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Estimates of Reynolds stress in a highly energetic shelf sea

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Abstract In recent years the use of Acoustic Doppler Current Profilers (ADCPs) to estimate Reynolds stresses, using the so-called variance method, has become popular; and although there was great effort in studying the uncertainties on this technique, there were no reports in the main literature of its validity using independent measurements. This work reports on the comparison of ADCP and Acoustic Current Velocimeter (ADV) estimates of Reynolds stresses. The comparison of the ADCP and ADV is encouraging during periods when no strong waves were present with both the explained variance of 0.8 and the slope of the regression being 0.97. Nevertheless, when strong waves are present the method breaks down and the comparison between ADCP and ADV is very poor with $R^2 = 0.04$.

Keywords Turbulence · Reynolds stresses · ADCP · Variance method

1 Introduction

To be able to model both the water column structure and momentum, it is necessary for numerical models to have an accurate representation of the vertical transfers due to turbulent diffusion. For this, modern numerical models use turbulence closure schemes which allow non-linear interactions between shear production and buoyancy fluxes. Most of these closure schemes involve explicit representations of the evolution of turbulent kinetic energy (TKE) (Umlauf and Burchard, 2003) including its production due to shear stress and buoyancy and its dissipation through which energy is converted to heat. Until recently testing these turbulence closure schemes with field data was not possible, but with developments to observing technologies, measurements of turbulent parameters can now be made in the field, e.g. turbulence dissipation (Simpson et al., 1996) and turbulence production (Rippeth, et al. 2002). This new capability should help realization of a better understanding and representation of turbulent processes.

In the quest to measure turbulence parameters, the use of Acoustic Doppler Current Profilers (ADCPs) to estimate Reynolds stresses and TKE production has become a common practice in recent years (Rippeth et al. 2002, Stacey et al. 1999). Great effort was put into studying the theoretical errors of the method (e.g. Williams and Simpson, 2004). Nevertheless, there has never been a clear validation experiment in which the ADCP Reynolds stresses are compared with estimates from other instrumentation, with the exception of the attempts of Howarth, 2003 involving comparisons with measurements by electromagnetic current meters (ECM). Unfortunately either the ECMs or the ADCP did not work properly and the measurements were located at different heights and separated by 1 km, so although the comparisons were encouraging they were not conclusive.

This work attempts to show the validity of using the ADCP variance method to measure Reynolds stresses through the water column, by comparing the results of the ADCP estimates with those of an Acoustic Doppler Velocimeter (ADV) located on the same mooring frame at about 1.5 m from the ADCP transducer head and the measuring volume at a height of 1.50 m above the bed, within the second bin of the ADCP. The results show that during most of the time the ADCP Reynolds stress estimates are in agreement with the ADV estimates, with exception when energetic waves are present.

2 Methods

The ADCP variance method to calculate Reynolds stresses is relatively simple and cheap, since bottom

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mounted ADCPs can be left for long deployments, in comparison with the use of shear profilers which need to have a ship present all the time. The Reynolds stresses are calculated following the variance method first explained by Lohrmann et al. (1990) and applied by van Haren et al (1994), Stacey et al. (1999), Lu and Lueck (1999) and Rippeth et al. (2002). The method is based on the fact that an ADCP has two pairs of opposing acoustic beams, and that each beam measures a velocity that is actually a weighted sum of the local horizontal and vertical velocities. So that, the velocities determined for each beam are given by:

$$\begin{aligned} u_1 &= v\sin\theta + w\cos\theta \\ u_2 &= -v\sin\theta + w\cos\theta \\ u_3 &= u\sin\theta + w\cos\theta \\ u_4 &= -u\sin\theta + w\cos\theta \end{aligned} \tag{1}$$

where θ is the angle of the acoustic beam from the vertical (20° in this case) and u, v and w are the horizontal and vertical velocity components (for schematic see figure 2 of Whipple et al. this volume). Separating the velocities into mean and fluctuating quantities and taking the difference between the two opposing beams it can be shown by combining the equation (1) and ensemble average them that

$$\overline{u'w'} = \frac{\overline{u_3'^2} - \overline{u_4'^2}}{4\sin\theta\cos\theta} \quad \text{and} \quad \overline{v'w'} = \frac{\overline{u_1'^2} - \overline{u_2'^2}}{4\sin\theta\cos\theta} \tag{2}$$

where the overbar indicates the temporal mean and prime indicates temporal fluctuations, for more information of the method and the errors associated with it see Stacey et al. (1999), Rippeth et al. (2002), Williams and Simpson (2004) and Lu and Lueck (1999).

The ADV follows a direct method of measuring the Reynolds stresses which involves rapid sampling of the three components of velocity in a small sampling volume, so that terms of the type $\overline{u'w'}$ can be calculated directly from the covariance of u and w. This approached was pioneered by Bowden and Fairbairn (1956), using electromagnetic current meters. In recent years, the use of ECMs has been substituted by using ADVs, which are simpler to use and can sample smaller water volumes, depending on frequency and brand. The ADVs used in these measurements were Sontek 5 MHz Ocean instruments which have a volume sample of about 2 cm^3 at sampling rates of about 25 Hz. These instruments are highly accurate (long term error of the order of 3×10^{-6} $\text{m}^2\ \text{s}^{-2}$) and have become the standard for boundary studies in the laboratory and field experiments (*e.g*. Williams and Bell, 2004 and Kim et al. 2000).

