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ESTIMATES OF RELIABILITY FOR THE NORMAL DISTRIBUTION

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1. INTRODUCTION

Let X be a normally $N(\mu, \sigma^2)$ distributed random variable, both μ and σ^2 unknown. Let c be a fixed real number. The probability

$$P(X > c) = \Phi\left(\frac{\mu - c}{\sigma}\right), \tag{1}$$

where Φ denotes the $N(0, 1)$ distribution function, is to be estimated from a random sample $X_1, \ldots, X_n$ from the parent population $N(\mu, \sigma^2)$.

Four different estimators will be studied: the minimum variance unbiased estimator R_1, the maximum likelihood estimator R_2, the Bayes estimator R_3 corresponding to a logarithmic a priori distribution, and the naive estimator R_4 given by the frequency of the event $\{X > c\}$.

Denoting as usual

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i, \quad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2,$$

we introduce the following statistics:

$$T = \frac{c - \bar{X}}{s}, \quad U_1 = \frac{\sqrt{(n)}}{n-1} T, \quad U_2 = \left(\frac{n}{n-1}\right) T, \quad U_3 = \sqrt{\left(\frac{n}{n+1}\right)} T.$$

The minimum variance unbiased estimator R_1 of (1) was found by Kolmogorov [4]; it may be expressed as

$$R_1 = \frac{1}{B(\frac{1}{2}, \frac{1}{2}(n-2))} \int_{U_1}^{1} (1 - u^2)^{(n-4)/2} \, du \quad \text{if} \quad -1 < U_1 < 1, \tag{2}$$

$$= 1 \quad \text{if} \quad U_1 \leqq -1,$$

$$= 0 \quad \text{if} \quad U_1 \geqq 1.$$

The maximum likelihood estimator R_2 is obtained by utilizing the invariance principle of the maximum likelihood estimates, which results in

(3) $$R_2 = \Phi(-U_2)\,.$$

Suppose now that μ and σ^2 are random variables; denoting $h = \sigma^{-2}$, assume that the a priori distribution of (μ, h) is given by the improper density function

$$g(\mu, h) = \frac{1}{h}\,, \quad -\infty < \mu < \infty\,, \quad h > 0\,.$$

Then the density of the a posteriori distribution is

$$g(\mu, h \mid x_1, \ldots, x_n) = \sqrt{\left(\frac{n}{\pi}\right)} \frac{1}{2^{n/2}\Gamma\left(\frac{1}{2}(n-1)\right)} \times$$

$$\times\ S^{n-1}\, h^{(n/2)-1} \exp\left\{-\frac{h}{2}\left[n(\mu - \bar{x}^2) + S^2\right]\right\},$$

where $S^2 = (n-1)\, s^2$. The Bayes estimator of (1), obtained as the expectation of $\Phi(\sqrt{(h)}\,(\mu - c))$ with respect to the above a posteriori distribution, is

(4) $$R_3 = \int_{U_3}^{\infty} w_{n-1}(u)\, \mathrm{d}u$$

where w_{n-1} denotes the density function of Student's t on $n - 1$ degrees of freedom.

Let Z_i be the indicator of the event $\{X_i > c\}$, i.e.,

$$Z_i = 1 \quad \text{if} \quad X_i > c\,,$$

$$= 0 \quad \text{otherwise.}$$

Then the naive estimator of (1) is

(5) $$R_4 = \frac{1}{n} \sum Z_i\,.$$

For further purposes, it is useful to express the estimates R_1, R_2, R_3 in an alternative form. After an appropriate transformation and some calculations we can write

(6) $$R_i = F_i(T)\,, \quad i = 1, 2, 3\,.$$

Here

(7) $$F_1(z) = \tfrac{1}{2} - B_1 \int_0^z (1 - k_1 t^2)^{(n-4)/2}\, \mathrm{d}t$$

$$\text{if} \quad -(n-1)/\sqrt{(n)} < z < (n-1)/\sqrt{(n)}\,,$$

$$= 1 \quad \text{if} \quad z \leqq -(n-1)/\sqrt{(n)}\,,$$

$$= 0 \quad \text{if} \quad z \geqq (n-1)/\sqrt{(n)}$$

where

$$B_1 = \frac{1}{B[\frac{1}{2}, \frac{1}{2}(n-2)]} \frac{\sqrt{(n)}}{n-1}, \quad k_1 = \frac{n}{(n-1)^2};$$

