Essays on Bayesian Estimation of DSGE Models with Indeterminacy

By

Bo Zhang

THESIS

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Abstract

This thesis comprises three self-contained papers on the Bayesian estimation of DSGE models with indeterminacy.

The first paper estimates a small open economy model of Australia with positive trend inflation while allowing for equilibrium indeterminacy. It shows that positive trend inflation can shrink the determinacy region especially when trend inflation rate or price stickiness is high. The estimation is conducted from 1983:I to 1993:I covering the pre-inflation-targeting regime and from 1993:II to 2007:III covering the inflation-targeting regime. It finds that Australian monetary policy before inflation targeting period made the economy more prone to multiple equilibria, whereas the inflation targeting policy pushed the economy towards stability.

The second paper estimates an artificial economy with financial market frictions. It shows that animal spirits are prime drivers of U.S. business cycle fluctuations. Animal spirits shocks account for well over a third of output fluctuations over the period from 1955 to 2014. Financial friction and technology shocks are considerably less significant in explaining the oscillations in aggregate real economic activity. It also finds that a substantial part of aggregate output's contraction during the Great Recession was caused by adverse shocks to expectations.

The third paper provides a quantitative assessment of an efficiency-wage model in which equilibrium can be indeterminate even without externalities or increasing returns. Indeterminacy in this model is linked to the degree of risk sharing between employed and unemployed workers. The theoretical model is estimated on U.S. data via full information Bayesian methods. The analysis

shows that the shirking model is capable of matching the stylized facts of the labor market. However, the data strongly favor a version of the artificial economy that is characterized by determinacy.

Declaration

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Chapter 1

Introduction

This thesis aims to contribute to our understanding of the role that animal spirits play in driving business cycles and testing for indeterminacy in a dynamic stochastic general equilibrium model via Bayesian methods. In the following three self-contained papers, I provide some channels by which the model can generate multiple equilibria and empirically test whether the data favors the determinate version of the model or the indeterminate one.

The first paper estimates a small open economy model for Australia with positive trend inflation while allowing for equilibrium indeterminacy. It is well known that passive monetary policy can give rise to multiple equilibria within the prototypical New Keynesian model. So far, most empirical works about the relationship between monetary policy and indeterminacy studied the U.S. economy and are based on closed economy models. Few studies, however, employ a small open economy model to test indeterminacy. Australia is a small open economy since the trade-to-GDP ratio is significantly high (40% in 2016). Australia underwent monetary policy switches from the non-inflation-targeting to inflation-targeting regime in the early 1990s. From then on, the volatility of nominal interest rate and inflation decreased sharply. Therefore, a natural question arises: is it likely that Australian economy switched from indeterminacy to determinacy after adopting the inflation-targeting regime? The reasoning why I incorporate positive trend inflation into the model is twofold. On the one hand, Australian inflation in the 1980s was apparently higher than zero; on the other hand, positive trend inflation can shift the boundary between determinacy and indeterminacy zones in the closed economy model. Therefore, it is worthwhile to see whether such phenomenon still holds in the small open economy model. The paper finds that positive trend inflation can shrink the determinacy region especially when the trend inflation rate or price stickiness is high. Besides, Australian monetary policy before the inflation targeting period made the economy more prone to multiple equilibria, whereas the inflation targeting policy pushed the economy towards stability.

The second paper pursues to identify the sources of business cycles for the post-Korean War American economy, allowing belief shocks to compete with fundamental shocks. To achieve this goal, this paper estimates an artificial economy with financial market frictions. The model is estimated by full information Bayesian methods using quarterly U.S. data covering the period from 1955:I to 2014:IV. The estimation results support the view that people's animal spirits play a significant role in the U.S. business cycle. In particular, variance decomposition suggests that animal spirits are behind around forty percent of output growth variations and they explain an even larger portion of fluctuations in investment spending. Disturbances that originate in the financial sector explain less than ten percent of output fluctuations. Moreover, belief shocks have played an essential role in the sharp contraction in economic activity of the Great Recession that began at the end of 2007. Finally, we compare the empirical fit of the model with determinacy versus indeterminacy. We find that the indeterminate model in which animal spirits play a significant role turns out to be empirically superior.

The third paper provides a quantitative assessment of an efficiency-wage model in which equilibrium can be indeterminate even without externalities or increasing returns. The efficiency wage theories have long been received much attention due to their potential to explain the presence of involuntary unemployment and the behavior of wages over the business cycle, phenomena failed to be described by the early real business cycle models. The efficiency-wage is similar to the standard one-sector neoclassical growth model except that firms do not perfectly measure the quantity or quality of workers' effort. The wage rate is, therefore, set above the market clearing one to prevent workers from shirking. When all firms behave this way, their demand for labor decreases and an involuntary unemployment is reached where unemployed workers willing to work at prevailing wages. Alexopoulous (2004) modified the standard efficiency-wage model by assuming punishing detected shirkers monetary instead of firing them and allowing for different unemployment insurance arrangements between agents. Nakajima (2006) showed that such version of the model could generate indeterminacy even without externalities which made a significant theoretical contribution. Although indeterminacy seems to arise in the efficiency-wage model for realistic parameterization, it is vital that the implications be supported by empirical evidence. Therefore, this paper goes a further step to conduct an empirical evaluation of the model. In particular, the theoretical model is estimated in both determinacy and indeterminacy regions by full information Bayesian methods using quarterly U.S. data covering the period from 1964:I to 2007:IV. The results show that the estimated model parameters are consistent with the existing evidence and the shirking model performs reasonably well on various unconditional second moments of the data. The paper also applies the methodology developed by Bianchi and Nicolò (2017) to compare the model fit under determinacy and indeterminacy. The exercise shows that versions of the model with determinacy empirically outperform the indeterminate counterparts.

Chapter 2

Monetary policy and macroeconomic stability in a small open economy with trend inflation: the Case of Australia

2.1 Introduction

The last two decades have witnessed a growing interest in monetary policy analysis involving nominal interest rate feedback rules as a description of central bank behavior. It is well known that, depending on the activeness of the policy, these rules can give rise to indeterminacy and endogenous instability, resulting in the destabilization of the economy and potentially to a deterioration of welfare.[1]

[1] Benhabib, Schmitt-Grohé and Uribe (2001) provide a number of examples in which, contrary to what is commonly believed, active monetary policy gives rise to multiple equilibria and passive monetary policy renders the equilibrium unique. The model in current paper is not among those examples.

Thus, there is a strong case for the central bank to follow an active role to bring the economy into determinacy.[2]

Since the seminal work of Lubik and Schorfheide (2004), most existing empirical studies test indeterminacy for the U.S. economy. Recent examples include Bhattarai, Lee, and Park (2012), Doko Tchatoka, et al. (2017) and Hirose, Kurozumi and Van Zandweghe (2017). However, only very few empirical investigations have taken indeterminacy to the framework of small open economy DSGE model.[3] Different from the above literature which is all based on the closed economy model, the current paper employs a small open economy model to test indeterminacy.

Compared to the U.S. economy, Australia is more like a small open economy (with the trade-to-GDP ratio as high as 40% in 2016). In history, Australia underwent significant monetary policy regime switches. After the breakdown of the Bretton-Woods system, Australian policymakers decided on a new nominal anchor for their monetary policy and initially opted for managed exchange-rate regimes but, over time, these regimes proved to be inefficient. RBA eventually adopted an inflation-targeting (henceforth IT) regime in 1993. This regime has many benefits and is helpful to reduce inflation volatility and the inflationary impact of shocks and to increase anchoring of inflation expectations. From then on, as can be seen from Figure 2.1, Australia's inflation and nominal interest rate became much less volatile.[4] Therefore, a natural question arises: is it likely

[2]Even though monetary policy follows the Taylor principle that brings the economy into (local) determinacy, chaotic dynamics may arise once the zero bound on nominal interst rates is taken into consideration. Benhabib, Schmitt-Grohé and Uribe (2002) propose several fiscal and monetary policies to rule out such liquidity trap.

[3]Recent exception is Zheng and Guo (2013) who investigate China's monetary policy by estimating a small open economy DSGE model with indeterminacy.

[4]Inflation in 2000Q3 stands out is due to the fact that Goods and Services Tax (GST) had a significant but transitory impact on inflation only in 2000Q3 when this new tax system was implemented.

that Australian economy switched from indeterminacy to determinacy after the regime switch?

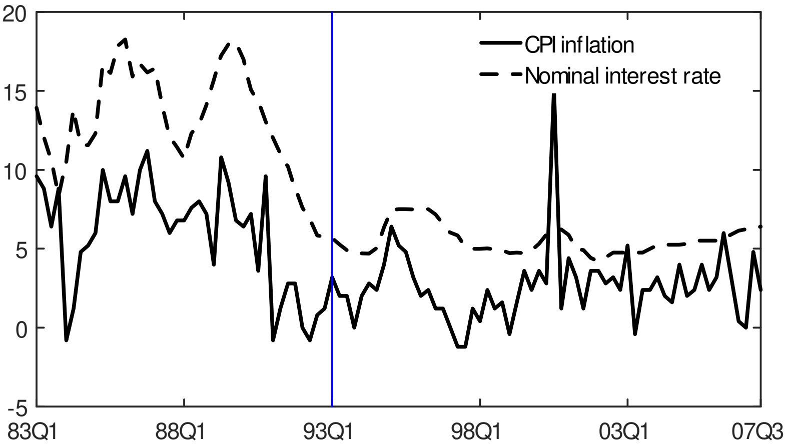

Figure 2.1: Annual CPI inflation and nominal interest rate

There is an extensive literature studying the economy in Australia. Nimark (2009) shows that a small micro-founded model of the Australian economy can capture the open economy dimensions quite well. Buncic and Melecky (2008), Jääskelä and Nimark (2011) and Rees, Smith, and Hall (2016) all identify the sources of Australian business cycle fluctuations. Justiniano and Preston (2010) explore optimal policy designs in Australia, Canada, and New Zealand. Abbas, Bhattacharya, Mallick, and Sgro (2016) investigate five different variants of the Gali-Monacelli new Keynesian Phillips curve for a small open economy using Australian data. However, none of the above takes positive trend inflation or indeterminacy into consideration. Lie and Yadav (2017) investigate whether the persistence and the time-varying nature of trend inflation can explain the persistence of inflation in Australia, but they also ignore multiple equilibria possibly resulted from monetary policy. The current paper is the first one to estimate a small open economy model with positive trend inflation while allowing for indeterminacy and also the first one to test whether the IT regime helped push the Australian economy towards stability.

The reasoning for positive trend inflation to be incorporated into the model is twofold: on the one hand, Australia experienced high levels of inflation before the IT period (henceforth Pre-IT) and targeted an inflation rate obviously higher than zero during the IT period; on the other hand, positive trend inflation can significantly affect the determinacy boundary (see for example Ascari and Ropele, 2009). The majority of the trend inflation literature has been conducted in a closed-economy context, and the analysis of the effect of trend inflation in a small open economy is quite limited. Kano (2016) and Junicke (2017) are the only papers that embed trend inflation into a two-country DSGE model. However, neither of them examines the effect of trend inflation on the determinacy properties. I find that as in a closed economy model, positive trend inflation can also shrink the determinacy region in a small open economy framework. This phenomenon is more obvious when trend inflation rate or price stickiness is high.

As for the empirical study, the estimation is conducted over two periods for Australia: Pre-IT from 1983:I to 1993:I and IT from 1993:II to 2007:III that excludes the global financial crisis period.[5] The main finding is that monetary policy in Australia before the IT regime could result in aggregate instability since data favors the indeterminacy version of the model, whereas a monetary policy regime switch to IT makes the determinacy model more supported. This result is robust for different measures of inflation.

The most closely related to this paper is the study by Lubik and Schorfheide (2007), who estimate small open economy models based on Galí and Monacelli (2005) for Australia, New Zealand, Canada, and the UK. The current paper

[5] Exchange rate policy in Australia shifted through several regimes before the Australian dollar was eventually floated in 1983, therefore, Pre-IT sample period starts from 1983:I in this paper.

performs a similar exercise based on their model but extends and complements their analysis in the following dimensions. First, it estimates the model log-linearized around a positive steady-state inflation. Second, it extends the parameter space allowing for indeterminacy and employs Lubik and Schorfheide (2003, 2004) methodology to estimate the model. Finally, it adopts the Sequential Monte Carlo (henceforth SMC) algorithm, wherein a particle approximation to the posterior is built iteratively through tempering the likelihood, developed by Herbst and Schorfheide (2014, 2015) instead of the Random-Walk Metropolis-Hastings (henceforth RWMH) algorithm in estimation.[6] The reason is that the SMC algorithm can generate more reliable estimates of model parameters for multi-modal than the widely used RWMH algorithm.

2.2 The Model

The small open economy model includes two economies, home (Australia) and foreign (rest-of-the-world). Domestic policy decisions do not have any impact on the rest of the world and both economies share identical preferences, technology, and market structure. The pass-through of exchange rates to import prices is complete and the law of one price holds. At home, there is a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. The current model incorporates indexation of prices to past inflation and trend inflation into the small open economy model developed by Lubik and Schorfheide (2007) and Del Negro and Schorfheide (2009), both of which are a simplified version of the model proposed by Galí and Monacelli (2005).

[6]Hirose, Kurozumi and Van Zandwaghe (2017) and Haque (2017) were the first to apply Bayesian estimation using SMC algorithm to test for indeterminacy using Lubik and Schorfheide's (2003, 2004) methodology.

2.2.1 Households

A representative domestic household seeks to solve the following decision problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/Z_t)^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$P_t C_t + E_t(Q_{t,t+1} D_{t+1} + \varepsilon_t Q^*_{t,t+1} D^*_{t+1}) \leq W_t N_t + D_t + \varepsilon_t D^*_t + \int \Omega_t(i) di$$

where $0 < \beta < 1$ is the discount factor; Z_t is the labor-augmenting technology; N_t denotes labor input; σ, $\varphi > 0$ are the inverse elasticities of intertemporal substitution and labor supply respectively; P_t is the nominal price level of the composite good; D_{t+1} (D^*_{t+1}) is the holding of a security that pays one unit of the domestic currency (foreign currency) in time t+1 and $Q_{t,t+1}$ ($Q^*_{t,t+1}$) is its current price in domestic currency (foreign currency); ε_t represents the nominal exchange rate (domestic currency/ foreign currency); W_t denotes the nominal wage and $\Omega_t(i)$ is nominal dividend earned from domestic firm i.

The first-order conditions can be written as

$$N_t^{\varphi} = W_t P_t^{-1} C_t^{-\sigma} Z_t^{\sigma-1},$$

$$C_t^{-\sigma} Z_t^{\sigma-1} = \beta E_t(R_t C_{t+1}^{-\sigma} Z_{t+1}^{\sigma-1} \pi_{t+1}^{-1}),$$

and

$$0 = E_t \left[(R_t - R^*_t e_{t+1}) C_{t+1}^{-\sigma} C_t^{\sigma} Z_{t+1}^{\sigma-1} Z_t^{1-\sigma} \pi_{t+1}^{-1} \right] ,$$

where $R_t = 1/E_t(Q_{t,t+1})$ and $R^*_t = 1/E_t(Q^*_{t,t+1})$ are nominal interest rates in domestic and foreign country, respectively; $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and $e_t = \varepsilon_t/\varepsilon_{t-1}$ is the gross depreciation rate.

2.2.2 Terms of Trade and the Real Exchange Rate

Let $P_{H,t}$ and $P_{F,t}$ be the domestic price of home- and foreign-produced goods, respectively. The terms of trade is defined as $Q_t \equiv \frac{P_{H,t}}{P_{F,t}}$. The law of one price holds so that $P_{F,t} = \varepsilon_t P^*_{F,t}$, where $P^*_{F,t}$ is the price of the foreign-produced good in the foreign country, measured in foreign currency. Following Del Negro and Schorfheide (2009), it is assumed that domestically produced goods have a negligible weight in foreign consumption. Specifically, let ϑ be the relative size of the domestic economy (defined more precisely below). $\vartheta \to 0$ so that $P^*_{F,t}$ will be approximately equal to the foreign consumer price index (CPI), P^*_t. Hence, the terms of trade $Q_t = \frac{P_{H,t}}{(\varepsilon_t P^*_t)}$. Let P_t be the domestic CPI. The real exchange rate is defined as $S_t = \frac{\varepsilon_t P^*_t}{P_t}$. Thus, the relative price can be expressed as $\frac{P_{H,t}}{P_t} = Q_t S_t$.

2.2.3 Firms

There are three types of firms in this artificial economy.

Firstly, perfectly competitive firms buy quantities $C_{H,t}$ and $C_{F,t}$ of the domestically-produced and foreign-produced goods and package them into a composite good that is used for consumption by the households. These firms solve the problem

$$\max P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \ ,$$

subject to

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \ ,$$

where $\alpha > 0$ is the share of foreign goods in the domestic consumption bundle and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods.

The first-order conditions and a zero-profit condition imply that

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \ ; \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \ ;$$

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \ .$$

Secondly, perfectly competitive firms buy the domestic intermediate goods $Y_t(i)$, package them, and resell the composite good to the firms that aggregate $C_{H,t}$ and $C_{F,t}$. These firms solve the following problem

$$\max P_{H,t}Y_t - \textstyle\int_0^1 P_{H,t}(i)Y_t(i)di \ ,$$

subject to

$$Y_t = \left[\textstyle\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \ .$$

where $\varepsilon > 1$ is the elasticity of substitution between types of differentiated domestic goods. The first-order conditions and a zero-profit condition lead to

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t; \qquad P_{H,t} = \left[\textstyle\int_0^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} \ .$$

Lastly, the producers of the domestic intermediate goods $Y_t(i)$ are monopolistic competitors. The firms' production function is linear in labour

$$Y_t(i) = Z_t N_t(i).$$

where $z_t = Z_t/Z_{t-1}$ follows AR(1) process. Therefore, output, consumption and wages are detrended according to $c_t = C_t/Z_t$, $y_t = Y_t/Z_t$ and $w_t = W_t/(P_t Z_t)$. In each period there is a fixed probability $1-\theta$ that a firm can re-optimize its nominal price, i.e., $\tilde{P}_{H,t}(i)$. With probability θ the firm automatically and costlessly adjusts its price according to an indexation rule that can depend both on the previous period inflation rate and/or on the trend inflation rate. Therefore, θ represents the degree of price stickiness.

Following Ascari and Sbordone (2014), the price setting problem becomes

$$\max_{\left\{\tilde{P}_{H,t}(i),Y_t(i)\right\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{t+j}(i) \left[\tilde{P}_{H,t}(i)(\bar{\pi}_H^{\chi j})^{1-\mu}(\Pi_{t-1,t+j-1}^{\chi})^{\mu} - MC_{t+j}^n\right] ,$$

subject to

$$Y_{t+j}(i) = \left[\frac{\tilde{P}_{H,t}(i)(\bar{\pi}_H^{\chi j})^{1-\mu}(\Pi_{t-1,t+j-1}^{\chi})^{\mu}}{P_{H,t+j}}\right]^{-\varepsilon} Y_{t+j} ,$$

and

$$\Pi_{t,t+j} = \begin{cases} \left(\frac{P_{H,t+1}}{P_{H,t}}\right)\left(\frac{P_{H,t+2}}{P_{H,t+1}}\right) \times \cdots \times \left(\frac{P_{H,t+j}}{P_{H,t+j-1}}\right) & for \quad j = 1, 2, ... \\ 1 & for \quad j = 0. \end{cases} ,$$

where $\bar{\pi}_H$ is steady state inflation for domestic goods. From $P_{H,t}/P_t = Q_t S_t$, it implies that $\bar{\pi}_H$ is equal to steady state of home country's inflation $\bar{\pi}$, which is the level of trend inflation. $Q_{t,t+j}$ is the stochastic discount factor and MC_{t+j}^n stands for the nominal marginal cost. This formulation is very general, because: (i) $\chi \in [0, 1]$ allows for any degree of price indexation. Under the full indexation $\chi = 1$, positive trend inflation will not appear in log-linearized system, therefore not affect model dynamics. (ii) $\mu \in [0, 1]$ allows for any degree of (geometric) combination of the two types of indexation usually employed in the literature: to steady state inflation and to past inflation rates. When $\mu = 0$, firms do not adjust prices according to past inflation rates.

The nominal marginal costs and the price chosen by firms that are able to reoptimize in terms of the price of the domestic goods are

$$mc_t^r = \frac{MC_t^n}{P_{H,t}} = \frac{W_t}{Z_t P_{H,t}} = w_t Q_t^{-1} S_t^{-1} \ ,$$

and

$$\tilde{p}_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}} \ .$$

Using the fact that $Q_{t,t+j} = \beta^j \frac{c_{t+j}{}^{-\sigma} P_t Z_t}{c_t{}^{-\sigma} P_{t+j} Z_{t+j}}$ and considering a symmetric equilibrium in which all firms solve the same problem and eliminating the index i, the firms' first-order condition can then be written as

$$\tilde{p}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} (\theta\beta)^j c_{t+j}{}^{-\sigma} \left[\frac{(\bar{\pi}_H^{\chi j})^{1-\mu} (\Pi_{t-1,t+j-1}^{\chi})^{\mu}}{\Pi_{t,t+j}} \right]^{-\varepsilon} S_{t+j} Q_{t+j} y_{t+j} mc_{t+j}^r}{E_t \sum_{j=0}^{\infty} (\theta\beta)^j c_{t+j}{}^{-\sigma} \left[\frac{(\bar{\pi}_H^{\chi j})^{1-\mu} (\Pi_{t-1,t+j-1}^{\chi})^{\mu}}{\Pi_{t,t+j}} \right]^{1-\varepsilon} S_{t+j} Q_{t+j} y_{t+j}} \ ,$$

The optimal relative price can be compactly rewriting recursively as

$$\tilde{p}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi_t}{\phi_t} \ ,$$

where

$$\psi_t = c_t^{-\sigma} S_t Q_t y_t mc_t^r + \theta\beta \bar{\pi}_H^{-\varepsilon(1-\mu)\chi} \pi_{H,t}^{-\mu\chi\varepsilon} E_t(\pi_{H,t+1}^{\varepsilon} \psi_{t+1}) \ ,$$

$$\phi_t = c_t^{-\sigma} S_t Q_t y_t + \theta\beta \bar{\pi}_H^{(1-\mu)(1-\varepsilon)\chi} \pi_{H,t}^{\chi\mu(1-\varepsilon)} E_t(\pi_{H,t+1}^{\varepsilon-1} \phi_{t+1}) \ ,$$

and $\pi_{H,t} = P_{H,t}/P_{H,t-1.}$ Moreover, the domestic good price level evolves according to

$$\tilde{p}_{H,t} = \left[\frac{1 - \theta \bar{\pi}_H^{(1-\varepsilon)(1-\mu)\chi} \pi_{H,t-1}^{\chi\mu(1-\varepsilon)} \pi_{H,t}^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \ .$$

2.2.4 International Risk Sharing

Under the assumption of complete securities markets, the first order condition analogous to $E_t\left[(R_t - R_t^* e_{t+1})C_{t+1}^{-\sigma} C_t^{\sigma} Z_{t+1}^{\sigma-1} Z_t^{1-\sigma} \pi_{t+1}^{-1}\right] = 0$ must also hold for the representative household in the rest-of-the-world. Therefore, the relationship between domestic and foreign consumption is

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} \pi_{t+1} = \left(\frac{c_{t+1}^*}{c_t^*}\right)^{\sigma} \pi_{t+1}^* e_{t+1} \ ,$$

where π_t^* is the inflation in the rest-of-the-world and it follows an AR(1) process and c_t^* is detrended total consumption in the foreign country. To obtain implications about the level of consumption in the two economies, let us assume that $S_0 = 1$ and set $\vartheta = C_0/C_0^*$, which implies

$$c_t = \vartheta S_t^{1/\sigma} c_t^* \ .$$

2.2.5 General Equilibrium

The market for domestically produced goods clears if the following condition in terms of variables detrended by Z_t is satisfied

$$y_t = c_{H,t} + c_{H,t}^* \ ,$$

where $c_{H,t}^*$ is detrended consumption of domestic produced good by foreign country. Moreover, let α^* be the share of imported goods in the foreign country and assume $\alpha^* = \vartheta\alpha$, so that $c_{H,t}^* = \vartheta\alpha\left(\frac{P_{H,t}/\varepsilon_t}{P_t^*}\right)^{-\eta} c_t^*$.

