Chen (2010) utilize Epstein and Zin (1989) preferences in order to increase the risk exposures of credit securities. My paper is different from these studies in that my dynamic model allows corporations to dynamically invest. Furthermore, at each point in time, firms have access to issuing multiple maturities of debt, both of which can change in book terms ("non-commitment").

This paper connects with the vast literature of dynamic models with endogenous investment. Many of these setups include firms that have a time-varying capital structure of equity and debt. A non-exhaustive list of papers includes Gomes (2001), Whited and Wu (2006), and Hennessy and Whited (2007). Hennessy and Whited (2005) were the first to discuss debt-equity tradeoffs in a business cycle model with idiosyncratic and aggregate shocks. Livdan et al. (2009) use a structural corporate model to discuss the relationship between firm constrainedness and asset prices. Perhaps closest to my work, Kuehn and Schmid (2014) develop a partial equilibrium firm model with endogenous investment, recursive preferences-based stochastic discount factor, and long-term debt. I extend their model to have an additional, short-term debt choice that is governed through a collateral constraint. My paper also relates to work by Covas and Den Haan (2011) and Jermann and Quadrini (2012) which discuss the fact that debt issuance is procyclical. In my model, I try to match the more granular fact that the share of long-term debt is positively correlated with output and investment. Crouzet (2015) discusses how multiple equilibria may arise in models with multiple debt maturities and investment. The main difference between the previous paper and my own is that within the scope of my model the short-term debt is assumed to be less costly, via the collateral constraint. This is a key driver of the pro-cyclical long-term debt share in my paper. Finally, a recent paper (Alfaro et al., 2016) discusses how economic uncertainty interacts with financial frictions to cause larger shifts in short-term debt than

<sup>&</sup>lt;sup>4</sup>The calculation of an optimal default boundary is done through a "smooth-pasting condition" that leaves the equityholder indifferent at the boundary, between continuing to operate and dissolving. In many papers, this is also known as "strategic default."

those in long-term debt. A key difference between their work and my own is that they use collateral constraints to maintain a risk free term structure. In my model long term debt, in particular, is risky.

In terms of work regarding maturity choice, Diamond (1991) suggests that cross-sectional heterogeneity of long-term debt shares can be linked to firm level signals in the form of credit ratings. More recently, Greenwood et al. (2010) try to understand the time-variation of corporate debt maturity choice. They suggest that the time series patterns of corporate debt maturity are linked to investor-related substitution effects between government and corporate debt. I provide evidence and construct a model that instead utilizes firm investment as a key driver for explaining the pro-cyclical long-term debt share. He and Milbradt (2016) discuss a dynamic debt rebalancing problem with short and long-term debt. Their model features classes of equilibria, one in which firms continually shorten the overall maturity of their debt and another in which they lengthen overall maturity. There are many ways in which my model differs from theirs, but one key difference again is that I allow for an endogenous choice of assets, which provides me the opportunity to discuss the investment-related impact of debt maturity. Finally, Chen et al. (2013) discuss the impact of multiple debt maturities in a structural credit model. They are able to show that the additional use of long-term debt cuts credit spreads. Through my model, I will be able to make a similar statement regarding debt prices and debt maturity, while also speaking to the endogenous asset side of the balance sheet.

A more recent literature discusses empirical evidence regarding debt refinancing. Using syndicate loan data from the FDIC, Mian and Santos (2011) provide evidence that creditworthy firms borrow at and extend existing loans to longer maturities when economic climates are positive in order to weather liquidity crises that might occur later. Similarly, Xu (2015) suggests that speculative grade firms issue or refinance longer maturity debt in favorable market conditions. Both of these papers support the general economic story my model displays. Other empirical work (see Ivashina and Scharfstein, 2010; Campello et al., 2011) have discussed how firms utilize alternative forms of liquidity (in particular, cash and credit lines) in order to buffer operations in the Great Recession period. As I calibrate my model to public market data, I focus my attention on tradeoffs between short and long-term debt and abstract away from alternative securities. Finally, a recent paper (Choi et al., 2016) examines the granuality or spread of debt maturity dispersion within corporations. Using data from Mergent's Fixed Income Securities Database (FISD) they suggest that younger and smaller firms have a debt maturity that is less diverse across corporate debt, while mature and larger firms display the exact opposite.

The sovereign default literature also discusses maturity tradeoffs for emerging market economies.

Arellano and Ramanarayanan (2012) explain how issuing long-term debt serves as a hedge to future movements in debt prices. Broner et al. (2013) suggest that emerging economies borrow sovereign debt at a maturity that lengthens in expansions of domestic business cycles. My economic story broadly agrees with this time-varying procyclical nature of debt maturity, in the context of US firms. On the computational front, I adopt techniques from this literature, first introduced in Chatterjee and Eyigungor (2012) and extended by Gordon and Guerron-Quintana (2016). In both of these papers, IID noise is introduced into the (effective) dividend flow payment to help smooth out the bond price calculation. This smoothing is very useful to handle the discrete jumps that come with the nature of default decisions. While I don't provide a proof for existence under the use of the IID noise (as is done in these papers), I do use this tool to help convergence greatly.

# 3.2. Procyclicality of Long Term Debt Ratio

In this section, I document the fact that the share of long-term debt is positively associated with business cycles. I present evidence at both the aggregate and firm levels.

# 3.2.1. Aggregate Dynamics

Using quarterly data at the U.S. Federal Reserve Financial Accounts going back to 1952, I construct a measure of the share of long-term debt, where long-term debt includes aggregated corporate, mortgage, and municipal debt on the balance sheet of non-financial corporations.<sup>5</sup> As discussed in Greenwood et al. (2010), this series contains a time-varying trend and I correct for it by extracting the cyclical component of the long-term debt share. In Figure 14, I provide a graph of this series, obtained through a Hodrick and Prescott (1997) filter. The grey bars indicate NBER recession dates and the left hand axis indicates changes in percentage points of the ratio. I find that the ratio decreases in recessions and increases in expansions of the business cycle.

This is made quantitatively clear in Figure 15. I compute cross correlation functions between cycle components of the long-term debt ratio and measures of the business cycle. All figures on the left hand side represent correlations between output growth and various cyclical measures of the long-term debt share, while those on the right hand side report correlations using investment growth. From top to bottom, the cyclical components are measured using Hodrick and Prescott, Baxter and King (1999), and Christiano and Fitzgerald (2003) filters, respectively. Across all three filters, I find that the contemparenous correlations between economic aggregates and the long-term debt share are significantly positive. Using the HP-filtered value, for example, it is at the order of 35% while its correlation with investment

<sup>&</sup>lt;sup>5</sup>The same measure is discussed in Chen et al. (2013). However they do not compute explicit correlations between the ratio, output, and investment growth.

growth is roughly 40%. Similar results hold for the BK-filtered debt share and the CF filter, particularly at lag zero. Altogether firms take on more long-term debt and extend the length of their maturity structure in business cycle expansions.

I seek to further understand the procyclical features of the long-term debt share measure, by examining predictive regressions. I project average, future output growth onto a set of controls and the HP-filtered long-term debt share, given here by  $LTDR^c$ :

$$\frac{1}{k} \sum_{i=1}^{k} \Delta y_{t+i} = \beta_0 + \beta_X' X_t + \beta_{LT} LT DR_t^c + error_{t+k}$$

The vector of controls,  $X_t$ , includes a wide array of lagged macroeconomic and financial variables known to have predictive power for business cycles, including lagged output growth, consumption growth, inflation, price-dividend ratios, credit spreads, and U.S. Treasury bond yields. The top panel of Table 12 reports the coefficients of the long-term ratio and its associated t-statistic after correcting for serial and autocorrelated errors using Newey and West (1987) adjustment. The long-term ratio predicts output at a very significant rate, up to four quarters out, even when controlling for a wide array of factors. Perhaps stemming from the higher raw correlation presented earlier, results are even stronger when I perform the same regression using investment growth as a dependent variable. In the bottom panel of the same table, I show that coefficients are larger and t-statistics are close to five in the first year. In terms of economic magnitudes, these results suggest that one percent of additional long-term debt share is associated with roughly .60% more output growth and 3% more investment growth at the annual basis. Putting both the contemparaneous and predictive facts together, the aggregate long-term debt ratio contains significant positive economic news.

# 3.2.2. Firm-Level Dynamics

Using firm-level data I seek to confirm facts shown at the aggregate level. I construct another measure of long-term debt, as that in Barclay and Smith (1995), with quarterly, Compustat data. I define the long-term debt ratio as the share of debt that is greater than one year at issuance. This measure includes any corporate bonds, mortgages and municipal debt that firms have on their balance sheet. It also includes long-term leases and wage contracts, but excludes long-term accounts payable. I do not provide full sample statistics of this measure, but in summary, the average firm holds close to 70% in the form of debt over one year. The fact that this number is so large suggests how important publically-issued long-term debt is. As financial, public, and utility firms are regulated in their capital structure behavior I remove these firms from the sample by way of their SIC codes. I

also remove firm-quarter observations if there are extreme quarterly movements in market leverage or the long-term debt ratio (in the bottom or top 1%). This is meant to remove the effects of capital structure shifts, due to merger and acquisition activity or divestitures. Due to data quality issues related to completeness, I run all tests using data following the first month of 1984.

As there is non-stationarity in many of the firm-level variables, I adjust firm quantities by their overall size and estimate the link between capital-adjusted profits  $(\pi_i/k_i)$  and the long-term debt share  $(LTDR_i)$ :

$$\frac{\pi_{it}}{k_{it}} = \beta_0 + \beta_X' X_{it} + \beta_{ld1} LTDR_{it} + error_{it}$$

where  $X_{it}$  indicates a vector of firm and aggregate level controls, depending on the specification. I provide the results of this regression in Table 13. From left to right, I test multiple specifications where I successively add (i) firm-level controls, (ii) macroeconomic controls, and (iii) firm fixed effects. Following the main feature in Gilchrist et al. (2014), I include a term accounting for the historical, four-quarter volatility in profitability. I also include contemporaneous leverage, lagged investment rate, lagged market to book ("average Q"), and the long-term debt ratio. Macroeconomic controls are similar to the past subsection and include quarterly growth rates of industrial production and the consumer price index, U.S. treasury yields, and the average aggregate credit spread.

The conclusion that is consistent across all specifications is that the long-term debt ratio, at the firm level, is significantly associated with higher levels of profitability, beyond the 1% confidence level. In terms of economic magnitudes, a one standard deviation movement in the long-term debt share would be associated with a roughly 6.5% increase in profitability, from the baseline average. I can also run the same regression using contemparenous rates of capital-adjusted investment. The results of this projection are provided in Table 14. Again, I find that the long-term debt ratio is significant in its association with investment. Economic magnitudes are similar here as a one standard deviation increase in the long-term debt share is associated with a roughly 6.8% increase in the capital-adjusted investment rate.

As in our aggregate regression results, we also check the power that firm specific long-term debt shares have in predicting profitability and investment rates. In Table 15, we display the results of both of these regressions. In the top panel, the left hand side displays average future profitability between one and four quarters out, while on the right hand side are the usual controls including the long-term debt share. It is evident that the predictive power is significant and (intuitively) declines from one quarter out to four quarters out. In economic magnitudes, a standard deviation movement in the current-long term debt share

correlates with 4.1% additional future profitability (at one quarter forward) and 1.5% more average profitability (four quarters). The bottom panel reports statistics for the investment regression, where conclusions are similar. There is significant predictive power up to four quarters.