(8) $$F_2(z) = \tfrac{1}{2} - k_2 \int_0^z \varphi(k_2 t)\,\mathrm{d}t$$

where φ is the $N(0, 1)$ density function and $k_2 = \sqrt{(n/(n-1))}$;

(9) $$F_3(z) = \tfrac{1}{2} - B_3 \int_0^z (1 + k_3 t^2)^{-n/2}\,\mathrm{d}t,$$

where

$$B_3 = \frac{1}{B[\frac{1}{2}, \frac{1}{2}(n-1)]} \sqrt{\left(\frac{n}{(n-1)(n+1)}\right)}, \quad k_3 = \frac{n}{(n-1)(n+1)}.$$

Although the F_i's and other symbols introduced depend on n as well, we have suppressed the subscript n in our notation.

2. ASYMPTOTIC PROPERTIES OF THE ESTIMATES

Let us denote

$$\theta = \frac{c - \mu}{\sigma}.$$

We first prove the asymptotic normality of the investigated estimates.

Theorem 1. *We have*

$$\sqrt{(n)}\,(R_i - \Phi(-\theta)) \xrightarrow{L} N(0, \varphi^2(\theta)\,(1 + \theta^2/2)), \quad i = 1, 2, 3,$$

$$\sqrt{(n)}\,(R_4 - \Phi(-\theta)) \xrightarrow{L} N(0, \Phi^2(\theta)\,(1 - \Phi(\theta))).$$

Proof. Without loss of generality we may suppose that $|\theta| < (n-1)/\sqrt{(n)}$ so that all the functions F_i, $i = 1, 2, 3$ admit continuous derivatives f_i in some neighbourhood of θ, where

(10) $$f_1(z) = -B_1(1 - k_1 z^2)^{(n-4)/2},$$

(11) $$f_2(z) = -k_2\,\varphi(k_2 z),$$

(12) $$f_3(z) = -B_3(1 + k_3 z^2)^{-n/2}.$$

Notice that the f_i's depend on n again, although not indicated by a subscript, and that

(13) $$\lim_{n\to\infty} f_i(z) = \varphi(z)$$

for $i = 1, 2, 3$ and all z. Let us write

(14) $$\sqrt{(n)}\,(T - \theta) = \frac{1}{s}\left[\sqrt{(n)}\,(\mu - \bar{X}) + \theta\sigma\sqrt{(n)}\,(1 - s^2/\sigma^2)(1 + s/\sigma)^{-1}\right].$$

The limiting distribution of $\sqrt{(n)}\,(1 - s^2/\sigma^2)$ is $N(0,2)$; further, $\theta\sigma(1 + s/\sigma)^{-1} \xrightarrow{P}$ $\xrightarrow{P} \frac{1}{2}\theta\sigma$, hence by [(x), 2c. 4] in [5]

$$\theta\sigma\sqrt{(n)}\,(1 - s^2/\sigma^2)(1 + s/\sigma)^{-1} \xrightarrow{L} N(0, \tfrac{1}{2}\theta^2\sigma^2).$$

Obviously, $\sqrt{(n)}\,(\mu - \bar{X}) \xrightarrow{L} N(0, \sigma^2)$, and $s^{-1} \xrightarrow{P} \sigma^{-1}$; hence

(15) $$\sqrt{(n)}\,(T - \theta) \xrightarrow{L} N(0, 1 + \tfrac{1}{2}\theta^2).$$

Finally, for $i = 1, 3$ we have

$$\sqrt{(n)}\,[F_i(\theta) - \Phi(-\theta)] \to 0 \quad \text{as} \quad n \to \infty.$$

Taking into account (13), (15), and utilizing (6a. 2.5) in [5] we obtain the assertion of the theorem for $i = 1, 3$. The assertion for R_2 is an immediate consequence of (6a. 2.1) in [5] and the case of R_4 is trivial. Q.E.D.