Since all state-contingent securities are in zero net supply and one can use the fact that $\int \Omega_t(i)di = P_{H,t}Y_t - W_t N_t$ derived from firm i's profit maximizing problem, the following global resource constraint from the budget constraints of the domestic and foreign households follows

$$c_t + S_t c_t^* = Q_t S_t y_t + S_t y_t^* \ ,$$

where y_t^* is detrended rest-of-the-world output that follows AR(1) process and c_t^* is approximately equal to y_t^*.

2.2.6 Monetary Policy

Finally, the monetary policy is described by an interest rate rule, where the central bank adjusts its instrument in response to movements in CPI inflation, output and nominal exchange rate depreciation

$$\begin{aligned}\log R_t &= \rho_R \log R_{t-1} + (1-\rho_R)[\log \bar{R} + \phi_\pi(\log \pi_t - \log \bar{\pi}) + \phi_y(\log y_t - \log \bar{y}) + \\ &\quad \phi_e(\log e_t - \log \bar{e})] + \varepsilon_t^R \ .\end{aligned}$$

where $\bar{R} \geq 1$ is the steady state of nominal interest rate; $\bar{y}$ is the steady state of detrended output and $\bar{e}$ is steady state of gross depreciation rate. The parameters ϕ_π, ϕ_y and ϕ_e govern the central bank's responses to inflation, output level and depreciation rate, respectively. $\rho_R \in [0, 1)$ is the degree of policy rate smoothing. ε_t^R is a zero-mean, serially uncorrelated innovation with standard deviation σ_R. The log-linearized system is shown in Appendix.

2.3 Trend inflation and indeterminacy zones

Figure 2.2 and 2.3 explore how trend inflation affects the determinacy properties of the model. The shaded areas indicate indeterminacy constellations.

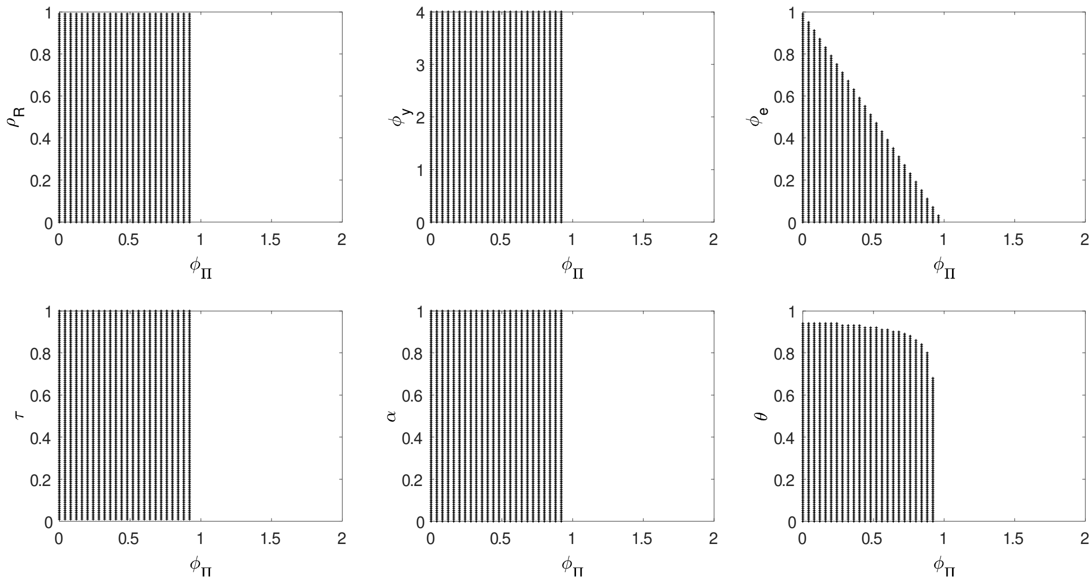

Figure 2.2: Indeterminacy region when trend inflation is equal to 0 percent.

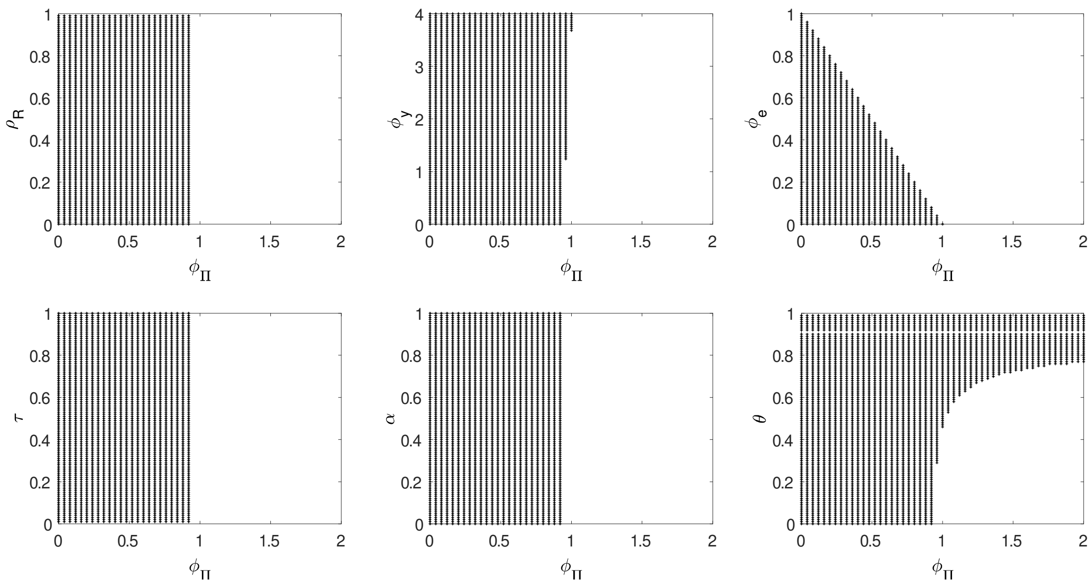

Figure 2.3: Indeterminacy region when trend inflation is equal to 4 percent.

All parameters except for the ones labeled in x-axis and y-axis are calibrated at the posterior mean (reported in more detail later) of the determinacy model estimated with IT period data from 1993:II to 2007:III. Figure 2.2 portrays the standard case of zero-inflation steady state where the economy boils down to a variant of the model in Lubik and Schorfheide (2007) and Del Negro and Schorfheide (2009). Figure 2.3 shows the case for positive trend inflation where steady state inflation is set to 4 percent. The downward sloping boundary for the combination of ϕ_π and ϕ_e in both figures shows that central bank's response to depreciation rate will shrink indeterminacy region and this finding is in line with Zheng and Guo (2013).[7] If compare the combination of ϕ_π and ϕ_y subplots in the two figures, under positive trend inflation rate, response to output level will slightly expand indeterminacy region. The most different case is the combination of θ and ϕ_π. Under the positive trend inflation, high stickiness (high θ) almost can make determinacy region disappear, thus the Taylor principle no longer holds.

2.4 Estimation Strategy and Data

This section explains the indeterminacy solution for linear rational expectations (henceforth LRE) models as well as the estimation strategy by using the SMC algorithm. Then the data and prior distributions used in the estimation are presented.

[7] Zheng and Guo (2013) analysis a zero-trend-inflation version of the current model and prove the boundary condition between determinacy and indeterminacy is $\phi_\pi = 1 - \phi_e - \frac{(1-\beta)(\lambda+\tau)}{\varkappa}\phi_y$, where $\varkappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$. Therefore, keeping ϕ_y fixed, the larger ϕ_e, the smaller ϕ_π is required to guarantte determinacy.

2.4.1 Rational expectations solutions under indeterminacy

To solve the model, the paper applies the method proposed by Lubik and Schorfheide (2003) in which case a full set of rational expectations solutions is of the form

$$\varrho_t = \Phi_\varrho(\xi)\varrho_{t-1} + \Phi_\varepsilon(\xi, \tilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\xi)\zeta_t,$$

where ϱ_t is a vector of model variables; ε_t denotes a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[8] $\Phi_\varrho(\xi)$, $\Phi_\varepsilon(\xi, \tilde{\mathbf{M}})$ and $\Phi_\zeta(\xi)$ are coefficient matrices and $\tilde{\mathbf{M}}$ is an arbitrary matrix.[9] The sunspot shock satisfies $\zeta_t \sim i.i.d.N(0, \sigma_\zeta^2)$. The solution shows two key features under indeterminacy. First, the dynamics of the LRE model under indeterminacy is driven not only by the fundamental shocks ε_t but also by the sunspot shock ζ_t; second, the solution cannot be unique since the matrix $\tilde{\mathbf{M}}$ is arbitrary. Following Lubik and Schorfheide (2004), $\tilde{\mathbf{M}}$ is replaced by $\mathbf{M}^*(\xi) + \mathbf{M}$ and the prior mean for $\mathbf{M}$ is set equal to zero. $\mathbf{M}^*(\xi)$ is selected to minimize the discrepancy between the impulse response of the endogenous variables to fundamental shocks under determinacy and under indeterminacy, using a least-squares criterion. Analytical solution for the boundary in this model is unavailable and hence, following Justiniano and Primiceri (2008) and Hirose (2014), this paper applies a numerical procedure to find the boundary by perturbing the parameter ϕ_π in the interest rate rule.

[8] Under determinacy, the solution boils down to $\varrho_t = \Phi_\varrho^D(\xi)\varrho_{t-1} + \Phi_\varepsilon^D(\xi)\varepsilon_t$.

[9] Lubik and Schorfheide (2003) express $\Phi_\zeta(\xi)$ as $\Phi_\zeta(\xi, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purposes, they impose a normalization such that $M_\zeta = 1$.

2.4.2 Bayesian estimation with Sequential Monte Carlo algorithm

The model is estimated using Bayesian methods and is tested for indeterminacy using posterior probabilities. I employ the SMC algorithm described in Herbst and Schorfheide (2014, 2015) which is particularly suitable to approximate irregular and non-elliptical posterior distributions. More importantly, unlike Markov Chain Monte Carlo (MCMC) which is the most commonly used algorithm in Bayesian estimation, the SMC does not require to find the mode of the posterior distribution, a task that can prove to be inefficient and time-consuming particularly under indeterminacy. Therefore, compared to MCMC, SMC can be more efficient in practice when estimating the indeterminacy model.

The priors are described by a density function of the form $p(\xi_\Delta|\Delta)$, where $\Delta \in \{D, I\}$. D and I stand for determinacy and indeterminacy respectively. ξ_Δ represents the parameter of the model Δ. The likelihood function $p(X^T|\xi_\Delta, \Delta)$ of the state-space model describes the density of the observed data X^T. Following the Bayes' Theorem, the posterior density is a combination of the prior density and the likelihood function

$$p(\xi_\Delta, X^T, \Delta) = \frac{p(X^T|\xi_\Delta, \Delta)p(\xi_\Delta|\Delta)}{p(X^T, \Delta)} = \frac{p(X^T|\xi_\Delta, \Delta)p(\xi_\Delta|\Delta)}{\int p(X^T|\xi_\Delta, \Delta)p(\xi_\Delta|\Delta)d\xi_\Delta} .$$

To approximate the posterior distribution, this paper builds a particle approximation of the posterior distribution through tempering the likelihood according to Herbst and Schorfheide (2014, 2015). A sequence of tempered posteriors is defined as

$$\varpi_n(\xi_\Delta) = \frac{[p(X^T|\xi_\Delta, \Delta)]^{\phi_n} p(\xi_\Delta|\Delta)}{\int [p(X^T|\xi_\Delta, \Delta)]^{\phi_n} p(\xi_\Delta|\Delta)d\xi_\Delta}, n = 1, ..., N_\phi$$

where ϕ_n is the tempering schedule that slowly increases from zero to one and is determined by $\phi_n = (\frac{n-1}{N_\phi - 1})^\kappa$ where κ controls the shape of the tempering schedule and N_ϕ is the number of stages.

Like the basic importance sampling algorithm, SMC generates weighted draws from the sequence of posteriors $\{\varpi_n(\xi_{_\Delta})\}_{n=1}^{N_\phi}$. The weighted draws are called particles. At each stage, the posterior distribution $\varpi_n(\xi_{_\Delta})$ is represented by a swarm of particles $\{\xi_n^i, w_n^i\}_{i=1}^N$ where w_n^i is the weight associated with ξ_n^i and N is the overall number of particles. For $n = 1, ..., N_\phi$, the algorithm sequentially updates the swarm of particles through importance sampling. Posterior inferences about estimated parameters are made based on the particles from the final importance sampling. The SMC algorithm-based approximation of the marginal data density is given by

$$p(\xi_{_\Delta}|\Delta) = \prod_{n=1}^{N_\phi} (\frac{1}{N}\sum_{i=1}^{N} \tilde{w}_n^i w_{n-1}^i) \ ,$$

where $\tilde{w}_n^i$ is the incremental weight defined by

$$\tilde{w}_n^i = [p(X^T|\xi_{n-1}^i, \Delta]^{\phi_n - \phi_{n-1}} \ .$$

The estimation uses $N = 10,000$ particles and $N_\phi = 200$ stage. The parameter controlling the tempering schedule is set at $\kappa = 2$, as in Herbst and Schorfheide (2015).

2.4.3 Data description

The system of the log-linearized equilibrium conditions is estimated using five time series: the real GDP growth rate ($100\Delta \log Y_t$), inflation ($400\log \pi_t$), nominal interest rates ($400\log R_t$), exchange rate changes ($100\Delta \log \varepsilon_t$) and terms of trade changes ($100\Delta \log Q_t$). All data are seasonally adjusted and at quarterly frequencies for the Pre-IT period 1983:I to 1993:I and the IT period 1993:II to 2007:III. Most of the data series are obtained from the Australian Bureau of Statistics (ABS) and Reserve Bank of Australia (RBA). The inflation series corresponds to the annualized quarterly log-difference in the consumer price index (all items). Nominal interest rate is measured as the average of 3-month Interbank Overnight Cash Rate (expressed in annualized percentages). Trade-weighted nominal exchange rate indices are obtained from the International Monetary Fund (IMF). The terms of trade is measured as the price of exports to imports which is available from ABS. Pre-IT data and IT data are demeaned separately prior to estimation. Figure 2.4 plots the observables before demeaned and reports the standard deviation (Std) of each observable in different sample periods. In the last figure Core CPI inflation is used to replace CPI inflation as a robust check and will be discussed in Section 2.5.5. All series are demeaned prior to estimation.

The corresponding measurement equation is given by

$$\begin{bmatrix} 100(\Delta \log Y_t - mean) \\ 400(\log \pi_t - mean) \\ 400(\log R_t - mean) \\ 100(\Delta \log \varepsilon_t - mean) \\ 100(\Delta \log Q_t - mean) \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\ 4\hat{\pi}_t \\ 4\hat{R}_t \\ \hat{e}_t \\ \hat{q}_t \end{bmatrix}$$

where $\hat{q}_t = \hat{Q}_t - \hat{Q}_{t-1}$. $\hat{X}_t$ denotes log deviations of a variable X_t from its steady state, $\bar{X}$: $\hat{X}_t = \ln(\frac{X_t}{\bar{X}})$.

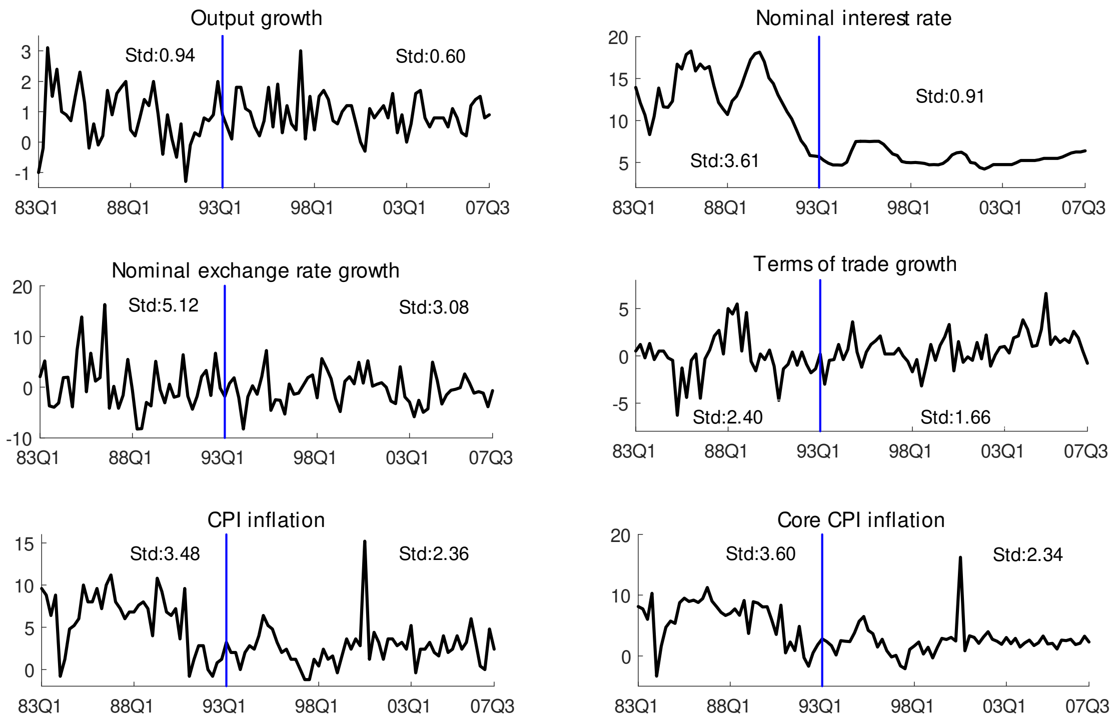

Figure 2.4: Data used in estimation (before demean)

2.4.4 Calibration and prior distributions

I calibrate the discount factor β to be 0.99, the steady-state markup at around ten percent (i.e. $\varepsilon = 11$), and the inverse of the labor supply elasticity equal to zero. Following Lubik and Schorfheide (2007), the parameter η that measures the substitutability between domestic and foreign goods is set to 1.[10] I set the indexation parameters χ and μ equal to zero. Trend inflation is calibrated with corresponding real inflation data so that $\bar{\pi}_H = 1.015$ for Pre-IT sample period while $\bar{\pi}_H = 1.006$ for IT sample period. All other parameters are estimated.

The specification of the prior distribution is summarized in Table 2.1.[11] The prior for the inflation coefficient ϕ_π follows a Gamma distribution centered at

[10] In the particular case of $\eta = 1$, the CPI takes the form $P_t = {P_{H,t}}^{1-\alpha}{P_{F,t}}^{\alpha}$, while the consumption index is given by $C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}}{C_{H,t}}^{1-\alpha}{C_{F,t}}^{\alpha}$.

[11] The prior for determinacy case drops the last six rows of table 2.1.

Table 2.1: Prior distributions

Name	Interpretation	Range	Density	Prior Mean	St. Dev
ϕ_π	Inflation coefficient	$(0,+\infty)$	Gamma	1.05	0.50
ϕ_y	Output level coefficient	$(0,+\infty)$	Gamma	0.25	0.13
ϕ_e	Depreciation rate coefficient	$(0,+\infty)$	Gamma	0.10	0.05
ρ_R	Interest rate smoothing	$[0,1)$	Beta	0.50	0.20
θ	Calvo probability	$[0,1)$	Beta	0.50	0.10
α	Import share	$[0,1)$	Beta	0.20	0.05
τ	Intertemporal substitution elasticity	$[0,1)$	Beta	0.50	0.20
ρ_q	Persistence of terms of trade shock	$[0,1)$	Beta	0.50	0.20
ρ_z	Persistence of technology shock	$[0,1)$	Beta	0.50	0.20
ρ_{y^*}	Persistence of world output shock	$[0,1)$	Beta	0.50	0.20
ρ_{π^*}	Persistence of world inflation shock	$[0,1)$	Beta	0.50	0.20
σ_R	Std.dev of monetary policy shock	$(0,+\infty)$	InvGam	0.50	0.20
σ_q	Std.dev of terms of trade shock	$(0,+\infty)$	InvGam	1.00	0.50
σ_z	Std.dev of technology shock	$(0,+\infty)$	InvGam	1.00	0.50
σ_{y^*}	Std.dev of world output shock	$(0,+\infty)$	InvGam	1.00	0.50
σ_{π^*}	Std.dev of world inflation shock	$(0,+\infty)$	InvGam	1.00	0.50
σ_ζ	Std.dev of sunspot shock	$(0,+\infty)$	InvGam	1.00	0.50
M_R	Propagation of monetary policy shock	$\mathbb{R}^+$	Normal	0.00	1.00
M_q	Propagation of terms of trade shock	$\mathbb{R}^+$	Normal	0.00	1.00
M_z	Propagation of technology shock	$\mathbb{R}^+$	Normal	0.00	1.00
M_{y^*}	Propagation of world output shock	$\mathbb{R}^+$	Normal	0.00	1.00
M_{π^*}	Propagation of world inflation shock	$\mathbb{R}^+$	Normal	0.00	1.00

Notes: The inverse gamma priors are of the form $p(\varkappa|v,s) \propto \varkappa^{-v-1}e^{-vs^2/2\varkappa^2}$ where v=4 and s=0.79.

1.05 with a standard deviation of 0.50 while the response coefficient to output ϕ_y is with mean 0.25 and standard deviation 0.13. The prior for ϕ_e is centered at 0.1 with standard deviation 0.05. I use a Beta distribution with mean 0.5 and standard deviation 0.1 for the Calvo probability θ and standard deviation 0.2 for the inverse of σ, which is denoted by τ. As for the indeterminacy, I follow Lubik and Schorfheide (2004) by having the coefficients of the vector $\mathbf{M}$ to follow standard normal distributions. All shocks innovations are inverse gamma distributions with mean 1.0 and 0.5 standard deviation. Due to the different calibration of trend inflation, the prior probability of determinacy for the Pre-IT and IT periods are 0.46 and 0.54, respectively, which suggests no prior bias towards determinacy or indeterminacy.

2.5 Results of Empirical Analysis

This section presents the results of the empirical analysis in detail. First, I report log data densities and model probabilities for different sample periods, then discuss the parameter estimates, forecast error variance decomposition as well as the alternative measure of inflation.

2.5.1 Testing for indeterminacy

The model is estimated using the SMC algorithm under the priors listed in Table 2.1. To assess the quality of the model's fit to the data over the two regions of the parameters space, Table 2.2 presents the log data densities and model probabilities. The posterior probabilities reveal striking differences between the two subsamples. The posterior probability of indeterminacy is around 0.95 for the Pre-IT sample while the IT sample concentrates almost all of its mass in the determinacy region. These results indicate that inflation targeting monetary policy appear to be reasonable and sufficiently active to rule out indeterminacy and the monetary policy regime switch in the early 1990s in Australia were helpful to stabilize the economy.

