# Estimating a Demand System with Nonnegativity Constraints: Mexican Meat Demand

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**Abstract**

A new information-based approach for estimating systems of many equations with nonnegativity constraints is presented. This approach, called generalized maximum entropy (GME), is more practical and efficient than traditional maximum likelihood methods. The GME method is used to estimate an almost ideal demand system for five types of meat using cross-sectional data from Mexico, where most households did not buy at least one type of meat during the survey week. The system of demands is shown to vary across demographic groups.

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**Mexican Meat Demand**

We present a new approach to efficiently estimate a system of many equations with binding nonnegativity constraints. Using this approach, we estimate a five-equation meat demand system based on data from a large cross-sectional survey of Mexican households in 1992. Most of these households did not purchase one or more of these meat products during the survey week. We compare our demand elasticity estimates for Mexico to estimates of those in wealthier countries. We also show how meat demands vary with demographic characteristics.

There have been relatively few previous attempts to estimate demand systems with binding nonnegativity constraints.[1] We know of only one study, Heien and Wessells (1990), that estimates a many-equation demand system with nonnegativity constraints with variable prices. They use a two-stage Amemiya (1974) approach to estimate an Almost Ideal Demand System, AIDS (Deaton and Muelbauer, 1980), for 11 food items with an emphasis on dairy products.

Although such two-step methods are consistent, they are not invariant to the choice of which good is dropped, and they are inefficient and require specific distributional assumptions. Flood and Tasiran (1990) find that the Amemiya two-stage estimator performs poorly compared to maximum likelihood (ML) in estimating a system of tobit equations with normal errors and that this inefficiency does not decrease with sample size.[2] Moreover, they find that the ML and the two-stage approaches perform poorly when the errors are not normal.

If the errors are normal (or another known distribution), greater efficiency can be achieved by using full-information ML techniques. Using standard ML techniques, however, is feasible only for systems consisting of a relatively small number of goods, say three.[3]

Wales and Woodland (1983) use ML techniques to estimate a demand system with nonnegativity constraints based on a random quadratic utility function for three goods (beef, lamb, and other meats) based on Australian data.[4] Because they observe no variation in prices, they estimate only the variation in demand due to differences in demographic characteristics.[5]

Ransom (1987) examined the relationship between the Wales and Woodland method and the Amemiya approach to estimating simultaneous tobits. He showed that the internal consistency condition for the Wales and Woodland model is equivalent to the second-order condition for systems of demand equations without binding quantity constraints. If prices are constant, Wales and Woodland's method and the simultaneous system with limited dependent variables of Amemiya are identical. If prices vary, the error terms are heteroscedastic.

Our objective is to develop a single-stage estimation procedure that allows us to recover the unknown parameters of a nonlinear, censored demand system with many goods without having to make distributional assumptions. Our approach has its roots in information theory and builds on the entropy-information measure of Shannon (1948), the classical maximum entropy (ME) principle of Jaynes (1957a, 1957b), which was developed to recover information from underdetermined systems, and the generalized maximum entropy (GME) theory of Golan, Judge, and Miller (1996).

The GME method allows us to consistently and efficiently estimate a demand system with nonnegativity constraints and a large number of goods without imposing restrictions on the error process. The GME estimates are robust even if errors are not normal and the exogenous variables are correlated. In this paper, we use the GME approach to estimate the nonlinear version of the Almost Ideal Demand System (AIDS), but our method could be applied to any demand system. In our empirical application, we concentrate on estimating the elasticities of demand and examining how the shares of each good vary with demographic characteristics of households.

## I. AIDS Model

The Almost Ideal Demand System (AIDS) is a flexible, complete demand system: It satisfies the adding up of budget shares, homogeneity, and symmetry. Throughout most of the paper, we assume that meat and all other goods are separable in the utility function and estimate a five-equation system of demand for meats only.[6]

The AIDS consists of a set of budget-share equations:

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln (E/P), \tag{1}$$

where $s_i$ ($\geq 0$) is the budget share of meat product $i$, $p_i$ is the price of product $i$, $E$ is the total expenditure on meats, $P$ is a price index, and $\gamma_{ij}$ and $\beta_i$ are constant parameters. We allow the intercept terms to vary with a matrix $X$ of $K$ exogenous demographic and geographic variables (the first column consists of ones), where

$$\alpha_i = \sum_{k=0}^{K} \rho_{ik} X_k , \tag{2}$$

the $\rho_{ik}$ are parameters, and $\rho_{i0}$ is the constant of the equation.

The corresponding nonlinear price index is

$$\ln P = \phi + \sum_{i=1}^{n} \alpha_i \ln p_i + ½ \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j , \tag{3}$$

where $\phi$ is a constant. Although we use the nonlinear price index, a common practice is to replace the nonlinear price index, Equation 3, with Stone's linear approximation:[7]

$$\ln P^* = \sum_{i=1}^{n} s_i \ln p_i . \tag{4}$$

We follow the standard practice of adding an error term, $\varepsilon_i$, to each budget-share equation. Thus, the model we estimate is[8]

$$s_i = \sum_{k=0}^{K} \rho_{ik} X_k + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln (E/P) + \varepsilon_i , \quad \text{for } s_i > 0 \tag{5}$$

$$s_i > \sum_{k=0}^{K} \rho_{ik} X_k + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln (E/P) + \varepsilon_i , \quad \text{for } s_i = 0. \tag{6}$$

## II. Estimation Approach

To estimate this system of censored demand equations, we generalize the GME method for estimating a single, censored equation given in Golan, Judge, and Perloff (1997). We start by providing some intuition as to how the traditional maximum entropy approach works. Then, we show how to estimate the AIDS using GME. The appendix discusses the properties of this estimator and derives the asymptotic variance-covariance matrix.