Another interesting data experiment to run would be to check how regression results change across firm size; in particular, do particular types of firms have stronger correlations between their long-term debt shares and fundamentals. In Table 16, we do exactly this, running pooled regressions after first sorting firms into size quintiles on a monthly basis. A conclusion that is borne out of this (effective) double sort is that smaller firms have a much larger sensitivity between their respective long-term debt share and fundamental statistic. In the top panel for example, the relationship between profitability and the long-term debt share for small firms is much more significant than that for large firms, where it is in fact insignificant. The same holds in the bottom panel with respect to the investment relationship.

Through all the analysis in this section, I am not claiming that the long-term debt ratio causes firm and aggregate conditions to change. Rather I display these facts in order to prove its positive correlation with the business cycle, and in particular investment rates.

#### 3.3. Economic Model

In order to better understand these empirical patterns, I introduce a dynamic, heterogeneous firm economy in which corporations maximize expected, discounted cash flows arising from endogenous investment. In order to finance their operations, they use funds from production, short-term and long-term debt issuance, and equity issuance. The optimal, simultaneous choice of investment, short and long-term debt issuance uniquely separates this framework from the literature. In the rest of this section I precisely lay out the structure of the economy and discuss the solution technique to the model.

# 3.3.1. Cash Flow Risks, Investment, and Production

The economy is populated by a large continuum of firms that are subject to both aggregate and idiosyncratic risks. The aggregate state of the economy is determined through consumption growth at t, denoted by  $\Delta c_t$ . It follows a first-order autoregressive process:

$$\Delta c_t = \mu_c + \rho_c \Delta c_{t-1} + \sigma_c \varepsilon_{ct} \tag{3.1}$$

While  $\Delta c_t$  governs the aggregate state of the cycle, the shocks that enter into firms' cash flows will be related to another variable,  $X_t$ , whose growth rate will be given by  $\Delta x_t$ :

$$\log\left(X_{t}/X_{t-1}\right) \equiv \Delta x_{t} = \mathbb{E}\left[\Delta c_{t}\right] + \lambda_{x}\left(\Delta c_{t} - \mathbb{E}\left[\Delta c_{t}\right]\right) \tag{3.2}$$

From the above equation,  $\Delta x_t$  will have the same mean as  $\Delta c_t$  however the volatility is larger for  $\lambda_x > 1$ . In terms of the model's performance, I increase the volatility of the aggregate portion of total firm productivity to increase the tie between default and the business cycle. Empirically as well, the growth rate of total factor productivity is multiple times more volatile than what is found in consumption growth data.

The firm is also exposed to idiosyncratic risks, which will create an endogenous cross-sectional distribution in quantities and prices. The idiosyncratic productivity will be given by a mean zero  $x_{i,t}$ :

$$x_{i,t} = \rho_x x_{i,t-1} + \sigma_x \varepsilon_{xt} \tag{3.3}$$

Going forward, I will denote all firm specific variables with the subscript i.

The cash flow shocks will enter into the operating profits of the firm,  $\pi_{it}$ . The profits are decreasing returns to scale in capital, with a factor  $0 < \alpha < 1$ , and taxed at a rate  $\tau$ . They are given by:

$$\pi_{it} = (1 - \tau) A_{it} k_{it}^{\alpha}$$
s.t.  $A_{it} = \exp(\bar{x} + x_{it} + (1 - \alpha) \log(X_t))$  (3.4)

where  $k_{it}$  measures the amount of capital the firm has on hand at the start of period t. The  $A_{it}$  term accounts for both aggregate and idiosyncratic productivity.  $\bar{x}$  is a constant, which scales the value function and does not change the core results of the model.<sup>6</sup> As  $\log(X_t)$  is a unit root variable that grows over time (implied by the consumption growth process), the model exhibits stochastic growth around a trend. When solving the model, we correct for the (time-varying) trend. There is more to be said about this when I discuss the solution technique to the model.

Firms make investment choices each period in response to economic conditions. Denote  $i_{it}$  as the amount of investment made at time period t. This will imply that next period capital,  $k_{i,t+1}$  is known today and given by:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + i_{it} \tag{3.5}$$

<sup>&</sup>lt;sup>6</sup>The scale,  $\bar{x}$ , is selected by analytically solving a no-leverage economy and equating steady state capital to one.

Existing capital depreciates at a rate  $\delta$ , which will factor into the steady state rate of investment.<sup>7</sup> Adjusting capital is not costless and these costs are given by:

$$\Phi_k(k_{it}, i_{it}) = \frac{\phi_k}{2} \left( \frac{i_{it}}{k_{it}} - \frac{i_{ss}}{k_{ss}} \right)^2 k_{it}$$
 (3.6)

where  $\phi_k$  is a constant parameter and  $\frac{i_{ss}}{k_{ss}}$  is the steady state investment-to-capital ratio in the model. I impose investment adjustment costs in order to slow the speed at which firms invest. In the literature investment adjustment costs are both empirically founded (Ramey and Shapiro, 2001) and help explain various features in asset pricing models. Zhang (2005), for example, suggests that a more costly downward adjustment of capital is crucial to rationalize the larger return on high book-to-market stocks.

#### 3.3.2. Discount Factor

A driving force behind much of the structural asset pricing literature is an adjustment to the physical probability measure in valuing cash flows. The stochastic discount factor in many models (see Campbell and Cochrane, 1999; Bansal and Yaron, 2004) values cash flows in bad states of the world at a higher rate relative to those in good states. I also embody this intuition in my model as the firm discounts its cash flows at a countercyclical rate, using an Epstein and Zin (1989) discount factor. The use of an Epstein and Zin pricing kernel is crucial to ensure that default events are properly priced into credit spreads, while keeping risk free rates reasonably low.<sup>8</sup>

At time t, each firm discounts its possible cash flows at t+1 using an aggregate discount factor,  $M_{t+1} = M(\Delta c_t, \Delta c_{t+1})$ . This pricing kernel must satisfy the following conditions:

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{PC_{t+1}+1}{PC_t}\right)^{\theta-1}$$

$$\mathbb{E}_t \left[M_{t+1} \left(\frac{C_{t+1}}{C_t}\right) \left(\frac{PC_{t+1}+1}{PC_t}\right)\right] = 1$$
(3.7)

where  $\frac{C_{t+1}}{C_t} = e^{\Delta c_{t+1}}$  and  $PC_t$  is the level of the price consumption ratio at time t. The time discount rate is  $\beta$ , risk aversion is given by  $\gamma$ , and  $\psi$  governs the intertemporal elasticity of substitution. As is common in the literature,  $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ . The second equation results from the first order condition of a household's consumption-savings problem. Using this Euler

<sup>&</sup>lt;sup>7</sup>As there is stochastic growth in this economy, the steady state investment to capital ratio will be given by  $\exp(\Delta x_{ss}) - (1 - \delta)$ . If there was no growth in steady state ( $\Delta x_{ss} = 0$ ) steady state investment to capital is  $\delta$ .

<sup>&</sup>lt;sup>8</sup>In cases where the Epstein and Zin friction is not present  $(\gamma = \frac{1}{\psi})$ , I find that the credit spread shrinks dramatically.

condition, we can solve for  $M_{t+1}$  via numerical techniques.

It is important to note that the discount factor is not part of a larger general equilibrium problem. There are no households (or investors) that maximize over equity holdings in the continuum of firms. Connecting an aggregate household to firms with heterogeneous capital structure is more challenging and beyond the scope of this paper. I leave this for future research.

## 3.3.3. Capital Structure

Every period, the firm can issue debt of two types – short (S) and long (L). Short term debt requires repayment the following period while long-term debt only requires a fractional payment and takes the form of an annuity. Both forms of debt also have a proportional coupon, c, that provides a tax advantage for debt. Meanwhile, firms also pay a fraction  $\kappa_L$  of outstanding long-term book debt each period. Suppose at time t, firms issue a new amount of debt in book value terms,  $w_{it}^S$  and  $w_{it}^L$ . This will imply that the new book debt outstanding at the start of t+1 are:

$$b_{i,t+1}^{S} = w_{i,t}^{S}$$

$$b_{i,t+1}^{L} = (1 - \kappa_L)b_{i,t}^{L} + w_{i,t}^{L}$$
(3.8)

To enforce that type L is indeed longer term at issue, I will set  $\kappa_L < 1$ . Hence, the average duration of long-term debt will be  $\frac{1}{\kappa_L}$ .

Modeling debt as an annuity helps us simplify the problem as I only need to keep track of the current book value of debt as a state variable. Nonetheless, this restriction still allows me to capture the basic intuition that long-term debt provides the opportunity to pay a smaller per-period payment. As a modeling assumption, this form of debt is not new either. It is the same as the sinking fund provision used in Leland and Toft (1996), Hackbarth et al. (2006), among many others. The key difference is again, that I allow the firm to choose between two types of debt at a dynamic rate.

The firm will face a collateral constraint on its short-term debt.<sup>9</sup> I impose that short-term debt to be paid off in the future, including the coupon, is no more than a fraction of capital,

<sup>&</sup>lt;sup>9</sup>It is true that collateralized short-term paper are mostly applicable for the low duration liabilities of financial firms (Kacperczyk and Schnabl, 2010), which is at odds with the non-financial data I calibrate the model to. That being said, many non-financial commercial paper contracts are associated with a standby line of credit, which helps reduce the paper's risk properties for potential investors (Coyle, 2002). A direct implication of the model, through the alternative mechanism of the constraint, will be a reduced risk profile for short-term debt.

net depreciation, next period:

$$(1+c)b_{i,t+1}^S \le s_0(1-\delta)k_{i,t+1} \tag{3.9}$$

where  $s_0 \leq 1$ . Furthermore I will assume that equity holders and long-term debt holders recognize that short-term debt holders are senior claimants upon default. The reason I do so is that the combination of these assumptions will imply that short-term debt will be a risk free claim. That is to say, upon default, the firm will always have enough capital on hand to service repayment of short-term debt. Hence the price of one dollar's worth of short-term debt,  $p_t^S$ , is the risk free discounted value of (1+c):

$$p_t^S = \mathbb{E}_t \left[ M_{t+1}(1+c) \right] \tag{3.10}$$

While the use of a collateral constraint and seniority are simplifying assumptions, it helps us model default risk solely in the long-term debt security. This makes the model much more computationally tractable to be taken to the data. It also has roots theoretically. Diamond (1993) suggests that the seniority and collateralization of short-term debt can serve as compensation for monitoring costs of short-term creditors. This compensation will make it incentive-compatible for short-term creditors to not run on the firm, allowing the scope for future debt issuance as well.

One might ask whether the seniority and risk free nature of the model's short-term debt hold in the data. Based on the Financial Accounts data discussed in the empirical section, a very large portion of short-term debt (on average, 95%) constitute of loans. To the extent these loans are extended by banks they are almost always senior, as discussed in Welch (1997). The relatively risk free nature of bank loans can also be corroborated by examining recovery rates. In Figure 16, I provide recovery rates across debt types, as provided by Moody's recovery database for non-financial corporations. In the twenty years prior to the financial crisis, the median recovery rate for bank loans was 100%. Contrastingly, in the same time period, the median recovery rates for corporate bonds ranged from 67% to 2%, depending on the seniority structure of the particular debt contract. The clear differences of recovery rates suggest to us that the risk-free rate assumption for short-term bank debt is not far from reality.

 $<sup>^{10}</sup>$ The data for recovery rates are taken from "Moody's Ultimate Recovery Database" (Emery et al. (2007)) and cover 3500 non-financial loans and bonds from 1987 – 2007. Recovery rates vary across industry, debt type, and seniority, among other categories.