Table 2.2: Determinacy versus Indeterminacy (CPI inflation)

	Log-data density		Probability	
Sample	Determinacy	Indeterminacy	Determinacy	Indeterminacy
Pre-IT	-470.44	-467.52	0.05	0.95
IT	-506.77	-510.29	0.97	0.03

2.5.2 Structural parameters

Table 2.3: Posterior distributions from data favoured regions

		Pre-IT		IT	
Name	Interpretation	Mean	St. Dev	Mean	St. Dev
ϕ_π	Inflation coefficient	0.84	0.09	1.44	0.27
ϕ_y	Output level coefficient	0.38	0.16	0.44	0.14
ϕ_e	Depreciation rate coefficient	0.05	0.02	0.06	0.02
ρ_R	Interest rate smoothing	0.47	0.09	0.64	0.08
θ	Calvo probability	0.36	0.07	0.15	0.04
α	Import share	0.15	0.04	0.19	0.05
τ	Intertemporal substitution elasticity	0.39	0.11	0.54	0.11
ρ_q	Persistence of terms of trade shock	0.34	0.10	0.23	0.06
ρ_z	Persistence of technology shock	0.28	0.12	0.07	0.04
ρ_{y^*}	Persistence of world output shock	0.63	0.15	0.88	0.04
ρ_{π^*}	Persistence of world inflation shock	0.21	0.10	0.21	0.08
σ_R	Std.dev of monetary policy shock	0.52	0.07	0.36	0.06
σ_q	Std.dev of terms of trade shock	2.12	0.21	1.55	0.14
σ_z	Std.dev of technology shock	0.71	0.15	0.60	0.06
σ_{y^*}	Std.dev of world output shock	1.46	0.72	0.87	0.33
σ_{π^*}	Std.dev of world inflation shock	3.80	0.40	2.90	0.27
σ_ζ	Std.dev of sunspot shock	2.58	1.31	–	–
M_R	Propagation of monetary policy shock	0.25	0.94	–	–
M_q	Propagation of terms of trade shock	-0.36	0.41	–	–
M_z	Propagation of technology shock	0.74	0.91	–	–
M_{y^*}	Propagation of world output shock	-0.95	1.06	–	–
M_{π^*}	Propagation of world inflation shock	-0.27	0.21	–	–

Notes: Results are based on 10,000 particles from final stage in SMC algorithm.

Table 2.3 reports posterior estimates for different sample periods conditional on the data-favored model: for the Pre-IT period, it is the indeterminacy model, while for the IT period, the determinacy model is favored by the data. As can be seen in the table, the key parameter ϕ_π is less than one under indeterminacy and larger than one under determinacy. The policy response to output level increases from 0.38 to 0.44 after the policy regime switch. In both sample periods, policy response of exchange rate depreciation keeps low, which is in line with Lubik and Schorfheide (2007) that it is more likely that the central bank of Australia does not target nominal exchange rate. The standard deviations of all shocks decline notably, implying that variables are less volatile during the IT period.

2.5.3 Variance decomposition

Table 2.4 reports the contribution of each shock, listed in the first column, to the variances of key observables used in estimation. For the Pre-IT sample, I report posterior conditional on indeterminacy. The posterior for the IT sample is conditional on determinacy. According to the posterior estimates, Australian output growth is primarily driven by technology shocks and to a lesser degree by (latent) world output for both sample periods, which is by and large consistent with Lubik and Schorfheide (2007) and Photphisutthiphong and Weder (2016). Yet, the contribution of technology shocks is negligible in explaining the fluctuations of other observables. Similar to Lubik and Schorfheide (2007), exchange rate movements are mainly determined by foreign inflation, and to a smaller degree by terms of trade shock in both sample periods. It is interesting to compare the difference in the relevance of shocks across sub-samples. In the Pre-IT period sunspot shocks play a considerable role as regards the inflation and interest rates, however, when moving to the IT period it appears notable difference:

the variance decomposition reveals that both monetary policy and world output shocks are important in explaining variation in inflation; moreover, for interest rate fluctuations, foreign output shocks contribute most fluctuation in interest rate. The terms of trade shock does not play a notable role in domestic business cycles, only explaining 15-20% of the change of exchange rate.

Table 2.4: Unconditional variance decomposition

	Output	Inflation	Interest rate	Exchange rate
		Pre-IT		
Monetary policy shock	8.53	19.55	9.70	1.08
Terms of trade shock	1.62	4.25	5.54	20.23
Technology shock	56.35	4.78	4.40	0.26
World output shock	32.93	13.20	14.83	0.73
World inflation shock	0.19	5.84	10.36	74.83
Sunspot shock	0.38	52.39	55.17	2.88
		IT		
Monetary policy shock	2.55	66.17	4.57	2.98
Terms of trade shock	0.78	0.93	0.37	15.12
Technology shock	83.15	0.22	0.46	0.01
World output shock	13.42	29.35	94.57	1.32
World inflation shock	0.09	3.34	0.02	80.56

2.5.4 Impulse response analysis

The model dynamics can be further studied by impulse response functions. Figure 2.5 represents impulse responses from indeterminacy model favored by Pre-IT data with parameters calibrated at the posterior mean. Contractionary monetary policy appreciates the currency and reduces inflation and output. An improvement in terms of trade raises output and lowers inflation on impact via a nominal appreciation. The decline in the exchange rate prompts the central bank to loosen policy, which has an additional expansionary effect on production. Both technology shock and sunspot shock raise output, inflation and interest rates and thereby depreciate the currency. Under a positive world output shock, domestic output declines along with an increase in inflation and the exchange rate, therefore central bank carries out expansionary policy by decreasing nominal interest rate to stimulate the economy. Shocks to import price inflation appreciate the currency. Hence domestic CPI inflation drops. Figure 2.6 reports impulse responses from determinacy model favored by IT data with parameters calibrated at the posterior mean. Unlike in Figure 2.5, both terms of trade shock and world inflation shock raise inflation, nominal interest rate, and world output shock also drives nominal interest rate up in Figure 2.6. Those differences between two figures are mainly due to the fact that the arbitrary matrix $\tilde{\mathbf{M}}$ which appears only under indeterminacy can affect model dynamics and propagation mechanism.

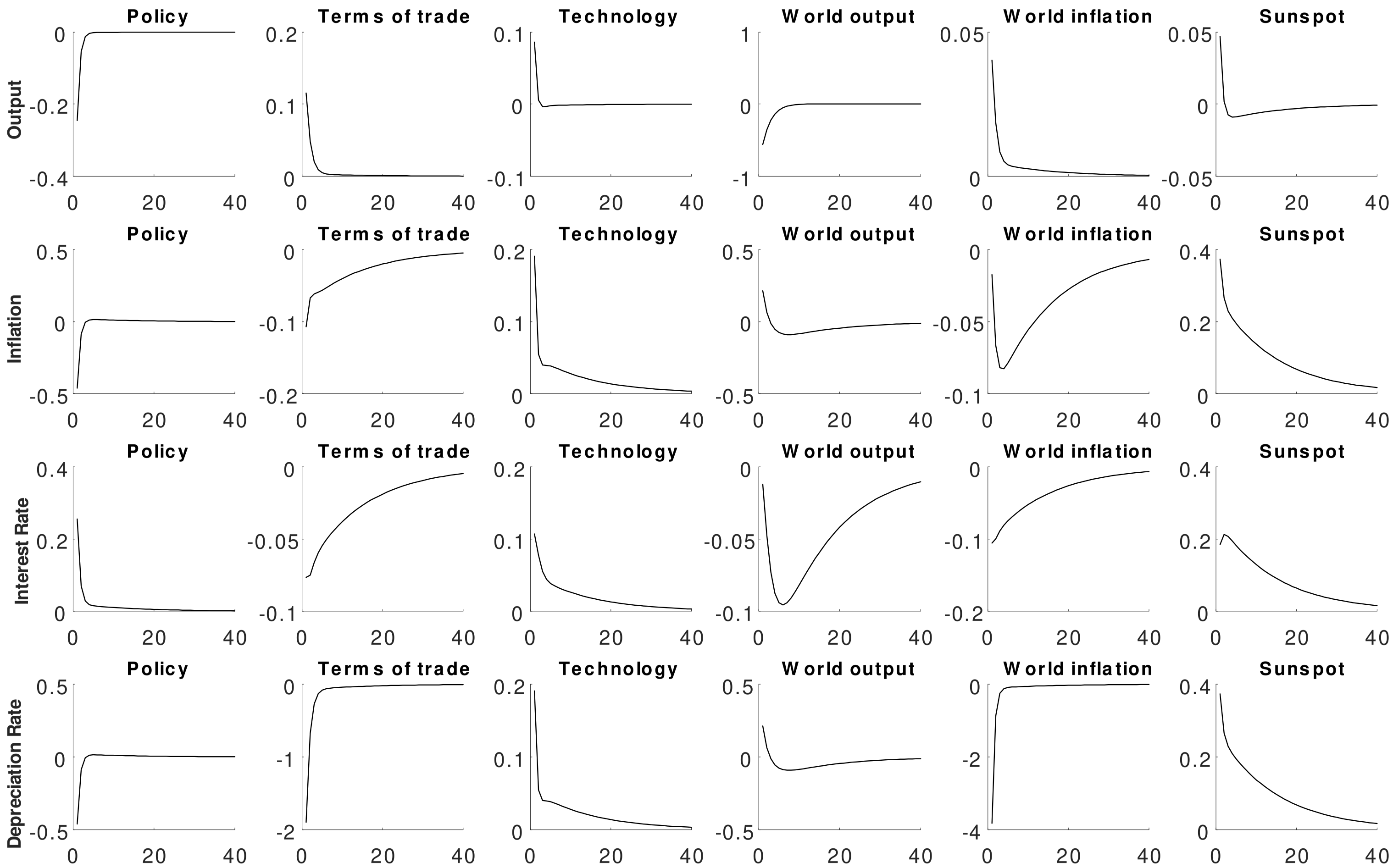

Figure 2.5: Impulse responses for pre-IT period. Figure depicts posterior means for impulse response of output, inflation, interest rate and exchange rate to one-standard deviation structural shocks.

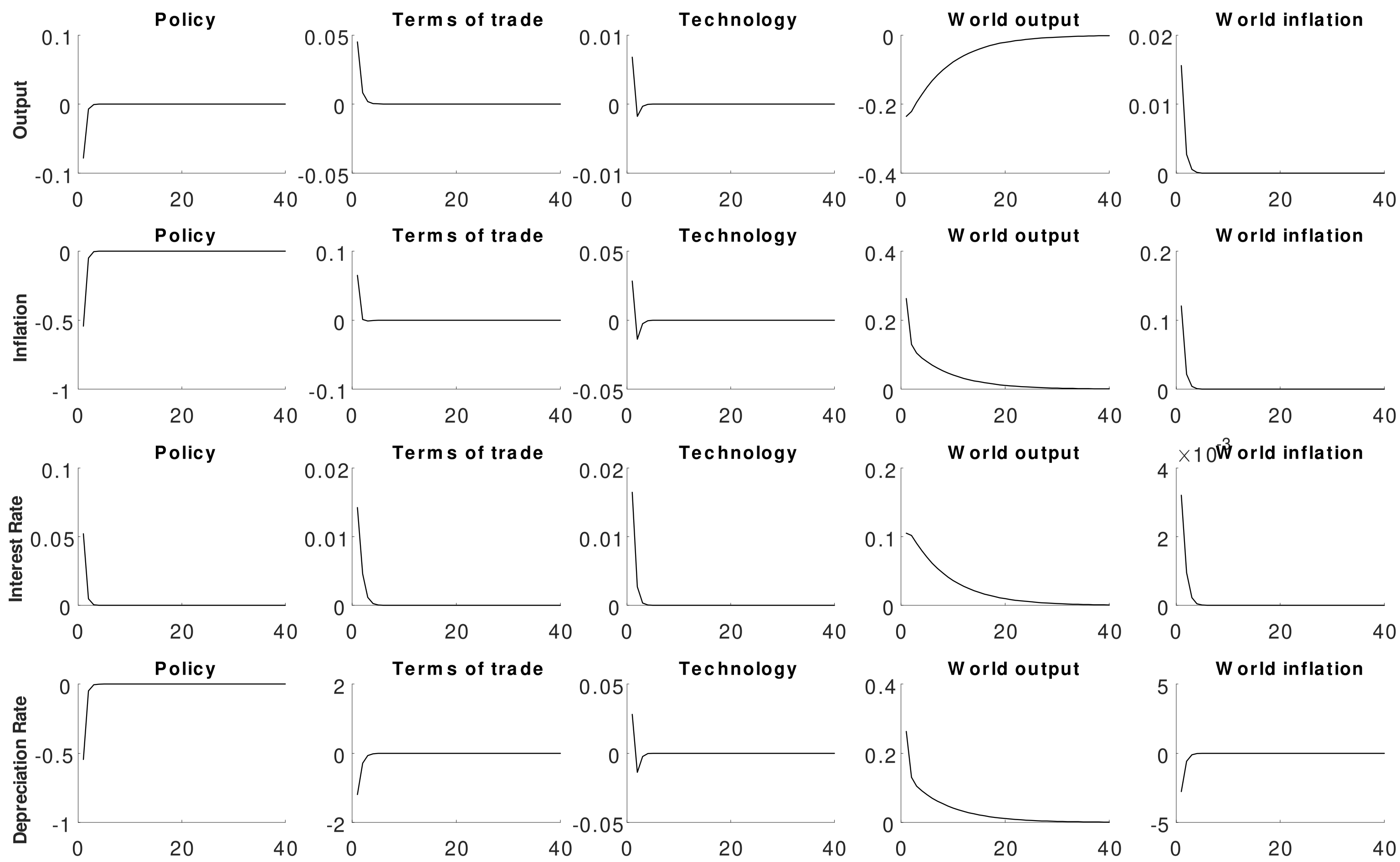

Figure 2.6: Impulse responses for IT period. Figure depicts posterior means for impulse response of output, inflation, interest rate and exchange rate to one-standard deviation structural shocks.

2.5.5 Alternative measure of inflation

In this section, I check the sensitivity of the above findings to alternative inflation data—Core CPI inflation, which excludes the food and energy prices and thus is less volatile than CPI inflation. The results presented in Table 2.5 suggest that the benchmark results remain unchanged: indeterminacy can be safely ruled out. The model under determinacy continues to dominate the indeterminate model with a posterior probability of 0.97.

Table 2.5: Determinacy versus Indeterminacy (Core CPI inflation)

	Log-data density		Probability	
Sample	Determinacy	Indeterminacy	Determinacy	Indeterminacy
Pre-IT	-473.60	-469.76	0.02	0.98
IT	-505.91	-509.51	0.97	0.03

2.6 Conclusion

Indeterminacy can arise if the central bank follows a Taylor-type rule and does not raise interest rates aggressively enough in response to changes in inflation. Although the indeterminacy is mostly concerned in the prototypical monetary policy model, it did not get sufficiently explored within the framework of small open economy DSGE model. This paper estimates a small open economy model with positive trend inflation while allowing for indeterminacy. Using Bayesian methods and the SMC algorithm, I test for indeterminacy for Australia covering the Pre-IT and IT periods, respectively. Results show that the positive trend inflation can shrink the determinacy region significantly, especially when trend inflation rate or price stickiness is high. Moreover, Australian monetary policy was not active enough in the Pre-IT period, resulting in multiple equilibria. During the inflation targeting years, policy reacted more aggressively towards inflation, leading to determinacy.

Admittedly, I have left out various aspects of the economy that could be considered relevant. For example, the assumption of complete pass-through of exchange rates is in stark contrast with the fact that there is an overwhelming failure of the law of one price for tradables. Besides, the current paper only analyzes the case of Australia, thus whether the monetary regime switches affected macroeconomic stability in other open economies is still untouched. I plan to address these issues in further research.

2.A Appendix

The log-linearized system is summarized by[12]

$$\hat{y}_t = E_t\hat{y}_{t+1} + \frac{\lambda}{\tau}E_t(\hat{y}^*_{t+1} - \hat{y}^*_t) - (\tau+\lambda)[\hat{R}_t - E_t(\hat{\pi}_{H,t+1} + \hat{z}_{t+1})] \tag{A.1}$$

$$\widehat{\tilde{p}}_{H,t} = \frac{\theta\bar{\pi}_H^{(1-\varepsilon)(\chi-1)}}{1-\theta\bar{\pi}_H^{(1-\varepsilon)(\chi-1)}}(\hat{\pi}_{H,t} - \mu\chi\hat{\pi}_{H,t-1}) \tag{A.2}$$

$$\widehat{\tilde{p}}_{H,t} = \hat{\psi}_t - \hat{\phi}_t \tag{A.3}$$

$$\hat{\psi}_t = [1-\theta\beta\bar{\pi}_H^{-\varepsilon(\chi-1)}](-\frac{1}{\tau}\hat{c}_t+\hat{y}_t+\hat{Q}_t+\hat{S}_t+\widehat{mc}^r_t)+\theta\beta\bar{\pi}_H^{-\varepsilon(\chi-1)}(-\mu\chi\varepsilon\hat{\pi}_{H,t}+\varepsilon\hat{\pi}_{H,t+1}+\hat{\psi}_{t+1}) \tag{A.4}$$

$$\hat{\phi}_t = [1-\theta\beta\bar{\pi}_H^{(1-\varepsilon)(\chi-1)}](-\frac{1}{\tau}\hat{c}_t+\hat{y}_t+\hat{Q}_t+\hat{S}_t)+\theta\beta\bar{\pi}_H^{(1-\varepsilon)(\chi-1)}[\mu\chi(1-\varepsilon)\hat{\pi}_{H,t}+(\varepsilon-1)\hat{\pi}_{H,t+1}+\hat{\phi}_{t+1}] \tag{A.5}$$

$$\hat{c}_t = \hat{y}^*_t - \tau(1-\alpha)\hat{Q}_t \tag{A.6}$$

$$\hat{S}_t = -(1-\alpha)\hat{Q}_t \tag{A.7}$$

$$\hat{Q}_t = -\frac{1}{\tau+\lambda}(\hat{y}_t - \hat{y}^*_t) \tag{A.8}$$

$$\hat{q}_t = -\frac{1}{\tau+\lambda}[(\hat{y}_t - \hat{y}_{t-1}) - (\hat{y}^*_t - \hat{y}^*_{t-1})] \tag{A.9}$$

$$\hat{\pi}_{H,t} = \hat{\pi}_t + \alpha\hat{q}_t \tag{A.10}$$

$$\hat{e}_t = -(1-\alpha)\hat{q}_t + \hat{\pi}_t - \hat{\pi}^*_t \tag{A.11}$$

$$\widehat{mc}^r_t = \varphi\hat{y}_t + \frac{1}{\tau}\hat{y}^*_t + \frac{1}{\tau+\lambda}(\hat{y}_t - \hat{y}^*_t) \tag{A.12}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t + \phi_e\hat{e}_t] + \varepsilon^R_t \tag{A.13}$$

[12] Let $\lambda = \alpha(2-\alpha)(1-\tau)$. A.9 shows the growth rate of terms of trade is endogenous in the model. Instead of imposing this condition, however, I follow the approach in Lubik and Schorfheide (2007) and specify an exogenous law of motion for the growth of terms of trade movements as in A.15.

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \tag{A.14}$$

$$\hat{q}_t = \rho_q \hat{q}_t + \varepsilon_t^q \tag{A.15}$$

$$\hat{y}_t^* = \rho_{y^*} \hat{y}_{t-1}^* + \varepsilon_t^{y^*} \tag{A.16}$$

$$\hat{\pi}_t^* = \rho_{\pi^*} \hat{\pi}_{t-1}^* + \varepsilon_t^{\pi^*} \tag{A.17}$$

Statement of Authorship

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Principal Author

Name of Principal Author (Candidate)	Bo Zhang
Contribution to the Paper	Set up the model for estimation. Solved the model. Wrote codes and estimated the model. (40%)
Overall percentage (%)	40%
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.
Signature	Date 30/11/17

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

i. the candidate's stated contribution to the publication is accurate (as detailed above);

ii. permission is granted for the candidate in include the publication in the thesis; and

iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Wei Dai
Contribution to the Paper	Estimated the model. Performed simulations. Wrote the manuscript and produced all figures and tables. (40%)
Signature	Date 30/11/17

Name of Co-Author	Mark Weder
Contribution to the Paper	Initialized the idea for the paper. Helped to solve the model and to write it up. Helped to circulate the paper and to collect feedback. Acted as corresponding author. (20%)
Signature	Date 30/11/17

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Chapter 3

Animal Spirits, Financial Markets and Aggregate Instability

3.1 Introduction

What are the shocks that cause macroeconomies to experience recurrent sequences of booms and slumps? The current paper pursues this question by presenting evidence on the sources of business cycles for the post-Korean War American economy. The results support the view that people's psychological motivations, a.k.a. animal spirits, provoke a significant portion of the fluctuations in aggregate real economic activity, causing well over one third of U.S. output volatility. This finding is demonstrated within an artificial economy of financial market frictions. Our exercise also suggests that it was chiefly adverse shocks to expectations that led to the Great Recession.

Models with credit market frictions have become popular since the Great Recession, reflecting the notion that disruptions to financial markets were the key factors behind this contraction. Building on earlier work, such as Kiyotaki and Moore (1997) as well as Bernanke et al. (1999), this research has shown how financial market frictions can amplify shocks to macroeconomic fundamentals by transforming small economic disturbances into large business cycles.[1] Christiano et al. (2015), for example, extend New Keynesian models by financial market frictions to explain some key aspects of the Great Recession.

We depart from the aforementioned works twofold. First, the parametric space of our model includes multiple equilibria. This multiplicity will be cleared up by people's animal spirits that select from the possible equilibrium outcomes. Second, unlike most existing work on such indeterminacy, the analysis concentrates on estimating the artificial economy: we focus on the empirical implications of the multiplicity by explicitly analyzing the business cycle variance contributions of animal spirits or belief shocks. The undertaking is implemented by building on a variant of Benhabib and Wang (2013).[2] Indeterminacy in this model is linked to the empirically observed countercyclical movement of financial market tightness. Figure 3.1 plots the cyclical pattern of financial market health. It measures financial health by the Baa Corporate Bond spread which is displayed on an inverted scale and is plotted opposite the fluctuations of per capita GDP. The shaded areas in the figure correspond to NBER recessions. They highlight that financial conditions are not only cyclical, but also deteriorate markedly during most slumps.

In the artificial economy, countercyclical financial health is a key mechanism

[1] See also Liu et al. (2013) and Nolan and Thoenissen (2009).

[2] Azariadis et al. (2016), Liu and Wang (2014) and Harrison and Weder (2013) are other models of various stripes that combine multiple equilibria and financial frictions.

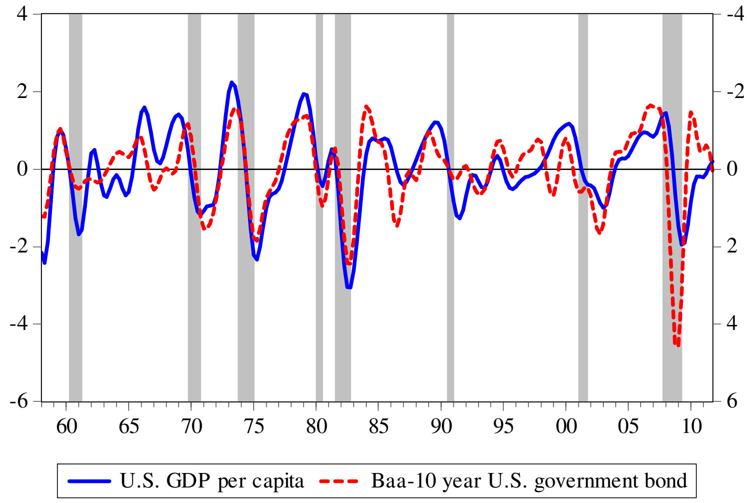

Figure 3.1: U.S. GDP and credit spread (on right-hand scale) at business cycle frequencies. Shaded areas indicate NBER recessions.

to multiplicity. It is the endogenous interaction of a time varying (flow) collateral constraint and a countercyclical markup that spawns equilibrium indeterminacy, a condition that allows aggregate fluctuations to be caused by extrinsic changes in people's expectations. Moreover, in addition to such animal spirits shocks, the economy is buffeted by an array of fundamental shocks. The model is estimated by full information Bayesian methods using quarterly U.S. data covering the period from 1955:I to 2014:IV. This approach follows for example Justiniano et al. (2011) as well as Schmitt-Grohé and Uribe (2012), who, however, only explore the role of fundamental shocks as the engines of business cycles. The key result that ensues from the Bayesian estimation is that animal spirits are important drivers of the repeated fluctuations of the U.S. macroeconomy. Specifically, by computing forecast error variance decompositions, we find that animal spirits account for about 40 percent of U.S. output variations and for about two thirds of the fluctuations in investment. Disturbances that originate in the financial sector explain less than ten percent of output fluctuations. Moreover, we show that belief shocks have played an important role in the sharp contraction in economic activity of the Great Recession that began at the end of 2007.