### *A. Maximum Entropy*

The traditional maximum entropy (ME) formulation is based on the entropy-information measure of Shannon (1948), as discussed in Golan, Judge, and Miller (1996). Shannon's entropy is used to measure the uncertainty (state of knowledge) we have about the occurrence of a collection of events. Letting $x$ be a random variable with possible outcomes $x_s$, $s = 1, 2, \ldots, N$, with probabilities $\pi_s$ such that $\Sigma_s \pi_s = 1$, Shannon defined the *entropy* of the distribution $\underline{\pi} = (\pi_1, \pi_2, ..., \pi_N)'$, as

$$S(\underline{\pi}) \equiv -\sum_{s=1}^{N} \pi_s \ln \pi_s \,, \tag{7}$$

where $0 \ln 0 \equiv 0$. The function $S$, reaches a maximum of $\ln(N)$ when $\pi_1 = \pi_2 = \ldots = \pi_N = 1/N$. It is zero when $\pi_s = 1$ for one value of $s$. To recover the unknown probabilities $\underline{\pi}$ that characterize $M$ moments of a given data set, Jaynes (1957a, 1957b) proposed maximizing entropy, subject to the available sample-moment information and the requirement that the probabilities be proper (add to one).

The basic axioms of the ME approach are well developed and discussed in the literature (see Golan, Judge, and Miller, 1996, for details). We now discuss its intuitive appeal. Suppose we have a sample of $T$ draws of an identically and independently distributed random variable $x$ that can take $N$ values, $x_1, x_2, ..., x_N$, with probabilities $\pi_1, \pi_2, ..., \pi_N$. Because the draws are independent, a list of the number of times each value occurs contains all of the information this experiment provides about the random variable (i. e., the order contains no information about the probabilities). We define the *outcome* of the experiment as a vector $\underline{f} = (f_1, f_2, ..., f_N)$, where $f_s$ is the number of times $x_s$ occurs and $\Sigma_s f_s = T$. A particular outcome may be obtained in a number of ways. For example, the outcome $(1, T\text{-}1, 0, 0, ..., 0)$ can occur in $T$ possible ways because $x_1$ may be observed in any of the $T$ draws. In contrast, the outcome $(T, 0, 0 ...0)$ can occur in only one way, where $x_1$ was drawn each time.

Define $v(\underline{f})$ as the number of ways that a particular outcome can occur. Suppose we have no information about the draws and are asked which outcome is the most likely. An "intuitively reasonable" response is that the outcome that can occur in the most number of ways, $\underline{f}^* \equiv \text{argmax } v(\underline{f})$, is the most likely outcome. Equivalently, we would consider it more likely to observe the frequency $\underline{f}^*/T$ than any other frequency. Shannon shows that, at the limit as $T \to \infty$, choosing $\underline{f}$ to maximize $v(\underline{f})$ is equivalent to choosing $\underline{\pi}$ to maximize the entropy measure, $S(\underline{\pi})$.

That is, the frequency that maximizes entropy is an intuitively reasonable estimate of the true distribution when we lack any other information. If we have information from the experiment, such as the sample moments, or nonsample information about the random variable, such as restrictions from economic theory, we want to alter our "intuitively reasonable" estimate. The ME method chooses the distribution that maximizes entropy, subject to the sample and nonsample information. That is, out of all the possible estimates or probability distributions that are consistent with the sample and nonsample data, the ME method picks the one that is most uninformed, i.e., closest to a uniform distribution. In this sense, the ME estimator is conservative.

### *B. Estimating an AIDS Model using GME*

In the traditional maximum entropy (ME) approach, sample information in the form of moment conditions is assumed to hold exactly. In contrast, the generalized maximum entropy approach (Golan, Judge, and Miller, 1996) uses each observation directly while allowing these conditions to hold only approximately by treating them as stochastic restrictions.

Further, the GME uses a flexible, dual-loss objective function: a weighted average of the entropy of the systematic part of the model and the entropy from the error terms. The ME is a special case of the GME where no weight is placed on the entropy of the error terms and where the data are represented in terms of exact moments. By varying the weight in the GME objective, we can improve either our precision or predictions. Here, we use a balanced approach where we give equal weight to both objectives.[9]

To write these two entropy measures, we need to express all the coefficients and errors in Equations 3, 5, and 6 in terms of proper probabilities. To transform $\gamma_{ij}$, for example, we start by choosing a set of discrete points, called the support space, $\underline{z}^{\gamma}_{ij} = (z^{\gamma}_{ij1}, z^{\gamma}_{ij2}, \ldots, z^{\gamma}_{ijD})'$ of dimension $D \geq 2$, that are at uniform intervals, symmetric around zero, and span the interval $[-a, a]$. We then introduce a vector of corresponding unknown weights $\underline{q}^{\gamma}_{ij} = (q^{\gamma}_{ij1}, q^{\gamma}_{ij2}, \ldots, q^{\gamma}_{ijD})'$ such that $\Sigma_d\, q_{ijd} = 1$ and $\Sigma_d\, z^{\gamma}_{ijd}\, q^{\gamma}_{ijd} = \gamma_{ij}$ for all $i$ and $j$. For example, if $D = 3$, then $\underline{z}^{\gamma}_{ij} = (-a, 0, a)'$, and there exists $q^{\gamma}_{ij1}$, $q^{\gamma}_{ij2}$, and $q^{\gamma}_{ij3}$ such that each $\gamma_{ij} = -aq^{\gamma}_{ij1} + aq^{\gamma}_{ij3}$. We index the number of discrete points (dimensions) in the support space for each unknown coefficient with $d = 1, 2, \ldots, D$. Each support space and the associated probability distribution can be of different dimension. We use the same approach for the β, ρ, and φ coefficients.

We treat the errors $\varepsilon_{it}$ as unknowns and define a transformation matrix $V$ that converts the possible outcomes for $\varepsilon_{it}$ to the interval [0, 1]. This transformation is done by defining a vector of $H \geq 2$ discrete points $\underline{v} = (v_1, v_2, \ldots, v_H)'$, distributed evenly and uniformly about zero, and a corresponding vector of proper unknown weights $\underline{w} = (w_{it1}, w_{it2}, \ldots, w_{itH})'$ such that $\Sigma_h v_h w_{ith} = \varepsilon_{it}$. No subjective information on the distribution of the probabilities is assumed. Substituting these reparameterized terms into the AIDS Equations 5 and 6, we obtain:

$$\begin{aligned} s_{it} &= \sum_{k=0}^{K} \sum_{d=1}^{D} z_{ikd}^{\rho} q_{ikd}^{\rho} X_{tk} + \sum_{j=1}^{n} \sum_{d=1}^{D} z_{ijd}^{\gamma} q_{ijd}^{\gamma} \ln(p_{tj}) \\ &+ \sum_{d=1}^{D} z_{d}^{\beta} q_{id}^{\beta} \ln(E_t / P_t) + \sum_{h=1}^{H} v_h w_{ith}, \qquad \text{for } s_{it} > 0 \end{aligned} \tag{8}$$

$$\begin{aligned} s_{it} &> \sum_{k=0}^{K} \sum_{d=1}^{D} z_{ikd}^{\rho} q_{ikd}^{\rho} X_{tk} + \sum_{j=1}^{n} \sum_{d=1}^{D} z_{ijd}^{\gamma} q_{ijd}^{\gamma} \ln(p_{tj}) \\ &+ \sum_{d=1}^{D} z_{d}^{\beta} q_{id}^{\beta} \ln(E_t / P_t) + \sum_{h=1}^{H} v_h w_{ith}, \qquad \text{for } s_{it} = 0. \end{aligned} \tag{9}$$