### 3.3.4. Default and Debt Valuation

The equity value of a firm accounts for the discounted stream of lifetime profits. Each period, after realizing both idiosyncratic  $(x_i)$  and aggregate (X) shocks, the corporation can choose whether to (a) continue operations or (b) default and transfer residual assets to bondholders. In the model, I define a default event occurring when the value from continuing operation is too low relative to a threshold. In terms of an equation, this means that:

$$\mathbb{1}_{\{\text{Default, } it\}} = \begin{cases} 1, & \text{if } V_{it} \le (\bar{V}X_{t-1}) \\ 0, & \text{otherwise} \end{cases}$$
 (3.11)

where  $V_{it}$  indicates the value from continuing operations and  $\bar{V}$  is a constant. Notice that this constant multiplies the business cycle shock indicating that the overall threshold value is time-varying and procyclical.

One way to interpret the default condition is that when  $\bar{V} > 0$  equity holders or managers have an outside option to consider. In Eisfeldt and Papanikolaou (2013), for example, a similar outside option exists where talented labor that manage a particular type of capital, have the ability to walk away from the firm. Another, more relevant interpretation, would be a Chapter 11 reorganization as discussed in Corbae and D'Erasmo (2016). Equityholders "re-organize" such that bankruptcy proceedings determine the fraction of firm value that short and long-term debt holders receive. Following this procedure, existing equity holders retain firm value and resume operations as an unlevered firm.<sup>11</sup>

When the firm goes into bankruptcy the bondholder will receive any remaining undepreciated capital and profits generated from the capital, net a repayment of the short-term debt holder. That is to say the payment given default at time t + 1 is:

$$X_{i,t+1}^{pd} = (1 - \xi) \left( \pi_{i,t+1} + (1 - \delta) k_{i,t+1} - (1 + c) b_{i,t+1}^{S} \right)$$
(3.12)

where  $\xi$  represents losses in default, which we can think to be related to legal and administrative fees paid out in bankruptcy.<sup>12</sup> At this point it is clear why the short-term debt holder will always be repaid in default. Because  $(1-\delta)k_{i,t+1} \geq (1+c)b_{i,t+1}^S$  due to the collateral constraint, there will always be enough capital on hand to repay the senior claimant. This will imply that the difference between the right two terms above is always greater than zero.

<sup>&</sup>lt;sup>11</sup>When simulating the model this is almost exactly the procedure that we conduct. The only difference however is that firm capital is reset upon default.

<sup>&</sup>lt;sup>12</sup>A similar form for payment given default is used in Hennessy and Whited (2007) and Kuehn and Schmid (2014)

Now I price the risky long-term debt. The equilibrium price, denoted by  $p_{it}^L$ , will equate total lent funds to total expected proceeds next period. In period t, the firm chooses a new amount of issuance,  $w_{it}^L$ , which brings him to a book value of  $b_{i,t+1}^L$ . The price on the new dollar of debt will reflect the total default risk of obtaining a new level of book debt. In order to obtain a level,  $b_{i,t+1}^L$  we will have:

$$p_{it}^{L}b_{i,t+1}^{L} = \mathbb{E}_{t} \left[ M_{t+1} (1 - \mathbb{1}_{\{\text{Default}, i, t+1\}}) \times \left( (\kappa_{L} + c)b_{i,t+1}^{L} + (1 - \kappa_{L})p_{i,t+1}^{L}b_{i,t+1}^{L} \right) \right] + \mathbb{E}_{t} \left[ M_{t+1} \left( \mathbb{1}_{\{\text{Default}, i, t+1\}} \right) \times X_{i,t+1}^{PD} \right]$$
(3.13)

The pricing equation can be understood in the following manner. The left hand side of the first line represents the total funds lent. On the right hand side of the first line are the payments that occur when the firm does not default. Again this includes both the effective coupon payment and the market value of remaining debt. The right hand side of the second line accounts for the payment upon default.

### 3.3.5. Equity Valuation

Shareholders seek to maximize the sum of discounted dividend payouts, taking into account the ability to potentially default in the future. Conditional on not defaulting, the firm will earn profits, choose investment, and issue short and long-term debt. The recursive formulation of each firm's problem is given by:

$$V_{it} = \max_{\{k_{i,t+1}, b_{i,t+1}^{S}, b_{i,t+1}^{L}\}} \{D_{it} - \Phi_{e}(D_{it}) + \mathbb{E}_{t}[M_{t+1}W_{i,t+1}]\}$$

$$D_{it} = \pi_{it} + \tau (\delta k_{it} + cb_{it}^{S} + cb_{it}^{L})$$

$$- i_{it} - \Phi_{k}(i_{it}, k_{it})k_{it}$$

$$+ p_{it}^{S} w_{it}^{S} + p_{it}^{L} w_{it}^{L} - (1+c)b_{it}^{s} - (\kappa_{L} + c)b_{it}^{L} - \Phi_{L}(w_{it}^{L})$$
(3.14)

$$W_{i,t+1} = \max\left\{V_{i,t+1}, \bar{V}X_t\right\}$$

s.t. 
$$(3.5), (3.8), (3.9), (3.10), (3.13)$$
 hold

Note that in the top equation, current firm value is comprised of a dividend payment  $(D_{it})$ , equity issuance costs in the case that firm dividends are negative  $(\Phi_e(\cdot))$ , and the dynamic continuation value of the firm  $(\mathbb{E}_t [M_{t+1}W_{i,t+1}])$ . The additional constraints that are referred to include the laws of motion for investment and long-term debt, the collateral constraint, and the pricing equations for short and long-term debt.

The dividend to the firm will consist of after-tax profits plus a tax shield for depreciation and debt-related coupon payments. It will also include an outflow for equilibrium investment and adjustment costs on capital. The terms on the final line of  $D_{it}$  represent debt proceeds and repayment on both debt contracts, as well as issuance costs  $(\Phi_L)$  in the case that the firm issues new long-term debt  $(w_{it}^L > 0)$ . Notice that the firm pays a fractional portion  $\kappa_L + c$  of long-term debt each period.

As the Bellman equation represents the value from continuing operations, the discounted future value must account for the chance of potential default in period t + 1. As a result,  $W_{i,t+1}$  is a maximum over continuing to operate next period and choosing to take the outside option.

# 3.3.6. Discussion of Capital Structure Tradeoffs

At its core this model discusses the tradeoffs among a number of securities that can be used to finance endogenous investment. Beyond operating cash flows, the firm has the opportunity each period to take upon new short-term and long-term debt, as well as equity issuance. How does this model break the irrelevance theorem stated in Miller and Modigliani (1959)? First, as firms take on more debt (both short and long) they receive a tax shield that is proportional to the coupon payments on debt. Inherently this tax advantage creates an incentive for leverage. Beyond the tax advantage, long-term debt embodies distress costs. If the recovery parameter,  $\xi > 0$ , then there will be a loss in firm value upon default. This will also create a deviation from capital structure irrelevance. Finally, the model features issuance costs in both long-term debt and equity issuance.

I now discuss how the model emits a partial pecking order. Suppose the corporation would like to raise additional funds for investment beyond those garnered from current production, net of debt-related payments. The costs and benefits to issuing the three possible securites are as follows:

- 1. <u>Short term debt</u>: the benefit consists of the discounted value of the tax advantage of the future coupon payment. The costs, however, are on net zero. There is no destruction of firm value implied by the bond pricing, due to the seniority and collateral constraint.
- 2. <u>Long term debt</u>: similar to short-term debt, the benefit consists of the discounted value of the tax advantage of the future coupon payment. Conditional on being in a default region, additional long-term debt increases the likelihood of bearing distress costs.

3. Equity issuance: there is no benefit to issuing additional equity while there are flotation costs that are positive ( $\Phi_e > 0$ ).

Among these securities, it is clear that short-term debt always provides a positive benefit. This implies that the firm takes upon as much short-term debt as it can and that the collateral constraint binds,  $(1+c)b_{i,t+1}^S = s_0(1-\delta)k_{i,t+1}$ .<sup>13</sup> Beyond short-term debt, it is difficult to definitively say whether long-term debt or equity will be preferred. This will be dependent on the tradeoff between floation costs and the time-varying net benefit of issuing corporate debt. As this will be specific to the quantitative behavior of the model, we will leave this discussion till later.

### 3.3.7. Model Solution

Due to stochastic growth over time, we solve a scaled version of the model where all time t variables are divided by the lagged level of the aggregate shock,  $X_{t-1}$ . In the discussion that follows,  $\hat{g}$  indicates the detrended value of a generic variable g. For more details on the exact system of equations that we iterate over, see Appendix 3.6.

The model emits four states,  $\{\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}_{it}^L\}$  and two controls,  $\{\hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L\}$ . The value and bond pricing functions will be of the form:

$$\hat{V}_{it}\left(\Delta c_{t}, x_{it}, \hat{k}_{it}, \hat{b}_{it}^{L}\right) = \max_{\{\hat{k}'_{i,t+1}, \hat{b}_{i,t+1}^{L'}\}} \left\{\hat{D}_{it} - \Phi_{e}\left(\hat{D}_{it}\right) + e^{(\Delta x_{t})} \mathbb{E}_{t}\left[M_{t+1}\hat{W}_{i,t+1}\right]\right\} 
\hat{W}_{i,t+1} = \max\left\{\bar{V}, \hat{V}_{i,t+1}\right\}$$
(3.15)

$$p_{it}^{L} = \mathbb{E}_{t} \left[ M_{t+1} \left( 1 - \mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}} \right) \left( \kappa_{L} + c + (1 - \kappa_{L}) p_{i,t+1}^{L} \right) \right] + \mathbb{E}_{t} \left[ M_{t+1} \left( \mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}} \right) \left( \frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^{L}} \right) \right]$$
(3.16)

A simple and intuitive algorithm to solve this system would be to start with a guess for prices and the value function. Using the guess for prices and an implied value for  $\hat{W}$ , I can compute a new value of  $\hat{V}$  that resulted from the maximization step of 3.15. I could then evaluate the right hand side of 3.16, using the previously computed  $\hat{V}$  to determine the default dummy variable and the original guess for prices, evaluated at the optimal policies.

As explained in great depth in Chatterjee and Eyigungor (2012), these sort of algorithms suffer convergence issues in models with long-term debt. The reason is the following. In

<sup>&</sup>lt;sup>13</sup>A binding collateral constraint aids the quantitative solution of the model. I am able to eliminate one state  $(b_{it}^S)$  and control variable  $(b_{i,t+1}^S)$ .

order to move from one iteration of bond prices to another I need to assume a value function and policy function. If the default decision switches, for a certain set of states, from the previous iteration to the current one, the resulting abrupt shift will create a large jump in the bond price. This jump then leads to a great shift in the next iteration of computing value and policy functions. In this pattern, I never reach joint convergence of price and value functions.

