

Research Article

Estimating Angstrom-Prescott Coefficients for India and Developing a Correlation between Sunshine Hours and Global Solar Radiation for India

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Received 21 June 2013; Accepted 16 July 2013

Academic Editors: B. Mwinyiwiwa and A. Stoppato

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The amount of solar energy that reaches the earth in one hour is sufficient to supply the world's energy needs for one year. Harvesting this energy efficiently is a huge challenge. In most countries including India, the number of observing stations is inadequate. Therefore, it is essential that some reliable mathematical models be developed to estimate the solar radiation for places where measurements are not carried out and for places where measurement records are not available. In this paper, Angstrom-Prescott model parameters are estimated for seven different sites in India, and a correlation is developed for India, which is found to be a good fit. Also a correlation is developed for predicting the solar radiation using only sunshine hour data.

1. Introduction

Solar radiation which we receive as heat and light can be converted to useful thermal energy or can be used for production of electricity by using solar photovoltaic cells or solar thermal power. Availability of reliable solar radiation data is vital for the success of solar energy installations in different sites of the country.

The kind of solar radiation data required depends on application and user. For example, monthly or daily averaged data are required for climatologic studies or to conduct feasibility studies for solar energy systems. Data for hourly (or shorter) periods are needed to simulate the performance of solar devices or during collector testing and other activities [1].

Knowledge of the local solar radiation is essential for the proper design of building energy systems, solar energy systems, and a good evaluation of thermal environment within buildings [2–7].

Systematic measurements of diffuse solar energy and the global (total) irradiation incident on a horizontal surface are usually undertaken by a national agency, which is the national meteorological office in many countries. There is a

relative abundance of sunshine duration data, and therefore it is a common practice to correlate the solar radiation with sunshine duration measurements.

2. Materials and Methods

In practical studies it is logical to consider that the solar radiation is directly proportional to the sunshine duration. The formulation of the proportionality can be derived from the measurements of the variables through scatter diagrams and most often by the application of statistical regression methods. The original models in solar energy expressed this relationship between solar radiation and the sunshine duration as linear. Such a pioneering relationship was presented by Angström in 1924 [8].

2.1. Angstrom Model. The original Angstrom-type regression equation related monthly average daily radiation to clear day radiation in a given location and average fraction of possible sunshine hours:

$$\frac{H}{H_c} = a + b \frac{n}{N}, \quad (1)$$

TABLE 1: Estimated angles of declination for various months in a year.

Month	d for <i>i</i> th day of the month	For the average day of the month		
		Date	d, day of the year	δ , angle of declination
January	<i>i</i>	17	17	-20.9
February	31 + <i>i</i>	16	47	-13.0
March	59 + <i>i</i>	16	75	-2.4
April	90 + <i>i</i>	15	105	9.4
May	120 + <i>i</i>	15	135	18.8
June	151 + <i>i</i>	11	162	23.1
July	181 + <i>i</i>	17	198	21.2
August	212 + <i>i</i>	16	228	13.5
September	243 + <i>i</i>	15	258	2.2
October	273 + <i>i</i>	15	288	-9.6
November	304 + <i>i</i>	14	318	-18.9
December	334 + <i>i</i>	10	344	-23.0

where H = monthly average daily global radiation ($\text{Wh}/\text{m}^2/\text{day}$), H_c = monthly average clear sky daily global radiation for the location in a given day, n = actual sunshine duration in a day, respectively, (hours), N = monthly average maximum possible bright sunshine duration in a day and also known as monthly mean length of the day in hours, a and b = empirical coefficients. These coefficients are location-specific coefficients referred to as fractions of extraterrestrial radiation on overcast days and on average days, respectively.

The ratio n/N is referred to as *cloudless index*. It gives information about atmospheric characteristics and conditions of the study area.