Remark 1. From the above theorem it follows that the estimates R_2 and R_3 are (weakly) asymptotically efficient, i.e. the variances of their asymptotic distribution are the same as the variance of the asymptotic distribution of the minimum variance unbiased estimate. Later we shall see that R_2 and R_3 are asymptotically efficient in the usual sense.

The estimate R_4 is not weakly asymptotically efficient as follows from the inequality

(16) $$\frac{\varphi^2(\theta)\,(1 + \frac{1}{2}\theta^2)}{\Phi(\theta)\,(1 - \Phi(\theta))} < 1,$$

valid for all θ. The inequality (16) may be verified by standard calculus. We omit the proof here.

Let us denote

$$P_k(z) = \int_0^z x^k\,\varphi(x)\,dx, \quad k = 0, 1, 2, \ldots.$$

Note that the estimated probability (1) is

(17) $$\Phi(-\theta) = \tfrac{1}{2} - P_0(\theta).$$

In the sequel, we will use the formulas

(18) $$P_2(z) = P_0(z) - z\,\varphi(z)\,,$$

(19) $$P_4(z) = 3P_0(z) - 3z\,\varphi(z) - z^3\,\varphi(z)$$

which may be deduced by integrating by parts.

Theorem 2. *For the expected values, variances, and expected squared errors of the estimates R_1, R_2, R_3, we have*

$$ER_1 = \Phi(-\theta)\,,$$

$$ER_2 = \Phi(-\theta) + \frac{1}{4n}\,\theta\,\varphi(\theta)\,(\theta^2 - 3) + O(n^{-2})\,,$$

$$ER_3 = \Phi(-\theta) + \frac{1}{2n}\,\theta\,\varphi(\theta)\,(\theta^2 + 1) + O(n^{-2})\,,$$

$$\operatorname{var} R_1 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{8n^2}\,\varphi^2(\theta)\,(4 + \theta^2 - 2\theta^4 + \theta^6) + O(n^{-5/2})\,,$$

$$\operatorname{var} R_2 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{8n^2}\,\varphi^2(\theta)\,(16 - 17\theta^2 - 10\theta^4 + 3\theta^6) + O(n^{-5/2})\,,$$

$$\operatorname{var} R_3 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{8n^2}\,\varphi^2(\theta)\,(-4 - 19\theta^2 - 2\theta^4 + 5\theta^6) + O(n^{-5/2})\,,$$

$$E(R_1 - \Phi(-\theta))^2 = \operatorname{var} R_1\,,$$

$$E(R_2 - \Phi(-\theta))^2 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{16n^2}\,\varphi^2(\theta)\,(32 - 25\theta^2 - 26\theta^4 + 7\theta^6) + O(n^{-5/2})\,,$$

$$E(R_3 - \Phi(-\theta))^2 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{16n^2}\,\varphi^2(\theta)\,(-8 - 34\theta^2 + 4\theta^4 + 14\theta^6) + O(n^{-5/2})\,.$$

Proof. We make use of Theorem 1 in [3] where we put $q = 3$. First we present the expansions of moments and covariances needed in the mentioned theorem:

(20) $$E(T - \theta) = \frac{3}{4n}\,\theta + O(n^{-2})$$

$$E(T - \theta)^2 = \frac{1}{n}\,(1 + \tfrac{1}{2}\theta^2) + O(n^{-2})$$

$$\operatorname{cov}[(T-\theta),\ (T-\theta)] = \frac{1}{n}(1+\tfrac{1}{2}\theta^2) + \frac{1}{n^2}(2+\tfrac{19}{8}\theta^2) + O(n^{-3})$$

$$\operatorname{cov}[(T-\theta),\ (T-\theta)^2] = \frac{1}{n^2}(\tfrac{9}{2}\theta + 2\theta^3) + O(n^{-3})$$

$$\operatorname{cov}[(T-\theta),\ (T-\theta)^3] = \frac{1}{n^2}(3+3\theta^2+\tfrac{3}{4}\theta^4) + O(n^{-3}).$$