The GME estimator maximizes the joint entropy of all the probabilities representing the signal (ρ, γ, β, φ) and the noise (ε), subject to the data (where $P_t$ is the nonlinear price index, Equation 3), and the adding up (including the probabilities), homogeneity, and symmetry conditions.

Letting $\underline{q} = (\underline{q}^{\rho\prime}, \underline{q}^{\gamma\prime}, \underline{q}^{\beta\prime}, \underline{q}^{\phi\prime})'$, the GME estimator is

$$\max_{\underline{q}, \underline{w}} S(\underline{q}, \underline{w}) = -\underline{q}' \ln \underline{q} - \underline{w}' \ln \underline{w}, \tag{10}$$

subject to budget-share Equations 8 and 9 (that include the nonlinear price index Equation 3), the GME adding-up conditions,

$$\sum_{d}^{D} q_{ikd}^{\rho} = \sum_{d}^{D} q_{ijd}^{\gamma} = \sum_{d}^{D} q_{id}^{\beta} = \sum_{d}^{D} q_{d}^{\phi} = \sum_{h}^{H} w_{ith} = 1, \tag{11}$$

the consumer-theory restrictions concerning adding-up, homogeneity, and symmetry, and the restriction for the shares to add to one:

$$\sum_{i=1}^{n} \rho_{i0} = 1, \tag{12}$$

$$\sum_{i=1}^{n} \rho_{ik} = 0, \quad \text{for } k = 1, \ldots, K, \tag{13}$$

$$\sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} = 0, \tag{14}$$

$$\gamma_{ij} = \gamma_{ji}, \tag{15}$$

$$\sum_{i=1}^{n} \sum_{h=1}^{H} v_h w_{ith} = 0. \tag{16}$$

The solution to this maximization problem is unique. Forming the Lagrangean and solving for the first-order conditions yields the optimal solution $\hat{q}$ and $\hat{w}$, from which we derive the point estimates for the AIDS coefficients:

$$\hat{\gamma}_{ij} = \sum_{d=1}^{D} z^{\gamma}_{ijd} \hat{q}^{\gamma}_{ijd}. \tag{17}$$

$$\hat{\rho}_{ik} = \sum_{d=1}^{D} z^{\rho}_{ikd} \hat{q}^{\rho}_{ikd}. \tag{18}$$

$$\hat{\beta}_{i} = \sum_{d=1}^{D} z^{\beta}_{id} \hat{q}^{\beta}_{id}. \tag{19}$$

$$\hat{\phi} = \sum_{d=1}^{D} z^{\phi}_{d} \hat{q}^{\phi}_{d}. \tag{20}$$

$$\hat{\varepsilon}_{it} = \sum_{h=1}^{H} v_{h} \hat{w}_{ith}. \tag{21}$$

That this GME estimator is consistent follows immediately by extending the proof in Golan, Judge, and Perloff (1997) that a censored GME estimator for a single equation is consistent (see the Appendix). The GME has several other desirable properties (see Golan, Judge, and Perloff, 1997, and Golan, Judge, and Miller, 1996), which we briefly summarize. The GME approach uses all the data points and does not require restrictive moment or distributional error assumptions. Thus, unlike the ML estimator, the GME is robust for a general class of error distributions. The GME estimator may be used when the sample is small, there are many covariates, and when the covariates are highly correlated. Moreover, using the GME method, it is easy to impose nonlinear and inequality constraints.

Most important for demand system estimation, the GME produces efficient estimates of a system of many censored equations. Using ML methods, one is practically restricted to estimating only a few equations or using relatively inefficient two-stage methods. The sampling experiments of Golan, Judge, and Perloff (1997) indicate that the balanced single-equation GME estimator is more efficient — has lower empirical mean square error — than the ML tobit estimator in small samples. These results hold even when the true underlying error distribution is normal, as is assumed in the tobit estimator. These experiments indicate that the GME is even more efficient relative to the ML when the error term is not normal.

## III. Data

The data set we use was provided by the World Bank. It derives from a cross-sectional Mexican household survey conducted in the last quarter of 1992 by the National Institute of Statistics, Geography and Informatics (INEGI), an agency of the Ministry of Budgeting and Programming in Mexico. A stratified and multi-stage sampling method was used to produce a representative sample for the entire population and for urban and rural households. The data cover 31 states and one Federal District.

The data base has detailed information about consumption during a one-week survey period and demographic characteristics by household. The survey recorded 581,027 observations of purchasing events by about 10,500 households.[10] At least 205 types of foods are separately reported.

We examine the quantities purchased for five aggregates of meat products: beef, pork, chicken, processed meat, and fish. The corresponding prices are also aggregates. For example, the price of beef is an expenditure weighted average of beef steak, pulp, bone, fillet, special cuts, and ribs and other.

The prices of various meats vary geographically. We conducted pairwise tests of the hypothesis that the prices are drawn from the same normal distribution across the 129 locations (urban and rural areas within states). Based on t-tests, we rejected the hypothesis at the 5% level that the average prices are homogeneous in 54% of the comparisons.

We are concerned only with households that bought at least one of five categories of meat. Of the 7,897 households that bought some meat during the sample week, 33% did not buy beef; 70% did not buy pork; 35%, chicken; 57%, processed meat; and 87%, fish. Because prices are only reported if purchases are made, we need measures of the prices for households that did not make purchases. We assume that those household face the average price level for that product in that particular geographic location: a rural or urban area in a particular state or Federal District.