Whatever the sensor used for this direct method, the Reynolds stress calculation is sensitive to the correct determination of the vertical component, but should be relatively insensitive to the presence of waves, since linear theory gives $\overline{u'w'}$ and $\overline{v'w'}$ zero for wave orbital velocities as the horizontal and vertical components are in quadrature (Howarth, 2003). The latter is not true in the case of the ADCP variance method as the presence of waves will lead to large along beam variances even near the bed, so that now accurate estimation of Reynolds stress would depend on finding a small difference between two large numbers. (In fact data from ADCP bins and ADV are used to estimate directional waves, using the PUV method Gordon and Lohrmann, 2001.)

3 Observations

The results from two deployments in the Irish Sea will be presented (figure 1)—the first demonstrates the close agreement between ADCP and ADV estimates of Reynolds stress, the second two areas of disagreement. The setup for both deployments was the same with the exception that at the first deployment ♦ (near Holyhead), a 600 kHz ADCP was deployed, because of the deeper water depth; whereas at the second ● (in Liverpool Bay), a 1.2 MHz ADCP was deployed. The ADCPs were operated using RDI rapid sampling mode 12 and set to record 8 subpings per second ensemble with a 1 m bin size near Holyhead and a 0.5 m bin size in the Liverpool Bay. The ADV was a 5 MHz Sontek Ocean Hydra system which was set to sample at 25 Hz. The instruments were mounted on the same bottom frame about 1.5 m apart, with the ADV mounted upwards on an arm about 0.85 m away from the frame to avoid any turbulent effect due to the frame. In the Liverpool Bay case, the data were recorded for 10 min. every hour, while in the Holyhead deployment, the instruments recorded for 20 min. every hour. This was to allow for the 40 day deployment. The measuring volume of the ADV and the centre of the second ADCP bin were co-located at about 1.5 m from the seabed. (At these heights the ADCP bins are about 0.7 m apart in the horizontal, for a 20° beam angle.)

Both the fast sample ADCP and current meter records were analyzed with a basic averaging period of 10 min. Values of Reynolds stresses are given in units of $\text{m}^2\ \text{s}^{-2}$ and should be multiplied by the water density (~1027 kg m^{-3}) for conversion to Pascals. For display purposes the data were rotated so that the $\overline{u'w'}$ component of the Reynolds stress was aligned with the major axis of the barotropic tidal current, which will be called the along-stream component, while $\overline{v'w'}$ will be the across-stream component. For quantitative comparison purposes the ADCP and ADV data were considered as vectors and compared using complex linear regression.

The Holyhead deployment took place from 23rd June to 26th July 2003 at 53° 23′ N 4° 42′ W, water depth 40 m, in a region of strong tidal currents (M_2 maximum amplitude 1.46 m s^{-1}). The correlation between the ADCP and ADV estimates of Reynolds stress is shown as a scatter plot in Fig. 2. The R^2 value (explained variance) is 0.84 and the slope of the linear regression is 0.90. The agreement between ADCP and ADV Reynolds stress estimates both in terms of correlation and amplitude is encouraging, suggesting that the ADCP along beam variances are being correctly evaluated in

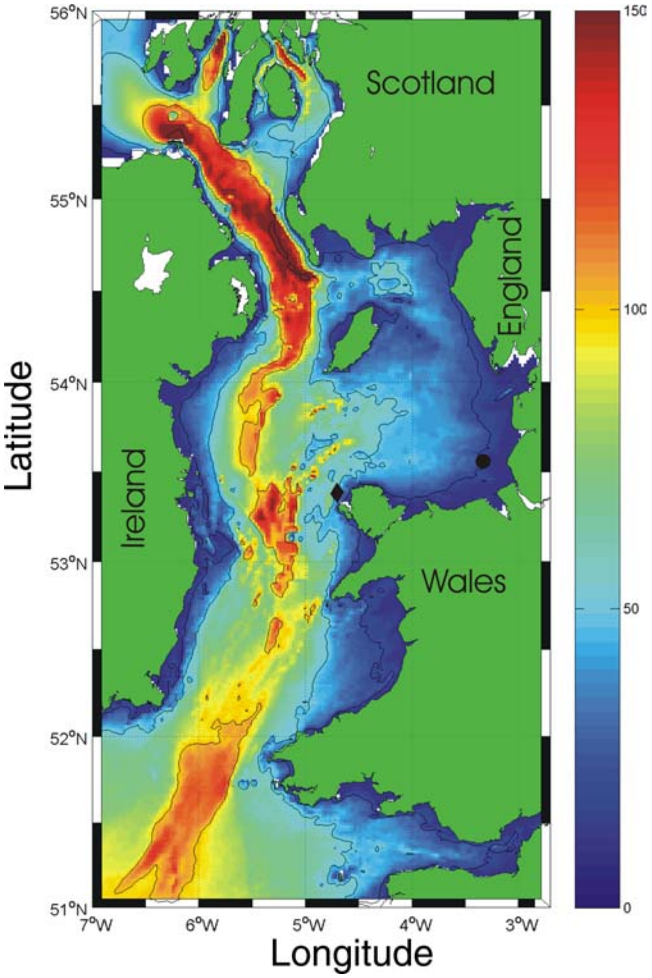


Fig. 1 Mooring location and bathymetry of the Irish Sea. ♦ is the Holyhead mooring and ● is the Liverpool Bay mooring

this high current speed environment despite the averaging implicit in the ADCP sampling (see also Howarth & Souza 2005, which includes comparisons at weaker tidal current sites). The quality of the agreement does not vary with current speed or with ebb and flood for the along stream component, Fig. 2, although the agreement is unfavourable in the (weaker) cross-stream component.