Previous work on multiple equilibria in real economies has overwhelmingly remained in the theoretical realm and estimation exercises have been rare. Farmer and Guo (1995) is an early attempt to estimate a sunspot model using classical simultaneous equations methods. It is only Pintus et al. (2016) and Pavlov and Weder (2017) who perform full-information Bayesian estimations as in the present paper. Pintus et al. (2016) build a model with financial market frictions and loan contracts that are arranged with variable-rates of interest. The model's indeterminacy affects the propagation mechanism in particular of (fundamental) financial shocks. These shocks then explain about one quarter of business cycles fluctuations. Financial markets are not featured in Pavlov and Weder (2017) and their study excludes the Great Recession. Lastly,while the exact definitions of confidence do not completely overlap, our result also parallels Angeletos et al. (2016) and Milani (2017) who maintain that sentiment swings drive a large fraction of U.S. aggregate fluctuations.

Next, we will lay out the artificial economy. This is followed by the presentation of the estimation, discussions of results and various robustness checks. Finally, we provide a theory of the Great Recession.

3.2 The Model

The artificial economy features credit frictions in the form of endogenous borrowing constraints in a model of monopolistic competition in which, as usual, perfectly competitive firms produce final output by combining a continuum of differentiated intermediate inputs. Intermediate goods producing firms are collateral-constrained in how much they can borrow to finance their working capital needs. We modify the original model by incorporating a set of fundamental shocks which are frequently considered as key drivers of business cycles. Time proceeds in discrete steps. The model's discussion will be relatively brief and it will concentrate on the alterations to Benhabib and Wang (2013).

3.2.1 Technology

A unit mass of monopolistic competitive firms has access to a constant returns technology that transforms capital services $\kappa_t(i)$ and labor hours $N_t(i)$ into intermediate, differentiated outputs $Y_t(i)$

$$Y_t(i) = \kappa_t(i)^{\alpha}(X_t N_t(i))^{1-\alpha} \qquad 0 < \alpha < 1.$$

Exogenous labor-augmenting technological progress X_t affects all firms equally. Its growth rate $\mu_t^x \equiv X_t/X_{t-1}$ evolves as a first-order autoregressive process

$$\ln \mu_t^x = (1-\rho_x)\ln \mu^x + \rho_x \ln \mu_{t-1}^x + \varepsilon_{x,t} \qquad 0 < \rho_x < 1$$

with $\varepsilon_{x,t} \backsim N(0, \sigma_x^2)$ and $\ln \mu^x$ is average growth rate. The firms rent the two factor services from the households at perfectly competitive prices w_t and r_t. Final output Y_t is a constant elasticity of substitution aggregator of a basket of intermediate inputs

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\lambda-1}{\lambda}} di\right)^{\frac{\lambda}{\lambda-1}} \qquad \lambda > 1.$$

Here λ denotes the elasticity of substitution between the differentiated varieties. The monopolistic competitive firms generate profits by charging a mark-up over marginal costs. Following Barth and Ramey (2001) who report that a substantial portion of U.S. firms raise working capital, we assume that firms' two variable inputs must be financed by short-run loans. Imperfect enforcement requires a process to constrain borrowing by the value of the collateral. Specifically, firm i's total amount of debt is an intraperiod loan $B_t(i)$ and it is constrained by the value of the collateral, which is the firms' pledge of the period-earnings, i.e.

$$B_t(i) = w_t N_t(i) + r_t \kappa_t(i) \leq \theta_t \xi_t P_t(i) Y_t(i).$$

Under this credit constraint, if there is a default event, the lender has the right to recover a fraction of the firm's end-of-period revenues $P_t(i)Y_t(i)$.[3] The model features two financial frictions and their product $\theta_t \xi_t$ represents the artificial economy's financial tightness. Concretely, ξ_t refers to an endogenous credit constraint: the borrowing constrictions vary with the aggregate state of economic activity which reflects creditors' ability to pay back loans. In particular, ξ_t is an increasing function of the deviation of actual output Y_t from balanced-growth output $\bar{Y}_t$

$$\xi_t = \tau \left(\frac{Y_t}{\bar{Y}_t} \right)^{\gamma}$$

in which we restrict the parameter to $0 < \tau < 1$ and $\gamma > 0$, an assumption in line with Figure 3.1. The parsimonious formulation of ξ_t entails many micro-founded makeups without the need to confine itself to a particular one.[4] For example, it can stand in for Benhabib and Wang's (2013) setup with fixed liquidation costs or ξ_t can also describe how market conditions determine the probability that lenders can recover as well as resell collateral. In addition to the endogenous component,

[3] Unlike in the original Benhabib and Wang (2013) model, our setup does not include fixed liquidation costs. Indeterminacy still holds. When we compare the two models using the Bayesian estimation method, we find that the model without fixed costs is favored by the data.

[4] Eisfeldt and Rampini (2006) offer some evidence about the cyclical properties of ξ_t.

exogenous disturbances θ_t affect financial health. These shocks originate in the financial sector as in Jermann and Quadrini (2012) or Liu et al. (2013). The exogenous collateral or financial shock θ_t evolves as

$$\ln \theta_t = (1-\rho_\theta)\ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t} \qquad 0 < \rho_\theta < 1$$

with $\varepsilon_{\theta,t} \backsim N(0, \sigma_\theta^2)$ and steady state value $\theta = 1$. The corresponding first-order conditions for the profit maximization problem involve

$$r_t \kappa_t(i) = \alpha \phi_t Y_t(i)$$

$$w_t N_t(i) = (1-\alpha)\phi_t Y_t(i)$$

and

$$\frac{\lambda-1}{\lambda} P_t(i) - \phi_t + \mu_t(i)\left[\theta_t \xi_t \frac{\lambda-1}{\lambda} P_t(i) - \phi_t\right] = 0 \tag{3.1}$$

where ϕ_t stands for monopolistic firms' marginal costs and $\mu_t(i)$ denotes the multiplier associated with the borrowing constraint.

3.2.2 Preferences

Households are represented by an agent with the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - \Lambda_t) - \varphi \frac{N_t^{1+\eta}}{1+\eta} \right) \qquad 0 < \beta < 1, \eta \geq 0 \text{ and } \varphi > 0$$

where β is the discount factor, C_t stands for consumption, and N_t for total hours worked. The functional form of the period utility ensures that the economy is consistent with balanced growth. The parameter φ denotes the disutility of working. The term Λ_t represents perturbations to the agent's utility of consumption that generate urges to consume, as in Baxter and King (1991) and Weder (2006). This element comes in two parts. One part grows with economy's consumption trend and the other one is a transitory shock that follows the autoregressive process

$$\ln \Delta_t = \rho_\Delta \ln \Delta_{t-1} + \varepsilon_{\Delta,t} \qquad 0 < \rho_\Delta < 1$$

with $\varepsilon_{\Delta,t} \backsim N(0, \sigma_\Delta^2)$. This shock is also one of the drivers of the economy's labor wedge, i.e. the gap between the marginal rate of consumption-leisure substitution and the marginal product of labor. Hence, our estimation will allow a wider interpretation than mere shocks to preferences. A more agnostic reading includes, for example, wage or price stickiness, changes to monetary policy, taxes, or labor market frictions. Households own the physical capital stock K_t and decide on its utilization rate, u_t, thus $\kappa_t = u_t K_t$. The agent faces the period budget constraint

$$C_t + A_t I_t + T_t = w_t N_t + r_t u_t K_t + \Pi_t$$

and the law of motion for capital is

$$K_{t+1} = (1 - \delta_t) K_t + I_t.$$

The term I_t is investment spending and A_t represents a non-stationary investment-specific technology shock which affects the transformation of consumption goods into investment goods. In the model, the concept corresponds to the relative price of new investment goods in terms of consumption goods. The shock's growth rate μ_t^a evolves as

$$\ln \mu_t^a = (1-\rho_a)\ln \mu^a + \rho_a \ln \mu_{t-1}^a + \varepsilon_{a,t} \qquad 0 < \rho_a < 1$$

with $\varepsilon_{a,t} \backsim N(0, \sigma_a^2)$, and $\ln \mu^a$ is the average growth rate. Lump-sum taxes are denoted by T_t. The rate of physical capital depreciation

$$\delta_t = \delta_0 \frac{u_t^{1+\nu}}{1+\nu} \qquad 0 < \delta_0 < 1 \text{ and } \nu > 0$$

is an increasing function in the utilization and $\nu > 0$ measures the elasticity of the depreciation rate with respect to capacity used. The first-order conditions are standard and delegated to the Appendix.

3.2.3 Government

The government purchases G_t units of the final output. G_t is neither productive nor does it provide any utility. The spending is financed by the lump-sum taxes. We model government's spending with a stochastic trend

$$X_t^G = (X_{t-1}^G)^{\psi_{yg}} (X_{t-1}^Y)^{1-\psi_{yg}} \qquad 0 < \psi_{yg} < 1$$

where ψ_{yg} governs the smoothness of the government spending trend relative to the trend in output. Then, detrended government spending is $g_t \equiv G_t / X_t^G$ and this follows the process

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \qquad 0 < \rho_g < 1$$

with the shock's variance σ_g^2.

3.2.4 Equilibrium

In symmetric equilibrium, $\kappa_t(i) = u_t K_t$, $N_t(i) = N_t$, $P_t(i) = P_t = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t u_t K_t$, hold and (3.1) becomes

$$\frac{\lambda - 1}{\lambda} - \phi_t + \mu_t \left[\theta_t \xi_t \frac{\lambda - 1}{\lambda} - \phi_t \right] = 0. \tag{3.2}$$

From (3.2), and if $\theta_t \xi_t \frac{\lambda-1}{\lambda} < \phi_t < \frac{\lambda-1}{\lambda}$, the financial constraint binds, thus, marginal costs equal

$$\phi_t = \theta_t \xi_t = \tau \theta_t \left(\frac{Y_t}{\bar{Y}_t} \right)^{\gamma}.$$

In the steady state, τ equals marginal costs ϕ, i.e. the inverse of the markup, thus it is not a free parameter.

3.2.5 Self-fulfilling dynamics

The detrended and linearized economy is solved numerically (using standard parameters as listed in Table 3.1). We assume a certain degree of market power such that the credit constraint is always binding, i.e. $\phi_t^{-1} > \frac{\lambda}{\lambda-1}$. Figure 3.2 maps the local dynamics' zones in the $\gamma - \phi^{-1}$−space. If the credit limit is close to constant, i.e. the parameter γ is small, the economy's dynamics are unique. However, combinations of market power and a procyclical credit limit delivers indeterminacy. The indeterminacy mechanism operates via an upwardly sloping wage-hours locus similar to many animal spirits models.[5] Then, how can, say, pessimistic expectations about the future create problems? The storyline would go as follows: if people believe that the future is worse, they will attempt to work more hours. In terms of the labor market equilibrium, this change in

[5] See for example, Farmer and Guo (1994) or Wen (1998).

expectations will shift the labor supply curve outwards. But their pessimistic expectations will also lead households to decrease the lending to firms. This contraction of credit will tighten the firms' borrowing constraints; given the cost structure, the markup will rise and the individual labor demand schedules move leftwards. As a consequence, the economy's wage-hours-locus is upwardly sloping. In equilibrium, the outward shift of labor supply will result in lower employment and in a drop in aggregate production. In sum, the low animal spirits will be self-fulfilling.

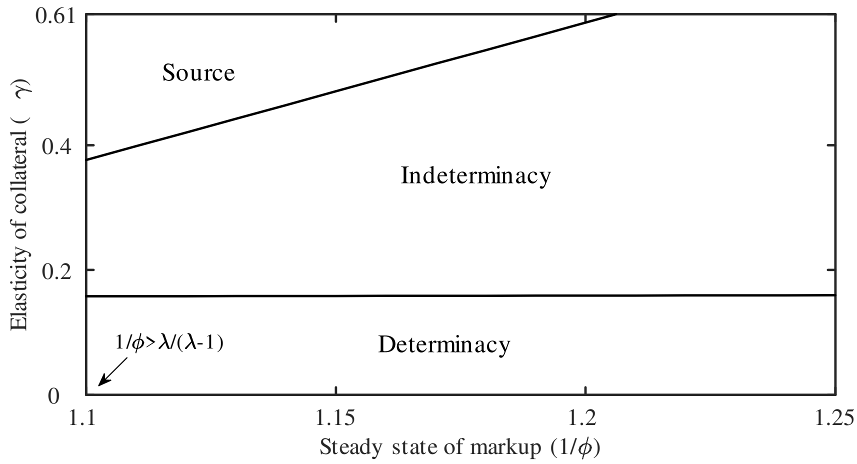

Figure 3.2: Parameter spaces of dynamics.

3.3 Estimation

Our next step is to discuss how animal spirits are introduced into the model, to present the data that is employed in the analysis, as well as to outline the full information Bayesian estimation of the artificial economy. We quantify the contribution of animal spirits shocks to business cycle fluctuations. Finally, we compare the estimated shocks to corresponding empirical measures.

3.3.1 Animal spirits in the rational expectations model

If there are many rational expectations equilibria in the model economy, this continuum is a device to introduce animal spirits. In fact, we treat them as quasi-fundamentals as they select from the many possible outcomes. Concretely, we break down the forecast error of output in the linearized model

$$\eta_t^y \equiv \hat{y}_t - E_{t-1}\hat{y}_t$$

(hats denote percentage deviations from steady states) into five fundamental and one non-fundamental components, as suggested by Lubik and Schorfheide (2003):

$$\eta_t^y = \Omega_x \varepsilon_t^x + \Omega_a \varepsilon_t^a + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \Omega_\theta \varepsilon_t^\theta + \varepsilon_t^b.$$

The parameters Ω_x, Ω_a, Ω_Δ, Ω_g and Ω_θ determine the effect of technological progress, investment-specific technology, preferences, government spending and collateral shocks on the expectations error. This break-down leaves the belief shock ε_t^b as a residual. The last equation then promulgates a strict definition of animal spirits: they are orthogonal to the other disturbances, thus independent of economic fundamentals.

3.3.2 Data and measurement equation

The estimation uses quarterly U.S. data running from 1955:I to 2014:IV and includes seven observable time series: (i) the log difference of real per capita GDP, (ii) real per capita consumption, (iii) real per capita investment, (iv) real per capita government spending, (v) the relative price of investment, (vi) the log difference of per capita hours worked from its sample mean, as well as (vii) the credit spread from its sample mean. We instrument financial market conditions by a credit spread similar to Christiano et al. (2014). In particular, Christiano et al. make use of the difference between the interest rate on Baa corporate bonds and the ten-year US government bond rate. The Appendix provides the full description of the data used and its construction. The corresponding measurement equation is

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln A_t I_t - \ln A_{t-1} I_{t-1} \\ \ln G_t - \ln G_{t-1} \\ \ln A_t - \ln A_{t-1} \\ \ln N_t - \ln \bar{N} \\ \text{credit spread} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}^y_t \\ \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}^y_t \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{\mu}^y_t \\ \hat{g}_t - \hat{g}_{t-1} + \hat{a}^g_t - \hat{a}^g_{t-1} + \hat{\mu}^y_t \\ \hat{\mu}^a_t \\ \hat{N}_t \\ -x * \phi * \hat{\phi}_t \end{bmatrix} + \begin{bmatrix} \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^a \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon^{me}_{y,t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon^{me}_{s,t} \end{bmatrix}$$

where $a^g_t \equiv X^G_t / X^Y_t = (a^g_{t-1})^{\psi_{yg}} (\mu^y_t)^{-1}$. In the last measurement equation, x is the scale parameter only appearing in the measurement equation to adjust the difference of the volatilities (that is, units) between the model frictions and the observable variable. Both output growth and credit spread are measured with errors $\varepsilon^{me}_{y,t}$ and $\varepsilon^{me}_{s,t}$ which are i.i.d. innovations with mean zero and standard deviation σ^{me}_y and σ^{me}_s, respectively. Allowing for a measurement error to output is a way to circumvent stochastic singularity (e.g. Schmitt-Grohé and Uribe, 2012). The measurement error to the spread can reconcile any mis-measurement in the data, especially since only a proxy is observed (e.g. Justiniano et al., 2011). Both measurement errors are restricted to absorb no more than ten percent of the variance of the corresponding observables. We estimate the model by allowing all fundamental and the animal spirits shocks to matter.

3.3.3 Calibrations and priors

We group the model parameters into two categories: calibrated and estimated. The first set of parameters is calibrated following the literature and is based on national accounts data averages. We only address some of these calibrations (all are listed in completion in Table 3.1). The elasticity of substitution parameter λ is set at ten, as in Dotsey and King (2005) and Cogley and Sbordone (2008). The average government spending share in GDP, G/Y, is calibrated at 21 percent, a number which matches national accounts average. The quarterly growth rates of per capita output μ^y and the relative price of investment μ^a are set equal to their sample averages of 1.0041 and 0.9949. Finally, the household's first-order conditions determine the elasticity of the depreciation rate from $\nu = (\mu^k/\beta - 1)/\delta$.

Table 3.1: Calibration

Parameters	Values	Description
β	0.99	Subjective discount factor
α	1/3	Capital share
η	0	Labor supply elasticity parameter
λ	10	Elasticity of substitution between goods
δ	0.0333	Steady-state depreciation rate
u	1	Steady-state capacity utilization rate
G/Y	0.21	Steady-state government expenditure share of GDP
μ^y	1.0041	Steady-state gross per capita GDP growth rate
μ^a	0.9949	Steady-state gross growth rate of price of investment

All other model parameters are estimated. Our prior assumptions are sum-

Table 3.2: Estimation

Estimated parameters	Prior distribution: Range	Prior distribution: Density[mean,std]	Posterior distribution: Mean	Posterior distribution: 90% Interval
Steady-state marginal cost, ϕ	[0.83,0.90]	Beta[0.88,0.01]	0.833	[0.831,0.834]
Elasticity of collateral, γ	[0.160,0.607]	Uniform	0.322	[0.315,0.329]
Gov. trend smoothness, ψ_{yg}	[0,1)	Beta[0.5,0.2]	0.965	[0.953,0.977]
Scale parameter, x	R^+	IGam[44,Inf]	47.33	[44.28,50.46]
AR technology shock, ρ_x	[0,1)	Beta[0.5,0.2]	0.025	[0.008,0.041]
AR investment shock, ρ_a	[0,1)	Beta[0.5,0.2]	0.029	[0.013,0.045]
AR preference shock, ρ_Δ	[0,1)	Beta[0.5,0.2]	0.984	[0.981,0.988]
AR government shock, ρ_g	[0,1)	Beta[0.5,0.2]	0.986	[0.982,0.989]
AR collateral shock, ρ_θ	[0,1)	Beta[0.5,0.2]	0.992	[0.990,0.994]
Belief shock volatility, σ_b	R^+	IGam[0.1,Inf]	0.640	[0.615,0.665]
SE technology shock, σ_x	R^+	IGam[0.1,Inf]	0.690	[0.646,0.733]
SE investment shock, σ_a	R^+	IGam[0.1,Inf]	0.562	[0.525,0.598]
SE preference shock, σ_Δ	R^+	IGam[0.1,Inf]	0.386	[0.364,0.407]
SE government shock, σ_g	R^+	IGam[0.1,Inf]	0.944	[0.896,0.992]
SE collateral shocks, σ_θ	R^+	IGam[0.1,Inf]	0.132	[0.121,0.143]
SE measurement error, σ_y^{me}	[0,0.29]	Uniform	0.290	[0.289,0.290]
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.28	[27.11,27.42]
Technology shock effect, Ω_x	[-3,3]	Uniform	-0.514	[-0.590,-0.438]
Investment shock effect, Ω_a	[-3,3]	Uniform	0.271	[0.176,0.367]
Preference shock effect, Ω_Δ	[-3,3]	Uniform	0.872	[0.756,0.994]
Government shock effect, Ω_g	[-3,3]	Uniform	0.256	[0.205,0.305]
Collateral shock effect, Ω_θ	[-3,3]	Uniform	0.999	[0.610,1.393]
Log-data density		4064.98		

marized in Table 3.2. The parameters estimated here include the steady state marginal cost ϕ (or equivalently the inverse of the mark-up), the elasticity of collateral γ, the scale parameter x, the parameters that describe the stochastic processes and the standard deviation of the measurement error. A beta distribution is adopted for the steady-state marginal cost ϕ and its value falls between 0.83 and 0.9, so that the steady-state markup varies from around eleven to twenty percent. The range of marginal costs is chosen for two reasons. First, the empirically estimated markup falls in this range (see for example Cogley and Sbordone, 2008, and De Loecker and Eeckhout, 2017). Second, the upper value of ϕ is further restricted by the inequality constraints $\xi\frac{\lambda-1}{\lambda} < \phi < \frac{\lambda-1}{\lambda}$ for the financial constraint to bind.[6] We set the prior mean for x to match the standard

[6] The prior distribution of γ guarantees that the complete indeterminacy region is covered. Since we concentrate on this region, during the MCMC, all proposed draws from the

deviation of the smoothed endogenous financial frictions in the model without any financial information (data and shock) and the standard deviation of the demeaned spread data. We adopt an inverse gamma distribution for the prior. For the persistence parameters we use a beta distribution and the standard deviations of the shocks follow an inverse gamma distribution. The prior distributions for the expectational parameters Ω_x, Ω_a, Ω_Δ, Ω_g and Ω_θ are uniform, thus agnostic about their values. Endogenous priors prevent overpredicting the model variances as in Christiano et al. (2011). We use the Metropolis-Hastings algorithm to generate one million draws from the posterior for each of the two chains, discard the initial half of the draws as burn-in, and adjust the scale in the jumping distribution to achieve a 25 to 30 percent acceptance rate for each chain.

3.3.4 Estimation results

The last two columns of Table 3.2 present the posterior means of the estimated parameters, along with their 90 percent posterior probability intervals. The parameters are precisely estimated as is evidenced by the percentiles. The estimated steady state of marginal cost implies a steady state markup of twenty percent. The table also reveals a significantly time-varying character of financial frictions. Disturbances to preference, government spending and collateral exhibit a high degree of persistence. The autocorrelation of the non-stationary technology shock is low, but it is not inconsistent with the moderate values commonly found in the literature.

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determinacy and source regions were discarded.

Table 3.3: Business cycle dynamics (band-pass filtered)

	Data			Model		
x	σ_x/σ_Y	$\rho(x,Y)$	ACF	σ_x/σ_Y	$\rho(x,Y)$	ACF
Y_t	1.00	1.00	0.93	1.00	1.00	0.91
C_t	0.58	0.85	0.92	0.63	0.75	0.90
I_t	3.25	0.89	0.94	3.09	0.88	0.92
G_t	0.99	0.01	0.94	0.96	0.21	0.90
N_t	1.24	0.87	0.94	1.01	0.98	0.92

Table 3.3 reports second moments of the main macroeconomic variables calculated using U.S. data and compares these moments to those obtained from model simulations at the posterior mean, both at business cycle frequencies. The model matches fairly well the relative standard deviations, autocorrelations and the variables' cross-correlations with output. Table 3.4 displays the contribution of each structural shock, which we list in the top row, to the variances of key macroeconomic variables. Through the lens of our theory, the decomposition suggests that animal spirits shocks ε_t^b are a major source of U.S. aggregate fluctuations. These shocks account for over 40 percent of output growth fluctuations. The ensemble of other aggregate demand shocks plays a lesser role and the contribution of the two technology shocks is small at no more than twenty percent. For investment, the vast majority of its variations comes from animal spirits suggesting that much of the spending is driven by entrepreneurial sentiments. The credit spread is mainly driven by stochastic financial factors as well as by the three demand side disturbances (i.e. animal spirits, preferences and government spending).[7] We re-ran the estimation, but halted in 2007:III,

[7] We estimate the model using loan data and animal spirits remain significant. Furthermore,

i.e. just before the onset of the Great Recession. This alteration does not affect the results as the parameter estimates as well as the variance decompositions remain virtually unchanged.