In order to remedy the convergence issues, Chatterjee and Eyigungor (2012) and Gordon and Guerron-Quintana (2016) add IID zero mean noise into the effective dividend flow. The purpose of this is two fold. First, because the IID noise enters monotonically into the dividend payout one can compute the policy functions and default decision perfectly as a function of the IID shock. Second, because the distribution of the IID noise is known perfectly, when integrating across potential default decisions in the future, as in equations 3.15 and 3.16, we can smooth out potential jumps using numerical integration techniques.<sup>14</sup>

In the same vein I add a concave function of IID noise,  $g(m_{it})$ , to my dividend payoff such that  $m_{it}$  is a truncated normal shock with a mean of zero and a very small variance. Furthermore g is chosen such that  $\mathbb{E}(g(m_{it})) = 0$ . This will imply that the value function becomes:

$$\hat{V}_{it}\left(\Delta c_{t}, x_{it}, \hat{k}_{it}, \hat{b}_{it}^{L}\right) = \max_{\{\hat{k}'_{i,t+1}, \hat{b}''_{i,t+1}\}} \left\{\hat{D}_{it} - \Phi_{e}\left(\hat{D}_{it}\right) + g(m_{it}) + e^{(\Delta x_{t})} \mathbb{E}_{t}\left[M_{t+1}\hat{W}_{i,t+1}\right]\right\}$$

Here the default decision will be a function of the noise and I compute the default rule perfectly with respect to thresholds of m. When computing the expectation of next period's continuation value, I account for the uncertainty of the noise in the expectation. To compute the expectation over m, I conduct a 15-interval numerical integration using properties of the truncated normal distribution. For a detailed description of the numerical algorithm see Appendix 3.7.

# 3.4. Results

In this section I document the quantitative results from the model, starting with an explanation of the quarterly calibration, followed by discussions on simulated results, model mechanisms, cross-sectional behavior, and other findings.

<sup>&</sup>lt;sup>14</sup>The implementation of IID noise in both of these papers is slightly different than in mine. In both of these papers the noise is added such that it enters into the *utility flow* of the representative household's Bellman equation. As a result optimal policies and default decisions are both affected by the noise. In my setup the noise is simply added to the (risk-neutral) dividend flow, which implies that the noise only factors into the optimal default decision. In this sense, my use of the IID noise purely helps smooth switches in the default decision. In the case of these two references, it can also help prove existence of equilibrium as policies are monotonic in the noise.

#### 3.4.1. Calibration

In Table 17, I display the parameters I calibrate the baseline version of the model to. The first three rows of parameters relate to the ? stochastic discount factor. The values for risk aversion ( $\gamma=2$ ) and the intertemporal elasticity of substitution ( $\psi=2$ ) would suggest the firm has a preference for a resolution of early uncertainty ( $\gamma>\frac{1}{\psi}$ ). As well known in the Long Run Risks literature, such preferences would suggest that shocks to current aggregate states will heavily influence future utility, which will then feed into the firm's discount factor. This creates a large "distortion" in state prices to help accurately capture default and credit spread patterns.<sup>15</sup>

The next three lines of the table relate to the production parameters of the model. I use a curvature parameter ( $\alpha = .65$ ) that is close in value to the estimates of Hennessy and Whited (2007). The depreciation parameter is standard in the literature ( $\delta = .025$ ). The capital adjusment parameter ( $\phi_k = 1$ ) is chosen to help curb investment rate volatility in the cross section. An additional criterion I use to set this parameter is that default rates are decreasing in  $\phi_k$ . As firms are more exposed to adjustment costs they are more cautious in adjusting their capital stock. These cautious adjustments make firms less susceptible to a large drop in equity value, in the event that an adverse productivity shock hits.

The model is calibrated to feature one quarter short-term debt and five year long-term debt. Setting  $\kappa_L = .05$  suggest that it will take 20 quarters, on average, to pay off long-term debt. The coupon rate (c = .01) is arbitrarily set and does not have a substantive effect on the results. As the collateral constraint will bind, the average ratio of short-term debt to total assets is  $s_0(1 - \delta)$ . I set  $s_0 = .08$  to capture the mean ratio in Compustat data. The next three parameters relate to issuance costs for both long-term debt and equity. The fixed cost parameter for long-term debt,  $\Phi_{L,a} = .006$  is set to target the frequency of long-term debt issuances in the cross section. Similarly the floation cost parameter,  $\Phi_{e,a} = .06$  is used to target the frequency of equity issuance. The parameter used for the proportional cost of equity,  $\Phi_{e,b} = .05$ , comes from Hennessy and Whited (2007).

The bottom set of numbers refer to productivity parameters. The productivity constant,  $\bar{x} = -2.50$ , is chosen to scale the economy such that detrended capital is roughly equal to one in a non-leverage economy. The autocorrelation and volatility of idiosyncratic factor productivity are taken from Kuehn and Schmid (2014). All parameters for consumption growth are set to match the mean, volatility, and autocorrelation of quarterly, real percapita consumption growth from NIPA tables. Finally, I set  $\lambda_x = 3.5$  to scale up aggregate

<sup>&</sup>lt;sup>15</sup>While  $\beta$  is not high enough to match the *level* of the risk free rate, all qualitative features of the model hold in this environment.

volatility in the firm TFP. More generally, it aids to induce a default that is more counter-cyclical. The last parameter in the table,  $\bar{V} = 1.425$  is set as an outside value to the firm. It is set to help match default rates as seen in the data.<sup>16</sup>

## 3.4.2. Model Fit

I solve the model for the previously described set of values and simulate the model. The simulation consists of a panel with 3000 firms over 500 quarters, including a burn-in period of 500 quarters. In the model results I describe, I remove defaulted firms each period. In Table 18, I provide cross-sectional statistics related to profitability, investment, debt, and default. The data comes from Compustat, onwards from 1984, and the numbers in parentheses represent time series bootstrapped standard errors.  $\mathbb{E}_t(\cdot)$  and  $\sigma_t(\cdot)$  refer to the cross sectional mean and volatility, respectively.

The model performs reasonably well with respect to investment and book leverage. There is a direct link in the model between book leverage and the long-term debt ratio. The portion of book leverage that is due to short-term debt is a fixed ratio  $-s_0(1-\delta)$ . Any additional book leverage beyond this is through long-term debt. The model does particularly well with respect to default (.970% in the model vs. 1.08% annually in the data). This results in a credit spread of 1.84% annually which is close to the empirical target. The key statistic that the model does poorly on is profitability. The likely reason why this occurs is the fact that I do not have fixed costs in production as used in Gomes (2001). As the profitability is too high this is probably causing the need for a positive outside option ( $\bar{V} > 0$ ) to induce default.

In Table 19 I display the aggregate statistics of the model. The first three rows provide the moments of consumption growth (mean, volatility, autocorrelation) which are set exogenously, in line with quarterly data. Aggregate investment growth and output growth move positively with consumption growth and are also close to data. Leverage in the model is also procyclical as firms issue more book debt in economic booms. Finally as desired, the model is able to match the stylized fact that the long-term debt ratio is strongly pro-cyclical. In the model, the aggregated long-term debt ratio has a correlation of .396 with consumption growth and .664 with output growth. In the next subsection, we will thoroughly discuss the mechanism that leads to this dynamic. As in the data, default rates vary negatively with the aggregate state of the economy. When firms become unproductive and have lower stocks of capital, this brings them closer to the default boundary. The smaller distance to

 $<sup>^{16}</sup>$ When I set  $\bar{V}=0$  ("optimal default"), an average firm never defaults. In this region the tradeoffs between short and long-term debt are not as clear. Absent of a collateral constraint which explicitly limits issuance of short-term debt, the firm is indifferent between both debt choices. Both debt contracts are risk free as well.

default then generates large credit spreads.

#### 3.4.3. Model Mechanism

The patterns that the model generates are best summarized by Figure 18. Here I display a set of aggregated series related to output, investment, book leverage, and the long-term debt ratio. The picture confirms the numerical evidence presented in the last sub-section. In particular, additions to book leverage, via long-term debt, are strongly connected to changes in investment. Moreover, firms take on more leverage to finance investment.<sup>17</sup>

This mechanism is particularly explained by Figure 19. Both panels represent the time series average of simulated data, across aggregate states – hence there are five bars. In the top figure, I describe what I call the funding deficit, which I define to be:

$$\sum_{i} \left( \hat{D}_{it} - p_{it}^{S} \hat{w}_{it}^{S} - p_{it}^{L} \hat{w}_{it}^{L} \right)$$

This deficit represents all dividends, net of short-term and long-term debt proceeds, or, how much the firm seeks to raise out of debt markets. This number is negative so I take its absolute value and index it to the median state. In terms of interpretation, it is clear that firms need to raise less debt in the first state (about 45% less) relative to the median state. In the fifth state, firms require much more, to the order of 70%. In particular the need for additional debt in the last state is driven by a higher marginal productivity of capital.

In the second panel I describe where this funding is obtained. In particular the figure displays the time-series average of:

$$\sum_{i} \left( p_{it}^L \hat{w}_{it}^L \right) / \sum_{i} \left( \hat{D}_{it} - p_{it}^S \hat{w}_{it}^S - p_{it}^L \hat{w}_{it}^L \right)$$

across states. This figure suggests that firms tend to fund more of their investment needs in good times using long-term debt proceeds. The reasoning for this is two-fold. First, in times when the marginal product of investment is high, firms are limited in funding through short-term debt markets, due to the collateral constraint.

The second reason why this occurs relates to the dynamic pecking order between long-term debt and equity. In order to obtain additional funding, firms can (i) issue additional long-term debt or (ii) issue equity, and the choice between the two is a choice between bearing expected distress costs and paying flotation costs, respectively. Certainly, issuing long-term

<sup>&</sup>lt;sup>17</sup>In this figure, there might seem to be a timing discrepancy between aggregate output and investment versus leverage and the long-term debt ratio. This discrepancy exists because the former group represents a set of choice variable decided at time t while leverage related variables are chosen at time t-1.

debt might provide additional default risk. However, as the firm is further away from default due to the state of the economy, the expected losses from distress are reduced. As a result, the firm ends up issuing more long-term debt.

Nonetheless, this preference changes with respect to the business cycle. The bottom panel also suggests that when aggregate conditions sour, the firm would rather buy back long-term debt using a combination of short-term debt and equity issuance. While I don't present the result here, for firms who do seek to issue equity, their issuance increases dramatically in low consumption growth states.

Moreover, the model endogenously generates a time-varying pecking order. Regardless of the state of the world, the firm first issues short-term debt. Any additional external financing will depend on consumption growth. In high consumption growth states, long-term debt will be preferred to equity. In lower growth states, firms will, if need be, prefer equity issuance.

## 3.4.4. Cross-Sectional Behavior and Rollover Risk

As the model features a set of heterogeneous firms, we can analyze characteristics across corporations. In Table 20, I sort simulated firms into quintiles each period by their detrended market value  $\hat{V}_{it}$ . For each statistic I build a panel time series of average statistics across quintiles. The chracteristics of the basic sort are given in the first line, where firms in quintile 1 are 15% smaller, in terms of market capitalization, than the median firm on average. Large firms are 20% larger.

Furthermore, I find that detrended capital varies monotonically with respect to market value. Firms in quintile 5 are 27% larger than quintile 3, while those in quintile 1 are 22% smaller. The most surprising result from this table is that while stable firms have more capital, they also have more leverage and long-term debt. This is surprising in that we would expect increased leverage to further decay firm value and increase credit spreads. The equilibrium cross-section in this model implies that capital is the largest driver for firm stability. In terms of key firm policy variables, profitability and investment increase as a function of market value. Low market cap firms are 3% less profitable and invest 3% less than high market cap firms. Economic stability is clearly reflected in credit spreads. Firms in quintile 1 average a cost of capital that is fifteen times larger than those in the top quintile.

In Figure 20, I plot firm behavior in the eight quarters that precede default, computed by taking the average of series generated across corporate default episodes in simulation. The bottom axis provides the number of quarters in relation to default. Firms that experience

default undergo a series of shocks that degrades the level of total productivity  $(\hat{A}_{it})$  by 40%. This is displayed in the first panel. As a result of the shocks, and the lower marginal productivity of capital, firms then dis-invest as shown in the second panel.