The ADCP and ADV were deployed in Liverpool Bay at 53° 32.0′ N 3° 21.9′ W, near the mouth of the river Mersey, in mean water depth of about 22 m from 23rd January to 6th March 2003. The deployment was for 3 spring-neap cycles, starting just after spring tides, with a depth-averaged M_2 tidal current of 0.5 m s^{-1} directed along 100° (flood)/280° (ebb). During the first 15 days of deployment there were two storms with winds from the northwest which generated waves with heights of more than 3 m and periods of more than 8 s.

Scatter plots of the Reynolds stress estimates from the second ADCP bin against ADV estimates during a period of calm sea, wave height less than 1 m and periods of about 2 s, are shown in Fig. 3. The explained variance (R^2) is 0.81 (nearly as good as the Holyhead case) and the slope is 0.97 with an intercept of 4×10^{-5} m^2 s^{-2} when the ADCP estimates are regressed against the ADV. Figure 4 shows the time series of Reynolds stresses for the regression used in Fig. 3. The along stream component of the Reynolds stress appears to be in good agreement between ADCP and ADV, while there appears to be clear over estimation of the across stream component of up to 2×10^{-4} m^2 s^{-2}. However, in this case, there is a repeatable, since we have made measurements at this site more than once showing the same pattern, an unexplained reduction of the slope of the correlation on flood tides that we have not seen on the other sites. What could cause a reduction in the turbulent length scale such that the ADCP-based estimate is reduced on flood tides? Two possibilities are an effect of stratification or possibly of asymmetric bedforms in a predominantly sandy sediment. The site is near the mouth of the river Mersey. For this deployment, the water column was always well mixed at high

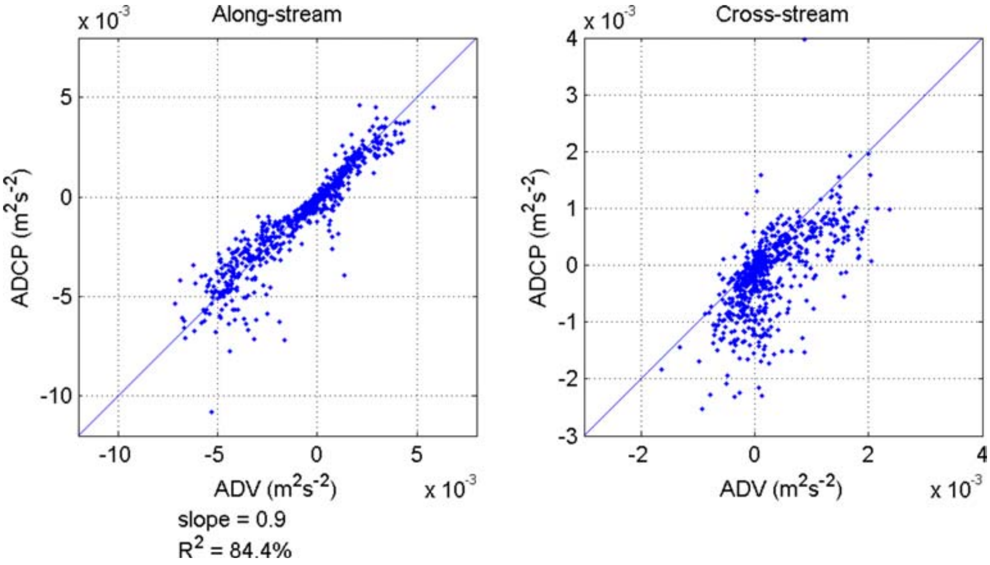


Fig. 2 Scatter plot ADCP vs ADV estimates of along stream and across stream Reynolds stress, for the Holyhead deployment