All the higher moments and covariances are $O(n^{-2})$ and $O(n^{-3})$, respectively. Because of the boundedness of both F_i and their derivatives such higher terms may be omitted. Thus in our case Theorem 1 from [3] takes the form

$$ER_i = F_i(\theta) + f_i(\theta)\,E(T-\theta) + \tfrac{1}{2}f_i'(\theta)\,E(T-\theta)^2 + O(n^{-2}), \tag{21}$$

$$\begin{aligned} \operatorname{var} R_i = {} & [f_i(\theta)]^2 \operatorname{cov}[(T-\theta),(T-\theta)] + \\ & + f_i(\theta)f_i'(\theta)\operatorname{cov}[(T-\theta),(T-\theta)^2] + \\ & + \tfrac{1}{3}f_i(\theta)f''_i(\theta)\operatorname{cov}[(T-\theta),(T-\theta)^3] + \\ & + \tfrac{1}{4}[f_i'(\theta)]^2\operatorname{cov}[(T-\theta)^2,(T-\theta)^2] + O(n^{-5/2}). \end{aligned} \tag{22}$$

Formulas (21) and (22) together with (20) imply that in the expansion of ER_i F_i appears up to $O(n^{-2})$, f_i and f_i' up to $O(n^{-1})$ and in that of var R_i $[f_i(\theta)]^2$ appears up to $O(n^{-2})$, $f_i(\theta)f_i'(\theta)$, $f_i(\theta)f_i''(\theta)$, and $[f_i'(\theta)]^2$ up to $O(n^{-1})$. Since $f_i(\theta) = -\varphi(\theta) + O(n^{-1})$ and $f_i'(\theta) = \theta\,\varphi(\theta) + O(n^{-1})$, only the leading term in (21) actually depends on i whereas the other terms coincide for $i = 2, 3$. Similarly, in the expansion (22) only the leading term actually depends on i whereas the other terms coincide for all i. With S.T. and C.T. standing for specific and common terms respectively, we have

$$\begin{aligned} \text{S. T. } ER_i &= F_i(\theta) \\ \text{C. T. } ER_i &= \text{sum of remaining terms in (21)} \\ \text{S. T. var } R_i &= [f_i(\theta)]^2 \operatorname{cov}[(T-\theta),(T-\theta)] \\ \text{C. T. var } R_i &= \text{sum of remaining terms in (22).} \end{aligned}$$

Now

$$ER_i = \text{S. T. } ER_i + \text{C. T. } ER_i \tag{23}$$

$$\operatorname{var} R_i = \text{S. T. var } R_i + \text{C. T. var } R_i. \tag{24}$$

We have

$$\text{C. T. } ER_i = \frac{1}{4n}\theta\,\varphi(\theta)(\theta^2-1) + O(n^{-2}). \tag{25}$$

Using (18) and (19), we obtain after some algebra

$$\text{S. T. } ER_2 = \tfrac{1}{2} - P_0(\theta) - \frac{1}{2n}\,\theta\,\varphi(\theta) + O\,(n^{-2})\,.$$

Let us note that

$$(26) \qquad B_3 = (2\pi)^{-1/2}\left(1 - \frac{3}{4n}\right) + O(n^{-2})\,.$$

Then we calculate

$$\text{S. T. } ER_3 = \tfrac{1}{2} - P_0(\theta) + \frac{1}{4n}\,\theta\,\varphi(\theta)\,(\theta^2 + 3) + O(n^{-2})\,.$$

This, together with (25) gives the expressions in the theorem. Analogously we continue with the variance. The common terms of var R_i are

$$(27) \qquad \text{C. T. var } R_i = \frac{1}{n^2}\,\varphi^2(\theta)\,(-1 - 4\theta^2 - \tfrac{3}{4}\theta^4 + \tfrac{3}{8}\theta^6) + O(n^{-5/2})\,.$$