We experimented with various sample sizes and found that our estimates were not very sensitive to sample size. In the following, our estimates are based on a random sample of 1,000 households (observations). Table 1 shows that the means and standard deviations of our 1,000 observations are virtually the same as for all 7,897 households. Table 1 also provides summary statistics for the consumption shares of the five meats, the corresponding prices, expenditures on meats, and the 12 demographic variables we use in our GME nonlinear AIDS Model.

## IV. Estimation

We obtained our GME estimates by maximizing the joint-entropy objective, Equation 10 subject to the AIDS Equations 8 and 9 (with the nonlinear price index), the GME adding-up restrictions, Equation 11, and the consumer-theory restrictions, Equations 12-16.

We set our support vectors $\underline{z}^l$, $l = \underline{\rho}, \underline{\gamma}, \underline{\beta}, \phi$, wide enough to include all the possible outcomes. The natural support vector for the error terms is $\underline{v} = (-1, 0, 1)$, because all the dependent variables are shares that lie between 0 and 1. In a variety of AIDS empirical studies [Heien and Pompelli (1988), Moschini and Meilke (1989), Heien and Wessells (1990), and Chalfant *et al.* (1991)], we found that the estimated coefficients on log prices were within the interval of (-0.2, 0.2) and the intercepts and coefficients on log expenditures were within the interval of (-1, 1). We chose support vectors that are 100 times wider than these intervals: (-20, 20) for the log price coefficients and (-100, 100) for the intercept and log expenditure coefficients. Making a moderately large change in these support vectors, while keeping the center of the support unchanged, has negligible effects on the estimated coefficients and elasticities.

The model was estimated using GAMS (Generalized Algebraic Modeling System), which is a nonlinear-optimization program.[11] Table 2 shows our GME estimates of the nonlinear AIDS demand system. The asymptotic standard errors are calculated using the method described in the Appendix.

We tested for symmetry and homogeneity. We tested for homogeneity using each equation separately (see Deaton, 1980). We estimated each equation with and without the homogeneity restrictions and then calculated the entropy-ratio statistic (developed in the Appendix), which has a limiting $\chi^2$ distribution. These tests fail to reject the homogeneity hypothesis at the 5% (or even 1%) significance level for all five goods.

The symmetry test is conducted for the entire system. The objective value with both symmetry and homogeneity imposed is 5,332.490. The corresponding objective value without symmetry is 5,332.800. Thus, on the basis of the entropy-ratio test developed, we fail to reject the symmetry hypothesis at the 5% (or 1%) significance level.[12]

Finally, we tested the hypothesis that the errors for each equation are IID. Given the estimated residuals, $\hat{\varepsilon}_{it}$, we used Wooldridge's (1990) robust test for heteroscedasticity (his

Equations 3.22 - 3.24). On the basis of this test, we cannot reject the hypothesis of homoscedastic errors for beef, pork, processed meat, and fish at the 5% significance level. The results for chicken were nonconclusive.

### *A. Prediction*

We can contrast a measure of the predictive power of these estimates to those from the two-step method of Heien and Wessells (1990). Table 3 shows the correlation between observed and predicted shares for various estimators including both the nonlinear AIDS model (using Equation 3) and the linear AIDS approximation (using Equation 4). Given that the estimates are based on a cross-section of households (with measurement errors in the price data), the correlations for all three estimators are surprisingly high.

Except for fish, the predicted shares of the Heien-Wessells two-step estimator are more highly correlated with the actual data than those obtained using single-equation tobit estimates (not shown in the table) or the least-square estimator that ignores the nonnegativity constraints. Apparently the information from the cross-equation restrictions in the two-step estimator more than compensate for the greater efficiency of the tobit ML approach for a single equation.

The GME predicted shares are more highly correlated with the actual data than are the two-step estimator for the entire system and for each type of meat (except for pork in the nonlinear specification and processed meat in the linear specification). Surprisingly, the linear approximation method predicts better than does the nonlinear model for the Heien-Wessells method (though not for the GME method).

*B. Elasticities*

We calculate expenditure elasticities and both Marshallian and Hicksian price elasticities at the sample means (indicated by a bar over a variable). The expenditure elasticities are

$$\eta_i = 1 + \beta_i/\bar{s}_i.$$

The Marshallian price elasticities are

$$\varepsilon_{ij}^{m} = \delta_{ij} + \frac{\gamma_{ij} - \beta_i(\bar{\alpha}_j + \sum_k \gamma_{ik} \ln \bar{p}_k)}{\bar{s}_i},$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise and $\bar{\alpha}_j = \sum_{k=0}^{K} \rho_{jk} \bar{X}_k$. The Hicksian price elasticities are

$$\varepsilon_{ij}^{h} = \varepsilon_{ij}^{m} + \eta_i \bar{s}_j,$$

where $\varepsilon_{ij}^{h} = \varepsilon_{ji}^{h}$.

Table 4 reports the Hicks-compensated price elasticities and expenditure elasticities for each type of meat and corresponding asymptotic standard errors. All the own-elasticities have the expected signs. Most of the elasticities are statistically significantly different from zero at the 0.05 level on the basis of asymptotic t-tests.

The first row of Table 5 shows our estimated Marshallian own-price elasticities. In that table, we compare our estimated elasticities to those from previous studies of meat demand. These studies used time-series data from wealthier countries. We can find no other estimates of meat-demand price elasticities for Mexico.[13] Nor have we found meat demand studies for other countries that use cross-sectional data.[14]

Our estimated Marshallian demand elasticity for processed meat is -0.78. None of the other studies calculate an elasticity for this meat. The estimated Marshallian elasticities for Mexican beef and chicken lie at the high-end and the pork elasticity at the low-end of the ranges over other countries. The Mexican fish elasticity is much more elastic than the few other existing estimates in other countries.

### *C. Demographic Effects*

Because our demand system uses demographic variables as well as prices to explain demand, we can examine how changes in various demographic characteristics affect the price and expenditure elasticities. Table 6 shows how the share of meat expenditures responds as we change one demographic variable at a time, while holding other demographic variables at the sample means or at 0 and 1 for the dummy variables.