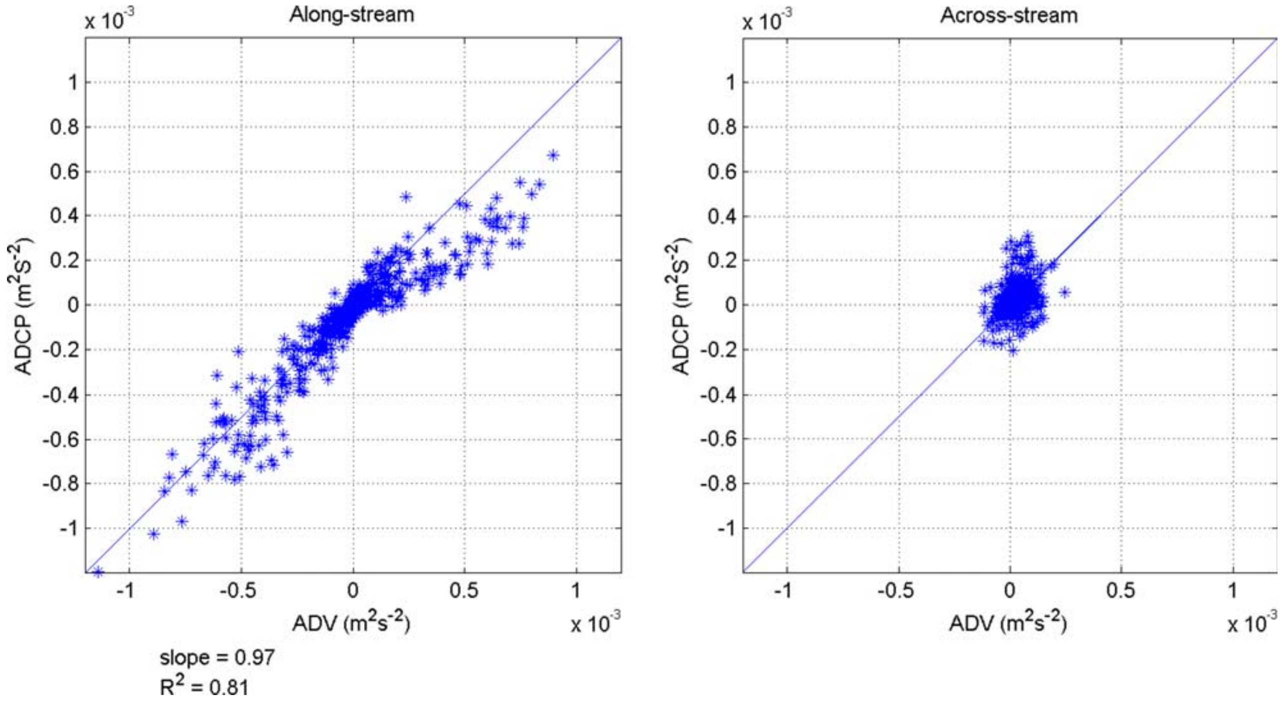


Fig. 3 Scatter plot ADCP vs ADV estimates of along stream and across stream Reynolds stress, for a calm weather period from 8th February to 5th March 2003 in Liverpool Bay

water and also for more than half, the tides at low water. For the remainder, it was weakly stratified at low water, generally by less than 0.5 kg m^{-3} but on seven tides by 1 kg m^{-3}, this switching between mixed and weakly stratified should be caused by tidal straining which could bring an extra production term due to convection and somehow be observed by the ADCP (Rippeth et al. 2001). The third possibility (the frame or instruments

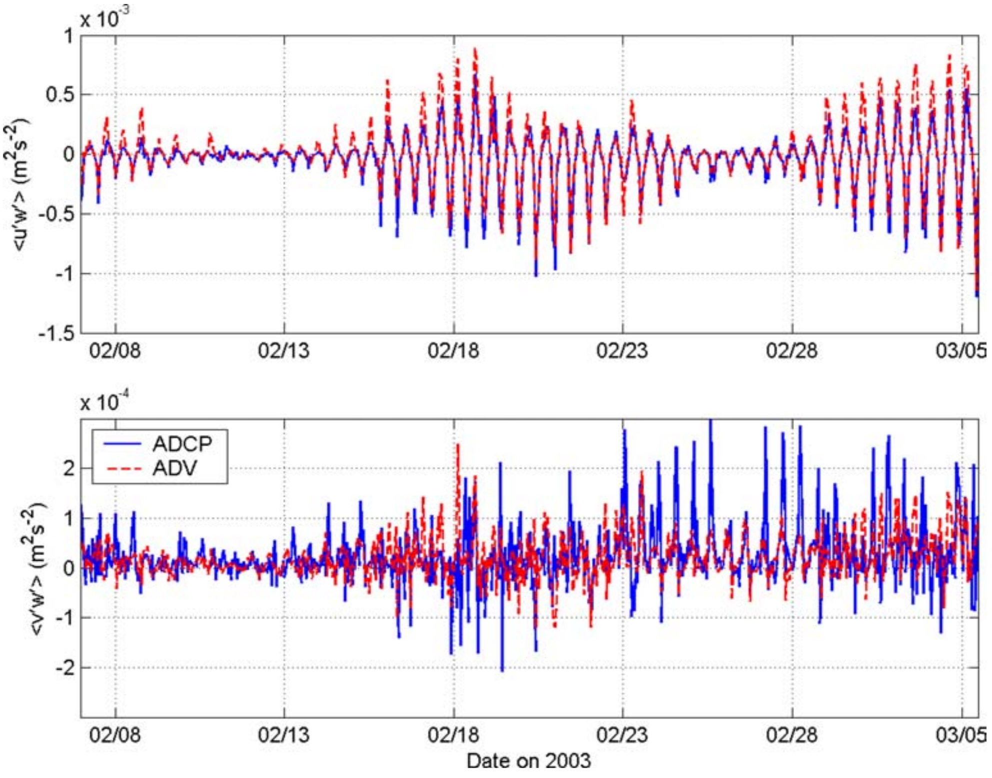


Fig. 4 Time series of Reynolds stresses in Liverpool Bay from 8th February to 5th March 2003, (*continuous blue line*) ADCP, (*dashed red line*) ADV. Note the different scales on the along- and cross-stream diagrams