The S. T. var R_1 may be directly calculated utilizing the formula

$$(28) \qquad B_1 = (2\pi)^{-1/2}\left(1 - \frac{1}{4n}\right) + O(n^{-2})\,.$$

Thus

$$\text{S. T. var } R_1 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{n^2}\,\varphi^2(\theta)\,(\tfrac{3}{2} + \tfrac{33}{8}\theta^2 + \tfrac{1}{2}\theta^4 + \tfrac{1}{4}\theta^6) + O(n^{-5/2})$$

which together with (27) gives the desired formula. Further,

$$\text{S. T. var } R_2 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{n^2}\,\varphi^2(\theta)\,(3 + \tfrac{15}{8}\theta^2 - \tfrac{1}{2}\theta^4) + O(n^{-5/2})$$

and using (26) again,

$$\text{S. T. var } R_3 = \frac{1}{n}\,\varphi^2(\theta)\,(1 + \tfrac{1}{2}\theta^2) + \frac{1}{n^2}\,\varphi^2(\theta)\,(\tfrac{1}{2} + \tfrac{13}{8}\theta^2 + \tfrac{1}{2}\theta^4 + \tfrac{1}{4}\theta^6) + O(n^{-5/2})\,.$$

From the last expressions and (27) the assertion immediately follows. The formulas for expected squared errors may be obtain by substituting the expansions ER_i and var R_i into the formula

$$E(R_i - \Phi(-\theta))^2 = [ER_i - \Phi(-\theta)]^2 + \text{var } R_i\,. \qquad \text{Q.E.D.}$$

Remark 2. Theorem 2 implies that the estimates R_2, R_3 are asymptotically efficient.

3. DEFICIENCY

To study the asymptotic behaviour of asymptotically efficient estimates more in details we use the concept of deficiency, see [1] or [2]. Let us denote the asymptotic

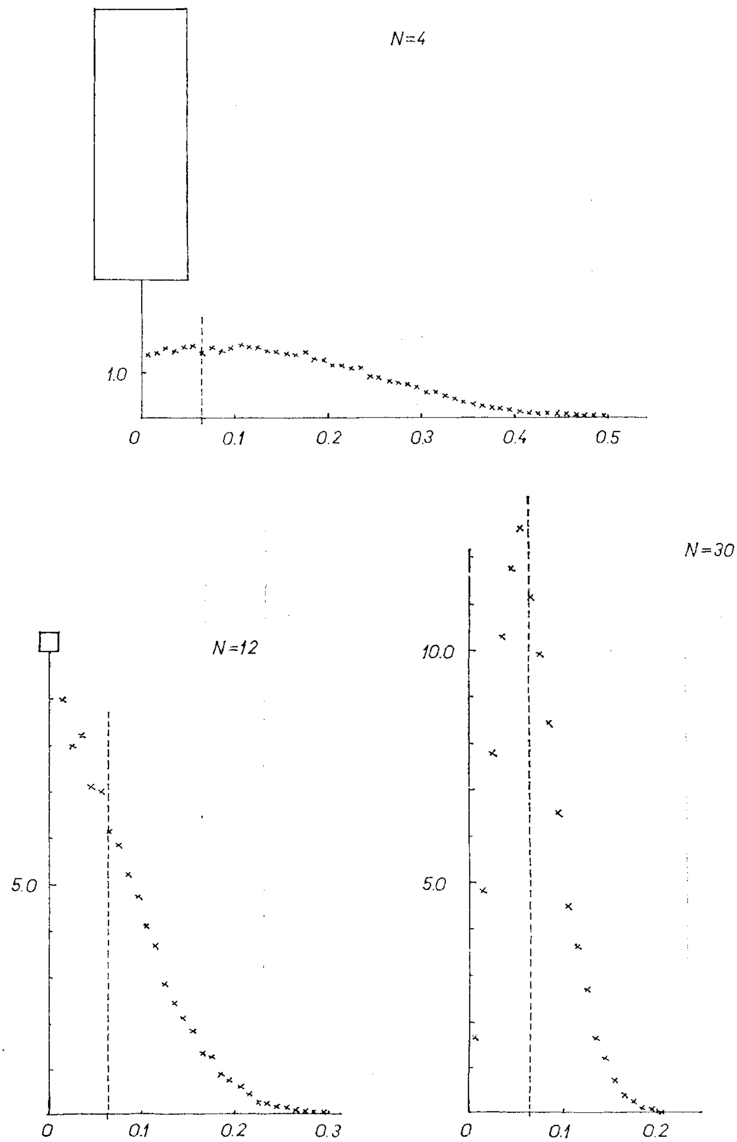

Fig. 1. Densities of R_1.