As productivity and capital both decrease firm value, making debt payments relatively more costly, corporations would like to buy back some of the long term debt on its balance sheet. However, this becomes difficult due to two reasons: (i) reduced capital and productivity result in less profits, which provides a shortage of internal funds to buy back debt and (ii) the collateral constraint, which is tied to the firm's decreasing capital stock, greatly limits the amount of short-term debt that can be used to liquidate long-term. As a result in panel 3, we see that the leverage ratio actually increases. It is clear that the combination of these endogenous state variables changing leads to the value function behavior. Furthermore, it is striking to see how much credit spreads react over the course of the default episode. Over eight quarters the credit spreads rise from  $\sim 0\%$  to  $\sim 220\%$ , when the firm is on the precipice of default. Moreover, this discussion confirms to us that the endogenous investment channel matters greatly for the model. As firms lower their capital due to a lower marginal product of capital, this has a substantial effect on firm value and the re-issuance costs of debt.

### 3.4.5. Further Considerations

While the model captures many salient features of investment behavior, corporate financing decisions, and asset prices there are a few places where it does not do as well. In particular, the model does not deliver the predictability results presented in the empirical section, between the long-term debt ratio and investment growth. The reason why this does not occur, in my estimation, is that investment growth rates in the model are not auto-correlated enough to begin with. In the data, the first order autocorrelation of investment growth is roughly .20 and slowly decreases to 0 over the course of three lags. In the model however, the first order autocorrelation is -.24 and is volatile. Moreover the level of aggregate investment is too strongly tied to the aggregate shock in the model, which has negative empirical consequences for the model's investment growth. One way to tackle these issues is to add adjustment costs to the growth rate of investment (see Christiano et al., 2005). This will directly induce an autocorrelation in investment growth. Additionally, the use of a "time-to-build" assumption (see Boldrin et al., 2001) will also generate the desired characteristics for investment growth. <sup>18</sup>

In reality firms have access to additional securities which allow them to potentially avoid default – in particular, credit lines. In the financial crisis it is well documented that firms

<sup>&</sup>lt;sup>18</sup>The literature on financial frictions (see ?Bernanke et al., 1999) suggests that financial intermediary related constraints on investment can create autocorrelation in investment growth.

drew down their credit lines in order to fund operations. For example, Campello et al. (2011) suggests via survey data that small, financially constrained firms drew close to 60% of their available lines of credit in 2009. In my model however, as firms effectively get priced out of public debt markets (due to low capital, high leverage, and rising long-term credit spreads), they are forced to default. One way to augment the model to address the "lack" of debt market choices is to add a cash asset, that allows firms to save for the rainy day. These savings would allow corporations to keep additional reserves on hand in the face of future productivity drops.<sup>19</sup> I leave this for future research.

#### 3.5. Conclusion

Corporate debt maturity is a time-varying phenomenon that is linked to business cycles. In this paper I study the extent of these linkages and show how they can arise in an economic model. I empirically document that firms extend their debt maturity in peaks of the business cycle, when aggregate output and investment are high, and corporate credit spreads are low. These results also extend to the corporation level, where investment rates and profitability are linked to firm-specific long-term debt ratios.

To understand why the ratio time varies and is linked to the business cycle requires a theoretically motivated explanation. I provide one by developing a dynamic heterogeneous firm economy where corporations trade off debt maturity choice in the face of investment opportunities. Long term debt inherits relatively more distress costs than short-term debt which creates an initial preference for lower duration liabilities. However, limits on short-term debt make the potential distress costs worth it, in order to take advantage of the pro-cyclical investment opportunities. The combination of investment and the collateral structure lead to the pro-cyclical long-term debt ratio. Moreover the model sheds light on the macro-economy as it implies that firms with higher amounts of long-term debt are more systemically important to aggregate fluctuations.

<sup>&</sup>lt;sup>19</sup>Mechanically, this would involve allowing  $b_{i,t+1}^S$  to drift to negative regions. This would be equivalent to implementing firm-level retained earnings, as in Livdan et al. (2009).

# 3.6. Appendix: Detrended Model Equations

As there is stochastic growth in the model, we normalize each variable,  $var_t$ , such that:

$$v\hat{a}r_t = \frac{var_t}{X_{t-1}}$$

The complete list of equations that govern the model consist of:

Exogenous Processes (outside of model solution):

$$\log(C_t/C_{t-1}) \equiv \Delta c_t = \mu_c + \rho_c \Delta c_{t-1} + \sigma_c \epsilon_{ct}$$

$$\log(X_t/X_{t-1}) \equiv \Delta x_t = \mathbb{E}(\Delta c_t) + \lambda_x \left(\Delta c_t - \mathbb{E}(\Delta c_t)\right)$$

$$x_{it} = \rho_x x_{i,t-1} + \sigma_x \epsilon_{xt}$$

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{PC_{t+1} + 1}{PC_t}\right)^{\theta - 1}$$

$$1 = \mathbb{E}_t \left[M_{t+1} \left(\frac{C_{t+1}}{C_t}\right) \left(\frac{PC_{t+1} + 1}{PC_t}\right)\right]$$

The above lines refer to equations (3.1), (3.2), (3.3), and (3.7) from the text.

Investment and Leverage Constraints:

$$\begin{array}{lcl} (1+c) \hat{b}_{i,t+1}^S & = & s(1-\delta) \hat{k}_{i,t+1} \\ e^{(\Delta x_t)} \hat{k}_{i,t+1} & = & (1-\delta) \hat{k}_{it} + \hat{i}_{it} \\ e^{(\Delta x_t)} \hat{b}_{i,t+1}^L & = & (1-\kappa) \hat{b}_{it}^L + \hat{\omega}_{it}^L \\ e^{(\Delta x_t)} \hat{b}_{i,t+1}^S & = & \hat{\omega}_{it}^S \end{array}$$

The above lines refer to equations (3.5), (3.8), and (3.9) from the text. The constraint for short-term debt binds due to the strict preference for short-term debt.

Firm Value:

$$\hat{V}_{it} \left( \Delta c_{t}, x_{it}, \hat{k}_{it}, \hat{b}_{it}^{L} \right) = \max_{\{\hat{k}'_{i,t+1}, \hat{b}_{i,t+1}^{L'}\}} \left\{ \hat{D}_{it} - \Phi_{e} \left( \hat{D}_{it} \right) + e^{(\Delta x_{t})} \mathbb{E}_{t} \left[ M_{t+1} \hat{W}_{i,t+1} \right] \right\} 
\hat{W}_{i,t+1} = \max \left\{ \bar{V}, \hat{V}_{i,t+1} \right\} 
\hat{D}_{it} = (1 - \tau) e^{(\bar{x} + x_{it} + (1 - \alpha)\Delta x_{t})} \hat{k}_{it}^{\alpha} + \tau \left( \delta \hat{k}_{it} + c \hat{b}_{it}^{S} + c \hat{b}_{it}^{L} \right) - \hat{i}_{it} - \Phi_{k} \left( \hat{i}_{it}, \hat{k}_{it} \right) \hat{k}_{it} 
+ p_{it}^{S} \hat{\omega}_{it}^{S} + p_{it}^{L} \hat{\omega}_{it}^{L} - (1 + c) \hat{b}_{it}^{S} - (\kappa + c) \hat{b}_{it}^{L} - \Phi_{L} \left( \hat{\omega}_{it}^{l} \right)$$

The above lines refer to equation (3.14) from the text.

Debt Pricing:

$$\begin{split} p_{it}^S &= & \mathbb{E}_t \left[ M_{t+1} \left( 1 + c \right) \right] \\ p_{it}^L \hat{b}_{i,t+1}^L &= & \mathbb{E}_t \left[ M_{t+1} \left( 1 - \mathbbm{1}_{\left\{ \hat{V}_{t+1} \leq \bar{V} \right\}} \right) \left( (\kappa_L + c) \hat{b}_{i,t+1}^L + (1 - \kappa_L) p_{i,t+1}^L \hat{b}_{i,t+1}^L \right) \right] \\ &+ & \mathbb{E}_t \left[ M_{t+1} \left( \mathbbm{1}_{\left\{ \hat{V}_{t+1} \leq \bar{V} \right\}} \right) \left( \hat{X}_{i,t+1}^L \right) \right] \\ \\ \hat{X}_{i,t+1}^L &= & (1 - \xi) \left( \hat{\pi}_{i,t+1} + (1 - \delta) \hat{k}_{i,t+1} - (1 + c) \hat{b}_{i,t+1}^S \right) \end{split}$$

The above lines refer to equations (3.10), (3.12), and (3.13) from the text.

## 3.7. Appendix: Numerical Solution

In this section, I outline the numerical solution that I use. As the model contains long-term debt, I use techniques from Chatterjee and Eyigungor (2012) to help convergence. The main difference is that in my model I have an additional choice variable (capital) which does not allow for the use of a monotonicity assumption. To get around this problem, I use a similar methodology as used in Gordon and Guerron-Quintana (2016), while also handling the additional short-term debt.<sup>20</sup>

The two key parts of the model are given by:

Equity Value:

$$\hat{V}\left(\Delta c_{t}, x_{it}, \hat{k}_{it}, \hat{b}_{it}^{L}, m_{it}\right) = \max_{\{\hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\}} \left\{\hat{D}_{it} - \Phi_{e}\left(\hat{D}_{it}\right) + g\left(m_{it}\right) + e^{(\Delta x_{t})}Z\left(\Delta c_{t}, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\right)\right\}$$

$$Z\left(\Delta c_{t}, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\right) = \mathbb{E}_{t}\left[M_{t+1} \times \max\left\{\bar{V}, \hat{V}_{i,t+1}\right\}\right]$$

Pricing of Long Term Debt:

$$p^{L}\left(\Delta c_{t}, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\right) = \mathbb{E}_{t}\left[M_{t+1}\left(1 - \mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}}\right)\left(\kappa_{L} + c + (1 - \kappa_{L})p_{i,t+1}^{L}\right)\right] + \mathbb{E}_{t}\left[M_{t+1}\left(\mathbb{1}_{\{\hat{V}_{t+1} \leq \bar{V}\}}\right)\left(\frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^{L}}\right)\right]$$

where  $g(\cdot)$  indicates a concave function of the noise,  $m_{it}$ . The noise is I.I.D. with a truncated  $\mathcal{N}(0,\sigma_m)$ , over support  $[-\bar{m},\bar{m}]$ . The algorithm broadly operates as follows:

- 0. Set aggregate grids and solve for the stochastic discount factor,  $M\left(\Delta c_t, \Delta c_{t+1}\right)$ , using iterative techniques on the price to consumption ratio in equation 3.7. Start with guesses for the expected continuation value,  $Z^0\left(\cdot\right)$  and bond pricing,  $p^{L0}\left(\cdot\right)$ . Neither is a function of the noise, m.
- 1. Input  $\{Z^0, p^{L0}\}$  into the right hand side of the equity value function and solve for one iteration of firm value. As the optimal policy is not dependent on m, we will receive two policy functions:  $\{\hat{k}^{1'}(\cdot), \hat{b}^{L1'}(\cdot)\}$ . The policy functions are specified over the entire state space.
- 2. For every state vector,  $S_{it} = \{\Delta c_t, x_{it}, \hat{k}_{it}, \hat{b}^L_{it}\}$ , compute the default decision (D) over the support of m. Because  $\hat{V}$  is monotonic in m, we check which of three possible cases hold:

<sup>&</sup>lt;sup>20</sup>There is one major difference in the way I compute my model relative to the above references. While I allow for the noise to impact the default decision, I do not allow it to affect policy functions, as it is only additive and separable in the Bellman equation. Because of this, the policy functions will not receive the smoothing benefits of the noise. The key reason I do not allow for the interaction between policies and the IID noise is because I would not like to change the concavity on the dividend flow. If I did change the concavity, this would alter the basic structure of the problem.

