

ESSAYS ON MACROECONOMICS AND
INTERNATIONAL FINANCE

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Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2013

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ABSTRACT
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This thesis addresses three topics in Macroeconomics and International Finance. Chapter 1 studies welfare implications of international financial integration in the presence of bank funding risks. Unregulated issuance of safe short-term liabilities by financial intermediaries leads to excessive reliance on this form of financing, which increases losses associated with financial crises. First, I show that integration increases the severity of potential financial crises in the countries that receive capital inflows. As a result, integration may reduce welfare for these countries. Second, I show that if macroprudential regulation of the banking sector is chosen by each country in an uncoordinated way, the outcome can be Pareto inefficient so that there is a role for global coordination of such policies. This effect arises because the macroprudential regulation that limits the overissuance of safe liabilities changes the international interest rate. The regulation may have an additional benefit from manipulating the interest rate. Third, the desire to manipulate the interest rate when regulating the local banking sector creates incentives to use two regulatory tools: macroprudential regulation of the banking sector and capital controls.

Chapter 2, written jointly with Emi Nakamura and Jón Steinsson, quantifies the importance of long-run risks—persistent shocks to growth rates and uncertainty—in a panel of long-term aggregate consumption data for developed countries. We identify sizable and highly persistent world growth-rate shocks as well as less persistent country-specific growth rate shocks. The world growth-rate shocks capture the productivity speed-up and slow-down many countries experienced in the second half of the 20th century. We also identify large and persistent world shocks to uncertainty. Our world uncertainty process captures the large but uneven rise and fall of volatility that occurred over the course of the 20th century. We find that negative shocks to growth rates are correlated with shocks that increase uncertainty. Our estimates based on macroeconomic data alone line up well with earlier calibrations of the long-run risks model designed to match asset pricing data. We document how these dynamics,

combined with Epstein-Zin-Weil preferences, help explain a number of asset pricing puzzles.

Chapter 3, written jointly with Neil R. Mehrotra, investigates the relationship between sector-specific shocks, shifts in the Beveridge curve, and changes in the natural rate of unemployment. We document a significant correlation between shifts in the US Beveridge curve in postwar data and periods of elevated sectoral shocks relying on a factor analysis of sectoral employment to derive our sectoral shock index. We provide conditions under which sector-specific shocks in a multisector model augmented with labor market search generate outward shifts in the Beveridge curve and raise the natural rate of unemployment. Consistent with empirical evidence, our model also generates cyclical movements in aggregate matching function efficiency and mismatch across sectors. We calibrate a two-sector version of our model and demonstrate that a negative shock to construction employment calibrated to match employment shares can fully account for the outward shift in the Beveridge curve. We augment our standard multisector model with financial frictions to demonstrate that financial shocks or a binding zero lower bound can act like sectoral productivity shocks, generating a shift in the Beveridge curve that may be counteracted by expansionary monetary policy.

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Acknowledgments

I benefited greatly from many people during my doctoral studies. I owe a special debt of gratitude to my advisers Michael Woodford, Emi Nakamura, Jón Steinsson and Ricardo Reis. They were incredibly generous with their time at every stage of my dissertation writing. Their advice had a deep influence on this work and my understanding of economics.

I would like to also thank other Columbia faculty members who played an important role in my research: Patrick Bolton, Andreas Mueller, Serena Ng, Tano Santos, Stephanie Schmitt-Grohe, Martin Uribe, and Pierre Yared.

I had the privilege of studying at Columbia with a number of talented economists. I want to thank Ryan Chahrour, Nicolas Crouzet, Mariana Garcia, Kyle Jurardo, Hyunseung Oh, María José Prados and Luminita Stevens for helpful feedback on my research. I wish to separately acknowledge the role of my classmate and co-author of the third chapter of this dissertation Neil R. Mehrotra. I am grateful to him for our endless research debates.

Part of the work on this dissertation was done at the Federal Reserve Bank of New York. I want to thank this institution for its financial support and many members of its staff for their hospitality. Specifically, I want to thank Stefania Albanesi, Nina Boyarchenko, Dong Beom Choi, Vasco Curdia, Fernando Duarte, Marc Giannoni, Bianca De Paoli, Ayşegül Şahin, Argia Sbordone and Tanju Yorulmazer for their stimulating conversations.

I would like to thank the Institute of Humane Studies and the Program for Economic Research at Columbia University for financial support.

Finally, I am deeply grateful to my family. My parents had the biggest influence on my life. Any of my accomplishments would be impossible without their love, encouragement and belief in me. Marrying Uliana Loginova is the happiest event that happened to me. Her love and patience supported me all these years.

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To my Uliana

Chapter 1

Financial Integration and Financial Instability

1.1 Introduction

The large increase in cross-border banking during the past decade has renewed interest in the effects of fluctuations in capital flows. The creation of the Eurozone in 1999 is a case in point. Capital account liberalization as a prerequisite for admission played a role in the increase of the cross-border capital flows. The cross-border assets of the Eurozone banks in domestic currency increased from \$2 trillion in 1999 to \$10 trillion in 2008, and the liabilities went up in the same period from \$2 trillion to \$8 trillion.¹ However, these flows were unevenly distributed across Eurozone countries. Slow-growing central countries were investing in fast-growing peripheral countries. For example, the net foreign asset positions of Spain decreased from -40 percent as a share of its GDP in 1999 to -80 percent in 2008 and continued falling after that.² More than half of the decline was associated with the banking sector increase in net foreign liabilities. A large fraction of these Spanish liabilities were held by surplus countries such as Germany and France. At the same time, bank lending to the foreign non-banking sector in the Eurozone did not show the same level of integration.³

The ongoing global financial crisis, which has had especially serious consequences in the Eurozone periphery, raises the question of whether increased financial integration may have played a role in exacerbating the negative effects of the crisis. Pre-crisis conventional wisdom suggested that financial integration leads to more efficient risk sharing by smoothing country-specific shocks and to capital reallocation from capital-abundant countries to capital-poor countries. However, in the presence of market imperfections, the benefits of financial integration may be mitigated or offset by exacerbated financial frictions.⁴

¹Data comes from BIS locational banking statistics, Table 5A. The BIS uses US dollars as the numeraire in its international banking statistics.

²The data comes from the International Financial Statistics Database.

³ECB (2012) presents the data on establishment and activity of foreign branches and subsidiaries across the euro area countries. The report concludes that the integration in cross-border retail banking market is limited.

⁴The argument that removing a distortion in an environment with other distortions may lead to a reduction in welfare goes back to at least Lipsey and Lancaster (1956). Hart (1975) presents an example in which adding a new market that does not make the market structure complete makes every agent in the economy worse off. Newbery and Stiglitz (1984) show that opening countries to international trade in goods can make agents worse off in participating economies in the absence of insurance markets.

In this paper I ask four questions. First, does the integration of bank short-term funding markets exacerbate financial crises? Second, can this lead to a decrease in social welfare? Third, what regulations should be put in place to neutralize the negative consequences that financial frictions have when funding markets are integrated? Finally, is it necessary for countries to coordinate to achieve optimal regulation?

I present a model of bank funding risk based on [Stein \(2012\)](#). Banks finance themselves by issuing risky and safe debt to invest in long-term risky projects. Entrepreneurs have liquidity preferences from holding safe debt. This makes safe debt a cheaper and therefore a preferable means of financing for banks in comparison to risky debt. Because there is more uncertainty in the long run, it is easier for banks to issue *short-term* safe debt. For short-term debt to be safe, the banks must have enough resources to honor their short-term liabilities in an adverse state. When outside funding is not available in the adverse state, banks have to sell their assets at a fire-sale price.⁵ Therefore, banks cannot issue more safe debt than the value of their assets in the adverse state. This implies that banks face endogenous collateral constraints on the issuance of safe debt. The banks do not internalize the fact that their choices of safe debt affect the collateral constraints of the other banks. This externality leads the banks to issue too much safe debt.

I embed this model of funding risk into a setting with two regions: the center and the periphery. Each region has entrepreneurs and banks. The entrepreneurs in the periphery have more productive marginal investment opportunities compared to the entrepreneurs in the center. The difference in productivities of marginal investment opportunities in the two regions leads to different returns on safe debt before the integration. The peripheral entrepreneurs create more risky projects (relative to the center) for the peripheral banks to buy. The banks need more funding to buy these assets which leads to a bigger safe debt issuance. Because entrepreneurs' liquidity preferences from holding safe debt have diminishing returns to scale, the interest that banks have to pay the safe debt holders is higher in the periphery than in

⁵There is a large literature on fire sales. See [Shleifer and Vishny \(1992\)](#), [Shleifer and Vishny \(1997\)](#), [Gromb and Vayanos \(2002\)](#), [Geanakoplos \(2001\)](#), [Brunnermeier and Pedersen \(2009\)](#) on theoretical models of fire sales. [Shleifer and Vishny \(2011\)](#) provide a recent survey of the literature.

the center.

The integration of banks' short-term liabilities funding markets leads to capital flows from the center to the periphery. As a result, the return on safe debt decreases in the periphery which increases the banks incentives to issue safe debt. More safe debt will lead to a larger fire-sale discount in the adverse state of the world in the periphery. At the same time, the return on safe debt increases in the center which decreases the banks incentives to issue safe debt. This results in a smaller fire-sale discount in the adverse state in the center.

I show that the center always benefits from integration while the periphery loses under certain conditions. There are two effects of the integration: capital reallocation and a change in the severity of welfare losses due to overissuance of safe debt. Consider the periphery. The inflow of resources from the center is a benefit because the banks in the periphery can issue safe debt cheaply. However, more safe debt leads to a larger fire-sale discount in the adverse state of the world which exacerbates the negative externality associated with overissuance of safe debt, leading to bigger welfare losses. I show that these welfare losses always dominate welfare benefits from having access to cheaper safe debt financing if the difference in the marginal productivities of investment opportunities across the two regions is not too large. However, the integration always increases welfare in the center. The banks in the center reduce issuance of safe debt, which decreases losses in the adverse state. In addition, agents in the center are able to invest their savings at a higher return in the periphery. Thus, both effects increase welfare in the center.

In a closed economy setting, a regulator wants to impose a tax on safe debt issuance to make banks internalize the social costs of fire sales. In the two-region model with two local regulators, I show that the regulators will choose inefficient tax rates on safe debt issuance. An increase in the tax level decreases the issuance of safe securities that in turn decreases the world equilibrium return on the securities. Because the periphery is a net supplier of safe debt, a decrease in the rate of return decreases the amount that bankers have to repay to the agents in the center. Hence, the regulator in this region chooses the level of taxes that is higher than needed to correct the externality in the banking sector. On the other hand, the

regulator in the center wants to increase the international interest rate because the center is the net buyer of safe debt. The Nash equilibrium outcome of the regulators' game can be Pareto improved.

Finally, the desire to manipulate the international interest rate when regulating the local banking sector creates incentives to use two regulatory tools—prudential taxes on the banking sector and capital controls—instead of just using prudential taxation in the banking sector.

The remainder of this paper is organized as follows. Section 1.2 presents the model. Section 1.3 studies the equilibrium properties. Section 1.4 analyses the welfare consequences of integration. Section 1.5 investigates how incentives to correct the externality changes with integration. Section 1.6 concludes. Formal proofs are presented in the Appendix.

1.2 Model

In this section, I describe a two-country model and derive agents' optimality conditions. I will use superscripts C (the center) and P (the periphery) to distinguish between country-specific variables. Each country is identical except for their marginal productivity of investment opportunities $A^C < A^P$ (see the description below).

The economic environment is based on [Stein \(2012\)](#) but adds modifications to allow for a two-country analysis. First, I assume that the liquidity preferences from holding safe securities have diminishing returns to scale. This assumption results in positive net capital flow after bank funding market integration of two asymmetric countries. Second, I assume that banks do not directly invest in the production of risky projects; instead, they buy the projects from entrepreneurs. This assumption allows me to consider the effects of the lending market integration at the end of the paper.

I will describe the model in terms of the periphery and then present a two-country equilibrium. The center description is identical. The economy goes on for three dates, $t = 0, 1, 2$, and there is a single consumption good that serves as the numeraire. The economy is populated by three types of agents: entrepreneurs, bankers, and outside investors. Each type of agent has measure 1. An entrepreneur has an endowment in period 0, and he chooses his consumption

plan, portfolio allocation, and investments in risky projects that he immediately sells to the bankers in period 0.⁶ A banker buys risky projects from the entrepreneurs and finances his purchases by issuing risky and safe debt to the entrepreneurs in period 0. The banker can sell his safe debt to entrepreneurs in both countries. The risky projects pay off in period 2. The uncertainty structure of the risky projects is presented in Figure 1.1.

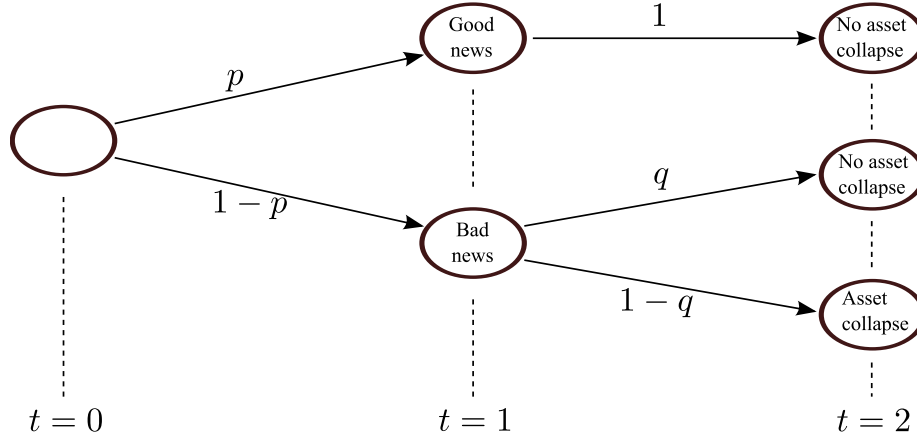


Figure 1.1: Aggregate uncertainty structure of risky projects. “No asset collapse” means that risky projects yield positive output in period 2 while “asset collapse” means that they are worthless.

In period $t = 1$ news about the future payoff of the projects arrives. With probability p there is good news, called the good state and denoted $s_1^P = G$, where subscript 1 denotes period 1, which ensures that the risky projects will yield a positive amount of consumption good in period $t = 2$. The corresponding state in period 2 is denoted by $s_2^P = G$. With probability $1 - p$ there is bad news, called the bad state and denoted $s_1^P = B$, informing that the risky projects will yield the same positive amount of consumption good in period 2 with probability q . I denote this state by $s_2^P = Bnc$, and 0 with probability $1 - q$. This state is denoted by $s_2^P = Bc$. The realizations of payoffs are common across different projects.

Bankers can sell their risky projects to outside investors in period 1. Outside investors have a fixed endowment of consumption goods in period 1, which they can invest in their late-arriving technology or the storage technology between period 1 and 2 or to buy bankers’

⁶I assume that the entrepreneurs can not insure that risky projects yield positive output at their completion. However, the bankers can guarantee that projects yield positive return in some future states if the bankers operate the risky projects.

assets. Only the outside investors have access to the storage technology.⁷

1.2.1 Entrepreneurs

An entrepreneur maximizes the following utility function

$$C_0^P + \beta \mathbb{E} C_2^P + v(D_d^P), \quad (1.1)$$

where C_0^P and C_2^P are consumption levels in period 0 and 2 respectively, which have to be non-negative.⁸ $v(D_d^P)$ represents the additional utility derived from holding safe claims on time 2 consumption, D_d^P is time 0 holdings of safe debt in units of period 2 consumption.^{9,10}

The budget constraint of the entrepreneur at $t = 0$ is

$$C_0^P + D_d^P P_D^P + \sum_{s_2^P} B^P(s_2^P) P_B^P(s_2^P) \leq Y + [P_0^P A^P F(I^P) - I^P], \quad (1.2)$$

where P_D is the price of safe debt (the return on the safe debt will be denoted $R_D = 1/P_D$). $\sum_{s_2^P} B^P(s_2^P) P_B^P(s_2^P)$ is the value of the entrepreneur's risky portfolio, where $B^P(s_2^P)$ is the repayment in state s_2^P and $P_B^P(s_2^P)$ is the price of a security that pays off one unit of consumption good in period 2 in state s_2^P , i.e., this is an Arrow-Debrew security price. I assume

⁷This assumption can be relaxed in two ways. I can allow all the agents to use the storage technology. In addition, I may allow the storage technology to operate between period 0 and 1. By allowing these additional opportunities, I would need to restrict my analysis to a specific range of parameters which guarantees that the bankers issue some amount of private safe debt.

⁸If I assume that discounting happens between period 0 and period 1 then the absence of consumption in period 1 is without loss of generality.

⁹Index d stands for demand. It will be useful later to differentiate it from supply of safe debt.

¹⁰The utility from holding safe securities captures the idea that safe securities provide transaction services. [Gorton and Pennacchi \(1990\)](#) and [Dang, Gorton, and Holmstrom \(2012\)](#) theoretically argue that private safe securities are useful for transactions because they eliminate the potential for adverse selection between transaction parties. Historically, demandable deposits were the main example of such securities. They pose a smaller threat to financial stability these days because demandable deposits are government-insured in most of the countries. Asset-backed securities (ABCP), repurchase agreements, short-term covered bonds are recent examples of private short-term safe securities, which, however, are not government insured. See, [Gorton and Metrick \(2012\)](#) and [Gorton \(2010\)](#) for a discussion of the pre-crisis developments in the unregulated banking in the U.S. [Sunderam \(2012\)](#) presents evidence that investors value safety of ABCP above their pecuniary return. [Krishnamurthy and Vissing-Jorgensen \(2012a\)](#) empirically show that investors value safety of US treasuries above and beyond their pecuniary returns. [Krishnamurthy and Vissing-Jorgensen \(2012b\)](#), using long-term US data, provide evidence that the supply of US treasuries is strongly negatively correlated with the amount of private safe securities outstanding, which is consistent with the idea that the two types of assets are substitutes.

that I^P units of investment immediately produce $A^P F(I^P)$ units of risky projects that are sold to the bankers. $P_0^P A^P F(I^P) - I^P$ is the profit from investing in the risky projects. I assume that $A^P F(I^P)$ is increasing, strictly concave and twice continuously differentiable in I^P .

The budget constraint of the entrepreneur in period 2 is given by

$$C_2^P(s_2^P) \leq D_d^P + B^P(s_2^P). \quad (1.3)$$

The entrepreneur takes prices as given and chooses consumption plan C_0^P , $C_2^P(s_2^P)$, amount of safe debt D_s^P , risky portfolio $\{B^P(s_2^P)\}$, and investment in the production of risky projects I^P . I assume that endowment Y is large enough so that non-negativity constraints on consumption do not bind.

The entrepreneur does not make any strategic decisions in period 1. The optimal interior choice of risky portfolio $\{B(s_2)\}$ leads to

$$\begin{aligned} P_B^P(G) &= \beta p, \\ P_B^P(Bnc) &= \beta(1-p)q, \\ P_B^P(Bc) &= \beta(1-p)(1-q). \end{aligned}$$

This immediately implies that the return on any risky security bought in positive amount is given by

$$R_B = \frac{1}{\beta}. \quad (1.4)$$

The optimal interior choice of the amount safe debt by the entrepreneur implies

$$R_D = \frac{1}{\beta + v'(D_d^P)}. \quad (1.5)$$

It is immediate that $R^P < R_B$ which represents the liquidity premium from holding safe debt.¹¹

¹¹ R_D and R_B do not have country index. The return on safe debt is not indexed because the market for safe

The optimal choice of investments in the production of risky projects implies

$$P_0^P A^P F'(I^P) = 1. \quad (1.6)$$

1.2.2 Outside Investors

An outside investor has endowment W of consumption good in period 1 that he can invest in his late technology and in the storage technology in period 1. The outside investor can issue safe securities backed by the storage technology output because the storage technology is safe. The outside investor can use these securities to buy bankers' risky projects. The price of the bankers' risky projects is Q^P if bad news arrives. I will call this price a fire-sale price. The late technology yields $g(x)$ units of consumption in case of success in period 2 and 0 in case of failure if x units of consumption are invested in period $t = 1$. Success and failure, which happens with probabilities δ and $1 - \delta$, respectively, are common across the outside investors. This is aggregate uncertainty.¹² I assume that $g(x)$ is increasing, strictly concave, twice continuously differentiable. I also assume that $\delta g'(W) > 1$. This assumption guarantees that the outside investor trades with the bankers only when bad news arrives. In addition, it guarantees that it is more profitable to invest in the late technology than in the storage technology. Imposing this assumption limits the number of uninteresting cases to consider.

If good news arrives the above assumption implies that the outside investor invests all his endowment in his late technology. If bad news arrives in period 1, the outside investor maximizes his revenue in period 2 from investing his endowment. This revenue equals his

debt is common for the two countries. The return on any risky security is not indexed because it is determined by the discount factor which is common across agents in both countries.

¹²This assumption will prevent the outside investors from issuing safe debt in period 1 backed by the proceeds of the late technology. This assumption is crucial to generate downward sloping demand curve for the bankers assets. Alternatively one can assume that $\delta = 1$, i.e., there is no uncertainty, but the outside investors cannot commit to keep their promises.

period 2 consumption. The problem in the bad state is

$$\begin{aligned} \max_{K_d^P, D_{OI}^P} \quad & qK_d^P + \delta g(W - D_{OI}^P) \\ \text{s.t.} \quad & Q^P K_d^P \leq D_{OI}^P, \end{aligned}$$

where D_{OI}^P is the amount of the endowment that the outside investor invests into the storage technology. The first term represents the expected payoff of the risky projects that the outside investor buys from the bankers. The second term represents the expected payoff of his investments in the late technology. The optimal choice implies

$$q = \delta Q^P g'(W - Q^P K_d^P) \tag{1.7}$$

Demand K_d^P decreases with Q^P because function $g(\cdot)$ is strictly concave. Intuitively, each additional unit of the bankers assets bought by the outside investor has a marginal benefit which equals q while the marginal cost, $\delta Q^P g'(W - Q^P K_d^P)$, increases with the price and the amount of the risky projects being purchased. Hence, the optimal level of K_d^P decreases with Q^P .

Notice also that the elasticity of the outside investor assets demand with respect to price Q^P is greater than 1. The marginal cost of buying the bankers' assets is more sensitive to price change than to a change in the quantity bought. To understand the intuition consider a 1 percent change in price Q^P . Assume that the outside investor decreases the risky projects demand by 1 percent. This does not change the marginal value of an additional unit of resources invested in the late technology, $g'(W - Q^P K_d^P)$. However, it increases the marginal cost of investing in the risky projects, $Q^P g'(W - Q^P K_d^P)$, which must be constant according to optimality condition (1.7). Thus, the outside investor must decrease his demand K_d by more than 1 percent.

Finally, the optimality condition (1.7) together with the assumption that $\delta g'(W) > 1$ implies

$$Q^P < q \tag{1.8}$$

Intuitively, whenever the outside investor chooses to buy bankers risky projects the fire-sale price is less than the risky project's fundamental value q .

1.2.3 Bankers

A banker buys risky projects from the entrepreneurs and raises funding by issuing debt to maximize his period-2 profits, which equals his consumption. The banker prefers to issue safe debt because it earns a liquidity premium. Because there is a positive probability for risky projects to become worthless in period 2, safe debt cannot be made long-term. However, the banker can issue some amount of safe debt by promising potential holders to repay them early (in period 1) with riskless claims on period-2 consumption if the bad state occurs.

The banker can issue “risky debt” in addition to the safe debt. Such debt promises repayment of a fixed amount in period 2, and gives the holders of the debt the following rights: (i) a claim to any resources in the hands of the banker in period 2, after safe debt has been repaid, up to the amount promised to be repaid in period 2 (i.e., the claims of the risky debt holders are junior to those of the holders of the safe debt); (ii) a right to prevent the banker from undertaking any transactions in period 1 that would reduce the value of the risky debt except the early repayment on safe debt.¹³

If the bad state occurs, the banker has to obtain riskless claims on period-2 consumption to repay his safe debt holders. I assume that the severe debt overhang problem prevents the banker from issuing securities that can be attractive to potential investors (Myers, 1977).¹⁴ Hence, the only way the banker can obtain riskless securities to fulfill the promise that he gave his safe debt holders is to sell some of his assets to the outside investors.

The banker's choice variables in period 0 are the quantity of risky projects to buy (Z^P), the quantity of safe debt to issue (measured by the face value D_s^P), and the quantity of the risky debt to issue (measured by the amount \bar{B}^P promised to repay in period 2). These three

¹³Restricting risky funding to risky debt may be optimal from the point of view of the entrepreneurs. This may prevent the bankers from borrowing more and wasting money in period 1 when the good state occurs. See, Hart (1993) and Hart and Moore (1995).

¹⁴New investors may not be willing to provide resources because the additional revenue that the banker gets from new funding will be paid off to senior investors.

quantities determine the state-contingent payout to the holders of the risky debt, in each of the three possible states in period 2. There is a well-defined asset-pricing kernel (taken as given by an individual banker because the financial markets are competitive) that determines the market value in period 0 of any type of risky debt that might be issued; this determines the market value of the risky debt as a function of the three quantities (Z^P, D_s^P, \bar{B}^P) chosen by the given banker.

To characterize the banker's problem, I first present his state-contingent profits. Denote the banker's state contingent profit as $\pi_B^P(s_2^P)$. In case of good news in period 1 there is no asset collapse in period 2, i.e., $s_2^P = G$, the banker collects risky projects payoff Z^P and pays out the holders of his safe debt D_s^P and risky debt holders \bar{B}^P . Thus, his profit is $\pi_B^P(G) = Z^P - D_s^P - \bar{B}^P$. If there is bad news and no asset collapse, state $s_2^P = Bnc$, the banker has to sell part of his risky projects (denoted K_s^P) to the outside investors in period 1 to make early repayment D_s^P to safe debt holders. The remainder of the risky projects $Z^P - K_s^P$ pays out at $t = 2$ and the banker repays risky debt holders. In this state, his profit is $\pi_B^P(Bnc) = QK_s^P - D_s^P + Z^P - K_s^P - \min\{\bar{B}^P, Z^P - K_s^P\}$. The last term takes into account the fact that the banker may end up having less output than the promised repayment on risky debt. Denote the last term as $B_s(Bnc)$. If there is bad news and assets collapse, state $s_2^P = \{Bc\}$, the banker has to sell K_s^P units of his risky projects in period 1, but then he gets nothing because his risky projects yield zero at $t = 2$. To summarize, the banker expected profits are

$$\begin{aligned} \mathbb{E}\pi_B^P &= p[Z^P - D_s^P - \bar{B}^P] \\ &\quad + (1-p)q[QK_s^P - D_s^P + Z^P - K_s^P - \min\{\bar{B}^P, Z^P - K_s^P\}] \\ &\quad + (1-p)(1-q)[Q^P K_s^P - D_s^P]. \end{aligned} \tag{1.9}$$

The value of the bankers' safe debt outstanding in period 0 is D_s^P/R_D , i.e., the face value of the safe debt is discounted with riskless discount factor $1/R_D$. The value of the risky debt in period 0 equals $V_B^P = P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}$. Hence, the banker

period-0 budget constraint is

$$P_0^P Z^P \leq \frac{D_s^P}{R_D} + P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}. \quad (1.10)$$

In addition to the budget constraint in period 0, the banker faces the resource constraint and the collateral constraint in period 1. The banker cannot sell more risky projects than he has on his balance sheet

$$K_s^P \leq Z^P. \quad (1.11)$$

For the safe debt to be safe, the value of the banker's assets in the bad state has to be more or equal to the value of safe debt

$$D_s^P \leq Q^P K_s^P \quad (1.12)$$

Let's now characterize the banker's optimality conditions. First, observe that constraint (1.12) is always binding. It is not optimal for the banker to sell more than it is required to service the safe debt. Thus, constraints (1.11) and (1.12) can be rewritten as a single constraint

$$D_s^P \leq Q^P Z^P, \quad (1.13)$$

which implies that the value of the safe debt cannot be greater than the value of all the assets on the banker's balance sheet in the bad state. Given the above analysis the banker maximizes (1.9) subject to (1.10) and (1.13) by choosing Z^P, D_s^P, \bar{B}^P .

It is easier to describe the banker's optimal behaviour after substituting out equilibrium prices. Thus, I turn to the definition of equilibrium.

1.3 Equilibrium

This section defines and characterizes the equilibrium of the model. I start by defining the equilibrium. Then I describe the bankers' optimality conditions in equilibrium which will allow me to characterize the equilibrium in closed and integrated worlds.

Equilibrium. An equilibrium in a two-country model is a collection of plans $\{C_0^P, C_2^P(s_2), D_d^P, I^P, D_s^P, B^P(s_2), B_s^P(s_2), Z^P, K_s^P, K_d^P\}$ in the periphery and a collection of plans $\{C_0^C, C_2^C(s_2), D_d^C, I^C, D_s^C, B^C(s_2), B_s^C(s_2), Z^C, K_s^C, K_d^C\}$ in the center and prices $\{P_0^P, P_0^C, R_D, P_B^P(s_2), P_B^C(s_2)Q^P, Q^C\}$ such that all the agents solve their problems taking the prices as given and all the markets clear, i.e.,

1. markets for risky projects in period 0 in both countries:

$$Z^P = A^P F(I^P) \text{ and } Z^C = A^C F(I^C),$$

2. risky projects fire-sale markets in period 1 in both countries:

$$K_d^P = K_s^P \text{ and } K_d^C = K_s^C,$$

3. risky debt markets in both countries:

$$B^P(s_2) = B_s^P(s_2),^{15}$$

4. integrated safe debt market:

$$D_d^P + D_d^C = D_s^P + D_s^C \tag{1.14}$$

1.3.1 Bankers Behavior

In the previous section, I presented the banker's problem. I can now conveniently characterize the solution to this problem by taking into account the equilibrium prices. The following lemma, which is proved in the Appendix, presents the optimal conditions for a banker in the periphery.

Lemma 1.1 *The banker's optimal choice of the risky projects, the amount of safe debt and*

¹⁵In general, it would not be enough for the risky debt markets clearing to require that period 0 value of risky bonds supplied to be equal to the amount of resources that the entrepreneurs pay for this value, because this would allow the entrepreneurs to demand a portfolio with different state-contingent payoffs than the supply by the bankers.

the face value of risky debt leads to the following conditions in equilibrium:

$$[p + (1 - p)q] - R_B P_0^P + \theta^P = 0, \quad (1.15)$$

$$\frac{R_B}{R_D} - \left[p + \frac{(1 - p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0, \quad (1.16)$$

where $\theta^P \geq 0$ is a shadow value of a unit of risky projects. The face value of the risky debt

$$\bar{B}^P = \begin{cases} \frac{R_B}{p+(1-p)q} \left[P_0^P Z^P - \frac{D_s^P}{R_D} \right] & \text{if no default in } s_2 = Bnc, \\ \frac{R_B}{p} \left[P_0^P Z^P - \frac{D_s^P}{R_D} \right] - \frac{(1-p)q}{p} \left[Z^P - \frac{D_s^P}{Q^P} \right] & \text{if default in } s_2 = Bnc. \end{cases} \quad (1.17)$$

Condition (1.15) states that the marginal return on a unit of risky projects equals the marginal cost when it is financed through risky debt. To see this, consider the following perturbation: the banker increases Z^P by one unit by increasing the issuance of the risky debt such that period 0 value of the risky debt goes up by 1 unit while keeping D_s^P constant. A unit increase in Z^P delivers additional $[p + (1 - p)q]$ units of period 2 consumption and relaxes the collateral constraint. A unit increase in value of the risky debt increases funding costs by R_B because the return on any risky security is R_B in equilibrium. This optimality condition makes clear that when constraint (1.13) binds, the banker wants to buy more risky projects relative to the case in which the constraint does not bind.

Condition (1.16) states that the banker is indifferent between risky and safe debt financing when he chooses his funding optimally. To see this, consider the following perturbation: the banker increases the face value of the safe debt by one unit but decreases period 0 value of the risky debt by $1/R_D$. This variation does not change the size of banker's balance sheet (it does not change the bankers budget constraint in period 0). However, it affects future repayments. First, it decreases the expected risky debt payments (the first term), which is a benefit for the banker. Second, it increases the expected payments on the safe debt (the second term), which is an additional cost to the banker. Third, it tightens the collateral constraint (1.13) (the third term), which is a loss to the banker if the constraint binds. This variation has no

affect on profits when the banker optimizes.

Equation (1.17) determines the optimal face value of the risky debt. The first line presents the face value in equilibrium when default on risky debt only occurs in the asset collapse state $s_2 = Bc$. The second line presents the face value for the case when the banker defaults in $s_2 = Bc$ and $s_2 = Bnc$ states.

1.3.2 Closed Economy Equilibrium

In this section I describe equilibrium properties of the economy. I start by considering the equilibrium in the periphery conditional on $D_s^C - D_d^C = 0$. The assumption is equivalent to assuming that the economy is closed. This allows me to study comparative statics which will be useful when I consider an integrated equilibrium.

An equilibrium can be of two types: (i) the collateral constraint (1.13) does not bind and (ii) the collateral constraint binds. If the collateral constraint does not bind, the optimality condition (1.16) pins down price Q^P as a function of the endogenous return on safe debt R_D

$$Q^P = \frac{1-p}{\frac{R_B}{R_D} - p} q. \quad (1.18)$$

The banker's optimality condition (1.15) and the entrepreneur optimal choice of his investments in the risky projects production determine the level of investments I^P

$$[p + (1-p)q] A^P F'(I^P) = R_B. \quad (1.19)$$

The last two equations, the outside investor optimality condition (1.7) and the entrepreneur demand for safe debt (1.5), fully characterize the solution. The solution is unique.¹⁶

When the collateral constraint binds, I can combine the banker's optimality conditions

¹⁶Solving the outside investor optimality condition (1.7) and equation (1.18) for D_s^P defines the safe debt supply function that depends negatively on R_D . The entrepreneur demand for safe debt (1.5) depends positively on R_D . The intersection of the supply and the demand determines uniquely R_D and D^P . Hence, the equilibrium is unique.

(1.15) and (1.16) with the entrepreneur's optimal choice of investments (1.6) to get

$$\left[\left(\frac{R_B}{R_D^P} - p \right) Q^P + p \right] A^P F' \left(F^{-1} \left(\frac{Z^P}{A^P} \right) \right) = R_B. \quad (1.20)$$

The equilibrium level of banker's risky project purchases Z^P depends on two endogenous variables: price Q^P and the return on safe debt R_D . To understand how Q^P affects the bankers consider the following intuition: an increase in Q^P raises the collateral value of Z^P , and it becomes more profitable to buy risky projects Z^P . This increases price P_0^P which increases the entrepreneurs incentives to invest in risky projects. As a result, investments in the risky projects I^P and production of risky projects Z^P go up. To understand how R_D affects the banker, consider the following intuition: an increase in R_D makes safe debt a less attractive mean of financing. This increases banker's financing costs and decreases the desire to buy risky projects Z^P . As a result, price P_0^P falls and the entrepreneur invests less in the risky technology. Thus, Z^P falls in equilibrium. Note also that the left-hand side of the above equation is an increasing function of A^P .¹⁷

Outside investor optimality condition (1.7) together with a market clearing condition $K_s^P = K_d^P$ and the fact that the collateral constraint binds, $D_s^P = Q^P Z^P$, imply

$$\delta g' (W - Q^P Z^P) = \frac{q}{Q^P}. \quad (1.21)$$

I can now solve equations (1.20) and (1.21) for $Q^P = Q^P(R_D)$ and $Z^P = Z^P(R_D)$ given R_D . The solution is graphically represented on the left panel of Figure 1.2. The line labeled as B corresponds to equilibrium condition (1.20). The line labeled as OI corresponds to equilibrium condition (1.21). The solution determines the supply of safe debt in the economy $D_s^P(R_D) = Q^P(R_D)Z^P(R_D)$.

To understand how the supply of safe debt changes with R_D consider an increase in R_D . This corresponds to a leftward shift in the B curve (see the intuition after equation (1.20)) and no shift in the OI curve. As a result, Q^P increases and Z^P decreases. Although, Q^P

¹⁷This is because $d \left[A^P F' \left(F^{-1} \left(\frac{Z^P}{A^P} \right) \right) \right] / dA^P = F'(I^P) - \frac{F''(I^P)F(I^P)}{F'(I^P)} > 0$.

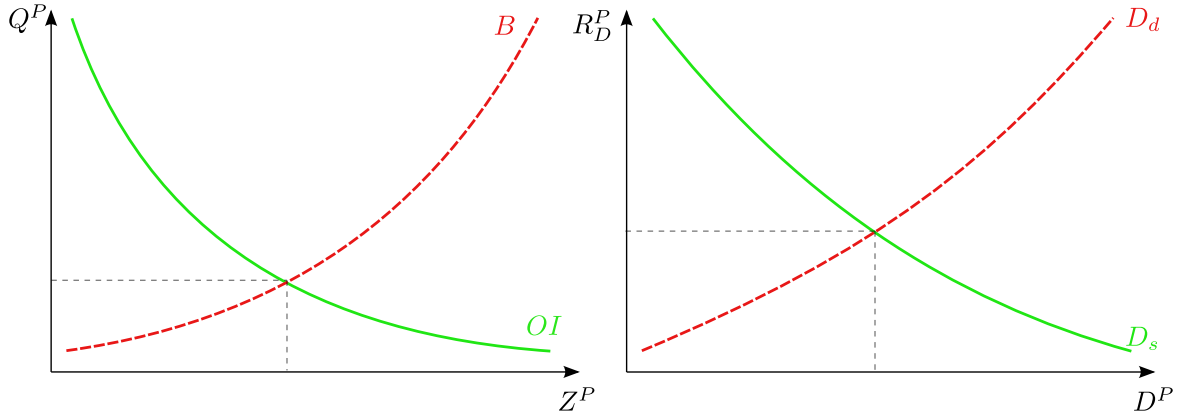


Figure 1.2: Equilibrium determination in a closed economy when collateral constraints bind: risky projects fire-sale market (left), safe debt market (right).

and Z^P change in the opposite directions we can still unambiguously determine the direction of a change in their product $Q^P Z^P = D^P$. Product $Q^P Z^P$ decreases because the elasticity of the outside investor demand for bankers assets with respect to price Q^P is greater than one. Thus, the supply of safe debt decrease with R_D , which is represented on the right panel of Figure 1.2 with downward sloping D_s curve. The upward sloping D_d curve represents the entrepreneur optimality condition (1.5). The intersection of these two curves determine the equilibrium level of D^P and R_D^P . The equilibrium level of R_D^P determines the position of B -curve on the left panel of the figure which in turn determines equilibrium Q^P and Z^P .

What happens to the equilibrium when the marginal productivity of investment opportunities A^P goes up? Given price P_0^P the entrepreneurs want to invest more I^P and sell more risky projects Z^P to the bankers. Price P_0^P will fall in equilibrium. The behavior of the rest of the equilibrium variables depends on whether the bankers collateral constraints bind or not. The following lemma, which is proved in the Appendix, summarizes the comparative statics.

Lemma 1.2 *There always exists \bar{A} such that for any $A^P < \bar{A}$ the collateral constraint binds, $\theta^P > 0$, and for any $A^P \geq \bar{A}$ the collateral constraint does not bind.*

- *If $A^P < \bar{A}$ then investment in risky projects I^P , amount of risky projects Z^P , safe debt D^P and return on safe debt R_D^P go up while price of risky projects P_0^P in period 0 and fire-sale price Q^P in period 1 go down after an increase in A^P . In addition the shadow*

value of risky projects θ^P strictly decreases.

- *If $A^P \geq \bar{A}$ then investment in risky projects I^P , amount of risky projects Z^P go up, price P_0^P goes down and all the other variables: D^P , Q^P , R_D^P , θ^P stay the same after an increase in A^P .*

The intuition behind this lemma is as follows. When A^P is sufficiently small, the amount of risky projects Z^P produced is small in equilibrium. Price Q^P is bounded by q from above. If the collateral constraint does not bind, then the amount of deposits D^P is smaller than $Z^P Q^P$. When the level of the safe debt is small, R_D^P is small. This creates strong incentives for the bankers to issue more safe debt. This eventually leads to a binding collateral constraint. When A^P is sufficiently high, this logic is reversed. Thus, the collateral constraints do not bind for high A^P . When the collateral constraints bind, an increase in marginal productivity A^P allows the entrepreneur to produce more and the bankers to buy more risky projects. To do that, the bankers increase their safe debt issuance. This leads to smaller price Q^P in the bad state and a higher return on safe debt. When the collateral constraints do not bind, the decision on the amount of the risky projects is decoupled from the safe debt issuance decision by the bankers because marginal projects don't serve as collateral in this case.

1.3.3 Open Economy Equilibrium

Now I remove assumption $D_s^P - D_d^P = 0$ and study the properties of the integrated economy. Specifically, I compare how the equilibrium allocations and prices under integration differ from those under autarky.

The effects of integration will depend on the type of equilibrium in each country before the integration. As I discussed in the previous subsection, each country can have one of the two possible types of equilibrium before the integration. There are four possibilities to consider when integrating two countries. However, Lemma 1.2 allows me to remove one possibility immediately. It is not possible for the collateral constraints to be binding in peripheral economy, with higher A^P , while the collateral constraints are not binding in the center, with

smaller A^C . It would contradict the fact that the shadow value of the risky projects decreases with an increase in A . This leaves three possibilities to consider.

The first case is the situation in which the collateral constraints do not bind in both countries. According to Lemma 1.2, the interest rate on safe debt is the same in both countries before the integration. This implies that opening up the two countries to trade in safe debt does not lead to changes in prices. Hence, none of the equilibrium variables change in both countries.

Consider the case in which the collateral constraints bind in both countries before the financial integration. This case is graphically illustrated in Figure 1.3, which is an extension of Figure 1.2 to a two-country model. The left column of plots represents the determination of equilibrium in the center; the right column presents the equilibrium in the periphery.

Let's first focus on autarky equilibria. Plot (c) of Figure 1.3 presents the fire sale of risky projects equilibrium in period 1 in the center. The green solid line, denoted as OI , is the outside investors' demand for the risky projects. The red-dashed line, denoted as $B(R_D^C(Aut), A^C)$, is the combination of the bankers and the entrepreneurs optimality conditions. This curve can be interpreted as the supply of the risky projects in period 1. $B(R_D^C(Aut), A^C)$ curve represents the supply conditional on the safe debt return $R_D^C(Aut)$ in autarky equilibrium. Plot (d) similarly presents the equilibrium on the fire sale of risky projects market in period 1 in the periphery. $B(R_D^P(Aut), A^P)$ line is shifted to the right on plot (d) relative to the respective line in plot (c). This is because of the difference in productivity of the risky projects' production, $A^P > A^C$, which makes the supply of the risky projects higher in the periphery (conditional on the same interest rate). Note that there is an opposing force: the interest rate on safe debt is higher in equilibrium in the periphery, which dampens the effect of the difference in productivity on the supply of the risky projects. However, the interest rate effect is always smaller (Lemma 1.2). Because the supply of the risky projects in period 1 in the bad state is higher in the periphery for a given value of safe return R_D , the supply of safe debt in period 0 by bankers is higher in the periphery compared to the center. This fact is represented on plots (a) and (b) of Figure 1.3: D_s^P curve is shifted

relative to D_s^C curve. However, the demand for safe debt is the same in both countries. As a result, the interest rate is higher in the periphery relative to the interest rate in the center before the integration.

Let's now consider the effects of integration. Arbitrage forces equalize the returns on safe debt in both countries $R_D^C = R_D^P$. As a result, the return in the center rises compared to the autarky case, while the return in the periphery falls. This leads to a flow of resources from the center to the periphery. One can see on plots (a) and (b) that the periphery is a net supplier of safe debt while the center is a net buyer of safe debt at new world interest rate R_D . A decline in the safe debt return in the periphery increases the supply of the risky projects in the bad state in period 1. This is indicated by the shift in the supply curve from $B(R_D^P(Aut), A^P)$ to $B(R_D, A^P)$ in plot (d). Consequently, there is a decline in the risky project's fire-sale price Q^P and a rise in equilibrium amount of the risky projects Z^P . The center experiences the opposite effects. An increase in the safe interest rate decreases the supply of the risky projects in period 1. This is indicated by the shift from $B(R_D^C(Aut), A^C)$ to $B(R_D, A^C)$ in plot (c). As a result, price Q^C increases and amount of the risky projects produced Z^C falls.

The following proposition summarizes the above analysis

Proposition 1.1 *The financial integration of the center and the periphery, when the collateral constraints bind before and after integration in both countries, leads to*

1. *return on safe debt R_D^C , fire-sale price Q^C , purchases of safe debt D_d^C increase, investments in risky projects I^C , production of risky projects Z^C , supply of safe debt D_s^C decrease after the integration in the center;*
2. *return on safe debt R_D^P , fire-sale price Q^P , purchases of safe debt D_d^P decrease, investments in risky projects I^P , production of risky projects Z^P , supply of safe debt D_s^P increase after the integration in the periphery.*

The third case is a situation in which the bankers' collateral constraints do not bind in the periphery ($\theta^P(Aut) = 0$); however, the constraints bind in the center ($\theta^C(Aut) > 0$). According to Lemma 1.2, the safe interest rate is higher in the periphery. Thus, financial

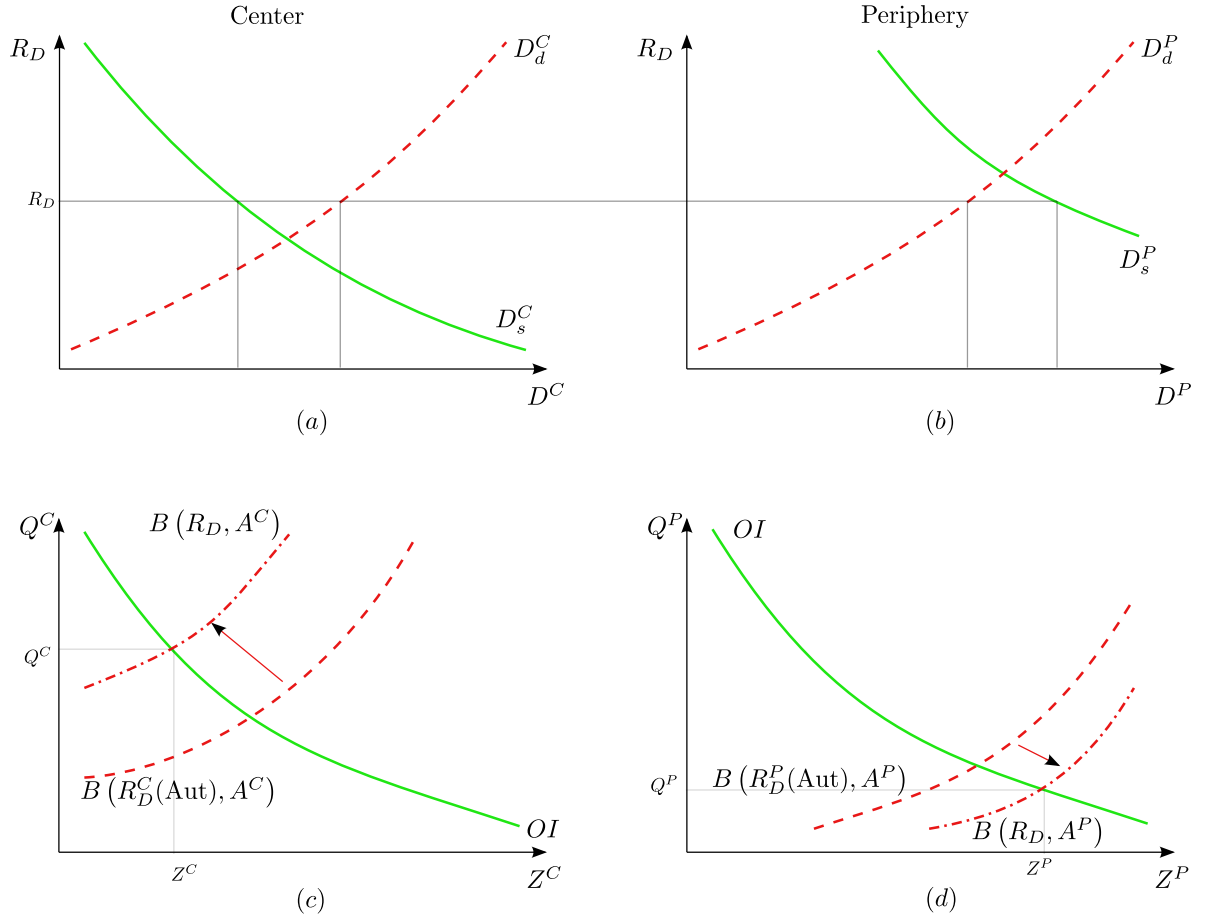


Figure 1.3: Open Economy Equilibrium.

integration leads to flows of resources from the center to the periphery. As a result, the equilibrium may have one of the following three types after integration: (i) the center collateral constraints bind ($\theta^C > 0$), while the periphery collateral constraints do not bind ($\theta^P = 0$); (ii) the collateral constraints bind in both countries ($\theta^P > 0$, $\theta^C > 0$); and (iii) the collateral constraints do not bind ($\theta^P = 0$, $\theta^C = 0$). However, independent of a type of equilibrium of an integrated economy, the effect of integration on equilibrium variables is qualitatively similar to the previous case. I do not provide the graphical characterization of each case but only summarize the effects of integration in the following lemma.

Lemma 1.3 *The financial integration of two countries with a higher productivity of investment opportunities in the periphery than in the center ($A^P > A^C$) and binding collateral constraints in the center but slack constraints in the periphery (before the integration) results*

in the following changes:

1. In the periphery, investment in the risky projects I^P , amount of risky projects Z^P , supply of safe debt D_s^P increase and interest rate R_D^P , risky projects price in period 1 in bad state Q^P and demand for safe debt D_d^P decrease;
2. In the center, investment in the risky projects I^C , production of risky projects Z^C , supply of safe debt D_s^C decrease and interest rate R_D^C , risky projects price in period 1 in bad state Q^C and demand for safe debt D_d^C increase.