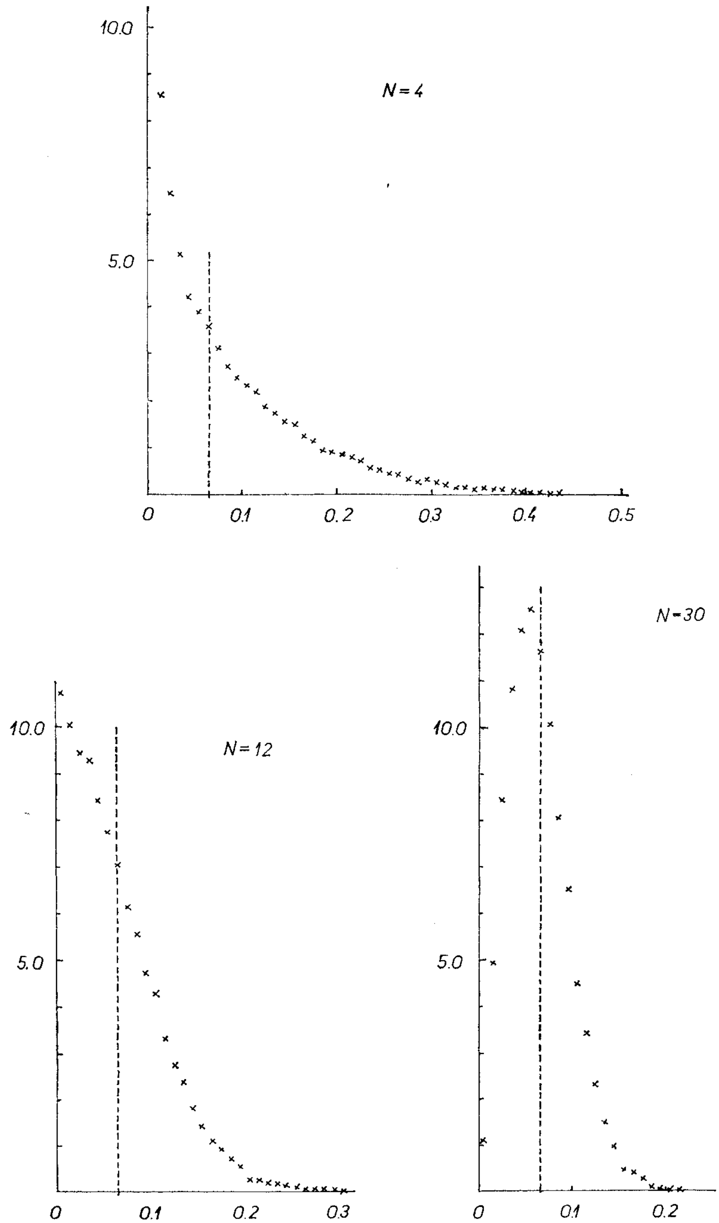

Fig. 2. Densities of R_2.