Table 3.4: Unconditional variance decomposition

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	43.43	11.17	5.72	15.70	9.93	6.71	6.80	0.00
$\ln(C_t/C_{t-1})$	6.18	40.42	2.76	39.84	1.96	8.82	0.00	0.00
$\ln(A_t I_t/A_{t-1} I_{t-1})$	66.53	2.41	7.06	9.34	7.09	7.57	0.00	0.00
$\ln(N_t/\bar{N})$	21.24	2.54	9.37	26.50	22.06	18.30	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.98	0.16	0.00	98.85	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	12.26	2.06	4.85	17.99	15.06	43.49	0.00	3.30

In sum, the estimation suggests that psychological motivations are behind a significant portion of the fluctuations in U.S. aggregate real economic activity. While the definitions of confidence shocks do not exactly overlap, this result parallels recent findings by Angeletos et al. (2016), Milani (2017) and Nam and Wang (2016) who, while arguing within theoretical frameworks that involve uniqueness, also find that bouts of optimism and pessimism are driving a large fraction of U.S. aggregate fluctuations.

3.3.5 Are shocks meaningfully labeled?

We identify the shocks by estimating in a system and it is thus fair to ask if the estimated shocks are meaningfully labelled. Specifically, do the shocks share resemblance with empirical series that are computed with orthogonal information

variance decompositions at business cycle frequencies deliver almost identical results.

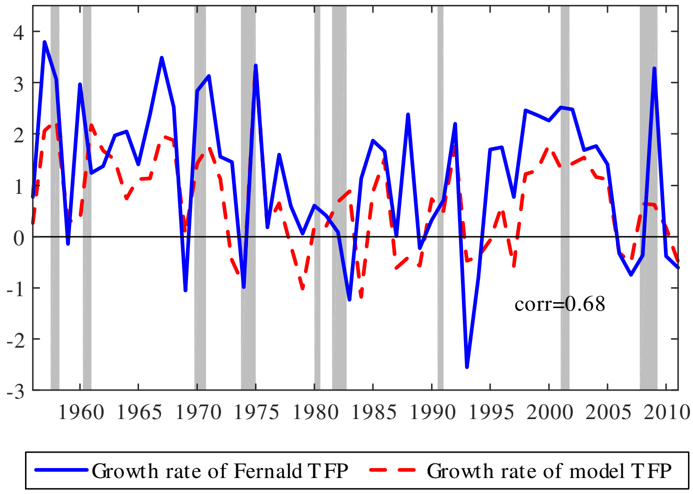

Figure 3.3: Fernald's vs model's total factor productivity (annual data).

sets? To begin with, the estimated model's total factor productivity (TFP) series is compared with Fernald's (2014) TFP series for the United States.[8] Fernald's TFP series are widely considered as the gold standard for this variable for which he adjusts for variations in factor utilization (labor effort and the workweek of capital) as well as labor skills. The results of this exogenous validation are reassuring as shown in Figure 3.3. Both productivity series not only have similar amplitudes, but their contemporaneous correlation comes in at 0.68. Hence, the model is successful in extracting productivity shocks. Next, Figure 3.4 compares the index of estimated confidence and the U.S. Business Confidence index (band-pass filtered to concentrate on the relevant frequencies). Clearly, the empirical confidence index is influenced by a raft of fundamentals and non-fundamentals, thus, it is not exactly clear how the empirical data would map our theoretical notion of animal spirits. Yet, one would expect that the animal spirits and confidence data display a certain similarity. In fact, the two sentiment series are strongly correlated and we interpret the relationship in Figure 3.4 as endorsing

[8] Growth of total factor productivity in our model is given by $(1-\alpha)(\hat{\mu}_t^x + \ln \mu^x)$.

our estimation and as supporting the case that estimated belief shocks reflect variations in people's expectations about the future path of the economy.[9]

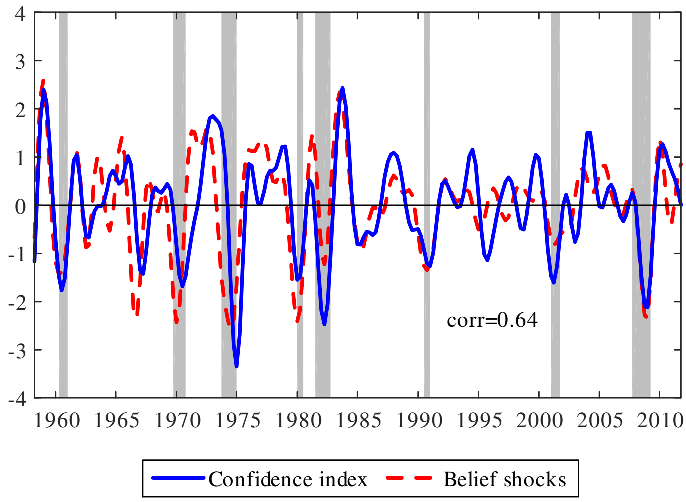

Figure 3.4: Business confidence index vs animal spirits shocks (normalized data).

3.4 Robustness checks

In this Section, we report several robustness checks. First, we leave Lubik and Schorfheide's (2003) representation of a belief shock and follow Farmer et al.'s (2015) formulation. Next, we go through alternative observables to measure financial markets' health. This is followed by adding Fernald's (2014) TFP data to the observables. We also replace permanent technology shocks by transitory shocks and consider the presence of shocks to the marginal efficiency of investment as in Justiniano et al. (2011).

We begin the chain of robustness checks by following the approach of Farmer et al. (2015) in which the animals spirits shock is simply the forecast error, i.e. $\eta_t^y = \varepsilon_t^b$, with a variance σ_η^2. Intuitively, since output is forward looking,

[9] The correlation of the estimated sunspot shocks and Fernald's TFP series is insignificant at 0.2.

Table 3.5: Posterior distribution comparison

	Model with $\eta_t^y = \varepsilon_t^b$			
	Prior distribution		Posterior distribution	
Parameters	Range	Density[mean,std]	Mean	90% Interval
ϕ	[0.83,0.90]	Beta[0.88,0.01]	0.833	[0.831,0.834]
γ	[0.160,0.607]	Uniform	0.322	[0.315,0.329]
ψ_{yg}	[0,1)	Beta[0.5,0.2]	0.965	[0.954,0.977]
x	R^+	IGam[44,Inf]	47.30	[44.18,50.35]
ρ_x	[0,1)	Beta[0.5,0.2]	0.025	[0.008,0.042]
ρ_a	[0,1)	Beta[0.5,0.2]	0.029	[0.014,0.045]
ρ_Δ	[0,1)	Beta[0.5,0.2]	0.984	[0.981,0.988]
ρ_g	[0,1)	Beta[0.5,0.2]	0.986	[0.982,0.989]
ρ_θ	[0,1)	Beta[0.5,0.2]	0.992	[0.990,0.994]
σ_η	R^+	IGam[0.1,Inf]	0.862	[0.821,0.902]
σ_x	R^+	IGam[0.1,Inf]	0.690	[0.647,0.733]
σ_a	R^+	IGam[0.1,Inf]	0.562	[0.525,0.598]
σ_Δ	R^+	IGam[0.1,Inf]	0.385	[0.364,0.407]
σ_g	R^+	IGam[0.1,Inf]	0.945	[0.897,0.993]
σ_θ	R^+	IGam[0.1,Inf]	0.132	[0.121,0.143]
σ_y^{me}	[0,0.29]	Uniform	0.290	[0.289,0.290]
σ_s^{me}	[0,27.42]	Uniform	27.28	[27.11,27.42]
$\rho(x, \eta^y)$	[-1,1]	Uniform	-0.406	[-0.465,-0.349]
$\rho(a, \eta^y)$	[-1,1]	Uniform	0.172	[0.110,0.233]
$\rho(\Delta, \eta^y)$	[-1,1]	Uniform	0.388	[0.338,0.438]
$\rho(g, \eta^y)$	[-1,1]	Uniform	0.275	[0.226,0.327]
$\rho(\theta, \eta^y)$	[-1,1]	Uniform	0.151	[0.091,0.213]
Log-data density		4066.02		

this expectation error should be correlated with fundamental shocks. Yet, it is also a sunspot shock, as it can cause movements in economic activity without any shifts to fundamentals. Assuming a uniform distribution, we thus estimate the correlations between η_t^y and the fundamental shocks. The priors for the other parameters are kept the same as in the baseline model. As can be seen by comparing Tables 3.2 and 3.5, our estimation results are robust to the formation of the expectation error. The posterior distributions are almost identical and the closeness of the log-data densities confirms that the goodness of fit between

Table 3.6: Unconditional variance decomposition (Baa-Aaa spread)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	45.46	11.34	5.34	15.63	9.12	6.31	6.80	0.00
$\ln(C_t/C_{t-1})$	6.67	41.08	2.65	38.98	1.84	8.78	0.00	0.00
$\ln(A_t I_t/A_{t-1} I_{t-1})$	68.22	2.32	6.45	9.04	6.24	7.73	0.00	0.00
$\ln(N_t/\bar{N})$	23.25	2.31	9.08	25.25	20.31	19.79	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.07	0.17	0.00	98.76	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	13.12	1.87	4.59	16.51	13.48	47.13	0.00	3.30

the models is equivalent.[10]

The next robustness check concerns the choice of the observed spread when instrumenting financial markets' conditions as we consider the sensitivity to using various alternative spreads. In particular, we ask if using the Baa-Aaa spread or the Baa-Federal funds rate spread leads to significantly different results in the estimation. We report the variance decompositions only. The results for the alternative spreads are documented in Tables 3.6 and 3.7. Animal spirits continue to stand out as the main driver of the business cycle.[11]

Table 3.7: Unconditional variance decomposition (Baa-FF spread)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	42.35	12.38	6.10	17.45	9.40	4.97	7.34	0.00
$\ln(C_t/C_{t-1})$	5.93	43.61	3.01	39.50	1.86	6.09	0.00	0.00
$\ln(A_t I_t/A_{t-1} I_{t-1})$	65.43	2.62	7.51	10.04	7.00	7.40	0.00	0.00
$\ln(N_t/\bar{N})$	22.11	2.33	10.53	26.72	22.55	15.76	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.02	0.17	0.00	98.81	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	14.32	2.19	6.08	20.38	17.16	34.61	0.00	5.26

[10] Second moments and variance decompositions are virtually identical and are not presented to conserve space.

[11] We considered other interest spreads and the results repeat.

Next, we add total factor productivity to the catalog of observables. Fernald's (2014) data is the natural series to choose from. Fernald adjusts for variations in factor utilization (labor and capital) and includes adjustment for quality or composition of inputs. Most of these influences are not part of the present artificial economy and we thus add one more measurement error on total factor productivity (at not more than ten percent). Table 3.8 shows that the previous results remain robust. Animal spirits continue to cause the bulk of U.S. output fluctuations. The technology shocks' contributions are lower, with a best point estimate near ten percent.

Table 3.8: Unconditional variance decomposition (Fernald TFP)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$	$\varepsilon_{tfp,t}^{me}$
$\ln(Y_t/Y_{t-1})$	39.02	10.35	5.10	12.63	9.13	17.01	6.77	0.00	0.00
$\ln(C_t/C_{t-1})$	4.63	38.01	2.18	34.21	1.49	19.48	0.00	0.00	0.00
$\ln(A_tI_t/A_{t-1}I_{t-1})$	59.56	2.09	6.31	8.56	6.12	17.36	0.00	0.00	0.00
$\ln(N_t/\bar{N})$	16.00	2.35	7.14	21.74	16.70	36.07	0.00	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.08	0.15	0.00	98.76	0.00	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
Credit spread	6.54	1.34	2.61	10.38	8.13	67.28	0.00	3.71	0.00
$\ln(TFP_t/TFP_{t-1})$	0.00	92.29	0.00	0.00	0.00	0.00	0.00	0.00	7.71

So far, we have assumed that technology follows a stochastic trend. We now replace permanent technology shocks by transitory shocks. Hence, the production technology is given by

$$Y_t = Z_t K_t^\alpha (\mu^t N_t)^{1-\alpha}$$

and the growth rate of labor augmenting technological progress is deterministic at the constant rate μ, as in King et al. (1988). We permit temporary changes in total factor productivity through Z_t, which follows a first-order autoregressive process

$$\ln Z_t = (1-\rho_z)\ln Z + \rho_z \ln Z_{t-1} + \varepsilon_{z,t} \qquad 0 < \rho_z < 1.$$

The presence of (one more) transitory shock will also make it (even) harder for animal spirits shocks to explain data's transitory fluctuations. Nevertheless, the model estimation delivers similar posterior means of the parameters as the baseline estimation and they are reported in the Appendix. Noteworthy is the estimate for ρ_z at 0.997 which is arguably very close to a unit root. While high, this number is consistent with Ireland (2001), for example. The variance decompositions of the stationary technology shocks model are reported in Table 3.9. Technology shocks account for about 17 percent of GDP volatility. Animal spirits remain the most critical driver of aggregate fluctuations and they continue to explain roughly 40 percent of output growth variations.[12]

Table 3.9: Unconditional variance decomposition (transitory TFP)

Series/shocks	ε_t^b	ε_t^z	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	39.18	16.79	5.28	15.64	8.21	8.69	6.22	0.00
$\ln(C_t/C_{t-1})$	3.78	43.19	2.19	40.98	1.13	8.73	0.00	0.00
$\ln(A_tI_t/A_{t-1}I_{t-1})$	57.92	11.81	6.28	10.25	5.64	8.10	0.00	0.00
$\ln(N_t/\bar{N})$	16.08	17.47	8.35	26.25	15.10	16.75	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.00	0.22	0.00	99.78	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	5.63	41.34	2.59	10.61	6.17	30.37	0.00	3.30

[12] The posterior means of the parameters in the model with transitory technology productivity are shown in the Appendix as Table 3.13. There, we also report an external validation as in Figures 3.3 and 3.4 and, again, estimated shocks are very similar to Fernald's series as well as U.S. confidence data.

The natural question arises which specification of technology is favored by data? This question is answered in Table 3.10 which compares the model fits of the two alternatively specified models. Data strongly prefers a version of the model in which total factor productivity has a stochastic trend.[13]

Table 3.10: Model comparison

	Baseline: permanent TFP	Alternative: transitory TFP
Log-data density	4064.98	3811.89

Justiniano et al. (2011) push for shocks that affect the production of installed capital from investment goods or the transformation of savings into the future capital input. This is an alternative way to model exogenous financial frictions. The concept of shocks to the marginal efficiency to investment (MEI) goes back to Greenwood et al. (1988) who formulate the ideas as

$$K_{t+1} = (1 - \delta_t)K_t + \nu_t I_t$$

where we abstract from adjustment costs to not mess with the indeterminacy properties of the artificial economy. The shock ν_t affects the marginal efficiency of capital and it follows an autoregressive process with persistence parameter ρ_ν. The MEI shocks are likely a

> "might proxy for more fundamental disturbances to the intermediation ability of the financial system." [Justiniano et al., 2011, 103]

We estimate the amended model and associate the observed spread with the value of the MEI to impose discipline on the inference of the shock as in

[13] We conduct a similar exercise with respect to the form of the preference shock. Data does strongly prefer the current setup over a version with a stochastic discount factor.

Justiniano et al. (2011).[14] Again, we add a measurement error to the spread equation. Table 3.11 shows, in line with our previous findings, that the animal spirits shocks remain a most prominent driver of U.S. output fluctuations.[15] An external validation exercise akin to Figures 3.3 and 3.4 finds that estimated shocks are again very similar to their empirical counterparts (see Appendix).

Table 3.11: Unconditional variance decomposition (MEI shock)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^{MEI}	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	46.82	10.15	5.51	15.76	11.18	2.08	8.49	0.00
$\ln(C_t/C_{t-1})$	8.77	40.93	2.92	43.77	2.96	0.66	0.00	0.00
$\ln(A_t I_t/A_{t-1} I_{t-1})$	69.61	2.35	6.77	9.82	8.68	2.77	0.00	0.00
$\ln(N_t/\bar{N})$	25.57	3.62	10.02	31.30	27.17	2.31	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.75	0.13	0.00	99.12	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	0.00	0.00	0.00	0.00	0.00	99.95	0.00	0.05

3.5 A closer look at the Great Recession

From 2007 to 2009, the U.S. economy was in a severe slump. The Great Recession was the single-worst economic contraction since the 1930s, with economic activity diving after various financial institutions collapsed. One of the aims of the recent financial friction models is to identify the sources of the crisis.

[14] Given the occurance of financial frictions in two places, we are only able to connect one model friction to the spread's measurement equation. The series of animal spirits remains highly correlation to earlier estimations, thus, our result is not the consequence of putting less restictions on the psychological shocks.

[15] We considered the hypothesis that sunspot shocks are in fact news shocks. In the spirit of Beaudry and Portier (2006), we looked into finding a relation of the belief shocks with future movements of technology. In particular, we compute the correlations of the estimated animal spirits with Fernald's TFP data at four to sixteen quarters out. The correlations are negligible at never more than 0.04.

To what extent can animal spirits explain the downturn in GDP observed in this recession?

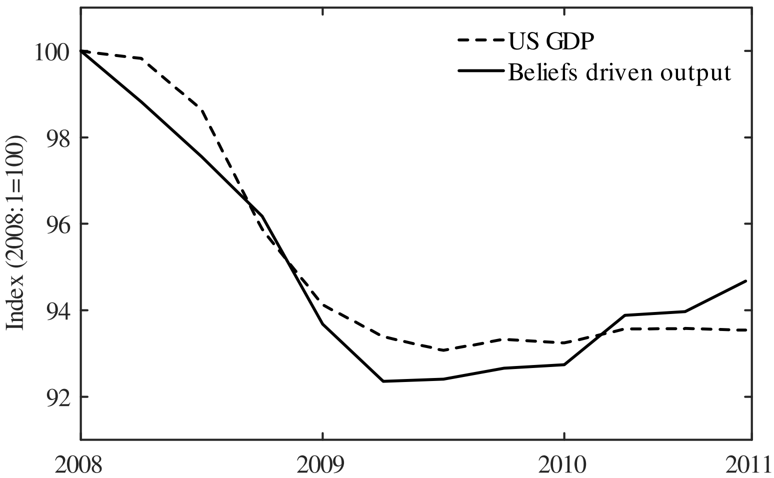

Figure 3.5: Counterfactual path of output, conditional on estimated belief shocks. Parameters are set at the posterior mean.

We begin with a counterfactual exercise in which we shut down all but the animal spirits shocks (using Section 3.3's model). Figure 3.5 plots the counterfactual path of output driven solely by these belief shocks along with the actual series over the Great Recession period. The U.S. data has been detrended by removing long-run productivity trend and also population growth, as we abstract from it in the model. We re-scale both model and U.S. data so that outputs are equal to 100 in 2008:I. The model economy virtually coincides in both timing and depth with the actual economy during the crisis period and the measured drop in confidence can account for most of the decline in output. The counterfactual exercise favors the interpretation that the fall of aggregate output during the Great Recession was closely associated with self-fulfilling beliefs. Our reading of events goes like this: adverse expectations led to a drop in aggregate demand which curbed lending and tightened credit (similar to Kahle and Stulz, 2013). This tightening occurred because people were expecting worsening business conditions and higher defaults. In other words, people became pessimistic and, as a consequence of the effect on financial markets, the reduced investment spending lowered productivity which then made pessimistic expectations self-fulfilled. Our results do not necessarily contradict Christiano et al.'s (2015) account of the Great Recession. Their study finds that the steep decline of aggregate economic activity was overwhelmingly caused by exogenous financial frictions. What our analysis suggests is, however, that it was a drop in people's animal spirits affected aggregate demand and then found its catalyst in financial markets. The endogenous reaction of the financial sector helped in propagating gloomy animal spirits into the full-blown crisis and macroeconomic collapse.

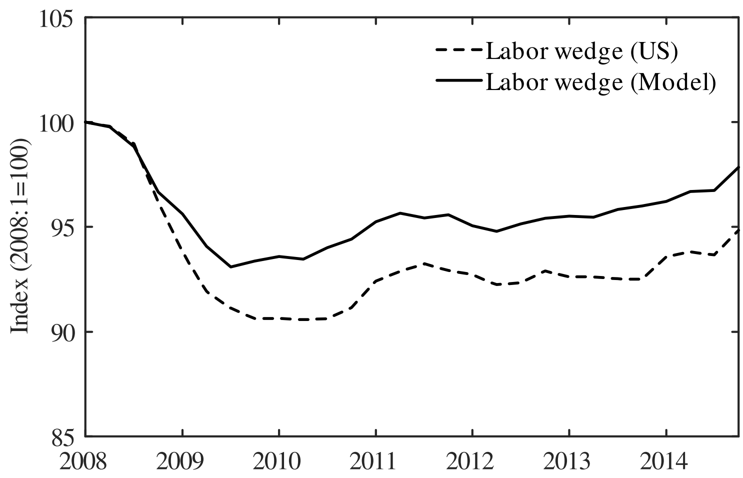

Figure 3.6: The artificial labor wedge during the Great Recession.

A useful way of thinking about the Great Recession is in terms of Chari et al.'s (2007) business cycle accounting framework which decomposes distortions in the economy into sets of residuals or wedges. When applying this framework, Brinca et al. (2016) assert that

> "[...] considering the period from 2008 until the end of 2011, [our] results imply that the Great Recession in the United States should be

thought of as primarily a labor wedge recession, with an important secondary role for the investment wedge." [Brinca et al., 2016, 1042]

This diagnostic finding leads to the question of what would the these wedges look like in the artificial economy? In a benchmark prototype economy, the labor wedge $1 - \tau_t^n$ shows up in the budget constraint as

$$... = (1 - \tau_t^n) w_t N_t + r_t u_t K_t$$

thus it is like a tax on labor services.[16] The labor wedge is plotted along with its data equivalent in Figure 3.6. Clearly, the two series show high conformity. The artificial wedge explains about three-fourths of the data wedge's plunge during 2008 and 2009 and it charts a tepid recovery over the 2010 to 2014 period. Our model estimation also suggests an important role for financial market imperfections. Thus, given Brinca et al.'s (2016) assertion, we report a wedge that measures these sort of distortions: it is like a tax on capital income as in Kobayashi and Inaba (2006) or Cavalcanti et al. (2008) and in a benchmark prototype economy it would show up on the right hand side of the budget constraint as $1 - \tau_t^k$:

$$... = w_t N_t + (1 - \tau_t^k) r_t u_t K_t.$$

Figure 3.7 maps out both the empirical and the model implied capital wedges next to the investment wedge as in Brinca et al. (2016). Note that we report the "τ_ts" rather than the full wedges. These distortions are shown alongside Romer and Romer's (2017) semi-annual index of financial stress which focusses

"on disruptions to credit supply, rather than on broader concep-

[16] In the Appendix, we describe the construction of wedges in terms of the artificial economy. Kobayashi and Inaba (2006) prove an equivalence of the capital wedge as well as the investment wedge.

tions of financial problems" [Romer and Romer, 2017, 3073].

We take three insights from this accounting. Firstly, capital and investment wedges display very similar patterns and they indeed point to a worsening of financial market health after 2007. This mirrors Romer and Romer's (2017) findings. Second, our model lines up well with Brinca et al.'s (2016) interpretation of the Great Recession in terms of both the labor as well as financial wedges. Thirdly, Romer and Romer's (2017) index suggests that financial distress in the U.S. ended by 2011 and this is at some odds with the pattern of both financial wedges which are significantly more persistent. Our take on this picture is that investment spending remained subdued for factors other than financial ones. From our analysis, it appears that the tepid spending reflects a lack of animal spirits, i.e. businesses were not confident about future demand to justify more investment.