The results presented in this section are related to the recent literature on the global imbalances. [Bernanke \(2005\)](#) argued that the US capital inflows and a decrease in the real interest rate can be both explained by excessive savings in many emerging and oil-exporting countries (“global saving glut” hypothesis)¹⁸. [Caballero, Farhi, and Gourinchas \(2008\)](#) and [Mendoza, Quadrini, and Rios-Rull \(2009\)](#) proposed that emerging economies financial systems cannot produce enough assets that can be used for savings, hence, capital flows to the countries with better developed financial systems, capable of generating more of these assets. In this paper, I do not assume differences in financial development across countries. Instead, capital flows are driven by the difference in productivities of marginal investment opportunities. In addition, I explicitly consider the presence of financial sectors and a possibility of financial crises.¹⁹ The equilibrium capital flows exacerbate potential crisis in the periphery while alleviating the consequences of a crisis in the center.

[Gourinchas, Rey, and Govillot \(2010\)](#) present evidence that, in addition to large capital inflows to the US prior to the recent crisis, there were sizable wealth transfers from the U.S. to the rest of the world during the crisis. They interpret this observation as evidence that the US provides insurance to the rest of the world. They build a model in which US agents have

¹⁸[Bernanke \(2011\)](#), [Shin \(2012\)](#) provide evidence on the heterogeneity of the rest of the world portfolio. Asian and oil-producing countries invest in US safe debt. However, there are large gross capital flows between the US and European countries. Many European banks raise funding in the US and then invest in the US assets. [Acharya and Schnabl \(2010\)](#) argue that difference in regulatory treatment of banks across countries may explain this behaviour.

¹⁹[Caballero and Krishnamurthy \(2009\)](#) study a model in which foreigners invests in the US safe debt issued by intermediaries, which leads to an increase in leverage of the financial system.

a lower risk-aversion than agents in the rest of the world. [Maggiore \(2012\)](#) builds a model which rationalizes different attitudes to risk in the US and the rest of the world by assuming different levels of financial development. In this model, an arrival of bad news in the periphery results in a fire-sale of banks assets. Both peripheral and center entrepreneurs receive claims on the outside investors in the periphery. As results, there is no capital outflow from the periphery to the center. However, this counterfactual assumption goes away if I assume that the entrepreneurs can also use storage technology between period 1 and 2. In this case, the entrepreneurs in the center can directly accept consumption good in period 1 in the bad state instead of claims on the outside investors in the periphery.

1.4 Welfare Effects of Integration

In this section, I study the welfare effects of short-term liabilities funding markets integration between the two countries. The section presents the main welfare result, which shows that financial integration leads to welfare decline in the periphery under certain conditions.

All of the agents are risk-neutral with respect to their consumption. I will evaluate social welfare in each country by adding consumption levels of all of the agents in each period.²⁰ The following lemma, which is proved in the Appendix, expresses the social welfare in equilibrium in the periphery.

Lemma 1.4 *The expectation of the social welfare in the periphery in period 0 is*

$$\begin{aligned} \mathbb{E}U^P &= Y - I^P + \frac{D_s^P - D_d^P}{R_D} \\ &+ v(D_d^P) \\ &+ \beta[p + (1 - p)q]A^P F(I^P) \\ &+ \beta \{ p [\delta g(W) - (D_s^P - D_d^P)] + (1 - p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)] \}. \end{aligned} \quad (1.22)$$

²⁰Alternatively I can assume that all three types of agents belong to the same large household with utility function similar to the entrepreneur utility function. To formally use this assumption I have to present the household problem rather than three separate problems. However, this does not change any of my conclusions. See [Lucas \(1990\)](#) on the exposition of the large family construct.

The first line represents the amount of goods available for consumption in period 0: Y is the initial endowment of the entrepreneurs. $(D_s^P - D_d^P)/R_D$ is the amount of goods that the entrepreneurs in the center pay to peripheral bankers to obtain $D_s^P - D_d^P$ units of safe debt. The second line represents the liquidity preferences from holding D_d^P units of safe debt. The third line represents the expected discounted output of the risky projects. The last line is the expected discounted revenue of the outside investors net of the repayments to the entrepreneurs in the center. Term $[g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]$ in the last line takes into account that in the bad state D_s^P units of the outside investors' endowment have to be invested in the storage technology rather than in the risky technology. The welfare in the center has the same form.

As demonstrated earlier, there are three possible types of equilibrium after integration. The first case, i.e., that the collateral constraints do not bind before the integration, is trivial. The integration has no effect on welfare. As I showed in the previous section, this result is due to the fact that the returns on safe debt are the same in both countries before the integration. The integration does not lead to a change in the safe debt return. Thus, the equilibrium allocations do not change. This implies that the social welfare is the same in both countries. The third case, i.e., the collateral constraints bind in the periphery but not in center (before integration), features similar effects that will be analyzed in the second case. I believe that the second case, i.e., the collateral constraints bind in both countries before integration, is the most interesting to analyze.

The next proposition summarizes the welfare effects of the integration.

Proposition 1.2 *The financial integration of the center and the periphery, with binding collateral constraints before financial integration, has the following effects on welfare:*

1. *The center always benefits from integration.*
2. *There always exists \hat{A} such that for all $A^P \in (A^C, \hat{A})$ the periphery loses from integration.*

See the Appendix for the proof of this proposition. The first part of the proposition states that the center always benefits from integration. The second part states that the periphery loses from integration if the difference in productivities is not very large.

There are two welfare effects of the financial integration: efficient capital reallocation and changes in welfare losses associated with a negative externality. The first effect is an efficient capital reallocation. Both countries benefit. The entrepreneurs in the center invest their resources in safe debt of the peripheral bankers and receive higher interest rate. Although this means that they invest less in their local banks, which implies smaller profits of the center entrepreneurs, the net effect is positive. The bankers in the periphery can fund themselves more cheaply after the integration. Although this effect is dampened by smaller holdings of safe debt by the periphery entrepreneurs, the net effect is positive.

Before explaining the source of welfare losses, I should comment on the nature of the externality. To simplify the exposition, I will focus on a closed economy equilibrium in which the collateral constraints bind. Consider the following perturbation: a banker decreases his issuance of safe debt by a small amount in period 0. Thus he will have to sell less risky projects in the bad state in period 1. This increases fire-sale price Q^P in a possible bad state. The marginal decrease in safe debt issuance has no effect on the banker profits because he optimizes in equilibrium. However, this perturbation has three effects through price Q^P that the banker does not internalize. First, an increase in Q^P is a benefit to the other bankers because they can get more on the fire-sale market for the same amount of risky projects. Second, an increase in Q^P is a loss to outside investors because they have to pay more for the same amount of the risky projects. Third, an increase in Q^P relaxes the collateral constraints of the other bankers in period 0. This allows them to issue more safe debt, which is a cheaper source of funding.

The first two effects cancel each other out from the perspective of the social welfare function used here. The fact that the bankers and the outside investors have the same marginal utility of consumption makes the transfer between them associated with an increase in Q^P welfare-

neutral.²¹ The third effect is a pure gain for the economy. However, because an individual banker does not internalize this positive effect from a smaller issuance of safe debt he overissues safe debt in equilibrium. In other words, there is a negative externality associated with binding collateral constraints.²²

Next, I continue discussing the effects of financial integration. The second effect of financial integration is due to changes in welfare losses associated with the negative pecuniary externality. The bankers in the periphery start issuing more safe debt after the integration. Because there is a wedge between social and private returns on issuing safe debt, an increase in issuance of safe debt increases losses because this wedge is multiplied by a larger amount of safe debt. Note that this is a first-order effect in the size of increase in the issuance of safe debt. On the other hand, because the center bankers issue smaller amount of safe debt, the loss becomes smaller because the wedge in the center now multiplies by a smaller amount of safe debt.

To formally see the influence of these two effects on the level of the social welfare in the periphery I can express the change in the welfare due to the integration as follows:

$$\begin{aligned}
 X^P &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\
 &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^P; \text{integration}) \\
 &= - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A}. \tag{1.23}
 \end{aligned}$$

$U^P(A^P, A^C; \text{integration})$ is a social welfare function where the first argument is the marginal productivity of investment opportunities in country P , the second argument is the marginal productivity of investment opportunities in country C , and the third argument is a dummy variable that indicates whether the two countries are integrated. In the proof of proposition

²¹Note that the generic inefficiency result in environments with incomplete markets described in [Geanakoplos and Polemarchakis \(1985\)](#) and [Greenwald and Stiglitz \(1986\)](#) deals with cases in which marginal utilities of agents are not equalized in equilibrium. That is, their inefficiency result stems from transfers associated with price changes. See also [Lorenzoni \(2008\)](#) who builds a model in which financial frictions prevent equalization of the marginal utilities of agents which leads to welfare losses associated with the pecuniary externality.

²²This externality has similar implications as the externality in [Bianchi and Mendoza \(2010\)](#). They present a model in which agents face borrowing constraints. However, in their model agents cannot borrow more than the current value of their collateral.

1.2, I show that

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

In the above formula, the first term is positive because $dQ^P/d\tilde{A} > 0$: an increase in \tilde{A} in the center leads to a larger supply of safe debt D_s^C which decreases the equilibrium issuance of safe debt in the periphery and increases price Q^P . When $\tilde{A} < A^P$, there is an inflow of resources to the periphery; hence, $D_s^P - D_d^P > 0$. Derivative $dR_D/d\tilde{A}$ is positive because the increase in the issuance of safe debt in the center leads to higher holdings of safe debt in both countries, which increases safe debt return R_D . Thus the second term of the above formula is positive. We can now see that

$$X^P = \int_{A^C}^{A^P} \underbrace{\left(-\beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \right)}_{<0} d\tilde{A} + \int_{A^C}^{A^P} \underbrace{\left(-\frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}} \right)}_{>0} d\tilde{A},$$

where the first term represents increased losses due to the negative pecuniary externality while the second term represents the efficient capital reallocation effect. In the proof of the proposition I show that dQ^P/\tilde{A} and $dR_D/d\tilde{A}$ are bounded from zero for all \tilde{A} ; however, because the second term features the difference $D_s^P - D_d^P$, the value of this term can be arbitrarily close to zero. This observation implies that the efficient capital reallocation benefit is smaller than the welfare losses associated with the pecuniary externality when the difference $A^P - A^C$ is small.

The same reasoning may be applied the center to show that both effects go in the direction of increasing the social welfare.

Example. Figure 1.4 presents a numerical example that shows the change in welfare for various values of A^P relative to A^C . First, observe when $A^P/A^C = 1$ the countries are identical ex-ante which prevents net flows from one country to the other with integration. Thus there are no welfare changes. For A^P slightly larger than A^C , integration leads to a net inflow of resources into the periphery. This leads to a decline in the welfare in the

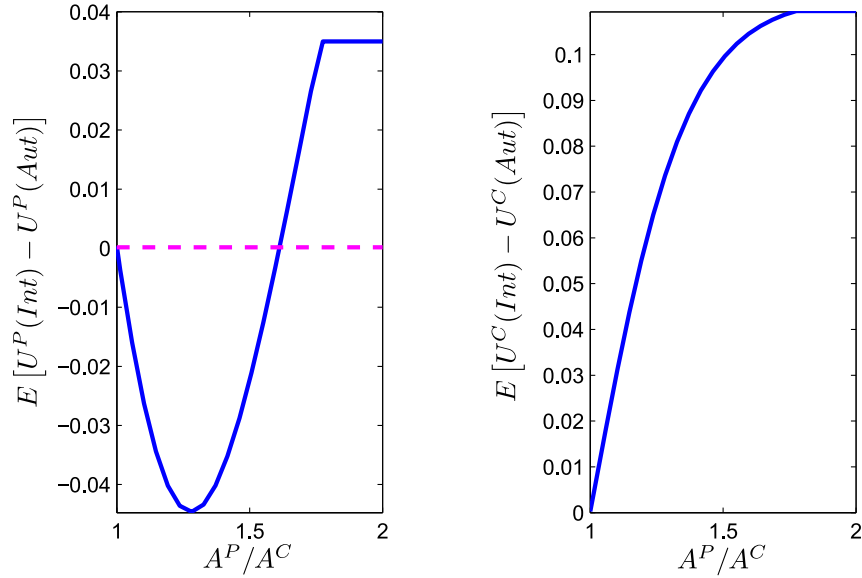


Figure 1.4: Change in the social welfare in the periphery $E[U^P(Int) - U^P(Aut)]$ and in the center $E[U^C(Int) - U^C(Aut)]$ as a function of the ratio of productivities A^P/A^C . The utility function from holding debt has the following form $v(D) = \gamma D^{\alpha_D}$, risky projects production function $F(I) = AI^{\alpha_F}$, the late technology $g(x) = Bx^{\alpha_G}$. Parameters: $\alpha_F = 0.8, \alpha_G = 0.65, \beta = 0.9, \gamma = 3, \alpha_M = 0.76, p = 0.8, q = 0.5, W = 5, A^C = 2, A^P \in [2, 4], B = 5, \delta = 0.99$.

periphery, negative $\mathbb{E}[U^P(Int) - U^P(Aut)]$, and an increase in welfare in the center, positive $\mathbb{E}[U^C(Int) - U^C(Aut)]$. For $A^P/A^C > 1.64$, the gain from efficient capital reallocation dominates the welfare loss due to the negative externality. Finally, when $A^P/A^C > 1.78$, productivity A^P is large enough so that the collateral constraints do not bind in the periphery in line with the results of lemma 1.2. When the collateral constraints do not bind before and after integration, the integration has only positive capital reallocation effect in the periphery.

The recent literature on global imbalances emphasized the welfare consequences of integration of countries with difference levels of financial development. Most closely related to this paper is [Mendoza, Quadrini, and Rios-Rull \(2007\)](#). They argue that financial flows that arise from different levels of financial development lead to an increase in welfare in a more financially developed country, that experiences financial inflows, and a decrease in welfare in a less financially developed country after integration. [Eden \(2012\)](#) studies the welfare effects of financial integration in the presence of the working capital constraints in a less financially developed country. The author concludes that the more financially developed country that does

not face working capital constraints and experiences capital inflows benefits, while the less financially developed country loses from financial integration. In contrast to this literature, the results of this section suggest that it is the source country that benefits from integration and the recipient country that loses from integration.

1.5 Regulation

This section studies policy. I first consider the optimal macroprudential taxes on the safe debt issuance. Then I show that both countries benefit from adding capital controls to their policy tools.

A number of recent papers suggested that a system of Pigouvian taxes can be used to bring financial sector incentives closer to social interests.²³ [Kashyap and Stein \(2012\)](#) and [Woodford \(2011\)](#) argue that such Pigouvian taxes can be implemented by the interest rate paid on reserves.²⁴ I start this section by studying the optimal policy in the presence of just one tool: safe debt taxes. Later, I investigate whether additional tools can help improve welfare.

1.5.1 Safe Debt Taxation and Interest Rate Manipulation

In this section, I consider the problem faced by a regulator in the periphery who maximizes the social welfare function in his country by choosing safe debt taxes given all of the equilibrium conditions and fixed behavior of the regulator in the other country. The regulator rebates the proceeds of the taxes to the entrepreneurs in a non-distortionary way.

I formally introduce safe debt taxes into the banker period 0 budget constraint as follows

$$P_0^P Z^P \leq V_B^P + \frac{D_s^P}{R_D}(1 - \tau^P),$$

²³See, for example, [Jeanne and Korinek \(2010\)](#), [Perotti and Suarez \(2010\)](#).

²⁴The effectiveness of this tool depends on the ability of the government to impose its reserve requirements on the issuance of assets that create systematic risk to the stability of financial system. For example, if the government can only impose reserve requirements on the traditional banking sector deposits, this may not be welfare increasing if deposits are already appropriately insured by the government.

where V_B^P is period 0 value of the risky debt. The optimal choice of safe debt funding D_s^P leads to

$$\frac{R_B}{R_D}(1 - \tau^P) - \left(p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0. \quad (1.24)$$

τ^P reduces the benefit from using cheaper safe debt financing. The optimal choice of the risky projects purchases Z^P is given (1.15) because the proportional taxes on safe debt do not affect this choice directly.

The regulator maximizes the peripheral social welfare (1.22), i.e., the sum of all agents consumption, by choosing τ^P subject to seventeen equilibrium conditions: bankers optimality conditions in the periphery (1.24) and (1.15), constraint on the issuance of safe debt (1.13), non-negativity of the Lagrange multiplier θ^P , the complementarity slackness condition, outside investor optimality condition (1.7), entrepreneurs optimality condition with respect to safe debt holdings (1.5), and entrepreneurs optimality condition with respect to investments into the risky projects, eight similar equations for the center and the safe debt market clearing condition. The proof of Lemma 1.5 states this problem explicitly and derives the first order necessary condition.

Lemma 1.5 *At the optimum of the periphery regulator problem, in which either $\theta^P > 0$ or $M_s^P < Q^P Z^P$, the following condition must hold:*

$$\frac{dU^P}{d\tau^P} = \beta \frac{A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left(\theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0.$$

This lemma states that if the regulator chooses to impose taxes τ^P on its bankers, then the above condition should hold for optimal choice of τ^P .²⁵ The condition that either $\theta^P > 0$ or $M_s^P < Q^P Z^P$ holds in the optimum rules out the case with $\theta^P = 0$ and $M_s^P = Q^P Z^P$. In this situation the welfare function derivative is not defined. If the left derivative of the welfare

²⁵Note that this condition is trivially satisfied if the regulator taxes out the issuance of safe debt in the periphery from existence. That is, the tax rate is sufficiently high so that the bankers do not issue any safe debt in the periphery. As a result, further changes in the tax rate can not alter the equilibrium variables, i.e., $dQ^P/d\tau^P = 0$ and $dR_D/d\tau^P = 0$.

function is positive while the right derivative is negative, then the optimum is attained at this kink.

The first term of this optimality condition represents two effects. On the one hand, an increase in τ^P has a positive effect because it mitigates the negative externality. Observe that this effect is only present when $\theta^P > 0$. On the other hand, an increase in τ^P makes it more expensive for the bankers to fund themselves, which leads to a lower production of the risky projects that yield less consumption in period 2. The second term $\frac{D_s^H - D_d^P}{R_D^2} \frac{dR_D}{d\tau^H}$ is due to manipulation of the international interest rate. If the periphery experiences an inflow of resources directed to investments in safe debt, then a decrease in the interest rate will benefit the bankers in the periphery because they will have to repay less in period 2 to the entrepreneurs in the center. The policy maker decreases the interest rate by taxing his bankers more than he would without the manipulation motive.

A symmetric condition holds for the regulator in the center

$$\frac{dU^C}{d\tau^C} = \beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^C} \left(\theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^C} = 0.$$

The only difference is that the last term is necessarily of the opposite sign relative to a similar term in the periphery. If the center experiences the outflow of resources $D_s^C - D_d^C < 0$, then the term is negative. This implies that the regulator sets lower taxes compared to the situation without the interest-manipulation motive.

Example. Figure 1.5 presents a numerical example that shows how the peripheral social welfare function depends on safe debt taxes. The parameters are chosen such that the periphery is a net issuer of safe debt when there is no regulation in this country which corresponds to the assumption that $A^P > A^C$ and the collateral constraints bind at least in the center. Several observations can be made looking at this example. First, a small, positive debt taxes level benefits the periphery. This is a combination of the interest-rate-manipulation benefit and the externality-correction benefit. At $\tau^P \approx 0.25$, the social welfare in the periphery attains its maximum. This point corresponds to the condition in Lemma 1.5. A further increase

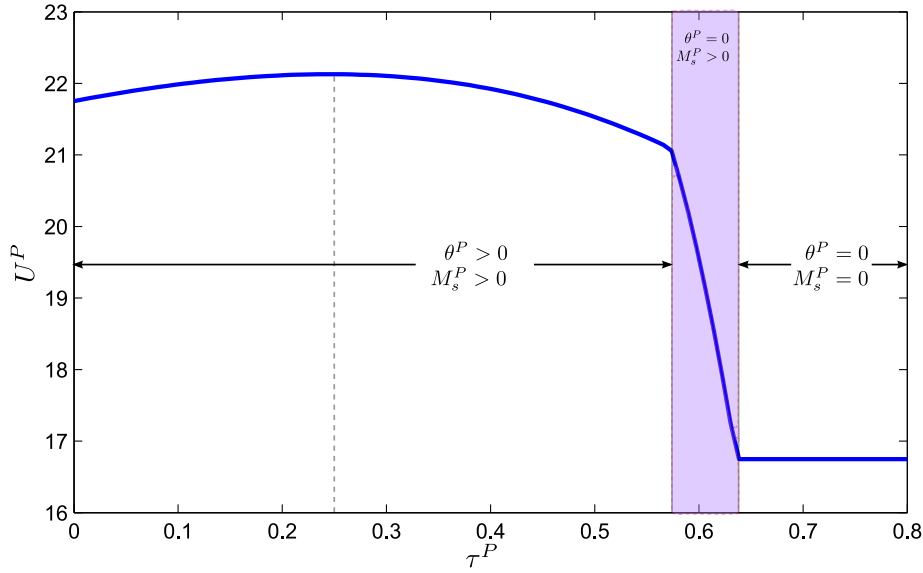


Figure 1.5: Ex ante social welfare in the periphery U^P as a function of the level of safe debt taxes τ^P . The utility function from the holding debt has the following form $v(D) = \gamma D^{\alpha_D}$, risky projects production function $F(I) = AI^{\alpha_F}$, the late technology $g(x) = Bx^{\alpha_G}$. Parameters: $\alpha_F = 0.8, \alpha_G = 0.65, \beta = 0.9, \gamma = 3, \alpha_M = 0.76, p = 0.8, q = 0.5, W = 5, A^C = 2, A^P = 2.3, B = 5, \delta = 0.99, \tau^C = 0$.

in the level of taxes makes losses from distortionary taxation dominate the benefits from the taxes. At $\tau^P \approx 0.57$, there is the first kink, i.e., the collateral constraints stop being binding in the periphery. It becomes costly enough for banks to issue safe debt that they decide to issue less safe debt than the value of their risky projects in the bad state. At $\tau^P \approx 0.64$, there is the second kink. It becomes extremely costly for bankers to issue safe debt, and they decide not to issue it at all.

Welfare. In the previous section I showed that the periphery may lose from financial integration because the negative effect associated with overissuance of safe debt may dominate the efficient capital reallocation effect. I now show that setting safe debt taxes optimally makes the integration beneficial if the center regulator is passive. Formally, I compare the welfare of the periphery before and after integration, assuming that the regulator in the periphery sets his taxes optimally. At the same time I assume that the regulator in the center is passive. That is, she does not change her taxes after the integration.

Proposition 1.3 *If the regulator in the periphery (i) chooses the levels of safe debt taxation optimally before and after the integration; (ii) the collateral constraints bind in both countries before and after integration, (iii) the periphery is a net supplier of safe debt after the integration then both countries benefit from the integration.*

When choosing taxes optimally, the regulator in the periphery offsets the negative welfare effect of debt overissuance. In addition, the regulator increases the welfare by manipulating the interest rate. It is important for this result to assume that the regulator in the center is passive. If the regulator in the center chooses her taxes optimally then the interaction of the two regulators has to be taken into account.²⁶ I turn to this issue next.

1.5.2 Non-Cooperative Safe Debt Taxation

I will now solve for a Nash equilibrium of the regulation game. A regulator in each country chooses the optimal level of taxes by taking the behavior of the other regulator as given. I will only focus on the case in which regulators optimal choices can be described using the first order necessary conditions from Lemma 1.5. In a Nash equilibrium, each regulator optimizes. Thus a marginal change in his policy has a second-order effect on the social welfare in his country. The next proposition shows that this marginal change leads to a first-order loss in the other country, which leaves the Nash equilibrium strictly inside the Pareto frontier.

Proposition 1.4 *A Nash equilibrium can be locally Pareto-improved if the periphery regulator decreases and the center regulator increases their taxes.*

To describe the effects at work, consider a marginal increase in taxes in the periphery and the corresponding reaction of the center social welfare:

$$\frac{dU^C}{d\tau^P} = \underbrace{\beta A^C \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \theta^C}_{<0} - \underbrace{\beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C}}_{<0} - \underbrace{\frac{D_s^C - D_d^C}{R_D^2}}_{<0} \underbrace{\frac{dR_D}{d\tau^P}}_{<0}$$

²⁶If the regulators choose their policies in uncoordinated way, the result in proposition 1.3 does not hold in general. There is a negative effect that the regulator in the center imposes on the welfare in the periphery which may lead to a decrease in welfare.

The first term is a loss due to the negative externality after a marginal increase in τ^P . Note that this term is present only when the collateral constraints bind in the center. An increase in τ^P decreases the supply of the safe debt in the periphery, which makes the world supply of safe debt smaller leading to an increase in the price (and decrease in returns) of safe debt $1/R_D$. As a result, the center bankers' incentives to issue safe debt go up. However, this leads to a more severe decline in the bankers' assets price in the bad state, i.e., $dQ^C/d\tau^P < 0$, which has negative consequences for welfare. The second term shows that the losses associated with distortional taxes in the periphery become smaller. The last term represents the loss for the entrepreneurs in the center who now receive smaller return on their purchases of safe debt from the periphery.

When choosing his optimal level of taxes, the regulator in the periphery does not internalize that he has the above three effects on the center economy. The proposition states that in a Nash equilibrium the net effect is negative. In addition, the net effect of a marginal increase in taxes in the center on the welfare in the periphery is positive.

These results are related to the literature that studies the international terms of trade manipulation. [Obstfeld and Rogoff \(1996\)](#) in a two-period and recently [Costinot, Lorenzoni, and Werning \(2011\)](#) in a dynamic model study how the incentives to install capital controls may arise because of the desire to manipulate the intertemporal terms of trade. In my paper, a regulator who only intends to limit the scope of the negative externality in the banking sector will inevitably affect the international interest rate. This creates the desire to manipulate the interest rate.

1.5.3 Safe Debt Taxation and Capital Controls

The regulators have incentives to use macroprudential safe debt taxation to manipulate the international interest rate. Thus, it is logical to add another tool to their policy choice sets. One such tool can be capital controls. By capital controls I mean a proportional tax or subsidy on capital flows. Consider the periphery. If the local interest rate on the safe debt equals R_D^P , then the agents in the center who invest in safe debt in the periphery will receive $(1 - \tau_f^P)R_D^P$

units of consumption good in period 2 for each unit invested in period 0.²⁷ A symmetric definition is applied to the center.

In the next lemma, I solve for the first-order necessary conditions of the regulator problem in the periphery, assuming that the other regulator is passive. The full problem that the regulator solves is defined in the proof of the lemma.

Proposition 1.5 *At an optimum of the regulator problem, the following condition must hold*

$$\tau^P = \theta^P \frac{\tilde{\epsilon}_g^P R_D^P}{Q^P R_B}, \quad (1.25)$$

$$\tau_f^P = \frac{-\frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}{1 - \frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}. \quad (1.26)$$

It is easy to see that $\tau^P, \tau_f^P \in [0, 1]$. The first condition states that the regulator does not use safe debt taxes to manipulate the interest rate. The second condition states that the capital control tax is proportional to the regulator's effect on the interest rate in the center, i.e., $dR_D^C(D_s^P - D_d^P)/dD_d^P$, and to the level of cross-border net safe debt $D_s^P - D_d^P$.

Jeanne and Korinek (2010) and Bianchi (2011) argue that negative externality associated with borrowing from abroad give rise to prudential capital controls.²⁸ In my paper, borrowing from abroad per se does not create inefficiencies. However, the incentives to manipulate the international interest rates, when regulating the local banking sector, will lead to the desire to use two tools—prudential taxes on banking sector and capital controls—instead of just using prudential taxation in the banking sector.

1.6 Conclusion

In this paper, I analyzed the effects of international financial integration in the presence of bank funding risk. The central feature of the analysis is the presence of negative pecuniary

²⁷Subscript f distinguishes this tool from the tax on safe debt.

²⁸Martin and Taddei (2012) build a model in which adverse selection problems leads to inefficient borrowing from abroad.

externality that bankers do not internalize. This leads to overissuance of safe debt that leads to inefficiently low price of bankers' assets in crises. The integration of the short-term safe funding markets leads to capital flows. As a result, the severity of possible financial crises increases in the region that experiences capital inflows, the periphery, but becomes smaller in the region that experiences capital outflows, the center. Thus, the integration leads to changes in the severity of this distortion.

I show that, in unregulated world, the periphery may lose from integration. The center always gains from the integration. There are two effects of the integration: efficient capital reallocation and changes in the welfare losses due to overissuance of safe debt. When the difference in the productivities of the marginal investment opportunities in the two regions are not large, the negative welfare effect always dominates efficient capital reallocation effect for the periphery. However, the two effects are positive for the center because the integration leads to less issuance of safe debt in the center.

A regulator in each country may want to correct the effects of the overissuance of safe debt by imposing macroprudential taxes on safe debt funding. In the integrated world, this macroprudential tool will have effect on the international price of safe debt. This creates incentives for the regulators to manipulate the interest rate. If the regulators set their policies in a non-cooperative manner then a resulting Nash equilibrium is not Pareto efficient. If the regulator in the periphery reduces his taxes while the regulator in the center increases her taxes the welfare of both countries can be Pareto improved.

Finally, I show that the regulators will have incentives to add capital controls to their policy tools. Using capital controls allows to correct the externality in the banking sector and to manipulate international interest rate more effectively.

The analysis in this paper was purely qualitative. It is important to quantify the effects discussed in the paper. I leave this for a future research.

Chapter 2

Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence

with Emi Nakamura and Jón Steinsson¹

¹We would like to thank Mariana Garcia and Channing Verbeck for excellent research assistance. We would like to thank Andrew Ang, Ravi Bansal, Geert Bekaert, Jaroslav Borovicka, John Campbell, John Cochrane, Lars Hansen, Ralph Koijen, Lars Lochstoer, Stavros Panageas, Bernard Salanie and seminar participants at various institutions for valuable comments and discussions. We thank the Columbia University Center for International Business Education and Research for financial support.

2.1 Introduction

A large recent literature has emphasized the importance of long-run risks—persistent shocks to growth rates and uncertainty—for explaining a variety of asset market phenomena. [Bansal and Yaron \(2004\)](#) demonstrated the importance of these features for explaining the high equity premium, high volatility of stock returns, low and stable risk-free rate and predictability of stock returns. Subsequent work has used these shocks to explain failures of the expectations hypothesis of the term structure and uncovered interest rate parity, the return premium on value stocks and small stocks, the term structure of equity returns, and the volatility of the real exchange rate.² A comparably large recent literature has focused on the macroeconomic consequences of these same types of shocks—news shocks about future growth rates and uncertainty shocks. [Beaudry and Portier \(2006\)](#) argue that news shocks about future growth rates are an important driver of business cycles.³ [Bloom \(2009\)](#) highlights the role of uncertainty shocks in generating recessions.⁴

[Bansal and Yaron \(2004\)](#) propose the following time-series model of consumption growth:

$$\begin{aligned}\Delta c_{t+1} &= \mu + x_t + \chi\sigma_t\eta_{t+1}, \\ x_{t+1} &= \rho x_t + \sigma_t\epsilon_{t+1}, \\ \sigma_{t+1}^2 &= \sigma^2 + \gamma(\sigma_t^2 - \sigma^2) + \sigma_\omega\omega_{t+1}.\end{aligned}\tag{2.1}$$

Relative to a simple, random-walk model for consumption, this model adds two novel features: 1) consumption growth is affected by a persistent process x_t , 2) the uncertainty about consumption growth varies over time in a persistent manner. A difficulty with empirically evaluating this model is that certain key parameters—e.g., the persistence of x_t and σ_t^2 and the volatility of the innovations to σ_t^2 —are difficult to estimate with 80 years of consumption

²Important papers include [Bansal and Shaliastovich \(2010\)](#), [Bansal, Dittmar, and Lundblad \(2005\)](#), [Hansen, Heaton, and Li \(2008\)](#), [Bonomo et al. \(2011\)](#), [Malloy, Moskowitz, and Vissing-Jorgensen \(2009\)](#), [Croce, Lettau, and Ludvigson \(2010\)](#), and [Colacito and Croce \(2011\)](#). See [Bansal, Kiku, and Yaron \(2012\)](#) for a more comprehensive review of this literature.

³See also [Aguar and Gopinath \(2007\)](#), [Jaimovich and Rebelo \(2009\)](#), [Barsky and Sims \(2010\)](#), [Schmitt-Grohe and Uribe \(2010\)](#), and [Blanchard, L’Huillier, and Lorenzoni \(2011\)](#).

⁴See also [Bloom et al., 2011](#), [Fernandez-Villaverde et al., 2011](#), and [Basu and Bundick \(2011\)](#).

data from a single country. This has led authors in the asset pricing literature to focus on calibrations of the long-run risks model designed to match asset pricing data (Bansal and Yaron, 2004; Bansal et al., 2012).⁵ A concern with this approach is that the asset pricing data may be driven by other factors such as habits, rare disasters and heterogeneous agents.⁶ More direct evidence for the mechanisms that the long-run risks model is based on would, therefore, strengthen the case for this model.

We quantify the importance of growth-rate and uncertainty shocks using recently assembled data on aggregate consumption for a panel of 16 developed countries. We assume that certain features of consumption dynamics are common across countries. This allows us to estimate key parameters more accurately. An advantage of our approach is that our estimates are based purely on macroeconomic data. We therefore avoid the concern that our estimates of growth-rate and uncertainty shocks are engineered to fit the asset pricing data, as opposed to being a fundamental feature of the aggregate consumption data.

Our empirical model augments Bansal and Yaron’s model to allow for common variation in growth-rates and uncertainty across countries as well as country-specific shocks to growth rates and uncertainty. We identify a substantial common component to expected growth rates in our panel of developed countries. This common variation in growth rates is highly persistent. It captures the productivity speed-up and slow-down in the second half of the 20th century as well as several world recessions, such as those of 1979-82, 1990 and 2008. The country-specific growth-rate processes we identify are less persistent, but nevertheless yields movements in consumption that differ substantially from a random walk.⁷

We also identify large and highly persistent common shocks to macroeconomic uncertainty. Our world uncertainty process captures the large but uneven rise and fall of volatility that occurred over the course of the 20th century. The “Great Moderation” identified by [McConnell](#)

⁵Several papers have also used a combination of macroeconomic and asset pricing data to estimate the parameters of the long-run risks model (e.g., [Bansal, Kiku, and Yaron, 2007](#); [Constantinides and Ghosh, 2009](#)).

⁶See [Campbell and Cochrane \(1999\)](#), [Barro \(2006\)](#) and [Constantinides and Duffie \(1996\)](#) for influential asset pricing models based on these features.

⁷These findings line up well with those of [Cogley \(1990\)](#), who finds that long-run growth rates of output are more highly correlated across countries than one-year growth rates for nine of the countries we study.

and [Perez-Quiros \(2000\)](#) is evident in our estimates. But we uncover several additional sharp swings in volatility, most recently a large increase associated with the “Great Recession.” We estimate substantial variation across countries in the timing and direction of uncertainty shocks. For example, uncertainty rose for several decades after World War II (WWII) in the U.K., while it fell in most countries over this period.

Another novel feature of our empirical model relative to earlier work is that we allow the growth-rate and uncertainty shocks to be correlated. We find that they are in fact substantially negatively correlated. In other words, negative shocks to growth rates tend to be associated with shocks that increase uncertainty. The 1960’s were both a period of high growth and low volatility, while in the 1970’s growth fell and uncertainty rose. More recently, during the recessions of 1990 and particularly 2008 growth fell and our estimates of uncertainty shot up.

Overall our empirical results based on macroeconomic data alone yield parameter values that are quite consistent with calibrations of the long-run risks model designed to match key asset pricing moments ([Bansal and Yaron, 2004](#); [Bansal, Kiku, and Yaron, 2012](#)). We analyze the asset pricing implications of our estimated model of consumption dynamics within the context of a representative agent endowment economy—following [Lucas \(1978\)](#) and [Mehra and Prescott \(1985\)](#)—and assume that agents have Epstein-Zin-Weil preferences ([Epstein and Zin, 1989](#); [Weil, 1990](#)). Our model can match the observed equity premium and risk-free rate if agents have a coefficient of relative risk aversion (CRRA) of roughly 6.5 and an intertemporal elasticity of substitution (IES) of 1.5. For the same utility function parameters, the model without growth-rate and uncertainty shocks generates an equity premium that is more than an order of magnitude smaller. [Bansal and Yaron \(2004\)](#) match the equity premium with a CRRA of 10. On this metric, our estimates, thus, yield more long-run risk than their original calibration. Our model also does well when it comes to other key asset pricing moments such as the volatility of stock returns, the volatility of the risk-free rate, the Sharpe ratio for equity, the volatility and persistence of the price-dividend ratio on stocks and predictability of stock returns based on the price-dividend ratio on stocks.

Uncertainty shocks play an important role in generating movements in asset prices in our

model. Shocks that raise expected future uncertainty lead stock prices to fall. And expected returns are predictably high following stock market declines provoked by such uncertainty shocks. Through this mechanism, our model is able to help explain the long-term predictability of stock returns (Campbell and Shiller, 1988; Fama and French, 1988; Hodrick, 1992; Cochrane, 2008; Binsbergen and Koijen, 2010). Our model also implies that price-dividend ratios should forecast volatility and consumption growth. We show that price-dividend ratios on stocks have substantial predictive power for future realized volatility of consumption growth in our sample of countries—extending earlier evidence by Bansal et al. (2005). We also extend related work by Lettau et al. (2004, 2008) that suggests that changes in macroeconomic volatility can explain a substantial fraction of low-frequency movements in price-dividend ratios on stocks. In the data, consumption growth is not forecastable by the price-dividend ratio (Beeler and Campbell, 2012).⁸ (Croce, Lettau, and Ludvigson, 2010) show that consumption is less forecastable in a long-run risks model in which investors don't have had full knowledge of the variation in growth prospects in real time .

We analyze the quantitative implications of growth-rate and uncertainty shocks under the assumption that the CRRA is 6.5. This value is substantially lower than the standard parameterization in the long-run risks literature of $CRRA = 10$. However, this degree of risk aversion is high relative to the values typically estimated in the microeconomics literature (Barsky et al., 1997; Chetty, 2006; Paravisini et al. 2010).⁹ Our findings, thus, leave ample “room” for additional factors, such as habit, heterogeneous agents, and rare disasters to play an important role in explaining the level and volatility of asset returns.

In addition to the work discussed above, our paper is related to several strands of work in macroeconomics and finance. A large body of work in macroeconomics has studied the long-run properties of output (Nelson and Plosser, 1982; Campbell and Mankiw, 1989; Cochrane, 1988; Cogley, 1990) and variation in the volatility of output growth (McConnell and Perez-

⁸However, Bansal, Kiku, and Yaron (2012) show that consumption growth is substantially forecastable in a VAR with the price-dividend ratio, the risk-free rate and consumption growth.

⁹In a static context, an agent with a CRRA of 6.5 would turn down a 50-50 gamble that either raised consumption by a factor of 1 million or lowered it by 12%. An agent with a CRRA of 10 would turn down a 50-50 gamble that either raised consumption by a factor of 1 million or lowered it by 8%.

Quiros, 2000; Blanchard and Simon, 2001; Stock and Watson, 2002b; Ursua, 2010). Our paper builds heavily on the large and growing literature on long-run risks as a framework for asset pricing pioneered by Kandel and Stambaugh (1990). We consider a simple representative agent asset pricing framework with known parameter values, taking the consumption process as given. Several theoretical papers extend on this framework, studying the production-based microfoundations for long run risks (e.g., Kaltenbrunner and Lochstoer, 2010; Kung and Schmid, 2011), the asset pricing implications of parameter learning (e.g., Collin-Dufresne, Johannes, and Lochstoer, 2012), deviations from the representative agent framework (e.g., Garleanu and Panageas, 2010), and frameworks where utility depends on more than just consumption (e.g., Uhlig, 2007).

The paper proceeds as follows. Section 2.2 discusses the data we use. Section 2.3 presents the empirical model. Section 2.4 discusses our estimation strategy. Section 2.5 presents our empirical estimates. Section 2.6 studies the asset-pricing implications of our model. Section 2.7 concludes.

2.2 Data

We estimate our model using a dataset on long-term consumer expenditures recently constructed by Robert Barro and Jose Ursua, and described in detail in Barro and Ursua (2008).¹⁰ Our sample includes 16 countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.¹¹ Our consumption data is an unbalanced panel with data for each

¹⁰One limitation of the Barro-Ursua data set is that it does not allow us to distinguish between expenditures on non-durables and services versus durables. Unfortunately, separate data on durable and non-durable consumption are not available for most of the countries and time periods we study. For the U.S., non-durables and services are about 70% as volatile as total consumer expenditures over the time period when both series are available. One way of adjusting our results would therefore be to scale down the volatility of the shocks we estimate by 0.7. Whether this adjustment is appropriate depends on the extent to which non-durables and services are less volatile at the longer horizons over which our long-run risks shocks are most important. For example, if durables and non-durables are cointegrated, the adjustment is likely to be smaller. The adjustment is also likely to be smaller for earlier points in our sample, when the role of durables in total consumer expenditures was much smaller.

¹¹We exclude countries in Southeast Asia and Latin America from our sample. Including these countries raises our estimates of the importance of long-run risks.

country starting between 1890 and 1914. Our sample period ends in 2009. Figure 2.1 plots our data series for France. We have drawn a trend line through the pre-WWII period and extended this line to the present. The figure strongly suggests that France has experienced very persistent swings in growth over the last 120 year. In analyzing the asset pricing implications of our model, we also make use of total returns data on stocks and bills as well as dividend yields on stocks from Global Financial Data (GFD) and data on inflation from Barro and Ursua (2008).

2.3 An Empirical Model of Growth-Rate and Uncertainty Shocks

Building on the work of Bansal and Yaron (2004), we model the logarithm of the permanent component of per capita consumption in country i at time $t + 1$ —denoted $\tilde{c}_{i,t+1}$ —as evolving in the following way:

$$\begin{aligned}\Delta\tilde{c}_{i,t+1} &= \mu_i + x_{i,t} + \xi_i x_{W,t} + \eta_{i,t+1}, \\ x_{i,t+1} &= \rho x_{i,t} + \epsilon_{i,t+1}, \\ x_{W,t+1} &= \rho_W x_{W,t} + \epsilon_{W,t+1}.\end{aligned}\tag{2.2}$$

Permanent consumption growth is governed by three shocks: a random-walk shock ($\eta_{i,t+1}$), and two shocks that have persistent effects on the growth rate of consumption—one of which is country specific ($\epsilon_{i,t+1}$) and one of which is common across all countries ($\epsilon_{W,t+1}$). The persistence of the effects of the last two of these shocks to consumption growth is governed by AR(1) processes $x_{i,t+1}$ and $x_{W,t+1}$, respectively. We allow the different countries in our sample to differ in their sensitivity to the world growth rate process. This differing sensitivity is governed by the parameter ξ_i .

The volatility of the three shocks affecting permanent consumption growth is time varying and governed by two AR(1) stochastic volatility processes:

$$\sigma_{i,t+1}^2 = \sigma_i^2 + \gamma(\sigma_{i,t}^2 - \sigma_i^2) + \omega_{i,t+1},\tag{2.3}$$

$$\sigma_{W,t+1}^2 = \sigma_W^2 + \gamma(\sigma_{W,t}^2 - \sigma_W^2) + \omega_{W,t+1},\tag{2.4}$$

where $\sigma_{i,t+1}^2$ is a country-specific component of stochastic volatility and $\sigma_{W,t+1}^2$ is a common component of stochastic volatility. We refer to the innovations to these stochastic volatility processes— $\omega_{i,t+1}$ and $\omega_{W,t+1}$ —as uncertainty shocks.¹²

The common component of stochastic volatility $\sigma_{W,t+1}^2$ affects the volatility of all three of the shocks to permanent consumption. The idea here is that when world uncertainty rises this affects the volatility of all shocks to permanent consumption. The country specific component of stochastic volatility $\sigma_{i,t+1}^2$, however, only affects the country specific shocks. Variation in this component, therefore, represents deviations in the uncertainty faced by a particular country from that faced by countries on average. More specifically, we assume that $\text{var}_t(\epsilon_{W,t+1}) = \sigma_{W,t}^2$, $\text{var}_t(\epsilon_{i,t+1}) = \sigma_{i,t}^2 + \sigma_{W,t}^2$, and $\text{var}_t(\eta_{i,t+1}) = \chi_i^2(\sigma_{i,t}^2 + \sigma_{W,t}^2)$, where χ_i governs the relative volatility of the two country specific shocks, $\epsilon_{i,t+1}$ and $\eta_{i,t+1}$.

We allow for correlation between the growth-rate shocks and the uncertainty shocks. This is meant to capture the possibility that times of high uncertainty may also tend to be times of low growth. Specifically, we allow the country-specific growth-rate shock $\epsilon_{i,t+1}$ and the country-specific uncertainty shock $\omega_{i,t+1}$ to be correlated with a correlation coefficient of λ . We also allow the world growth-rate shock $\epsilon_{W,t+1}$ and the world uncertainty shocks $\omega_{W,t+1}$ to be correlated with a correlation coefficient of λ_W .

To summarize, we assume the following distributions for the random-walk, growth-rate and uncertainty shocks:

$$\eta_{i,t+1} \sim N(0, \chi_i^2(\sigma_{i,t}^2 + \sigma_{W,t}^2)), \quad (2.5)$$

$$\begin{bmatrix} \epsilon_{i,t+1} \\ \omega_{i,t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{i,t}^2 + \sigma_{W,t}^2 & \lambda\sigma_\omega\sqrt{\sigma_{i,t}^2 + \sigma_{W,t}^2} \\ \lambda\sigma_\omega\sqrt{\sigma_{i,t}^2 + \sigma_{W,t}^2} & \sigma_\omega^2 \end{bmatrix} \right), \quad (2.6)$$

¹²We could alternatively model $\log \sigma_{i,t+1}^2$ and $\log \sigma_{W,t+1}^2$ as following AR(1) processes. This would allow us to avoid truncating the uncertainty shocks (see below). We have experimented with this specification. However, with this specification, the volatility of σ^2 drops to very low levels when σ^2 is small implying that σ^2 can “get stuck” close to zero for a very long time. It is not clear to us that the data support this feature. Also, our MCMC estimation algorithm runs into trouble in this case since the likelihood function is very flat when $\log \sigma^2$ becomes sufficiently negative (σ^2 sufficiently small). In this region very large movements in $\log \sigma^2$ correspond to tiny movements in σ^2 . This leads the MCMC algorithm to get stuck.

$$\begin{bmatrix} \epsilon_{W,t+1} \\ \omega_{W,t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{W,t}^2 & \lambda_W \sigma_{W,t} \sigma_{\omega,W} \\ \lambda_W \sigma_{W,t} \sigma_{\omega,W} & \sigma_{\omega,W}^2 \end{bmatrix} \right). \quad (2.7)$$

To avoid negative variances, we truncate the process for $\sigma_{W,t+1}^2$ at a small positive value ζ and we truncate the process for $\sigma_{i,t+1}^2$ such that $\sigma_{i,t+1}^2 > \zeta - \sigma_{W,t}^2$.¹³

We allow parameters to vary across countries whenever our data contains enough information to make this feasible. For example, we allow σ_i^2 to differ across countries. This allows some countries to have permanently higher or lower volatility of macroeconomic shocks than others. However, as [Bansal and Yaron \(2004\)](#) emphasize, some of the key parameters of the long-run risks model are difficult to estimate precisely using data from a single country, even with over 100 years of data. For these parameters, we rely on the panel structure of the data set and assume that they are equal for all countries in our data set. The parameters we make this pooling assumption for are: the persistence of the growth-rate components ρ and ρ_W , the persistence of the stochastic volatility processes γ , the volatility of the uncertainty shocks σ_ω^2 and $\sigma_{W,\omega}^2$, the average volatility of the world stochastic volatility process σ_W^2 , and the correlations between the growth-rate and uncertainty shocks λ and λ_W .¹⁴

We allow measured consumption—denoted $c_{i,t}$ —to differ from permanent consumption $\tilde{c}_{i,t}$ because of two transitory shocks:

$$c_{i,t+1} = \tilde{c}_{i,t+1} + \nu_{i,t+1} + I_{i,t+1}^d \psi_{i,t+1}^d. \quad (2.8)$$

The first of these shocks $\nu_{i,t+1}$ is mainly meant to capture measurement error. We assume that this shock is distributed $N(0, \sigma_{i,t,\nu}^2)$, where the volatility of this shock is allowed to differ before and after 1945. By incorporating this break in the volatility of $\nu_{i,t+1}$ we can capture potential changes in national accounts measurement around this time ([Romer, 1986](#); [Balke](#)

¹³For world stochastic volatility, this means that when an $\omega_{W,t+1}$ is drawn that would yield a value of $\sigma_{W,t+1}^2 < \zeta$, we set $\sigma_{W,t+1}^2 = \zeta$. This implies that the innovations to the $\sigma_{W,t+1}^2$ have a positive mean when $\sigma_{W,t+1}^2$ is close to ζ . For the estimated values of the parameters of our model (baseline estimation), $\sigma_{W,t+1}^2 = \zeta$ about 9.2% of the time. We incorporate this truncation in our asset pricing analysis in section 2.6.

¹⁴Notice also, that we assume that the same parameter (γ) governs the persistence of both the common and country-specific components of stochastic volatility. We do this because there is insufficient information in our dataset to estimate a separate parameter for the persistence of world volatility.

and Gordon, 1989). This is empirically important since it avoids the possibility that our estimates of the high persistence of macroeconomic uncertainty arise spuriously from these changes in measurement procedures.

The second shock $I_{i,t+1}^d \psi_{i,t+1}^d$ captures transitory variation in consumption due to disasters.¹⁵ We do not estimate the timing of disasters in this paper. Instead, the dummy variable $I_{i,t}^d$ is set equal to one in periods identified as disaster periods by Nakamura et al. (2010) and during a two year recovery period after each such episode and zero otherwise.¹⁶ The disaster shock $\psi_{i,t}^d$ is distributed $N(\mu_d, 1)$. We fix the variance of $\psi_{i,t}^d$ at 1 (a large value), to ensure that this shock “soaks up” all transitory variation in consumption during the disaster periods. Were we to exclude the disaster shock, we would estimate substantially higher volatilities of the stochastic volatility processes $\sigma_{i,t+1}^2$ and $\sigma_{W,t+1}^2$.