The top of the table shows the effect of changing each dummy variable from 0 to 1. The share of beef is 4.9 percentage points higher for a household in an urban area than a comparable rural household. Urban dwellers also eat relatively more chicken and fish and less pork and processed meats. Female-headed households eat relatively less beef, pork, and chicken, and relatively more processed meats and fish. Households headed by someone with a college degree eat much more processed meats and much less pork than those headed by people who did not complete their elementary education.

The bottom part of the table shows the effect of changing a household's age composition. If a family adds a child under the age of 5 to a family of five, the share of beef increases by 1.4 percentage points, the share of pork falls by 2.4 percentage points, and the other shares change by smaller amounts. In general, changes in age composition have the largest effects for the two youngest and the oldest age groups.

## V. Conclusions

Our generalized maximum entropy (GME) approach is a practical way to estimate systems of many equations with nonnegativity constraints. The GME approach has several advantages over traditional maximum likelihood (ML) methods.

First, because no assumptions about the error structure need be made to use the GME estimator and because it uses all the data, it is more robust and efficient than are ML estimators. The sampling experiments of Golan, Judge, and Perloff (1997) indicate that the balanced single-equation GME estimator is more efficient — has lower empirical mean square error — than the ML tobit estimator in small samples regardless of whether errors are normal or not. In particular, the predictive power of the GME estimator is greater than that of two-stage estimators or single-equation maximum likelihood estimators for the data set we examined.

Second, imposing inequality (nonnegativity) constraints, equality (various consumer theory) constraints, and nonlinear constraints is straight forward. In general, theoretical and other nonsample information may be directly imposed on the GME estimates much easier than with classical ML or Bayesian techniques. The resulting one-stage GME procedure is easy to implement.

Third, GME performs well in both well-posed and ill-posed (small data sets, under-determined problems, high levels of collinearity, and so forth) problems. Fourth, the GME objective gives equal weight to precision and prediction. Fifth, the GME approach can be used with a larger number of censored equations than is practical to estimate with standard full-information maximum likelihood approaches. This method of estimating simultaneous censored equations can be applied to many other problems as well, such as cost and production studies.

We employed our GME method to estimate the demand for five types of meat using cross-sectional data from Mexico, where most households did not buy at least one type of meat during the survey week. Our estimates of the Marshallian elasticities of demand for Mexico are similar to estimates based on aggregate, time-series data from other, wealthier countries except for fish, where Mexican demand is more elastic. We show that demands vary across demographic groups.

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**Appendix: Properties of the AIDS Generalized Maximum Entropy Estimator**

Given mild conditions, our AIDS GME estimator is consistent and asymptotically normal. These conditions are that (i) the errors' support $\underline{v}$ is symmetric around zero, (ii) the support $\underline{z}_\delta$ spans the true values for each one of the unknown parameters [$\underline{\delta} = (\underline{\rho}, \underline{\gamma}, \underline{\beta}, \phi)'$] and has finite lower and upper bounds, (iii) the errors are independently and identically distributed with variance-covariance matrix $\Sigma$ for each equation, and (iv) plim $(1/T)(\partial \underline{f}/\partial \underline{\delta})'(\partial \underline{f}/\partial \underline{\delta})$, where $s_i = f_i(X, p_j, E, P; \underline{\rho}_i, \underline{\gamma}_i, \underline{\beta}_i, \varepsilon_i)$, exists and is nonsingular. The proofs of consistency and asymptotic normality follow immediately from those in Golan, Judge, and Miller (1996), Golan, Judge, and Perloff (1997), and Mittelhammer and Cardell (1996):

*Theorem 1*: Under the four assumptions (i) - (iv), the restricted GME-AIDS estimator $\hat{\underline{\delta}}$ is consistent and asymptotically normally distributed

$$\sqrt{T}\,(\hat{\underline{\delta}} - \underline{\delta}) \overset{d}{\rightarrow} \mathrm{N}(\underline{0},\ \Omega_R). \tag{A1}$$

The asymptotic-covariance matrix for the five-goods nonlinear system with constraints is

$$\hat{\Omega}_R = \Omega - \Omega R'(R\Omega R')^{-1}R\Omega, \tag{A2}$$

where

$$\Omega = \text{plim}\ T^{-1}\left[\left(\frac{\partial \underline{f}'}{\partial \underline{\delta}_R}\right)\left(\hat{\Sigma}^{-1} \otimes I\right)\left(\frac{\partial \underline{f}}{\partial \underline{\delta}_R}\right)\right]^{-1}, \tag{A3}$$

the derivatives are evaluated at $\hat{\delta}_R$ (the restricted estimator), $R\underline{\delta} - \underline{r} = \underline{0}$ is the set of equality restrictions 12 - 16 for the $Q \times 1$ dimensional vector $\underline{\delta} = (\underline{\rho}, \underline{\gamma}, \underline{\beta}, \phi)'$, $\underline{f}$ are the nonlinear Equations 5 and 6 or 8 and 9 that include the nonlinear price index $P$ (defined in Equation 3), and $\partial \underline{f}/\partial \underline{\delta}$ is a $T \times Q$ matrix of partial derivatives with respect to $\underline{\delta}$ evaluated at $\hat{\underline{\delta}}$. The elements of $\Sigma$ can be consistently estimated by

$$\hat{\sigma}_{ij} = \frac{1}{T}\sum_t \hat{\varepsilon}_{it}\hat{\varepsilon}_{jt}, \tag{A4}$$

where $i, j = 1,..., 5$ (for the five meats) and $\hat{\varepsilon}_{it} \equiv \sum_h v_h \hat{w}_{ith}$.

We now derive the entropy-ratio statistic for the various parameters of the unknown distribution generating the data, which has a limiting $\chi^2$ distribution and may be used to obtain confidence intervals. Let $S_U$ be the objective (total entropy) value for the complete AIDS-GME model where *none of the parameters* $\underline{\delta} = (\underline{\zeta}, \underline{\gamma}, \underline{\beta}, \phi)'$ *is constrained*. Thus, $S_u$ is just the optimal value of Equation (10). Next, let $S_M$ be the entropy value of the constrained problem where all the parameters are constrained to be zero (or at the *center* of their supports). Thus, $S_M$ is the maximum possible value of the joint entropies (objective function). It can be obtained by maximizing Equation (10) subject to no constraints except for the requirements that all distributions are proper (effectively ignoring the data). Doing so yields the total entropy value of the two discrete, uniform distributions $\underline{q}$ and $\underline{w}$, so that

$$S_M = K \ln(D) + nT \ln(H), \qquad \text{(A5)}$$

where $K$ is the total number of parameters to be estimated, $D$ is the dimension of the support space for each one of the $K$ parameters (taken here to be the same for all $k$ = 1,2,...$K$, to simplify exposition), $n$ is the number of data equations, $T$ is the total number of observations, and $H$ is the dimension of the support space, $\underline{v}$, for each error $\varepsilon_i$.