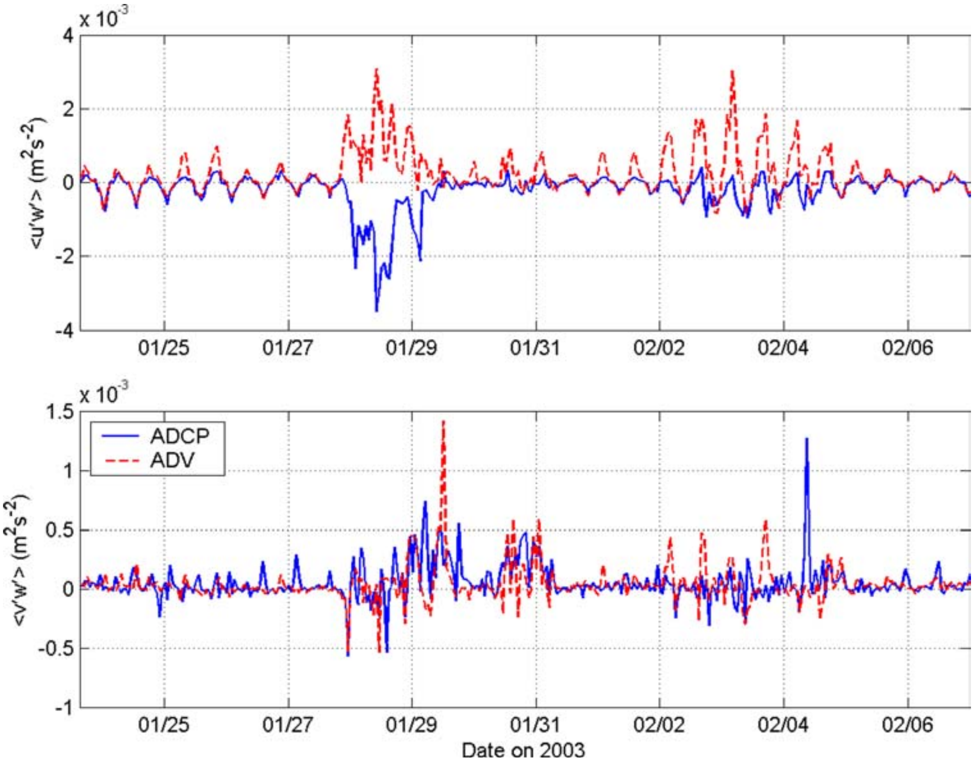


Fig. 5 Time series of Reynolds stresses in Liverpool Bay in the presence of high energy surface waves from 23rd January to 7th February 2003, (*continuous blue line*) ADCP, (*dashed red line*) ADV. Note the different scales on the along- and across-stream diagrams

affecting the flow) is unlikely since the same pattern was seen on several deployments.

In complete contrast, observations over the period in which strong wave effects were present show that the method completely breaks down. Figure 5 shows the time series of Reynolds stress estimates between the 23rd January and 7th February 2003, within this period there were two storms. The first storm was between the 28th and 31st January with the highest and longest waves between the 28th and 29th January—significant wave

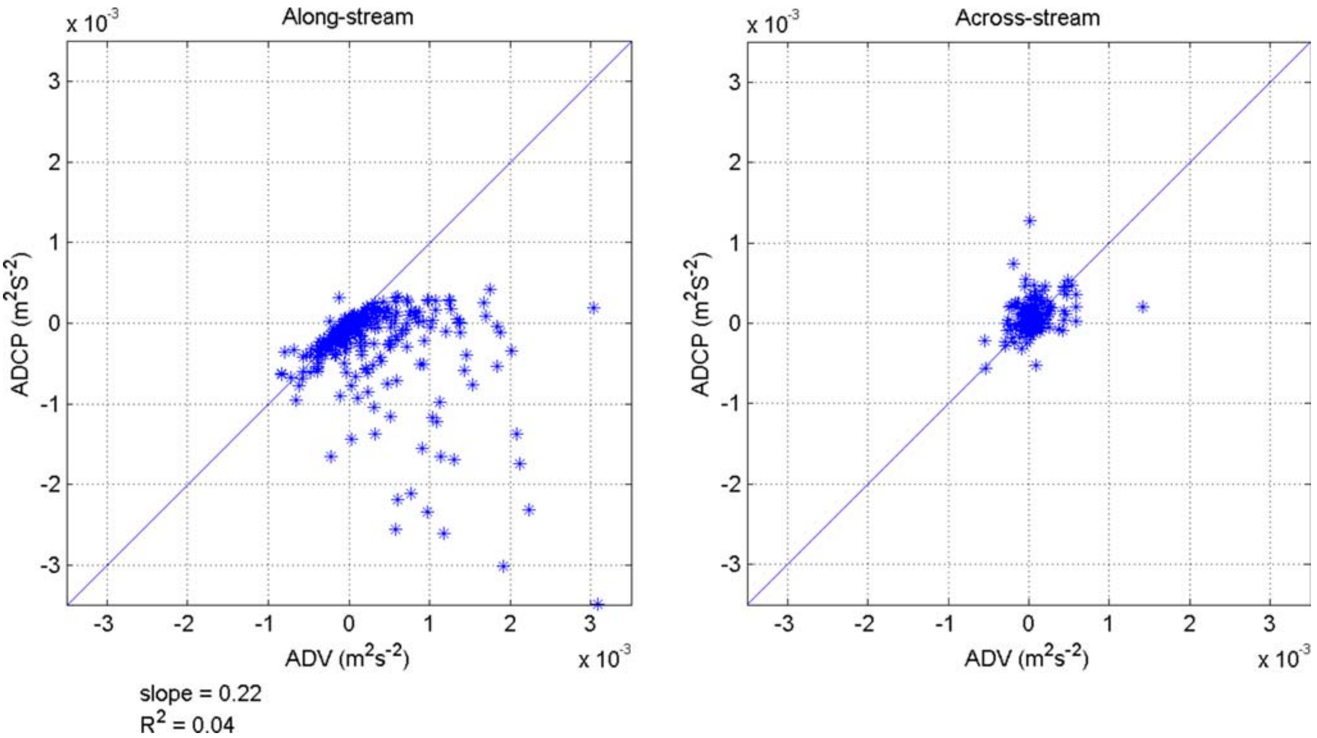


Fig. 6 Scatter plot ADCP vs ADV estimates of along stream and across stream Reynolds stress, in Liverpool Bay, for a high wave energy period from 23rd January to 7th February 2003