2.4 Estimation

The model presented in section 2.3 contains a large number of unobserved state variables, since it decomposes consumption into several unobserved components. We estimate the model using Bayesian MCMC methods.¹⁷ To carry out our Bayesian estimation we need to specify a set of priors on the parameters of the model. We choose highly dispersed priors for all the main parameters of the model to minimize their effect on our inference. The full set of priors we

¹⁵The permanent effects of disasters are captured by $\eta_{i,t+1}$, $\epsilon_{i,t+1}$, and $\epsilon_{W,t+1}$.

¹⁶Nakamura et al. (2010)’s results indicate that there is unusually high growth after disasters—i.e., recoveries—but that this unusually high growth dies out rapidly—it has a half-life of 1 year. By allowing for a two year recovery period after disasters, we allow the disaster shocks in our model to capture the bulk of the unusually high growth after disasters and avoid having this growth variation inflate our estimates of long-run risks.

¹⁷Our algorithm samples from the posterior distributions of the parameters and unobserved states using a Gibbs sampler augmented with Metropolis steps when needed. This algorithm is described in greater detail in appendix B.1. The estimates discussed in section 2.5 for the three versions of the model, are based on four independent Markov chains each with 5 million draws or more with the first 450,000 draws from each chain dropped as “burn-in”. To assess convergence, we employ Gelman and Rubin’s (1992) approach to monitoring convergence based on parallel chains with “over-dispersed starting points” (see also Gelman, 2004, ch. 11).

use is:

$$\begin{aligned}
\rho &\sim \text{U}(0.005, 0.995), & \rho_W &\sim \text{U}(0.005, 0.995), \\
\gamma &\sim \text{U}(0.005, 0.98), & \sigma_W^2 &\sim \text{U}(10^{-8}, 10^{-2}), \\
\sigma_\omega^2 &\sim \text{U}(10^{-10}, 10^{-6}), & \sigma_{W,\omega}^2 &\sim \text{U}(10^{-10}, 10^{-6}), \\
\lambda &\sim \text{U}(-0.995, 0.995), & \lambda_W &\sim \text{U}(-0.995, 0.995), \\
\xi_i &\sim \text{U}(10^{-4}, 10), & \chi_i^2 &\sim \text{U}(10^{-4}, 25), \\
\sigma_i^2 &\sim \text{U}(10^{-8}, 10^{-2}), & \sigma_{\nu,i}^2 &\sim \text{U}(10^{-8}, 10^{-2}), \\
\mu_i &\sim \text{N}(0.015, 1), & \mu_d &\sim \text{N}(0, 1),
\end{aligned}$$

except that we normalize $\xi_{US} = 1$ to identify the scale of the world stochastic volatility process. We assume that the initial values of $x_{i,t}$, $x_{W,t}$, $\sigma_{i,t}$ and $\sigma_{W,t}$ are drawn from their unconditional distributions. We assume that the initial value of \tilde{c}_{it} for each country is drawn from a highly dispersed normal distribution centered on the initial observation for $c_{i,t}$. It can be shown that the model is formally identified except for a few special cases in which multiple shocks have zero variance.

2.5 Empirical Results

Our baseline empirical results are for the full model described in section 2.3 for the full sample period 1890-2009. We also report results for a shorter post-WWII sample period and for a simplified version of the model in which we shut down the world growth-rate and volatility components as well as the correlation between the country-specific growth-rate and volatility shocks. We refer to this latter model as the “simple model.” Tables 2.1-2.3 present parameter estimates for these three cases. For each parameter, we present the prior and posterior mean and standard deviation. We refer to the posterior mean of each parameter as our point estimate for that parameter.

We estimate a highly persistent world growth-rate process in our baseline model. The autoregressive coefficient for the world growth-rate component is estimated to be $\rho = 0.83$, implying a half-life of 3.8 years. The country specific growth-rate process is estimated to be less persistent. The autoregressive coefficient for the country-specific growth-rate component

is estimated to be $\rho = 0.56$, implying a half-life of 1.2 years. Table 2.2 compares these estimates to the calibration of the persistence of the growth-rate process in [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2012\)](#). The persistence of the growth-rate process in these papers is in-between that of our world and country-specific growth-rate processes. In the simple model, the persistence of the (country-specific) growth-rate component is estimated to be $\rho = 0.68$, which implies a half-life of 1.8 years. This illustrates that allowing for a world growth-rate component is important in capturing the low-frequency variation in growth in our dataset.

Figure 2.2 plots the impulse response of consumption to our estimated growth-rate processes as well as to the random-walk shock. The figure shows clearly that despite the relatively modest half-lives of the growth-rate shocks, their effects on output are very different from those of the random-walk shock. After a country-specific growth-rate shock, consumption continues to grow for several periods and eventually rises by more than two times the initial size of the shock with the bulk of the growth occurring in the first 5 years. After a world growth-rate shock, continuing growth in subsequent periods leads the eventual impact of the shock on consumption to be six times its initial impact with roughly a third of that growth occurring more than 5 years after the shock.

Figure 2.3 presents our estimate of the world growth-rate process. The most striking feature of this process is its high values in the 1950's, 60's and early 70's. This captures the persistently high growth seen in many countries in our sample in the 3rd quarter of the 20th century.¹⁸ The world growth-rate process also captures several major recessions such as the 1979-82 recession following the spike in oil prices that accompanied the Iranian Revolution as well as the tightening of U.S. monetary policy, the recession of 1990 following, among other events, the Persian Gulf War, the unification of Germany, and the accompanying tightening of German monetary policy, and the 2008 recession following the sharp fall in house prices in several countries, associated collapse of major financial institutions and turmoil in financial markets.

¹⁸It is intriguing that this growth spurt so closely followed World War II. It is tempting to infer that this high growth is due to post-war reconstruction. However, for most countries, the vast majority of the unusually high growth during this period occurred in years when consumption (and output) had surpassed its pre-WWII trend-adjusted level (see, e.g., [Figure 2.1](#)).

We estimate large and very persistent shocks to economic uncertainty. Table 2.1 reports that our estimate of the autoregressive coefficient for the uncertainty processes in the baseline estimation is $\gamma = 0.970$. This implies that uncertainty shocks in the baseline case have a half-life of 22.8 years (Table 2.2). This estimate lies between the 4.4 year calibration of [Bansal and Yaron \(2004\)](#) and the 57.7 year calibration of [Bansal et al. \(2012\)](#). Uncertainty shocks are also estimated to be highly persistent in the simple model and in the post-WWII sample. For these cases, we estimate half-lives of 13.5 years and 18.2 years, respectively.

Figure 2.4 presents our estimates of the evolution of the world stochastic volatility process ($\sigma_{W,t}$). We estimate that world volatility was high in the early post-WWII period and has been on an uneven downward trend since then. World volatility fell a great deal in the 1960's, but was high again in the 1970's and early 1980's. It fell sharply in the mid-to-late 1980's but was relatively high in the early 1990's. From 1995 to 2007 the world experienced a long period of relative tranquility with volatility falling sharply towards the end of this period to record lows. At the end of our sample period, world volatility rose sharply once again. In studying this figure, it is important to keep in mind that our model attributes much of the volatility in the first half of our sample to our disaster and measurement error shocks.

Comparing Figures 2.3 and 2.4, it is evident that the world growth-rate process and the world stochastic volatility process are negatively correlated. Our model allows explicitly for a correlation between shocks to these processes (λ_W). Table 2.1 reports that our estimate of this correlation is -0.25. We also estimate a common correlation between the country-specific growth-rate and uncertainty shocks in our data and find this correlation to also be -0.40. Our estimates, thus, strongly suggest that periods of high volatility are also periods of low growth.

We estimate a substantial amount of heterogeneity in the evolution of volatility across countries. Figure 2.5 presents our estimates of the evolution of the volatility process for the U.S., the U.K. and Canada— $(\sigma_{i,t}^2 + \sigma_{W,t}^2)^{1/2}$ in our notation. For the United States our results reflect the “long and large” decline in macroeconomic volatility documented by [Blanchard and Simon \(2001\)](#) and well as the rather abrupt decline in volatility in the mid-1980's documented by [McConnell and Perez-Quiros \(2000\)](#) and [Stock and Watson \(2002b\)](#). The experience of

the U.K. is quite different. Volatility in the U.K. was lower in the early part of the 20th century (excluding disasters), but then rose substantially over the first three decades after WWII. Volatility in the U.K. began falling only around the time Margaret Thatcher came to power and has remained elevated relative to volatility in the U.S. ever since 1960. In contrast, volatility in Canada fell much more abruptly in the 1950's and early 1960's than volatility in the U.S. and was substantially below U.S. volatility in the 1960's, 1970's and early 80's at which point U.S. volatility converged down to similarly low levels.

One feature of our results that differs markedly from the calibrations of the long-run risks model used in [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2012\)](#) is that the growth-rate shocks we estimate are substantially more volatile. Recall that the parameter χ_i governs the relative volatility of the random-walk shock ($\eta_{i,t}$) and the growth-rate shock ($\epsilon_{i,t}$). Estimates for this parameter as well as other country-specific parameters are reported in [Table 2.3](#). For the median country, we estimate χ_i to be 0.81, while we estimate a value of 1.16 for the United States.¹⁹ Our estimates thus imply that the growth-rate shocks and the random-walk shocks are roughly equally volatile. [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2012\)](#) calibrate the growth-rate shock to be only about 5% as volatile as the random-walk shock.

We allow countries to differ in their sensitivity to the world growth-rate process. The parameter ξ_i governs this sensitivity. We fix $\xi_{US} = 1$, implying that for other countries this parameter can be interpreted as their sensitivity to world shocks relative to the sensitivity of the U.S. to these shocks. For the median country, our estimate of $\xi_i = 1.51$. In particular, many continental European countries have values of ξ_i that are substantially larger than one (see [Table 2.9](#)). This heterogeneity in sensitivity to the world-growth rate shock is one source of heterogeneity in risk-premia across countries in our asset-pricing calculations in [section 2.6](#). We estimate a substantial decline in the volatility of transitory shocks $\sigma_{\nu,i}$ after 1945 in most countries. This change likely reflects in part changes in national accounts measurement, as we discuss in [section 2.3](#).²⁰

¹⁹Estimates for all 16 countries for our baseline case are presented in the appendix ([Table 2.9](#)).

²⁰[Ursua \(2010\)](#) argues—based on methods developed by [Romer \(1986\)](#)—that this change also reflects changes in macroeconomic fundamentals. Since transitory shocks turn out to be relatively unimportant for asset pricing, the choice of whether to treat this change as a consequence of measurement or fundamental shocks plays a

One potential concern with our results is that they might be influenced by our treatment of disasters in the early part of our sample. Another potential concern is that the quality of the data for the period before World War II may be lower than for the more recent period. To address these concerns, we estimate our model on data starting in 1950. This yields results that are very similar to our baseline estimation along most dimensions. The main deviation is that in this case we estimate a smaller and less volatile world stochastic volatility process and larger values of the sensitivity to the world growth-rate shock for most countries. Also, the posterior standard deviation of several key parameters increases substantially—in particular, the standard deviation of the sensitivity to the world growth-rate—reflecting the much smaller sample. For the median country, the degree to which consumption growth is driven by the world growth-rate shock rises since the increase in the sensitivity to the world growth-rate shock is larger than the decrease in the volatility of the world growth-rate shocks.

2.5.1 Autocorrelations, Cross-Country Correlations and Variance Ratios

Given that we estimate large persistent components to consumption growth, one might worry that our estimated model implies too much autocorrelation of consumption growth relative to the data. Table 2.4 presents estimates of autocorrelations, cross-country correlations and variance ratios in the data and in the model. We report statistics for the median country in our dataset and for the United States. Both for the data and the model, we exclude transitory variation in consumption due to disasters.²¹

Consider first the autocorrelations of consumption growth. In the data, the autocorrelations for the median country are positive but small at all horizons; for the US, they oscillate around zero. The model also generates modest autocorrelations at all horizons. The 95% probability intervals generated by the model contain the corresponding empirical statistics in almost all cases.²² Despite assigning an important role to long-run risks, our estimated

small role in our asset pricing analysis.

²¹For the real world data, we do this by subtracting from the raw data our estimate of the transitory disaster shock. This yields series for consumption that smoothly “interpolate” through disasters. For the simulated data from our model, we simulate the model without the transitory disaster shock.

²²Estimated on the post-WWII sample, the autocorrelations for the U.S. oscillate less and are slightly

model yields modest short-term autocorrelation in the growth rate of consumption because the model also features transitory shocks to the level of consumption, which generate an offsetting negative correlation in short-term growth rates.

The cross-country correlation of consumption growth for the median country is estimated to be substantial and to grow with the horizon of the growth rates. The median one-year cross-country correlation is 0.23, while it is 0.44 at the five year horizon and 0.56 at the ten year horizon. The model is able to capture both the magnitude and the increasing pattern of this cross-country correlation through the world growth-rate process. The correlation of the U.S. with other countries in our sample is somewhat smaller than for the median country both in the data and in the model.

Table 2.4 also reports estimates of variance ratios for consumption growth and the volatility of consumption growth at the 15 year horizon for the median country and for the United States. Variance ratios above one indicate reduced form evidence for positive autocorrelation of consumption growth and volatility. The definition and intuition for these statistics is discussed in more detail in appendix B.2. In the data, the variance ratio for consumption growth for the median country is 1.69, substantially above one. The average across countries is even higher at 2.28. For the U.S. it is somewhat smaller but still above one.²³ These high variance ratios provide reduced form evidence for positive autocorrelation of growth rates. Our model captures this well. For the median country, the model generates a 15-year variance ratio of 2.69. The variance ratio of realized volatility is substantially larger than one both in the median country and in the United States. Again, our model is able to capture this feature of the data well.

negative at horizons longer than one year.

²³We have also calculated these variance ratios including disasters and they are lower both in the data and in the model. Excluding disasters raises the variance ratio of consumption growth because disasters are typically followed by significant recoveries (Kilian and Ohanian, 2002; Nakamura et al., 2010). Ursua (2010) presents a related analysis. Rather than filtering the data the way we do, he excludes “outlier” growth observations. This simpler procedure also yields substantially larger variance ratios than raw consumption growth in his broader sample.

2.6 Asset Pricing

We analyze the asset pricing implications of the model of aggregate consumption described in section 2.3 within the context of a representative consumer endowment economy with Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990). For this preference specification, Epstein and Zin (1989) show that the return on an arbitrary cash flow is given by the solution to the following equation:

$$E_t \left[\beta^\theta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{(-\theta/\psi)} R_{c,t,t+1}^{-(1-\theta)} R_{i,t,t+1} \right] = 1, \quad (2.9)$$

where $R_{i,t,t+1}$ denotes the gross return on an arbitrary asset in country i from period t to period $t + 1$, $R_{c,t,t+1}$ denotes the gross return on the agent's wealth, which in our model equals the endowment stream. The parameter β represents the subjective discount factor of the representative consumer. The parameter $\theta = \frac{1-\gamma}{1-1/\psi}$, where γ is the coefficient of relative risk aversion (CRRA) and ψ is the intertemporal elasticity of substitution (IES), which governs the agent's desire to smooth consumption over time.

We begin by calculating asset prices for two assets: a risk-free one-period bond and a risky asset we will use to represent equity. The risk-free one-period bond has a certain pay-off of one unit of consumption in the next period. We follow Bansal, Kiku, and Yaron (2012) in modeling equity as having a levered exposure to the stochastic component of permanent consumption. Specifically, the growth rate of dividends for our equity claim is

$$\Delta d_{t+1} = \mu + \phi(x_{i,t} + \xi_i x_{W,t} + \eta_{i,t}), \quad (2.10)$$

where ϕ is the leverage ratio on expected consumption growth (Abel, 1999). We base our analysis on the posterior mean estimates for the baseline case from section 2.5. We therefore abstract from learning, doubt and fragile beliefs (Timmermann, 1993; Pastor and Veronesi, 2009; Hansen, 2007; Hansen and Sargent, 2010). These issues are potentially important in our context, given the difficulty of estimating long-run risks, both for the econometrician, and the economic agent (see, e.g., Croce et al., 2010).

The asset-pricing implications of our model with Epstein-Zin-Weil (EZW) preferences cannot be derived analytically. We solve for asset prices in our model using standard grid-based numerical methods of the type used, e.g., by [Campbell and Cochrane \(1999\)](#) and [Wachter \(2005\)](#).²⁴ We choose a subjective discount factor of $\beta = 0.990$ to fit the observed average risk-free rate in our baseline specification. We choose a CRRA of $\gamma = 6.5$ to match the U.S. equity premium in our baseline specification. We follow the long-run risks literature in choosing an IES of $\psi = 1.5$ ([Bansal and Yaron, 2004](#); [Bansal et al., 2012](#)).²⁵ We follow [Bansal and Yaron \(2004\)](#) in setting leverage of $\phi = 3$. We calculate asset prices for a consumption process that ignores the transitory disaster shock in our model.²⁶ We do this to focus attention on the asset-pricing implications of long-run risks. Allowing for transitory drops in consumption due to disasters would further raise the equity premium we estimate (or equivalently allow us to match the equity premium with a lower value of the CRRA) but at considerable cost in terms of computational complexity. The asset pricing implications of disaster risk have been the focus of a large recent literature (see, e.g., [Barro, 2006](#), and [Nakamura, et al., 2010](#)). We present asset pricing results for the post-WWII estimation of our model—a sample without major disasters in our sample of countries—in an appendix ([Table 2.10](#)).

2.6.1 The Effects of Long-Run Risks on Asset Prices

[Figure 2.6](#) presents impulse responses for the return on equity and the risk-free rate to a world growth-rate shock. A positive world growth-rate shock yields a large positive return on equity

²⁴We solve the integral in [equation \(2.9\)](#) on a grid. Specifically, we start by solving for the price-dividend ratio for a consumption claim. In this case we can rewrite [equation \(2.9\)](#) as $PDR_t^C = E_t[f(\Delta C_{t+1}, PDR_{t+1}^C)]$, where PDR_t^C denotes the price dividend ratio of the consumption claim. We specify a grid for PDR_t^C over the state space. We then solve numerically for a fixed point for PDR_t^C as a function of the state of the economy on the grid. We can then rewrite [equation \(2.9\)](#) for other assets as $PDR_t = E_t[f(\Delta C_{t+1}, \Delta D_{t+1}, PDR_{t+1}^C, PDR_{t+1})]$, where PDR_t denotes the price dividend ratio of the asset in question and ΔD_{t+1} denotes the growth rate of its dividend. Given that we have already solved for PDR_t^C , we can solve numerically for a fixed point for PDR_t for any other asset as a function of the state of the economy on the grid.

²⁵There is little agreement in the macroeconomics and finance literatures on the appropriate value for the IES. [Hall \(1988\)](#) and [Campbell \(1999\)](#) estimate the IES to be close to zero. However, [Hansen and Singleton \(1982\)](#), [Bansal and Yaron \(2004\)](#), [Gruber \(2006\)](#), [Hansen et al. \(2007\)](#) and [Nakamura et al. \(2010\)](#) argue for values of the IES above one.

²⁶Recall that the permanent effects of disasters on consumption are captured by the random-walk and growth rate shocks in our model.

on impact. This positive return reflects the balance of two opposing forces. On the one hand, the shock raises expected future dividends on equity, which pushes up stock prices. On the other hand, since consumption growth is expected to be high for some time, agents' desire to save falls, which pushes down all asset prices. If agents are sufficiently willing to substitute consumption over time ($IES > 1$), as we assume, the first of these effects is stronger than the second for equity and the price of equity rises on impact. In the periods after the shock, returns on equity and the risk-free rate are higher than average because of agents' reduced desire to save.

Figure 2.7 presents impulse responses for the return on equity and the risk-free rate to an uncertainty shock. A shock that increases economic uncertainty yields a large negative return on equity on impact. As with the growth-rate shock, there are two opposing forces that together determine the response of stock prices. The increase in economic uncertainty makes stocks riskier, which raises the equity premium. This tends to depress their value. However, the increase in uncertainty also increases the desire of agents to save. This tends to raise the price of all assets. With $CRRA > 1$ and $IES > 1$, the first force is stronger than the second and the price of stocks falls on impact (Campbell, 1993). In the periods after the shock, the equity premium remains elevated because uncertainty has risen. A one standard deviation shock to $\omega_{W,t}$ raises the equity premium by roughly 0.6% in the period after the shock.

Notice that in our model neither the growth-rate shock nor the uncertainty shock affect consumption growth on impact. For an agent with power utility, these shocks would therefore not affect marginal utility on impact. This implies that agents with power utility would not demand a risk premium on stocks as compensation for exposure to these shocks. With EZW utility, however, marginal utility depends not only on current consumption but also on news about future consumption.²⁷ In equation (2.9), this is captured by the presence of the return on wealth— $R_{c,t+1}$. Since negative growth-rate shocks and shocks that increase uncertainty imply negative returns on wealth on impact, they increase marginal utility. Households are, thus, willing to pay a premium for assets that provide insurance against growth-rate and

²⁷This implication of EZW preferences is illustrated elegantly by the decomposition developed by Borovicka et al. (2011).

uncertainty shocks. Conversely, they demand a risk premium for holding assets that expose them to these shocks.

2.6.2 Risk-Premia and Return Volatility

Two key features of the asset pricing data are the equity premium and the large volatility of equity returns. Long-run risks shocks make the world a riskier place, leading both the level and the volatility of equity returns to rise relative to the risk free rate. Table 2.5 presents key asset pricing statistics in the data and for our baseline specification of the model. The table presents results for the U.S. and for the median country in our sample.

Our model matches the observed equity premium for the United States with a CRRA of 6.5, a value that is an order of magnitude lower than in a model without long-run risks (Mehra and Prescott, 1985; Tallarini, 2000). On this metric, we find more long-run risks than the original calibrations of the long-run risks model, which require a CRRA of 10 to fit the equity premium. Our model also generates highly volatile returns on equity. The standard deviation of equity returns for the U.S. is 18% in the model versus 17% in the data. Finally, the model generates large and persistent swings in the price-dividend ratio, similar to those observed in the data. For the U.S., the standard deviation of the price-dividend ratio in the model is 0.3 and its first-order autocorrelation is 0.85, while these statistics are 0.4 and 0.9 in the data, respectively.²⁸

One might worry that the model would generate counterfactually large variation in the risk-free rate owing to fluctuations in households' desire to save associated with the long-run risk shocks. This is, however, not the case. The standard deviation of the risk-free rate generated by our model is 1.6%. The standard deviation of ex post real returns on U.S. T-bills, our empirical measure of this statistic, is 3.3%. Since the model does not incorporate inflation risk, it is appropriate that the model yields a lower number than the data along this

²⁸Table 2.10 presents analogous results to Table 2.5 for our two alternative specifications. The simple model (without world components) generates a slightly smaller equity premium than the baseline case—roughly 4%. This specification matches the equity premium for a CRRA of 10. A higher CRRA is required because the simple model doesn't capture the persistent world component of consumption growth. The post-WWII case generates very similar results for the U.S. but a larger equity premium for the median country. This arises because the median country becomes more sensitive to the world growth-rate shock in this specification.

dimension.

Roughly half of the increase in the equity premium in our model results from the growth-rate shocks and the other half from the uncertainty shocks. This can be seen from Table 2.6. The table presents results on the equity premium and the risk free rate for all 16 countries in our sample. The full model generates equity premia ranging from 8-23% with an average equity premium of 13.7%. The second case preserves the baseline parameter values of the full model, but turns off the uncertainty shocks. This “constant volatility” model yields equity premia that are roughly half as large as the full model. The third case eliminates all long-run risks and re-calibrates the volatility of the random-walk shocks to match the volatility of $\Delta\tilde{c}_{i,t}$. This case corresponds closely to the model considered by [Mehra and Prescott \(1985\)](#). It generates equity premia that are more than an order of magnitude smaller than the full model.

Our estimated model tends to generate a higher equity premium in countries in which the equity premium in the data has been higher. The correlation between the equity premium in our model and the equity premium in the data is 0.25. Spain is an outlier and if we exclude it this correlation rises to 0.50. In our model, cross-country variation in the equity premium is driven by variation in the extent to which growth-rates in different countries load on the world growth-rate process (ξ_{it}) and also because of variation in the average level of volatility across countries (σ_i^2).

We estimate a negative correlation between growth-rate and uncertainty shocks—i.e., negative growth-rate shocks tend to be associated with shocks that raise economic uncertainty. Since negative growth-rate shocks and shocks that increase uncertainty both raise marginal utility, being hit by both at the same time is particularly painful for the representative agent. This implies that the negative correlation between these two shocks contributes positively to the equity premium in our model. We have calculated asset prices for a case with $\lambda = \lambda_W = 0$ but keeping other parameters unchanged. This yields an equity premium that is 0.8 percentage points smaller for the U.S. than our baseline case.

Finally, we analyze the term structure implications of our model. We approximate long-

term bonds by a perpetuity with coupon payments that decline over time by 10% per year. This yields a bond with a duration similar to that of 10-year coupon bonds. In our model, the term-premium for this real long-term bond is -2.0%. [Piazzesi and Schneider \(2006\)](#) document that the real yield curve in the United Kingdom has been downward sloping, while it has been mostly upward sloping in the United States. They caution, however, that this evidence is hard to assess because of the short sample and poor liquidity in the U.S. TIPS market.²⁹

2.6.3 Predictability of Returns, Consumption and Volatility

A large literature in finance has argued that a high price-dividend ratio predicts low stock returns ([Campbell and Shiller, 1988](#); [Fama and French, 1988](#); [Hodrick, 1992](#); [Cochrane, 2008](#); [Binsbergen and Koijen, 2010](#)).³⁰ Leading asset pricing models differ in their implications about return predictability. In the long-run risks model, uncertainty shocks cause variation in the price-dividend ratio on stocks that forecasts stock returns. More generally, variation in the price-dividend ratio on stocks comes from two sources in the long-run risks model: growth-rate shocks and uncertainty shocks. Consequently, the price-dividend ratio on stocks should forecast not only future returns on stocks but also future volatility and future consumption growth.

Table 2.7 presents results on the predictability of five-year excess returns on equity, realized volatility and consumption growth in our estimated models. We estimate equations of the

²⁹Building on Alvarez and Jermann's (2005) analysis of the implication of the term structure for the properties of the stochastic discount factor, Koijen et al. (2010) emphasize that the positive autocorrelation of growth rates in the long-run risk model implies that the model has a downward sloping term structure of real bond yields. Binsbergen et al. (2010a,b) show that short term dividend strips on the aggregate stock market have substantially higher expected returns than the stock market as a whole. (The price of a k-year dividend strip is the present value of the dividend paid in k years.) They point out that this fact is difficult to match using the original calibration of the long-run risk model proposed by [Bansal and Yaron \(2004\)](#). [Croce, Lettau, and Ludvigson \(2010\)](#) show that a model with long-run risk shocks that agents do not observe directly but must instead learn about over time can generate high excess returns on short-term assets relative to long-term assets.

³⁰The statistical significance of return predictability has been hotly debated (see, e.g., [Stambaugh, 1999](#); [Ang and Bekaert, 2007](#)). Recent work by [Lewellen \(2004\)](#) and [Cochrane \(2008\)](#) has exploited the stationarity of price-dividend ratios and the lack of predictability of dividend growth to develop more powerful tests of return predictability. These tests reject the null of no predictability of returns at the 1-2% level.

following form

$$y_{i,t+5} = \alpha_i + \beta_i pd_{i,t} + \epsilon_{i,t+5}, \quad (2.11)$$

where $pd_{i,t}$ denotes the logarithm of the price-dividend ratio on equity and $y_{i,t+5}$ is one of three things: five-year excess returns on stocks, five-year realized volatility or five-year consumption growth.³¹ We estimate these regressions in the data for the countries in our sample, and we run the same regressions on simulated datasets of the same length (120 years) from our baseline estimation and our simple model. We report the median from 1000 such simulations, as well as the 2.5% and 97.5% quantiles. For comparison, Table 2.7 also presents the degree of predictability of these variables in the models of [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2012\)](#).

The first panel of Table 2.7 presents results on the predictability of excess returns. Our point estimates imply a large degree of predictability of returns in the U.S. data. The regression coefficient on the price-dividend ratio is -0.41 and the R-squared of the regression is 0.24. We estimate less predictability of returns for the median country in our sample—the regression coefficient is -0.30 and the R-squared is 0.11. Our baseline case generates a median regression coefficient of -0.40 and a median R-squared of 0.10. The simple model yields similar results. Our model can thus account for a large fraction of the predictability of excess 5-year stock returns seen in the data. Our estimated model generates more predictability of excess stock returns than do the calibrations of the long-run risks model in [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2012\)](#).

The second panel of Table 2.7 presents results on the predictability of volatility. We find that the price-dividend ratio on stocks has substantial predictive power for realized volatility of consumption growth. For the U.S., the regression coefficient is -0.81 and the R-squared 0.32. For the median country in our sample, predictability of volatility is smaller, but nevertheless substantial—the regression coefficient is -0.38 and the R-squared is 0.19. These results are in line with earlier results by [Bansal et al. \(2005\)](#). Our model generates predictability of

³¹We follow [Bansal et al. \(2005\)](#) in using the absolute value of the residual from an AR(1) regression for consumption growth as our measure of realized volatility and summing this over five years.

volatility that lines up well with the data. The regression coefficients for our baseline case is -0.37 and the R-squared is 0.07 , while for the simple model we get a regression coefficient of -0.91 and an R-squared of 0.12 . The values for the U.S. and for the median country are well within the 95% probability interval we construct.

Our model also implies a low frequency link between stock prices and macroeconomic uncertainty. Figure 2.8 plots our estimate of the evolution of economic uncertainty in the U.S. along with the dividend-price ratio on stocks. The figure illustrates the comovement between economic uncertainty and the value of the stock market emphasized by Lettau et al. (2008). Figure 2.9 in the appendix presents analogous plots for all countries in our sample. This extends the results of Lettau et al. (2004) by including more countries and longer sample periods for several countries. The comovement of economic uncertainty and stock prices varies across countries and time. It is not very strong for most countries before 1970, but is stronger after this.

The third panel of Table 2.7 presents results on the predictability of consumption growth. The price-dividend ratio on stocks has little predictive power for consumption growth both in the U.S. or in the median country in our sample. These results extend earlier work by Beeler and Campbell (2012). Our estimated version of the long-run risks model generates somewhat more predictability of consumption growth than we see in the data. In the data, the regression coefficients for this regression is less than 0.05 . In the model, the median regression coefficient across model runs is 0.19 in our baseline case. However, the empirical value lies within the 95% probability interval generated by our model. Our estimated model generates a degree of predictability of consumption growth that is intermediate between that in Bansal and Yaron (2004) and Bansal et al. (2012).

2.6.4 The Volatility of Real Exchange Rates

An important finding from our empirical analysis is that there is a large amount of comovement of growth-rates and uncertainty across countries. This has important implications for real exchange rates. In a world with complete markets, the log change in the real exchange rate

between two countries is

$$\Delta e_t = m_t^* - m_t, \quad (2.12)$$

where e_t denotes the log real exchange rate (home goods price of foreign goods), and m_t and m_t^f are the logarithm of the home and foreign stochastic discount factors, respectively. The annual standard deviation of changes in real exchange rates has been roughly 10% in the post-Bretton Woods period (see Table 2.8). However, Hansen and Jagannathan (1991) show that $\sigma(M_t)R_t^f \geq E(R_t^e)/\sigma(R_t^e)$, where M_t is the level of the stochastic discount factor and R^e is the excess return on the stock market. From Table 2.5 we can see that $R^f \simeq 1.01$, $E(R_t^e) \simeq 7\%$, and $\sigma(R_t^e) \simeq 18\%$, which implies $\sigma(M_t) \geq 40\%$. Brandt, Cochrane, and Santa-Clara (2006) point out that this logic combined with equation (2.12) implies that either m_t and m_t^* are highly correlated—i.e., there is a high degree of international risk sharing—or exchange rates are not as volatile as the theory predicts. In addition, the low degree of comovement of consumption growth across countries at short horizons suggests that stochastic discount factors are not highly correlated. Colacito and Croce (2011) refer to this as the international equity premium puzzle.

The common components of growth-rates and uncertainty that we estimate have the potential to resolve this puzzle. They generate comovement in the stochastic discount factors across countries that is not evident from the short-run comovement of consumption growth. Table 2.8 presents the standard deviation implied by our estimated model of annual changes in the bilateral real exchange rate versus the United State for each country in our sample. The table also presents a counter-factual for this statistic based on the same simulated data from our estimated model but ignoring the correlation between the stochastic discount factors of each country and the United States that is implied by our model—i.e., simply adding the variances of the two stochastic discount factors and taking a square root. We see that the presence of common long-run risk shocks in our model lowers the volatility of the real exchange rate by roughly a factor of two relative to what it would be if the stochastic discount factors were uncorrelated. Our model can therefore account for a large part of the discrepancy between the observed volatility of the real exchange rate and the volatility implied by a model

in which marginal utility across countries is uncorrelated. Our results complement those of Colacito and Croce (2011), who carry out a related exercise for the exchange rate of the U.S. versus the U.K.

2.7 Conclusion

The long-run risks model is one of the leading frameworks of consumption-based asset pricing. It is difficult to obtain precise estimates of the key parameters of this model using even 100 years of macroeconomic data from a single country. As a consequence, previous work has used a combination of macroeconomic and asset price data to estimate the model. Our model of consumption dynamics allows for country-specific variation in the average level of volatility across countries, but pools across countries in estimating the persistence of growth-rate and uncertainty shocks as well as the volatility of shocks to uncertainty. This allows us to estimate long-run risk parameters using macroeconomic data alone. We can thereby avoid relying on a particular asset pricing model, and the concern that our estimates derive from a need to fit the asset pricing data.

Our estimates suggest that growth-rate and uncertainty shocks play an important role in asset pricing. We identify a large and persistent world growth-rate component and a less persistent country-specific growth-rate process. Shocks to uncertainty are highly persistent and yield substantial variation in uncertainty over time. With EZW preferences, current marginal utility depends not only on current consumption growth but also on news about future growth and uncertainty. With a $CRRA > 1$ and $IES > 1$, shocks that lower future expected growth or raise future economic uncertainty raise current marginal utility and cause stock prices to fall. This generates a substantial equity premium, high volatility of equity returns, and predictability of returns based on the price-dividend ratio.

2.8 Appendix: Figures and Tables

Table 2.1: Estimates for Pooled Parameters

	Prior	Baseline	Simple Model	Post-WWII
<i>Persistence:</i>				
Country-Specific Growth-Rate Shocks (ρ)	0.500 (0.286)	0.565 (0.046)	0.682 (0.038)	0.622 (0.060)
World Growth-Rate Shocks (ρ_w)	0.500 (0.286)	0.832 (0.077)	--	0.832 (0.093)
Stochastic Volatility (γ)	0.493 (0.281)	0.970 (0.011)	0.950 (0.028)	0.963 (0.024)
<i>Standard Deviations:</i>				
Mean of World Stoch. Vol. Process (σ_w)	0.0667 (0.0236)	0.0053 (0.0028)	--	0.0032 (0.0025)
Country-Specific Stoch. Vol. Shock (σ_ω)	0.000667 (0.000236)	0.000028 (0.000007)	0.000054 (0.000015)	0.000037 (0.000011)
World Stoch. Vol. Shock ($\sigma_{\omega,w}$)	0.000667 (0.000236)	0.000024 (0.000009)	--	0.000007 (0.000007)
<i>Correlations:</i>				
Country-Specific (λ)	0.00 (0.57)	-0.40 (0.17)	--	-0.34 (0.20)
World (λ_w)	0.00 (0.57)	-0.25 (0.28)	--	-0.32 (0.32)

The table reports prior and posterior means of the parameters with prior and posterior standard deviations in parentheses. The "Baseline" case is for our full model estimated on data from 1890-2009. The "Simple Model" case is for our simple model estimated on data from 1890-2009. The "Post-WWII" case is for our full model estimated on data from 1950-2009.

Table 2.2: Half-Life of Growth-Rate and Uncertainty Shocks

	Half-Life in Years		
	Growth-Rate Process		Uncertainty Process ($\sigma_{i,t}^2$ and $\sigma_{w,t}^2$)
	Country-Specific ($x_{i,t}$)	World ($x_{w,t}$)	
Baseline	1.2	3.8	22.8
Simple Model	1.8	--	13.5
Post-WWII	1.5	3.8	18.2
Bansal and Yaron (2004)	2.7	--	4.4
Bansal, Kiku and Yaron (2012)	2.3	--	57.7

Table 2.3: Estimates for Country-Specific Parameters

	Prior	Baseline		Simple Model		Post-WWII	
		Median	U.S.	Median	U.S.	Median	U.S.
Rel. St. Dev. of Random Walk Shock (χ_i)	3.38 (1.18)	0.81 (0.45)	1.16 (0.44)	0.87 (0.45)	0.99 (0.50)	1.09 (0.60)	1.09 (0.60)
Sensitivity to Common Shocks (ξ_i)	5.00 (2.89)	1.51 (0.55)	1.00 (0.00)	--	--	4.65 (1.62)	1.00 (0.00)
Average Growth (μ_i)	0.015 (1.00)	0.016 (0.008)	0.018 (0.006)	0.016 (0.004)	0.017 (0.004)	0.022 (0.015)	0.021 (0.006)
<i>Standard Deviations:</i>							
Average Stochastic Volatility (σ_i)	0.067 (0.024)	0.009 (0.004)	0.009 (0.004)	0.013 (0.005)	0.012 (0.005)	0.011 (0.004)	0.011 (0.004)
Post-1945 Transitory Shock (σ_{vi})	0.067 (0.024)	0.004 (0.002)	0.003 (0.001)	0.004 (0.002)	0.004 (0.002)	0.003 (0.002)	0.003 (0.002)
Pre-1945 Transitory Shock (σ_{vi})	0.067 (0.024)	0.024 (0.005)	0.023 (0.005)	0.022 (0.005)	0.023 (0.005)	--	--

The table reports prior and posterior means of the parameters with prior and posterior standard deviations in parentheses. The "Baseline" case is for our full model estimated on data from 1890-2009. The "Simple Model" case is for our simple model estimated on data from 1890-2009. The "Post-WWII" case is for our full model estimated on data from 1950-2009. "Median" refers to the median country. In other words, we report the value of each statistic - both means and standard deviations - for the country that has the median value of that statistic.

Table 2.4: Properties of Consumption Growth

	Median Country			United States		
	Data	Model Median	Model [2.5%, 97.5%]	Data	Model Median	Model [2.5%, 97.5%]
AC(1)	0.13	0.25	[0.01,0.51]	-0.08	0.07	[-0.21,0.35]
AC(2)	0.14	0.28	[0.12,0.50]	0.15	0.18	[-0.06,0.40]
AC(3)	0.04	0.21	[0.06,0.42]	-0.21	0.12	[-0.11,0.35]
AC(4)	0.07	0.15	[0.01,0.36]	0.28	0.09	[-0.15,0.31]
AC(5)	0.00	0.11	[-0.02,0.32]	-0.09	0.07	[-0.16,0.28]
AC(10)	0.12	0.01	[-0.13,0.17]	0.12	0.01	[-0.21,0.22]
CrossC(1)	0.23	0.34	[0.17,0.56]	0.18	0.25	[0.08,0.47]
CrossC(5)	0.44	0.65	[0.37,0.84]	0.43	0.54	[0.18,0.79]
CrossC(10)	0.56	0.73	[0.42,0.90]	0.54	0.64	[0.20,0.87]
VR(15) ΔC	1.62	2.57	[1.18, 5.17]	1.29	1.70	[0.56, 4.14]
VR(15) Vol	2.14	1.76	[1.01, 3.04]	1.80	1.95	[0.68, 4.48]

The Table reports autocorrelations, cross-country correlations and variance ratios for the real-world data and simulated data from the model (excluding disasters in both cases). The first through sixth rows present the autocorrelation of one year through five year and ten year consumption growth. The next three rows present cross-country correlations of one, five and ten year consumption growth. The last two rows present the fifteen year variance ratio of consumption growth and the realized volatility of consumption growth. For the cross-country correlations, the median country results are the median of the 120 cross-country correlations across our 16 countries. For the results based on data from the model, we simulate 1000 datasets from the model of the same size as the actual data. For each such simulation we calculate the median across countries as well as the value for the U.S. for each statistic. We then report the median along with the 2.5% and 97.5% quantiles across simulations for each of these statistic.

Table 2.5: Asset Pricing Statistics

	Data		Baseline Model	
	Median	U.S.	Median	U.S.
$E(R_m - R_f)$	6.87	7.10	11.07	7.41
$\sigma(R_m - R_f)$	21.82	17.37	21.69	17.88
$E(R_m - R_f) / \sigma(R_m - R_f)$	0.32	0.41	0.51	0.41
$E(R_m)$	9.10	8.23	11.83	8.83
$\sigma(R_m)$	21.99	17.89	21.65	17.84
$E(R_f)$	1.43	1.13	0.66	1.41
$\sigma(R_f)$	4.57	3.33	2.26	1.74
$E(p-d)$	3.30	3.30	2.58	2.94
$\sigma(p-d)$	0.41	0.40	0.36	0.30
AC1(p-d)	0.85	0.90	0.85	0.85

Columns labeled as "Median" report the result for the median country for each statistic. Columns labeled as "U.S." report these statistics for the United States. For returns the statistics we report are the unconditional average of the level of the ex-post real net return in percentage points (i.e., multiplied by 100). R_m denotes the return on equity (the market), while R_f denotes the return on a short term nominal government bond (risk-free rate). The last three rows report statistics for the logarithm of the price-dividend ratio on equity. For the model, these results are for a $CRRRA = 6.5$, $IES = 1.5$, and subjective discount factor of $\beta = 0.99$, and are calculated using a sample of length 1 million years.

Table 2.6: The Equity Premium and Risk-Free Rate Across Countries and Models

	Equity Premium				Risk-Free Rate	
	Data	Full Model	Constant Volatility	Mehra-Prescott	Data	Full Model
Australia	0.090	0.083	0.036	0.005	0.007	0.010
Belgium	0.068	0.137	0.064	0.006	0.012	0.000
Canada	0.061	0.098	0.045	0.008	0.013	0.009
Denmark	0.043	0.094	0.045	0.005	0.028	0.010
Finland	0.128	0.193	0.105	0.014	-0.001	-0.006
France	0.078	0.123	0.056	0.006	-0.018	0.005
Germany	0.101	0.110	0.053	0.008	-0.027	0.007
Italy	0.061	0.153	0.088	0.008	-0.008	0.000
Netherlands	0.067	0.147	0.072	0.007	0.007	0.001
Norway	0.058	0.111	0.052	0.007	0.013	0.008
Portugal	0.089	0.187	0.094	0.016	0.001	-0.005
Spain	0.051	0.221	0.116	0.011	0.006	-0.011
Sweden	0.073	0.099	0.046	0.004	0.019	0.011
Switzerland	0.056	0.084	0.037	0.002	0.009	0.009
United Kingdom	0.054	0.104	0.048	0.005	0.013	0.007
United States	0.075	0.074	0.033	0.005	0.009	0.014
Average	0.072	0.126	0.062	0.007	0.005	0.004
Median	0.067	0.111	0.053	0.006	0.008	0.007

The table presents asset pricing statistics based on simulated data from our model as well as from the historical data. The "Constant Volatility" model is a version of the full model where we "turn off" the stochastic volatility by setting the volatility of the uncertainty shocks ω and ω_w to zero but keep other parameters at their estimated values for the full model. For the "Mehra-Prescott" model we "turn off" both the stochastic volatility and the growth-rate shocks and then we recalibrate the random-walk shocks based on the volatility of permanent consumption in the full model. These results are for a CRRA = 6.5, IES = 1.5 and subjective discount factor of $\beta = 0.99$.

Table 2.7: Predictability Regressions

	Data		Baseline (U.S.)		Simple Model (U.S.)		BY	BKY
	Median	U.S.	Median	95% Prob. Int.	Median	95% Prob. Int.	Median	Median
<u>5 Year Excess Returns on Price Dividend Ratio</u>								
β	-0.30	-0.41	-0.40	[-1.01, 0.18]	-0.44	[-1.06, 0.19]	-0.23	-0.39
R^2	0.11	0.24	0.10	[0.00, 0.39]	0.09	[0.00, 0.40]	0.03	0.05
<u>5 Year Realized Volatility on Price-Dividend Ratio</u>								
β	-0.38	-0.81	-0.37	[-1.35, 0.48]	-0.91	[-2.25, 0.25]	-0.10	-0.83
R^2	0.19	0.32	0.06	[0.00, 0.35]	0.12	[0.00, 0.44]	0.02	0.13
<u>5 Year Consumption Growth on Price-Dividend Ratio</u>								
β	0.03	0.02	0.19	[0.01, 0.36]	0.18	[-0.03, 0.39]	0.35	0.12
R^2	0.04	0.02	0.26	[0.00, 0.64]	0.18	[0.00, 0.55]	0.32	0.08

The table reports results from regressions of excess returns, consumption growth and realized volatility at a 1, 3 and 5 year horizon on the price-dividend ratio. Our measure of realized volatility is the absolute value of the residual from an AR(1) model for consumption growth. The first two columns report results using data from our 16 country sample and the U.S., respectively. The first column is the median across countries of the statistic in question. The next two columns report results from our model. The last two columns report results for the models of Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2012). The results for the Bansal-Yaron model are taken from Beeler and Campbell (2009). We use the end of year convention for the timing of consumption, whereby time t consumption is assumed to occur at the end of year t .

Table 2.8: World Long-Run Risks and Real Exchange Rate Volatility

	Exchange Rate Volatility		
	Data	Baseline Estimation	Ignoring Correlation
Australia	0.09	0.37	0.79
Belgium	0.11	0.51	1.06
Canada	0.05	0.40	0.85
Denmark	0.10	0.38	0.87
Finland	0.10	0.71	1.22
France	0.10	0.45	1.00
Germany	0.10	0.42	0.94
Italy	0.10	0.58	1.13
Netherlands	0.10	0.56	1.11
Norway	0.08	0.42	0.94
Portugal	0.10	0.69	1.21
Spain	0.11	0.88	1.43
Sweden	0.11	0.38	0.89
Switzerland	0.11	0.34	0.82
United Kingdom	0.09	0.39	0.91
Average	0.10	0.50	1.01
Median	0.10	0.42	0.94

The table presents the standard deviation of the log change in the real exchange rate of each country with the United States. First, it presents results based on historical data from 1975-2009. Second, it presents results based on simulated data from our baseline estimates. The last column calculates counterfactual exchange rates based on the simulated data from our estimated model but ignoring the correlation between the stochastic discount factors of the two countries in question.

Table 2.9: Estimates of Country-Specific Parameters

	Rel. St. Dev. Random Walk Shock (γ_i)		Sensitivity to Common Shocks (ξ_i)		Average St. Dev. Stoch. Vol. (σ_i)		St. Dev. Transitory Shock (σ_{η}) pre-1945				Average Growth (μ_i)	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Australia	1.73	0.61	1.05	0.42	0.008	0.004	0.004	0.002	0.035	0.009	0.015	0.006
Belgium	1.01	0.50	1.95	0.56	0.007	0.003	0.004	0.002	0.020	0.008	0.012	0.009
Canada	1.96	0.63	1.17	0.41	0.009	0.004	0.003	0.001	0.028	0.009	0.018	0.006
Denmark	0.79	0.41	1.29	0.58	0.012	0.004	0.007	0.003	0.012	0.003	0.016	0.007
Finland	2.00	1.23	2.30	0.83	0.012	0.006	0.004	0.002	0.024	0.007	0.022	0.012
France	0.83	0.42	1.73	0.46	0.007	0.004	0.002	0.001	0.027	0.004	0.015	0.008
Germany	0.63	0.34	1.54	0.53	0.012	0.004	0.003	0.001	0.012	0.004	0.015	0.008
Italy	0.58	0.30	2.16	0.65	0.011	0.004	0.003	0.002	0.015	0.003	0.017	0.011
Netherlands	0.59	0.32	2.09	0.63	0.010	0.004	0.003	0.002	0.023	0.004	0.016	0.010
Norway	1.17	0.57	1.49	0.60	0.009	0.004	0.006	0.003	0.006	0.003	0.019	0.008
Portugal	2.59	0.80	2.27	0.68	0.008	0.004	0.005	0.003	0.030	0.009	0.021	0.011
Spain	0.76	0.45	3.24	0.86	0.010	0.004	0.002	0.001	0.048	0.008	0.019	0.016
Sweden	0.72	0.47	1.36	0.52	0.010	0.004	0.004	0.002	0.024	0.005	0.018	0.007
Switzerland	0.64	0.44	1.21	0.42	0.009	0.004	0.001	0.001	0.043	0.006	0.011	0.006
United Kingdom	0.63	0.30	1.48	0.49	0.010	0.004	0.004	0.002	0.006	0.002	0.013	0.008
United States	1.16	0.44	1.00	0.00	0.009	0.004	0.003	0.001	0.023	0.005	0.018	0.006
Average	1.11	0.51	1.71	0.54	0.010	0.004	0.004	0.002	0.024	0.006	0.017	0.009
Median	0.81	0.45	1.51	0.55	0.009	0.004	0.004	0.002	0.024	0.005	0.016	0.008

The table presents our estimates of the posterior mean and standard deviation of the country-specific parameters in our full model.

Table 2.10: Asset Pricing Statistics

	Data		Baseline		Simple Model		Post-WWII	
	Median	U.S.	Median	U.S.	Median	U.S.	Median	U.S.
$E(R_m - R_f)$	6.87	7.10	11.07	7.41	4.13	3.98	20.74	6.39
$\sigma(R_m - R_f)$	21.82	17.37	21.69	17.88	13.76	13.53	31.38	16.99
$E(R_m - R_f) / \sigma(R_m - R_f)$	0.32	0.41	0.51	0.41	0.30	0.29	0.66	0.38
$E(R_m)$	9.10	8.23	11.83	8.83	5.68	5.68	19.94	8.13
$\sigma(R_m)$	21.99	17.89	21.65	17.84	13.81	13.55	31.40	16.86
$E(R_f)$	1.43	1.13	0.66	1.41	1.54	1.71	-0.77	1.74
$\sigma(R_f)$	4.57	3.33	2.26	1.74	1.41	1.34	3.51	1.50
$E(p-d)$	3.30	3.30	2.58	2.94	3.45	3.52	2.08	3.10
$\sigma(p-d)$	0.41	0.40	0.36	0.30	0.20	0.19	0.48	0.26
AC1(p-d)	0.85	0.90	0.85	0.85	0.79	0.79	0.84	0.81

Columns labeled as "Median" report the result for the median country for each statistic. Columns labeled as "U.S." report these statistics for the United States. For returns the statistics we report are the unconditional average of the level of the ex-post real net return in percentage points (i.e., multiplied by 100). R_m denotes the return on equity (the market), while R_f denotes the return on a short term nominal government bond (risk-free rate). The last three rows report statistics for the logarithm of the price-dividend ratio on equity. For the model, these results are for a CRRA = 6.5, IES = 1.5, and subjective discount factor of $\beta = 0.99$, and are calculated using a sample of length 1 million years.