Then, the entropy-ratio statistic for testing the null hypothesis that $H_0$: $\underline{\delta} = \underline{r}$ (a vector of constants) is

$$\mathscr{E}(\underline{\delta} = \underline{0}) = 2 \left| S_M \Big|_{\underline{\delta} = \underline{r}} - S_U \right|. \qquad \text{(A6)}$$

In Equation A6, we are testing the hypothesis that the coefficients equal $\underline{r}$ by comparing the restricted entropy value $S_M$ to the optimal value of the entropy $S_U$ when the coefficients are unrestricted. Under the four mild assumptions we made above (or see Gallant and White, 1988), $\mathscr{E}(\underline{\delta} = \underline{0}) \xrightarrow{d} \chi^2(K)$ when $H_0$ is true, or, similarly,

$$\mathscr{E} = 2 \left| S_M - S_U \right| \xrightarrow{d} \chi^2(m), \qquad \text{(A7)}$$

where $S_M$ is the unrestricted solution, $S_U$ is the restricted solution, and $m$ is the number of restrictions. We can test any other hypothesis of the form $H_0$: $\underline{\delta} = \underline{\delta}_0$ for all, or any subset, of the parameters. For example, let $n(n - 1)/2$ be the number of symmetry requirements on $\underline{\gamma}$ (Equation 15), then $H_0$, $\underline{\gamma}_{ij} = \underline{\gamma}_{ji}$. The entropy-ratio statistic is

$$\mathscr{E}(\gamma_{ij} = \gamma_{ji}) = 2\,|\,S_{\mathrm{M}} - S_{\mathrm{U}}\Big|_{\gamma_{ij} = \gamma_{ji}}\,|\,, \tag{A8}$$

where the restrictions are *not* imposed in $S_{\mathrm{M}}$ and are imposed in $S_{\mathrm{U}}$. If $H_0$ is true,

$$\mathscr{E}(\gamma_{ij} = \gamma_{ji}) \overset{d}{\rightarrow} \chi^2(n[n-1]/2)^2\,. \tag{A9}$$

We use the same line of reasoning as above (each constraint, or data point, represents additional potential information that may lower the value of the objective function but can never increase it) to derive a "goodness of fit" measure for our estimator:

$$R^* = 1 - S^*(\hat{\underline{q}})\,, \tag{A10}$$

where $R^* = 0$ implies no informational value of the data set, $R^* = 1$ implies perfect certainty or perfect in-sample prediction, and $S^*(\hat{\underline{q}})$ is the normalized entropy of the signal $\underline{q}$ (it does not include the noise component $\underline{w}$), which is defined as $S(\hat{\underline{q}})/[K \ln(D)]$.

**Table 1**
**Summary Statistics**

| | *All 7,897 Observations* | | *Random Sample of 1,000 Observations* | |
|---|---|---|---|---|
| | *Mean* | *SD* | *Mean* | *SD* |
| Expenditure Share of Beef Consumption | 0.388 | 0.342 | 0.369 | 0.335 |
| Expenditure Share of Pork Consumption | 0.123 | 0.237 | 0.127 | 0.245 |
| Expenditure Share of Chicken Consumption | 0.301 | 0.322 | 0.318 | 0.328 |
| Expenditure Share of Processed Consumption | 0.142 | 0.247 | 0.136 | 0.235 |
| Expenditure Share of Fish Consumption | 0.046 | 0.143 | 0.049 | 0.150 |
| Natural Log of Price of Beef | 9.552 | 0.272 | 9.544 | 0.278 |
| Natural Log of Price of Pork | 9.433 | 0.236 | 9.432 | 0.227 |
| Natural Log of Price of Chicken | 8.947 | 0.305 | 8.952 | 0.304 |
| Natural Log of Price of Processed Meat | 9.484 | 0.297 | 9.483 | 0.295 |
| Natural Log of Price of Fish | 9.365 | 0.467 | 9.352 | 0.459 |
| Natural Log of Expenditure on Meats | 16.025 | 0.891 | 16.030 | 0.871 |
| Household Lives in Urban Area | 0.622 | 0.485 | 0.611 | 0.488 |
| Household Head is Female | 0.128 | 0.334 | 0.120 | 0.325 |
| Household Head is in School | 0.021 | 0.142 | 0.025 | 0.156 |
| Household Head Attended: | | | | |
| Primary School | 0.516 | 0.500 | 0.548 | 0.498 |
| Secondary School | 0.176 | 0.381 | 0.173 | 0.378 |
| Preparatory or Vocational School | 0.078 | 0.268 | 0.068 | 0.252 |
| College | 0.095 | 0.294 | 0.082 | 0.275 |
| Share of Household Members | | | | |
| Between 0 and 5 Years Old | 0.139 | 0.170 | 0.141 | 0.173 |
| Between 6 and 15 | 0.212 | 0.209 | 0.222 | 0.214 |
| Between 16 and 28 | 0.257 | 0.247 | 0.260 | 0.244 |
| Between 29 and 45 | 0.206 | 0.204 | 0.199 | 0.194 |
| Between 46 and 60 | 0.104 | 0.196 | 0.106 | 0.185 |