heights up to 4 m and peak periods as high as 9 s - exactly when the Reynolds stress estimates from ADV and ADCP are uncorrelated. The second, slightly weaker, storm was between the 2nd and 6th February with the longest and highest waves around the 3rd February when there appears to be more differences between the ADCP and ADV estimates. The scatter plot for this period (Fig. 6) shows very poor correlation with an R^2 of 0.04 and a slope function of 0.22.

4 Discussion and conclusion

When the estimates of Reynolds stress with and without waves are compared, we have observed that the maximum value of Reynolds stress measured by an ADV under waves is about 3×10^{-3} m^2 s^{-2}, even during neap tides. While in the absence of waves, it is about 0.8×10^{-3} m^2 s^{-2} during spring tides. The estimates of Reynolds stresses during the two storms showed a very different behaviour. Note, here we are looking at the differences between two sets of measurements—during a storm, the Reynolds stresses will have contributions both from currents and from waves. During the first storm, the ADV Reynolds stress estimates still contained a tidal signal although with no reversal, the stress always being directed offshore (positive). In contrast, the ADCP estimates lost the tidal signal and were directed onshore (negative). During the second storm, the ADV record was dominated by the tidal signal which was of the right order of magnitude while the ADCP estimate was also tidal but with magnitudes too large, especially in the flood direction.

However, there were only small differences between the two storms. The first storm was slightly stronger, with winds measured near by (24 km away) at Bidston Observatory peaking at 17 m s^{-1}, and hence wave heights and periods were slightly larger; while the second storm had winds of up to 13 m s^{-1} and waves of about 3 m high with periods of about 8 s (Fig. 7). The wave directions were broadly similar: the first storm (28th to 31st January) had waves propagating mainly towards 165° azimuthal; the waves during the second storm propagated mainly towards 135°. (These winds, from between west and north, have the longest fetch at the site and hence generate the largest waves.) These differences imply that the wavelengths would have been slightly longer during the first storm and hence the near bed wave orbital velocities might have been larger (maybe by a factor of 1.4). The first storm occurred during neap