Figure 2.1: Log per Capita Consumption in France

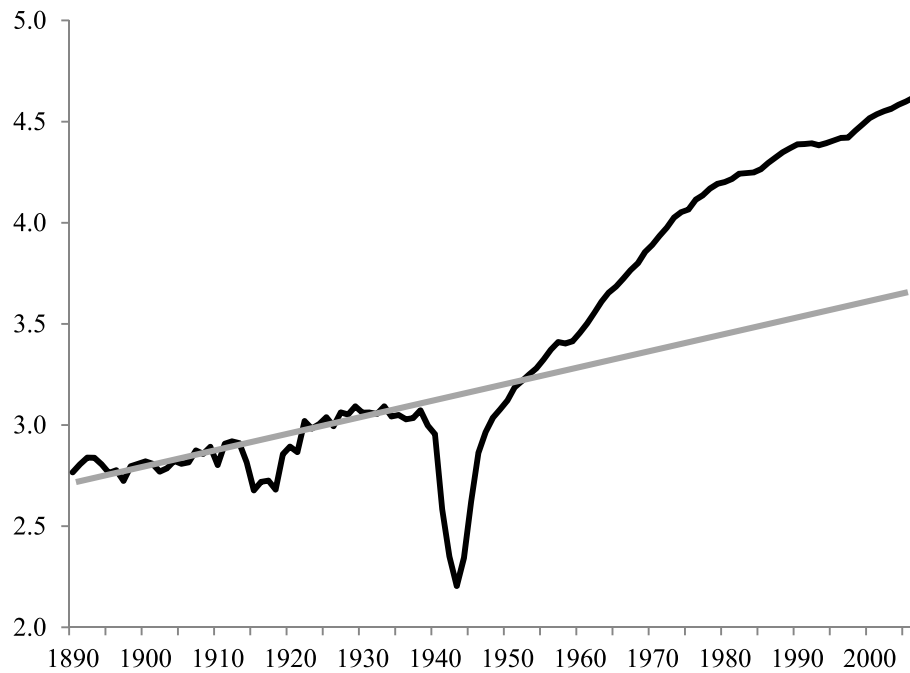


Figure 2.2: Response of Consumption to Growth-Rate and Random-Walk Shocks

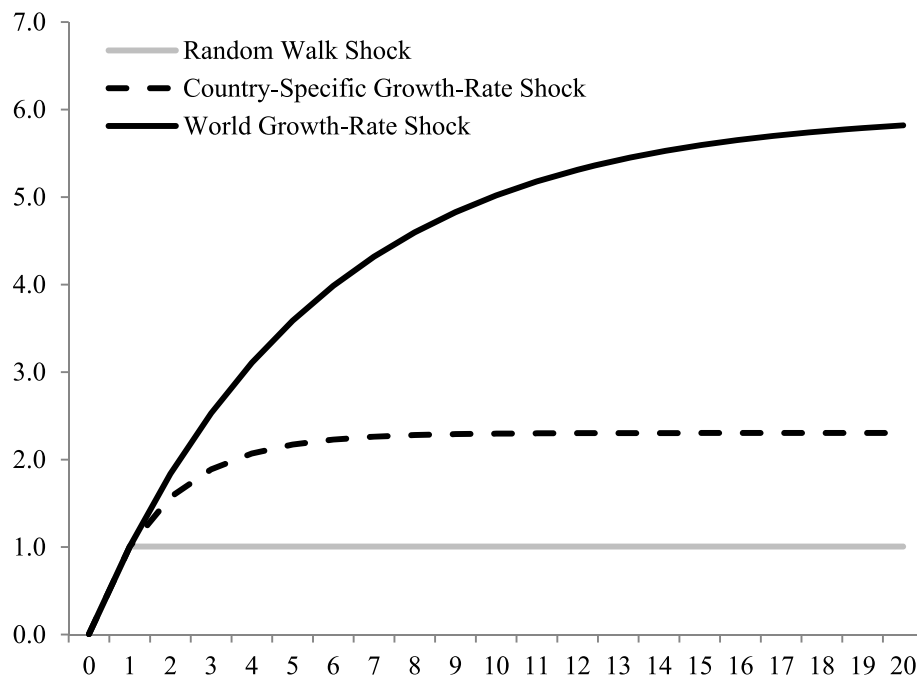


Figure 2.3: The World Growth-Rate Process.

The figure plots the posterior mean value of $x_{w,t}$ for each year in our sample.

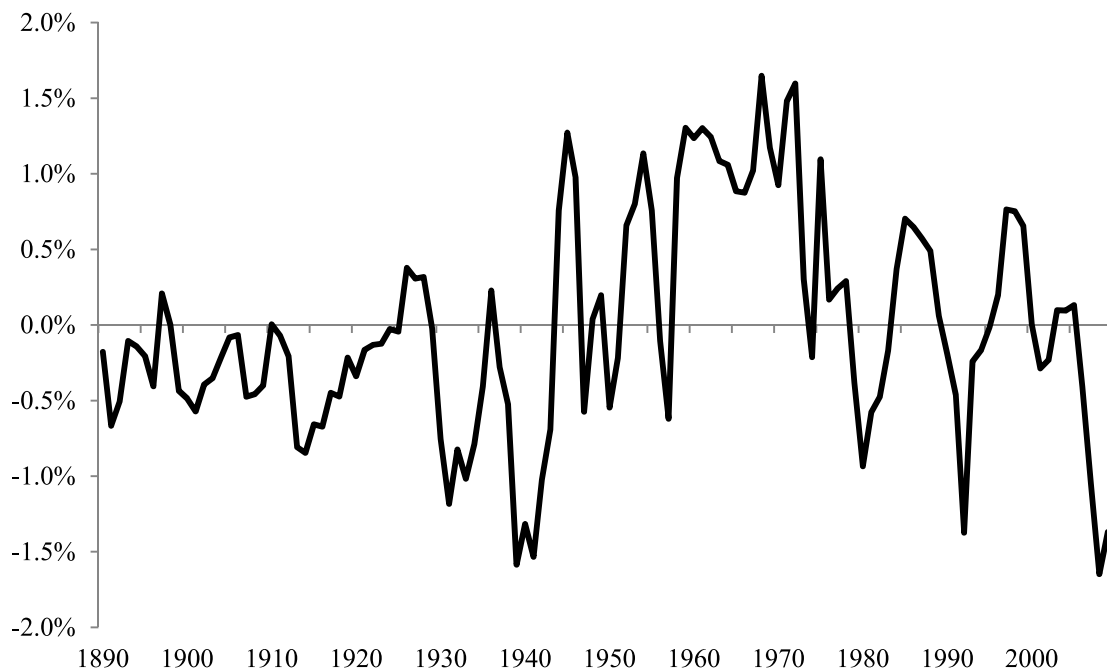


Figure 2.4: World Stochastic Volatility.

The figure plots the posterior mean value of $\sigma_{w,t}$ for each year in our sample.

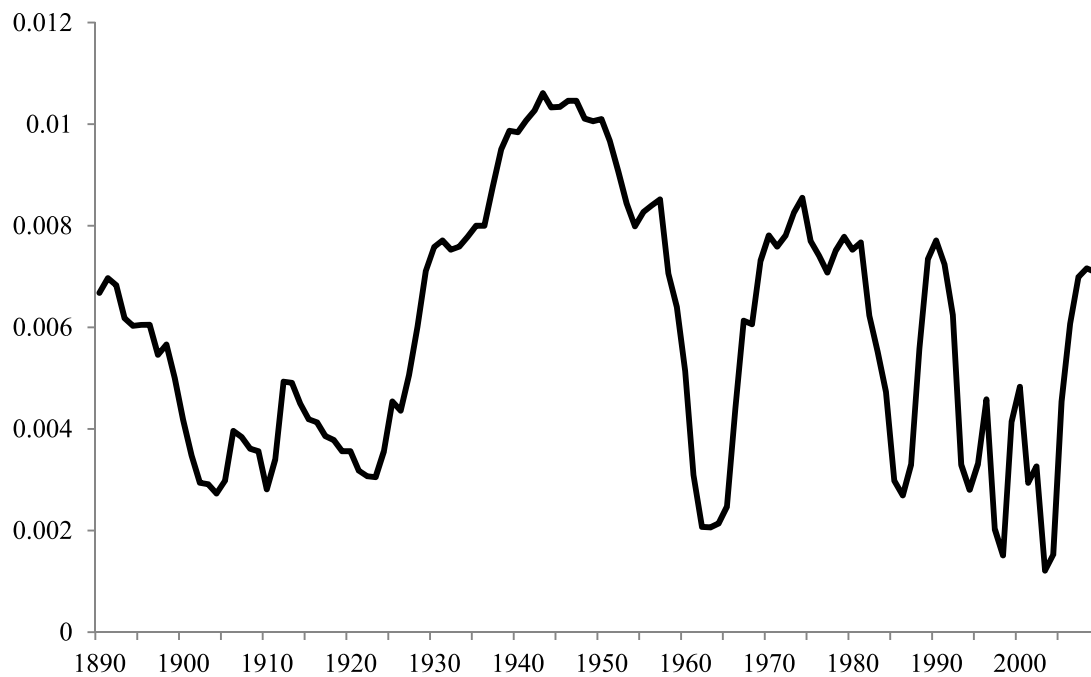


Figure 2.5: Stochastic Volatility for the United States, the United Kingdom and Canada

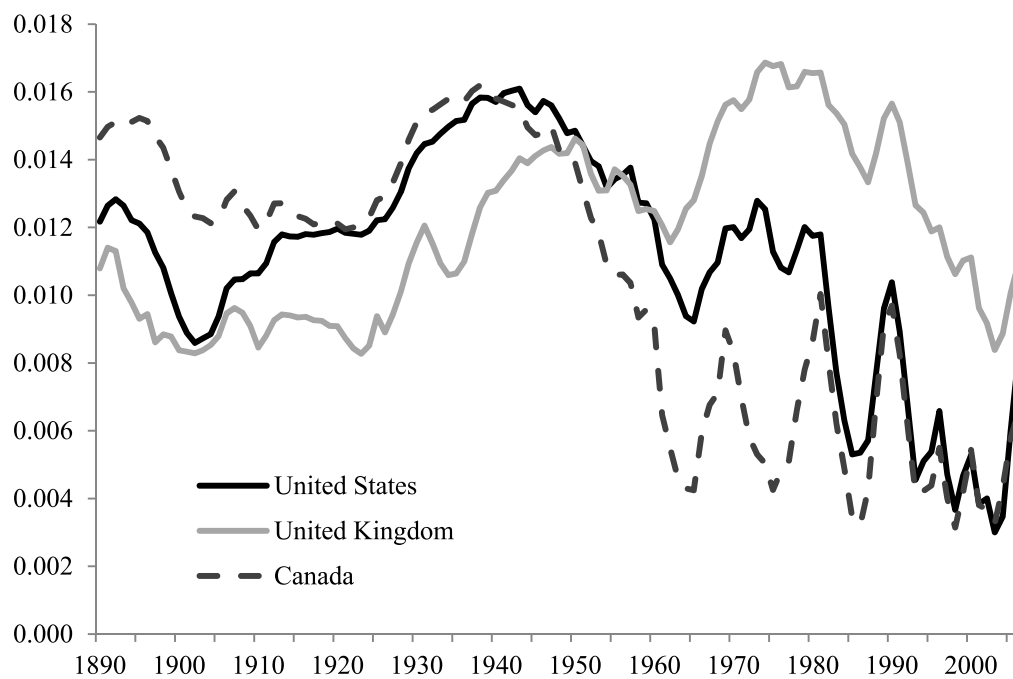


Figure 2.6: Asset Returns in Response to a World Growth-Rate Shock.
 Response of asset returns to a one standard deviation shock in $\epsilon_{W,t}$ starting from the models steady state.

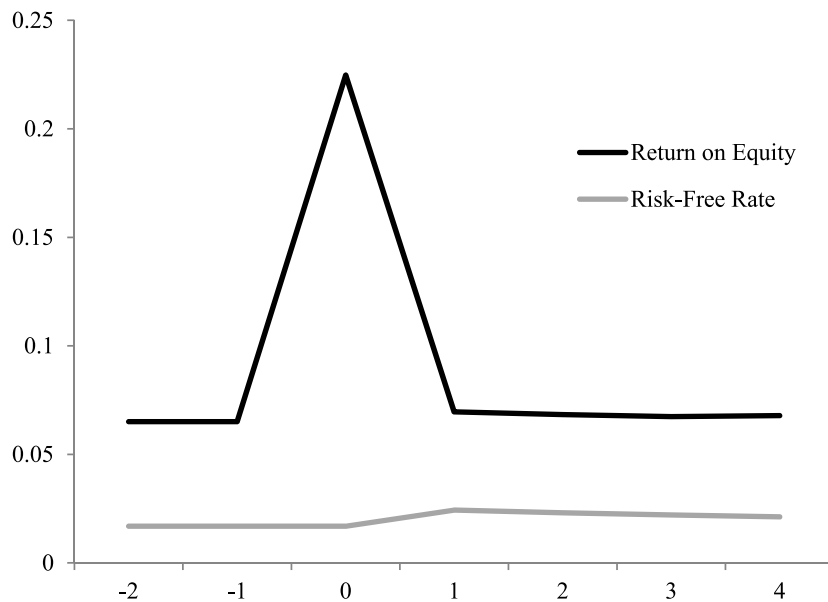


Figure 2.7: Asset Returns in Response to a World Uncertainty Shock.
 Response of asset returns to a one standard deviation shock in $\omega_{W,t}$ starting from the models steady state.

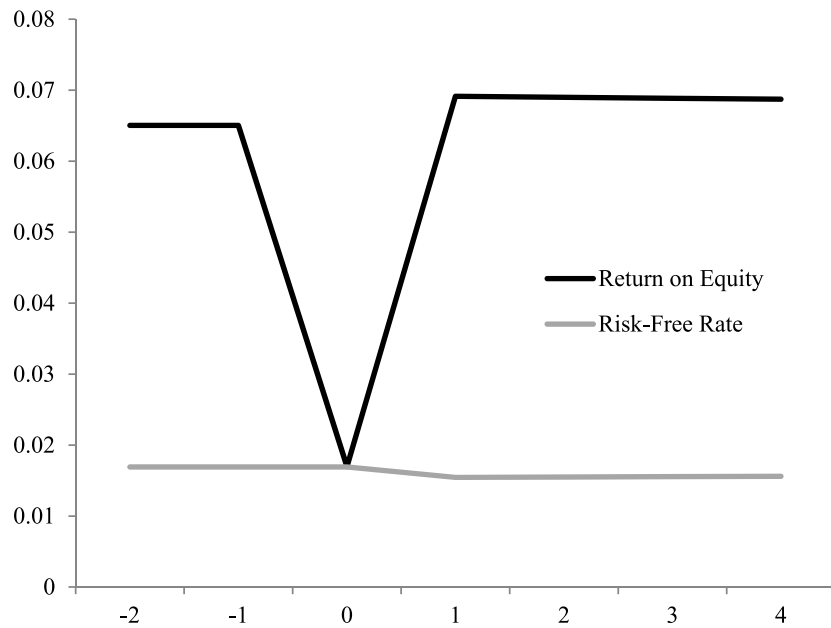


Figure 2.8: Stock Prices and Economic Uncertainty for the United States

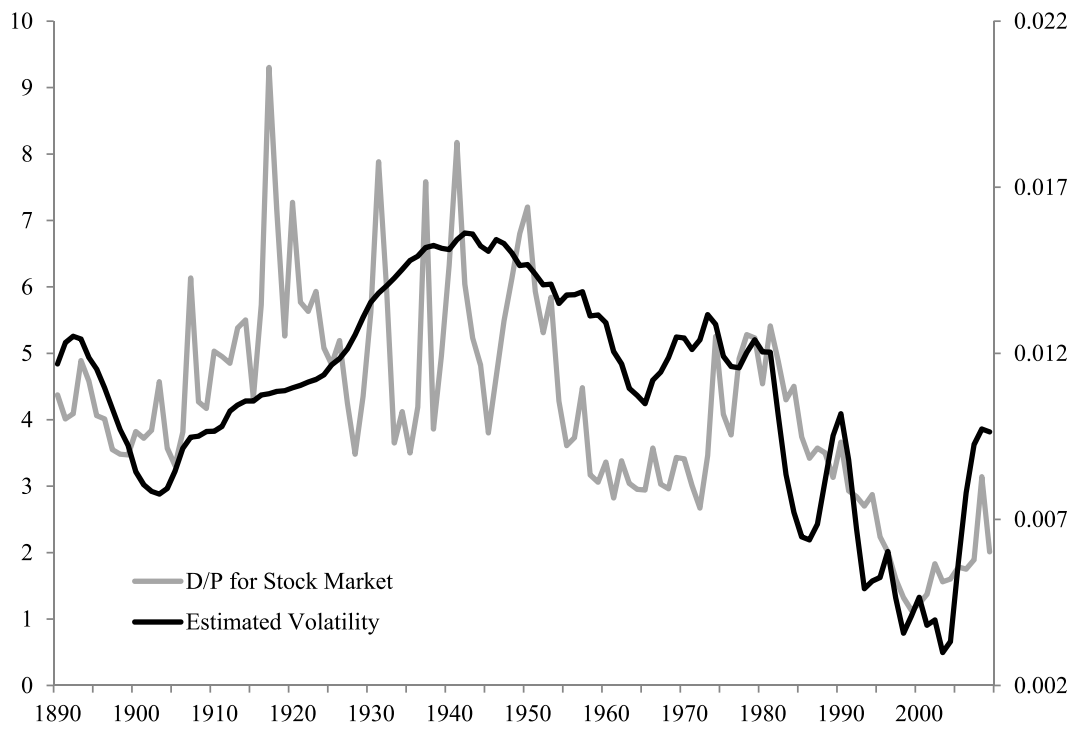
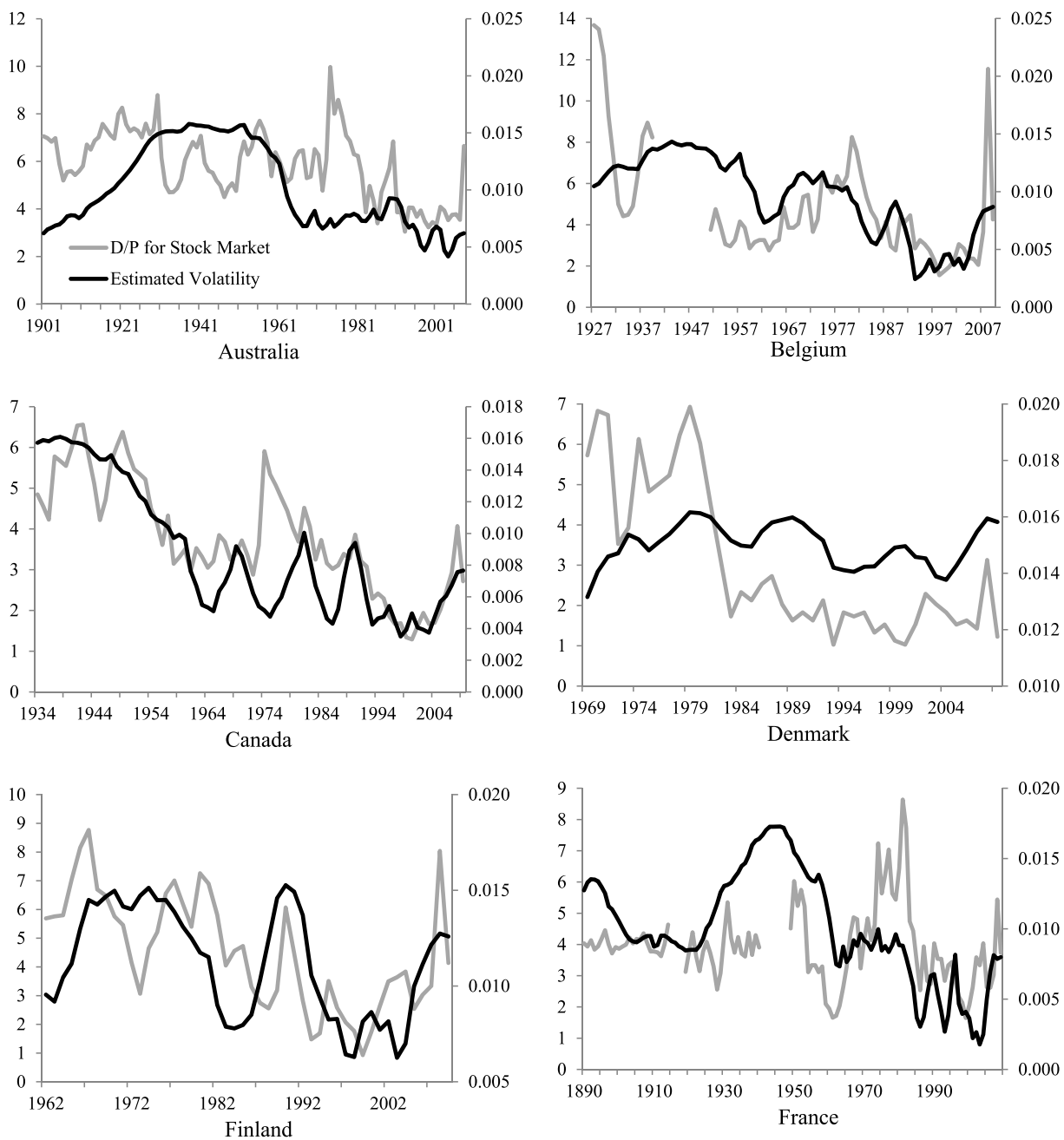


Figure 2.9: Dividend-Price Ratio for Stocks and Economic Uncertainty



Chapter 3

Sectoral Shocks, the Beveridge Curve and Monetary Policy

with Neil R. Mehrotra¹

¹We would like to thank Andreas Mueller, Ricardo Reis, Jón Steinsson and Michael Woodford for helpful discussions and Nicolas Crouzet, Hyunseung Oh, Andrew Figura, Emi Nakamura, Serena Ng, Bruce Preston, Stephanie Schmitt-Grohe, Luminita Stevens, Martin Uribe, Gianluca Violante, and Reed Walker for useful comments.

3.1 Introduction

You can't change the carpenter into a nurse easily, and you can't change the mortgage broker into a computer expert in a manufacturing plant very easily. Eventually that stuff will work itself out . . . [M]onetary policy can't retrain people. Monetary policy can't fix those problems.

Charles Plosser, President of the Federal Reserve Bank of Philadelphia

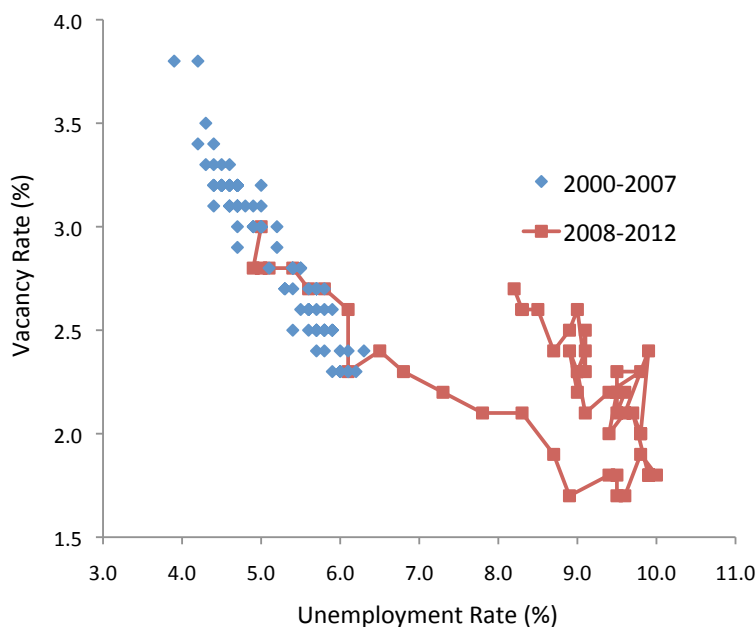
Though the Great Recession ended in the middle of 2009, the US labor market remains weak three years later with an unemployment rate near 8%. Some have speculated that a slow recovery is inevitable as the labor force must reallocate from housing-related sectors to the rest of the economy. Proponents of this view have cited the shift in the US Beveridge curve as evidence for sectoral shocks leading to labor reallocation.² The view that Beveridge curve shifts reflect sectoral disruptions and periods of increased labor reallocation was first elucidated by [Abraham and Katz \(1986\)](#) and [Blanchard and Diamond \(1989\)](#). Figure 3.1 displays unemployment and vacancies since 2000 using vacancy data from the Job Openings and Labor Turnover Survey (JOLTs). The Beveridge curve has shifted during the recovery period with the unemployment rate rising 1.5-2 percentage points at each level of vacancies.³ Vacancy rates in 2012 are consistent with an unemployment rate of less than 6% on the pre-recession Beveridge curve. The observed shift in the Beveridge curve has prompted disagreement on what implications, if any, this shift may have for monetary policy. [Kocherlakota \(2010\)](#) and [Plosser \(2011\)](#) suggest that, if sectoral shocks require labor reallocation and that process is costly and prolonged, then the natural rate of unemployment has risen, implying that further monetary easing would be inflationary.

We investigate the relationship between sector-specific shocks, shifts in the Beveridge curve, and changes in the natural rate of unemployment. In particular, we address three

²See [Kocherlakota \(2010\)](#), and [Plosser \(2011\)](#)

³See [Barnichon, Elsby, Hobijn, and Şahin \(2010\)](#) for measurement of the shift in the empirical Beveridge curve using JOLTs data. Exact size of the shift depends on the definition of the vacancy rate: job openings rate used in JOLTs is $V/(N + V)$ or alternative is vacancy to labor force ratio V/L (analogous to the unemployment rate).

Figure 3.1: US Beveridge curve, 2000-2012



questions: Has the US labor market experienced sector-specific disruptions? Can sectoral shocks account for the shift in the Beveridge curve? Do sectoral shocks raise the natural rate of unemployment? We build a measure of sector-specific shocks using a factor analysis of sectoral employment and augment a standard multisector model with labor market search to analyze the relationship between sector-specific shocks, the Beveridge curve, and the natural rate of unemployment.

Our first contribution is a new index of sector-specific shocks that measures the dispersion of the component of sectoral employment not explained by an aggregate employment factor. Our measure is distinct from the [Lilien \(1982\)](#) measure of employment dispersion and addresses the [Abraham and Katz \(1986\)](#) critique that asymmetric responses of sectoral employment may be attributable to differing sensitivities of sectors to aggregate shocks. We confirm that the recovery from the Great Recession is characterized by a substantial increase in sectoral shocks that matches the timing of the shift in the Beveridge curve. Moreover, we show that shifts in the US Beveridge curve in postwar data are correlated with periods in which sector-specific shocks are elevated as measured by our index.

Our second contribution is to define the Beveridge curve in a multisector model and examine its behavior in the presence of sectoral shocks. The Beveridge curve is defined as the set of unemployment and vacancy combinations traced out by changes in real marginal cost, which captures the effect of a variety of aggregate disturbances. We show that sectoral productivity or demand shocks will, in general, shift the Beveridge curve. Sectoral shocks shift the Beveridge curve through two channels: a composition effect and a mismatch effect. The former channel is operative if a sectoral shock shifts the distribution of vacancies towards a sector with greater hiring costs, thereby increasing unemployment for any given aggregate level of vacancies. The latter channel stems from decreasing returns to the matching function and costly reallocation: a sectoral shock that leaves overall vacancies unchanged raises unemployment because the reduction in vacancies in one sector increases unemployment by more than the corresponding fall in unemployment in the other sector. Our model validates our empirical strategy and verifies the hypothesized relationship between our sector-specific shock index and shifts in the Beveridge curve.

Our third contribution is to clarify the relationship between the Beveridge curve and the natural rate of unemployment. In the baseline model with exogenous sectoral productivity or demand shocks, shifts in the Beveridge curve necessarily imply a movement in the natural rate of unemployment in the same direction as the shift in the Beveridge curve. However, the converse need not hold: for example, a negative aggregate productivity shock raises the natural rate of unemployment without shifting the Beveridge curve. Changes in the natural rate affect monetary policy by changing the inflation-employment tradeoff for the central bank.

We calibrate a two-sector version of our model to data on the construction and non-construction sectors of the US labor market to quantify the effect of sectoral shocks on the Beveridge curve and the natural rate of unemployment. A sector-specific shock to construction of sufficient magnitude to match movements in construction's employment share generates a shift in the Beveridge curve that quantitatively matches the shift observed in the US. Moreover, the shock to construction raises the natural rate of unemployment by 1.4 percentage

points - insufficient to fully explain the rise in unemployment observed in the current recession and of similar magnitude to the estimates in [Sahin, Song, Topa, and Violante \(2010\)](#) who examine the contribution of mismatch to overall unemployment.

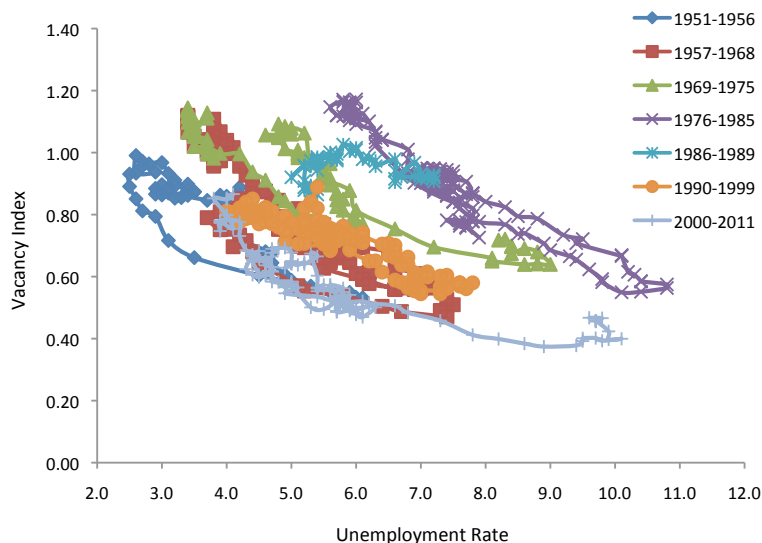
Our final contribution is an extension of the model to incorporate financial frictions. In this environment, it is no longer the case that a Beveridge curve shift implies a change in the natural rate. We show that financial shocks or systematic changes in monetary policy increase mismatch in the same way as a sector-specific productivity or demand shocks. Events like a binding zero lower bound could act like a sector-specific shock, generating a shift in the Beveridge curve while not implying any change in the natural rate of unemployment. Given our analysis, we conclude that a Beveridge curve shift is not sufficient to draw any conclusions about the behavior of the natural rate of unemployment.

Our paper is organized as follows. Section [3.2](#) describes our method for constructing a long-run sector-specific shock index and its correlation with historic shifts in the Beveridge curve. Section [3.3](#) lays out our baseline model: a sticky price multisector model augmented with labor market search within sectors and costly reallocation across sectors. Analytical results establishing the relationship between sectoral shocks, labor reallocation, and the Beveridge curve along with implications for the natural rate are described in Section [3.4](#). Section [3.5](#) describes our calibration strategy and shows the effect of sectoral productivity shocks in a two-sector model. Section [3.6](#) extends the multisector model to incorporate financial frictions and illustrates how financial frictions and changes in the monetary policy rule can act as sectoral shocks and shift the Beveridge curve. Section [3.7](#) concludes.

3.2 Empirics on Sectoral Shocks and the Beveridge Curve

To examine the relationship between sectoral shocks and the Beveridge curve, we construct the long-run US Beveridge curve and build a summary measure of sector-specific shocks. Since vacancies data from the JOLTS survey is only available after 2000, the Conference Board's Help-Wanted Index is frequently used as a proxy for the vacancy rate prior to 2000. Figure [3.2](#) displays the Beveridge curve using the Help-Wanted Index (HWI) normalized by the labor

Figure 3.2: US Beveridge curve, 1950-2011



force as a proxy for the vacancy rate.⁴ Figure 3.2 shows that the historic Beveridge curve exhibits periods when the vacancy-unemployment relationship is stable and periods when it appears to shift.

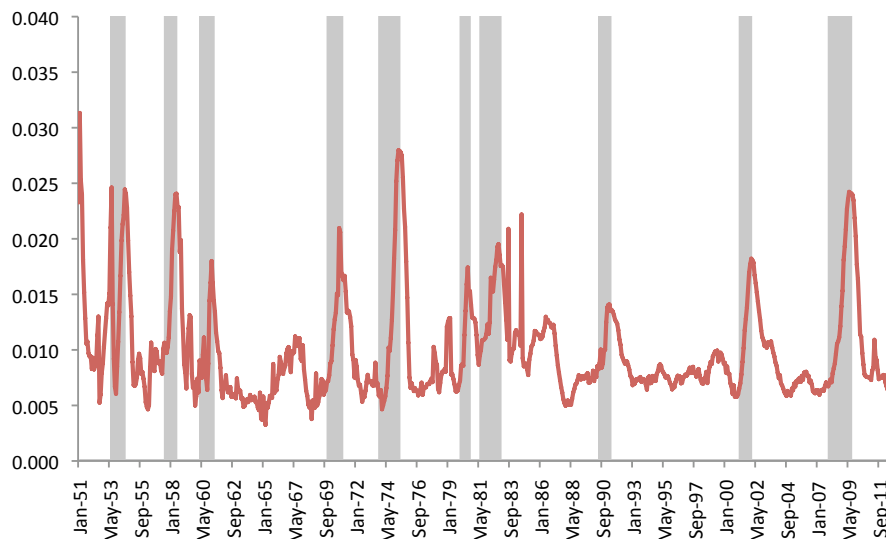
Historic shifts in the US Beveridge curve are documented in [Bleakley and Fuhrer \(1997\)](#) and [Valletta and Kuang \(2010\)](#). Importantly, shifts in the Beveridge curve are not a business cycle phenomenon with some recessions accompanied by shifts but other shifts occurring during expansions - the behavior of vacancies and unemployment in the mid 1980s provides a good example. Like the Beveridge curve obtained using JOLTs data, the composite HWI Beveridge curve exhibits an upward shift since 2009.

3.2.1 Existing Measures of Sector-Specific Shocks

Lilien (1982) proposed the dispersion in sectoral employment growth as a measure for sector-specific shocks, arguing that these shocks are an important driver of the business cycle given the strong countercyclical behavior of his measure. Figure 3.3 plots the Lilien measure using

⁴After 1996, the HWI is the composite index derived in [Barnichon \(2010\)](#) and updated to 2011, which adjusts for the shift away from newspaper advertising of vacancies to online advertising.

Figure 3.3: Lilien measure of dispersion in employment growth



monthly sectoral employment data.⁵ The figure demonstrates the strongly countercyclical behavior of the series including most recent recessions that have featured a slower recovery in the labor market in comparison to past recessions. In the current recession, the Lilien measure peaks in the summer of 2009 at the recession trough.

Abraham and Katz (1986) questioned the Lilien measure by arguing that increases in the dispersion of employment growth could be attributed to differences in the elasticity of sectoral employment to aggregate shocks. As an alternative, Abraham and Katz argued that sector-specific shocks should result in periods in which vacancies and unemployment are both rising and showed that the Lilien measure does not comove positively with vacancies.

3.2.2 Constructing Sector-Specific Shock Index

To derive a measure of sector-specific shocks, we conduct a factor analysis of sectoral employment. The factor analysis addresses the Abraham and Katz critique by allowing sectoral employment to respond differently to aggregate shocks.

⁵The Lilien measure is: $\sigma_t = \left(\sum_{i=1}^K (g_{it} - g_t)^2 \right)^{1/2}$ where g_{it} is the growth rate of employment in sector i and g_t is the growth rate of aggregate employment.

We estimate the following approximate factor model:

$$n_t = \epsilon_t + \lambda F_t,$$

where n_t is a $N \times 1$ vector of employment by sector, ϵ_t is a $N \times 1$ vector of mean-zero sector-specific shocks, F_t is a $K \times 1$ vector of factors, and λ is a $N \times K$ matrix of factor loadings.

As is standard in the approximate factor model discussed in [Stock and Watson \(2002a\)](#), we assume that n_t and F_t are covariance stationary processes, with $Cov(F_t, \epsilon_t) = 0$. As shown by Stock and Watson (2002), the approximate factor model allows for serial correlation in F_t , ϵ_t , and weak cross-sectional correlation in ϵ_t - the variance-covariance matrix of ϵ_t need not be diagonal. The factor analysis implicitly identifies the sector-specific shock by assuming that loadings on the aggregate factor are invariant over time; that is, sectoral employment responds in a similar manner over the business cycle to aggregate fluctuations.

The sectoral residual ϵ_{it} represents the sector-specific shock, and we construct an index to examine the time variation in sector-specific shocks by measuring cross-sectional dispersion, squaring the sectoral employment residuals from our factor analysis:

$$S_t^{dis} = \frac{1}{K} \left(\sum_{i=1}^K \epsilon_{it}^2 \right)^{1/2}.$$

Given that variances are normalized to unity before estimating, the sector specific shocks need not be weighted by their employment shares. We also construct an alternative measure of employment dispersion as the sum of the absolute values of the residuals from our factor analysis:

$$S_t^{abs} = \frac{1}{K} \sum_{i=1}^K |\epsilon_{it}|.$$

This measure of sector-specific shocks is always positive and weights all sectors equally.

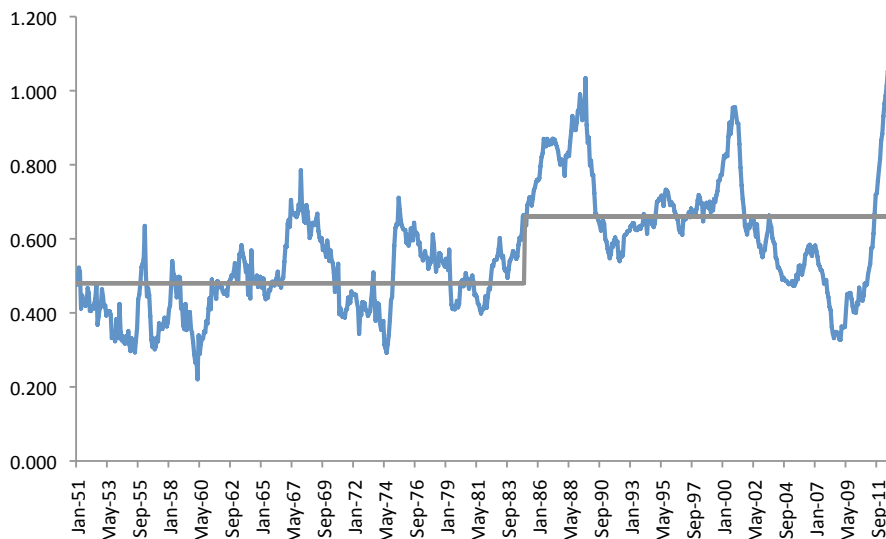
3.2.3 Data

To estimate the sectoral shock index, we use long-run US data on sectoral employment. These data are available for the US from January 1950 to July 2012 on a monthly basis for 14 sectors that represent the first level of disaggregation for US employment data. Due to its relatively small share of employment, we drop the mining and natural resources sector. The sectoral data is taken from the Bureau of Labor Statistics establishment survey. While, in principle, we could use sectoral data on variables like real output, relative prices, or relative wages, employment data offers the longest available history at the highest frequency and is presumably measured with the least error. The principal concern with this data set is the small number of cross-sectional observations relative to the number of observations in the time dimension. While traditional factor analyses draw on highly disaggregated price, output, or employment data, these series are not available before the 1970s. Given our aim of investigating shifts in the Beveridge curve and the relative infrequency of these events, we try to construct the longest possible series for sector-specific shocks.

The log of monthly sectoral employment is detrended to obtain a mean-zero stationary series and the variance of each series is normalized to unity. This normalization ensures that no series has a disproportionate effect on the estimation of the national factor.

We detrend employment in each sector by means of a cubic deterministic trend. The underlying trend in sectoral employment differs substantially among sectors, and employment shares are nonstationary over the postwar period. For example, manufacturing employment falls as a share of total employment over the whole period, but even decreases in absolute terms starting in the 1980s. Sectors, such as construction and information services show a general upward trend in levels characterized by very large and long swings in employment that are longer than simple business cycle variation. Higher-order deterministic trends fit certain sectors much better than a simple linear or quadratic trend. Moreover, most of the sectoral employment series obtained by removing a linear or quadratic trend fail a Dickey-Fuller test at standard confidence levels. For robustness, as will be shown in the next section, we also consider detrending by first-differences, computing quarter-over-quarter or year-over-

Figure 3.4: Sector-specific shock index



year growth rates for each sector, normalizing variances, and then estimating the factor model.

Given that our full sample from 1950-2012 has a small number of cross-section observations relative to the time dimension, we also estimate the same model using a larger cross-section of 85 sectoral employment series at the 2-digit NAICS level available monthly since 1990. We find the same pattern for the shock index as in our larger sample.

3.2.4 Sectoral Shock Index and Shifts in the Beveridge Curve

The sector-specific shock index shown in Figure 3.4 displays several notable features. First, the shock index rises rapidly in late 2009. The rise in the shock index occurs at the beginning of the recovery, not at the beginning of the recession, matching the timing of the shift in the Beveridge curve.⁶ Second, the sector-specific shock index is not a business cycle measure. Its correlation with various monthly measures of the business cycle is highlighted in Table 3.1, with all correlations below 0.15. Third, the sectoral shock index displays a low and negative

⁶The rise in the index in the recovery period after the Great Recession is also consistent with the elevated dispersion in labor market conditions highlighted by [Barnichon and Figura \(2011\)](#) and sectoral dispersion measures computed by [Rissman \(2009\)](#).

Table 3.1: Correlation of shock index with business cycle measures

	Business Cycle Measures			Lilien measure	Beveridge Curve
	Industrial production growth	Employment growth	Unemployment rate		
<u>Detrend with time trend</u>					
Cubic	-0.02	0.12	0.00	-0.18	0.36
Quartic	-0.05	0.08	0.01	-0.06	0.18
<u>Detrend with growth rates</u>					
Quarter over quarter	-0.17	-0.12	-0.07	0.28	-0.06
Year over year	-0.32	-0.36	0.09	0.57	-0.09

correlation with the Lilien measure.⁷ Finally, the average level of the shock index is higher in the Great Moderation period as shown by the gray line in Figure 3.4. This behavior is consistent with the behavior of sectoral employment documented in [Garin, Pries, and Sims \(2010\)](#).

Just as the current shift in the Beveridge curve coincides with a rise in the sector-specific shock index, historic shifts in the Beveridge curve are also correlated with elevated levels of sector-specific shocks. We illustrate this correlation between shifts in the Beveridge curve and the sector-specific shock index by plotting the shock index against the intercept of a 5-year rolling regression of vacancies on unemployment (five-year trailing window). Absent any shifts in the Beveridge curve, the intercept should be constant. Therefore, variation in the intercept series captures movements in the Beveridge curve. Figure 3.5 shows a clear correlation between movements in the intercept of the Beveridge curve and the sector-specific shock index. This correlation in monthly data calculated from 1956-2012 is 0.363 and is shown in the last column in Table 3.1. This result is robust to the use of a 4th order trend, though somewhat weaker. Our evidence provides support for the mechanism described by Abraham and Katz where sector-specific shocks generate a shift the Beveridge curve.

To examine the robustness of this correlation, we also estimate the Beveridge curve aug-

⁷For the index obtained using growth rates, the correlation with business cycle measures and the Lilien measure is markedly higher than the time trend specifications. This correlation is driven by the behavior of the index in the first half of the sample. The correlation of the sectoral shock index with the Lilien measure drops to 0.18 from 0.56 in the Great Moderation period.

mented with our sector-specific shock index:

$$v_t = c + \beta(L) u_t + \gamma(L) S_t + \eta_t$$

where v_t is log vacancies, u_t is log unemployment, $\beta(L)$ and $\gamma(L)$ are lag polynomials, c is a constant, and η_t is a mean zero error term.⁸ The Beveridge curve is estimated with four lags of unemployment to control for the persistence of both vacancies and unemployment and with Newey-West standard errors (4 lags) to account for serial correlation in η_t . We consider several variants of our sector-specific shock index using both the dispersion measure (Panel A) and the absolute-value measure (Panel B). Employment is detrended with either time trends and growth rate trends. Given the persistence exhibited by the sector-specific shock indices obtained from time detrending, we estimate specifications both with and without an additional lag of the shock index.

Figure 3.5: Correlation of Beveridge curve shifts and sector-specific shocks

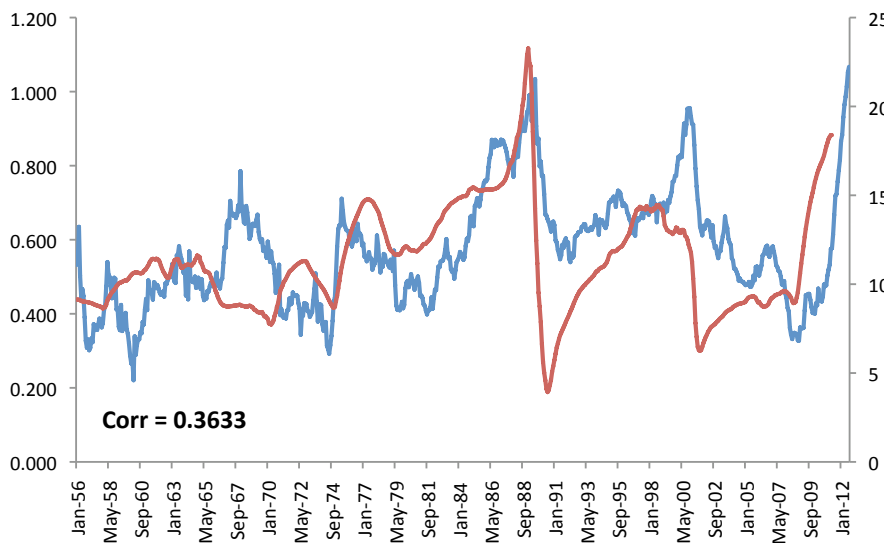


Table 3.2 displays the estimates for the coefficient γ on the sector-specific shock index. This coefficient enters significantly for most of the time trend specifications we consider. Our

⁸An earlier version of this paper estimates the Beveridge curve using vacancies and unemployment rates in levels. Given the nonlinear nature of the Beveridge curve, the log specification is preferred. However, the use of log or levels does not greatly affect the estimation.

baseline cubic detrending is highlighted in bold in the table with positive and statistically significant coefficients in all cases. The shock index based on growth rate detrending delivers a significant negative coefficient in the case of the year-over-year specification. While our reduced form model makes no prediction about the sign of the coefficient γ , we show in section 3.4.3 that our model-implied measure of Beveridge curve shifts delivers coefficients that are consistent in sign across all specifications. We defer further discussion until then.

Table 3.2: Effect of shock index on Beveridge curve intercept

Panel A: Dispersion Index			Panel B: Absolute Value Index		
	Coeff	<i>t</i> -stat		Coeff	<i>t</i> -stat
<u>Detrend with time trend</u>			<u>Detrend with time trend</u>		
Cubic**	0.82	2.64	Cubic**	0.33	3.35
Cubic w/1 lag**	0.80	2.58	Cubic w/1 lag**	0.33	3.31
Quartic**	0.64	2.11	Quartic	0.21	1.92
Quartic w/1 lag**	0.62	2.07	Quartic w/1 lag	0.21	1.89
<u>Detrend with growth rates</u>			<u>Detrend with growth rates</u>		
Quarter over quarter	-0.08	-0.45	Quarter over quarter	-0.09	-1.24
Year over year**	-1.36	-4.18	Year over year**	-0.53	-4.76

Tables indicate coefficients of sectoral shock index in Beveridge curve estimation under alternative detrending procedures. W/1 lag specification includes lagged value of index with coefficient expressed as sum of contemporaneous and lagged effect of sectoral shock index on vacancies. ** indicates significance at 5% level. The number of time series observations is $T = 726$.

3.3 Multisector Model with Labor Reallocation

We augment a multisector sticky-price model as in Aoki (2001) or Carvalho and Lee (2011) with search and matching frictions in the labor market as in Shimer (2010).

There model features four types of agents: continuum of identical households, intermediate good producers, wholesale firms and retailers. The households hold preferences over consumption and leisure, they trade state-contingent assets, own all firms, and provide different types of workers to intermediate goods producers through a frictional labor market.

The intermediate goods producers are competitive and hire labor to produce an intermediate good. Each intermediate firm operates in one of several sectors. A firm working in

a particular sector hires sector-specific workers and produces a sector-specific intermediate good. Their production is subject to sector-specific productivity shocks. The intermediate goods producers sell their output to the wholesale firms.

Wholesale firms are competitive and combine sector-specific intermediate goods into a homogenous final good. Their production is subject to sector-specific demand shocks, which affects the relative demand of the wholesale firms for different intermediate goods. The wholesale firms sell their final goods to retailers.

Retailers are monopolistically competitive. A retailer buys final goods from the wholesale firms, costlessly differentiates these goods, and sells their good to a household exploiting their market power to set prices in excess of marginal cost. In a sticky price version of the model, we assume that prices for these differentiated goods are updated à la Calvo.

The labor market is sector-specific and subject to search and matching frictions. Each sector has a pool of unemployed workers who search for jobs in this sector. Intermediate goods producers from this sector search for workers in the unemployment pool of this sector. The households can reallocate its unemployed workers among different sectors. Sectors may, in principle, conform to geographies, industries, occupations or other dimensions of worker heterogeneity. Households reallocate their workers across sectors subject to a utility cost of changing the distribution of the labor force.