**Table 2**
**GME Estimates of Nonlinear AIDS Meat Demand System**

| | *Beef* | *Pork* | *Chicken* | *Processed Meat* | *Fish* |
|---|---|---|---|---|---|
| Intercept | -0.8330* | -0.0435 | 1.2382* | 0.7290* | -0.0907 |
| Beef Price | -0.1348* | -0.0019 | 0.0728* | 0.0372 | 0.0267 |
| Pork Price | -0.0019 | 0.0537 | -0.0123 | -0.0110 | -0.0285* |
| Chicken Price | 0.0728* | -0.0123 | 0.0128 | -0.0674* | -0.0059 |
| Processed Meat Price | 0.0372 | -0.0110 | -0.0674* | -0.0241 | 0.0653* |
| Fish Price | 0.0267 | -0.0285* | -0.0059 | 0.0653* | -0.0576* |
| Expenditure | 0.1126* | 0.0190 | -0.0812* | -0.0625* | 0.0121* |
| Household is Urban | 0.0487* | -0.0453* | 0.0038 | -0.0203 | 0.0131 |
| Household Head is Female | -0.0250 | -0.0116 | -0.0177 | 0.0837* | -0.0294* |
| Household Head is in School | -0.0455 | -0.0397 | 0.1016 | -0.0560 | 0.0397 |
| Household Head Attended: | | | | | |
| Primary | 0.0388 | -0.0577 | 0.0109 | 0.0206 | -0.0126 |
| Secondary School | 0.0059 | -0.0820* | -0.0259 | 0.1043* | -0.0024 |
| Preparatory or Vocational School | 0.0452 | -0.1120* | 0.0055 | 0.0793* | -0.0181 |
| College | 0.0005 | -0.1075* | -0.0488 | 0.1577* | -0.0018 |
| Share of Household Members | | | | | |
| Between 0 and 5 Years Old | -0.2546* | -0.0077 | 0.1801* | 0.0733 | 0.0089 |
| Between 6 and 15 | -0.1572* | 0.0177 | 0.0899 | 0.0661 | -0.0165 |
| Between 16 and 28 | -0.0437 | 0.1403 | -0.0861 | 0.0246 | -0.0351 |
| Between 29 and 45 | -0.0886 | 0.0132 | -0.0222 | 0.1189* | -0.0212 |
| Between 46 and 60 | -0.0984 | -0.0138 | 0.0290 | 0.0757 | 0.0075 |
| $\phi$ | | | -4.3838 | | |

* We can reject the null-hypothesis of no effect at the 5% level: |asymptotic t-statistic| ≥ 1.96.
*Note*: Sample size is 1,000 observations.

**Table 3**
**Correlations between Observed and Predicted Shares**

| | *GME* | | *2-Step Estimator Heien and Wessells (1990)* | | *Least Squares** |
|---|---|---|---|---|---|
| | *Nonlinear* | *Linear* | *Nonlinear* | *Linear* | *Nonlinear* |
| Beef | 0.347 | 0.339 | 0.273 | 0.263 | 0.214 |
| Pork | 0.198 | 0.199 | 0.200 | 0.194 | 0.193 |
| Chicken | 0.286 | 0.282 | 0.209 | 0.202 | 0.172 |
| Processed Meat | 0.261 | 0.260 | 0.230 | 0.308 | 0.219 |
| Fish | 0.163 | 0.160 | 0.127 | 0.158 | 0.028 |
| Entire Demand System | 0.489 | 0.487 | 0.315 | 0.468 | 0.245 |

* Nonlinear least squares estimate of the AIDS model: Nonnegativity constraint is ignored.
*Note*: Sample size is 1,000 observations.

**Table 4**
**Estimated Hicks Price and Expenditure Elasticities**

| | *Beef* | *Pork* | *Chicken* | *Processed Meat* | *Fish* |
|---|---|---|---|---|---|
| Beef Price | -0.596 | 0.187 | 0.228 | 0.015 | 0.166 |
| | (0.138) | (0.068) | (0.077) | (0.064) | (0.041) |
| Pork Price | 0.550 | -0.418 | 0.081 | -0.059 | -0.153 |
| | (0.365) | (0.255) | (0.196) | (0.150) | (0.113) |
| Chicken Price | 0.263 | 0.034 | -0.402 | 0.111 | -0.006 |
| | (0.130) | (0.073) | (0.104) | (0.064) | (0.043) |
| Processed Meat Price | 0.041 | -0.052 | 0.255 | -0.706 | 0.462 |
| | (0.316) | (0.163) | (0.176) | (0.168) | (0.093) |
| Fish Price | 1.236 | -0.400 | -0.034 | 1.285 | -2.088 |
| | (1.132) | (0.436) | (0.725) | (0.330) | (0.280) |
| Expenditure | 1.305 | 1.149 | 0.745 | 0.542 | 1.247 |
| | (0.037) | (0.082) | (0.043) | (0.072) | (0.130) |

*Note*: The elasticities are calculated at the sample means. Asymptotic standard errors are reported in the parentheses.

**Table 5**
**Estimated Marshallian Own-Price Elasticities in Selected Meat Demand Studies**

| | *Period* | *Country* | *Beef* | *Pork* | *Chicken* | *Fish* |
|---|---|---|---|---|---|---|
| GME Model* | 1992 | Mexico | -1.08 | -0.56 | -0.64 | -2.15 |
| Cashin (1991) | 1960-90 | Australia | -1.24 | -0.83 | -0.47 | NA |
| Chalfant, Gray, & White (1991) | 1960-88 | Canada | -0.96 | -0.73 | -0.91 | -0.20 |
| Hayes, Wahl, & Williams (1990) | 1965-86 | Japan | -1.89 | -0.76 | -0.59 | -0.70 |
| Capps (1994) | 1960-88 | S. Korea | -0.939 | -0.647 | -0.470 | NA |
| Capps (1994) | 1968-91 | Taiwan | -1.158 | -0.919 | -0.278 | NA |
| Chalfant (1987) | 1947-87 | U. S. | -0.37 | -0.67 | -0.51 | -0.23 |
| Moschini & Meilke (1989) | 1967-87 | U. S. | -1.05 | -0.84 | -0.10 | -0.20 |
| Thurman (1989) | 1955-83 | U. S. | -0.11 | -0.73 | -0.41 | NA |
| Dahlgran (1989) | 1950-85 | U. S. | -0.66 | -0.58 | -0.60 | NA |

*Notes*:

• Only our GME study uses the nonlinear AIDS model, cross-sectional data, and imposes nonnegativity constraints. The other studies use maximum-likelihood and employ LA/AIDS models based on annual data unless otherwise stated.

• The asymptotic standard errors for the four GME elasticities shown are 0.114, 0.237, 0.124, and 0.234. The GME Marshallian elasticity for processed meat is -0.78 with an asymptotic standard error of 0.19.

• Cashin: Quarterly data. Elasticities are for 1985:4. Elasticities vary little over time. Lamb elasticity is -1.326.