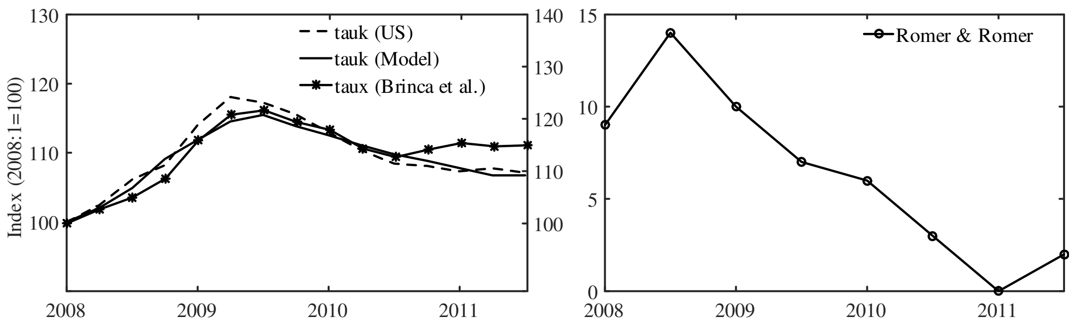

Figure 3.7: Financial wedges during the Great Recession: the initial observations have been normalized to 100 (capital wedges measured on left-hand axis). Right-hand panel shows Romer and Romer (2017) index.

3.6 Does data prefer indeterminacy?

So far we have restricted the estimation to the parameter space with multiple equilibria, yet a natural question arises: does data in fact favor a model with indeterminacy? To answer this question, we now estimate the economy over the entire parameter space using the methodology proposed in Bianchi and Nicolò (2017).[17] Their procedure can be implemented without knowing the analytical expressions for the boundaries between the three dynamic regions (recall Figure 3.2).

Table 3.12: Determinacy versus Indeterminacy

	Determinacy	Indeterminacy
Model prior probabilities	0.52	0.47
	Permanent TFP	
Log-data density	3470.07	4065.42
Model posterior probability	0.00	1.00
	Transitory TFP	
Log-data density	3441.67	3812.86
Model posterior probability	0.00	1.00
	MEI	
Log-data density	3601.05	4305.71
Model posterior probability	0.00	1.00

The estimation process begins by setting the priors so that determinacy, indeterminacy and source probabilities are at 52:47:1 (in percent). To do this, we adjust the prior of the elasticity of the collateral γ, which is now beta-distributed, to being centered at 0.17 with a standard deviation of 0.1 and truncated to be no more than 0.61.[18] All parameters that pertain to the solution under indeterminacy are restricted to be zero when the estimation for draws is taking

[17] The Appendix explains their methodology in more detail.

[18] All other priors are as above. Details of the estimation procedure are delegated to the Appendix 3.A.3.

place in the determinacy region of the model. Draws from the source region were discarded. In line with Bianchi and Nicolò (2017), we follow the approach proposed in Farmer et al. (2015) and construct the forecast errors of output η_t^y as a belief shock with variance σ_η^2 and allow the expectation errors to be correlated with the fundamental shocks. As would be reasonable, for these correlations we assume flat priors that are uniform between -1 and 1. Table 3.12 presents the results for model versions discussed earlier involving i) permanent technology shocks, ii) transitory technology shocks and iii) shocks to the marginal efficiency to investment. The observable variables are the same as in Sections 3.3 and 3.4. The log data densities in Table 3.12 suggest that U.S. data strongly favours the indeterminacy model over all three versions of the economy in which animal spirits cannot play a role.

Three further observations are worthwhile mentioning. First, the estimated parameters under indeterminacy that arise when we implement the methodology developed in Bianchi and Nicolò (2017) are essentially equivalent to our previous results. Thus, estimating via their procedure leaves results unaffected and the implications regarding the important role of animal spirits carry over (see for example Table 3.14 in the Appendix). Second, in addition to being favored by data, the indeterminacy model is superior in identifying shocks for which empirical counterparts exist. For example, the model-based technology shocks track the empirical TFP series better under indeterminacy: when comparing the estimated sequence as done in the external validation of Figure 3.3, then the contemporaneous correlation with Fernald's series drops slightly from 0.68 to 0.65 under determinacy. Third, the key difference in the parameter estimates across the two regions applies to the parameter γ that controls the endogenous component of credit market tightness: γ approaches zero for the determinacy versions of the model. The endogenous aspect of the collateral constraint disappears.

How can we make sense of the finding that the indeterminacy model is preferred by U.S. data? The absence of the endogenous feedback of financial market conditions to the state of the economy implies that other fundamental shocks' amplification mechanisms are curtailed and movements of the collateral constraint (and of marginal costs) are determined by the exogenous financial friction shocks. For example, as is shown in the Appendix' Table 3.15, under determinacy the MEI shock explains about thirty percent of output fluctuations and the spread's variations in almost their entirety. These numbers are quite similar to Justiniano et al. (2011, Table 4) while at somewhat different frequencies. However, the rigid collateral constraints imply that the other fundamental shocks are no longer able to contribute towards the procyclical variations of financial health. In other words, the pattern that was reported in Figure 3.1 – namely that financial conditions are cyclical and deteriorate during basically all slumps – is more effortlessly accommodated by an artificial economy with an endogenously varying collateral constraint, however, this then implies that the economy becomes indeterminate and, consequently, animal spirits are assigned an important role.

3.7 Concluding remarks

This paper has presented evidence on the sources of U.S. aggregate fluctuations over the period 1955 to 2014. We perform a Bayesian estimation of a financial accelerator model which features an indeterminacy of rational expectations equilibria. Indeterminacy in the model is linked to the empirically observed countercyclical movement of financial market tightness. The artificial economy is driven both by fundamental shocks as well as by animal spirits. U.S. data favours the indeterminacy model over versions of the economy in which sunspots do not play a role. The estimation supports the view that people's animal spirits play a significant role for the U.S. business cycle. Variance decompositions suggest that animal spirits are behind a substantial fraction of output growth variations and they explain an even larger portion of fluctuations in investment spending. Technology shocks and financial frictions shocks are significantly less important in explaining the oscillations in aggregate real economic activity. The 2007-2009 recession appears to have been chiefly caused by adverse confidence shocks.

Admittedly, we have left out various aspects of the economy that could be considered relevant. For example, the economy is real and nominal variables are absent. Thus, we exclude the potential effects of price stickiness and any influence of a monetary authority. Also, the absence of monetary policy as well as the exogenous character of the fiscal side precludes from addressing how policy could potentially influence the dynamics of this economy. The small-scale character of our model, however, provides the advantage of tractability specifically when conducting the various robustness exercises. This being said, mentioned extensions are beyond the scope and the goals of the current paper, but we plan to work out a medium-scale version of the indeterminacy model in the future.

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3.A Appendix

The Appendix sets out the complete model, a discussion of the wedges, and it lists the data sources and definitions. We begin with collecting the model's equations.

3.A.1 Model equations and equilibrium dynamics

The first-order conditions for the household's optimization problems are

$$\varphi N_t^{\eta} = \frac{1}{C_t - \Lambda_t} W_t$$

$$r_t = A_t \delta_0 u_t^{\nu}$$

and

$$\frac{A_t}{C_t - \Lambda_t} = \beta E_t \left[\frac{1}{C_{t+1} - \Lambda_{t+1}} (r_{t+1} u_{t+1} + A_{t+1}(1 - \delta_{t+1})) \right].$$

In the model, output, consumption, and real wage fluctuate around the same stochastic growth trend $X_t^Y = X_t A_t^{\alpha/(\alpha-1)}$, the growth rate of which is $\mu_t^y \equiv$

$X_t^Y/X_{t-1}^Y = \mu_t^x(\mu_t^a)^{\frac{\alpha}{\alpha-1}}$. The trend in capital stock, which is also the trend in investment equals $X_t^K = X_t^Y/A_t$, the growth rate of which is $\mu_t^k \equiv X_t^K/X_{t-1}^K = \mu_t^x(\mu_t^a)^{\frac{1}{\alpha-1}}$. Besides, the government expenditure fluctuates around its own trend X_t^G. There is no growth trend in hours, utilization and marginal cost. We first derive the detrended dynamic equilibrium equations and then log-linearly approximate them around the deterministic steady state. Let $y_t = Y_t/X_t^Y$, $c_t = C_t/X_t^Y$, $w_t = W_t/X_t^Y$, $i_t = I_t/X_t^K$, $k_t = K_t/X_{t-1}^K$, $g_t = G_t/X_t^G$, $\Delta_t = \Lambda_t/X_t^Y$ and $y_t/\bar{y}$ approximately equal to $Y_t/\bar{Y}_t$, where $\bar{y}$ represents the steady state of detrended output. The log-linearized system is summarized by

$$\hat{y}_t = \alpha\hat{k}_t + \alpha\hat{u}_t - \alpha\hat{\mu}_t^k + (1-\alpha)\hat{N}_t$$

$$\hat{y}_t = [1 - \frac{\alpha\phi(\mu^k - 1 + \delta)}{\delta(1+\nu)} - \frac{G}{Y}]\hat{c}_t + \frac{\alpha\phi(\mu^k - 1 + \delta)}{\delta(1+\nu)}\hat{\imath}_t + \frac{G}{Y}(\hat{a}_t^g + \hat{g}_t)$$

$$\hat{y}_t = (1+\eta)\hat{N}_t + \hat{c}_t - \hat{\Delta}_t - \hat{\phi}_t$$

$$\hat{y}_t = (1+\nu)\hat{u}_t + \hat{k}_t - \hat{\phi}_t - \hat{\mu}_t^k$$

$$\hat{k}_{t+1} = \frac{(1-\delta)}{\mu^k}(\hat{k}_t - \hat{\mu}_t^k) + \frac{(\mu^k - 1 + \delta)}{\mu^k}\hat{\imath}_t - \frac{\delta(1+\nu)}{\mu^k}\hat{u}_t$$

$$\hat{c}_{t+1} = \hat{c}_t - \hat{\Delta}_t - [1 - \frac{\beta\delta(1+\nu)}{\mu^k}]\hat{\mu}_{t+1}^k + \hat{\Delta}_{t+1} + \frac{\beta\delta(1+\nu)}{\mu^k}(\hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\phi}_{t+1} - \hat{u}_{t+1})$$

and

$$\hat{\phi}_t = \gamma\hat{y}_t + \hat{\theta}_t.$$

In these equations, variables without time subscripts refer to steady state values while the hatted variables denote percent deviations from their corresponding steady-state, e.g., $\hat{y}_t \equiv \log(y_t/\bar{y})$. The last equation shows that if $\gamma \to 0$, then marginal cost and the credit constraint are determined by the exogenous financial shocks only.

The following table shows the estimation results for transitory technology shocks.

Table 3.13: Estimation (transitory TFP)

	Prior distribution		Posterior distribution	
Estimated parameters	Range	Density[mean,std]	Mean	90% Interval
Steady-state marginal cost, ϕ	[0.83,0.90]	Beta[0.88,0.01]	0.832	[0.831,0.833]
Elasticity of collateral, γ	[0.160,0.607]	Uniform	0.296	[0.291,0.301]
Gov. trend smoothness, ψ_{yg}	[0,1)	Beta[0.5,0.2]	0.953	[0.932,0.975]
Scale parameter, x	R^+	IGam[44,Inf]	44.38	[42.62,46.24]
AR technology shock, ρ_z	[0,1)	Beta[0.5,0.2]	0.997	[0.996,0.998]
AR investment shock, ρ_a	[0,1)	Beta[0.5,0.2]	0.020	[0.008,0.032]
AR preference shock, ρ_Δ	[0,1)	Beta[0.5,0.2]	0.979	[0.974,0.983]
AR government shock, ρ_g	[0,1)	Beta[0.5,0.2]	0.981	[0.976,0.987]
AR collateral shock, ρ_θ	[0,1)	Beta[0.5,0.2]	0.992	[0.991,0.994]
Belief shock volatility, σ_b	R^+	IGam[0.1,Inf]	0.662	[0.640,0.685]
SE technology shock, σ_z	R^+	IGam[0.1,Inf]	0.321	[0.306,0.334]
SE investment shock, σ_a	R^+	IGam[0.1,Inf]	0.564	[0.527,0.600]
SE preference shock, σ_Δ	R^+	IGam[0.1,Inf]	0.467	[0.445,0.488]
SE government shock, σ_g	R^+	IGam[0.1,Inf]	0.943	[0.894,0.992]
SE collateral shocks, σ_θ	R^+	IGam[0.1,Inf]	0.145	[0.133,0.156]
SE measurement error, σ_y^{me}	[0,0.29]	Uniform	0.290	[0.289,0.290]
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.29	[27.12,27.42]
Technology shock effect, Ω_z	[-3,3]	Uniform	1.054	[0.924,1.187]
Investment shock effect, Ω_a	[-3,3]	Uniform	0.277	[0.188,0.371]
Preference shock effect, Ω_Δ	[-3,3]	Uniform	0.729	[0.644,0.818]
Government shock effect, Ω_g	[-3,3]	Uniform	0.255	[0.203,0.305]
Collateral shock effect, Ω_θ	[-3,3]	Uniform	1.546	[1.186,1.931]

Figure 3.8 and 3.9 show the estimated model's total factor productivity series compared with Fernald's (2014) total productivity series, as well as the index of

estimated confidence compared with the U.S. Business Confidence index for the estimation with transitory technology shock.

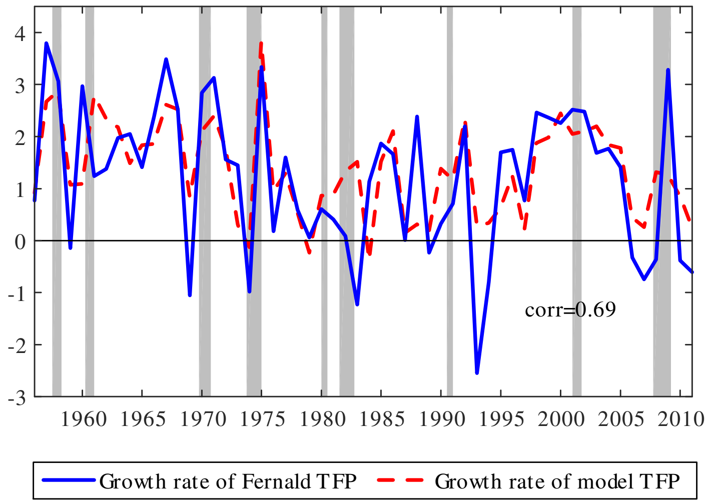

Figure 3.8: Fernald's vs model's total factor productivity (annual data).

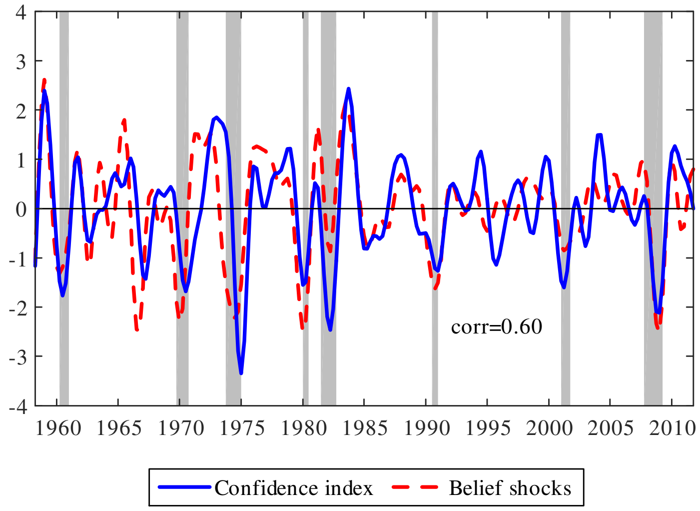

Figure 3.9: Business confidence index vs animal spirits shocks (normalized data).

Figure 3.10 and 3.11 show the estimated model's total factor productivity series compared with Fernald's (2014) total productivity series, as well as the index of estimated confidence compared with the U.S. Business Confidence index for the estimation with MEI shock.

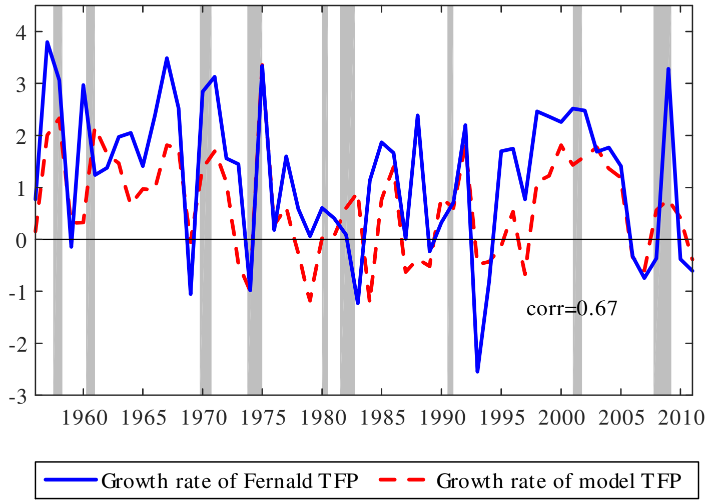

Figure 3.10: Fernald's vs model's total factor productivity (annual data).

3.A.2 Wedges

Business cycle accounting has been introduced by Chari et al. (2007). Brinca et al.'s (2016) interpretation of the Great Recession in terms of both the labor as well as financial wedges (denoted by τ_t^x). In terms of a benchmark prototype economy, the labor wedge is introduced via the household's period budget constraint

$$... = (1 - \tau_t^n) w_t N_t + r_t u_t K_t.$$

hence it is like a tax on labor services. The labor wedge $1 - \tau_t^n$ is constructed from the intratemporal first-order condition that is a wedge between the marginal rate of substitution and the marginal product of labor. In log-linear form, it would write as

$$\underbrace{(\eta \widehat{N}_t + \widehat{c}_t)}_{\text{MRS}_{C,l}} - \underbrace{\left(\widehat{y}_t - \widehat{N}_t\right)}_{\text{MPL}} = \frac{\tau^n}{\tau^n - 1} \widehat{\tau}_t^n.$$

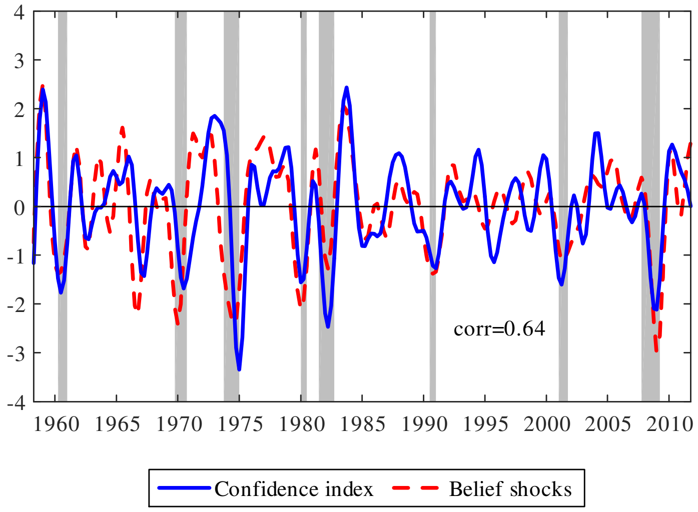

Figure 3.11: Business confidence index vs animal spirits shocks (normalized data).

The model's labor wedge is driven by fluctuations of both the markup as well as stochastic preferences. Chari et al. (2007) introduce in their business cycle accounting framework an investment wedge to measure distortions that would occur capital and financial markets. It is like a tax on investment. As the relative price (that we use as observable) maps exactly into this wedge in our artificial economy, we decided to turn to a slightly different measure of capital market distortions as do Kobayashi and Inaba (2006) as well as Cavalcanti et al. (2008).[19] The capital wedge τ_t^k is introduced via the household's period budget constraint

$$... = w_t N_t + (1 - \tau_t^k) r_t u_t K_t.$$

[19]In fact, Kobayashi and Inaba (2006) prove an equivalence of the capital wedge as well as the investment wedge.

Hence it is like a tax on capital services. This then implies from capital utilization's first order condition that

$$1 - \tau_t^k = \frac{\delta_0}{\alpha} A_t u_t^{1+\nu} K_t / Y_t$$

which allows to compute the empirical wedge from available data of the right hand side variables (rather than using the intertemporal Euler equation). In terms of our original model, the capital wedge equals the inverse of the markup. In a log-linearized world, we have a relation of the artificial wedge $\widehat{\tau}_t^{m,k}$ and marginal costs $\widehat{\phi}_t$ as

$$\widehat{\tau}_t^{m,k} = -\frac{1 - \tau^{m,k}}{\tau^{m,k}} \widehat{\phi}_t.$$

In the steady state, $1 - \tau^{m,k}$ equals ϕ which, of course, is the inverse of the markup. Given data on the relative price, utilization rates, output and capital constructed using

$$K_{t+1} = \left(1 - \delta_0 \frac{u_t^{1+\nu}}{1+\nu}\right) K_t + I_t$$

as well as a parameter calibration, one can compute an empirical series for the capital wedge. We then use the estimated model and the implied series for $\widehat{\tau}_t^{m,k}$ to construct a series of the model-wedge $\tau_t^{m,k}$. The model wedge replicates the overall empirical pattern as well as the depth of the distortions associated with the market of capital. The investment wedge in Figure 3.7 is computed from the original Chari et al. (2007) formulation, that is the wedge shows up as

$$\frac{1}{1 + \widetilde{\tau}_t^x}.$$

From this we construct a series for $1 - \tau_t^x \equiv \left(1 + \widetilde{\tau}_t^x\right)^{-1}$ and report the realizations for τ_t^x in Figure 3.7. While, by construction, not identical, the two series – $\left\{\tau_t^k\right\}$

and $\{\tau_t^x\}$ – are very similar.