Next we provide a detailed description of the agents problems followed by analysis of wage setting and the definition of equilibrium.

3.3.1 Households

Households supply labor across K distinct sectors and invest in a full-set of state-contingent securities. While hiring in each sector is subject to search frictions, the household is free to reallocate workers across sectors subject to a utility cost of changing the distribution of labor. This utility cost captures costs associated with worker retraining, relocation, or the loss of industry-specific skills. With costly reallocation, the household's problem differs from the standard labor market search model since the household has an active margin of adjustment

by reallocating the pool of available workers across sectors. As a result, the initial distribution of the labor force is a state variable for the household in addition to the last period distribution of employment.

Household behavior can be expressed by the following optimization problem:

$$\max_{\{C_t, B_{t+1}, \mathbf{L}_t, \mathbf{N}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, N_t) - \sum_{i=1}^K R(L_{i,t-1}, L_{i,t}) \right\}, \quad (3.1)$$

$$\begin{aligned} \text{s.t.} \quad P_t C_t &= \sum_{i=1}^K W_{i,t} N_{i,t} + B_t - E_t Q_{t,t+1} B_{t+1} - T_t \\ &\quad + \Pi_t^f + \sum_{i=1}^K \Pi_{i,t}^{Int} + \int_{l=0}^1 \Pi_t^{ret}(l) dl, \end{aligned} \quad (3.2)$$

$$N_{i,t} = (1 - \delta_i) N_{i,t-1} + p_{i,t} U_{i,t}, \quad (3.3)$$

$$L_{i,t} = N_{i,t-1} + U_{i,t}, \quad (3.4)$$

$$\sum_{i=1}^K L_{i,t} = 1, \quad (3.5)$$

where $\mathbf{N}_t = \{N_{i,t}\}_{i=1}^K$ and $\mathbf{L}_t = \{L_{i,t}\}_{i=1}^K$ for $t \geq 0$ are $K \times 1$ vectors of sectoral employment and the sectoral distribution of the labor force respectively,

$$N_t = \sum_{i=1}^K N_{i,t}, \quad (3.6)$$

and C_t is an index of the household's consumption of the differentiated goods. The initial conditions for this problem are $\{B_0, \mathbf{N}_{-1}, \mathbf{L}_{-1}\}$. The household maximizes utility net of reallocation costs subject to a standard budget constraint (3.2), where P_t is an index of the prices of the differentiated goods, $W_{i,t}$ is the nominal wage that workers receive when working in sector i , Π_t^f , $\Pi_{i,t}^{Int}$, $\Pi_t^{ret}(l)$ represent wholesale, intermediate and retailer firms nominal profits distributed to households, B_t are nominal payments from state contingent securities and $Q_{t,t+1}$ is an asset-pricing kernel.⁹ For each sector, sectoral employment $N_{i,t}$ evolves by

⁹The existence and uniqueness of the asset-pricing kernel is guaranteed by the absence of arbitrage opportunities in equilibrium.

a law of motion (3.3) where $p_{i,t}$ is the job-finding rate in sector i and δ_i is a sector-specific separation rate. Sectoral unemployment is the difference between the labor force allocated in that sector $L_{i,t}$ and last period sectoral employment (3.4). The total labor force of the household is normalized to unity (3.5). The household takes the sectoral job-finding rate and profits from firms as exogenous.

Following Dixit and Stiglitz (1977), we assume that the index C_t is a constant-elasticity-of-consumption aggregator

$$C_t = \left[\int_0^1 C_t(l)^{(\zeta-1)/\zeta} dl \right]^{\zeta/(\zeta-1)}$$

with $\zeta > 1$, and P_t is the corresponding price index

$$P_t = \left[\int_0^1 P_t(l)^{1-\zeta} dl \right]^{1/(1-\zeta)}.$$

We assume that the cost of reallocation of a worker from sector i to sector $j \neq i$ depends on the current and past labor force in sector i and on the current and the past employment in sector j . The function $R(\cdot, \cdot)$ is assumed to be continuous and differentiable in its arguments and minimized when $L_{i,t-1} = L_{i,t}$ for any sector i .

The optimal choice of assets purchases and consumption implies the following relation determining the nominal one-period interest rate:

$$1 + i_t = \beta^{-1} \left\{ \mathbb{E}_t \left[\frac{u_c(C_{t+1}, N_{t+1})}{u_c(C_t, N_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1}. \quad (3.7)$$

See Appendix C.1.1 for a detailed discussion of the household optimality conditions.

Optimal choice of the allocation of the labor force across sectors implies

$$p_{i,t} \lambda_{2,t,i} = \lambda_{3,t} + R_2(L_{i,t-1}, L_{i,t}) + \beta \mathbb{E}_t R_1(L_{i,t}, L_{i,t+1}), \quad (3.8)$$

where $\lambda_{2,t,i}$ is a Lagrange multiplier on constraint (3.3), $\lambda_{3,t}$ is a Lagrange multiplier on constraint (3.5). $\lambda_{2,t,i}$ represents the utility value of an additional employed worker in sector

i for the household given the equilibrium path for wages $\{W_{i,t}\}_{t=0}^{\infty}$, while $\lambda_{3,t}$ represents the utility value of an increase in the labor force by one worker for the household. This first order condition states that the household equalizes the costs and benefits when choosing to allocate an additional worker to sector i . The left-hand side represents the utility benefit of an additional worker employed $\lambda_{2,t,i}$ weighted by the probability of finding a job $p_{i,t}$. The right-hand side is the cost of an additional worker in sector i , which is the sum of the shadow value of a person for the household $\lambda_{3,t}$ plus the adjustment costs of the labor force in sector i : $R_2(L_{i,t-1}, L_{i,t})$ gives the immediate costs of adjustment while the term $\beta\mathbb{E}_t R_1(L_{i,t}, L_{i,t+1})$ takes into account the affect on future adjustment costs.

Optimality with respect to $N_{i,t}$ gives a recursive formula for $\lambda_{2,t,i}$

$$\lambda_{2,t,i} = u_N(C_t, N_t) + \frac{W_{i,t}}{P_t} u_c(C_t, N_t) + \beta\mathbb{E}_t [(1 - \delta_i - p_{i,t+1})\lambda_{2,t+1,i}]. \quad (3.9)$$

This expression states that the value of an additional employed worker equals the sum of the disutility from working $u_n(C_t, N_t)$, the utility value of the nominal wage $W_{i,t}$, and the expected discounted value from having this worker employed in the next period weighted by the probability of retaining a job $\beta\mathbb{E}_t [(1 - \delta_i)\lambda_{2,t+1,i}]$ less the expected discounted value that the worker could be worth next period if he was not employed in the current period $\beta\mathbb{E}_t [p_{i,t}\lambda_{2,t+1,i}]$.

It will prove useful to introduce a variable closely related to $\lambda_{2,t,i}$ that will be used to determine workers wages. Let $J_{i,t}(\widetilde{W})$ denote the marginal utility for a household at the equilibrium level of employment of having one additional worker employed at a wage \widetilde{W} in period t rather than unemployed and with the wage returning to an equilibrium sequence from the next period for this worker. We can express this new variable as follows:

$$J_{i,t}(\widetilde{W}) = \lambda_{2,t,i} + \frac{u_c(C_t, N_t)}{P_t} (\widetilde{W} - W_{i,t}).$$

This expression states that the value of an additional worker employed at wage \widetilde{W} equals the value of a worker employed at the equilibrium wage, the first term, plus a gain from receiving

wage \widetilde{W} rather than $W_{i,t}$ expressed in units of marginal utility, the second term.

Two extreme cases for labor reallocation will prove useful in our analysis and are defined here.

Definition 3.1

- *Costless reallocation:* $R(L_{i,t-1}, L_{i,t}) = 0$ for all $L_{i,t-1}, L_{i,t} \geq 0$, $i = 1, 2, \dots, K$ and $t \geq 0$.
- *No labor reallocation:* $R(L_{i,t-1}, L_{i,t}) = \infty$ for any $L_{i,t-1} \neq L_{i,t} \geq 0$, $i = 1, 2, \dots, K$ and $t \geq 0$.

If reallocation is costless, then the right-hand side of equation (3.8) is always equalized across sectors to $\lambda_{3,t}$. Alternatively, if there is no reallocation the labor force is fixed across sectors and equation (3.8) becomes redundant.

Also, for reference in later sections, we define the case of no wealth effects on labor supply.

Definition 3.2 *Let*

$$-u_n(C_t, N_t)/u_c(C_t, N_t) = f(N_t)$$

*for some function f . That is, the marginal rate of substitution does not depend on consumption C_t . Then, labor supply does not exhibit wealth effects.*¹⁰

3.3.2 Retailers

The consumption goods are sold to households by a set of monopolistically competitive retailers who can costlessly differentiate the single final good purchased from wholesale firms. These retailers periodically set prices à la Calvo at a markup to marginal cost, which is the real cost of the final good P_{ft}/P_t , where P_{ft} is the nominal price of the final good. The

¹⁰The standard search and matching model, see, for example, [Mortensen and Pissarides \(1994\)](#), assumes neither wealth effects nor any variable disutility of labor supply. This conforms to the case of $f(N) = z$ for some constant reservation wage z .

retailers problem is standard to any New Keynesian model:

$$\begin{aligned} \max_{P_t(l)} \Pi_t^{ret}(l) &= \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} [P_t(l) - P_{fT}] Y_T(l), \\ \text{s.t. } Y_T(l) &= Y_T \left(\frac{P_t(l)}{P_T} \right)^{-\zeta}, \end{aligned}$$

where $P_t(l)$ is the nominal price chosen by a retailer that sells differentiated good l and who faces a downward sloping demand schedule and discounts future profits by the nominal stochastic discount factor $Q_{t,T}$. Parameter χ is the Calvo parameter governing the degree of price stickiness. The optimality condition for price-setting is given by:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^\zeta Y_T \left(P_t^*(l) - \frac{\zeta}{\zeta-1} P_{fT} \right) = 0,$$

Which implies

$$\frac{P_t^*(l)}{P_t} = \frac{K_t}{F_t}. \quad (3.10)$$

where

$$\begin{aligned} K_t &= \frac{\zeta}{\zeta-1} \mathbb{E}_t \sum_{T=t}^{\infty} u_c(C_t, N_t) (\beta\chi)^{T-t} \frac{P_{fT}}{P_t} \left(\frac{P_T}{P_t} \right)^{\zeta-1} Y_T, \\ F_t &= E_t \sum_{T=t}^{\infty} u_c(C_t, N_t) (\beta\chi)^{T-t} \left(\frac{P_T}{P_t} \right)^{\zeta-1} Y_T. \end{aligned}$$

The last two relations can be expressed in recursive form:

$$K_t = \frac{\zeta}{\zeta-1} u_c(C_t, N_t) \frac{P_{ft}}{P_t} Y_t + \beta\chi \mathbb{E}_t \Pi_{t+1}^\zeta K_{t+1}, \quad (3.11)$$

$$F_t = u_c(C_t, N_t) Y_t + \beta\chi \mathbb{E}_t \Pi_{t+1}^{\zeta-1} F_{t+1}. \quad (3.12)$$

where $\Pi_t \equiv P_t/P_{t-1}$. The inflation rate is derived from the Calvo assumption with a fraction $1 - \chi$ of firms resetting their prices to $P_t(l)/P_t$:

$$P_t = \left\{ \chi P_{t-1}^{1-\zeta} + (1-\chi) (P_t^*)^{1-\zeta} \right\}^{\frac{1}{1-\zeta}}.$$

The last equation implies:

$$\frac{1 - \chi \Pi_t^{\zeta-1}}{1 - \chi} = \left(\frac{K_t}{F_t} \right)^{1-\chi} \quad (3.13)$$

At the zero inflation steady state, a log-linearization of these equilibrium conditions delivers the standard New Keynesian Phillips curve.

3.3.3 Wholesale Firms

The final good purchased by retailers is sold by wholesale firms who purchase an intermediate output good produced by firms in each sector. We assume a finite set of sectors that produce an intermediate good that is transformed into the final good using a constant elasticity of substitution aggregator:

$$\begin{aligned} \Pi_t^f &= \max_{Y_{i,t}} P_{ft} Y_t - \sum_{i=1}^K P_{i,t} Y_{i,t}, \\ \text{s.t.: } Y_t &= \left(\sum_{i=1}^K \phi_{i,t}^{\frac{1}{\eta}} Y_{i,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \end{aligned} \quad (3.14)$$

where $\phi_{i,t}$ represents a relative preference shock (or relative demand shock) and η is the elasticity of substitution among intermediate goods. Optimization by final good firms provides demand functions for each intermediate good:

$$Y_{i,t} = \phi_{i,t} Y_t \left(\frac{P_{i,t}}{P_{ft}} \right)^{-\eta}. \quad \forall i = 1, 2, \dots, K \quad (3.15)$$

For $\eta = 1$, the CES aggregator is Cobb-Douglas and intermediate goods are neither complements nor substitutes. If $\eta < 1$, intermediate goods are complements, while if $\eta > 1$, intermediate goods are substitutes. The aggregate price index for the final good can be expressed as follows

$$\frac{P_{ft}}{P_t} = \left\{ \sum_{i=1}^K \phi_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \quad (3.16)$$

3.3.4 Intermediate Good Firms

Intermediate goods are produced by competitive firms in each sector who hire labor and post vacancies subject to a linear production function and a law of motion for firm employment. The production function has linear form:

$$Y_{i,t} = A_{i,t}N_{i,t}, \quad (3.17)$$

where $A_{i,t}$ is sector-specific productivity. Firms in each sector take sectoral productivity shocks, wages, separation rates, and a job-filling rate as given. The firm solves the following problem:

$$\Pi_{i,t}^{Int} = \max_{\{V_{i,T}, N_{i,T}\}_{T=t}^{\infty}} \mathbb{E}_t \sum_{T=0}^{\infty} Q_{t,T} [P_{i,T}A_{i,t}N_{i,t} - W_{i,T}N_{i,T} - \kappa V_{i,T}P_T], \quad (3.18)$$

$$\text{s.t.: } N_{i,t} = (1 - \delta_i)N_{i,t-1} + q_{i,t}V_{i,t}. \quad (3.19)$$

where $q_{i,t}$ is the vacancy yield or job-filling rate. Optimal choice of vacancies is determined as follows:

$$q_{i,t}\lambda_{4,t,i} = \kappa, \quad (3.20)$$

where $\lambda_{4,t,i}$ is the Lagrange multiplier on (3.19) expressed in real terms; this multiplier can be interpreted as the value of an additional hired worker in period t at the equilibrium wage. This condition states that the cost of posting a vacancy, the right-hand side, equals the value that it brings if the firm meets a worker with probability $q_{i,t}$. Optimality with to employment leads to:

$$\lambda_{4,t,i} = \frac{P_{i,t}}{P_t}A_{i,t} - \frac{W_{i,t}}{P_t} + E_t Q_{t,t+1} (1 - \delta_i) \lambda_{4,t+1,i} \quad (3.21)$$

where $Q_{t,t+1}$ is the stochastic discount factor of the representative household between period t and $t + 1$. The condition states that the value of an additional employed worker equals the revenue this worker brings net of wage costs plus the future value of the worker tomorrow conditional on not separating.

It will prove useful to introduce a variable closely related to $\lambda_{4,t,i}$ that determines workers wages. Let $J_{i,t}^{Int}(\widetilde{W})$ be the value for an intermediate goods firm at equilibrium employment levels of having one additional worker employed at a wage \widetilde{W} in period t and with the wage returning to an equilibrium sequence from the next period for this worker. We can express the new variable as follows:

$$J_{i,t}^{Int}(\widetilde{W}) = \lambda_{4,t,i} - \frac{(\widetilde{W} - W_{i,t})}{P_t}.$$

This expression states that the value of an additional worker employed at wage \widetilde{W} equals the value of a worker employed at the equilibrium wage net of a gain from paying wage \widetilde{W} rather than $W_{i,t}$.

3.3.5 Labor Market and Wages Determination

Hiring is mediated by a sectoral matching function that depends on the level of vacancies and unemployment in each sector. We allow sectoral matching functions to differ in matching function productivity, but require the matching function to display constant returns to scale and share a common matching function elasticity α . The job-filling rate can be defined as follows:

$$q_{it} \equiv \frac{H_{it}}{V_{it}} = \varphi_i \left(\frac{V_{it}}{U_{it}} \right)^{-\alpha} \quad (3.22)$$

The job-finding probability is taken as exogenous by the household and is determined in equilibrium by the sectoral matching function and the level of vacancies and unemployed persons in each sector:

$$p_{it} \equiv \frac{H_{it}}{U_{it}} = \varphi_i \left(\frac{V_{it}}{U_{it}} \right)^{1-\alpha} \quad (3.23)$$

Wages are determined via Nash bargaining in each sector. Assuming that there are gains from trade, i.e., $J_{i,t}^{Int}$, $J_{i,t} \geq 0$, the bargained wage solves the following problem

$$\max_{\widetilde{W}} \left[J_{i,t}^{Int}(\widetilde{W}) \right]^{1-\nu} \left[J_{i,t}(\widetilde{W}) \right]^{\nu}$$

Nash-bargaining implies that the sectoral wage satisfies the following condition:

$$\nu J_{i,t}^{Int}(\widetilde{W}) = (1 - \nu) \frac{J_{i,t}(\widetilde{W})}{u_c(C_t, N_t)}.$$

In equilibrium it will be true that $\widetilde{W} = W_{i,t}$ which implies that $J_{i,t}^{Int}(\widetilde{W}) = \lambda_{4,t,i}$ and $J_{i,t}(\widetilde{W}) = \lambda_{2,t,i}$. Hence,

$$\nu \lambda_{4,t,i} = (1 - \nu) \frac{\lambda_{2,t,i}}{u_c(C_t, N_t)}. \quad (3.24)$$

3.3.6 Shocks

Our model features both aggregate and sector-specific shocks. We consider two types of sector-specific shocks: sectoral productivity shocks $A_{i,t}$ and sectoral preferences (or demand) shocks $\phi_{i,t}$. Fluctuations in government purchases G_t provide an aggregate demand shock, though, as we will show, other types of demand shocks like preference shocks or monetary shocks could be considered without affecting our conclusions. A uniform change in $\{A_{i,t}\}_{i=1}^K$ can be an example of aggregate productivity shock.

Since our model features a finite number of sectors, it is necessary to account for the aggregate component of variation in $A_{i,t}$ and $\phi_{i,t}$. In the absence of productivity shocks and assuming a uniform level of productivity, i.e., $A_{i,t} = A_{h,t} = A_t$ for $i, h = 1, 2, \dots, K$, the only sector-specific shock is the product share $\phi_{i,t}$ in the CES aggregator. Naturally, a sector-specific shock is any change in the distribution of $\phi_{i,t}$ subject to the restriction that $\sum_{i=1}^K \phi_{i,t} = 1$. However, given that sectors have nonzero mass, an increase in sectoral productivity will have aggregate effects if not offset by declines in sectoral productivity elsewhere. Moreover, the size of the offsetting shock depends on the degree of substitutability for goods across sectors. For example, if goods are perfect complements and productivity is initially equalized across sectors, a negative shock to one sector shifts in the production possibilities frontier of the economy even if offset by a corresponding positive shock to another sector. We address this issue by *redefining* aggregate productivity and sectoral shocks as follows:

Definition 3.3

1. **aggregate productivity** is given by $A_t \equiv \left\{ \sum_{i=1}^K \phi_{i,t} A_{i,t}^{\eta-1} \right\}^{\frac{1}{\eta-1}}$,
2. **sector-specific productivity** is given by $\tilde{A}_{i,t} \equiv A_{i,t}/A_t$,
3. **sector-specific demand** is given by $\tilde{\phi}_{i,t} \equiv \phi_{i,t} \tilde{A}_{i,t}^{\eta-1}$.

Note that $\tilde{A}_{i,t}$ and $\tilde{\phi}_{i,t}$ are functions of the underlying sectoral shocks $A_{i,t}$ and $\phi_{i,t}$. Also note that $A_t, \{\tilde{A}_{i,t}\}_{i=1}^K, \{\tilde{\phi}_{i,t}\}_{i=1}^K$ represent only $2K - 1$ independent variables because of the restrictions $\sum_{i=1}^K \tilde{\phi}_{i,t} = 1$ and $\sum_{i=1}^K \tilde{\phi}_{i,t} / \tilde{A}_{i,t}^{\eta-1} = 1$. Let these independent variables be $\{A_t, \{\tilde{A}_{i,t}\}_{i=1}^{K-1}, \{\tilde{\phi}_{i,t}\}_{i=1}^{K-1}\}$, where we removed $\tilde{A}_{K,t}$ and $\tilde{\phi}_{K,t}$.

This definition of aggregate and sector-specific shocks is motivated by a simple decomposition of the CES aggregator where output can be expressed in terms of aggregate productivity, aggregate employment, and a misallocation term that reflects the deviation from the equilibrium allocation in the absence of the labor market frictions.¹¹ Formally (omitting time subscripts),

$$\begin{aligned} Y &= \left\{ \sum_{i=1}^K \phi_i^{\frac{1}{\eta}} Y_i^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} = \left\{ \sum_{i=1}^K \phi_i^{\frac{1}{\eta}} (A_i N_i)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ &= AN \left\{ \sum_{i=1}^K \left[\phi_i \left(\frac{A_i}{A} \right)^{\eta-1} \right]^{\frac{1}{\eta}} \left(\frac{N_i}{N} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \leq AN, \end{aligned}$$

where the last inequality follows from the fact that both $\phi_i (A_i/A)^{\eta-1}$ and N_i/N must sum to one.¹² When the distribution of productivity is uniform, a sector-specific preference shock satisfies the typical CES condition that product shares sum to one.

¹¹In the absence of sectoral reallocation costs and a search-and-matching friction the following condition $\phi_{i,t} A_{i,t}^{\eta-1} = \text{const} \cdot N_{i,t}$ for $i = 1, 2, \dots, K$ holds in equilibrium. This implies that the output of the wholesale firms can be expressed as $Y_t = A_t N_t$.

¹²The fact that $\phi_i (A_i/A)^{\eta-1}$ and N_i/N sum to one follows directly from the definition of A_t and N_t . The inequality formally follows from the application of Holder's inequality: $\sum_{i=1}^K x_i^{1/p} y_i^{1/q} \leq \left(\sum_{i=1}^K x_i \right)^{1/p} \left(\sum_{i=1}^K y_i \right)^{1/q}$ with $x_i, y_i \geq 0$ and $1/p + 1/q = 1$, see, for example, [Kolmogorov and Fomin \(1970\)](#). In our case $x_i = \phi_i (A_i/A)^{\eta-1}$, $y_i = N_i/N$, $p = \eta$, $q = \eta/(\eta - 1)$.

3.3.7 Government Sector

We assume that the central bank can control riskless short-term nominal interest rate i_t ¹³ and the zero lower bound on i_t never binds.¹⁴ The central bank follows the variant of Taylor rule

$$\log(1 + i_t^d) = \log(1 + \bar{i}_d) + \phi_\pi \log(\Pi_t) + \phi_y \log\left(\frac{Y_t}{\bar{Y}}\right) \quad (3.25)$$

The fiscal authority chooses a sequence of government purchases G_t . We assume that the fiscal authority insures intertemporal government solvency regardless of the monetary policy chosen by the central bank.

3.3.8 Equilibrium

A competitive equilibrium is a set of aggregate allocations $\{Y_t, N_t, C_t, K_t, F_t, \lambda_{3,t}\}_{t=0}^\infty$, sectoral allocations $\left\{\{Y_{i,t}, N_{i,t}, U_{i,t}, V_{i,t}, L_{i,t}, \lambda_{2,t,i}, \lambda_{4,t,i}\}_{i=1}^K\right\}_{t=0}^\infty$, sectoral prices $\left\{\{W_{i,t}/P_t, P_{i,t}/P_t\}_{i=1}^K\right\}_{t=0}^\infty$ and aggregate prices $\{P_{ft}/P_t, i_t^d, \Pi_t\}_{t=0}^\infty$, job-finding and job-filling rates $\left\{\{p_{it}, q_{it}\}_{i=1}^K\right\}_{t=0}^\infty$, initial values of sectoral employment, unemployment, and the labor force $\{N_{i,-1}, U_{i,-1}, L_{i,-1}\}_{i=1}^K$, exogenous processes $\left\{G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\}_{i=1}^{K-1}\right\}_{t=0}^\infty$ that jointly satisfy:

1. (3.3) - (3.9) (household optimization)
2. (3.11) - (3.13) (retailers optimization and inflation dynamics equation),
3. (3.14), (3.15) (wholesale firms optimization),
4. (3.17), (3.20), (3.21) (intermediate goods firms optimization),
5. (3.22), (3.23) (job-filling and job-finding rates),
6. (3.24) (wages are determined by Nash bargaining),

¹³See Woodford (2003) for the analysis of monetary policy in the absence of the demand for central bank liabilities.

¹⁴See Eggertsson and Woodford (2003) for the analysis of the consequences of the binding zero lower bound constraint on short-term nominal interest rate.

7. (3.25) (monetary policy rule),
8. $Y_t = C_t + \sum_{i=1}^K \kappa V_{i,t} + G_t$ (goods-market clearing),

3.4 Sectoral Shocks, the Beveridge Curve and Unemployment Rate

In this section, we characterize the Beveridge curve in a multisector model and provide analytical results relating sectoral shocks, the Beveridge curve, and the natural rate of unemployment.

3.4.1 Preliminaries

The definition of equilibrium implies that the economy is characterized by $11K + 9$ endogenous variables with $11K + 9$ equilibrium conditions, $2K + 1$ exogenous shocks and K initial values for $\{N_{i,-1}\}_{i=1}^K$. The aggregate productivity shock is derived from the sectoral shocks using Definition 3.3.

Substituting the relation determining Nash wages (3.24) into the dynamic equation for the household value of an additional worker (3.9), we can express the wage in terms of the job-filling rate and job-finding rates in each sector:

$$W_{i,t} = -\frac{u_n(C_t, N_t)}{u_c(C_t, N_t)} + \frac{\nu}{1 - \nu} \kappa \left[\frac{1}{q_{i,tt}} - E_t Q_{t,t+1} (1 - \delta_i - p_{itt+1}) \frac{1}{q_{itt+1}} \right] \quad (3.26)$$

While the optimality condition for worker reallocation (3.8) may appear cumbersome, the costless reallocation limit is instructive. When reallocation is costless or in the nonstochastic steady state, the right hand side of the reallocation condition is equalized across sectors and household surpluses are equalized for all sectors. In particular, this condition implies the Jackman-Roper condition that labor market tightness must be equalized across sectors.¹⁵

¹⁵The condition that labor market tightness be equalized across sectors was postulated in Jackman and Roper (1987) as a benchmark for measuring the degree of structural unemployment.

Proposition 3.1 *Let $R(L_{i,t-1}, L_{i,t}) = 0$ for all $L_{i,t-1}, L_{i,t} \geq 0$. Then, for any sectors i and j , $\theta_{i,t} = \theta_{j,t}$ where $\theta_{i,t} = V_{i,t}/U_{i,t}$.*

Proof. Observe that for any two sectors, household optimality and Nash-bargaining imply:

$$p_{i,t}J_{i,t} = p_{j,t}J_{j,t} \Rightarrow \kappa \frac{\nu}{1-\nu} \frac{p_{i,t}}{q_{i,t}} = \kappa \frac{\nu}{1-\nu} \frac{p_{j,t}}{q_{j,t}} \Rightarrow \frac{V_{i,t}}{U_{i,t}} = \frac{V_{j,t}}{U_{j,t}},$$

where the first equality follows from the relation of firm surplus and household surplus from Nash-bargaining and the second equality follows from the definition of $p_{i,t}$ and $q_{i,t}$. ■

This result requires bargaining power and flow vacancy costs to be equalized across sectors but places no restriction on the parameters of the matching function or separation rates. In contrast to the environment considered by Jackman and Roper (1987), our results show that this condition continues to hold in a fully dynamic setting and allowing for greater heterogeneity in hiring costs across sectors. More generally, if bargaining power or vacancy posting costs differ across sectors, a generalized Jackman-Roper condition will obtain where sectoral tightness will be equalized up to a wedge term reflecting differences in bargaining power and vacancy costs. This condition is analogous to the generalized Jackman-Roper condition derived in [Sahin, Song, Topa, and Violante \(2010\)](#).

When reallocation is costly, the probability-weighted household surplus will generally fail to be equalized across sectors and the household will have an incentive to transfer workers to sectors with a higher surplus or a greater job-finding rate. In the no reallocation limit with a fixed labor force distribution, tightness across sectors will generically depart from the Jackman-Roper condition.

3.4.2 Defining the Beveridge Curve

For the US, labor market flows are large and vacancies and unemployment quickly converge to their flow steady state. To derive the Beveridge curve, we treat the sectoral equations determining vacancies, unemployment and employment as steady state conditions. In particular, in the analysis that follows, equations (3.3) - (3.5), (3.21) and (3.26) are assumed to be at

their flow steady state.¹⁶

In the standard one-sector model (i.e., $K = 1$), the Beveridge curve is a single equation defining the relationship between unemployment and vacancies and given by the steady state of the employment flow equation (3.3):

$$\delta(1 - U) = \varphi U^\alpha V^{1-\alpha}.$$

Only changes in the separation rate δ and matching function productivity φ shift the Beveridge curve, while other shocks like aggregate productivity shocks simply move unemployment and vacancies along the pair of points defined by this equation. This relation also explains why the one-sector Beveridge curve is the same irrespective of real or demand-driven business cycles.

In a multi-sector model, an analytical relationship between U and V does not exist, and the aggregate steady state Beveridge curve is an equilibrium object. It is useful to construct the multisector analog of the one-sector steady state employment flow equation. Summing over sectoral employment in equation (16), we obtain a single equation relating sectoral vacancies and sectoral unemployment:

$$L - U = \sum_{i=1}^K \frac{\varphi_i}{\delta_i} U_i^\alpha V_i^{1-\alpha} \Rightarrow \frac{L - U}{V} \theta^\alpha = \sum_{i=1}^K \frac{\varphi_i}{\delta_i} \left(\frac{\theta_i}{\theta} \right)^{-\alpha} \frac{V_i}{V}$$

where $\theta = V/U$ is aggregate labor market tightness and $\theta_i = V_i/U_i$ is sectoral labor market tightness. The left-hand side is an expression solely in terms of aggregate unemployment and vacancies but the right-hand side will generally depend on both the type of aggregate shocks and the distribution of sectoral shocks. This term is the source of shifts in the Beveridge curve.

In a solution to our model, aggregate vacancies and unemployment are a function of the exogenous shocks: government purchases, aggregate productivity and the full set of sectoral

¹⁶Impulse responses for the multisector model calibrated to monthly data show that unemployment and vacancies converge to the log-linearized Beveridge curve within 3 months. The rapid convergence of the labor market to the steady state Beveridge curve explain the high correlation of vacancies and unemployment in the calibration exercise in [Shimer \(2005\)](#).

productivities $\tilde{A}_{i,t}$ and preferences $\tilde{\phi}_{i,t}$:

$$U = U \left(G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\} \right),$$

$$V = V \left(G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\} \right),$$

The full set of equations that determine unemployment and vacancies are listed at the beginning of Appendix C.2. We use variations in G_t as the variable that traces out the Beveridge curve and drop time subscripts:

Definition 3.4 *The Beveridge curve is a function $f(\cdot)$ given by $V \left(G; A, \{\tilde{A}_i, \tilde{\phi}_i\} \right) = f \left(U \left(G; A, \{\tilde{A}_i, \tilde{\phi}_i\} \right) \right)$, where G is the parameter varying U and V , holding constant aggregate productivity, sectoral productivity and preferences: A , $\{\tilde{A}_i\}$ and $\{\tilde{\phi}_i\}$.*

Aggregate Shocks and the Beveridge Curve

To separate movements along the Beveridge curve from shifts in the Beveridge curve, it is necessary to choose a single shock as the source of business cycles. Indeed, in the absence of any other aggregate or sectoral shocks, the Beveridge curve in a multisector model never shifts. However, in the presence of several different types of aggregate and sectoral shocks, the Beveridge curve could be equally well-defined as the locus of points in the U-V space traced out by aggregate productivity shocks or shocks to any given sector.

While our definition of the Beveridge curve as the locus of points in the U-V space traced out by government purchases shocks may seem fairly restrictive, a variety of real and nominal shocks trace out the same Beveridge curve. In the absence of wealth effects on labor supply, the equations that determine aggregate vacancies and unemployment and the sectoral distribution of vacancies and unemployment can be decoupled from the remaining equations that determine other endogenous variables.

Proposition 3.2 *Assume no wealth effects and either costless labor reallocation or no reallocation. For any value of government spending shock G , there exists an A such that*

$$V\left(G, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K\right) = V\left(1, A, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K\right) \text{ and } U\left(G, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K\right) \\ = U\left(1, A, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K\right).$$

Proof. See Appendix C.2.1. ■

This proposition shows that an aggregate productivity shock traces out the same Beveridge curve as a government purchases shock. Moreover, the same proposition applies to other types of demand shocks like monetary policy shocks not specified in our model. Indeed, any shock, real or nominal, that does not enter the steady state labor market equations that determine vacancies and unemployment, traces out the same Beveridge curve.

In the absence of wealth effects, holding constant sectoral productivity and preferences, aggregate vacancies and unemployment can be parameterized by real marginal cost times aggregate productivity: $A_t P_{ft}/P_t$. Real marginal cost, an endogenous variable, is the only link between the block of equations that determine aggregate vacancies and unemployment and the rest of the model equations. Under no wealth effects on labor supply (as in [Shimer \(2005\)](#) or [Hagedorn and Manovskii \(2008\)](#)), our multisector model effectively generalizes the behavior of the one-sector Beveridge curve under aggregate shocks.

Moreover, given the results on aggregate productivity shocks in Proposition 3.2, our conclusions about the relationship between sectoral shocks and shifts in the Beveridge curve continue to hold in a model without sticky prices where business cycle fluctuations are driven by real shocks instead of demand shocks.

Neutrality of Sector-Specific Shocks

As our derivation of the Beveridge curve suggests, sectoral shocks can shift the Beveridge curve if these shocks alter the distribution of vacancies or generates mismatch across sectors. However, as showed earlier, when labor reallocation is costless, the Jackman-Roper condition obtains and tightness is equalized across sectors. In this case, we can once again obtain an aggregate Beveridge curve that is identical to the one-sector Beveridge curve:

Proposition 3.3 *If labor reallocation is costless across sectors and separation rates and*

matching function efficiencies are the same across sectors (i.e. $\delta_i = \delta, \varphi_i = \varphi$), then sector-specific shocks do not shift the Beveridge curve.

Proof. Under costless labor reallocation, the Jackman-Roper condition holds and labor market tightness across sectors is equalized: $V_{i,t}/U_{i,t} = V_{h,t}/U_{h,t}$ for all $i, h = 1, 2, \dots, K$. Summing over the steady state sectoral Beveridge curves (steady state version of equation (3.3)):

$$\sum_{i=1}^K N_i = \sum_{i=1}^K \frac{\varphi}{\delta} \theta^{-\alpha} V_i \Rightarrow 1 - U = \frac{\varphi}{\delta} \left(\frac{V}{U} \right)^{-\alpha} V$$

as required. ■

As a result, neither aggregate nor sector-specific shocks generate a shift in the Beveridge curve, providing a useful benchmark for our analysis of the effects of sector-specific shocks when reallocation is costly.

The conditions that recover the aggregate Beveridge curve in Proposition 3.3 highlight the two channels through which sector-specific shocks shift the Beveridge curve: the mismatch channel and the composition channel. If sectors share identical hiring technologies and separation rates, a sector-specific shock can only shift the Beveridge curve by changing the distribution of θ_i/θ - in other words, by generating mismatch. When labor market reallocation is costly, a sector-specific shock increases tightness in one sector while decreasing tightness in the other. Because of the decreasing returns to scale of the matching function, the rise in vacancies for the sector experiencing a positive shock exceeds the fall in vacancies for the sector with a negative shock. In contrast, an aggregate shock depresses tightnesses more or less uniformly, lowering vacancies in all sectors. The composition effect is present even when labor reallocation is costless. If some sectors feature greater hiring frictions, a shock favoring those sectors will shift the distribution of vacancies toward that sector, raising overall vacancies relative to a shock that leaves the distribution unchanged. Together, these two channels account for the effect of sector-specific shocks on the Beveridge curve.

3.4.3 Model-Implied Measures of Sectoral Shocks and Beveridge Curve Shifts

Our multisector model provides a useful framework for assessing the validity of empirical measures that rely on the labor market to measure sector-specific disturbances. As discussed earlier, [Lilien \(1982\)](#) argued that sector-specific shocks could be measured by dispersion in employment growth across sectors, with [Abraham and Katz \(1986\)](#) countering that increases in employment growth dispersion could be generated by aggregate shocks if sectors feature asymmetric responses to aggregate shocks.

Our model verifies that the Lilien measure is a biased measure of sector-specific shocks validating the Abraham and Katz critique. To a log-linear approximation, sectoral employment can be expressed as a function of sectoral shocks and aggregate output. Below, we express sectoral employment under the polar cases of no reallocation n_{it}^{nr} and costless reallocation n_{it}^r respectively, assuming no wealth effects on labor supply:

$$n_{it}^{nr} = \underbrace{\lambda_i [\phi_{it} - (1 - \eta) a_{it}]}_{\epsilon_{it}} + \lambda_i \underbrace{[y_t - (1 - \eta) a_t]}_{F_t},$$

$$n_{it}^r = [\phi_{it} - (1 - \eta) a_{it}] + y_t - (1 - \eta) a_t - \eta [s_i \varphi_i + (1 - s_i) \alpha] \theta_t,$$

$$\text{where } \lambda_i = \left\{ 1 + \eta [s_i \varphi_i + (1 - s_i) \alpha] \frac{\bar{L}_i / \bar{U}_i}{1 - \alpha} \right\}^{-1},$$

where φ_i is a macro Frisch elasticity that reflects the dependence of the Nash-bargained sectoral wages on labor market tightness and $1 - s_i$ is the steady state size of the surplus.¹⁷ This parameter is a function of steady-state job-finding rates and vacancy-filling rates along with others parameters of the model such as the sectoral separation rate, etc. These expressions for sectoral employment are not materially changed by allowing for wealth effects or convex disutility of labor supply, which would simply add linear functions of y_t and n_t to each expression.

These expressions for sectoral employment show that both sector-specific shocks and ag-

¹⁷Specifically, $s_i = \bar{W}_i / \bar{P}_i \bar{A} \bar{A}_i$.

gregate shocks will increase employment dispersion in both the costless reallocation and no reallocation cases. In the case of the latter, the sensitivity of a sector to aggregate and sector-specific shocks increases with the elasticity λ_i which is larger for sectors with a lower Frisch elasticity. For example, if household's bargaining power is zero, wages are set at a constant level and $\varphi_i = 0$ for all sectors. Then sectors with a lower surplus display greater sensitivity to aggregate shocks consistent with the volatility of employment in a one-sector search model as discussed by [Hall \(2005\)](#) and [Hagedorn and Manovskii \(2008\)](#).

Since sector-specific shocks are generally correlated with output, our model shows that the assumptions underlying our factor analysis in Section 3.2 will generally not be satisfied. In short, simply allowing for differential elasticities to aggregate shocks is insufficient to identify sector-specific shocks. However, following the procedure in [Foerster, Sarte, and Watson \(2011\)](#), we can conduct a structural factor analysis by using a calibrated version of the model to correct for the endogeneity problem. For simplicity, assume only sectoral productivity shocks a_{it} and assume that aggregate productivity shocks are simply a linear combination of sectoral productivity shocks. Let $a_t = (a_{1t}, \dots, a_{Kt})'$ be the vector of sectoral productivity shocks taken as exogenous. Assume a factor decomposition of this exogenous process such that:

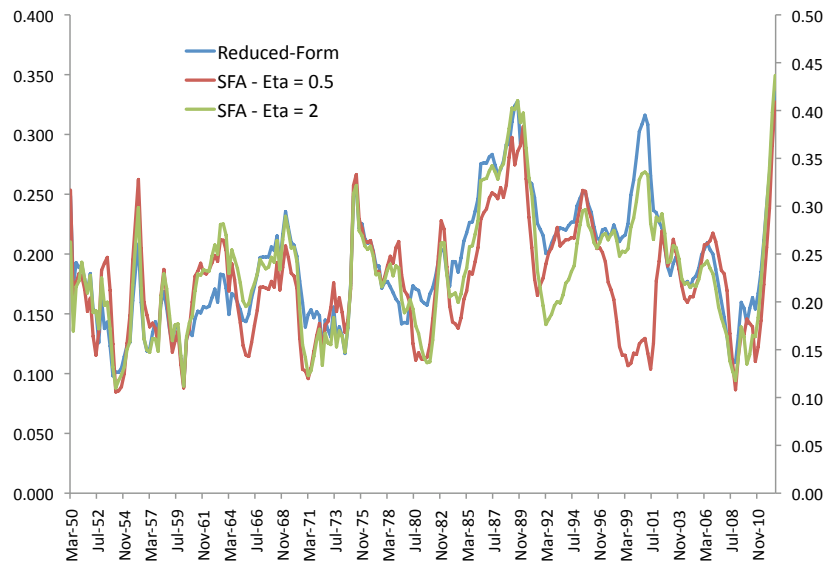
$$a_t = \Phi z_t + \epsilon_t,$$

where ϵ_t is a $K \times 1$ vector of sector-specific productivity shocks and z_t is a scalar defined as the aggregate productivity shock with $cov(z_t, \epsilon_t) = 0$. Combining the expressions for sectoral employment and output, sectoral employment is a function of the vector of sectoral productivity shocks:

$$Mn_t = Ha_t,$$

where M is a nondiagonal matrix with $1/\lambda_i - \gamma_i$ as its diagonal elements and $-\gamma_j$ as its off-diagonal elements. Similarly H is a nondiagonal matrix with $\eta - 1 + \gamma_i$ as its diagonal elements and γ_j as its off diagonal elements. The coefficient $\gamma_i = \bar{\phi}_i^{1/\eta} (\bar{Y}_i/\bar{Y})^{\frac{\eta-1}{\eta}}$ - the steady state share of output for each sector - enters the solution for sectoral employment since

Figure 3.6: Sectoral shock index using structural factor analysis



$y_t = \sum_{i=1}^K \gamma_i (a_{it} + n_{it})$. Unless M is diagonal, a factor analysis of n_t will not accurately identify the sectoral shocks ϵ_t . However, for higher degrees of substitutability, the off-diagonal elements of M and H are dominated by the diagonal elements and the endogeneity correction becomes less important. In the limit, when goods are perfect substitutes, the reduced-form analysis in Section 3.2 is the correct procedure for identifying sector-specific shocks.

Proposition 3.4 *Assume the case of no labor reallocation and let $\eta \rightarrow \infty$. Then $n_t = H a_t$ and a factor analysis of employment identifies the sector-specific shock ϵ_t .*

Proof. See Appendix C.2.2. ■

To correct for possible endogeneity in our estimates of sector-specific shocks, we calibrate our model to derive the rotation matrix M , apply this rotation to sectoral employment data, and then perform a factor analysis on this rotation of the data. The calibration used to derive the matrix M is discussed in the Appendix. Our structural factor analysis follows the same procedure as in Section 3.2 with the exception of applying the rotation M to the data and using quarterly data instead of monthly data before removing the first principal component and computing the sector-specific shock index.¹⁸ As shown in Figure 3.6, the model-implied

¹⁸We use quarterly data instead of monthly data since, in our model, we assume the labor market is in its

sectoral shock index displays a strong correlation with our reduced form shock index. As hypothesized, the correlation is stronger when goods are moderate substitutes (the case of $\eta = 2$) because the off-diagonal elements of M are less important. Table 3.3 provides the correlation for alternative specifications of the sector-specific shock index obtained using 4th order detrending or year-over-year growth rates.

Table 3.3: Reduced-form and structural sectoral shock index correlation

Detrending	Index Type	$\eta = 0.5$	$\eta = 2$
Cubic Trend	Absolute value index	0.59	0.91
Cubic Trend	Dispersion index	0.65	0.91
Quartic Trend	Absolute value index	0.81	0.95
Quartic Trend	Dispersion index	0.83	0.96
Growth rates (year-over-year)	Absolute value index	0.92	0.95
Growth rates (year-over-year)	Dispersion index	0.89	0.96

Correlation of model-implied shock index with reduced form shock index computed under different detrending procedures and alternative summary measures. See Section 2.

Sectoral Shock Index and Shifts in the Beveridge Curve

In Section 3.2, we correlated our sector-specific shock index with movements in the Beveridge curve intercept and showed that the index appears significant in explaining variation in vacancies controlling for the the variation explained by unemployment. Our model can also be used to think about the relationship between sector-specific shocks and movements in the Beveridge curve.

Under the assumption of no reallocation across sectors and log-linearizing around a steady state with $\bar{\theta}_i = \bar{\theta}_h$ for all $i, h = 1, 2, \dots, K$, we can derive an expression for the Beveridge curve augmented with sectoral dispersion:

$$v_t = -\frac{1}{1-\alpha} \left(\alpha + \frac{\bar{U}}{\bar{N}} \right) u_t + \frac{1}{1-\alpha} \sum_{i=1}^K \left(\frac{\bar{U}_i}{\bar{U}} - \frac{\bar{N}_i}{\bar{N}} \right) n_{it}$$

where α is the matching function elasticity, and the weights on sectoral employment are difference between the unemployment share and employment share in each sector. When matching

flow steady state.

function parameters are identical, these weights are all zero, and we obtain a standard log-linearized Beveridge curve relating vacancies and unemployment. Positive shocks to sectors with a higher share of employment than unemployment shift in the Beveridge curve since these sectors have lower search frictions while the opposite happens to sectors with a lower employment share than unemployment share.

Table 3.4: Regression analysis for reduced-form and model-implied index

<u>Panel A: Dispersion Index (Full Sample)</u>			<u>Panel B: Model-Implied Index (Full Sample)</u>		
	Coeff	<i>t</i> -stat		Coeff	<i>t</i> -stat
<u>Detrend with time trend</u>			<u>Detrend with time trend</u>		
Cubic	0.60	1.27	Cubic**	-0.72	-2.43
Cubic w/1 lag	0.58	1.21	Cubic w/1 lag**	-0.73	-2.40
Quartic	0.53	1.09	Quartic	0.17	0.37
Quartic w/1 lag	0.51	1.04	Quartic w/1 lag	0.19	0.40
<u>Detrend with growth rates</u>			<u>Detrend with growth rates</u>		
Quarter over quarter	-0.11	-0.35	Quarter over quarter***	0.58	2.90
Year over year*	-0.99	-1.93	Year over year***	1.10	4.09
<hr/>			<hr/>		
<i>T</i> = 242					
<u>Panel C: Dispersion Index (After 1980)</u>			<u>Panel D: Model-Implied Index (After 1980)</u>		
	Coeff	<i>t</i> -stat		Coeff	<i>t</i> -stat
<u>Detrend with time trend</u>			<u>Detrend with time trend</u>		
Cubic***	2.24	3.98	Cubic	1.38	1.56
Cubic w/1 lag***	2.19	3.77	Cubic w/1 lag	1.33	1.47
Quartic**	1.78	2.37	Quartic***	1.93	7.78
Quartic w/1 lag**	1.74	2.32	Quartic w/1 lag***	1.95	7.70
<u>Detrend with growth rates</u>			<u>Detrend with growth rates</u>		
Quarter over quarter	0.29	0.50	Quarter over quarter***	1.39	4.18
Year over year	-1.01	-1.21	Year over year***	2.36	7.14

Estimation of Beveridge curves using reduced-form and model-implied measures of sectoral shocks. Left panel uses reduced form dispersion index computed in Section 2, right panel uses model-implied measure computed as described in text. Top panel uses full sample from 1950-2011, bottom panel uses subsample 1980-2011. *** indicates significance at 1% level, ** indicates significance at 5% level, * indicates significance at 10% level. The number of time series observations is $T=726$.

Using our calibration described in the Appendix C.3, we compute the model-based distribution of unemployment and run a regression of vacancies on unemployment and the model-based measure of shifts in the Beveridge curve. Log vacancies (measured by the HWI) and log unemployment are quarterly from 1951 to 2011. We replicate the regression in Section 3.2

using quarterly instead of monthly data.

Our results are presented in Table 3.4. The top panels A and B compute the Beveridge curve estimate using the reduced form shock index from Section 3.2 and the model-implied index respectively using the full sample. In quarterly data, the reduced-form regressions are similar to the regressions presented in Table 3.2 but feature higher standard errors. Panel B shows that sectoral employment detrended with time trends displays coefficients that are negative and often insignificant, inconsistent with the predictions of our model. However, in the case of growth rate detrending, coefficients are positive and significant.

Panel D shows that the negative coefficients on the specifications using detrending via time trends are driven by the early part of the sample. If we consider a sample only after 1980, the coefficients are positive, consistent with our model, and frequently greater than one as predicted by the model. Given that our model is a log-linearization around a steady state and that our calibration relies on unemployment and employment weights computing averages in the last decade, our model-implied measure is likely to be less accurate farther back in time. Given the large movements in employment share across sectors over time, our model-implied measure should fit better in more recent data. It is also worth noting that our model-implied measure delivers positive coefficient across all detrending procedures in Panel D, in contrast to the reduced-form measure considered in Section 3.2.

3.4.4 Beveridge Curve and the Natural Rate of Unemployment

We define the natural rate of unemployment as the unemployment rate at which inflation is stabilized. This is a policy-relevant variable for a central bank that seeks to lower unemployment to a point at which inflation remains stable.