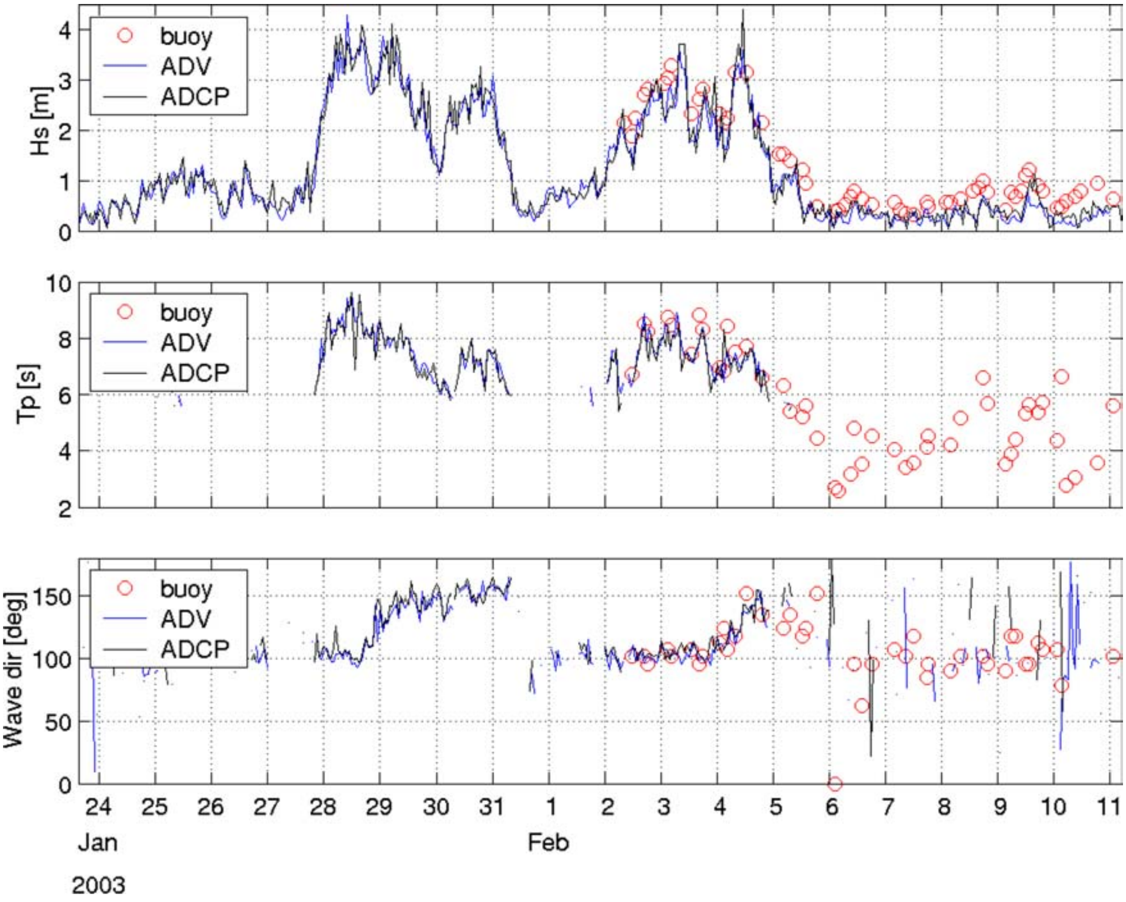


Fig. 7 Wave characteristics in Liverpool Bay as derived from ADCP, ADV and wave rider. (a) Significant wave height, (b) Peak wave period and (c) Peak wave direction

tides (maximum tidal currents 0.3 – 0.4 m s^{-1}) and the second during spring tides (maximum tidal currents about 0.5 m s^{-1}) hence the ratio between tidal currents during the two storms was about 1.4, small for a spring/neap ratio, or a factor of two for the tidal Reynolds stresses. During both storms there was a weak offshore residual current, of order 0.05 m s^{-1}, but not sufficient to stop the tidal reversal. All the above factors might contribute to the different behaviour in the ADCP estimates of Reynolds stresses, as they will contribute to produce different spatial distribution in the along beam velocity variances specially in the case of the first storm which had longer waves and smaller currents, which could lead to the fact that each of the two along beams will be measuring a different part of the wave but it seems surprising that such small differences in circumstance can lead to such large differences in behaviour.

Estimates of Reynolds stress during strong waves using ADVs can be unreliable. This is because, if the instrument is not accurately aligned in the vertical, the measured vertical velocity will be dominated by a spurious 'horizontal' contribution. In the case of the system used, the tilt accuracy is 0.1°, so the error from this misalignment with waves of 4 m height and 9 s period is 4.5×10^{-4} m^2 s^{-2}, based on linear theory. This is three times the value of bottom stress expected at neap tides when the maximum velocity was 0.25 ms^{-1}. A misalignment of only 0.6° will lead to a spurious Reynolds stresses of about 2.75×10^{-3} m^2 s^{-2}, as observed during the presence of waves. The wave effect on the Reynolds stresses can be overcome if we have a pair of ADVs at different heights and then use the technique described by Trowbridge (1998) and modified to be used with ADCPs by Whipple et al. (this volume) or using the spectral method (Kim et al. 2000; Voulgaris and Trowbridge, 1998).

Although this paper has concentrated on Reynolds stresses, another turbulent parameter of interest is the turbulent kinetic energy $(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$. However, this cannot be calculated from standard ADCP data without an extra piece of information such as the anisotropy, $\overline{w'^2}/(\overline{u'^2} + \overline{v'^2})$, although it could be calculated from ADV records. Values for continental shelf seas seem to average about 0.2 but there is a wide scatter at any site. Fig. 8, shows the data from the Holyhead and Liverpool Bay deployments, with values form zero to about 0.25. The Holyhead data shows a parabolic behaviour with speed, while there is no apparent behaviour for the Liverpool Bay data.

This study together with Howarth and Souza (2005) builds confidence on the use of ADCP Reynolds stress estimates in shelf seas at least during periods of calm weather. Nevertheless, it should raise awareness that in the presence of waves the methods break down, so that we should be very careful when interpreting the data. A useful guide if co-located ADV measurements are not available is to compare the ADCP estimates of Reynolds stress against a quadratic drag law, $\mathbf{u}|\mathbf{u}|$.

A last note of warning is that, although the ADCP estimates of Reynolds stresses appear to be in good agreement with the ADV estimates when waves are not present, this does not mean that they are correct in the entire water column. In Fig. 9, we have shown the time evolution of (a) acoustic backscatter and (b) TKE production. It shows a band of high values of acoustic backscatter and production between 13 m and 16 m above the bed, which is clearly due to multiple reflections from the surface. This problem arose from the fact