3.A.3 Bianchi and Nicolò (2017)

We briefly set out the methodology that we apply in Section 3.6. It closely follows Bianchi and Nicolò (2017) and it does not require to know the (analytical solution) of the boundaries of the determinacy region.[20] The parameters of the log-linearized benchmark model are contained in the vector

$$\Theta \equiv [\alpha, \phi, \mu^y, \mu^a, \mu^k, \delta, \nu, \eta, \beta, \gamma, G/Y, \rho_x, \rho_a, \rho_\Delta, \rho_g, \rho_\theta, \sigma_x, \sigma_a, \sigma_\Delta, \sigma_g, \sigma_\theta].$$

The linear rational expectations (LRE) model can be rewritten in the canonical form

$$\Gamma_0(\Theta)s_t = \Gamma_1(\Theta)s_{t-1} + \Psi(\Theta)\varepsilon_t + \Pi(\Theta)\eta_t, \tag{3.3}$$

where

$$s_t = [\hat{y}_t, \hat{c}_t, \hat{\imath}_t, \hat{N}_t, \hat{k}_{t+1}, \hat{u}_t, \hat{\phi}_t, E_t(\hat{y}_{t+1}), E_t(\hat{c}_{t+1}), E_t(\hat{\phi}_{t+1}), E_t(\hat{u}_{t+1}), \hat{a}_t^g, \hat{\mu}_t^y, \hat{\mu}_t^k, \hat{\mu}_t^x, \hat{\mu}_t^a, \hat{\Delta}, \hat{g}_t, \hat{\theta}_t]'$$

is a vector of endogenous variables, $\varepsilon_t = [\varepsilon_t^x, \varepsilon_t^a, \varepsilon_t^\Delta, \varepsilon_t^g, \varepsilon_t^\theta]'$ is a vector of exogenous shocks, and $\eta_t = [\eta_t^y, \eta_t^c, \eta_t^\phi, \eta_t^u]'$ collects the one-step ahead forecast errors for the expectational variables of the system. Since our model can generate at most one degree of indeterminacy, Bianchi and Nicolò suggest to append the original linear rational expectations model (3.3) with the autoregressive process

$$\omega_t = \varphi^* \omega_{t-1} + \upsilon_t - \eta_{f,t} \tag{3.4}$$

[20] Bianchi and Nicolò (2017) show that their characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2003).

where υ_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast errors vector η_t. We choose $\eta_{f,t} = \eta_t^y$. The variable φ^* belongs to the interval (-1,1) when the model is determinate or it is outside the unit circle under indeterminacy. Under determinacy the Blanchard-Kahn condition is satisfied and the absolute value of φ^* is inside the unit circle since the number of explosive roots of the original LRE model in (3.3) already equals the number of expectational variables in the model. Then the autoregressive process ω_t does not affect the solution for the endogenous variables s_t. On the other hand, under indeterminacy the Blanchard-Kahn condition is not satisfied. The system is characterized by one degree of indeterminacy and it is necessary to introduce another explosive root to fulfill the Blanchard-Kahn condition – the absolute value of φ^* falls outside the unit circle. Denoting the newly-defined vector of endogenous variables $\hat{s}_t \equiv (s_t, \omega_t)'$ and the vector of exogenous shocks $\hat{\varepsilon}_t \equiv (\varepsilon_t, \upsilon_t)'$, then the system (3.3) and (3.4) can be condensed into

$$\hat{\Gamma}_0 \hat{s}_t = \hat{\Gamma}_1 \hat{s}_{t-1} + \hat{\Psi}\hat{\varepsilon}_t + \hat{\Pi}\eta_t,$$

where

$$\hat{\Gamma}_0 \equiv \begin{bmatrix} \Gamma_0(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \; \hat{\Gamma}_1 \equiv \begin{bmatrix} \Gamma_1(\Theta) & \mathbf{0} \\ \mathbf{0} & \varphi^* \end{bmatrix}$$

and

$$\hat{\Psi} \equiv \begin{bmatrix} \Psi(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \; \hat{\Pi} \equiv \begin{bmatrix} \Pi_n(\Theta) & \Pi_f(\Theta) \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}.$$

The matrix $\Pi(\Theta)$ in (3.3) is partitioned as $\Pi(\Theta) = [\Pi_n(\Theta) \quad \Pi_f(\Theta)]$ without loss of generality. Figure 3.2 shows that the model's (in-)determinacy regions. To start with, the prior probability of determinacy or indeterminacy is set. The prior probability for determinacy, indeterminacy and source is 52:47:1 in percent.

All priors are as in benchmark cases with the exception of the prior for the elasticity of the collateral constraint γ which is now beta-distributed, centered at 0.17 with standard deviation 0.1 and we truncate it to be no more than 0.61. Following Bianchi and Nicolò (2017), the determinacy model is estimated by fixing the parameter φ^* to a value smaller than one (e.g. 0.5) in a way that the model is solved only under determinacy while the indeterminacy model is estimated by fixing φ^* greater than one (e.g. 1.5) in a way that the model is solved only under indeterminacy. All parameters that pertain to the solution under indeterminacy are restricted to zero when we estimate the determinacy model. Lastly, we report the estimation results for the two versions of the model. The "Indeterminacy" column shows that using the alternative estimation method has only a very small effect on the paper's main results in regards to parameter estimates.

3.A.4 Determinacy versus indeterminacy

Table 3.14 shows, the estimated parameters that arise from applying Bianchi and Nicolò (2017) are essentially equivalent to our previous results (e.g. Table 3.2) and thus the implications regarding the important role of animal spirits persist.

Table 3.15 shows the variance decomposition for the determinacy model with technology and MEI shocks.

Table 3.14: Estimation (Determinacy vs Indeterminacy)

		Determinacy		Indeterminacy	
Parameters	Density[mean,std]	Mean	90% Interval	Mean	90% Interval
ϕ	Beta[0.88,0.01]	0.891	[0.884,0.899]	0.833	[0.831,0.834]
γ	Beta[0.17,0.10]	0.001	[0.000,0.002]	0.322	[0.315,0.329]
ψ_{yg}	Beta[0.5,0.2]	0.997	[0.996,0.998]	0.965	[0.953,0.977]
x	IGam[44,Inf]	10.48	[9.57,11.34]	47.37	[44.24,50.43]
ρ_x	Beta[0.5,0.2]	0.042	[0.031,0.053]	0.025	[0.008,0.042]
ρ_a	Beta[0.5,0.2]	0.083	[0.073,0.092]	0.029	[0.013,0.045]
ρ_Δ	Beta[0.5,0.2]	0.961	[0.955,0.966]	0.984	[0.981,0.988]
ρ_g	Beta[0.5,0.2]	0.935	[0.923,0.946]	0.986	[0.982,0.989]
ρ_θ	Beta[0.5,0.2]	0.982	[0.978,0.985]	0.992	[0.990,0.994]
σ_η	IGam[0.1,Inf]	—	—	0.862	[0.823,0.904]
σ_x	IGam[0.1,Inf]	0.546	[0.520,0.572]	0.690	[0.645,0.733]
σ_a	IGam[0.1,Inf]	0.544	[0.510,0.578]	0.562	[0.525,0.598]
σ_Δ	IGam[0.1,Inf]	0.608	[0.582,0.633]	0.386	[0.364,0.407]
σ_g	IGam[0.1,Inf]	1.106	[1.049,1.166]	0.945	[0.896,0.993]
σ_θ	IGam[0.1,Inf]	0.258	[0.245,0.270]	0.132	[0.121,0.143]
σ_y^{me}	Uniform	0.290	[0.289,0.290]	0.290	[0.289,0.290]
σ_s^{me}	Uniform	27.40	[27.37,27.42]	27.28	[27.10,27.42]
$\rho(x,\eta^y)$	Uniform	—	—	-0.406	[-0.466,-0.347]
$\rho(a,\eta^y)$	Uniform	—	—	0.173	[0.112,0.234]
$\rho(\Delta,\eta^y)$	Uniform	—	—	0.387	[0.336,0.437]
$\rho(g,\eta^y)$	Uniform	—	—	0.275	[0.225,0.326]
$\rho(\theta,\eta^y)$	Uniform	—	—	0.151	[0.090,0.212]

Table 3.15: Unconditional variance decomposition (Determinacy, MEI shock)

Series/shocks	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^{MEI}	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	25.93	11.24	16.37	10.60	30.49	5.36	0.00
$\ln(C_t/C_{t-1})$	44.73	2.73	49.00	0.99	2.56	0.00	0.00
$\ln(A_tI_t/A_{t-1}I_{t-1})$	18.65	17.13	5.87	6.08	52.28	0.00	0.00
$\ln(N_t/\bar{N})$	2.57	3.79	7.51	13.40	72.73	0.00	0.00
$\ln(G_t/G_{t-1})$	19.39	3.53	0.00	77.08	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	0.00	0.00	0.00	0.00	93.87	0.00	6.13

3.A.5 Data description

This appendix is to describe the details of the source and construction of the data used in estimation. The sample period covers the first quarter of 1955 through the fourth quarter of 2014:

1. Real Gross Domestic Product. Billions of Chained 2009 Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Government Consumption Expenditure. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

8. Government Gross Investment. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

9. Nonfarm Business Hours. Index 2009=100, Seasonally Adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

10. Relative Price of Investment Goods. Index 2009=1, Seasonally Adjusted. Source: Federal Reserve Economic Data, Series Id: PIRIC.

11. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

12. Confidence: Business Tendency Survey for Manufacturing, Composite Indicators, OECD Indicator for the United States, Series Id: BSCICP03USM665S.

13. Total Factor Productivity. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity", retrieved from

http://www.frbsf.org/economicresearch/economists/john-fernald/.

14. Moody's Seasoned Baa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

15. Moody's Seasoned Aaa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

16. 10 Year Treasury Constant Maturity Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

17. Effective Federal Funds Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

18. Capacity Utilization: Total Industry (TCU), Percent of Capacity, Seasonally Adjusted, Source: Board of Governors of the Federal Reserve System.

19. GDP deflator= (2)/(1).

20. Real Per Capita Output, $Y_t = (1)/(11)$.

21. Real Per Capita Consumption, $C_t = [(3) + (4)]/(19)/(11)$.

22. Real Per Capita Investment, $I_t = [(5) + (6)]/(19)/(11)$.

23. Real Per Capita Government Expenditure, $G_t = [(7) + (8)]/(19)/(11)$.

24. Per Capita Hours Worked, $N_t = (9)/(11)$.

25. Credit spread $= (14) - (16)$.

Chapter 4

A Bayesian Evaluation of an Efficiency-wage Model with Indeterminacy

4.1 Introduction

The work of Benhabib and Farmer (1994) has triggered an interest in formulating of business cycle models with sunspot equilibria. The primary reason for this is that the multiplicity of equilibria can give rise to fluctuations driven by extrinsic uncertainty (a.k.a. sunspots or animal spirits) in the class of dynamic stochastic general equilibrium model thereby providing an alternative source of aggregate fluctuations. The Benhabib and Farmer condition for indeterminacy has been widely criticized as unrealistic because the required degree of increasing returns seems to be significantly larger than empirical estimates (see Basu and Fernald, 1997). Later developments in literature, however, tried to address this criticism by exploring the mechanism to bring the degree of returns to scale required for indeterminacy down to an empirically plausible range.[1]

More recently, Lubik (2016) estimated the aggregate returns to scale in U.S. production via Bayesian methods and found strong evidence for the case of constant returns. Therefore, alternative avenues than production externalities are necessary for the theory of sunspot-driven business cycles. A couple of studies take a different approach by looking at a source of indeterminacy different from externalities. Examples include, among others, the model with non-separable utility by Bennett and Farmer (2000); the model with multiple sectors by Benhabib, Meng and Nishimura (2000); the efficiency wage model with unemployment insurance by Nikajima (2006); the model with endogenous borrowing constraints by Benhabib and Wang (2013) and Liu and Wang (2014). All these works imply that returns to scale are entirely irrelevant and unnecessary for generating indeterminacy.

Many modern labor markets are characterized by involuntary unemployment. Involuntarily unemployed people, by definition, would like to offer his labor at less than the market wage rate but is unable to find a buyer. The presence of such unemployment raises the question of why firms do not cut wages to clear labor markets. The early real business cycle models of the kind proposed by Kydland and Prescott (1982) have been criticized for failing to account for the existence of unemployed workers. The shortcoming has lead researchers to consider alternative models that incorporate labor market frictions, such as efficiency wages and costly job search. The efficiency wage theories have long been received much concern due to their potential to explain the presence of involuntary unemployment and the behavior of wages over the business cycle

[1] For example, Benhabib and Farmer (1996), Wen (1998), Weder (2000), Guo and Harrison (2001).

(see, e.g., Shapiro and Stiglitz, 1984; Yellen, 1984; Katz, 1986; Danthine and Donaldson, 1990). The efficiency-wage model is similar to the standard one-sector neoclassical growth model except that firms do not perfectly measure the quantity or quality of workers' effort. The wage rate is, therefore, be set more than the market average to prevent workers from shirking. When all firms behave this way, their demand for labor decreases, an involuntary unemployment is reached where there are unemployed workers willing to work at prevailing wages. Alexopoulous (2004) modified the standard efficiency-wage model by assuming punishing detected shirkers monetary instead of firing them and allowing for different unemployment insurance arrangements between agents. Nakajima (2006) showed that such version of the model could generate indeterminacy even without externalities which made an important theoretical contribution. Although indeterminacy seems to arise in the efficiency-wage model for realistic parameterization, it is vital that the implications be supported by empirical evidence. Therefore, this paper goes a further step to conduct an empirical evaluation of the model.

Specifically, the theoretical model is estimated in both determinacy and indeterminacy regions by full information Bayesian methods using quarterly U.S. data covering the period from 1964:I to 2007:IV. The results show that the estimated model parameters are consistent with the existing evidence and the shirking model performs fairly well on various unconditional second moments of the data. The paper also applies the methodology developed by Bianchi and Nicolò (2017) to compare the model fit under determinacy and indeterminacy. The exercise shows that version of the model with determinacy empirically outperforms the indeterminate counterpart.

The rest of this paper proceeds as follows. Section 4.2 outlines the modified model. Section 4.3 presents the estimation and discusses the results. Section 4.4 concludes.

4.2 The Model

The artificial economy is an efficiency-wage model in which firms imperfectly observe workers' effort levels so that they set the wage rate in a way to prevent workers from shirking on the job. Based on the wage rate set, firms choose the number of employed workers according to their labor demand, which generates unemployment in equilibrium. Multiple equilibria in this model rely on the risk sharing between employed and unemployed workers—the less unemployment insurance is, the more likely equilibrium is to be indeterminate. I modify the original model by incorporating a number of fundamental shocks. Time proceeds in discrete steps. The economy features three agents: a representative household, which consists of a unit-measure continuum of individuals, a representative perfectly competitive firm, and the government.

4.2.1 The Household

The household owns all the capital good K_t and rents it to the firm at the rate r_t. After paying lump-sum taxes, Tax_t, and purchasing new investment goods I_t from renting capital income, the household distributes the left proceeds, C_t^h, equally to each individual. C_t^h, which is also the minimum level of income guaranteed for each individual is given by

$$C_t^h \equiv r_t K_t - I_t - Tax_t. \tag{4.1}$$

Capital is accumulated according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{4.2}$$

where δ is the depreciation rate.

At period t, the number of employed workers is N_t while $1 - N_t$ are unemployed. There are four types of household members at each point in time: (a) employed workers who do not shirk; (b) employed workers who shirk but not get caught; (c) employed workers who shirk and are caught and (d) unemployed individuals.

$$\text{Household members}\begin{cases}\text{Employed}\begin{cases}\text{Not shirk}\\ \text{Shirk}\begin{cases}\text{Not caught}\\ \text{Caught}\end{cases}\end{cases}\\ \text{Unemployed}\end{cases}$$

The household sets up a fully funded unemployment insurance scheme for the family members sharing the risk of unemployment. F_t is assumed to be the intra-household transfer from working household members to the unemployed. The total amount of the unemployment insurance, N_tF_t, is distributed equally among unemployed members.

4.2.1.1 Individuals

Firms offer a contract to the employed members by specifying a fixed number of working hours, h, an effort level e_t and a wage rate W_t. The detected shirker receives only the wage rate sW_t, where $s \in (0, 1)$ is an exogenous parameter.

Following Alexopoulos (2004), I define the income insurance F_t as

$$F_t = \sigma(1 - N_t)hW_t, \tag{4.3}$$

where $\sigma \in (0, 1]$ measures the degree of income insurance (risk sharing).

An employed individual who does not shirk or who shirks but is not detected has the same level of consumption C_t which is given by

$$C_t = C_t^h + hW_t - F_t. \tag{4.4}$$

The detected shirker's consumption C_t^s is

$$C_t^s = C_t^h + shW_t - F_t, \tag{4.5}$$

and the consumption of an unemployed individual C_t^u is

$$C_t^u = C_t^h + \frac{N_t F_t}{1 - N_t}. \tag{4.6}$$

With full income insurance ($\sigma = 1$), unemployed individuals receive the same income as employed members who are not detected shirking, $C_t^u = C_t$. In such a case, the shirking model is observationally equivalent to the standard growth model with utility linear in leisure developed by Hansen (1985) and Rogerson (1988).[2] When income insurance is partial ($\sigma < 1$), $C_t^u < C_t$. We will see later that limiting the amount of income insurance available to individuals in the shirking model might generate indeterminacy even without externalities. The

[2] Unemployment is voluntary in the full income insurance scheme because an unemployed individual's utility is higher than that of an employed person's. To avoid individuals turning down the job offers, it is assumed that the job offers can be observed and individuals who receive but refuse jobs would be worse off since they are ineligible for any intra-family transfers. Thus, the individual rationality constraint is satisfied and non-binding.

consumption of each type of individuals thus is given by

$$C_t^s = C_t^h + [s - \sigma(1 - N_t)]\, hW_t, \tag{4.7}$$

$$C_t = C_t^h + [1 - \sigma(1 - N_t)]\, hW_t, \tag{4.8}$$

$$C_t^u = C_t^h + \sigma N_t hW_t. \tag{4.9}$$

The instantaneous utility of each type of workers is given by

$$\begin{cases} U(C_t, e_t) = \ln(C_t) + \theta \ln(T - \xi - he_t) & \text{employed who does not shirk,} \\ U(C_t, 0) = \ln(C_t) + \theta \ln T & \text{a shirker who is not caught,} \\ U(C_t^s, 0) = \ln(C_t^s) + \theta \ln T & \text{a shirker who is caught,} \\ U(C_t^u, 0) = \ln(C_t^u) + \theta \ln T & \text{unemployed,} \end{cases}$$

where $\theta > 0$, T is the time endowment and ξ is the fixed cost if the worker provides any positive level of effort. The firm cannot perfectly observe the worker's behavior so that a shirker is only caught with probability $d \in (0, 1)$. Thus a shirker's expected utility is $(1 - d)U(C_t, 0) + dU(C_t^s, 0)$.

4.2.1.2 The household's problem

The presentative household chooses the sequences of minimum level of income C_t^h and investment I_t to maximize his lifetime utility[3]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ N_t \left[\ln(C_t - \Delta_t) + \theta \ln(T - \xi - he_t) \right] + (1 - N_t) \left[\ln(C_t^u - \Delta_t) + \theta \ln T \right] \right\},$$

subject to equations (4.1) and (4.2). The functional form of the period utility ensures that the economy is consistent with balanced growth. We introduce

[3]Since it is not profitable for firms to allow worker to shirk, they sign a contract to make the number of employed individuals who shirk equal to zero.

preference shock Δ_t into agent's utility function and the term Δ_t represents shocks to the agent's utility of consumption that generate urges to consume, as in Baxter and King (1991) and Weder (2006). This element comes in two parts. One part grows with economy's consumption trend and the other one is a transitory shock that follows the autoregressive process

$$\ln d_t = \rho_d \ln d_{t-1} + \varepsilon_{d,t} \qquad 0 < \rho_d < 1$$

with $\varepsilon_{d,t} \sim N(0, \sigma_d^2)$.

4.2.2 The Firm

Firms are assumed to operate in perfectly competitive product and factor markets. The production function is buffeted by a nonstationary productivity shock. Formally,

$$Y_t = A_t K_t^{\alpha} \left(he_t X_t N_t\right)^{1-\alpha}$$

$$A_t = \bar{K}_t^{\alpha\eta}(h\bar{e}_t \bar{X}_t \bar{N}_t)^{(1-\alpha)\eta}$$

where $0 < \alpha < 1$. A_t represents the aggregate externalities and bars over variables denote average economy-wide levels. Deviations from constant returns to scale are measured by η. X_t measures the labor-augmenting technology. Its growth rate $g_t^x \equiv X_t/X_{t-1}$ evolves as a first-order autoregressive process

$$\ln g_t^x = (1-\rho_x)\ln g^x + \rho_x \ln g_{t-1}^x + \varepsilon_{x,t} \qquad 0 < \rho_x < 1$$

with $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$. All non-stationary variables fluctuate around the same stochastic growth trend X_t^{γ}, where $\gamma = [(1-\alpha)(1+\eta)]/[1-\alpha(1+\eta)]$. The firm maximizes its profit subject to the incentive compatibility (IC) constraint:

$$\ln(C_t-\Delta_t)+\theta\ln(T-\xi-he_t) \geq (1-d)\ln(C_t-\Delta_t)+d\ln(C_t^s-\Delta_t)+\theta\ln T \quad (4.10)$$

This inequality constraint ensures that workers are indifferent between shirking and providing the effort required in the period's contract. In equilibrium, the IC constraint holds with equality so that the effort e_t can be solved as a function of $\frac{C_t-\Delta_t}{C_t^s-\Delta_t}$:

$$e_t = \frac{T-\zeta}{h} - \frac{T}{h}\left(\frac{C_t-\Delta_t}{C_t^s-\Delta_t}\right)^{-\frac{d}{\theta}}. \quad (4.11)$$

Since a firm does not believe it could influence the value of F_t, C_t^h and Δ_t, it takes them as given. Therefore, the firm views $\frac{C_t-\Delta_t}{C_t^s-\Delta_t}$ as a function of W_t:

$$\frac{C_t-\Delta_t}{C_t^s-\Delta_t} = \frac{hW_t + C_t^h - F_t - \Delta_t}{shW_t + C_t^h - F_t - \Delta_t}.$$

The first-order conditions of the firm for profit maximization are

$$\frac{e'(W_t)W_t}{e_t} = 1, \quad (4.12)$$

$$(1-\alpha)\frac{Y_t}{N_t} = hW_t, \quad (4.13)$$

and

$$\alpha\frac{Y_t}{K_t} = r_t. \quad (4.14)$$

Equation (4.12) is the classic Solow condition that implies firms chooses the wage rate W_t to minimize the cost per unit effort. It also implies that the firm sets the real wage rate to make the consumption ratio $\frac{C_t-\Delta_t}{C_t^s-\Delta_t}$ constant over time:

$$\frac{C_t-\Delta_t}{C_t^s-\Delta_t} = \chi \qquad \forall\ t. \quad (4.15)$$

The key feature of the efficiency-wage model is that the "no-shirking condition" plays the same role as the labor supply function. From Equation (4.15), we can get the no-shirking condition

$$hW_t = \frac{(\chi - 1)}{1 - s\chi + \sigma(\chi - 1)(1 - N_t)}(C_t^h - \Delta_t). \tag{4.16}$$

Combining Equations (4.7), (4.9) and (4.16), the ratio of the consumption of the (no-shirking) employed to that of the unemployed is given by

$$\frac{C_t - \Delta_t}{C_t^u - \Delta_t} = \mu \equiv \frac{(1 - s)\chi}{1 - s\chi + \sigma(\chi - 1)}.$$

Note that with full income insurance ($\sigma = 1$), unemployed individuals consume the same level as employed individuals, thus $\mu = 1$. With partial income insurance ($\sigma < 1$), unemployed individuals consume less than employed counterparts so that $\mu > 1$.

4.2.3 Government

The government purchases G_t units of the final output. This spending is financed by the lump-sum taxes, Tax_t. Thus the government's period budget constraint is

$$G_t = Tax_t.$$

The detrended government spending $g_t \equiv G_t / X_t^\gamma$ follows the process

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \qquad 0 < \rho_g < 1$$

with the shock's variance σ_g^2.

4.2.4 Equilibrium

Letting Λ_t be the multiplier associated with the household's utility maximization, a competitive equilibrium in this model is characterized by the following necessary conditions:

$$\frac{N_t}{C_t - \Delta_t} + \frac{1 - N_t}{C_t^u - \Delta_t} = \Lambda_t \tag{4.17}$$

$$hW_t = \frac{\chi - 1}{(1 - s)\chi} \left[\mu - (\mu - 1)N_t\right] \frac{1}{\Lambda_t} \tag{4.18}$$

$$Y_t = C_t^{all} + I_t + G_t \tag{4.19}$$

$$C_t^{all} = N_t C_t + (1 - N_t) C_t^u \tag{4.20}$$

$$(1 - \alpha)\frac{Y_t}{N_t} = hW_t \tag{4.21}$$

$$\alpha \frac{Y_t}{K_t} = r_t \tag{4.22}$$

$$Y_t = \left[K_t^\alpha \left(he_t X_t N_t\right)^{1-\alpha}\right]^{1+\eta} \tag{4.23}$$

$$\Lambda_t = \beta \Lambda_{t+1} \left(r_{t+1} + 1 - \delta\right) \tag{4.24}$$

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{4.25}$$

where $C_t^u = \left(C_t + \mu\Delta_t - \Delta_t\right)/\mu$. C_t^{all} is defined as the household's total consumption. Equation (4.18) is the Frisch no shirking condition which is obtained from (4.7), (4.16) and (4.17). The log-linearized system is presented in Appendix 4.A.1.