Definition 3.5 *The natural rate of unemployment is unemployment rate when $P_{ft}/P_t = 1$.*

Undistorted Initial State

A useful benchmark for assessing the relationship between sector-specific shocks, Beveridge curve shifts, and the natural rate is the case of an undistorted initial state with no misallo-

cation of output and no differences in labor market tightness across sectors. The household's marginal rate of substitution is assumed to be constant at $z < 1$. If sectors share the same separation rates δ and matching function efficiencies φ , then hiring costs are equalized, relative prices P_i/P are equalized and determined by the inverse markup. In this setting, the model admits a symmetric solution with $Y = AN$, $A_i P_i/P = \mu^{-1} A$, $N_i = \tilde{\phi}_i N$ where $\tilde{\phi}_i$ is the productivity-adjusted product share defined in Section 3.3.6 and $\mu = \zeta/(\zeta - 1)$ is a markup. Aggregate employment N and labor market tightness θ are implicitly defined by a common vacancy posting condition and labor market clearing:

$$\mu^{-1} A = z + \frac{\kappa}{\varphi} g(\theta), \quad (3.27)$$

$$N = \frac{\varphi \theta^{1-\alpha}}{\delta + \varphi \theta^{1-\alpha}}, \quad (3.28)$$

where g is an increasing and concave function of labor market tightness θ . Total employment is simply the job-finding rate over the sum of job-finding rate and the separation rate. Moreover, the distribution of labor market variables: employment, unemployment, vacancies and the labor force all equal the productivity-adjusted product share $\tilde{\phi}_i$.

Proposition 3.5 *Assume costless labor reallocation and for $i = 1, 2, \dots, K$, $\delta = \delta_i$ and $\varphi = \varphi_i$. Then a sector-specific demand or productivity shock does not change the natural rate of unemployment and does not shift the Beveridge curve.*

Proof. The first result follows from the solution for the undistorted steady state and the joint determination of employment and tightness in the equations (3.27) and (3.28). Observe that sector-specific productivity and preferences shares do not enter these equilibrium conditions implying that total employment is determined independently of any sector-specific shock. The second result is an application of Proposition 3. ■

With costless reallocation, a sector-specific shock results in an immediate redistribution of the labor force. Because the cost of hiring is equalized across sectors, a sector-specific shock does not shift the production possibilities frontier leaving aggregate tightness and employment unchanged. Thus, both the Beveridge curve and the natural rate after left unchanged by a

sector-specific shock. While the Beveridge curve does not shift under sectoral or aggregate shocks (due to Proposition 3.3), the natural rate of unemployment may change under real aggregate shocks. A negative productivity shock raises the natural rate, but an increase in markups due to a negative aggregate demand disturbance will leave the natural rate of unemployment unchanged. This provides a simple instance in which changes in the natural rate do not imply a shift in the Beveridge curve.

However, the neutrality of sector-specific shocks for both the Beveridge curve and the natural rate of unemployment hinge on the assumption of costless labor reallocation.

Proposition 3.6 *Assume no reallocation of labor with $\delta_i = \delta$ and $\varphi_i = \varphi$ for $i = 1, 2, \dots, K$. Then a sector-specific demand or productivity shock such that $L_i \neq \tilde{\phi}_i$ raises the natural rate of unemployment and shifts the Beveridge curve outward (i.e. for any level of unemployment, aggregate vacancies rise).*

Proof. See Appendix. ■

In this case, shifts in the Beveridge curve and changes in the natural rate are tightly connected, with an outward shift in the Beveridge curve implying an increase in the natural rate of unemployment. Our proof relies on the properties of convex functions to show how mismatch raises the unemployment rate. Intuitively, a sector-specific shock generates mismatch since labor must be reallocated across sectors to ensure that employment shares equals the product shares. If the labor force cannot be reallocated, tightness rises in the sector where desired employment rises and falls in the other sector. This causes aggregate employment to fall since hiring costs rise faster in the sector that is positively impacted relative to the fall in costs for the sector that is negatively impacted. Similarly, due to the convexity of the matching function, vacancies in the sector with a positive shock rise more than the fall in vacancies in the sector that is negatively hit.

Distorted Initial State

When separation rates or matching function efficiency differ across sectors, the relationship between shifts in the Beveridge curve and changes in the natural rate are not as straightforward.

ward. Assuming that labor market reallocation is costless, the steady state of the two-sector version of the model can be summarized in three equations:

$$\mu^{-1}A = \left[\tilde{\phi}g_A(\theta)^{1-\eta} + (1 - \tilde{\phi})g(\theta)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (3.29)$$

$$1 = N \left[1 + \theta^{\alpha-1} \left(n_A \frac{\delta_A}{\varphi_A} + (1 - n_A) \frac{\delta_B}{\varphi_B} \right) \right] \quad (3.30)$$

$$\frac{n_A}{1 - n_A} = \frac{\tilde{\phi}}{1 - \tilde{\phi}} \left[\frac{g_A(\theta)}{g_B(\theta)} \right]^{-\eta} \quad (3.31)$$

where labor market tightness θ , total employment N , and employment share n_A are the endogenous variables. The function g_i measures hiring costs (inclusive of wages) and is increasing and concave in labor market tightness. Without loss of generality, if sector A has a higher relative matching function efficiency or lower relative separation rate, then $g_A < g_B$ for $\theta > 0$.

Differences in hiring frictions across sectors imply that even in the absence of sectoral shocks, employment shares respond asymmetrically to changes in labor market tightness as can be discerned from equation (3.31). If sector A has lower hiring costs, it follows that $n_A > \tilde{\phi}$ since relative prices are distorted by the asymmetry in hiring costs. Effectively, sector A has higher productivity than sector B and the competitive allocations of labor are distorted toward that sector. A sector-specific shock favoring sector A lowers hiring costs and shifts out the production possibilities frontier for the economy thereby reducing the natural rate of unemployment. Moreover, this reduction in the natural rate is accompanied by a decrease in the aggregate quantity of vacancies needed to attain a particular level of employment. Since labor market tightness is equalized, shifts in the Beveridge curve due to sectoral shocks in this case stem from a composition channel. Moreover, shifts in the Beveridge curve and changes in the natural rate of unemployment move in the same direction; the Beveridge curve may shift inward or outward depending on whether or not the sector-specific shock favors the sector with lower hiring costs. The following proposition summarizes this result:

Proposition 3.7 *Consider the two-sector version of the model with costless labor reallocation and zero bargaining power for households $\nu = 0$. Without loss of generality, assume that*

$\varphi_A > \varphi_B$ and $\delta_A = \delta_B$ or vice versa (i.e. sector A has lower hiring costs than sector B). Then, a positive sector-specific shock to sector A lowers the natural rate of unemployment (i.e. if $\tilde{\phi}_A < \tilde{\phi}'_A \Rightarrow N < N'$) and shifts the Beveridge curve inward.

Proof. See Appendix. ■

The assumption of zero bargaining power simply guarantees that the ratio g_A/g_B as a function of θ is monotonic. For moderate values of θ , the ratio of hiring costs will be locally monotonic with nonzero bargaining power as confirmed in numerical experiments.

With the combination of costly reallocation and asymmetric hiring costs, the connection between the direction of Beveridge curve and the natural rate of unemployment appears to hold in our numerical examples. However, we cannot analytically rule out cases in which a sector-specific shock lowers the natural rate but shifts out the Beveridge curve or vice versa. The analysis here however suggests that this would be the exception rather than the rule.

3.4.5 Alternative Labor Market Measures and Sectoral Shocks

Our model also provides a framework for assessing how well alternative labor market measures capture sector-specific shocks and shifts in the Beveridge curve.

Aggregate Matching Function Efficiency and Mismatch

Recent papers by [Sedlacek \(2011\)](#) and [Barnichon and Figura \(2011\)](#) perform a decomposition analysis of the matching function analogous to measuring the Solow residual in a growth accounting exercise. Constructing measures of unemployment, vacancies, and hires, these authors measure aggregate matching function efficiency as the residual relating these variables

$$\varphi = \frac{H}{U^\alpha V^{1-\alpha}}$$

and show that aggregate matching function efficiency is procyclical. In our multisector model, aggregate matching function efficiency can be expressed in terms of mismatch and the distri-

bution of vacancies:

$$H = \sum_{i=1}^K \varphi_i U_i^\alpha V_i^{1-\alpha} \Rightarrow \frac{H}{\bar{\varphi} U^\alpha V^{1-\alpha}} = \sum_{i=1}^K \frac{\varphi_i}{\bar{\varphi}} \left(\frac{\theta_{it}}{\theta} \right)^{-\alpha} \frac{V_i}{V},$$

where $\bar{\varphi}$ is the average level of matching function efficiency. Changes in mismatch and the distribution of vacancies will lead to variations in measured aggregate matching function efficiency. To a log-linear approximation, mismatch is a function of sectoral employment in our model:

$$\theta_{it} = \frac{1 + \frac{\bar{N}_i}{\bar{U}_i} n_{it}^{nr}}{1 - \alpha}.$$

Since sectoral employment is a function of both aggregate and sector-specific shocks, dispersion in mismatch will also be subject to the Abraham and Katz critique. Therefore, fluctuations in matching function efficiency are not, as such, an indicator of either sector-specific shocks or shifts in the Beveridge curve. For a suitably long time series, if the relationship between matching function efficiency and aggregate shocks is stable, then sector-specific shocks could be identified as periods where movements in matching function efficiency are not explained by the business cycle. Unlike a one-sector model with constant matching function efficiency, our multisector model with costly reallocation is consistent with the empirical observation of movements over the cycle in aggregate matching efficiency.

Similarly, work by [Sahin, Song, Topa, and Violante \(2012\)](#) and [Lazear and Spletzer \(2012\)](#) construct mismatch indices by industry, region and occupation to examine whether mismatch has increased in the current recession. Like measurements of matching function efficiency, our model shows that variation in these measures over the cycle is not sufficient to identify sector-specific shocks or Beveridge curve shifts. Instead, these measures are evidence of the feature in our model that generates mismatch: costly labor reallocation. These empirical mismatch indices rely on direct measures of labor market tightness with vacancies data from either the JOLTs or from online vacancy postings collected by the Conference Board. Measures of sectoral or regional unemployment are constructed from the Current Population Survey (CPS). Data availability limits the time series dimension of these measures, with the mismatch

indices beginning in either 2001 or 2006. Since, mismatch can be driven by either aggregate or sectoral shocks, the cyclical increase in mismatch shown in [Sahin, Song, Topa, and Violante \(2012\)](#) is consistent with either aggregate or sectoral shocks.

Labor Productivity

[Garin, Pries, and Sims \(2010\)](#) document systematic changes in the behavior of labor productivity in post Great Moderation recessions. Our model supports the view that measured labor productivity behaves differently under sectoral shocks than aggregate shocks. To a log-linear approximation, measured labor productivity is a function of sectoral employment:

$$y_t - n_t = a_t + \sum_{i=1}^K (\gamma_i - \bar{N}_i/\bar{N}) n_{it}$$

where γ_i is the share of sector i 's output in total output and \bar{N}_i/\bar{N} is sector i 's employment share. In an undistorted state where these shares are equalized, measured labor productivity equals true productivity, but if these shares are not equalized, measured labor productivity will be a biased indicator of labor productivity and sectoral shocks can both raise or lower labor productivity depending on whether the sector experiencing a positive shock has a larger output share than its employment share. To the extent that sector-specific shocks contribute more to business cycles in the Great Moderation, labor productivity's correlation with the business cycle will be weakened.

Okun's Law

Our multisector model provides a straightforward relationship between output and the unemployment rate. The typically stable relationship between output growth and the changes in the unemployment rate is labeled as Okun's Law and, like the Beveridge curve, is a reduced form relationship that occasionally breaks down. Combining the CES aggregator with our definition of sector-specific shocks and total employment, a structural relationship between

output and unemployment can be obtained:

$$\begin{aligned} Y_t &= A_t N_t \left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ &= A_t (1 - U_t) \left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \end{aligned}$$

where the last term reflects the effect of labor misallocation on output.

The misallocation term is maximized at one - any misallocation must reduce output holding constant the level of unemployment. In this case, sector-specific shocks can disrupt the Okun's law relationship between output growth and changes in the unemployment rate. If the economy is typically characterized by some steady state level of misallocation, then sectoral shocks can shift Okun's law relationship in either direction. For example, a sectoral shock that improves the allocation of labor raises output for any level of unemployment - as shown in Proposition 3.7, this case would conform to an inward shift in the Beveridge curve. However, without a direct measure of aggregate productivity, it is not clear how to separate the misallocation channel from changes in aggregate productivity.

3.4.6 Reservation Wage Shocks and Implications for Structural Change

We can readily extend our model to consider the effect of exogenous shocks to the reservation wage with no wealth effects. Now, a solution for vacancies and unemployment is a function of the reservation wage z in addition to the other exogenous shocks described earlier.

Proposition 3.8 *Assume no reallocation and no wealth effects. Assume that $A_i = A_j = A$, $\delta_i = \delta_j$, $\varphi_i = \varphi_j$ for $i, j = 1, 2, \dots, K$. For any value of the government spending shock G , there exists a z such that $V(G, z_0, A, \phi_i) = V(1, z, A, \phi_i)$ and $U(G, z_0, A, \phi_i) = U(1, z, A, \phi_i)$.*

Proof. See Appendix. ■

A uniform increase in the reservation wage reduces the surplus in each sector in the same way as a productivity or demand shock leaving aggregate vacancies and unemployment on the same Beveridge curve. This proposition shows that, to the extent that unemployment

benefits act as an increase in the household's reservation wage, extensions in the duration of unemployment insurance cannot generate a shift in the Beveridge curve.

With some assumptions on functional forms, our multisector model can be augmented to address the effect of structural change in the long-run on labor market variables and employment shares. Structural change refers to the long-run trends in employment and output shares across sectors. Over the postwar period, employment in manufacturing has steadily dropped from nearly 1/3 of total employment to less than 10%. Over the same period, sectors like education, health care and professional services have all steadily grown. Alternatively, sectors like construction have displayed highly persistent fluctuations without any clear time trend. A recent literature highlighted by [Acemoglu and Guerrieri \(2008\)](#) and [Ngai and Pissarides \(2007\)](#) consider the implications of structural change for aggregate growth rates in models without labor market search. Our model extends these models to allow for consideration of structural change on unemployment and vacancies.

Under the assumption of balanced growth preferences (i.e., King-Plosser-Rebelo) and vacancy posting costs that are proportional to the household's marginal rate of substitution, our model admits a balanced growth path with constant unemployment and vacancy rates and constant growth rates for employment. Wages and output grow at the same rate as aggregate productivity, though, aggregate productivity growth is only asymptotically constant if sectors diverge in their growth rates of productivity. The assumption that vacancy posting costs are proportional to the household's MRS is a natural one if hiring is an activity that requires labor. Similar assumptions in [Blanchard and Gali \(2010\)](#) and [Michaillat \(2012\)](#) on vacancy costs are justified by assuming that the cost of hiring is proportional to the wage paid to workers.

Proposition 3.9 *Consider the K sector flexible-price version of the model with costless labor reallocation and identical separation rates and matching function parameters. Additionally, assume that vacancy posting costs are proportional to the households marginal rate of substitution: $\kappa_t = -\chi_c U_n(C_t, N_t) / U_c(C_t, N_t)$, preferences are King-Plosser-Rebelo: $U(C, N) = \log(C) - v(N)$, and the number of households grows at a constant rate g_l with*

each household supplying a unit measure of labor inelastically. Then, in the labor market steady state:

1. Employment shares equal product shares: $N_{it}/N_t = \tilde{\phi}_i$
2. Unemployment rates U_t/L_t and vacancy rates V_t/L_t are constant
3. Employment growth $\Delta N/N$ equals labor force growth g_l
4. Aggregate output $\Delta Y/Y$ and consumption growth $\Delta C/C$ is equal to productivity plus labor force growth: $g_y = g_c = g_A + g_l$
5. Wage growth equals productivity growth: $g_w = g_A$

If initial productivity is equalized across sectors and grows at the same rate or if $\eta = 1$, then g_A is constant and equal to input-share average of productivity growth across sectors. If sectors grow at different rates, productivity growth is asymptotically constant with $g_A = \gamma_j$ where $j = 1, 2, \dots, K$ is the sector with the highest growth rate if $\eta > 1$ or j is the sector with the lowest growth rate if $\eta < 1$.

Proof. See Appendix. ■

Under KPR preferences and symmetry across sectors in hiring costs, the household reallocates labor to mirror the movements in productivity-adjusted product shares. Since the cost of labor is equalized across sectors, relative prices are equalized and an aggregate vacancy posting condition obtains. The assumption that vacancy posting costs are proportional to the household's MRS ensures that market tightness and employment have no trend. If real vacancy posting costs did not change over time, productivity growth would result in a downward trend for unemployment. In contrast, US unemployment exhibits, if anything, a slight upward trend. In general, if sectors exhibit persistent differences in matching function efficiency or separation rates, unemployment, vacancies and employment would not exhibit constant growth rates. However, the proposition presented here establishes a useful benchmark for thinking about long-run trends in unemployment and vacancies.

3.5 Quantitative Predictions of the Model

To examine whether a sector-specific shock can account for the observed shift in the Beveridge curve and the rise in the unemployment rate in the Great Recession, we calibrate a two-sector version of our model. In this recession, the construction sector is the largest contributor to the sector-specific shock index and is frequently identified as the sector where the employment dislocation has been most severe and persistent. We calibrate the two-sector model to match various moments on employment, unemployment and vacancies across construction and non-construction sectors. Since construction displays a far higher job-filling rate than the rest of the economy, our calibration requires that construction either feature markedly lower hiring costs or reduced labor market tightness relative to the non-construction sector. We consider each explanation in turn.

3.5.1 Calibration Strategy

The economy is partitioned into construction and non-construction sectors with initial labor market tightness equalized across sectors as would be the case in the model steady state. Several standard parameters in search models are chosen exogenously: the discount rate $\beta = 0.96^{1/12}$ to target an annual interest rate of 4%, and the matching function elasticity $\alpha = 0.5$ is assumed to be the same across sectors consistent with evidence from [Petrongolo and Pissarides \(2001\)](#). We also assume that sectoral productivity is equalized and normalized to unity along with the price markup.¹⁹

Parameters unique to our model determine hiring costs in each sector: the sectoral separation rates δ_c and δ_{nc} , sectoral matching function efficiencies φ_c and φ_{nc} , the cost of posting vacancies κ , the reservation wage z , and the household's bargaining power ν . Moreover, we must also choose parameters in the CES aggregate - namely the input share of construction ϕ in the CES aggregator and the elasticity of substitution η that determines the degree of complementarity or substitutability across goods. We fix $\eta = 0.5$ so that construction and

¹⁹A positive markup has no effect on our calibration other than changing the average price of each good. Alternatively, if the fiscal authority provides a production subsidy to retailers, the markup will be fully offset in steady state with the price index equal to unity.

non-construction goods are moderate complements. However, we consider other values of η in our robustness checks.

Separation rates are set using the 2001-2006 averages of employment-weighted sectoral separation rates in the Job Openings and Labor Turnover survey; construction exhibits a significantly higher separation rate than other sectors. Bargaining power is set at $\nu = 0$ to deliver real wage rigidity as in Hall (2005) to ensure large employment effects from small changes in markups or aggregate productivity. As Hagedorn and Manovskii (2008) emphasize, the key variable determining the variability of employment is the size of the surplus rather than the bargaining power. Moreover, since bargaining power is the same across sectors, the level of bargaining power does not affect the mismatch channel by which sector-specific shocks shift the Beveridge curve.

The remaining five parameters - matching function efficiencies, reservation wage, vacancy posting cost, and product share - are jointly chosen to match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, construction's share in total employment $N_c/N = 5.7\%$, construction's share in total vacancies $V_c/V = 3.7\%$, and a product share-weighted average accounting surplus of 10% as in Monacelli, Perotti, and Trigari (2010) and close to the surplus delivered in the calibration of Hagedorn and Manovskii (2008).²⁰ The construction share of employment is chosen to match the peak of construction employment in 2007 and the vacancy share is the average level of vacancies from 2001-2006. Parameter values and targets are summarized in the Table 3.5.

Under the assumption that labor market tightness is equalized across sectors, the model generates a lower unemployment rate for construction relative to non-construction sectors, 3.3% vs. 5.1%. Because hiring costs are considerably lower in the construction sector under this calibration, the household allocates fewer worker to the construction sector to search in order to equalize labor market tightness, 5.6% vs. 94.4% in non-construction sector. However, using sectoral unemployment shares calculated in the CPS, the level of unemployment in the construction sector appears counterfactually low. Nevertheless, the correspondence

²⁰The surplus is defined as $A_i P_i / P - z$, the difference between the marginal product of labor and the household's marginal rate of substitution.

Table 3.5: Summary of Calibration Parameters

<u>Aggregate Parameters</u>		<u>Value</u>
Discount rate	β	$0.96^{1/12}$
Bargaining power	v	0
Matching function elasticity	α	0.5
Elasticity of substitution	η	0.5
Reservation wage	z	0.9
Construction share	ϕ	0.056
Vacancy posting cost	κ	3.49
<u>Construction</u>		
Monthly separation rates	δ_c	6.00%
Matching function efficiency	φ_c	2.48
<u>Non-Construction</u>		
Monthly separation rates	δ_{nc}	3.60%
Matching function efficiency	φ_{nc}	0.95

between the CPS measure of sectoral unemployment and the economic concept of sectoral unemployment in the model is unclear. The CPS measures sectoral unemployment by assigning workers to sectors based on the industry of previous employment with those workers outside the labor force or entering the labor force unassigned to any sector. In the model, a worker is unemployed in sector i if that worker is searching for jobs in sector i . The CPS measure may not accurately capture the sector in which a worker is searching, particularly among those workers transiting between participation and non-participation. In any case, in the next section, we show that an alternative calibration matching the unemployment and vacancy shares of workers does not substantially alter our results.

3.5.2 Experiment

We depict the shift in the steady state Beveridge curve generated by a permanent shock to the construction share ϕ that reduces the share to $\phi' = 0.04$. This reduction in construction share is chosen to match the observed drop in construction employment shares from a pre-recession peak of 5.7% to its 2012 level of 4.1%. The pre-shock Beveridge curve traces out the locus of aggregate vacancies and unemployment rates for different levels of real marginal cost, while the post-shock Beveridge curve traces the same locus with $\phi = \phi'$ leaving the distribution of the labor force either unchanged (in the case of no reallocation) or shifting the distribution

to ensure equalized labor market tightness across sectors (in the case of perfect reallocation).

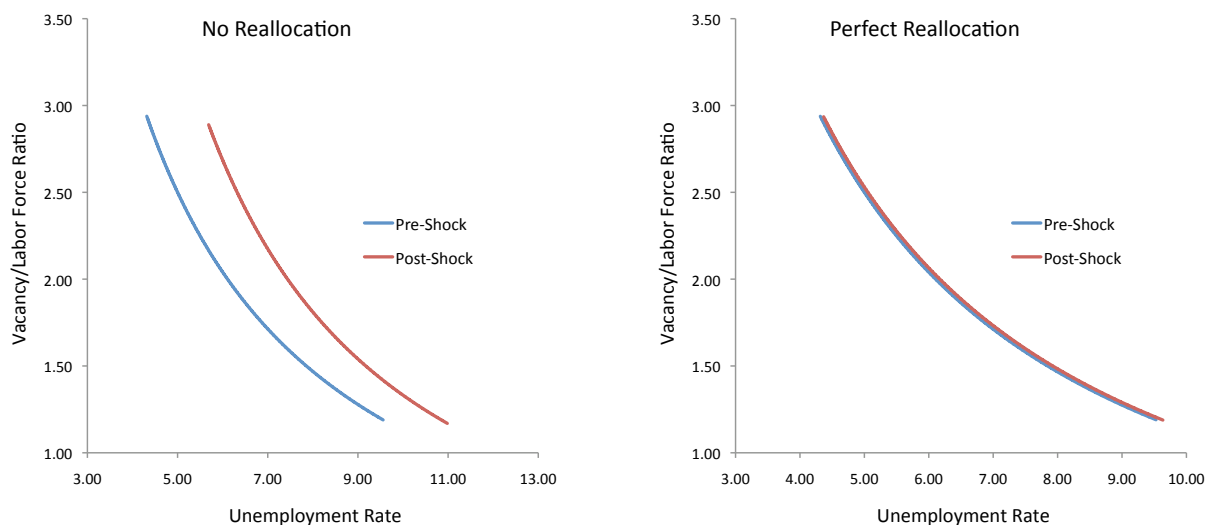
Figure 3.7 illustrates our main quantitative results. We show that, in the absence of reallocation (left-hand panel of Figure 3.7), a sector-specific shock to the construction sector generates a shift in the Beveridge curve of about 1.3% (horizontal shift - the rise in the unemployment rate at each level of vacancies). This matches the observed shift in the US data on unemployment rates and the vacancy to labor force ratio. A comparison of simple trend lines of V/L on U/L before and after 2009 (using data from December 2001-November 2011) reveals a shift in the horizontal intercept of 1.4%. While analyses using the job-openings rate (a slightly different measure of vacancies than the vacancy to labor force ratio) reveal a somewhat larger shift of 2%, the shift generated in our baseline calibration with no labor reallocation explains a substantial fraction of the observed shift in either case.

In contrast, when reallocation is costless, the Beveridge curve is essentially unchanged after the sector-specific shock. We take each case as bounds on the shift in the Beveridge curve and, as we will argue, the case of no reallocation is both a good approximation for the short-run behavior of the Beveridge curve and will continue to hold over the medium run given evidence on the costs of labor reallocation for displaced workers. So long as the labor force does not overshoot its long-run distribution, vacancies and unemployment along the transition path will lie in the region between these curves.²¹

In our model, employment shares vary with both changes in the markup and sector-specific shocks, though the movement in employment shares for aggregate shocks is quite small. For a markup shock, employment shares in construction drop because the surplus in construction is lower than that of the non-construction sectors. Lower hiring costs ensure a smaller surplus and, therefore, a greater decrease in the relative surplus for the construction sector. While construction shares displayed somewhat larger cyclical movements in employment shares before 1984, construction shares did not fall in the last recession and recovered quite slowly after the 1990s recession. Our calibration is consistent with small cyclical effects of aggregate shocks on employment shares consistent with evidence in the past three recessions where shares show

²¹Numerical simulations using a quadratic cost of reallocation in a two-sector model show that the labor force moves monotonically after a permanent shock towards the labor force distribution that equates tightnesses.

Figure 3.7: Model-implied Beveridge curves



little systematic movement in recessions. In this experiment, construction's employment share falls to 4.2% when overall unemployment is at 9% and predicts that the share would only rise to 4.3% at a 5% unemployment rate (with no labor reallocation). Once reallocation takes place, this sector-specific shock lowers construction's employment share further to 4.1%.

3.5.3 Distorted Initial State and Substitutability

As mentioned, the restriction that initial labor market tightness is equalized across sectors results in a counterfactual sectoral unemployment rate and labor force distribution using measures of these moments from the CPS. If we relax the assumption of equalized labor market tightness, an alternative calibration matches the distribution of employment, unemployment and vacancies. As before, five parameters - matching function efficiencies, the reservation wage, the vacancy posting cost and the product share - are jointly chosen to match the same targets as in Section 3.5.1. For consistency, we modify the targeted employment share of construction at 5.3%, its 2000-2006 average. As Table 3.6 shows, aside from the matching function efficiencies, the remaining parameters are largely unchanged.

Figure 3.8 shows the shift in the Beveridge curve for a preference shock that reduces the construction share to $\phi' = 0.04$. This shock generates a shift in the Beveridge curve slightly

Table 3.6: Calibration for $\theta_c \neq \theta_{nc}$

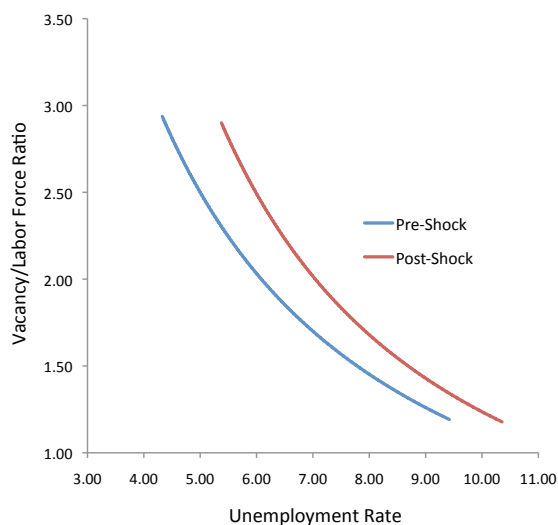
Parameters		Value
Reservation wage	z	0.9
Construction share	ϕ	0.052
Vacancy posting cost	κ	3.49
<u>Construction</u>		
Monthly separation rates	δ_c	6.00%
Matching function efficiency	φ_c	1.38
<u>Non-Construction</u>		
Monthly separation rates	δ_{nc}	3.60%
Matching function efficiency	φ_{nc}	0.99

smaller than the previous calibration with an average 1% shift in the unemployment rate at each vacancy rate. At higher levels of unemployment, the shift is mitigated since the sector-specific shock favors the non-construction sector which has a lower cost of hiring for a given level of market tightness. Even though job-filling rates are similar under both calibrations, the reasons for the higher job-filling rate for construction in each calibration are quite different. In our baseline calibration, job-filling rates in the construction sector are higher solely due to higher matching function productivity (even after accounting for the higher separation rate). However, in the distorted steady state calibration, job-filling rates are higher because of lower labor market tightness in the construction sector - effectively the labor force is misallocated with too many workers in construction. Absent labor reallocation, the sector-specific shock still shifts the Beveridge curve outward because a negative sector-specific shock worsens the mismatch between construction and non-construction sectors.

In addition to generating a similar shift in the Beveridge curve, employment shares exhibit somewhat greater volatility under aggregate shocks, though the overall volatility remains low. Since the initial level of mismatch is elevated in this case, the surplus is lower in construction than in non-construction sectors. As a result, aggregate shocks have a greater effect on employment and generate larger increases in mismatch and movement in employment shares.

The behavior of employment shares under aggregate shocks is also affected by the degree of complementarity among goods. When goods are complements, aggregate shocks generate relatively small movements in employment shares. This is due to the limited effect of prices

Figure 3.8: Effect of SS-Shock in Distorted Initial State

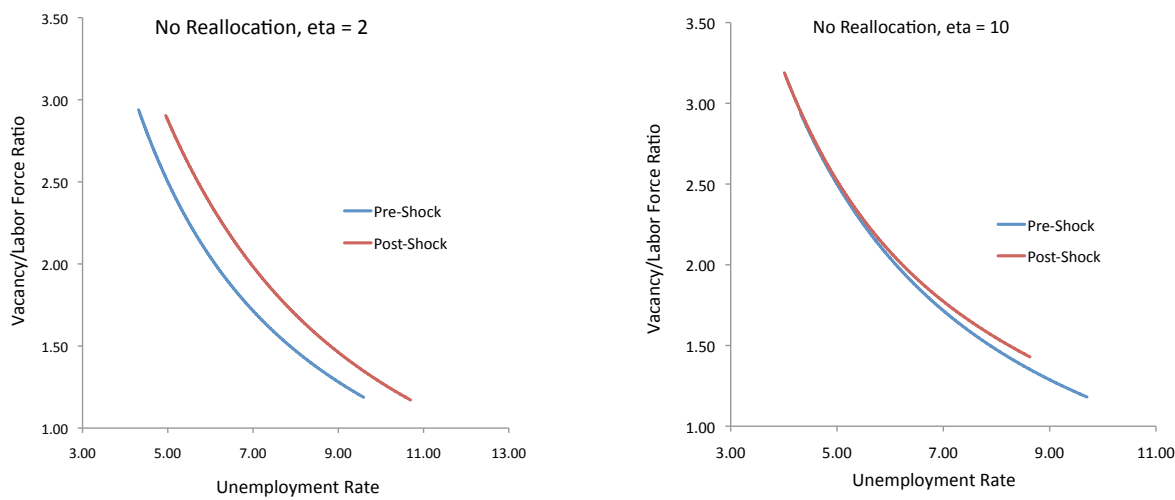


on relative employment shares and can be seen by combining input demand conditions:

$$\frac{N_A}{N_B} = \frac{\phi}{1 - \phi} \left(\frac{P_A}{P_B} \right)^{-\eta}.$$

In the limit, when $\eta \rightarrow 0$, goods are perfect complements and employment shares are constant irrespective of any aggregate shocks. For higher levels of substitutability, employment shares exhibit greater variation with aggregate shocks, but the magnitude of the shift in the Beveridge curve induced by a sector-specific shock decreases. Figure 3.9 displays the shift in the Beveridge curve when $\eta = 2$ and $\eta = 10$ - moderate and high degrees of substitutability. For the alternative values of η , we recalibrate the five parameters discussed earlier to maintain the same aggregate and distributional targets. With a higher degree of substitutability, sectors exhibit greater variation in employment shares over the business cycle but show a smaller shift in the Beveridge curve conditional on a sector-specific shock that delivers the same movement in employment shares from 5.3% to about 4% after labor reallocation. However, in the absence of labor reallocation, sector-specific shocks do not match the observed fall in construction employment shares. With $\eta = 2$, construction's employment share is 4.5% at an unemployment rate of 8% - too high relative to the data. Similarly, for $\eta = 10$, construction's share is 5.3%.

Figure 3.9: Effect of Substitutability



Aside from counterfactually high employment shares in the short-run, high degrees of substitutability imply business cycle variation in employment shares inconsistent with evidence in the Great Moderation period. Aside from trends, employment shares across sectors are typically stable over the cycle with durable goods and service sectors displaying the strongest business cycle movements (durables are countercyclical while services are countercyclical). While construction's share of employment fell in the early 1990s recession, the construction share remained stable in the 2001 recession before rising and falling with the housing bubble. This suggests that the assumption of mild complementarity or substitutability is not unreasonable in the current recession. Moreover, evidence cited in the growth literature and in studies of durable versus nondurable goods do not support very high levels of substitutability in the CES aggregator.²² In short, our conclusions that a sector-specific shock to construction account for over 2/3 of the shift in the Beveridge curve hold under alternative assumptions of labor market tightness and for reasonable values of the degree of substitutability.

3.5.4 Natural Rate of Unemployment

The experiments considered here also allows for an examination of the quantitative relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment.

²²See [Acemoglu and Guerrieri \(2008\)](#), [Carvalho and Lee \(2011\)](#), and [Monacelli \(2009\)](#).

Table 3.7 shows the natural rate of unemployment before and after a sector specific shock for various specifications of our model. The baseline calibration, which fully accounts for the shift in the Beveridge curve, finds a rise of 1.4 percentage points in the natural rate of unemployment to 6.4%. Once labor reallocation takes place, the sectoral shock to construction has a trivial effect on the unemployment rate, raising the rate to 5.06%. The initial rise in the natural rate of unemployment is similar in magnitude to the estimate in [Sahin, Song, Topa, and Violante \(2012\)](#) of the contribution of mismatch unemployment in the Great Recession. The absence of labor reallocation is responsible for most of the rise in the unemployment rate, while the composition effect accounts for the increase in the unemployment rate once reallocation takes place. This slight long-run rise in the unemployment rate is due to the fact that a sectoral shock shifts employment away from the sector with lower hiring costs. For higher degrees of substitutability, sectoral shocks that deliver the same employment share once reallocation takes place imply similar long-run unemployment rates but also a lower rise in the natural rate even in the absence of labor reallocation. In each case, a higher degree of substitutability implies less movement in employment shares as agents tolerate greater deviations of employment shares from product shares leading to a smaller rise in the natural rate of unemployment. Greater substitutability also generates a smaller shift in the Beveridge curve.

Table 3.7: Natural Rate and BC Shift

	No Reallocation		Full Reallocation
	BC shift	Unemployment Rate	Unemployment Rate
<u>Undistorted state</u>			
$\eta = 0.5$	1.3	6.40	5.06
$\eta = 2$	0.9	6.00	5.06
$\eta = 10$	0.4	5.12	5.06
<u>Distorted state</u>			
$\eta = 0.5$	0.9	6.04	4.88
$\eta = 2$	0.5	5.50	4.88
$\eta = 10$	-0.2	4.92	4.87

Beveridge curve shift measured as average increase in unemployment rate at any given level of vacancies. Size of shift and change in unemployment rate measured for various degrees of substitutability among intermediate goods.

As the second panel of Table 3.7 illustrates, in the presence of some initial degree of mismatch, the quantitative relationship between the natural rate and the shift in the Beveridge curve is somewhat weaker. In the baseline case of $\eta = 0.5$, both the shift in the Beveridge curve and the rise in the natural rate are somewhat lower than the undistorted case with a somewhat larger increase in the natural rate than implied by the shift in the Beveridge curve. Moreover, once reallocation takes place, the natural rate actually falls to 4.88% relative to the initial unemployment rate. This reduction in the long-run unemployment rate differs from the undistorted case because hiring costs are now greater in the construction sector relative to the non-construction sector. Therefore, the sectoral shock favors the sector with lower costs. For higher levels of substitutability, movements in the natural rate are attenuated, consistent with the smaller shifts in the Beveridge curve.

Our experiment reveals an approximate one-to-one relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment. Moreover, when labor reallocation is complete, the natural rate of unemployment returns to approximately the same level despite a permanent sector-specific shock and differences across sectors in hiring costs and matching function technology. However, the one-to-one link between Beveridge curve shifts and the natural rate of unemployment does not hold under extensions of the model considered in Section 3.6.

3.5.5 Labor Reallocation

As our quantitative results have emphasized, the ability of sector-specific shocks to explain the shift in the Beveridge curve and generate any economically significant fluctuations in the natural rate of unemployment depends crucially on the speed of labor reallocation across sectors. The available evidence supports slow labor reallocation in the short-run (1-2 years) but evidence on the pace of labor reallocation over the medium-run (2-8 years) is more mixed. We review the available evidence on labor reallocation in both the short-run and medium-run.

Costless labor reallocation is likely to be a poor approximation for the short-run behavior of the labor market. Given the quantitatively small role played by composition effects, costless

reallocation would imply no mismatch across sectors and nearly constant aggregate matching function efficiency over the business cycle. However, the empirical measures constructed in [Sahin, Song, Topa, and Violante \(2010\)](#), [Barnichon and Figura \(2011\)](#), and [Sedlacek \(2011\)](#) show that these variables fluctuate significantly over the business cycle. Moreover, observed vacancy to unemployment ratios using JOLTs and CPS data are not equalized across sectors, which is also inconsistent with the view that labor market reallocation is costless.

Table 3.8: Reallocation Rates by Education Level

Education Level	Industry			Occupation		
	LF	Emp	Unemp	LF	Emp	Unemp
HS Dropout	2.83	2.85	2.63	3.09	3.13	2.67
HS Graduate	2.24	2.21	2.71	2.58	2.56	3.03
Some College	2.15	2.14	2.40	2.52	2.52	2.56
College Graduate	1.86	1.84	2.25	2.07	2.07	2.26
Advanced Degrees	1.28	1.27	1.80	1.30	1.29	1.78

Average monthly transition rates in Current Population Survey, 2000-2006. LF is % of workers in the labor force recording an industry or occupation transition over consecutive months, while Emp and Unemp are transition rates for employed and unemployed workers respectively.

However, to explain a persistent shift in the Beveridge curve, labor reallocation must also be costly over the medium run. Transition rates for workers across sectors suggest large rates of reallocation, while evidence for displaced workers suggest substantial and persistent barriers to reallocation. The most natural measure of reallocation rates across sectors is monthly transition rates for employed and unemployed workers in the CPS. Since the CPS features a rotating panel design, households are tracked for four consecutive months and interviewed again a year later for another four consecutive months. Using matched CPS data from 2003-2006, we measure monthly reallocation rates for both employed and unemployed workers across major industries and major occupations. These monthly transition rates averaged 2.1% and 2.4% for the industry and occupation reallocation rates respectively. As [Table 3.8](#) shows, reallocation rates are decreasing with educational attainment and are generally higher for workers who are currently unemployed. Interestingly, for workers with less than a high school degree, reallocation rates drop for unemployed workers relative to employed workers. This fact may be salient for construction workers since construction exhibits the lowest skill

attainment of any major industry. The left-hand column of Table 3.9 gives the fraction of workers in each industry who are college graduates or higher.

Given our interest in labor reallocation out of construction, we examine transitions for only workers in the construction sector over the same period. Table 3.9 also shows the distribution of transitions from construction to other industries both unconditionally and conditional on the initial skill level. As Table 3.9 reveals, low-skilled construction workers reallocate toward other low skill industries like retail trade and leisure and hospitality. Service-sector industries - like education and health services, financial activities, and government - which account for a significant share of aggregate employment, are relatively underrepresented. While a significant fraction of transitions take place into professional and business services, these transitions may reflect movements into low skilled jobs like janitorial services and office support rather high-skilled occupations like lawyers, scientists, and managers which both belong to this sector.

Table 3.9: Skill Distribution and Reallocation for Construction

Industry	% College +	Construction		Some College
		Transition	College +	or less
Agriculture	14%	3%	2%	3%
Mining	17%	1%	0%	1%
Construction	11%	n/a	n/a	n/a
Manufacturing	23%	18%	14%	19%
Wholesale and Retail Trade	19%	17%	12%	18%
Transportation and Utilities	17%	9%	6%	10%
Information Services	41%	2%	4%	2%
Financial Activities	40%	5%	11%	5%
Professional and Business Services	42%	18%	23%	17%
Education and Health Services	46%	6%	14%	6%
Leisure and Hospitality	14%	9%	4%	10%
Other Services	20%	8%	5%	8%
Public Administration	39%	3%	6%	2%

First column is the percentage of workers within a sector with a college degree or higher. Second column is the sectoral distribution of transitions from construction to non-construction sectors. The third and fourth columns show the sectoral distributions for workers with less than a college degree and for workers with more than a college degree respectively. All values are averages from the Current Population Survey, 2000-2006

The aggregate industry and occupation transition rates reported here are similar in magnitude to the rates documented in Moscarini and Thomsson (2007) who examine transition rates at a higher level of disaggregation across occupations instead of industries. However,

[Kambourov and Manovskii \(2004\)](#) argue that classification errors are significant in year-to-year transitions in the CPS, leading to spurious transitions. Indeed, measurement error always biases transition rates upwards since a transition is recorded for any consecutive change in recorded industry. Moreover, these transition rates are silent on whether newly transitioned workers are a good substitute for existing workers with industry experience. Therefore, while the raw transition rates suggest large flows across sectors, these transitions are subject to significant measurement error and may not capture whether workers who reallocate are screened by firms. Measurement error and the absence of any measures of match quality may also bias the mismatch measures of [Sahin, Song, Topa, and Violante \(2012\)](#), who record a sharp fall in mismatch in the recovery period despite the shift in the Beveridge curve.

High rates of reallocation in the medium term are also inconsistent with evidence from the literature on displaced workers, which documents persistent effects of job loss on wages and labor force outcomes. [Davis and von Wachter \(2011\)](#) show that, in periods of high unemployment, wage loss is up to three years of pre-displacement earnings. This study and related work relies on higher quality longitudinal data from administrative records that accurately track worker outcomes for extended periods. To the extent that wages accurately reflect a worker's marginal product, the steep decline in wages suggests that, conditional on finding employment, displaced workers are not as well suited for their new jobs. The most recent Displaced Workers Survey - a occasional supplement to the CPS - shows that 62% of long-tenured displaced workers (i.e. workers employed for over 3 years) from 2007-2009 came from construction, manufacturing, wholesale and retail trade, or professional and business services.²³ These are precisely the same sectors into which construction workers reallocate suggesting that weak labor market conditions in these sectors make them unlikely to absorb transitions from construction. Moreover, in the latest wave of the Displaced Workers Survey, displaced construction workers exhibit among the lowest rate of reemployment in another industry at 23.9% - second lowest next to education and health services at 19.4%. In short,

²³Construction workers alone account for 13% of long-tenured displaced workers with a total of 6.8 million workers displaced over the 2007-2009 period. These findings are also supported by [Charles, Hurst, and Notowidigdo \(2012\)](#).

evidence on displaced workers suggests significant costs to reallocation over the medium term.

3.5.6 Skilled and Unskilled Labor

While evidence on the degree of labor reallocation across sectors is mixed, one dimension along which workers cannot readily reallocate is skill level. In this section, we extend our baseline model to include skilled and unskilled workers and show that sector-specific shocks can still shift the Beveridge curve even when industry reallocation is costless. Firms in all sectors now hire both skilled and unskilled workers using a fixed proportions technology to produce sectoral output. Workers at a given skill level can freely reallocate across sectors, but workers cannot reallocate across skill levels.

The intermediate good firm's problem from Section 3.3.4 is modified as follows:

$$\begin{aligned} \Pi_{i,t}^{Int} = & \max_{\{V_{i,T}^s, N_{i,T}^s, V_{i,T}^u, N_{i,T}^u\}_{T=t}^{\infty}} \mathbb{E}_t \sum_{T=0}^{\infty} Q_{t,T} (P_{i,T} Y_{i,T} - W_{i,T} N_{i,T}^u - W_{i,T} N_{i,T}^s - \kappa P_T V_{i,T}^u - \kappa P_T V_{i,T}^s), \\ \text{s.t.: } & N_{i,t}^u = (1 - \delta_i) N_{i,t-1}^u + q_t^u V_{i,t}^u, \\ & N_{i,t}^s = (1 - \delta_i) N_{i,t-1}^s + q_t^s V_{i,t}^s, \\ & Y_{i,t} = A_{i,t} \min \left\{ N_{i,t}^s, \nu_i N_{i,t}^u \right\}. \end{aligned}$$

Relative to the baseline model, firms in each sector i hire both skilled workers $N_{i,t}^s$ and unskilled workers $N_{i,t}^u$ subject to a fixed proportions technology where a unit of effective labor requires a constant sector-specific combination of skilled and unskilled labor ν_i . Firms post vacancies $V_{i,t}^s$ and $V_{i,t}^u$ for both types of workers with skill-specific job-filling rates q_t^s and q_t^u . Given costless reallocation within skill cohorts, the job-filling rates are the same across sector for a given skill level. Wages may differ across skill levels but vacancy posting costs are assumed to be the same.

Optimizing behavior by firms implies a single vacancy posting condition for hiring a fixed

proportion of workers across skill levels:

$$\frac{P_{it}}{P_t} A_{it} = \frac{W_{it}^s}{P_t} + \frac{1}{\nu_i} \frac{W_{it}^u}{P_t} + \frac{\kappa}{q_t^s} + \frac{\kappa}{q_t^u \nu_i} - E_t Q_{t,t+1} \frac{\kappa}{q_t^s} (1 - \delta_i) - E_t Q_{t,t+1} \frac{\kappa}{q_t^u \nu_i} (1 - \delta_i),$$

$$N_{it}^s = \nu_i N_{it}^u.$$

This vacancy posting condition generalizes the standard vacancy posting condition. For sectors with a higher ratio of skilled to unskilled labor, wages and search costs for skilled workers account for a larger share of the marginal product of labor. Changes in the marginal product for a sector characterized by a relatively high skill workforce have a greater effect on skilled worker employment than unskilled worker employment.

The household problem is left largely unchanged with households free to assign skilled and unskilled workers to search across sectors but unable to transform unskilled workers into skilled workers or vice versa. At each skill level, workers search in sectors to equate their probability-weighted surplus from finding a job - the same condition as in Section 3.3.1. This optimality condition implies the Jackman-Roper condition with labor market tightness equated across sectors for a given skill level.

We calibrate a two-sector version of this model to demonstrate that sector-specific shocks to the low-skilled sector can generate a quantitatively significant shift in the Beveridge curve. Following the discussion in Section 3.5.4, we partition the economy into two sectors and two skill levels, segmenting workers as either college graduates or workers with less than a four-year college degree. As noted in Table 3.8, sectors differ markedly in the skill composition of their workforce. We define the low-skilled sector as construction, mining, leisure and hospitality, trade and transportation, and other services, assigning all remaining sectors to a composite high-skilled sector. The employment weighted ratio of college graduates to non-college graduates is 0.193 for the low-skilled sectors while this ratio is 0.64 for the other sector and determines the value for the parameter ν_i .

For the remaining parameters, our calibration strategy largely follows our strategy described in Section 3.5.1. Bargaining power ν , matching function elasticity α , the elasticity of

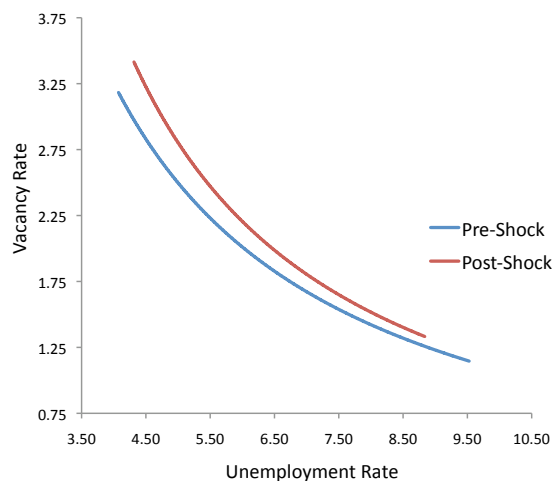
Table 3.10: Skilled-Unskilled Model Parameters

Parameters		Value
Discount rate	β	$0.96^{1/12}$
Bargaining power	ν	0
Matching function elasticity	α	0.5
Elasticity of substitution	η	0.5
Reservation wage	z	0.17
Reservation wage	z_s	0.31
Low-skilled share	ϕ	0.56
Vacancy posting cost	κ	0.8
<u>Low-Skilled</u>		
Monthly separation rates	δ_{ls}	5.10%
Matching function efficiency	φ_{ls}	0.63
Labor share	L_{ls}	70%
Skill ratio	ν_{ls}	0.19
<u>High-Skilled</u>		
Monthly separation rates	δ_s	3.10%
Matching function efficiency	φ_s	1.85
Labor share	L_s	30%
Skill ratio	ν_s	0.64

substitution η across goods, and the discount rate β are the same as in Section 3.5.1. Sectoral separation rates are chosen to match the employment weighted separation rates (2000-2006 averages) reported from JOLTs. The remaining parameters to be chosen are the matching function efficiencies for skilled workers φ_s and unskilled workers φ_u , the reservation wages for skilled workers z_s and unskilled workers z , the cost of posting vacancies κ , and the preference for the low-skilled sector's good ϕ . These parameters are chosen to jointly match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, employment share of low-skilled sector $N_{ls}/N = 38.9\%$, vacancy share of low-skilled sector $V_{ls}/V = 37.1\%$, skill premium $z_s/z = 1.82$, and share-weighted average accounting surplus of 10%. The calibration target for employment shares is 2003-2006 average from the BLS establishment survey, while the calibration target from vacancy shares is the average share of vacancies for low-skilled sectors from the JOLTs data over the same period. The skill premium is chosen from estimates in Goldin and Katz (2007), while the share-weighted average accounting surplus is the same as the baseline calibration.²⁴ The labor share for the skilled sector $L_s = 30\%$ matches

²⁴See Table A8.1, data for 2005.