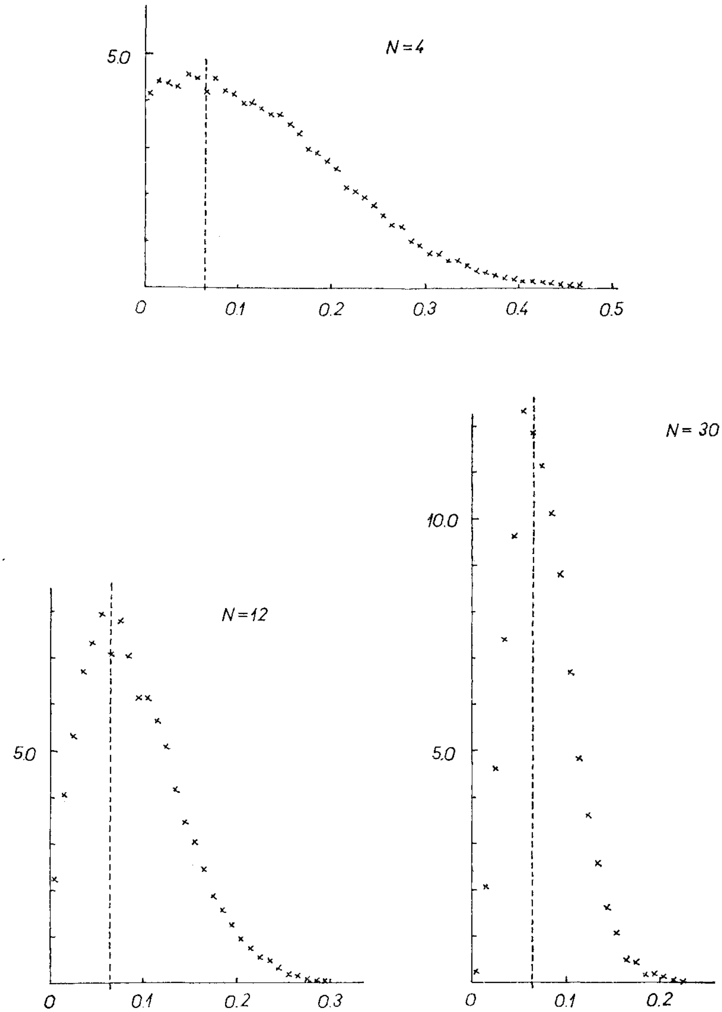

Fig. 3. Densities of R_3.

deficiency of R_i with respect to R_j by d_{ij}, $i, j = 1, 2, 3$. This means, roughly speaking, that to attain the same value of the expected squared errors of R_i and R_j we need d_{ij} additional observations for the calculation R_i.

Theorem 3. *Put* $\varkappa(\theta) = \left(1 + \frac{1}{2}\theta^2\right)^{-1}$. *Then*

$$d_{21} = \tfrac{1}{16}\,\varkappa(\theta)\left(24 - 27\theta^2 - 22\theta^4 + 5\theta^6\right),$$

$$d_{31} = \frac{1}{16}\varkappa(\theta)(-16 - 36\theta^2 + 8\theta^4 + 12\theta^6),$$

$$d_{32} = \frac{1}{16}\varkappa(\theta)(-40 - 9\theta^2 + 30\theta^4 + 7\theta^6)$$

hold.

Proof. The proof follows immediately from the formulas for $E(R_i - \Phi(-\theta))^2$ given in Theorem 2. Q.E.D.

Some numerical values of d_{ij} for various values θ are given in Table 1.

TABLE 1

Deficiencies

θ	0·0	0·5	1·0	1·5	2·0	2·5	3·0
d_{21}	1·50	0·89	−0·83	−2·68	−2·42	3·28	18·68
d_{31}	−1·00	−1·35	−1·33	2·36	15·33	45·47	102·91
d_{32}	−2·50	−2·24	−0·50	5·04	17·75	42·20	84·23

4. MONTE CARLO STUDY

In order to gain an idea about the distribution of R_1, R_2 and R_3 for small n some simulations were done. Necessary computations were made on the high-speed computer ICL 4–72 at the University Regional Computer Centre in Prague.

As a generator of random standard normal deviates the standard software generator based on the sum of 12 uniform random numbers was used. The latter were produced by a multiplicative congruential method. The integrals in (7) and (9) were calculated numerically using the Gaussian twelve-point formula. All calculations were programmed in FORTRAN IV with double precesion arithmetic.