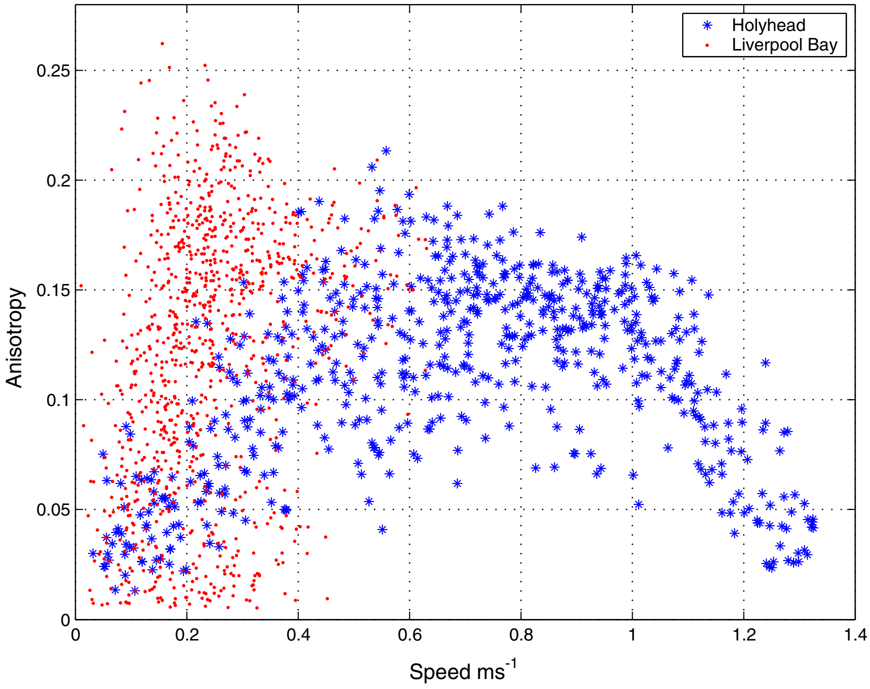
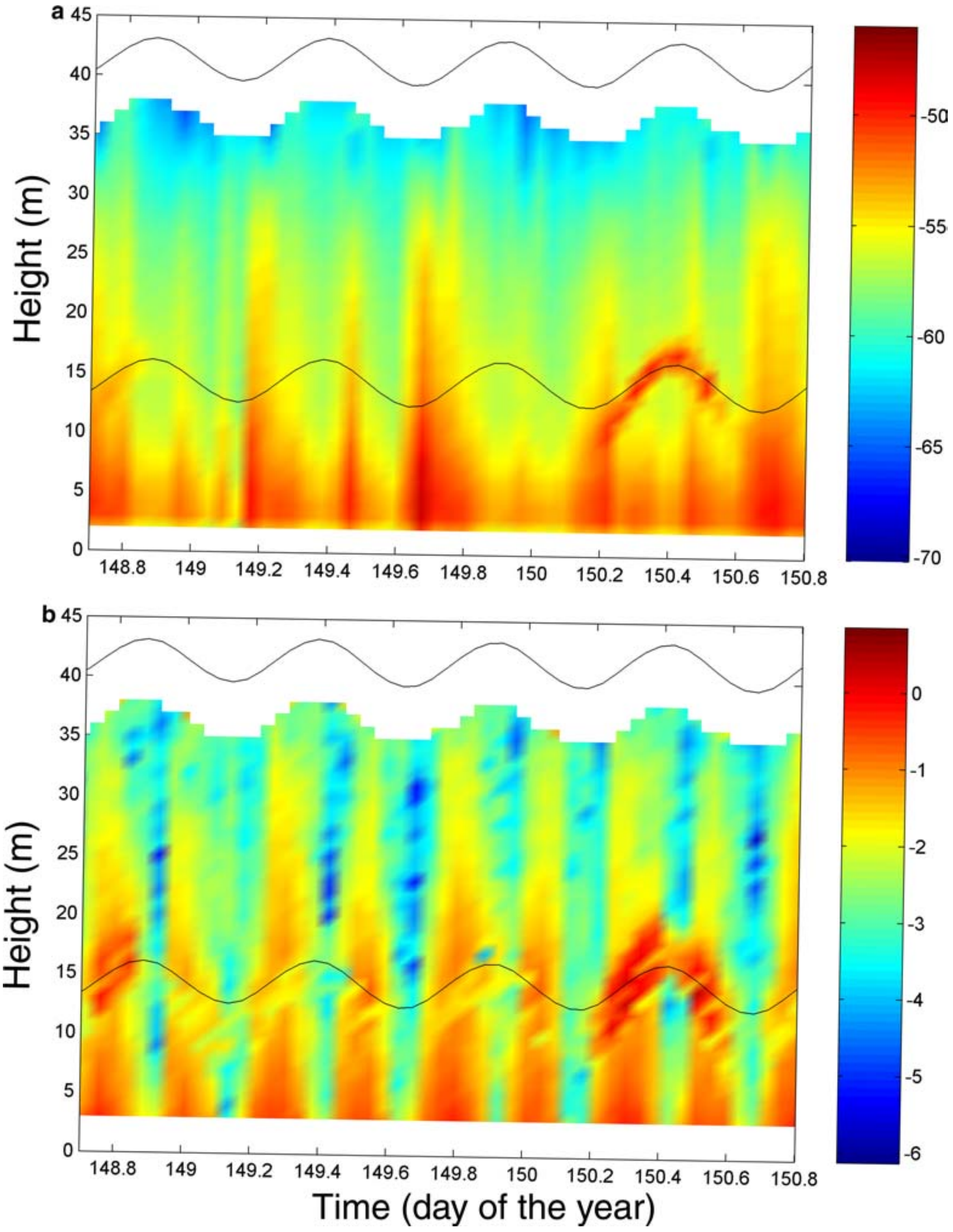


Fig. 8 Anisotropy against speed for Holyhead (blue) and Liverpool Bay (red)

Fig. 9 Time series of backscatter (a) in decibels and TKE production (b) in log10(W m^3) profiles from the 600 kHz ADCP at Holyhead, the line at mid water is a copy of the sea surface displaced 27 m to highlight the multiple reflection effect



that in our eagerness to reduce the error on the estimates of Reynolds stresses and TKE production (see Williams and Simpson, 2004), we tried to get as many sub-pings as possible in a 1 second ping, without taking in consideration the possible ping-to-ping interference. To avoid this data contamination, we advise using a spacing between sub-pings of at least 40 milliseconds for a 1200 kHz ADCP in 20 m of water and 0.5 m bins and 60 milliseconds for a 600 kHz ADCP in 50 m of water and 1 m bins, for more information see RDI (2002).

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