4.2.5 Self-fulfilling dynamics

The detrended and linearized economy is solved numerically (using standard parameters as listed in Table 4.1). Figure 4.1 maps the local dynamics' zones in the $\mu - \eta-$space. If the unemployment insurance is perfect, i.e. the ratio of consumption of employed to that of unemployed μ equals one and there are no externalities, i.e. $\eta = 0$, the economy's dynamics are unique. However, with partial unemployment insurance, the higher inequality in consumption between the employed and the unemployed (large μ) or larger externalities (high η) makes indeterminacy more likely to occur.

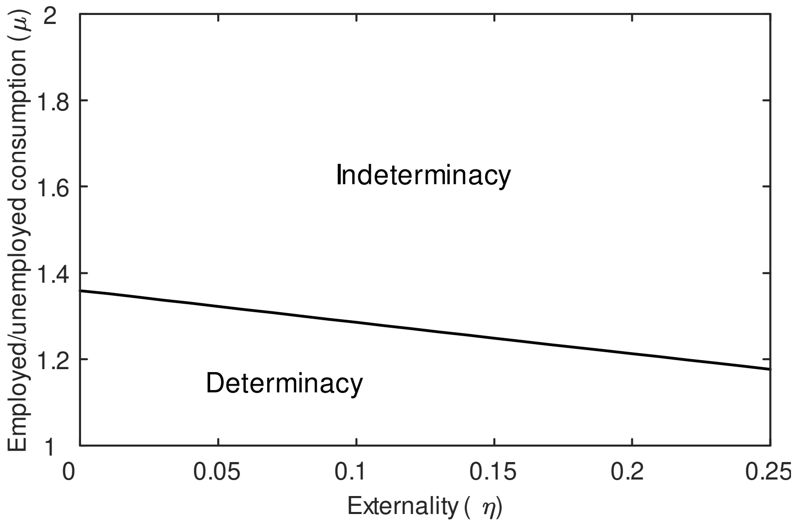

Figure 4.1: Parameter space for dynamics.

4.3 Empirical analysis

The previous section has theoretically shown that indeterminacy is likely to arise in an efficiency-wage model with partial unemployment insurance without externalities. Next, I employ Bianchi and Nicolò (2017)'s method to estimate both determinate and indeterminate versions of the shirking model to evaluate their performance.

4.3.1 Methodology

Bianchi and Nicolò (2017) propose a novel approach to estimate linear rational expectation (LRE) models that easily accommodates both determinacy and indeterminacy cases. It does not require to know the analytical solution of the boundaries of the determinacy region so that one can estimate the economy in standard software packages, such as Dynare. The characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2003) and Farmer et al. (2015).

This paper closely follows Bianchi and Nicolò (2017) and briefly set out the methodology that it applies. The parameters of the loglinearized benchmark model are contained in the vector

$$\Theta = \left[\mu,\, \bar{N},\, \alpha,\, g^x,\, \delta,\, \beta,\, G/Y,\, \gamma,\, \eta,\, \rho_x,\, \rho_d,\, \rho_g,\, \sigma_x,\, \sigma_d,\, \sigma_g\right].$$

The LRE model can be rewritten in the canonical form

$$\Gamma_0(\Theta)s_t = \Gamma_1(\Theta)s_{t-1} + \Psi(\Theta)\varepsilon_t + \Pi(\Theta)\eta_t, \tag{4.26}$$

where

$$s_t = \left[\hat{y}_t,\, \hat{c}_t,\, \hat{\imath}_t,\, \hat{N}_t,\, \hat{\lambda}_t,\, \hat{c}_t^{all},\, \hat{w}_t,\, \hat{r}_t,\, \hat{k}_t,\, E_t(\hat{\lambda}_{t+1}),\, E_t\left(\hat{r}_{t+1}\right),\, \hat{g}_t^x,\, \hat{d}_t,\, \hat{g}_t\right]$$

is a vector of endogenous variables, $\varepsilon_t = [\varepsilon_t^x,\, \varepsilon_t^d,\, \varepsilon_t^g]'$ is a vector of exogenous shocks, and $\eta_t = [\eta_t^r,\, \eta_t^\lambda]'$ collects the one-step ahead forecast errors for the expectational variables of the system with $\eta_t^r = \hat{r}_t - E_{t-1}\hat{r}_t$ and $\eta_t^\lambda = \hat{\lambda}_t - E_{t-1}\hat{\lambda}_t$. $\Gamma_0(\Theta), \Gamma_1(\Theta), \Psi(\Theta)$ and $\Pi(\Theta)$ are appropriately defined coefficient matrices. Since the model can generate at most one degree of indeterminacy, Bianchi and

Nicolò suggest to append the original linear rational expectations model (4.26) with the autoregressive process

$$\omega_t = \varphi^* \omega_{t-1} + \upsilon_t - \eta_{f,t} \tag{4.27}$$

where υ_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast errors vector η_t. The forecast error is attached to interest rate so that $\eta_{f,t} = \eta_t^r$. The variable φ^* belongs to the interval (-1,1) when the model is determinate or is outside the unit circle under indeterminacy. Under determinacy the Blanchard-Kahn condition is satisfied and the absolute value of φ^* is inside the unit circle since the number of explosive roots of the original LRE model in (4.26) already equals the number of expectational variables in the model. Then the autoregressive process ω_t does not affect the solution for the endogenous variables s_t. On the other hand, under indeterminacy the Blanchard-Kahn condition is not satisfied. The system is characterized by one degree of indeterminacy and it is necessary to introduce another explosive root to fulfill the Blanchard-Kahn condition – the absolute value of φ^* falls outside the unit circle. Denoting the newly-defined vector of endogenous variables $\hat{s}_t \equiv (s_t, \omega_t)'$ and the vector of exogenous shocks $\hat{\varepsilon}_t \equiv (\varepsilon_t, \upsilon_t)'$, then the system (4.26) and (4.27) can be condensed into

$$\hat{\Gamma}_0 \hat{s}_t = \hat{\Gamma}_1 \hat{s}_{t-1} + \hat{\Psi} \hat{\varepsilon}_t + \hat{\Pi} \eta_t,$$

where

$$\hat{\Gamma}_0 \equiv \begin{bmatrix} \Gamma_0(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \; \hat{\Gamma}_1 \equiv \begin{bmatrix} \Gamma_1(\Theta) & \mathbf{0} \\ \mathbf{0} & \varphi^* \end{bmatrix}$$

and

$$\hat{\Psi} \equiv \begin{bmatrix} \Psi(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \ \hat{\Pi} \equiv \begin{bmatrix} \Pi_n(\Theta) & \Pi_f(\Theta) \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}.$$

The matrix $\Pi(\Theta)$ in (26) is partitioned as $\Pi(\Theta) = [\Pi_n(\Theta) \quad \Pi_f(\Theta)]$ without loss of generality, where the matrices $\Pi_n(\Theta)$ and $\Pi_f(\Theta)$ are respectively of dimension number of endogenous variables $\times$ (number of forecast errors $-$ degree of indeterminacy) and number of endogenous variables $\times$ degree of indeterminacy, therefore both 14×1 in the model.

4.3.2 Estimation

4.3.2.1 Data

The model is estimated via Bayesian method using the quarterly real per capita growth rate of GDP, consumption, the log difference of employment rate from its sample mean and real growth rate of wage from 1964:I to 2007:IV as observables. Consistent with the model, I measure population by the labor force. Since the artificial economy excludes financial friction in its setup, the series is truncated right before the Great Recession to avoid the possible effects arising from financial markets. The data series are described in Appendix 4.A.2. The corresponding measurement equation is

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t^{all} - \ln C_{t-1}^{all} \\ \ln N_t - \ln \bar{N} \\ \ln W_t - \ln W_{t-1} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \gamma \hat{g}_t^x \\ \hat{c}_t^{all} - \hat{c}_{t-1}^{all} + \gamma \hat{g}_t^x \\ \hat{N}_t \\ \hat{w}_t - \hat{w}_{t-1} + \gamma \hat{g}_t^x \end{bmatrix} + \begin{bmatrix} \gamma \ln g^x \\ \gamma \ln g^x \\ 0 \\ \gamma \ln g^x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_{w,t}^{me} \end{bmatrix}.$$

I assume real wage growth is measured with error $\varepsilon^{me}_{w,t}$ which is i.i.d. innovation with mean zero and standard deviation σ^{me}_{w}. Allowing for a measurement error to wage is a way to circumvent stochastic singularity and this measurement error is restricted to absorb not more than ten percent of the variance of the corresponding observable.

4.3.2.2 Calibration and priors

Table 4.1: Calibration

Parameter	Value	Description
β	0.99	Discount factor
α	0.3	Capital share
η	0	Externality
δ	0.025	Steady-state depreciation rate
$\bar{N}$	0.935	Average employment rate
G/Y	0.2	Steady-state government expenditure share of GDP
$(g^x)^\gamma$	1.0038	Steady-state gross per capita GDP growth rate

Prior to the estimation, a subset of the model parameters are calibrated as listed in Table 4.1. I set the discount factor β to 0.99, capital share α to 0.3 and capital depreciation rate δ to 0.025. Since the artificial economy can generate indeterminacy without externality, I set the externality parameter η equal to zero so that γ equal to one. The average government spending share in GDP, G/Y, is calibrated at 20 percent, a number that is taken from national accounts. The quarterly growth rates of per capita output $(g^x)^\gamma$ is set equal to its sample average of 1.0038. The remaining parameters are estimated and the specification of the prior distribution is summarized in Table 4.2.

Table 4.2: Prior distribution of parameters

Name	Range	Density	Prior Mean	St. Dev
μ	[1.28,1.45]	Normal	1.35	0.1
ρ_x	[0,1)	Beta	0.5	0.2
ρ_d	[0,1)	Beta	0.5	0.2
ρ_g	[0,1)	Beta	0.5	0.2
σ_η	R^+	IGam	0.1	∞
σ_x	R^+	IGam	0.1	∞
σ_d	R^+	IGam	0.1	∞
σ_g	R^+	IGam	0.1	∞
σ_w^{me}	[0,0.15]	Uniform	0.075	0.043
$\rho(x, \eta^r)$	[-1,1]	Uniform	0	0.577
$\rho(d, \eta^r)$	[-1,1]	Uniform	0	0.577
$\rho(g, \eta^r)$	[-1,1]	Uniform	0	0.577

Notes: Inf implies two degrees of freedom for the inverse gamma distribution. Standard deviations are in percent terms.

The estimated parameters include the ratio of consumption of employed to that of unemployed, μ, the parameters that describe the stochastic processes, i.e., ρ_x, ρ_d, ρ_g, σ_η ,σ_x, σ_d, σ_g and the measurement error standard deviation σ_w^{me}. A normal distribution is adopted for μ with a mean of 1.35 and a standard deviation of 0.1. This value implies that the consumption of an unemployed individual is approximately 74 percent of that of an employed individual which is based on Eusepi and Preston (2015) who set this value to 1.30 (the consumption of the unemployed is about 77 percent of that of the employed). The priors for shocks are standard. The paper follows the approach proposed by Farmer et al. (2015) to introduce the sunspot shock as a forecast error, i.e., η_t^r, with variance σ_η^2. Intuitively, since the rate is forward-looking, this expectation error should be correlated with fundamental shocks. Yet, it is also a sunspot shock, as it can cause movements in economic activity without any shifts to fundamentals. Assuming a uniform distribution in the range [-1,1], I thus estimate the correlations between η_t^r and the fundamental shocks.

Figure 4.1 shows that the artificial economy has one determinacy region and one indeterminacy region. Bianchi and Nicolò's procedure can be implemented without knowing the boundaries between the dynamic regions. The choice of the prior leads to a prior probability of determinacy of 0.49, which is quite even and suggests no prior bias toward either determinacy or indeterminacy. The determinacy model is estimated by fixing the parameter φ^* to a value smaller than one (e.g. 0.5) in a way that the model is solved only under determinacy while the indeterminacy model is estimated by fixing φ^* greater than one (e.g. 1.5) in a way that the model is solved only under indeterminacy. All parameters that pertain to the solution under indeterminacy are restricted to zero when I estimate the determinacy model.

The paper follows Christiano et al., (2011) by choosing endogenous priors in Dynare to prevent over-predicting the model variances. The random walk Metropolis-Hasting algorithm is used to obtain 500,000 draws from the posterior distribution for each of the two chains. I discard the first 250,000 draws and adjust the scale in the jumping distribution to achieve an acceptance ratio between 25 and 30 percent.

4.3.2.3 Estimation findings

Table 4.3 presents the posterior means of the estimated parameters, along with their 90 percent posterior probability intervals for both determinacy and indeterminacy model. The estimated μ in indeterminacy model is 1.431, implying that the consumption of an unemployed is about 70 percent of that of an employed. Preference and government spending shocks exhibit a high degree of persistence in both models. The autocorrelation of the non-stationary technology shock is low, but it is not inconsistent with the moderate values commonly found in the literature.

Table 4.3: Posterior distribution of parameters

	Determinacy		Indeterminacy	
Name	Mean	90% Interval	Mean	90% Interval
μ	1.280	[1.280,1.281]	1.434	[1.425,1.444]
ρ_x	0.065	[0.055,0.074]	0.006	[0.001,0.009]
ρ_d	0.950	[0.940,0.960]	0.975	[0.970,0.980]
ρ_g	0.991	[0.988,0.995]	0.989	[0.986,0.992]
σ_η	—	—	0.620	[0.590,0.650]
σ_x	0.650	[0.620,0.680]	0.520	[0.500,0.540]
σ_d	0.370	[0.350,0.390]	0.400	[0.380,0.410]
σ_g	1.170	[1.090,1.260]	0.680	[0.620,0.740]
σ_w^{me}	0.150	[0.150,0.150]	0.150	[0.150,0.150]
$\rho(x,\eta^r)$	—	—	0.820	[0.797,0.844]
$\rho(d,\eta^r)$	—	—	0.222	[0.178,0.264]
$\rho(g,\eta^r)$	—	—	0.471	[0.376,0.558]

A classic challenge faced by researchers in macroeconomics is to explain ob-

Table 4.4: Business cycle dynamics

U.S. economy	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.86	1.00	0.20
$\ln(C_t^{all}/C_{t-1}^{all})$	0.53	0.57	0.22
$\ln(N_t/\bar{N})$	3.61	0.05	0.98
$\ln(W_t/W_{t-1})$	0.61	0.05	0.07
Shirking, partial insurance (determinacy)	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	1.33	1.00	0.08
$\ln(C_t^{all}/C_{t-1}^{all})$	0.65	0.77	0.05
$\ln(N_t/\bar{N})$	4.09	0.24	0.93
$\ln(W_t/W_{t-1})$	0.53	-0.25	-0.02
Shirking, partial insurance (indeterminacy)	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.99	1.00	0.67
$\ln(C_t^{all}/C_{t-1}^{all})$	0.57	0.45	0.08
$\ln(N_t/\bar{N})$	6.76	0.25	0.98
$\ln(W_t/W_{t-1})$	0.53	-0.23	0.47
Shirking, full insurance (indivisible labor)	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	1.04	1.00	0.06
$\ln(C_t^{all}/C_{t-1}^{all})$	0.64	0.75	0.06
$\ln(N_t/\bar{N})$	3.81	0.17	0.97
$\ln(W_t/W_{t-1})$	0.52	0.42	0.07

served variations in labor market facts, such as relatively little cyclical variation and a weak cyclical pattern of real wage. Next, I formally evaluate the shirking model's performance in accounting for the important labor market phenomena. Table 4.4 shows the second moments of the U.S. data and the estimated artificial economy. With full income insurance, the shirking model is equivalent to the standard indivisible labor model. Table 4.4 reveals that all shirking models (partial and full unemployment insurance) are able to capture the high volatility of employment and low volatility of real wages. Moreover, all models are fairly well in matching the autocorrelations and the variables' cross-correlations with output growth. In the data the real wage is acyclical, and the two partial insurance shirking models predict that the real wage growth is mildly countercyclical while full income insurance model generates strongly procyclical real wage behavior.

The shocks are estimated in a system, and it is thus fair to ask if they are meaningfully labeled. In particular, I compare the estimated model's total factor productivity series with Fernald's (2014) total factor productivity series for the United States.[4] Since the model economy is absent of variable capacity utilization, Fernald's non-factor utilization adjusted series is served as the benchmark. Even though we did not use Fernald's TFP in the estimation of the model, the theoretical shocks turn to be highly correlated with their empirical counterparts as shown in Figure 4.2. Both series are very similar as evinced by a contemporaneous correlation 0.91.

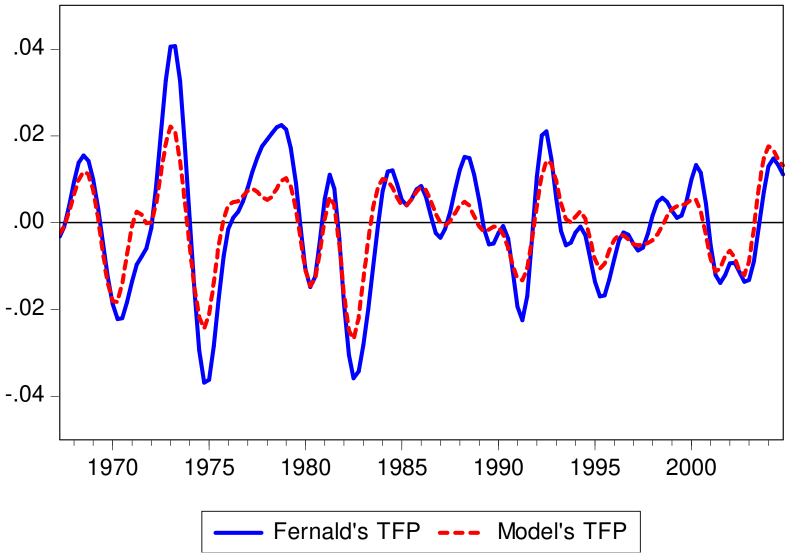

Figure 4.2: Fernald's vs Model's total factor productivity (band-pass filtered).

4.3.3 Determinacy versus Indeterminacy

The previous section has shown that both the determinate and indeterminate versions of the shirking model can explain the stylized facts of the labor market. A natural question arises: does data favor a model with determinacy or indeterminacy? To assess the quality of the model's fit to the data over the two

[4]Growth of total factor productivity in the model is given by $(1-\alpha)((1+\eta)\hat{g}_t^x + \ln(g^x)^\gamma)$.

regions of the parameter space, we present marginal data densities and posterior model probabilities in Table 4.5. The posterior concentrates all its mass in the determinacy region. Therefore, our estimation results suggest that U.S. data strongly favors the determinacy model over versions of the economy in which the equilibrium is indeterminate.

Table 4.5: Determinacy versus Indeterminacy

	Prior probability		Log-data density		Posterior probability	
Externality	Det	Ind	Det	Ind	Det	Ind
$\eta = 0$	0.49	0.51	562.02	419.98	1	0
$\eta = 0.05$	0.47	0.53	593.23	530.76	1	0
$\eta = 0.1$	0.46	0.54	609.39	579.76	1	0
$\eta = 0.15$	0.47	0.53	625.51	602.11	1	0
$\eta = 0.2$	0.48	0.52	636.96	609.51	1	0

Notes: Det is short for Determinacy and Ind stands for Indeterminacy.

4.4 Conclusion

This paper empirically evaluates the theoretical model of Nakajima (2006). In particular, the paper performs a Bayesian estimation of an efficiency-wage model which can generate indeterminacy of rational expectations equilibria. Indeterminacy in this model is linked to the degree of risk sharing between employed and unemployed workers. The estimation is conducted in both determinate and indeterminate versions of the model, and the results show that the estimated model parameters are consistent with the existing evidence and the shirking model performs fairly well on various unconditional second moments of the data. When comparing the model fit of the data, the data favors a version of the artificial economy that is characterized by determinacy.

4.A Appendix

The Appendix sets out the complete linearized system and lists the data sources and definitions.

4.A.1 Model equations

In the model, all non-stationary variables fluctuate around the same stochastic growth trend X_t^γ. The detrended dynamic equilibrium equations are firstly derived and then log-linearly approximated around the deterministic steady state. Let $y_t = Y_t/X_t^\gamma$, $c_t = C_t/X_t^\gamma$, $i_t = I_t/X_t^\gamma$, $g_t = G_t/X_t^\gamma$, $w_t = W_t/X_t^\gamma$, $\lambda_t = \Lambda_t X_t^\gamma$, $d_t = \Delta_t/X_t^\gamma$ and $k_t = K_t/X_{t-1}^\gamma$. The log-linearized system is summarized by

$$-\frac{(\mu-1)\bar{N}}{\mu-(\mu-1)\bar{N}}\hat{N}_t - \hat{c}_t + \hat{d}_t = \hat{\lambda}_t$$

$$\hat{w}_t = \hat{c}_t - \hat{d}_t$$

$$\hat{y}_t = \left[1 - \frac{\alpha\left[(g^x)^\gamma - 1 + \delta\right]}{\frac{(g^x)^\gamma}{\beta} + \delta - 1} - \frac{G}{Y}\right]\hat{c}_t^{all} + \frac{\alpha\left[(g^x)^\gamma - 1 + \delta\right]}{\frac{(g^x)^\gamma}{\beta} + \delta - 1}\hat{\imath}_t + \frac{G}{Y}\hat{g}_t$$

$$\hat{c}_t^{all} = \hat{c}_t + \frac{\bar{N}(\mu-1)}{\bar{N}(\mu-1)+1}\hat{N}_t + \frac{(1-\bar{N})(\mu-1)}{\bar{N}(\mu-1)+1}\hat{d}_t$$

$$\hat{w}_t = \hat{y}_t - \hat{N}_t$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \gamma\hat{g}_t^x$$

$$\hat{y}_t = \alpha\eta\hat{k}_t + (1-\alpha)\eta\hat{N}_t - \alpha\eta\gamma\hat{g}_t^x$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \frac{(g^x)^\gamma + \beta(\delta-1)}{(g^x)^\gamma}\hat{r}_{t+1} - \gamma\hat{g}_{t+1}^x$$

$$\hat{k}_{t+1} = \frac{1-\delta}{(g^x)^\gamma}\hat{k}_t + \frac{(g^x)^\gamma + \delta - 1}{(g^x)^\gamma}\hat{\imath}_t + \frac{(\delta-1)\gamma}{(g^x)^\gamma}\hat{g}_t^x$$

In these equations, variables without time subscripts refer to steady state values while the hatted variables denote percent deviations from their corresponding steady-state, e.g., $\hat{y}_t \equiv \log(y_t/\bar{y})$.

4.A.2 Data description

This appendix is to describe the details of the source and construction of the data used in estimation. The sample period covers the first quarter of 1964 through the fourth quarter of 2007:

1. Real Gross Domestic Product. Billions of Chained 2009 Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Personal Consumption Expenditures, Durable Goods. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Residential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Gross Private Domestic Investment, Fixed Investment, Nonresidential.

Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

8. Hours of Wage and Salary Workers on Nonfarm Payrolls: Total. Seasonally Adjusted. Source: Bureau of Labor Statistics.

9. Nonfarm Business Sector: Compensation Per Hour. Index 2009=100, Seasonally Adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006103.

10. Civilian Labor Force. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNS11000000Q.

11. GDP deflator= (2)/(1).

12. Real Per Capita Output, $Y_t = (1)/(10)$.

13. Real Per Capita Consumption, $C_t = [(3) + (4)]/(10)/(11)$.

14. Real Per Capita Investment, $I_t = [(5) + (6) + (7)]/(10)/(11)$.

15. Real Wage, $W_t = (9)/(11)$.

16. The employment series N_t was constructed using (8) and (10). Employment in the shirking models is defined as

$$N_t = \frac{\text{total hours}}{\text{total labor force} \times \text{hours worked per person}}$$

to get the fraction of the people employed. The number of hours worked per person used was chosen so that the implied employment rate from this series matched the average employment rate from the employment series, 0.935.

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