The value of θ was chosen to be 1·514102 corresponding to the estimated reliability $\Phi(-\theta) = 0{\cdot}065$. Since the distribution of the estimates in question depends on the parameters μ and σ^2 only through θ, we put $\mu = 0$ and $\sigma^2 = 1$ for simplicity, so that $c = 1{\cdot}514102$. Monte Carlo values for the statistics R_1, R_2, R_3 were obtained for $n = 4$, 12, and 30 for which the numbers of samples were $N = 50\,000$, 20 000, and 10 000, respectively. The range [0, 1] was divided into 1000 equal intervals and the frequencies of the values of R_i in these intervals were registered. From this the empirical densities of R_1, R_2 and R_3 were obtained. Their plots are in Figures 1, 2 and 3, respectively. The distribution of R_1 is of mixed continuous-discrete type. The relative frequency of the zero value is represented by the area of the rectangle. The broken vertical line indicates the estimated value 0·065. Monte Carlo means, variances, and mean squared errors are given in Table 2.

Table 2

Monte Carlo means, variances, and mean squared errors (MSE)

	n	Mean	Variance	MSE
	4	0·065058	0·010387	0·010387
R_1	12	0·064850	0·002936	0·002936
	30	0·065335	0·001134	0·001134
	4	0·060223	0·006288	0·006311
R_2	12	0·063036	0·002400	0·002404
	30	0·064220	0·001063	0·001064
	4	0·130822	0·008516	0·012849
R_3	12	0·090008	0·002827	0·003452
	30	0·075394	0·001132	0·001240

5. CONCLUSIONS

The estimates R_2 and R_3 are biased, in general. Maximum likelihood estimate R_2 is "almost" unbiased (up to the order $O(n^{-2})$) for $\theta = 0$ and $\theta = \pm\sqrt{(3)}$. Bayes estimate R_3 possesses a similar property for $\theta = 0$ only. It follows from Theorem 2 that R_2 is, up to the order $O(n^{-2})$, positively biased for $\theta > \sqrt{(3)}$ or $-\sqrt{(3)} < \theta < 0$ ane negatively biased for $\theta < -\sqrt{(3)}$ or $0 < \theta < \sqrt{(3)}$. The estimate R_3 is positively biased for $\theta > 0$ and negatively biased for $\theta < 0$. Numerical calculations show and Monte Carlo experiments confirm that the bias of R_3 is rather large for the most frequently used values of θ even for large n, and for smaller n the bias makes the estimate R_3 practically inapplicable. For $n = 4$ and $\theta = 1{\cdot}514102$ the bias exceeds 100 per cent. The bias of R_2 in comparison with that of R_3 is not so drastic.

Numerical analysis of deficiencies shows that R_2 is superior to R_1 for θ approximately from the interval $(1, 2)$. For increasing θ R_2 becomes worse, however. Bayes estimate R_3 is better than R_1 for θ close to zero, for larger θ it is much worse than R_1. Similar conclusions remain valid for the comparison of R_3 with R_2.

It follows from the above that the best results may be expected when using the minimum variance unbiased estimate R_1. In the worst for R_1 case about three observations are lost. On the other hand, for a wide range of θ values the use of R_1 is without any risk. A little more complicated computation which requires tables of B – distribution or a computer might be of some disadvantage. If it is necessary to avoid complicated calculations it is possible to use simple R_2. Its use is somewhat risky, however, particularly if there is no imagination of potential values of θ. The Bayes estimate is not generally recommendable.

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Souhrn

ODHADY SPOLEHLIVOSTI V NORMÁLNÍM ROZDĚLENÍ

JAN HURT

Jsou studovány čtyři odhady funkce spolehlivosti normálního rozdělení s neznámými parametry, a to nejlepší nevychýlený, maximálně věrohodný, bayesovský a neparametrický. Je dokázána jejich asymptotická normalita a odvozeny asymptotické rozvoje středních hodnot a středních čtvercových odchylek (SČE). Pomocí rozvojů SČE jsou odhady porovnány z hlediska deficience. Nejlepší výsledky dává nejlepší nevychýlený odhad. Na závěr je uvedena rozsáhlá studie Monte Carlo, ve které jsou studovány vlastnosti odhadů pro malé výběry.

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