• Chalfant et al.: Estimates of the AIDS model without concavity restriction imposed.

• Hayes et al.: Elasticity for Wagyu, Japanese domestic beef. Imported-beef elasticity is -0.46. Test that the domestic beef is a perfect substitute for the import beef is strongly rejected. Estimation period is not clear from the paper.

• Capps: Rotterdam Demand Model.

• Moschini-Meilke: Post-structural change, time-varying coefficients, based on quarterly data.

• Thurman: Quad-log demand system. Estimated price elasticities are for 1983 observations, with symmetry restriction imposed.

• Dahlgran: Rotterdam Demand Model.

**Table 6**
**Percentage Point Change in Share**
**Due to a Change in an Exogenous Variable**

| | *Beef* | *Pork* | *Chicken* | *Processed Meats* | *Fish* |
|---|---|---|---|---|---|
| *Change from 0 to 1* | | | | | |
| Urban | 4.9 | -4.5 | 0.4 | -2.0 | 1.3 |
| Female | -2.5 | -1.2 | -1.8 | 8.4 | -2.9 |
| Household Head is in School | -4.6 | -4.0 | 10.2 | -5.6 | 4.0 |
| Primary | 3.9 | -5.8 | 1.1 | 2.1 | -1.3 |
| Secondary | 0.6 | -8.2 | -2.6 | 10.4 | -0.2 |
| Preparatory | 4.5 | -11.2 | 0.6 | 7.9 | -1.8 |
| College | 0.1 | -10.8 | -4.9 | 15.8 | -0.2 |
| *Increase of 1 Person** | | | | | |
| Age < 5 | 1.4 | -2.4 | -0.6 | 1.3 | 0.3 |
| 5 ≥ Age < 15 | -2.8 | -0.7 | 3.1 | -0.1 | 0.4 |
| 15 ≥ Age < 28 | -1.2 | -0.3 | 1.7 | -0.2 | 0.01 |
| 28 ≥ Age < 45 | 0.7 | 1.7 | -1.2 | -0.9 | -0.3 |
| 45 ≥ Age < 60 | -0.04 | -0.4 | -0.2 | 0.7 | -0.07 |
| Age ≥ 60 | 1.4 | -0.6 | 0.2 | -1.3 | 0.3 |

* The typical family has five people. Initially, the family consists of one child less than 5 years old, one child between 5 and 15, two adults between 28 and 45, and one grandparent between 45 and 60 years old. We then calculate the change in shares resulting from adding one more person in one age group.

**Footnotes**

1. Deaton and Irish (1984), Keen (1986), and Blundell and Meghir (1987) use models based on the discrepancy between observed expenditure and actual consumption. We (and the other papers discussed here) concentrate on actual purchases.

2. Their experiments suggest, however, that the Nelson and Olsen two-stage estimator (which does not drop the constrained observations) performs reasonably well compared to ML. In contrast, Lee (1978) shows that, for a system of equations, the Amemiya two-stage estimator is more efficient than the Nelson and Olsen or Heckman two-stage estimators when the normality hypothesis is maintained.

3. It may be possible to estimate larger systems using a general method of moments estimator, however, this approach has not been used in a demand study and requires more a priori information than does the approach developed here.

4. Lee and Pitt (1986) propose using the dual of Wales and Woodland's method to transform binding nonnegativity constraints into nonbinding constraints based on virtual (shadow) prices. They estimate a three-input energy demand system using a translog cost function.

5. Other studies that examine how demands vary across demographic groups include Pollak and Wales (1992) and Blundell, Pashardes, and Weber (1993).

6. Alston and Chalfant (1987) contrast using expenditure or total income to estimate a separable meat demand systems using Australian data. Based on non-nested tests, they favored using expenditure. Their results are mixed on whether separability holds. Moschini, Moro, and Green (1994) find support for separability between meat and other foods in a Rotterdam model.

7. One reason why many earlier studies that used maximum likelihood estimate the linear model is that it is difficult to estimate the constant term in Equation 3 and achieve convergence with the nonlinear specification. We had no trouble with either of these problems when using GME to estimate the nonlinear specification.

8. To estimate an AIDS system with censored data, Deaton (1990) substituted the double-logarithmic demand functions that relate budget shares to the logarithm of the prices. But his new approach is not derived from utility theory but rather is an econometric necessity.

9. In our empirical application, the results are not very sensitive to the weight. Indeed, raising the weight from 0.5 to 0.9 on the systematic measure causes the estimated coefficients and the correlation between actual and estimated values to changes by less than 1%.

10. The quantity measures reported below also include own-produced and consumed goods as well as purchased goods.

11. We are very grateful to Michael Ferris and GAMS Corporation for helping us in converting our primal nonlinear maximization problem to a dual one, thereby substantially decreasing computational time. This method is described in Dirkse and Ferris (1995), and Ferris and Horn (1998). The reason we use a subsample of the data is that there is a space limitation on our computers when using the GAMS program. However, this limitation will be removed soon when GAMS makes available a more efficient nonlinear solver. This limitation can also be overcome by adding RAM (we had only 180 MB on a Sparc Ultra) or using an alternative solver such as MATLAB.

12. Explicitly, the $\chi^2$ values for the homogeneity test statistics are 0.516, 0.682, 0.180, 0.178, and 0.014 for beef, pork, chicken, processed meat, and fish respectively. None of these values are statistically significant and therefore we cannot reject the null hypothesis of

homogeneity. The symmetry test is (2 × 5,332.80) - (2 × 5,332.49) = 0.62 < $\chi^2_{10}(.01)$ = 23.209.

13. Heien, Jarvis, and Perali (1989) examine a nine-commodity food demand system (including meat) for Mexico. They also examine poultry, pork, and beef in more detail. As they lack price variation data, however, they cannot estimate elasticities. They describe their estimates as a demographically augmented Engel curve analysis.

14. Deaton (1988) uses cross-sectional data to estimate price elasticities for beef, other meat, cereal, and starches for the Ivory Coast. He does not, however, break down the other meat category in further detail. Wales and Woodland's (1983) study of meat demand using cross-sectional Australian data shows only how demand varies with demographic characteristics. As with Heien, Jarvis, and Perali (1989), Wales and Woodland could not estimate price effects because they did not observe variations in price.