Figure 3.10: Beveridge Curve for Skilled/Unskilled Model



the 2003-2006 average share of college graduates in the CPS. The model generates an unemployment share of 51% for the low-skilled sector (versus 50% in the CPS) and unemployment rates by skill level of 4.5% and 5.2% for high skilled and low-skilled workers respectively. Our calibration is summarized in Table 3.10.

The experiment we conduct is a preference shock that reduces the share of low-skilled employment from 38.9% to 38% corresponding to the reduction observed in the current recession. This fall in employment share is driven largely by construction and partially offset by increases in the other constituent sectors classified as low-skilled. A shock that reduces the input share to $\phi' = 0.547$ reduces the employment share to 38%, raises the unemployment rate to 5.12% and raises vacancies from 2.5% to 2.72% accounting for a sizable outward shift in the Beveridge curve. As seen in Figure 3.10, this shock increases the unemployment rate by 0.5 percentage points holding vacancies constant, explaining a bit over 1/3 of the observed shift in the Beveridge curve. For higher levels of unemployment, the shift is smaller analogous to the shape of the Beveridge curve observed in the calibration with a distorted initial state. Moreover, in contrast to the construction/non-construction calibration, the sector-specific shock in this calibration delivers an increase in the natural rate of unemployment that is just a quarter of the shift in the Beveridge curve confirming that the size of Beveridge curve shifts and changes in the natural rate are not necessarily one for one.

3.6 Financial Disruptions as Sectoral Shocks

In this section, we extend our baseline model to illustrate how sector-specific shocks could be represented as financial shocks. If financial shocks are responsible for the shift in the Beveridge curve, then Beveridge curve shifts no longer necessarily imply any changes in the natural rate of unemployment. In particular, it is now possible for monetary easing to counteract any shift in the Beveridge curve since changes in the conduct of monetary policy in and of itself could generate a shift in the Beveridge curve. We show that a binding zero lower bound on the policy rate - effectively a departure from the unconstrained monetary policy rule - operates as a financial shock that disproportionately impacts the financially constrained sector.

3.6.1 Financial Frictions on the Firm Side

To model the effect of financial shocks on the production side, we now assume that some sectors face a working capital constraint of the form considered in [Christiano, Eichenbaum, and Evans \(2005\)](#). Financially constrained firms have to borrow to pay wages and the cost of posting vacancies.²⁵ For these firms, their optimization problem is slightly modified from the baseline model by introducing a borrowing rate i_t^b :

$$\Pi_{i,t}^{Int} = \max_{\{V_{i,T}, N_{i,T}\}_{T=t}^{\infty}} \mathbb{E}_t \sum_{T=0}^{\infty} Q_{t,T}^b \left[P_{i,T} Y_{i,T} - \left(1 + i_T^b\right) (W_{i,T} N_{i,T} - \kappa P_T V_{i,T}) \right], \quad (3.32)$$

$$\text{s.t. } N_{i,t} = (1 - \delta_i) N_{i,t-1} + q_{i,t} V_{i,t}, \quad (3.33)$$

$$Y_{i,t} = A_t N_{i,t}. \quad (3.34)$$

Financially constrained firms' vacancy posting condition now includes the borrowing rate and changes in expected future borrowing rates:

$$\frac{P_{i,t}}{P_t} \frac{A_t}{1 + i_t^b} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} - E_t Q_{t,t+1}^b (1 - \delta_i) \frac{\kappa}{q_{i,t+1}} \frac{1 + i_{t+1}^b}{1 + i_t^b}. \quad (3.35)$$

²⁵To introduce financial frictions, we now assume that firms are operated by a distinct set of agents with stochastic discount factor Q_t^b . Given our focus on labor market steady states, the entrepreneur's stochastic discount factor does not enter into the steady state vacancy posting condition.

In steady state, the second term with expected future borrowing rates drops out and changes in the borrowing rate are isomorphic to a negative sector-specific productivity shock as in equation (3.21). We show in the appendix that a collateral constraint as opposed to a working capital constraint would imply the exact same vacancy posting condition. In [Curdia and Woodford \(2010\)](#), and [Mehrotra \(2012\)](#), the borrowing rate is endogenous to monetary policy as the sum of the nominal deposit rate - the instrument of monetary policy - and an exogenous credit spread less changes in expected inflation:

$$1 + i_t^b = (1 + \omega_t) (1 + i_t^d) / E_t \Pi_{t+1}.$$

While the credit spread is exogenous, the borrowing rate is not and credit spread shocks may be offset by a reduction in the policy rate or increases in inflation expectations. The presence of a working capital constraint (or other type of financial friction) creates a channel for increasing labor market mismatch between financially constrained and unconstrained sectors, while the effect of the deposit rate on the borrowing rate renders movements in mismatch partially endogenous.

3.6.2 Financial Frictions on the Household Side

Analogous to the production side, financial frictions on the household side can generate the same change in relative prices as a sector-specific preference shock does in our baseline model. The most realistic financial friction on the household side involves costs of borrowing for purchasing durable goods as modeled in [Campbell and Hercowitz \(2005\)](#) or [Monacelli \(2009\)](#). Since durable goods are lumpy purchases, households typically borrow to make these purchases.

However, the correspondence between sector-specific preference shocks and financial frictions on the household side can be established in a simpler cash-in-advance type setting. We modify our existing model with two types of households and incomplete markets. Assume that a subset of patient households enjoys a fixed share of national income and carries positive wealth from period to period (in the form of government debt). These households provide

loanable funds in our setup. The impatient households in our economy supply labor (subject to the search frictions and reallocation frictions detailed earlier) and carry zero wealth from period to period since they are subject to a nonnegative wealth constraint that will bind in the steady state. The impatient household consume two types of goods: C_t and D_t , but the impatient household must borrow at the beginning of the period to purchase D_t , and repay this loan at the end of the period out of income earned from working.

In this setting, the impatient household faces a static optimization problem (in addition to the labor allocation decision detailed in Section 3.3.1):

$$\begin{aligned} \max_{C_t, D_t, B_t} \quad & u(C_t, D_t), \\ \text{s.t.} \quad & \frac{P_{ct}}{P_t} C_t + (1 + i_t^b) B_t = \sum_{i=1}^K (W_{it} N_{it} + \Pi_{it}), \\ & \frac{P_{dt}}{P_t} D_t = B_t, \end{aligned}$$

where i_t^b is the net borrowing rate and the last constraint requires that borrowing inclusive of interest be repaid in full by the end of the period. Instead of a single set of retailers selling a continuum of differentiated goods, we now assume retailers for both types of goods as in [Monacelli \(2009\)](#). These retailers are identical implying the same markup in each sector.

The optimality conditions for the impatient household determine the relative demand for each good. Under the assumption that $u(C_t, D_t)$ is separable:

$$\begin{aligned} \lambda_t u_c(C_t) &= P_{ct}/P_t, \\ \lambda_t u_d(D_t) &= (1 + i_t^b) P_{dt}/P_t. \end{aligned}$$

Relative consumption demand is now a function of both prices and the borrowing rate:

$$\frac{u_c(C_t)}{u_d(D_t)} = \frac{P_{ct}}{(1 + i_t^b) P_{dt}}.$$

Under log utility and a Cobb-Douglas aggregator for C_t and D_t , we have a relative demand

condition that is analogous to the relative employment condition in our baseline model. When patient households relative demand for consumption goods is small or is very similar to that of the impatient household, it follows that a shock to the borrowing rate changes relative employment shares in the same manner as a sector specific shock to ϕ :

$$\frac{N_{ct}}{N_{dt}} \approx \frac{C_t}{D_t} = \frac{\phi}{1 - \phi} (1 + i_t^b) \left(\frac{P_{ct}}{P_{dt}} \right)^{-1}.$$

While a shock to the borrowing rate is not isomorphic to a preference shock ϕ , changes in borrowing rates shift employment shares and, in the presence of costly labor reallocation, will increase mismatch across sectors.

3.6.3 Phillips Curve and Mismatch

Since financial frictions on the firm side fits most naturally into our existing model, we illustrate how a change in the monetary policy rule increases mismatch thereby shifting the Beveridge curve. We log-linearize a two-sector version of model where firms in the financially constrained sector are subject to a working capital constraint and there is no reallocation of labor across sectors. When reservation wages are constant, the firms' log-linearized vacancy-posting conditions are given as follows:

$$\begin{aligned} p_{ct} &= i_t^b + s_c \alpha \theta_{ct}, \\ p_{ut} &= s_u \alpha \theta_{ut}, \end{aligned}$$

where c indexes the financially constrained sector, u indicates the unconstrained sector and $1 - s_i$ is the surplus in sector i . When $s_c = s_u$, the borrowing rate constitutes a wedge between relative prices; an increase in the borrowing rate drives up the prices disproportionately in the financially constrained sector.

An aggregate Phillips curve is obtained by combining the price index and the log-linearized equilibrium conditions of the retailers, with the latter delivering the standard New Keynesian

Phillips curve along with equations defining aggregate output and relative employment:

$$\pi_t = \kappa \left\{ \nu \left[i_t^d + \omega_t + \frac{\alpha}{1-\alpha} s_c (1 + \epsilon_c) n_{ct} \right] + (1 - \nu) \frac{\alpha}{1-\alpha} s_u (1 + \epsilon_u) n_{ut} \right\} + \beta E_t \pi_{t+1}, \quad (3.36)$$

$$y_t = \gamma n_{ct} + (1 - \gamma) n_{ut}, \quad (3.37)$$

$$n_{ct} - n_{ut} = -\eta \left[i_t^d + \omega_t + s_c (1 + \epsilon_c) \frac{\alpha}{1-\alpha} n_{ct} - s_u (1 + \epsilon_u) \frac{\alpha}{1-\alpha} n_{ut} \right], \quad (3.38)$$

where γ is the steady state share of output for the constrained sector, ν is the steady state share of the price index for the constrained sector, and ϵ_i is the ratio of employment to unemployment in each sector. The three equations summarize the supply block of the two-sector model with financial frictions where labor markets are in their flow steady state. If the initial steady state is distorted (i.e. $P_{ct} \neq P_{ut}$), output and price level shares need not be equalized. Moreover, these shares will generally differ from employment shares and vacancies shares. In the special case where $\gamma = \nu = \phi$, these three equations simplify to two equations:

$$\pi_t = \kappa \left[\nu \left(i_t^d + \omega_t \right) + \frac{\alpha}{1-\alpha} s (1 + \epsilon) y_t \right] + \beta E_t \pi_{t+1}, \quad (3.39)$$

$$n_{ct} - n_{ut} = -\frac{\eta}{1 + \eta s (1 + \epsilon) \frac{\alpha}{1-\alpha}} \left(i_t^d + \omega_t \right) \quad (3.40)$$

and the inflation/output tradeoff is decoupled from the determination of employment shares.

The model is closed by adding the household's aggregate IS condition and specifying a monetary policy rule. We assume that the exogenous credit shock also affects some subset of borrower households as described in the model of [Mehrotra \(2012\)](#). In that setting, an increase the credit spread delivers a business cycle: a decrease in output, inflation, consumption, and employment. Monetary policy is assumed to follow a standard Taylor rule:

$$y_t = E_t y_{t+1} - \sigma \left(i_t^d + E_t \pi_{t+1} \right) - \sigma_b \omega_t, \quad (3.41)$$

$$i_t^d = \phi_\pi \pi_t + \phi_y y_t, \quad (3.42)$$

where σ is the average intertemporal elasticity of substitution and σ_b is the elasticity of substitution for the borrower household. A solution to this five equation system (3.36) - (3.38) and (3.41) - (3.42) is a process for $\{n_{ct}, n_{ut}, y_t, \pi_t, i_t^d\}$ as a function of the exogenous shock ω_t .

To see how a change in the monetary policy rule shifts the Beveridge curve, it is useful to fix the level of employment n_t and observe that equation (3.38) determines the distribution of employment conditional on the response of monetary policy. To a log-linear approximation, steady state employment is $n_t = \tau n_{ct} + (1 - \tau) n_{ut}$ where employment shares τ need not match output or price level shares in a distorted steady state. Employment in each sector is given by the expressions:

$$n_{ct} = \frac{1}{\tau} [n_t - (1 - \tau) n_{ut}], \quad (3.43)$$

$$n_{ut} = \frac{\frac{1+\eta\lambda_c}{\tau} n_t + \eta (i_t^d + \omega_t)}{(1 + \eta\lambda_c) \frac{1-\tau}{\tau} + (1 + \eta\lambda_u)}, \quad (3.44)$$

where λ_i is composite of the other parameters like the sectoral surplus s_c . A weaker policy response (decrease in i_t^d) to the increase in spreads ω_t will increase the share of employment at unconstrained firms so long as similar size shocks ω_t are needed to deliver the same level of employment under each policy.²⁶ This change in the distribution of employment shifts the Beveridge curve since total vacancies are also a function of the distribution of employment. As shown below, vacancies are equal to:

$$\begin{aligned} v_t &= \frac{\bar{V}_c}{\bar{V}} \frac{1 + \alpha\epsilon_c}{1 - \alpha} n_{ct} + \frac{\bar{V}_u}{\bar{V}} \frac{1 + \alpha\epsilon_u}{1 - \alpha} n_{ut}, \\ &= \frac{\bar{V}_c}{\bar{V}} \frac{1 + \alpha\epsilon_c}{1 - \alpha} \frac{1}{\tau} n_t + \left(\frac{\bar{V}_u}{\bar{V}} \frac{1 + \alpha\epsilon_u}{1 - \alpha} - \frac{1 - \tau}{\tau} \frac{\bar{V}_c}{\bar{V}} \frac{1 + \alpha\epsilon_c}{1 - \alpha} \right) n_{ut}, \end{aligned}$$

where the second equality is obtained by expressing employment in the constrained sector in terms of total employment and employment in the unconstrained sector. So long as uncon-

²⁶In particular, instead of a Taylor rule, assume that monetary policy keeps the borrowing rate constant: $i_t^d = -\omega_t$. Then at the zero lower bound, monetary policy cannot offset the rise in the credit spread and the share of employment in the unconstrained sector rises.

strained firms face a tighter labor market or account for a disproportionate share of vacancies (relative to their employment share), the coefficient on n_{ut} will be positive and the increase at vacancies at these firms will more than offset the fall in vacancies at the constrained firms shifting the Beveridge curve outward.

In addition to offering an explanation for the shift in the Beveridge curve, the interaction of the zero lower bound and financial frictions at the firm level also offers a potential explanation for the relative stability of inflation in the US despite persistently high unemployment. A credit shock, by affecting firms' costs of production, raises marginal costs for constrained firms. This rise in costs for constrained firms partially offsets the fall in marginal costs from decreasing employment. The financial frictions channels dampens downward pressure on prices, limiting the degree of deflation and, depending on the relative strength of these channels, possibly generating higher inflation. Standard ZLB models in the spirit of [Eggertsson and Woodford \(2003\)](#) have difficulty generating long-lasting zero lower bound episodes without predicting counterfactually high levels of deflation (see [Mehrotra \(2012\)](#)). While extreme downward rigidity in wages could also explain the absence of outright deflation, the presence of a supply-side channel for financial frictions offers another realistic channel to account for stable inflation at the zero lower bound.

3.7 Conclusion

Discussions about the slow recovery in the US following the Great Recession have raised the possibility of sectoral shocks. Proponents of this view have cited the disproportionate impact of the recession on housing-related industries and the shift in the Beveridge curve as evidence of sector-specific shocks. We investigate the role of sector-specific shocks and their impact on the Beveridge curve empirically and theoretically.

On the empirical side, a factor analysis of sectoral employment in the postwar data is used to isolate sector-specific shocks while addressing the Abraham and Katz critique. We derive a sector-specific shock index and show that this index is elevated in the current period and distinct from the business cycle or the Lilien measure of sectoral shocks. Moreover, we show

that this measure of sector-specific shocks is elevated in those periods when the Beveridge curve shifts.

On the theoretical side, we build a multisector model with labor market search to investigate how sector-specific shocks affect equilibrium variables like the aggregate Beveridge curve and the level of employment. Our model shows that sector-specific shocks generally shift the Beveridge curve through a composition channel due to differences in hiring costs and hiring technology across sectors and a mismatch channel due to segmentation in labor markets. We show analytically that, through the composition effect, sectoral shocks can raise or lower the natural rate of unemployment, while the mismatch effect always raises the natural rate of unemployment. Moreover, in our baseline model, sectoral shocks that shift the Beveridge curve must also change the natural rate of unemployment.

We calibrate a two-sector version of our model and show that a negative preference shock to the construction sector that matches the distribution of employment shares at the recession trough generates a shift in the Beveridge curve that matches the magnitude of the shift observed in the data. This shock raises the natural rate of unemployment by a quantitatively similar level as the shift in the Beveridge curve - the natural rate rises 1.4 percentage points and results are robust if goods are moderate substitutes instead of complements.

Finally, we show that financial shocks act like sector-specific shocks and can also generate a shift in the Beveridge curve if a subset of firms is financially constrained. In this richer setting, a change in the conduct of monetary policy can generate a shift in the Beveridge curve by magnifying the effect of financial constraints. For example, if monetary policy switches from a Taylor rule to a fixed nominal rate due to a binding zero lower bound, financial constraints will lead to a higher level of mismatch across sectors. These changes in mismatch due to a binding zero lower bound can still be addressed through unconventional monetary policy such as price level targets or credit easing.

As noted in our quantitative results, the assumption of costly or no labor reallocation is crucial in generating the observed persistence of the shift in the Beveridge curve. Existing evidence suggests somewhat contradictory findings on the pace of labor reallocation. Ob-

served transition rates in the CPS and the size of cross-sector flows suggest relatively frequent transitions across sectors. However, evidence from the Displaced Worker Survey and an extensive literature studying labor market outcomes after job loss point to fairly high costs to reallocation. Future research will seek to reconcile these findings to determine the business cycle cost of labor reallocation and dimensions of heterogeneity along which workers do not readily transition.

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Appendix A

Financial Integration and Financial Instability

A.1 Proof of Lemma 1.1

A banker in the periphery solves the following problem

$$\begin{aligned} \max_{Z^P, D_s^P, \bar{B}^P} \mathbb{E}\pi_B^P &= [p + (1-p)q] Z^P - \left(p + \frac{(1-p)q}{Q^P} \right) D_s^P \\ &\quad - \left[p\bar{B}^P + (1-p)q \min\{\bar{B}^P, Z^P - D_s^P/Q^P\} \right], \\ \text{s.t. } P_0^P Z^P &\leq \frac{D_s^P}{R_D} + P_B^P(G)\bar{B}^P + P_B^P(Bnc) \min\{\bar{B}^P, Z^P - D_s^P/Q^P\}, \\ D_s^P &\leq Q^P Z^P. \end{aligned}$$

Define θ^P such that the Lagrange multiplier on the collateral constraint is θ^P/Q^P , denote the Lagrange multiplier on the budget constraint as η .

I consider two different cases depending on whether the banker defaults or not in state $s_2^P = Bnc$.

Default. If the banker defaults then $\bar{B}^P > Z^P - D^P/Q^P$. The optimal interior choice of Z^P leads to

$$p - \eta[P_0^P - P_B^P(Bnc)] + \theta^P = 0. \tag{A.1}$$

It is clear that the banker chooses positive Z^P in equilibrium because otherwise P_0^P would be zero. This implies an infinite gain for the banker from choosing small positive Z^P . The optimal choice of \bar{B}^P leads to

$$-p + \eta P_B^P(G) = 0 \quad (\text{A.2})$$

If the banker chooses positive amount of safe debt financing this implies

$$\frac{R_B}{R_D} - \left[p + \frac{(1-p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} + \eta \left[\frac{1}{R_D} - \frac{P_B^P(Bnc)}{Q^P} \right] = 0. \quad (\text{A.3})$$

In a closed economy the banker always chooses positive amount of safe debt in equilibrium. However, in open economy there can be parameter values that imply zero safe debt issuance in one of the countries. Because I am interested in analyzing situations in which the collateral constraints bind in both countries I assume here that $D_s^P > 0$.

In equilibrium the state prices equal $P_B^P(G) = \beta p$, $P_B^P(Bnc) = \beta(1-p)q$, $P_B^P(Bc) = \beta(1-p)(1-q)$. From (A.2) I get $\eta = 1/\beta$. Then (A.1) implies

$$[p + (1-p)q] - R_B P_0^P + \theta^P = 0. \quad (\text{A.4})$$

Finally, (A.3) implies

$$\frac{R_B}{R_D} - \left[p + \frac{(1-p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0. \quad (\text{A.5})$$

The budget constraint implies $P_0^P Z^P = D_s^P/R_D + \beta p \bar{B}^P + \beta(1-p)q (Z^P - D_s^P/Q^P)$. Thus, the face value of the risky debt is

$$\bar{B} = \frac{R_B}{p} \left[Z^P P_0^P - \frac{D_s^P}{R_D} \right] - \frac{(1-p)q}{p} \left[Z^P - \frac{D_s^P}{Q^P} \right].$$

Using this equation, the default condition $\bar{B}^P > Z^P - D^P/Q^P$ can be rewritten as

$$Z^P (R_B P_0^P - (1-p)q - p) > D_s^P \left(\frac{R_B}{R_D} - \frac{(1-p)q}{Q} - \frac{p}{Q} \right).$$

No default. If the banker does not default then $\bar{B}^P \leq Z^P - D^P/Q^P$. The optimal interior

choice of Z^P leads to

$$\eta[P_B^P(G) + P_B^P(Bnc) - P_0^P] + \theta^P = 0$$

The optimal choice of \bar{B}^P leads to

$$\eta[P_B^P(Bnc) + P_B^P(G)] - p - (1-p)q = 0$$

If the banker chooses positive amount of safe debt financing this implies

$$\frac{\eta}{R_D} - \left[p + \frac{(1-p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0.$$

Taking into account the equilibrium prices I obtain $\eta = 1/\beta$ and conditions identical to (A.4) and (A.5). The budget constraint implies $P_0^P Z^P = D_s^P/R_D + \beta[p + (1-p)q]\bar{B}^P$. Thus, the face value of the risky debt is

$$\bar{B} = \frac{R_B}{p + (1-p)q} \left[Z^P P_0^P - \frac{D_s^P}{R_D} \right].$$

Using this equation, the default condition $\bar{B}^P \leq Z^P - D^P/Q^P$ can be rewritten as

$$Z^P (R_B P_0^P - (1-p)q - p) \leq D_s^P \left(\frac{R_B}{R_D} - \frac{(1-p)q}{Q} - \frac{p}{Q} \right). \blacksquare$$

A.2 Proof of Lemma 1.2

Step 1. Denote the level of investment productivity by for which $\theta^P = 0$ and $D^P = Z^P Q^P$ by \bar{A} . Let's show that such a level exists. The bankers optimality condition in equilibrium with $\theta^P = 0$ and the investors optimality condition when $D^P = Z^P Q^P$

$$\frac{Q^P}{q} = \frac{1-p}{\frac{R_B}{R_D} - p} \quad \text{and} \quad \frac{q}{Q^P} = \delta g'(W - D^P)$$

imply

$$\delta g'(W - D^P) = \frac{\frac{R_B}{R_D} - p}{1-p}.$$

Using the fact that the entrepreneur optimal choice of safe debt D^P leads to $1/R_D = \beta + v'(D^P)$, I can rewrite the last equation

$$\delta g'(W - D^P) = \frac{1 - p + v'(D^P)/\beta}{1 - p}.$$

Because $g(\cdot)$ is strictly concave, the left-hand side (LHS) of the last equation is increase in D^P . Because $v(D^P)$ is strictly concave, the right-hand side (RHS) of the above equation is increasing in D^P . If the value of the RHS is higher than the value of the LHS in $D^P = 0$ then the equation always has a solution. If $1 + v'(0)/[\beta(1 - p)] > \delta g'(W)$ then the RHS is greater than the LHS at $D^P = 0$. This condition holds when $v(\cdot)$ satisfies $\lim_{D \rightarrow 0} v'(D) = \infty$. Denote the solution to the last equation by \bar{D} .

Equilibrium condition $\bar{D} = Q^P Z^P = q/[\delta g'(W - \bar{D})]A^P F(I^P)$ determines a negative relation between I^P and A^P , $I^P = \phi(A^P)$.

The bankers optimal choice of Z^P and the entrepreneurs optimal choice of I^P when $\theta^P = 0$ imply $[p + (1 - p)q]A^P F'(I^P) = R_B$. This determines a positive relation between I^P and A^P , $I^P = \psi(A^P)$.

Because $F(\cdot)$ satisfied the Inada conditions the solution to equation $\phi(A^P) = \psi(A^P)$ always exists, unique and equals \bar{A} . I will distinguish all the equilibrium endogenous variables corresponding to \bar{A} with a bar.

Step 2. Consider $A < \bar{A}$. Let's show by contradiction that $\theta^P > 0$. Assume that $\theta^P = 0$ for this A . If $D^P = Q^P Z^P$ then all the equilibrium variables should be equal to equilibrium variables under \bar{A} . This is not possible because $A < \bar{A}$. If $D^P < Q^P Z^P$ then $D^P(A) = \bar{D}$, which is a result of a reasoning similar to the one in the beginning of Step 1. Fire-sale price $Q^P = q/[\delta g'(W - D^P)] = q/[\delta g'(W - \bar{D})] = \bar{Q}$. Because $D^P < Q^P Z^P$ we have

$$\bar{A}F(\bar{I})\frac{q}{\delta g'(W - \bar{D})} = \bar{D} = D^P < Z^P Q^P = AF(I^P)\frac{q}{\delta g'(W - D^P)} = AF(I^P)\frac{q}{\delta g'(W - \bar{D})}$$

Comparing the first and last terms in this equation I get $\bar{A}F(\bar{I}) < AF(I^P)$. $A < \bar{A}$ implies $I^P > \bar{I}$.

The bankers optimal choice of Z^P and the entrepreneurs optimal choice of I^P when $\theta^P = 0$ imply $[p + (1 - p)q] A^P F'(I^P) = R_B$. Thus, $A F'(I^P) = \bar{A} F'(\bar{I})$. Because $I^P > \bar{I}$ I can write

$$\bar{A} F'(\bar{I}) = A F'(I^P) < A F'(\bar{I}) < \bar{A} F'(\bar{I}),$$

which is a contradiction. Hence, $\theta^P > 0$ for all $A < \bar{A}$.

Step 3. In this step I prove the comparative statics statements in the lemma for $A < \bar{A}$. From (1.20), the banker and entrepreneur optimal choices, I know that

$$\left[\left(\frac{R_B}{R_D^P} - p \right) Q^P + p \right] A^P F' \left(F^{-1} \left(\frac{Z^P}{A^P} \right) \right) = R_B.$$

Given properties of $F(\cdot)$

$$\frac{\partial \left[A^P F' \left(F^{-1} \left(\frac{Z^P}{A^P} \right) \right) \right]}{\partial A^P} = F'(I^P) - \frac{F''(I^P) F(I^P)}{F'(I^P)} > 0.$$

This implies that for a given Z^P, R_D^P a marginal increase in A leads to a decrease in Q^P . This corresponds to a shift in the B curve to the right on the left panel of Figure 1.2. Conditional on R_D^P the equilibrium value of Z^P goes up. Because the elasticity of the outside investor demand is greater than 1 the supply of safe debt $D^P = Q^P Z^P$ increases. This increases the amount of safe debt issued and the return on safe debt in equilibrium. An increase in R_D^P has the opposite effect on Z^P and Q^P relative to the direct effect of changes in A . However, the indirect effect is weaker than the direct effect.

The shadow value of risky projects for the bankers is

$$\theta^P = \left[1 - p + \frac{v'(D^P)}{\beta} \right] Q^P - (1 - p)q.$$

Because D^P increases and Q^P decreases as a result of increase in A it is clear that θ^P falls.

Step 4. Consider $A > \bar{A}$. I need to show that $\theta^P = 0$. Assume that $\theta^P > 0$. Then, by Step 2 of this proof this implies that $\theta^P(\bar{A}) > 0$ which is a contradiction. Thus, $\theta^P = 0$ for all $A > \bar{A}$.

Step 5. For $A > \bar{A}$ $D^P, Q^P, \theta^P, R_D^P$ are all determined independently of A . To see this observe that the optimal choice of D_s^P by the banker is decoupled from optimal choice of Z^P which depends on price P_0^P which in turn depends on A . The bankers optimal choices of Z^P and the entrepreneurs optimal investments I^P imply $[p + (1 - p)q] A^P F'(I^P) = R_B$. Hence, I^P and Z^P are negatively related to A in equilibrium. ■

A.3 Proof of Lemma 1.4

The social welfare function equals

$$U = C_0^P(E) + \beta \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] + v(D_d^P), \quad (\text{A.6})$$

where $C_0^P(E)$ and $C_2^P(E)$ represent consumption of the entrepreneurs in the periphery, $C_2^P(B)$ is consumption of the bankers, $C_2^P(OI)$ is consumption of the outside investors. I use agents' budget constraints to express consumption levels. The entrepreneurs and the bankers budget constraints in period 0 are

$$C_0^P(E) = Y + P_0^P A^P F(I^P) - I^P - \frac{D_d^P}{R_D} - \sum_{s_2^P} B(s_2^P) P_B^P(s_2^P),$$

$$P_0^P Z^P = V_B^P + \frac{D_s^P}{R_D}.$$

The market clearing conditions imply $V_B^P = \sum_{s_2^P} B(s_2^P) P_B^P(s_2^P)$ and $Z^P = A^P F(I^P)$. Thus,

$$C_0^P(E) = Y + \frac{D_s^P - D_d^P}{R_D} - I^P.$$

Next, consider period 2. Because the bankers consume their profits and the outside investors consume their revenues, I can write

$$\begin{aligned}
& \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] \\
&= \mathbb{E} [C_2^P(E) + \pi_B^P + \pi_{OI}^P] \\
&= D_d^P + \mathbb{E} \min\{\bar{B}^P, Z^P - \frac{D_s^P}{Q^P}\} && \text{entrepreneur} \\
&+ [p + (1-p)q] Z^P - \left(p + \frac{(1-p)q}{Q^P}\right) D_s^P - R_B \left(P_0^P Z^P - \frac{D_s^P}{R_D}\right) && \text{banker} \\
&+ p\delta g(W) + (1-p) \left[q \frac{D_s^P}{Q^P} + \delta g(W - D_s^P)\right] && \text{outside investor}
\end{aligned}$$

Note that in equilibrium $\mathbb{E} \min\{\bar{B}^P, Z^P - \frac{D_s^P}{Q^P}\} = R_B \left(P_0^P Z^P - \frac{D_s^P}{R_D}\right)$. Thus,

$$\begin{aligned}
& \mathbb{E} [C_2^P(E) + C_2^P(B) + C_2^P(OI)] \\
&= [p + (1-p)q] Z^P + p [g(W) - (D_s^P - D_d^P)] + (1-p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)],
\end{aligned}$$

where $Z^P = A^P F(I^P)$. Combining the above results, the social welfare is

$$\begin{aligned}
U &= Y + \frac{D_s^P - D_d^P}{R_D} - I^P + v(D_d^P) + \beta [p + (1-p)q] A^P F(I^P) \\
&+ \beta p [g(W) - (D_s^P - D_d^P)] + \beta(1-p) [\delta g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]. \blacksquare
\end{aligned}$$

A.4 Proof of Proposition 1.2

I first prove that welfare in the center unambiguously goes up.

Step 1. Let's denote the social welfare in country C by $U^C = U^C(A^C, A^P; \cdot)$, where the first argument is the marginal productivity of investment opportunities in country C , the second argument is the marginal productivity of investment opportunities in country P , the third argument is a dummy variable that indicates if the two countries are integrated. We

are interested in computing the following difference

$$X^C = U^C (A^C, A^P; \text{integration}) - U^C (A^C, A^P; \text{autarky})$$

I can express the social welfare in country C as follows

$$U^C (A^C, A^P; \text{integration}) = \int_{A^C}^{A^P} \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} + U^C (A^C, A^P; \text{integration})$$

Observe that if the two countries have the same level of A then there is no gains from integration. Formally, $U^C (A^C, A^P; \text{integration}) = U^C (A^C, A^P; \text{autarky})$. Thus, the variable of interest X^C can be expressed as follows

$$X^C = \int_{A^C}^{A^P} \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \quad (\text{A.7})$$

Step 2. I now show that $dU^C (A^C, \tilde{A}; \text{integration}) / d\tilde{A} > 0$. Thus, from (A.7) I will get that $X^C > 0$. In words, country P unambiguously benefits from integration when $A^P > A^C$.

The social welfare function in country C is

$$\begin{aligned} U^C (A^C, \tilde{A}; \text{integration}) &= Y - I^C + \frac{D_s^C - D_d^C}{R_D} + v(D_d^C) + \beta[p + (1-p)q]A^C F(I^C) \\ &\quad + \beta\{p[g(W) - (D_s^C - D_d^C)] \\ &\quad + (1-p)[g(W - D_s^C) + D_s^C - (D_s^C - D_d^C)]\}. \end{aligned}$$

Rearranging last equation I get

$$\begin{aligned} U^C (A^C, \tilde{A}; \text{integration}) &= Y - I^C + (D_s^C - D_d^C) \left(\frac{1}{R_D} - \beta \right) + v(D_d^C) \\ &\quad + \beta \{ [p + (1-p)q]A^C F(I^C) + pg(W) + (1-p)[g(W - D_s^C) + D_s^C] \} \end{aligned}$$

Now, I take the full derivative of the above expression with respect to \tilde{A}

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= -\frac{dI^C}{d\tilde{A}} + \left(\frac{dD_s^C}{d\tilde{A}} - \frac{dD_d^C}{d\tilde{A}} \right) \left(\frac{1}{R_D} - \beta \right) \\ &\quad - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} + v'(D_d^C) \frac{dD_d^C}{d\tilde{A}} \\ &\quad + \beta \left\{ [p + (1-p)q] A^C F'(I^C) \frac{dI^C}{d\tilde{A}} + (1-p) [1 - g'(W - D_s^C)] \frac{dD_s^C}{d\tilde{A}} \right\} \end{aligned}$$

Rearranging I get

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= \left\{ \beta [p + (1-p)q] A^C F'(I^C) - 1 \right\} \frac{dI^C}{d\tilde{A}} \\ &\quad - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} \\ &\quad + \left[-\frac{1}{R_D} + \beta + v'(D_d^C) \right] \frac{dD_d^C}{d\tilde{A}} \\ &\quad + \left\{ \frac{1}{R_D} - \beta + \beta(1-p) [1 - g'(W - D_s^C)] \right\} \frac{dD_s^C}{d\tilde{A}} \quad (\text{A.8}) \end{aligned}$$

If $\tilde{A} > A^C$ which is the case of interest then $D_s^C < D_s^P$. It is also true that $dQ^C/d\tilde{A} > 0$, $dR_D/d\tilde{A} > 0$, $dI^C/d\tilde{A} < 0$, $dD_d^C/d\tilde{A} > 0$ and $dD_s^C/d\tilde{A} < 0$.¹ Before I simplify the above formula it useful to interpret all the terms to understand the effects of the marginal increase in \tilde{A} . Consider the **first line** of (A.8). An increase in \tilde{A} leads to decrease in investment in country C . This has two effects. First, the expected revenue of the bankers projects goes down which is represented by the first term in curly brackets. Second, the entrepreneurs in country C have now more endowment in period $t = 0$ to consume which is represented by the second term. When the collateral constraint binds the net effect of these two effects is positive. This is because in equilibrium the marginal product of investment is smaller than the marginal financing cost of investment. This is because a unit of risky projects has additional benefit of increasing the amount of collateral for the bankers. See the first line of (A.9). Consider the **second line**. Because $D_s^C < D_s^P$ country C is net lender of resources to

¹I don't show this formally here but it can be simply obtained by differentiating the integrated market equilibrium conditions with respect to A^P .

country P in period $t = 0$. An increase in \tilde{A} leads to an increase in R_D which means that the entrepreneurs in country C have to lend less to banks in country H to get 1 unit return in the future. This is a benefit. Consider the **third line**. An increase in \tilde{A} increases demand for riskless securities in country C . This has a cost $-1/R_D$ because the entrepreneurs give part of their endowment to buy the securities. It has two benefits: (i) the entrepreneurs get a unit of consumption at period $t = 2$ but discount this at rate β ; (ii) the entrepreneurs benefit from using more riskless securities in their transactions which is captured by $v'(D_d^C)$. Observe that in equilibrium these two benefits exactly equal to the cost. This follows from the entrepreneurs optimality condition (1.5). Thus, the third line equals zero. Consider the **fourth line**. An increase in \tilde{A} leads to a decrease in riskless securities issuance D_s^C . A unit decrease in D_s^C have several effects on the welfare in country C . First, it decreases the amount of resources that the bankers in country C use to invest by $1/R_D$. Second, it decreases the amount of consumption goods that has to be paid out by the bankers in period $t = 2$ which adds β to the welfare. Third, it decreases the reallocation of resources from the outside investors to the bankers in the bad state which has the following effect on welfare $-\beta(1-p)[1-g'(W-D_s^C)]$. That's, the output of projects that are run by the outside investors increases by $g'(W-D_s^C)$ in the bad state while the bankers get 1 unit less of consumption goods. The bankers optimality condition with respect to riskless securities (1.16) can be used to simplify the fourth line. See the third line of (A.9).

$$\begin{aligned} \frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} &= -\beta\theta^C A^C F'(I^C) \frac{dI^C}{d\tilde{A}} \\ &\quad - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}} \\ &\quad + \beta \frac{\theta^C}{Q^C} \frac{dD_s^C}{d\tilde{A}} \end{aligned} \tag{A.9}$$

Next, I use that $\theta^C[D_s^C - Q^C A^C F(I^C)] = 0$ to combine the first and third line of the equation above to get

$$\frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^C A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tilde{A}} - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

The first term in the above formula is positive while the second one is negative which makes the overall expression positive. This completes the proof that the center benefits from the integration.

I consider the periphery next. The proof of this result uses the same idea as the proof of the previous result.

Step 1. Let's denote the social welfare in country P by $U^P = U^P(A^P, A^C; \cdot)$, where the first argument is the marginal productivity of investment opportunities in country P , the second argument is the marginal productivity of investment opportunities country C , the third argument is a dummy variable that indicates if the two countries are integrated. We are interested in computing the following difference

$$\begin{aligned} X^P &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\ &= - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A}. \end{aligned} \quad (\text{A.10})$$

Step 2. Repeating calculations in Step 2 of the previous proof I get

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

Because $dQ^P/d\tilde{A} > 0$ the first term of this expression is positive. Because for $\tilde{A} < A^P$ it is true that $D_s^P > D_d^P$ and $dR_D/d\tilde{A} > 0$ the second term is negative (taking into account the sign in front of this term). If I plug the above expression into (A.10) and take into account the negative sign in front of the integral the effects of the two terms in the above formula reverses. However, because the two terms have the opposite effects the net effect can be either

negative or positive.

Step 3. Consider a case in which the marginal productivity of investment opportunities in the two countries are as follows $(A^P, A^C) = (A + \epsilon, A)$ where A is some positive number and ϵ is small and positive number. In this case I can write

$$X^P \approx -\frac{dU^P(A, A; \text{integration})}{d\tilde{A}} \cdot \epsilon = -\beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \Big|_{(A^P, A^C)=(W, W)} \cdot \epsilon < 0$$

By continuity there exists $\bar{A} > A^C$ such that for all $A^P \in (A^C, \bar{A})$ it is true that $X^P < 0$. ■

A.5 Proof of Lemma 1.5.

$$\begin{aligned} \max_{\tau^P} & Y - I^P + \frac{D_s^P - D_d^P}{R_D} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\ & + \beta \{p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\}, \end{aligned}$$

subject to the following system of equilibrium conditions

$$\left. \begin{aligned} \frac{R_B}{R_D}(1 - \tau^P) - \left(p + \frac{(1-p)q}{Q^P}\right) - \frac{\theta^P}{Q^P} &= 0, \\ [p + (1-p)q] - R_B P_0^P + \theta^P &= 0, \\ g'(W - D_s^P) &= \frac{q}{Q^P}, \\ D_s^P \leq Q^P A^P F(I^P), \theta^P &\geq 0, \\ \theta^P (D_s^P - Q^P A^P F(I^P)) &= 0, \\ R_D &= \frac{1}{\beta + v'(D_d^P)}, \\ P_0^P &= \frac{1}{A^P F'(I^P)}, \end{aligned} \right\} \text{periphery eq-um}$$

$$\left. \begin{aligned}
& \frac{R_B}{R_D}(1 - \tau^C) - \left(p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0, \\
& [p + (1-p)q] - R_B P_0^C + \theta^C = 0, \\
& g'(W - D_s^C) = \frac{q}{Q^C}, \\
& D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0, \\
& \theta^C (D_s^C - Q^C A^C F(I^C)) = 0, \\
& R_D = \frac{1}{\beta + v'(D_d^C)}, \\
& P_0^C = \frac{1}{A^C F'(I^C)},
\end{aligned} \right\} \text{center eq-um}$$

$$D_d^P + D_d^C = D_s^P + D_s^C. \quad \left. \right\} \text{safe debt mkt clearing}$$

This system of fourteen equations and four constraints uniquely defines a mapping from τ^P to fourteen variables $P_0^P(\tau^P), P_0^C(\tau^P), I^P(\tau^P), I^C(\tau^P), Q^P(\tau^P), Q^C(\tau^P), D_s^P(\tau^P), D_s^C(\tau^P), D_d^P(\tau^P), D_d^C(\tau^P), R_D, \theta^P(\tau^P), \theta^C(\tau^P)$. The uniqueness comes from the analysis similar to the one presented in section 1.3. The mapping is differentiable for any τ^P except τ^P for which the collateral constraints change from being binding to not being binding. Given an implicit mapping of τ^P to all the equilibrium variables I can write the first order necessary condition by differentiating the welfare function with respect to τ^P

$$\begin{aligned}
\frac{dU^P}{d\tau^P} = & -\frac{dI^P}{d\tau^P} + \left(\frac{dD_s^P}{d\tau^P} - \frac{dD_d^P}{d\tau^P} \right) \left(\frac{1}{R_D} - \beta \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + v'(D_d^P) \frac{dD_d^P}{d\tau^P} \\
& + \beta \left\{ [p + (1-p)q] A^P F'(I^P) \frac{dI^P}{d\tau^P} + (1-p) [1 - g'(W - D_s^P)] \frac{dD_s^P}{d\tau^P} \right\} = 0.
\end{aligned}$$

Rearranging I get

$$\begin{aligned} \frac{dU^P}{d\tau^P} &= \left\{ \beta[p + (1-p)q]A^P F'(I^P) - 1 \right\} \frac{dI^P}{d\tau^P} \\ &\quad - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} \\ &\quad + \left[-\frac{1}{R_D} + \beta + v'(D_d^P) \right] \frac{dD_d^P}{d\tau^P} \\ &\quad + \left\{ \frac{1}{R_D} - \beta + \beta(1-p) [1 - g'(W - D_s^P)] \right\} \frac{dD_s^P}{d\tau^P} = 0. \end{aligned}$$

After plugging in the bankers and the entrepreneurs optimality conditions the regulator first condition can be written as follows

$$\begin{aligned} \frac{dU^P}{d\tau^P} &= -\beta\theta^P \left(A^P F'(I^P) \frac{dI^P}{d\tau^P} - \frac{1}{Q^P} \frac{dD_s^P}{d\tau^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + \frac{R_B}{R_D} \tau^P \frac{dD_s^P}{d\tau^P} \\ &= \beta\theta^P \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} + \frac{R_B}{R_D} \tau^P \frac{dD_s^P}{d\tau^P} \\ &= \beta \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left(\theta^P - \tau^P \frac{R^B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0, \end{aligned}$$

where the second line uses the observation that the derivative of $\theta^P [D_s^P - Q^P A^P F(I^P)] = 0$ with respect to τ^P equals

$$\theta^P \left[\frac{dD_s^P}{d\tau^P} - \frac{dQ^P}{d\tau^P} A^P F(I^P) - Q^P A^P F'(I^P) \frac{dI^P}{d\tau^P} \right] = 0,$$

and the third line uses

$$\frac{1}{D_s^P} \frac{dD_s^P}{d\tau^P} = -\frac{1}{\tilde{\epsilon}_g^P Q^P} \frac{dQ^P}{d\tau^P}. \blacksquare$$

A.6 Proof of Proposition 1.3

Consider the periphery. The social welfare function change after integration equals

$$\begin{aligned} X^P &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \\ &= U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^P; \text{autarky}) \\ &= - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \end{aligned}$$

The derivative of the social welfare function with respect to the marginal productivity of investment opportunities \tilde{A} in the center is

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \left(\theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tilde{A}}$$

Using optimality condition of the regulator in country P from lemma 1.5 I obtain

$$\frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \frac{D_s^P - D_d^P}{R_D^2} \left[\frac{dR_D}{d\tau^P} \frac{dQ^P/d\tilde{A}}{dQ^P/d\tau^P} - \frac{dR_D}{d\tilde{A}} \right]$$

where $D_s^P - D_d^P > 0$, $dR_D/d\tau^P < 0$, $dQ^P/d\tilde{A} > 0$, $dQ^P/d\tau^P > 0$, $dR_D/d\tilde{A} > 0$. This implies that $dU^P(A^P, \tilde{A}; \text{integration})/d\tilde{A} < 0$. Thus, $X^P > 0$.

Hence, $X^C > 0$. ■

A.7 Proof of Proposition 1.4

From lemma 1.5 the optimal level of taxes in country P satisfies

$$\beta \frac{A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left(\theta^P - \tau^P \frac{R_B Q^P}{R_D \tilde{\epsilon}_g^P} \right) - \frac{D_s^P - D_d^P}{R_D^2} \frac{dR_D}{d\tau^P} = 0. \quad (\text{A.11})$$

Similar equation holds for country C

$$\beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^C} \left(\theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^C} = 0. \quad (\text{A.12})$$

Let's denote a solution to these equations, a Nash equilibrium, as $(\hat{\tau}^C, \hat{\tau}^P)$.

Next I consider the effect of the marginal change in τ^P on the social welfare function in country C evaluated at a Nash equilibrium $(\hat{\tau}^C, \hat{\tau}^P)$. Repeating the algebra from Lemma 1.5

I obtain

$$\frac{dU^C}{d\tau^P} = \beta \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \left(\theta^C - \tau^C \frac{R_B Q^C}{R_D \tilde{\epsilon}_g^C} \right) - \frac{D_s^C - D_d^C}{R_D^2} \frac{dR_D}{d\tau^P}. \quad (\text{A.13})$$

This formula is key to understanding the coordination failure result. A marginal increase in the taxes in the periphery has three effect: (i) it makes the welfare losses from the externality bigger (first term in the brackets); (ii) it decreases country C tax-induced bank funding costs; (iii) it decreases interest rate which makes entrepreneurs gain from investing in peripheral safe debt smaller.

Taking into account the optimality condition (A.12) I can rewrite the previous equation as follows

$$\frac{dU^C}{d\tau^P} \Big|_{(\hat{\tau}^C, \hat{\tau}^P)} = \frac{D_s^C - D_d^C}{R_D^2} \left[\frac{dR_D}{d\tau^C} \frac{dQ^C}{dQ^P/d\tau^P} - \frac{dR_D}{d\tau^P} \right] < 0. \quad (\text{A.14})$$

This expression is negative because (i) by the assumption of the proposition the center is a net buyer of safe debt $D_s^C - D_d^C < 0$, (ii) an increase in the tax level in the periphery decreases the level of safe debt in the world making it more expensive which implies $dR_D/d\tau^P < 0$, (iii) analogously $dR_D/d\tau^C < 0$, (iv) an increase in taxes τ^P increases the issuance of safe debt in the center (because the return on safe debt falls) which implies more severe fire-sale price decline (relative to fundamental value of the risky projects q) $dQ_c^C/d\tau^P < 0$, however, at the same time the fire-sale price in the periphery rises $dQ_c^F/d\tau^F > 0$. Negative sign in (A.14) implies that there is a gain for agents in country C from a marginal decrease in taxes in country P .

I can analogously compute the marginal effect of change in τ^C on U^P .

$$\frac{dU^P}{d\tau^C} \Big|_{(\hat{\tau}^C, \hat{\tau}^P)} = \frac{D_s^P - D_d^P}{R_D^2} \left[\frac{dR_D}{d\tau^C} \frac{dQ^C/d\tau^C}{dQ^P/d\tau^C} - \frac{dR_D}{d\tau^C} \right] > 0.$$

This expression is positive because $D_s^P - D_d^P > 0$, $dR_D/d\tau^C < 0$, $dR_D/d\tau^P < 0$, $dQ_c^P/d\tau^C < 0$ and $dQ^P/d\tau^C > 0$. Positive sign of this expression implies that there is gain for agents in country P from a marginal increase in taxes in country C .