(a) 
$$\left\{\hat{V}\left(S_{it}, \bar{m}\right) \leq \bar{V}\right\}$$

In this case the firm will always default over the support. The optimal decision will be given by:

$$D\left(S_{it}, m_{it}\right) = 1 \ \forall m_{it}$$

(b) 
$$\left\{ \hat{V}\left(S_{it}, -\bar{m}\right) \leq \bar{V} \quad and \quad \hat{V}\left(S_{it}, \bar{m}\right) > \bar{V} \right\}$$

In this case, we know that the firm switches its default decision over the support of m. Because of the concave nature of g, we know there exists an  $\tilde{m}$  in the support of m such that:

$$\hat{V}\left(S_{it}, \tilde{m}\right) = \bar{V}$$

With  $\tilde{m}$  defined in such a manner the default decision will be given by:

$$D(S_{it}, m_{it}) = \begin{cases} 1, & \text{if } m_{it} \le \tilde{m} \\ 0, & \text{otherwise} \end{cases}$$

(c) 
$$\left\{\hat{V}\left(S_{it}, -\bar{m}\right) > \bar{V}\right\}$$

In this case the firm will never default over the support. The optimal decision will be given by:

$$D(S_{it}, m_{it}) = 0 \quad \forall m_{it}$$

Note that in order to compute values across different m, we use the optimal policies computed in step 1 of the procedure. Another implication of this procedure is that, if applicable,  $\tilde{m} = \tilde{m}(S_{it})$ .

- 3. To this point we have computed optimal policies which matter in the case firms do not default, as well as indicator variables for default. In some cases the default decision may vary over m, so we have also computed default breakpoints. Using this information we can compute our next iteration of expected continuation value and bond price,  $\{Z^1, p^{L1}\}$ .
  - (a) Computing Expected Continuation Value. The discounted, continuation value will be given by:

$$\mathbb{E}_{t}\left[M_{t+1} \times \max\left\{\bar{V}, \hat{V}_{i,t+1}\right\}\right] = \mathbb{E}_{t}\left[M_{t+1} \times \underbrace{\mathbb{E}\left[\max\left\{\bar{V}, \hat{V}_{i,t+1}\right\} \middle| \Delta c_{t+1}, x_{i,t+1}\right]}_{\text{reflects expectation solely over } m}\right]$$

Using the law of iterated expectations as shown above, we can focus on first computing the expectations of the continuation value over  $m_{i,t+1}$  and then taking an expectation

over uncertainty regarding the fundamental states. For the purposes of my work, the second expectation is taken using a standard discrete-state approach. Hence, I will focus on explaining the bracketed expectation below.

$$\mathbb{E}\left[\max\left\{\bar{V},\hat{V}_{i,t+1}\right\} \middle| \Delta c_{t+1},x_{i,t+1}\right] = \mathbb{E}\left[D\left(S_{i,t+1},m_{i,t+1}\right)\bar{V} + (1-D\left(S_{i,t+1},m_{i,t+1}\right))\hat{V}_{i,t+1}\left(S_{i,t+1},m_{i,t+1}\right)\right] + \mathbb{E}\left[D\left(S_{i,t+1},m_{i,t+1}\right)\bar{V}_{i,t+1}\right] + \mathbb{E$$

where  $S_{i,t+1}$  encorporates the relevant future productivity states. To compute the right hand side expectation, for each given state  $(S_{i,t+1})$ , there will be two cases:

I. There is **no switch** in default over m. In the case where the firm always defaults, the expectation will be given by  $\bar{V}$ . In the case where the firm always continues to operate, the expectation will be given by:

$$\int_{-\bar{m}}^{\bar{m}} \hat{V}\left(S_{i,t+1}, m_{i,t+1}\right) dm = \sum_{i=1}^{14} \left[ \hat{V}\left(S_{i,t+1}, \frac{m_j + m_{j+1}}{2}\right) \times Pr\left(m_j < m_{i,t+1} < m_{j+1}\right) \right]$$

where Pr denotes the probability. Notice above that we approximate the integral using a numerical integral with 14 intervals. All  $m_j$  refer to elements from an equally spaced vector over the support of m,  $[m_1, m_2, \ldots, m_{14}, m_{15}]$ .

II. There is a switch in default over m, which occurs at  $\tilde{m}$ . We now write the expectation from above as:

$$\int_{-\bar{m}}^{\bar{m}} \max \left\{ \bar{V}, \hat{V}\left(S_{i,t+1}, m_{i,t+1}\right) \right\} dm = \int_{-\bar{m}}^{\bar{m}} \bar{V} dm + \int_{\bar{m}}^{\bar{m}} \hat{V}_{i,t+1}\left(S_{i,t+1}, m_{i,t+1}\right) dm \\
= \bar{V} \times Pr\left(m_{i,t+1} < \tilde{m}\right) \\
+ \int_{\bar{m}}^{\bar{m}} \hat{V}_{i,t+1}\left(S_{i,t+1}, m_{i,t+1}\right) dm$$

Without loss of generality, suppose that  $\tilde{m}$  falls between  $m_{k-1}$  and  $m_k$ . Then the last integral will be computed as:

$$\begin{split} \int_{\tilde{m}}^{\tilde{m}} \hat{V}_{i,t+1} \left( S_{i,t+1}, m_{i,t+1} \right) dm &= \hat{V}_{i,t+1} \left( S_{i,t+1}, \frac{\tilde{m} + m_k}{2} \right) \times Pr \left( \tilde{m} < m_{i,t+1} < m_k \right) \\ &+ \sum_{j=k}^{14} \left[ \hat{V} \left( S_{i,t+1}, \frac{m_j + m_{j+1}}{2} \right) \times Pr \left( m_j < m_{i,t+1} < m_{j+1} \right) \right] \end{split}$$

After computing this at a given state, we are left with  $\tilde{Z}^1(S_{i,t+1})$ . Using this object we can proceed and compute our desired object using standard discrete methods:

$$Z^{1}\left(\Delta c_{t}, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\right) = \mathbb{E}_{t}\left[M_{t+1}\tilde{Z}^{1}\left(S_{i,t+1}\right)\right]$$

(b) Computing the Bond Price. Similar to the computation of the expected continuation value the bond price will be a function of a default portion and a non-default portion.

$$p^{L}\left(\Delta c_{t}, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L}\right) = \mathbb{E}_{t}\left[M_{t+1}\left(1 - D_{i,t+1}\right)\left(\kappa_{L} + c + (1 - \kappa_{L})p_{i,t+1}^{L}\right)\right] + \mathbb{E}_{t}\left[M_{t+1}D_{i,t+1}\left(\frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^{L}}\right)\right]$$

where  $D_{i,t+1}$  is the potential default decision at time t+1. It is important to realize that the future bond price,  $p_{i,t+1}^L$  is a function of a number of future policies:

$$p_{i,t+1}^{L} = p^{L} \left( \Delta c_{t+1}, x_{i,t+1}, \hat{k}_{i,t+2}, \hat{b}_{i,t+2}^{L} \right) = p^{L} \left( \dots, \hat{k}' \left( \Delta c_{t+1}, x_{i,t+1}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^{L} \right), \hat{b}^{L'} \left( \dots \right) \right)$$

where  $\hat{b}^{L'}$  is a function of the same variables as  $\hat{k}'$ . In the course of the algorithm we use the policy functions computed in step 1 to express the future price as a function of  $S_{i,t+1}$ . All of this being said we are left with a familiar problem:

$$p^{L}(\cdot) = \mathbb{E}_{t}\left[M_{t+1} \times ((1 - D(S_{i,t+1}, m_{i,t+1}))f_{1}(S_{i,t+1}) + D(S_{i,t+1}, m_{i,t+1})f_{2}(S_{i,t+1}))\right]$$

$$f_1(S_{i,t+1}) = \kappa_L + c + (1 - \kappa_L)p^L(S_{i,t+1})$$

$$f_2(S_{i,t+1}) = \frac{\hat{X}_{i,t+1}^{PD}}{\hat{b}_{i,t+1}^L} = xb(S_{i,t+1})$$

Because the inner piece that multiples  $M_{t+1}$  is also dependent on the realization of the noise, we again use law of iterated expectations to focus on computing the inner expectation over m, given by:

$$\tilde{p}^{L1}(S_{i,t+1}) = \mathbb{E}\left[\left((1 - D(S_{i,t+1}, m_{i,t+1}))f_1(S_{i,t+1}) + D(S_{i,t+1}, m_{i,t+1})f_2(S_{i,t+1})\right)\right]$$

The computation of this expectation will be conceptually identical to the technique used in part (a). As a result I won't go into further detail here. The final derived bond price will be:

$$p^{L1}\left(\Delta c_t, x_{it}, \hat{k}_{i,t+1}, \hat{b}_{i,t+1}^L\right) = \mathbb{E}_t\left[M_{t+1}\tilde{p}^{L1}(S_{i,t+1})\right]$$

4. Having computed expected continuation value and bond prices we check convergence:

$$\varepsilon = \max\left\{\|Z^0 - Z^1\|, \|p^{L0} - p^{L1}\|\right\}$$

If  $\varepsilon$  is small enough we are done. Otherwise, choose a new starting guess based on the recent outcomes:

$$Z^{NEW} = \xi_z Z^0 + (1 - \xi_z) Z^1$$
  
$$p^{NEW} = \xi_p p^{L0} + (1 - \xi_p) p^{L1}$$

Let  $Z^0 = Z^{NEW}, p^{L0} = p^{NEW}$  and go back to step 1. In practice, I set  $\xi_z = 0$  and choose  $\xi_p = .95$ .

Table 12: Aggregate Predictive Regressions

k = 1Q	2	3	4	5	6	7	8

$$\frac{1}{k} \sum_{i=1}^{k} \Delta y_{t+i} = \beta_0 + \beta_X' X_t + \beta_{LT} LT DR_t^c + error_{t+k}:$$

$$\frac{1}{k} \sum_{i=1}^{k} \Delta i_{t+i} = \beta_0 + \beta_X' X_t + \beta_{LT} LTDR_t^c + error_{t+k}$$
:

This table examines the relationship between average future, real per-capita output growth and investment growth, and the HP-filtered cyclical component of the aggregate long-term debt share  $(LTDR_t^c)$ . Each panel displays the results of regressing an economic aggregate on a vector of controls,  $X_t$ , and the cyclic component. The first line of each panel measures the sensitivity,  $\beta_{LT}$ , with respect to the LTDR measure; the second line measures the t-statistic when accounting for Newey-West standard errors; the final row represents adjusted  $R^2$  measures. The column heading provides the horizon of the average future dependent variable, in terms of k quarters. Controls are at time t and include: real per-capita GDP growth, real per-capita consumption growth, CPI inflation, log PD ratios, the difference between Moody's BAA and AAA interest rates, the yield on the 3M US Treasury bill, the difference between the 10Y US Treasury bond and 3M Treasury bill yields, and the growth rate of aggregate debt. All economic growth variables are in log terms, quarter over quarter, while financial prices are in level terms, from 1954 onwards. The number of stars correspond to double-sided significance at the 90%, 95%, and 99%\*\*\* confidence levels.

Table 13: Firm-Level Profitability and Long Term Debt

Explanatory Variable	(1)	(2)	(3)

$$\frac{\pi_{it}}{k_{it}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LTDR_{it} + error_{it}:$$

$\sigma(\Pi_i/K_i)$	-3.65***	-3.66***	-3.12***
t-statistic	-18.3	-18.3	-15.0
$B_i/K_i$ t-statistic	031***	030***	034***
	-29.7	-29.5	-31.2
$I_{i,-1}/K_{i,-1}$ t-statistic	.068***	.062***	.045***
	11.0	9.91	7.11
$Q_{i,-1}$ t-statistic	.003** 12.1	.003*** 11.5	.003*** 13.3
$LTDR_i / 100$ t-statistic	.475***	.497***	.290***
	7.13	7.50	4.22
No. of Firms  No. of Observations  Adjusted $R^2$ Firm Level Controls  Macro Controls  Firm Fixed Effects	10,872 217,967 .028 Yes No	10,872 217,967 .032 Yes Yes No	10,872 217,967 .014 Yes Yes Yes

This table regresses firm profitability onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current book leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%\*, 95%\*\*, and 99%\*\*\* confidence levels.

Table 14: Firm-Level Investment and Long Term Debt

Explanatory Variable	(1)	(2)	(3)

$$\frac{i_{it}}{k_{it}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LT D R_{it} + error_{it}:$$

$\sigma(\Pi_i/K_i)$	080*	112***	.028
t-statistic	-1.89	-2.62	.550
$B_i/K_i$ t-statistic	007***	007***	013***
	-21.3	-23.2	-26.7
$I_{i,-1}/K_{i,-1}$ t-statistic	.393*** 60.4	.384*** 59.0	.267*** 38.5
$Q_{i,-1}$ t-statistic	.001*** 17.9	.001*** 16.8	.001*** 19.7
$LTDR_i / 100$ t-statistic	.334***	.362***	.161***
	16.9	18.4	6.50
No. of Firms  No. of Observations  Adjusted $R^2$ Firm Level Controls  Macro Controls  Firm Fixed Effects	10,598 204,211 .312 Yes No	10,598 204,211 .314 Yes Yes No	10,598 204,211 .254 Yes Yes

This table regresses capital-adjusted firm investment onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current book leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%\*, 95%\*\*, and 99%\*\*\* confidence levels.

Table 15: Predicting Firm-Level Variables using Long-Term Debt

Explanatory Variable	k = 1	2	3	4

$$\frac{1}{k+1} \sum_{j=0}^{k} \frac{\pi_{i,t+j}}{k_{i,t+j}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LT D R_{it} + error_{it}:$$

$LTDR_i / 100$	.317***	.244***	.179***	.113***
t-statistic	11.8	9.79	7.57	4.87
No. of Firms  No. of Observations  Adjusted $R^2$ Firm Level Controls  Macro Controls  Firm Fixed Effects	10,377	9.880	9,427	8,946
	207,166	196,785	186,852	177,613
	.012	.005	.003	.001
	Yes	Yes	Yes	Yes
	Yes	Yes	Yes	Yes
	No	No	No	No

$$\frac{1}{k+1} \sum_{j=0}^{k} \frac{i_{i,t+j}}{k_{i,t+j}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LT D R_{it} + error_{it}$$
:

$LTDR_i / 100$	.298***	.242***	.191***	.164***
t-statistic	20.5	17.4	14.1	12.2
No. of Firms	9,943	9,143	8,620	8,066
No. of Observations	185,836	169,889	157,363	146,032
Adjusted $R^2$	.337	.311	.295	.269
Firm Level Controls	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No
	•			

This table regresses average, future profitability and investment onto firm and macro level variables. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%\*, 95%\*\*, and 99%\*\*\* confidence levels.

Table 16: Firm-Level Variables and Long Term Debt, by Book Size

Explanatory Variable	Small	2	3	4	Large

$$\frac{\pi_{it}}{k_{it}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LTDR_{it} + error_{it}:$$

$LTDR_i / 100$	.587***	.247***	.419***	.112	.034
t-statistic	3.98	2.17	4.15	1.12	.270
Average Log Size	2.57	4.11	5.22	6.37	8.32
No. of Firms	4,158	4,469	3,960	3,013	1,756
No. of Observations	46,030	$45,\!697$	44,819	43,723	37,697
Adjusted $R^2$	.105	.009	.064	.151	.265
Firm Level Controls	Yes	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No	No
	'	•		•	'

$$\frac{i_{it}}{k_{it}} = \beta_0 + \beta_X' X_{it} + \beta_{LT} LTDR_{it} + error_{it}:$$

$LTDR_i / 100$	.342***	.397***	.382***	.322***	.101**
t-statistic	9.05	10.5	9.28	7.34	2.00
Average Log Size	2.57	4.11	5.22	6.37	8.32
No. of Firms	3,998	4,308	3,782	2,882	1,693
No. of Observations	43,026	42,897	42,138	41,133	35,016
Adjusted $\mathbb{R}^2$	.186	.298	.354	.399	.399
Firm Level Controls	Yes	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	No	No

This table regresses profitability and investment onto firm and macro level variables, running separate pooled regressions for firms of different size quintile. Five size quintiles are computed period by period, based on the book value of assets. Robust standard errors are clustered at the firm level. Firm-level controls include historical one year volatility on profitability, current leverage, lagged investment rate, lagged ratio of market value to book value of assets, and the long-term debt share. Macro controls include the quarterly growth rates of industrial production and the consumer price index, the level of the 3M treasury bill, the difference between the 10Y treasury bond and 3M bill, and the difference between Moody's measures of BAA and AAA corporate bond yields. All data is from 1984 onwards. The number of stars correspond to double-sided significance at the 90%\*, 95%\*\*, and 99%\*\*\* confidence levels.

Table 17: Calibration Parameters for Baseline Model

Parameter	Value	Note
rarameter	varue	Note
0	00	
eta	.98	Time Discount
$\gamma$	7.5	Risk Aversion
$\psi$	2	Intertemporal Elasticity of Substitution
$\delta$	.025	Depreciation of Capital
$\alpha$	.65	Production Exponent on Capital
$\phi_k$	1	Coefficient on Capital Adjustment Costs
$\kappa_L$	.05	Governs Average Maturity of Debt
c	.01	Coupon Rate
$s_0$	.08	Selected to hit $b^S/k$ ratio in the data
30		Soldered to inv o //w ladio in the date
$\Phi_{L,a}$	.006	Fixed Issuance Cost of Long Term Debt
$\Phi_{e,a}$	.06	Fixed Issuance Cost of Equity
$\Phi_{e,b}$	.05	Proportional Issuance Cost of Equity
$\Psi_{e,b}$	.00	1 roportional issuance Cost of Equity
$ar{x}$	-2.50	Productivity constant
		Productivity constant  Firm Productivity Autocompletion
$ ho_z$	.85	Firm Productivity Autocorrelation
$\sigma_z$	.15	Firm Productivity Conditional Volatility
$\mu_c, \rho_c, \sigma_c$		Set to match mean, volatility, and autocorrelation of $\Delta c_t$
$\lambda_x$	3.50	
_		
$ar{V}$	1.425	Chosen to hit default rate

This table provides the calibrated parameters for the baseline version of the model. The calibration is at a quarterly basis. For a discussion of the specific parameters see the main text.

Table 18: Model Versus Data: Firm-Level Statistics

Variable	Description	Model	Data (2.5, 97.5%)
$\mathbb{E}\left(\mathbb{E}_{t}\left(rac{\pi_{it}}{k_{it}} ight) ight) \\ \mathbb{E}\left(\sigma_{t}\left(rac{\pi_{it}}{k_{it}} ight) ight)$	Cross-Sec Mean of Profitability Cross-Sec Stdev of Profitability	.066 .017	.022 (.020, .024) .050 (.047, .053)
$\mathbb{E}\left(\mathbb{E}_t\left(rac{i_{it}}{k_{it}} ight) ight) \\ \mathbb{E}\left(\sigma_t\left(rac{i_{it}}{k_{it}} ight) ight)$	Mean of Investment Rate Stdev of Investment Rate	.029	.040 (.035, .045) .057 (.050, .064)
$\frac{\mathbb{E}\left(\mathbb{E}_t\left(\frac{b_{it}^S + b_{it}^L}{k_{it}}\right)\right)}{\mathbb{E}\left(\sigma_t\left(\frac{b_{it}^S + b_{it}^L}{k_{it}}\right)\right)}$	Mean of Book Leverage Stdev of Book Leverage	.193 .079	.249 (.238, .263) .183 (.180, .185)
$\mathbb{E}\left(\mathbb{E}_t\left(rac{b_{it}^L}{b_{it}^S+b_{it}^L} ight) ight) \ \mathbb{E}\left(\sigma_t\left(rac{b_{it}^L}{b_{it}^S+b_{it}^L} ight) ight)$	Mean of Long Debt Ratio Stdev of Long Debt Ratio	.520 .225	.694 (.669, .719) .323 (.314, .331)
$400 \times \mathbb{E}\left(\mathbb{E}_{t}\left(\frac{\kappa_{L}+c}{p_{it}^{L}} - \frac{\kappa_{L}+c}{p_{it}^{*L}}\right)\right)$ $400 \times \mathbb{E}\left(\sigma_{t}\left(\frac{\kappa_{L}+c}{p_{it}^{L}} - \frac{\kappa_{L}+c}{p_{it}^{*L}}\right)\right)$	Mean of Credit Spread Stdev of Credit Spread	1.84 12.30	1.25 (.909, 1.65)
$400 \times \mathbb{E} \left( \mathbb{E}_t \left( \mathbb{1}_{\mathrm{Default},it} \right) \right)$	Mean of Default Rate	.968	1.08 (.422, 1.68)

This table provides firm-level statistics generated from a panel simulation of 3000 firms across 500 quarters. For variable  $x_{it}$ ,  $\mathbb{E}\left(\mathbb{E}_t\left(x_{it}\right)\right)$  refers to the time series mean of the cross-sectional mean. Meanwhile,  $\mathbb{E}\left(\sigma_t\left(x_{it}\right)\right)$  refers to the time series mean of the cross-sectional standard deviation of  $x_{it}$ . All model computations remove defaulted firms. Quarterly data for profitability, investment rates, book leverage, and the long-term debt ratio are taken from Compustat, 1984 – 2015. The credit spread is defined as the difference between the Moody's BAA and AAA corporate bond yields. The annual default rate series is also Moody's. The numbers in parentheses represent bootstrapped time series errors at the 2.5% and 97.5% bounds.