Thus, the following perturbation $d(\tau^C, \tau^P) = (-\Delta^C, \Delta^P)$, where Δ^C and Δ^P are small and positive numbers, increases the social welfare functions in both countries. Hence, if the policy makers could coordinate on their decisions they could achieve higher welfare than in a Nash equilibrium by decreasing taxes in country P and increasing taxes in country C . ■

A.8 Proof of proposition 1.5

I start by defining the problem of the regulator in the periphery

$$\begin{aligned} \max_{\tau^P, \tau^C} Y - I^P + \frac{D_s^P - D_d^P}{R_D^F} + v(D_d^P) + \beta[p + (1-p)q]A^P F(I^P) \\ + \beta \{ p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)] \}, \end{aligned}$$

subject to the following system of equilibrium conditions

$$\frac{R_B}{R_D^P}(1 - \tau^P) - \left(p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0, \quad (\text{A.15})$$

$$[p + (1-p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0, \quad (\text{A.16})$$

$$g'(W - D_s^P) = \frac{q}{Q^P},$$

$$D_s^P \leq Q^P A^P F(I^P), \theta^P \geq 0,$$

$$\theta^P (D_s^P - Q^P A^P F(I^P)) = 0,$$

$$R_D^P = \frac{1}{\beta + v'(D_d^P)}, \quad (\text{A.17})$$

$$\frac{R_B}{R_D^C}(1 - \tau^C) - \left(p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0,$$

$$[p + (1-p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) = 0,$$

$$g'(W - D_s^C) = \frac{q}{Q^C},$$

$$D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0,$$

$$\theta^C (D_s^C - Q^C A^C F(I^C)) = 0,$$

$$R_D^C = \frac{1}{\beta + v'(D_d^C)},$$

$$D_d^P + D_d^C = D_s^P + D_s^C,$$

$$R_D^C = (1 - \tau_f^P) R_D^P.$$

Instead of solving this problem I propose to solve less constrained problem and then show that the solution satisfies omitted constraints. The less constrained problem looks as follows

$$\begin{aligned} \max_{D_d^P, D_s^P, I^P} & Y - I^P + \frac{D_s^P - D_d^P}{R_D^F} + v(D_d^P) + \beta [p + (1-p)q] A^P F(I^P) \\ & + \beta \{ p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)] \}, \end{aligned}$$

subject to the following subset of the equilibrium conditions

$$\begin{aligned}
g'(W - D_s^P) &= \frac{q}{Q^P}, \\
D_s^P &\leq Q^P A^P F(I^P), \\
\frac{R_B}{R_D^C} (1 - \tau^C) - \left(p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} &= 0, \\
[p + (1-p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) &= 0, \\
g'(W - D_s^C) &= \frac{q}{Q^C}, \\
D_s^C &\leq Q^C A^C F(I^C), \theta^C \geq 0, \\
\theta^C (D_s^C - Q^C A^C F(I^C)) &= 0, \\
R_D^C &= \frac{1}{\beta + v'(D_d^C)}, \\
D_d^P + D_d^C &= D_s^P + D_s^C.
\end{aligned}$$

Observe that the regulator can directly affect the first two conditions. All the remaining conditions are affected through changes in $D_s^P - D_d^P$ (because of the safe debt market clearing condition). These remaining conditions determine the equilibrium in the center conditional on $D_s^P - D_d^P$. Because only one variable from the center the peripheral welfare function and the first to constraints the only thing we need to know about the remaining conditions is how R_D^C depends on $D_s^P - D_d^P$. Hence, the problem can be written as follows

$$\begin{aligned}
\max_{D_d^P, D_s^P, I^P} Y - I^P + \frac{D_s^P - D_d^P}{R_D^C(D_s^P - D_d^P)} + v(D_d^P) + \beta [p + (1-p)q] A^P F(I^P) \\
+ \beta \{ p [g(W) - (D_s^P - D_d^P)] + (1-p) [g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)] \},
\end{aligned}$$

subject to

$$\begin{aligned}
g'(W - D_s^P) &= \frac{q}{Q^P}, \\
D_s^P &\leq Q^P A^P F(I^P).
\end{aligned}$$

The optimal choice of I^P leads to

$$[p + (1-p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0. \quad (\text{A.18})$$

The optimal choice of D_s^P leads to

$$\frac{R_B}{R_D^P} - \left(p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = -\theta^P D_s^P \frac{g''(W - D_s^P)}{q} + \frac{D_s^P - D_d^P}{R_D^P} \frac{R_B}{R_D^P} \frac{dR_D^C(D_s^P - D_d^P)}{dD_s^P}. \quad (\text{A.19})$$

The optimal choice of D_d^P leads to

$$\frac{1}{R_D^C} = \beta + v'(D_d^P) - \frac{D_s^P - D_d^P}{R_D^P} \frac{R_B}{R_D^P} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}. \quad (\text{A.20})$$

Note that

$$\frac{dR_D^C(D_s^P - D_d^P)}{dD_s^P} + \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P} = 0.$$

Finally, the complementarity slackness conditions should be satisfied

$$\theta^P [D_s^P - Q^P A^P F(I^P)] = 0.$$

I now show that the optimality conditions of the less constrained problem satisfy the condition omitted from the more constrained problem. Pick τ_f^P such that

$$\tau_f^P = \frac{-\frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}{1 - \frac{R_D^P}{R_D^C} \frac{D_s^P - D_d^P}{R_D^C} \frac{dR_D^C(D_s^P - D_d^P)}{dD_d^P}}. \quad (\text{A.21})$$

This τ_f^P together with (A.20) implies (A.17). Next, (A.21) together with (A.19) and the following choice of τ^P

$$\tau^P = \theta^P \frac{\tilde{\epsilon}_g}{Q^P} \frac{R_D^P}{R_B}, \quad (\text{A.22})$$

imply (A.15). Next, (A.18) implies (A.16). Thus, I showed that the less constrained problem optimum is feasible under the more constrained problem optimum. ■

Appendix B

Growth-Rate and Uncertainty

Shocks in Consumption:

Cross-Country Evidence

B.1 Appendix: Model Estimation

We employ a Bayesian MCMC algorithm to estimate our model. More specifically, we employ a Metropolized Gibbs sampling algorithm to sample from the joint posterior distribution of the unknown parameters and variables conditional on the data. The full probability model we employ may be denoted by

$$f(Y, X, \Theta) = f(Y, X|\Theta)f(\Theta),$$

where $Y \in \{c_{i,t}, I_{i,t+1}^d\}$ is the set of observable variables for which we have data,

$$X \in \{z_{i,t}, x_{i,t}, x_{W,t}, \sigma_{i,t+1}^2, \sigma_{W,t+1}^2\}$$

is the set of unobservable variables, and

$$\Theta \in \{ \rho, \rho_W, \gamma, \sigma_W^2, \sigma_\omega^2, \sigma_{W,\omega}^2, \lambda, \lambda_W, \xi_i, \chi_i, \sigma_i^2, \sigma_{\nu,i}^2, \mu_i, \mu_d, \}$$

is the set of parameters. From a Bayesian perspective, there is no real importance to the distinction between X and Θ . The only important distinction is between variables that are observed and those that are not. The function $f(Y, X|\Theta)$ is often referred to as the likelihood function of the model, while $f(\Theta)$ is often referred to as the prior distribution. Both $f(Y, X|\Theta)$ and $f(\Theta)$ are fully specified in sections 2.3 and 2.4 of the paper. The likelihood function may be constructed by combining equations (2.2)-(2.4) and (2.8), the distributional assumptions for the shocks in these equations detailed in section 2.3 and the assumptions about the distributions of $z_{i,t}$, $x_{i,t}$, $x_{W,t}$, $\sigma_{i,t}$, and $\sigma_{W,t}$ for the initial period for each country that are detailed in section 2.4. The prior distributions are described in detail in section 2.4.

The object of interest in our study is the distribution $f(X, \Theta|Y)$, i.e., the joint distribution of the unobservables conditional on the observed values of the observables. For expositional simplicity, let $\Phi = (X, \Theta)$. Using this notation, the object of interest is $f(\Phi|Y)$. The Gibbs sampler algorithm produces a sample from the joint distribution by breaking the vector of unknown variables into subsets and sampling each subvector sequentially conditional on the value of all the other unknown variables (see, e.g., Gelman et al., 2004, and Geweke, 2005). In our case we implement the Gibbs sampler as follows.

1. We derive the conditional distribution of each element of Φ conditional on all the other elements and conditional on the observables. For the i th element of Φ , we can denote this conditional distribution as $f(\Phi_i|\Phi_{-i}, Y)$, where Φ_i denotes the i th element of Φ and Φ_{-i} denotes all but the i th element of Φ . In most cases, $f(\Phi_i|\Phi_{-i}, Y)$ are common distributions such as normal distributions or gamma distributions for which samples can be drawn in a computationally efficient manner. In cases where the Gibbs sampler cannot be applied, we use the Metropolis algorithm to sample values of $f(\Phi_i|\Phi_{-i}, Y)$.¹

¹The Metropolis algorithm samples a proposal Φ_i^* from a proposal distribution $J_i(\Phi_i^*|\Phi_i^{t-1})$. This proposal distribution must be symmetric, i.e., $J_i(x_a|x_b) = J_i(x_b|x_a)$. The proposal is accepted with probability $\min(r, 1)$

2. We propose initial values for all the unknown variables Φ . Let Φ^0 denote these initial values.
3. We cycle through Φ sampling Φ_i^t from the distribution $f(\Phi_i|\Phi_{-i}^{t-1}, Y)$ where

$$\Phi_{-i}^{t-1} = (\Phi_1^t, \dots, \Phi_{i-1}^t, \Phi_{i+1}^{t-1}, \dots, \Phi_d^{t-1})$$

and d denotes the number of elements in Φ . At the end of each cycle, we have a new draw Φ^t . We repeat this step N times to get a sample of N draws for Φ .

4. It has been shown that samples drawn in this way converge to the distribution $f(\Phi|Y)$ under very general conditions (see, e.g., Geweke, 2005). We assess convergence and throw away an appropriate burn-in sample.

In practice, we run four such “chains” starting two from one set of initial values and two from another set of initial values. We choose starting values that are far apart in the following way: For one chain, we set the initial values of $x_{i,t} = 0$ for all i and t . For the other chain, we set the initial values of $x_{i,t} = \Delta c_{i,t}$ for all i and t .

Given a sample from the joint distribution $f(\Phi|Y)$ of the unobserved variables conditional on the observed data, we can calculate any statistic of interest that involves Φ . For example, we can calculate the mean of any element of Φ by calculating the sample analogue of the integral

$$\int \Phi_i f(\Phi_i|\Phi_{-i}^{t-1}, Y) d\Phi_i.$$

B.2 Appendix: Variance Ratios

Variance ratios are a simple tool to quantify the persistence of shocks to aggregate consumption (Cochrane, 1988). The k -period variance ratio for consumption growth is defined as

where $r = f(\Phi_i^*|\Phi_{-i}, Y)/f(\Phi_i^{t-1}|\Phi_{-i}, Y)$. If the proposal is accepted, $\Phi_i^t = \Phi_i^*$. Otherwise $\Phi_i^t = \Phi_i^{t-1}$. Using the Metropolis algorithm to sample from $f(\Phi_i|\Phi_{-i}, Y)$ is much less efficient than the standard algorithms used to sample from known distributions such as the normal distribution in most software packages. Intuitively, this is because it is difficult to come up with an efficient proposal distribution. The proposal distribution we use is a normal distribution centered at Φ_i^{t-1} .

the ratio of the variance of k -period consumption growth and 1-period consumption growth divided by k :

$$\text{VR}_{i,k} = \frac{1}{k} \frac{\text{var}\left(\sum_{j=0}^{k-1} \Delta c_{i,t-j}\right)}{\text{var}(\Delta c_{i,t})}. \quad (\text{B.1})$$

The intuition for this statistic comes from the fact that for a simple random-walk process $\text{var}(c_{i,t} - c_{i,t-k})$ is equal to k times $\text{var}(c_{i,t} - c_{i,t-1})$, implying that the variance ratio for such a process is equal to one for all k . For a trend-stationary process, the variance ratio is less than one and falls toward zero as k increases. However, for a process that has persistent growth-rate shocks—i.e., positively autocorrelated growth rates—the variance ratio is larger than one.

[Bansal and Yaron \(2004\)](#) introduce a variance ratio statistic for assessing the persistence of shocks to volatility. They first compute the innovations to consumption growth $u_{i,t}$ as the residuals from an AR(5) regression and use the absolute value of these innovations $|u_{i,t}|$ as a measure of realized volatility of consumption growth. They then construct variance ratios for $|u_{i,t}|$,

$$\text{VR}_{i,k}^u = \frac{1}{k} \frac{\text{var}\left(\sum_{j=0}^{k-1} |u_{i,t-j}|\right)}{\text{var}(|u_{i,t}|)}. \quad (\text{B.2})$$

This statistic provides a rough measure of the persistence of stochastic volatility. As with the variance ratio for consumption growth, if this variance ratio is above one, it indicates that uncertainty shocks have persistent effects on volatility—i.e., high volatility periods are “bunched together” leading to a high value of the variance in the numerator.

Appendix C

Sectoral Shocks, the Beveridge Curve and Monetary Policy

C.1 Model Details

C.1.1 Household's Problem

$$\begin{aligned}
 V_0(B_0, \mathbf{N}_{-1}) &= \max_{\{C_t, B_{t+1}, L_t, \mathbf{N}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, N_t) - \sum_{i=1}^{K-1} R(L_{i,t-1}, L_{i,t}) \right\}, \\
 \text{s.t.} \quad P_t C_t &= \sum_{i=1}^K (W_{i,t} N_{i,t} + \Pi_{i,t}) + B_t - E_t Q_{t,t+1} B_{t+1}, & (\lambda_{1,t}) \\
 N_{i,t} &= (1 - \delta_i) N_{i,t-1} + p_{i,t} (L_{i,t} - N_{i,t-1}), & (\lambda_{2,t,i}) \\
 \sum_{i=1}^K L_{i,t} &= 1. & (\lambda_{3,t})
 \end{aligned}$$

The Lagrangian for this problem is

$$\begin{aligned} \mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, N_t) - \sum_{i=1}^{K-1} R(L_{i,t-1}, L_{i,t}) - \lambda_{1,t} \left[P_t C_t - \sum_{i=1}^K (W_{i,t} N_{i,t} + \Pi_{i,t}) - B_t \right. \right. \\ \left. \left. + E_t Q_{t,t+1} B_{t+1} \right] - \sum_{i=1}^K \lambda_{2,t,i} [N_{i,t} - (1 - \delta_i) N_{i,t-1} - p_{i,t} (L_{i,t} - N_{i,t-1})] \right. \\ \left. - \lambda_{3,t} \left[\sum_{i=1}^K L_{i,t} \right] \right\}. \end{aligned}$$

The optimality condition with respect to consumption index is

$$u_c(C_t, N_t) = \lambda_{1,t} P_t. \quad (\text{C.1})$$

The first order condition with respect to B_{t+1} in any possible state at $t+1$, taking into account the previous expression, leads to

$$\beta \frac{u_c(C_{t+1}, N_{t+1})}{P_{t+1}} = \frac{u_c(C_t, N_t)}{P_t} Q_{t,t+1}. \quad (\text{C.2})$$

The riskless one-period nominal interest rate can be expressed as follows

$$\frac{1}{1 + i_t} = \mathbb{E}_t Q_{t,t+1}.$$

The optimality condition with respect to labor force in sector i is

$$\lambda_{2,t,i} p_{i,t} = \lambda_{3,t} + R_2(L_{i,t-1}, L_{i,t}) + \beta \mathbb{E}_t R_1(L_{i,t}, L_{i,t+1}) \quad (\text{C.3})$$

The optimality condition with respect to the employed labor in sector i can be written as follows

$$\lambda_{2,t,i} = u_N(C_t, N_t) + \lambda_{1,t} W_{i,t} + \beta \mathbb{E}_t [(1 - \delta_i - p_{i,t+1}) \lambda_{2,t+1,i}]. \quad (\text{C.4})$$

Note that $\lambda_{2,t,i}$ represents the utility value of additional employed worker in sector i for the household conditional on the equilibrium path of wages $\{W_{i,t}\}_{t=0}^{\infty}$.

C.1.2 Retailer's Problem

The retailers problem is similar to the standard specification in [Woodford \(2003\)](#). Monopolistically competitive retailers set prices to maximize profits:

$$\begin{aligned} \max_{P_t(l)} \Pi_t^{ret}(l) &= \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} [P_t(l) - P_{fT}] Y_T(l), \\ \text{s.t. } Y_T(l) &= Y_T \left(\frac{P_t(l)}{P_T} \right)^{-\zeta}, \end{aligned}$$

where $P_t(l)$ is the nominal price chosen by retailer that sells differentiated good l and who faces a downward sloping demand schedule and discount future profits by the nominal stochastic discount factor $Q_{t,T}$. Parameter χ is the Calvo parameter governing the degree of price stickiness. The optimality condition for price-setting is given by:

$$\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^\zeta Y_T \left(P_t^*(l) - \frac{\zeta}{\zeta - 1} P_{fT} \right) = 0.$$

Which implies

$$\frac{P_t^*(l)}{P_t} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_{fT} P_T^\zeta Y_T}{\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^\zeta Y_T}.$$

The inflation rate is derived from the Calvo assumption with a fraction $1 - \chi$ of firms resetting their prices to $P_t(l)/P_t$:

$$P_t = \left\{ \chi P_{t-1}^{1-\zeta} + (1 - \chi) (P_t^*)^{1-\zeta} \right\}^{\frac{1}{1-\zeta}}.$$

In a zero inflation steady state, a log-linearization of these equilibrium conditions delivers the standard New Keynesian Phillips curve.

C.2 Additional Proofs

For several proofs, we will refer repeatedly to the equilibrium conditions that determine the steady state Beveridge curve and the natural rate of unemployment in the K -sector model. A solution of the multisector model with “fast-moving” labor markets is a value for aggregate output Y_t , real marginal cost P_{ft}/P_t , consumption C_t , state-variables in the retailers pricing problem K_t, F_t and sectoral prices and quantities $\{Y_{i,t}, N_{i,t}, U_{i,t}, V_{i,t}, P_{i,t}/P_t, W_{i,t}/P_t, p_{i,t}, q_{i,t}\}_{i=1}^K$ that satisfy the following static equilibrium conditions

$$Y_t = \left\{ \sum_{i=1}^K \phi_{i,t}^{\frac{1}{\eta}} Y_{i,t}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \Rightarrow Y_t = A_t \left\{ \sum_{i=1}^K \tilde{\phi}_{i,t}^{\frac{1}{\eta}} N_{i,t}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \quad (\text{C.5})$$

$$\frac{P_{ft}}{P_t} = \left\{ \sum_{i=1}^K \phi_i \left(\frac{P_{i,t}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}}, \quad (\text{C.6})$$

$$Y_{i,t} = \phi_{i,t} Y_t \left(\frac{P_{i,t}}{P_{ft}} \right)^{-\eta} \Rightarrow Y_{i,t} = \tilde{\phi}_{i,t} \tilde{A}_{i,t}^{1-\eta} Y_t \left(\frac{P_{i,t}}{P_{ft}} \right)^{-\eta}, \quad (\text{C.7})$$

$$Y_{i,t} = A_{i,t} N_{i,t} \Rightarrow Y_{i,t} = A_t \tilde{A}_{i,t} N_{i,t}, \quad (\text{C.8})$$

$$\frac{P_{i,t}}{P_t} A_{i,t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} [1 - \beta (1 - \delta_i)], \quad (\text{C.9})$$

$$\frac{W_{i,t}}{P_t} = f(N_t) + \frac{\nu}{1-\nu} \frac{\kappa}{q_{it}} [1 - \beta (1 - \delta_i - p_{it})], \quad (\text{C.10})$$

$$\delta_i N_{i,t} = \varphi_i U_{i,t}^\alpha V_{i,t}^{1-\alpha}, \quad (\text{C.11})$$

$$q_{i,t} = \varphi_i \left(\frac{V_{i,t}}{U_{i,t}} \right)^{-\alpha}, \quad (\text{C.12})$$

$$p_{i,t} = \varphi_i \left(\frac{V_{i,t}}{U_{i,t}} \right)^{1-\alpha} \quad (\text{C.13})$$

and the following dynamic conditions:

$$1 = \vartheta \Pi_t^{\zeta-1} + (1 - \vartheta) \left(\frac{K_t}{F_t} \right)^{\zeta-1}, \quad (\text{C.14})$$

$$K_t = \frac{\zeta}{\zeta - 1} u_c(C_t, N_t) \frac{P_{ft}}{P_t} Y_t + \beta \chi \mathbb{E}_t \Pi_{t+1}^{\zeta} K_{t+1}, \quad (\text{C.15})$$

$$F_t = u_c(C_t, N_t) Y_t + \beta \chi \mathbb{E}_t \Pi_{t+1}^{\zeta-1} F_{t+1}, \quad (\text{C.16})$$

$$1 = \beta E_t \frac{u_c(C_{t+1}, N_{t+1})}{u_c(C_t, N_t)} (1 + i_t^d) / \Pi_{t+1}, \quad (\text{C.17})$$

$$Y_t = C_t + \sum_{i=1}^K \kappa V_{it} + G_t, \quad (\text{C.18})$$

in terms of the exogenous variables: aggregate productivity A_t , government spending G_t , and sector-specific productivity and demand $\left\{ \tilde{A}_{i,t}, \tilde{\phi}_{i,t} \right\}_{i=1}^{K-1}$. We consider either the case of no reallocation or the case of costless reallocation. With no reallocation

$$L_{i,t} = N_{i,t-1} + U_{i,t} \quad (\text{C.19})$$

and with costless reallocation

$$1 = N_t + U_t, \quad (\text{C.20})$$

$$V_{i,t}/U_{i,t} = V_{j,t}/U_{j,t}, \text{ for } i, j = 1, 2, \dots, K. \quad (\text{C.21})$$

C.2.1 Proof of Proposition 3.2

To that aggregate productivity shocks A_t trace out the same Beveridge curve as government spending shocks G_t , we must show that for any value of the government spending shock G_t , there exists an aggregate productivity shock A_t that implies the same level of aggregate vacancies and unemployment holding constant $\left\{ \tilde{A}_{i,t}, \tilde{\phi}_{i,t} \right\}_{i=1}^{K-1}$.

Observe that equations (C.5) and (C.7) - (C.9) can be combined to derive the following

modified sectoral demand conditions and vacancy posting conditions:

$$A_t \tilde{A}_{i,t} N_{i,t} = \tilde{A}_{i,t}^{1-\eta} \tilde{\phi}_{i,t} A_t \left\{ \sum_{i=1}^K \tilde{\phi}_{i,t}^{\frac{1}{\eta}} N_{i,t}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \left(\frac{P_{i,t}}{P_{ft}} \right)^{-\eta}, \quad (\text{C.22})$$

$$\frac{P_{i,t}}{P_{ft}} \frac{P_{ft}}{P_t} A_t \tilde{A}_{i,t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} [1 - \beta (1 - \delta_i)]. \quad (\text{C.23})$$

Collectively, equations (C.10) - (C.13) for K sectors, the K equations (C.19) (or (C.20) and (C.21)), and the K equations in (C.22) and (C.23) define the quantities $\{N_{i,t}, U_{i,t}, V_{i,t}, P_{i,t}/P_{ft}, W_{i,t}/P_t, p_{i,t}, q_{i,t}\}_{i=1}^K$ as a function of $\{P_{ft}/P_t, A_t\}$. Thus, aggregate vacancies and unemployment are the same conditional on the same combinations of P_{ft}/P_t , an endogenous variable, and A_t , an exogenous variable. The absence of wealth effects on labor supply is important, otherwise household consumption C_t would tie these equations back to the rest of the equilibrium conditions.

Since vacancies and unemployment are functions solely of $A_t P_{ft}/P_t$, any combinations of G_t and A_t that implies the same value for marginal cost times aggregate productivity implies the same values for vacancies and unemployment. Define the function $\frac{P_f}{P}(G, A)$ as the endogenous value of real marginal cost for different combinations of the aggregate shocks holding sectoral shocks constant. Choose, $A_t = \bar{A}$ such that $\frac{P_f}{P}(\bar{G}, 1) = \frac{P_f}{P}(G_0, \bar{A})$.

Then, it follows that:

$$\begin{aligned} V(\bar{G}, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K) &= V(G_0, \bar{A}, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K), \\ U(\bar{G}, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K) &= U(G_0, \bar{A}, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K). \end{aligned}$$

■

C.2.2 Proof of Proposition 3.4

We show that under perfect substitutability, sectoral employment has a factor representation in terms of the exogenous sectoral productivity process. Under perfect reallocation, the relative price of goods across sectors must be equalized. From equation (C.6), $P_i/P = \mu^{-1}$

for all sectors $i = 1, 2, \dots, K$. For simplicity, assume no aggregate demand shocks and set $\mu^{-1} = 1$. The firm's vacancy posting condition is given by equation (50):

$$A_i = W_i + \frac{\kappa}{q_i} [1 - \beta (1 - \delta_i)].$$

Log-linearizing equations (50) - (55) and combining, we have:

$$a_{i,t} = (1 - s_i) \tilde{\alpha}_i \frac{\bar{L}_i / \bar{U}_i}{1 - \alpha} n_{it},$$

where s_i is the steady state surplus and $\tilde{\alpha}_i$ is a composite parameter that depends on the matching function elasticity α and other matching function parameters when bargaining power is nonzero. The diagonal matrix H is obtained by simply inverted the expression to solve for sectoral employment. ■

C.2.3 Proof of Proposition 3.6

In this proof, we show that sector-specific shocks raise the natural rate of unemployment and shift outward the Beveridge curve in the absence of labor market reallocation. We begin by listing the equilibrium conditions that determine aggregate employment. Under the assumption of no heterogeneity in matching function efficiencies or separation rates, the system of equations determining employment are given by the following conditions where time subscripts are dropped for simplicity:

$$\begin{aligned} Y &= A \left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} N_i^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ AN_i &= \tilde{\phi}_i Y A^\eta g(\theta_i)^{-\eta} \\ \delta N_i &= \varphi \theta_i^{1-\alpha} (L_i - N_i) \end{aligned}$$

where g is an increasing and concave function of sectoral labor market tightness. These $2K + 1$ equations determine equilibrium output Y , sectoral employment N_i , and sectoral labor market

tightness θ_i in terms of the labor force distribution L_i taken as given and constant, sectoral shocks $\tilde{\phi}_i$, and an aggregate productivity shock A that traces out the Beveridge curve.

To prove that the natural rate of unemployment must increase, we normalize $A = 1$ and eliminate θ_i :

$$\begin{aligned} Y &= \left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} N_i^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ N_i &= \tilde{\phi}_i Y g \left(\left(\frac{\delta}{\varphi} \frac{N_i}{L_i - N_i} \right)^{1/(1-\alpha)} \right)^{-\eta} \end{aligned}$$

Eliminating Y , rearranging and summing across sectors, we have:

$$\left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} n_i^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} = \sum_{i=1}^K n_i g \left(x_i^{-\frac{1}{1-\alpha}} \right)^{\eta} = \sum_{i=1}^K n_i h(x_i) \quad (\text{C.24})$$

where $x_i = \frac{L_i/N - n_i}{n_i}$, an expression of aggregate employment N , sectoral employment shares n_i , and the distribution of the labor force L_i . The function h is defined in terms of the function g : $h = g \left(x^{-\frac{1}{1-\alpha}} \right)^{\eta}$ where g is given by:

$$g(\theta) = z + \frac{1}{1-\nu} \frac{\kappa}{\varphi} \theta^{\alpha} (1 - \beta(1 - \delta)) + \frac{\nu}{1-\nu} \kappa \beta \theta$$

It is readily shown that h is a decreasing and strictly convex function for standard assumptions on the matching function parameters which ensure the coefficients on the polynomial terms of θ in the function g are positive.

Let N_0 be the level of employment when $L_i = \tilde{\phi}_i$ and let N_1 be the level of employment when $L_i \neq \tilde{\phi}_i$. When $L_i = \tilde{\phi}_i$, labor market tightness θ_i is equalized across sectors and the left-hand side of equation (66) is equal to unity. Therefore, N_0 is implicitly defined by the function h :

$$1 = h \left(\frac{1}{N_0} - 1 \right)$$

Using our definitions of x_i and the fact that h is a convex function, we have:

$$\begin{aligned} \left\{ \sum_{i=1}^K \tilde{\phi}_i^{\frac{1}{\eta}} n_i^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} &= \sum_{i=1}^K n_i h(x_i) \\ &> h\left(\sum_{i=1}^K n_i x_i\right) \\ &= h\left(\frac{1}{N_1} - 1\right) \end{aligned}$$

where the first strict inequality follows from the strict convexity of h and the fact for some sectors i and j , it must be the case that $x_i \neq x_j$. The second equality follows from the definition of x_i .

The left-hand side of equation (66) is bounded above by 1. This can be shown by considering the cases of $\eta < 1$ and $\eta > 1$ separately, and applying the properties of convex or concave functions. If $\eta < 1$, then:

$$\begin{aligned} \sum_{i=1}^K n_i \left(\frac{\tilde{\phi}_i}{n_i}\right)^{1/\eta} &\geq 1 \\ \Rightarrow \left(\sum_{i=1}^K n_i \left(\frac{\tilde{\phi}_i}{n_i}\right)^{1/\eta}\right)^{\frac{\eta}{\eta-1}} &\leq 1 \end{aligned}$$

and vice versa in the case of $\eta > 1$.

Thus, we conclude that:

$$\begin{aligned} h\left(\frac{1}{N_0} - 1\right) &> h\left(\frac{1}{N_1} - 1\right) \\ \Rightarrow \frac{1}{N_0} - 1 &< \frac{1}{N_1} - 1 \\ \Rightarrow N_0 &> N_1 \end{aligned}$$

and the natural rate of unemployment must rise in the case that $L_i \neq \tilde{\phi}_i$ as required.

It can be readily verified that when $L_i = \tilde{\phi}_i$, then $N_i = \tilde{\phi}_i N$ with aggregate tightness and

employment implicitly defined by the following equations:

$$\begin{aligned} A &= g(\theta) \\ N &= \frac{\varphi\theta^{1-\alpha}}{\varphi\theta^{1-\alpha} + \delta} \end{aligned}$$

Since sectoral shocks do not appear in these equations, aggregate shocks keep tightness equalized across sectors even if reallocation is costly. To show that vacancies rise under a sector-specific shock, we derive an expression for aggregate vacancies in terms of aggregate employment and sectoral tightness:

$$V = \frac{\delta}{\varphi} N \sum_{i=1}^K (\theta_i)^\alpha \frac{N_i}{N}$$

In the case of aggregate shocks, tightness is equalized across sectors and given by the expression:

$$\theta = \left(\frac{\delta}{\varphi} \frac{N}{1-N} \right)^{\frac{1}{1-\alpha}}$$

Let $N' = N(1, \tilde{\phi}'_i)$ and $\bar{N} = N(A', \tilde{\phi}_i)$. Define the share of employment in a given sector under the sectoral shock as $n_i = N'_i/N'$ and ratio of labor to employment as $l_i = L_i/N' = \tilde{\phi}_i/N'$. Then, sectoral tightness is given by:

$$\begin{aligned} \theta_i &= \left(\frac{\delta}{\varphi} \frac{n_i}{l_i - n_i} \right)^{\frac{1}{1-\alpha}} \\ &= \left(\frac{\varphi}{\delta} \frac{l_i - n_i}{n_i} \right)^{-\frac{1}{1-\alpha}} \end{aligned}$$

Define $V' = V(1, \tilde{\phi}'_i)$ and $\bar{V} = V(A', \tilde{\phi}_i)$. Then:

$$\begin{aligned} V' &= \frac{\delta}{\varphi} N' \sum_{i=1}^K \left(\frac{\varphi}{\delta} \frac{l_i - n_i}{n_i} \right)^{-\frac{\alpha}{1-\alpha}} n_i > \frac{\delta}{\varphi} N' \left(\sum_{i=1}^K \frac{\varphi}{\delta} \frac{l_i - n_i}{n_i} n_i \right)^{-\frac{\alpha}{1-\alpha}} \\ &= \frac{\delta}{\varphi} N' \left(\frac{\delta}{\varphi} \frac{N'}{1-N'} \right)^{\frac{\alpha}{1-\alpha}} = \bar{V} \end{aligned}$$

where the first inequality follows from the strict convexity of the inverse labor market tightness and the last equality follows from the fact that $N' = \bar{N}$, which follows from the assumption that unemployment is equalized. ■

C.2.4 Proof of Proposition 3.7

Under costless reallocation, the equations determining the steady state Beveridge curve in a two-sector version of the model can be summarized by the following equations:

$$\mu^{-1} = \left\{ \tilde{\phi} g_A(\theta)^{1-\eta} + (1 - \tilde{\phi}) g_B(\theta)^{1-\eta} \right\}^{1/1-\eta} \quad (\text{C.25})$$

$$\frac{n_A}{1 - n_A} = \frac{\tilde{\phi}}{1 - \tilde{\phi}} \left(\frac{g_A(\theta)}{g_B(\theta)} \right)^{-\eta} \quad (\text{C.26})$$

$$1 = N \left(1 + \theta^{\alpha-1} \left(n_A \frac{\delta_A}{\varphi_A} + (1 - n_A) \frac{\delta_B}{\varphi_B} \right) \right) \quad (\text{C.27})$$

where $g_i(\theta) = z + \frac{1}{1-\nu} \frac{\kappa}{\varphi_i} \theta^\alpha (1 - \beta(1 - \delta_i))$. Hiring costs are increasing and strictly concave in θ . Moreover, the condition $\varphi_A > \varphi_B$ (or $\delta_A < \delta_B$) is a sufficient condition for $g_A \leq g_B$ for $\theta \geq 0$ with $g_A = g_B$ at $\theta = 0$. The ratio $g_A/g_B = 1$ at $\theta = 0$ and $\lim_{\theta \rightarrow \infty} g_A/g_B = \varphi_B/\varphi_A < 1$.

We must first show that, if $\tilde{\phi}' > \tilde{\phi}$, then $N' > N$. For equation (67), if $\tilde{\phi} \rightarrow \tilde{\phi}'$, then holding constant θ , the RHS of equation (67) falls. Thus $\theta' > \theta$. If we show that $n'_A > n_A$, then it must be the case that $N' > N$. Since g_A/g_B is monotonic and decreasing, if $\theta \rightarrow \theta'$, then $n'_A > n_A$ using equation (68) since the ratio g_A/g_B is less than 1 and falling and $\tilde{\phi}' > \tilde{\phi}$. Thus, we conclude that $N' > N$ and the natural rate of unemployment falls.

To show that the Beveridge curve shifts, we consider the implied level of vacancies for a markup shock and a sector-specific shock that deliver the same level of employment: $N(\tilde{\phi}', 1) = N(\tilde{\phi}, \mu')$. Observe from equation (69), if both shocks deliver the same level of employment, then:

$$\theta^{\alpha-1} \left(n_A \frac{\delta_A}{\varphi_A} + (1 - n_A) \frac{\delta_B}{\varphi_B} \right) = \bar{\theta}^{\alpha-1} \left(\bar{n}_A \frac{\delta_A}{\varphi_A} + (1 - \bar{n}_A) \frac{\delta_B}{\varphi_B} \right)$$

where the bar superscript signifies the sector-specific shock. It cannot be the case that $\theta = \bar{\theta}$, since equation (68) would not be satisfied. If $\theta < \bar{\theta}$, then $n_A > \bar{n}_A$. Taking ratios of equation (68), it must be the case that:

$$\begin{aligned} \left(\frac{g_A/g_B}{\bar{g}_A/\bar{g}_B} \right)^{-\eta} &< 1 \\ \Rightarrow \left(\frac{\bar{g}_A/\bar{g}_B}{g_A/g_B} \right)^{\eta} &> 1 \end{aligned}$$

which is a contradiction since the ratio g_A/g_B is decreasing in tightness. Therefore, it must be the case that $\theta > \bar{\theta}$ and $n_A < \bar{n}_A$. Under costless reallocation, vacancies simply $V = \theta(1 - N)$. Since $N = \bar{N}$, but $\theta > \bar{\theta}$, $V(\tilde{\phi}', 1) < V(\tilde{\phi}, \mu')$ and the Beveridge curve shifts inward. ■

C.2.5 Proof of Proposition 3.8

Holding constant $\{A_i, \phi_i\}_{i=1}^K$, we define $V(G, z)$ and $U(G, z)$ as aggregate vacancies and unemployment for given values of the government spending shock G and the common reservation wage z . We wish to show that $\forall \bar{G} > 0$, there exists a \bar{z} such that $V(\bar{G}, z_0) = V(1, \bar{z})$ and $U(\bar{G}, z_0) = U(1, \bar{z})$.

The government spending shock only affect vacancies and unemployment via the real marginal cost. Let $\bar{\mu}^{-1} = \frac{P_f}{P}(\bar{G})$. Relative prices are equalized in steady state since sectoral productivities and hiring costs are equalized. Therefore, the surplus in each sector is the same:

$$\begin{aligned} \frac{P_i}{\bar{P}} A_i &= z + g(\theta_i) \\ \mu^{-1} A - z &= g(\theta) \end{aligned}$$

For each sector $\theta = g(\mu^{-1} A - z)^{-1}$ where g is an increasing and concave function. If $\mu = \bar{\mu}$, then $\bar{z} = A - (\bar{\mu}^{-1} A - z)$ ensures the same labor market tightness in each sector when $\mu^{-1} = 1$, which is the case of no government spending shocks, and tightness is invariant to combinations of μ and z . As a result, aggregate vacancies and unemployment are equalized as required.

If labor reallocation is costless, then Proposition 3 applies. However, in the absence of

labor reallocation, sectoral shocks will shift the same Beveridge curve as shown in Proposition 6. Therefore, the fact that two aggregate shocks, government spending shocks and reservation wage shocks, trace out the same Beveridge curve does not follow because no shocks shift the Beveridge curve. ■

C.2.6 Proof of Proposition 3.9

We compute the balanced growth path of the multisector model with costless reallocation. We proceed by stating the equilibrium conditions and solving the model. Under the assumptions in Proposition 9, the model equilibrium conditions given by (46) - (54) and (60) or (61) - (62) can be simplified as follows:

$$\begin{aligned}
Y_t &= A_t N_t \left\{ \sum_{i=1}^K \tilde{\phi}_{it}^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\} \\
1 &= \left\{ \sum_{i=1}^K \tilde{\phi}_{it} \left(\frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \\
A_t N_{it} &= \tilde{\phi}_{it} Y_t \left(\frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \\
\frac{\tilde{P}_{it}}{P_t} A_t &= W_t + \frac{\kappa_t}{\varphi} \theta_t^\alpha (1 - \beta (1 - \delta)) \\
W_t &= v'(N_t) C_t + \frac{\nu}{1-\nu} \frac{\kappa_t}{\varphi} \theta_t^\alpha (1 - \beta (1 - \delta - \varphi \theta_t^{1-\alpha})) \\
Y_t &= C_t + \kappa_t V_t \\
\theta_t &= V_t / U_t \\
1 &= N_t + U_t
\end{aligned}$$

where $\tilde{P}_{it}/P_t = P_{it} A_{it}/A_t$ is the productivity-adjusted relative price of sector i 's output and $A_t = \left\{ \sum_{i=1}^K \phi_{it} A_{it}^{\eta-1} \right\}^{\frac{1}{\eta-1}}$

Since hiring costs are equalized, it must be the case that $\tilde{P}_{it}/P_t = 1$ and $N_{it}/N_t = \tilde{\phi}_{it}$. Combining the vacancy-posting condition, Nash-bargained wages and the assumption for

vacancy costs, we obtain the following:

$$\begin{aligned} A_t &= v'(N_t) C_t \left(1 + \frac{\chi}{\varphi} h(\theta_t)\right) \\ \Rightarrow 1 &= v'(N_t) \frac{N_t}{1 + \chi V_t v'(N_t)} \left(1 + \frac{\chi}{\varphi} h(\theta_t)\right) \end{aligned}$$

This vacancy posting condition combined with labor market clearing and the definition of market tightness jointly determine labor market variables N , U , V , θ where the time subscript is dropped since none of these variables is a function of exogenous variables that change over time: namely A_t and L_t .

Since growth in the labor force is modeled as a net addition of new households, the labor market variables have a per capita interpretation and each variable grows at the rate $g_l = \Delta L/L$. Thus, the unemployment rate, vacancy rate, and employment rate are constant. It is straightforward to compute the growth rates of per household output, consumption and wages given the resulting expressions:

$$\begin{aligned} Y_t &= A_t N \\ Y_t &= C_t + \kappa_t V \\ W_t &= v'(N) C_t + \frac{\nu}{1 - \nu} \frac{\kappa_t}{\varphi} \theta_t^\alpha (1 - \beta (1 - \delta - \varphi \theta^{1-\alpha})) \end{aligned}$$

with $g_y = g_c = g_w = g_A$.

However, these growth rates are constant only in the special case when sectoral productivities are equalized and grow at the same rates. Since the expression for aggregate productivity is a sum, different growth rates across sectors will generally change the growth rate of aggregate productivity. Moreover, changes in preference shares over time will also alter productivity growth rates. If all structural change is driven by changes in product shares, all per capita growth rates are zero and all aggregates grow only with the labor force. Employment shares will mirror their productivity-adjusted product shares along the growth path.

More generally, if sectoral TFP growth rates differ, then output, consumption and wage

growth will be asymptotically constant. If $\eta > 1$, then $\lim_{t \rightarrow \infty} \Delta A/A = \gamma_{max}$ where γ_{max} is the TFP growth rate of the fastest growing sector. Alternatively, if $\eta < 1$, then the opposite holds and TFP growth converges to the growth rate of the slowest growing sectors. These results are analogous to the asymptotic growth rates computed in Acemoglu and Guerrieri (2008). If $\eta = 1$, the TFP aggregator is Cobb-Douglas and the aggregate TFP growth rate is a weighted average of each sector's TFP growth rate. ■

C.3 Calibration and Model-Based Measures

C.3.1 Structural Factor Analysis

To a log-linear approximation, sectoral employment can be expressed by solving the equations that determine the steady state Beveridge curve in our model (shown at the beginning of the appendix):

$$Mn_t = Ha_t = H(\Phi z_t + \epsilon_t)$$

where $n_t = (n_{1,t}, \dots, n_{K,t})'$ is the vector of log-linearized sectoral employment expressed in terms of the exogenous variables, the vector $a_t = (a_{1,t}, \dots, a_{K,t})'$ of sectoral productivity shocks. As argued, the exogenous sectoral productivity process can be decomposed into its first principal component and a vector of sectoral shocks $\epsilon_t = (\epsilon_{1,t}, \dots, \epsilon_{K,t})'$ with $cov(z_t, \epsilon_{i,t}) = 0$ for $\forall i = 1, 2, \dots, K$.

The matrix M is determined by the model parameters and the steady state values of labor market variables. To compute this matrix, it is necessary to choose parameters and solve for the model steady state. We calibrate an 11-sector version of our model where the sectors conform to the NAICS supersectors for which there is readily available data on employment, unemployment and vacancies. Our reduced form sector-specific shock index was computed using 13 NAICS sectors, but we use only 11 sectors since retail trade, wholesale trade, transportation and utilities are combined into a single sector in the data on unemployment and vacancies from the CPS and JOLTs respectively.

To calibrate the 11 sector version of the model, some parameters are chosen directly while some parameters are chosen to match targets. As in the calibrations shown earlier, the household's discount rate β , matching function elasticity α , and bargaining power ν are all set to the values described in Section 3.5.1. Separation rates for the 11 sectors are set to match the 2000-2006 average in the JOLTs data. We chose matching function efficiencies φ_i , CES product shares ϕ_i , reservation wage z , and the vacancy posting cost κ to match the following targets: the distribution of vacancies V_i/V , the distribution of employment N_i/N , an unemployment rate $U/L = 5\%$, a vacancy rate $V/L = 2.5\%$, and a share-weighted accounting surplus of 10%. Vacancy shares and employment shares are set using 2000-2006 averages from the JOLTs and payroll survey respectively. Initial labor market tightness is equalized across sectors so that unemployment shares match vacancy shares. The table below summarizes the calibration targets, parameters, and components of the matrix M that is used to rotate the sectoral employment data. We consider two possible values for the elasticity of substitution η , with $\eta = 0.5$ and $\eta = 2$. Table C.1 summarizes the calibration for the case of complementary goods:

When goods are substitutes the product shares, output shares, and diagonal elements of M are changed. For brevity, the employment shares, vacancy shares, separation rates, and matching function efficiencies are omitted from this table as they are the same as in Table C.1. These new steady state values are summarized in Table C.2.

C.3.2 Relation of Sector-Specific Shock Index and the Beveridge Curve

Consider a steady state where $\bar{\theta}_i = \bar{\theta}_h$ for all sectors $i, h = 1, 2, \dots, K$. In the absence of labor market mismatch, it follows that unemployment shares and vacancy shares are equalized. To a log linear approximation, aggregate vacancies, unemployment and employment are given by the following equations:

$$v_t = \sum_{i=1}^K \frac{\bar{V}_i}{\bar{V}} v_{it}, \quad u_t = \sum_{i=1}^K \frac{\bar{U}_i}{\bar{U}} u_{it}, \quad n_t = \sum_{i=1}^K \frac{\bar{N}_i}{\bar{N}} n_{it}.$$

Table C.1: Calibration

Panel A							
Parameters		Value					
Discount rate	β	0.96 ^{1/12}					
Bargaining power	v	0					
Matching function elasticity	α	0.5					
Elasticity of substitution	η	0.5					
Vacancy posting cost	κ	3.26					
Reservation wage	z	0.9055					

Panel B							
Sectors	Employment Share	Vacancy Share	MFE ϕ	Separations δ	Product Share ϕ	Output Share γ	Diag M
Construction	5.3%	3.6%	2.47	6.2	5.2%	5.2%	1.92
Durables Goods	7.2%	4.6%	1.15	2.8	7.1%	7.0%	1.98
Education and Health Services	12.6%	17.7%	0.53	2.8	12.8%	13.1%	1.94
Financial Activities	6.1%	6.9%	0.66	2.8	6.1%	6.2%	1.95
Government	16.3%	10.7%	0.62	1.5	16.1%	15.9%	2.05
Information Services	2.5%	2.7%	0.77	3.1	2.5%	2.5%	1.94
Leisure and Hospitality	9.4%	12.4%	1.42	7	9.5%	9.6%	1.89
Nondurable Goods	4.3%	2.6%	1.37	3.1	4.2%	4.2%	1.97
Other Services	4.1%	4.3%	1	3.9	4.1%	4.1%	1.93
Professional and Business Services	12.6%	17.9%	1.07	5.7	12.8%	13.0%	1.89
Trade, Transportation and Utilities	19.6%	16.5%	1.35	4.2	19.5%	19.3%	1.93

Employment shares and vacancy shares are 2000-2006 averages from the CES and JOLTs respectively. Separation rates are 2000-2006 averages from JOLTs. Matching function efficiency is chosen to equate labor market tightness across sectors and target overall vacancy rate of 2.5%. Product shares are chosen to match the sectoral distribution of employment. Output shares and diagonal entries of the M matrix are model-implied values.

A log-linear approximation to the sectoral Beveridge curve provides the following expression:

$$n_{it} = \alpha u_{it} + (1 - \alpha) v_{it}$$

Using the expressions for aggregate vacancies and unemployment and the fact that $\bar{U}_i/\bar{U} = \bar{V}_i/\bar{V}$, we have:

$$\sum_{i=1}^K \frac{\bar{U}_i}{\bar{U}} n_{it} = \alpha u_t + (1 - \alpha) v_t$$

Adding and subtracting aggregate employment and rearranging, we obtain the following relation:

$$v_t = \frac{1}{1 - \alpha} \left\{ - \left(\alpha + \frac{\bar{U}}{\bar{N}} \right) u_t + \sum_{i=1}^K \left(\frac{\bar{U}_i}{\bar{U}} - \frac{\bar{N}_i}{\bar{N}} \right) n_{it} \right\}$$

Table C.2: New Parameters: $\eta = 2$

Panel A			
Parameters		Value	
Elasticity of substitution	η	2	
Vacancy posting cost	κ	3.26	
Reservation wage	z	0.9049	

Panel B			
Sectors	Product Share ϕ	Output Share γ	Diag M
Construction	5.0%	5.2%	4.67
Durables Goods	6.8%	7.0%	4.9
Education and Health Services	13.6%	13.1%	4.76
Financial Activities	6.3%	6.2%	4.8
Government	15.5%	15.9%	5.22
Information Services	2.6%	2.5%	4.77
Leisure and Hospitality	9.8%	9.6%	4.56
Nondurable Goods	4.0%	4.2%	4.87
Other Services	4.1%	4.1%	4.71
Professional and Business Services	13.5%	13.0%	4.58
Trade, Transportation and Utilities	19.0%	19.3%	4.73

Product shares, output shares and diagonal values of the M matrix recomputed using higher elasticity of substitution. Other calibration values unchanged from Table 11

It is worth noting that in our numerical calibration, the aggregate component of n_{it} approximately cancels out, and we are left with an expression in terms of the sectoral shocks:

$$v_t = \frac{1}{1 - \alpha} \left\{ - \left(\alpha + \frac{\bar{U}}{\bar{N}} \right) u_t + \sum_{i=1}^K \left(\frac{\bar{U}_i}{\bar{U}} - \frac{\bar{N}_i}{\bar{N}} \right) \epsilon_{it} \right\}$$

C.4 Collateral Constraint

Our result demonstrating an equivalence between sector-specific shocks and shocks to the borrowing rate in a model with a working capital constraint can be generalized to other types of financial shocks. A common shock considered in the literature is a Kiyotaki and Moore type shock to the value of collateral. We modify the problem of the intermediate goods producer to include a time-varying collateral constraint that limits the ability of the firm to borrow to

finance the wage bill and the cost of posting vacancies:

$$\begin{aligned} \Pi_{i,t}^{Int} &= \max_{\{V_{i,T}, N_{i,T}\}_{T=t}^{\infty}} \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \left[P_{iT} Y_{iT} - \left(1 + i_T^b\right) (W_{iT} N_{iT} - \kappa V_{iT} P_T) \right], \\ \text{s.t. } N_{i,t} &= (1 - \delta_i) N_{i,t-1} + q_{i,t} V_{i,t}, \\ Y_{it} &= A_t N_{it}, \\ \lambda_t \bar{K} &\geq W_{i,t} N_{i,t} + \kappa V_{i,t} P_t. \end{aligned}$$

Fluctuation in λ_t can represent a tightening of lending standards by financial institutions or a fall in the value of collateral like real estate or other forms of capital. For simplicity, we continue to assume that labor is the only variable factor of production and that constrained firms have some fixed endowment of capital. The vacancy posting condition in this setting is similar to the vacancy posting condition (3.21) and can be written as follows:

$$\frac{P_{i,t}}{P_t} \frac{A_t}{1 + \varphi_t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} - \mathbb{E}_t \left[Q_{t,t+1} (1 - \delta_i) \frac{\kappa}{q_{i,t+1}} \frac{1 + \varphi_{t+1}}{1 + \varphi_t} \right],$$

where φ_t is the Lagrange multiplier on the collateral constraint and replaces the interest rate on borrowed funds. In steady state, the Lagrange multiplier on the constraint enters as a sector-specific productivity shock for any sector that faces a working capital constraint. A decrease in the value of λ_t tightens the constraint and raises the Lagrange multiplier. Therefore, our choice of modeling the financial shock as an interest rate shock instead of a shock to collateral values has no qualitative effects on the behavior of firms.