Table 19: Model Versus Data: Aggregate Statistics

Variable	Description	Model	Data (2.5, 97.5%)	
$\mathbb{E} (\Delta c_t)$ $\sigma (\Delta c_t)$ $\rho (\Delta c_t, \Delta c_{t-1})$	Mean Cons. Growth Stdev Cons. Growth AR(1) Cons. Growth	.446 .454 .499	.493 (.397, .590) .466 (.413, .510) .450 (.345, .536)	
$ \rho\left(\Delta c_t, \Delta y_t\right)  \rho\left(\Delta c_t, \Delta i_t\right)  \rho\left(\Delta c_t, \frac{\sum_i (b_{it}^S + b_{it}^L)}{\sum_i k_{it}}\right) $	Corr(Cons. Growth, Output Growth) Corr(Cons. Growth, Investment Growth) Corr(Cons. Growth, Leverage)	.741 .444 .400	.804 (.750, .863) .596 (.460, .743)	
$\rho\left(\Delta c_t, \frac{\sum_i (b_{it}^L)}{\sum_i (b_{it}^S + b_{it}^L)}\right) \\ \rho\left(\Delta y_t, \frac{\sum_i (b_{it}^L)}{\sum_i (b_{it}^S + b_{it}^L)}\right)$	Corr(Cons. Growth, Agg. LTDR ) Corr(Output Growth, Agg. LTDR )	.396 .664	.262 (.151, .356) .323 (.264, .388)	
$\rho\left(AggDefault_{t}, \Delta c_{t}\right) \\ \rho\left(MeanCreditSpread_{t}, \Delta c_{t}\right)$	Corr(Agg. Default Rate, Cons. Growth) Corr(Agg. Credit Spread, Cons. Growth)	365 325	223 (315,053) 505 (659,345)	

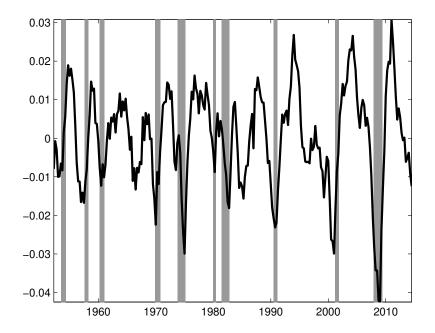
This table provides aggregate statistics generated from the aggregation of a panel simulation of 3000 firms across 500 quarters. All model computations remove defaulted firms. Data for consumption and output series are taken from NIPA accounts, from 1954 onwards. The data counterpart of the long-term debt series comes from the HP filtered version of the Flow of Funds measure. The credit spread is defined as the difference between the Moody's BAA and AAA corporate bond yields. The annual default rate series is also Moody's. The numbers in parentheses represent bootstrapped time series errors at the 2.5% and 97.5% bounds.

Table 20: Sorting on Distance to Default

25 (37 + 11	0 1 11 1			1 4	
Mean of Variable	Quintile 1	2	3	4	5
Detrended Value $(\hat{V}_{it})$ $Index$	2.33 86.51	2.51 94.60	2.65 100.0	2.81 106.0	3.10 119.4
Detrended Capital $(\hat{k}_{it})$ Index	1.07 78.25	1.24 92.14	1.35 100.0	1.45 108.0	1.67 126.6
Profitability $\left(\frac{\pi_{it}}{k_{it}}\right)$	.052	.058	.065	.071	.082
Investment Rate $\left(\frac{i_{it}}{k_{it}}\right)$	.012	.026	.031	.036	.042
Book Leverage $\left(\frac{b_{it}^S + b_{it}^L}{k_{it}}\right)$	.156	.186	.205	.213	.204
Book Long Term Debt Ratio $\left(\frac{b_{it}^L}{b_{it}^S + b i_{it}^L}\right)$	.417	.521	.553	.557	.530
Long Term Credit Spread (%, Annual)	6.56	.901	.686	.583	.468

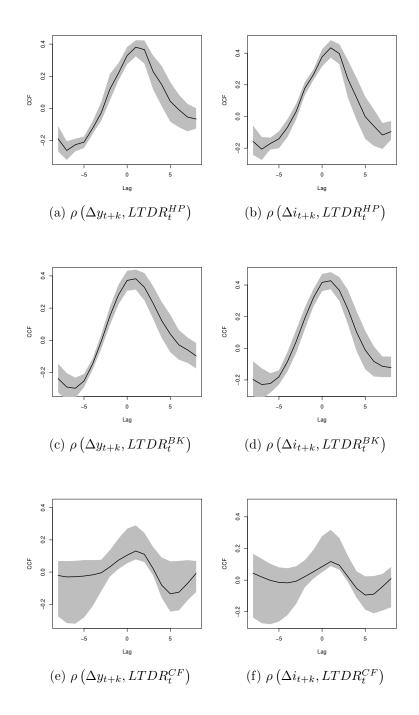
This table provides cross-sectional model statistics generated from a panel simulation of 3000 firms across 500 quarters. Each period I sort non-defaulted firms by their detrended value ( $\hat{V}_{it}$ ) into five quintiles. After generating five time series for each variable I compute the average which is reported above. Under deterended value and capital I also include values that are indexed to the middle quintile.

Figure 14: Cyclical Component of the Long Term Debt Share



This figure displays the time series of the cyclical component of the long-term debt share, using a Hodrick and Prescott (1997) filter. Grey bars indicate NBER-defined recession dates. Data related to the long-term debt share comes from the Federal Reserve Flow of Funds. The frequency is quarterly from 1952Q2 through 2014Q2.

Figure 15: Cross Correlation of Economic Aggregates and LTDR Cycles



This figure displays cross correlation functions between cyclical components of the long-term debt share and economic aggregates. Figures in the left column correlate aggregate output growth and the cyclical component of the long-term debt share while those in the right column test investment growth. From top to bottom, we examine filtered values of the long-term debt share using: (1) Hodrick and Prescott  $(LTDR^{HP})$ , (2) Baxter and King band-pass  $(LTDR^{BP})$  and (3) Christiano and Fitzgerald band-pass  $(LTDR^{CF})$  filters. The x-axis provides the number of forward lags for each economic aggregate while the y-axis provides the cross correlation. All data is quarterly from 1952Q2 through 2014Q2. Bootstrapped 95% confidence intervals are computed at each lag and given by the gray bands.

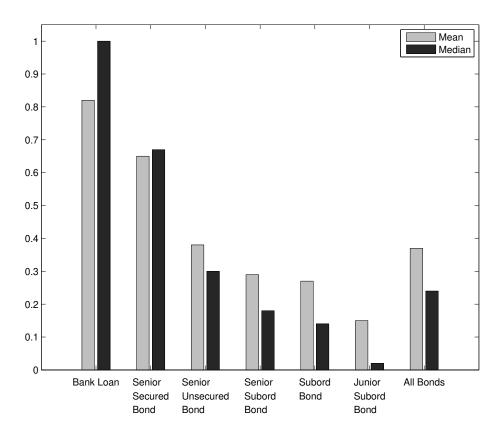


Figure 16: Recovery Rates Across Debt Types

This figure displays the mean and median recovery rates across different loan and bond seniority types. All data is from "Moody's Ultimate Recovery Database" and spans approximately 3500 loans and bonds over 720 US non-financial corporate default events. All data refers to the twenty years preceding the financing crisis (1987 - 2007). From left to right, bars represent statistics related to: bank loans, senior secured corporate bonds, senior unsecured, senior subordinated, subordinated, junior subordinated, and all corporate bonds.

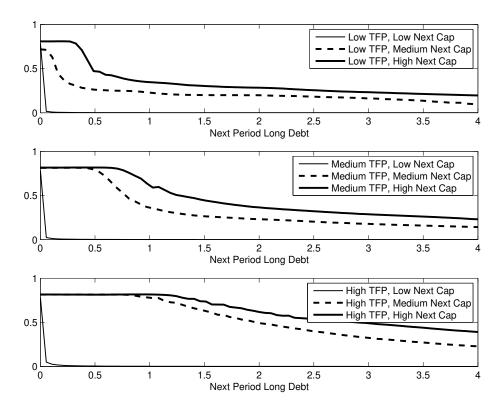


Figure 17: Model-Implied Bond Prices

This figure displays the equilibrium value for  $p_{it}^L(\hat{A}_{it},\hat{k}_{i,t+1},\hat{b}_{i,t+1}^L)$  across states, where  $\hat{A}_{it}$  indicates the joint aggregate and idiosyncratic productivity states. The top panel focuses on the low joint TFP state (low aggregate and low idiosyncratic). The second panel focuses on the medium aggregate and idiosyncratic, while the bottom panel relates to high productivity states. In each panel the different lines (from bottom to top) represent different choices for capital next period  $(\hat{k}_{i,t+1})$ . The bottom axis represents the choice of next period long-term debt  $(\hat{b}_{i,t+1}^L)$ .

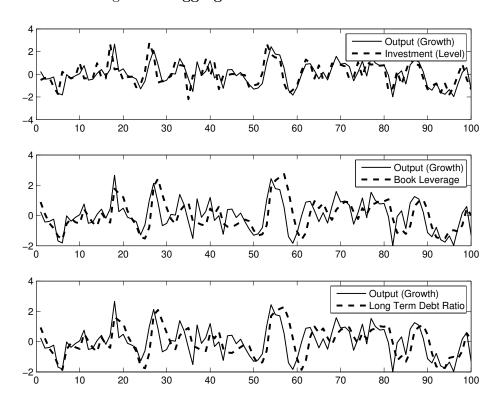
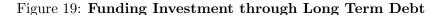


Figure 18: Aggregate Behavior of the Model

This figure displays the aggregate behavior of the model in a sample simulation of 100 quarters. In all three graphs the solid black line represents output growth. From top to bottom, the dashed line represents investment, book leverage, and the long-term debt ratio. All values are standardized and provided as z-scores.



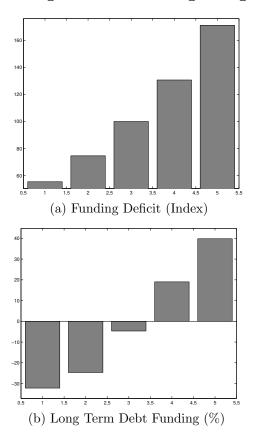


Figure (a) displays the time series average of the simulated aggregate funding deficits across five aggregate states. The left most bar relates to the lowest consumption growth state while the right most to highest. All of the state values are indexed to the middle state. To build the funding deficit we simulate the model to compute the gap between total dividend and the amount raised through short and long-term debt  $(\hat{D}_{it} - p_{it}^S \hat{w}_{it}^S - p_{it}^L \hat{w}_{it}^L)$ . To receive the aggregate number, we sum across non-defaulted firms. Figure (b) displays the time series average of the percentage of the funding deficit that comes from the long-term debt proceeds  $(p_{it}^L \hat{w}_{it}^L)$ . For cases where this percentage is negative, this suggests that long-term debt was puchased back.

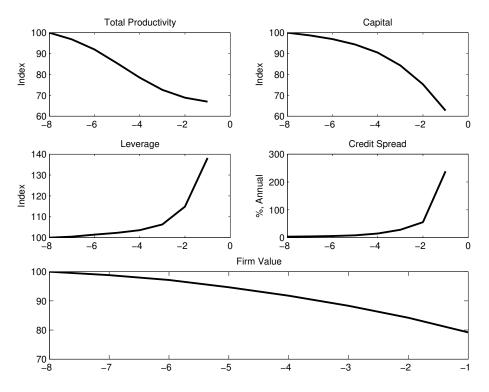


Figure 20: Firm Behavior Preceding Default

This figure displays the average behavior of the firm eight quarters in advance of default. The panels represent, from left to right, total productivity  $(\hat{A}_{it})$ , de-trended capital  $(\hat{k}_{it})$ , leverage  $\left(\frac{\hat{b}_{it}^S+\hat{b}_{it}^L}{\hat{k}_{it}}\right)$ , the credit spread  $\left(\frac{\kappa_L+c}{p_{it}^L}-\frac{\kappa_L+c}{p_{it}^L}\right)$ , and detrended firm-value  $(\hat{V}_{it})$ . The bottom axis provides the number of quarters in relation to default which occurs at time 0. Productivity, capital, leverage, and firm value are all indexed to the initial value, 8 quarters in advance of default, while the credit spread is expressed in annual percentage terms.

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