2.2. Angstrom-Prescott Model. A basic difficulty with (1) lies in the definition of the terms n/N and H_c . Because there may be problems in calculating clear sky radiation accurately, this model was modified by Prescott [9] in 1940 by replacing clear sky radiation with extraterrestrial radiation H_0 . This model popularly known as *the Angstrom-Prescott model* is the most commonly used model and is given by

$$\frac{H}{H_0} = a + b \frac{n}{N}, \quad (2)$$

where H_0 is the monthly average daily extraterrestrial radiation.

The ratio of solar radiation at the surface of the Earth (H) to extraterrestrial radiation (H_0), that is, H/H_0 , is called the *Clearness Index*. Values of the monthly average daily extraterrestrial radiation (H_0) is calculated from the following [10]:

$$H_0 = \frac{24 \times 3600 \times I_{sc}}{\pi} \times \left[1 + 0.033 \cos \left(\frac{360 \times d}{365} \right) \right] \times \left[\cos \phi \cos \delta \sin \omega + \frac{\pi \omega}{180} \sin \phi \sin \delta \right], \quad (3)$$

TABLE 2: Location of the selected meteorological stations [19].

Meteorological station	Latitude ($^{\circ}\text{N}$)	Longitude ($^{\circ}\text{E}$)	Altitude (m)
New Delhi	28° 39' 42"	77° 18' 32"	207
Jaipur	25° 23' 02"	72° 30' 32"	269
Kolkata	22° 34' 11"	88° 22' 11"	12
Jodhpur	26° 18' 00"	73° 01' 00"	224
Mumbai	18° 58' 30"	72° 49' 33"	27
Ahmedabad	23° 2' 00"	72° 35' 00"	53
Pune	18° 32' 00"	73° 52' 00"	560

where I_{sc} = solar constant with a value of 1367 Wm^{-2} , d = day of the year from January 1 to December 31 taking January 1st as 1, ϕ = latitude of the location, δ = declination angle, ω = sunset hour angle, and $\omega = \cos^{-1}(-\tan \phi \tan \delta)$.

Declination angle can be obtained by the equation given by Cooper in 1969 [11]:

$$\delta = 23.45 \sin \left[360 \left(\frac{284 + d}{365} \right) \right]. \quad (4)$$

Klein [12] in 1977 recommended average days of the various months and corresponding angle of declination as in Table 1.

The maximum possible sunshine duration N (hours) for a horizontal surface is given by

$$N = \frac{2\omega}{15}. \quad (5)$$

Many authors like Fritz and MacDonald [13], Black et al. [14], Mateer [15], and Samuel [16] estimated the values of “ a ” and “ b ” for different sites.

2.2.1. Estimating Parameters of Angstrom-Prescott Model for Different Sites in India Using Monthly Radiation Values. India is situated at the north of the equator between $8^{\circ}4'$ and $37^{\circ}6'$ north latitude and $68^{\circ}7'$ and $97^{\circ}25'$ east longitude [17]. In order to determine the value of H in (2) for India, sunshine hours data recorded over the period 1986–2000 are used for seven selected meteorological stations, namely, New Delhi, Jaipur, Kolkata, Jodhpur, Mumbai, Ahmedabad, and Pune [18]. Meteorological parameters for these locations are listed in Table 2.

In this work values of H_0 and N are calculated for each month using (3) and (5), respectively. A regression line is fit between H/H_0 and n/N using scatter diagram (see Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, and 14). The regression coefficients of the plotted line are the parameters “ a ” and “ b ” of the Angstrom-Prescott model. Here “ a ” is the intercept on the H/H_0 axis and “ b ” is the slope of the regression line. Results obtained are given in Table 3.

2.2.2. Estimates of Annual Global Radiation for Different Stations in India. Regression equations obtained previously for different meteorological stations are calculated using monthly average of daily solar radiation and sunshine data.

TABLE 3: Regression lines for selected meteorological stations in India.

Meteorological station	Angstrom-Prescott model
New Delhi	$H/H_0 = 0.6043 + 0.131 (n/N)$
Jodhpur	$H/H_0 = 0.1257 + 0.6002 (n/N)$
Jaipur	$H/H_0 = 0.3359 + 0.3328 (n/N)$
Ahmedabad	$H/H_0 = 0.1212 + 0.5822 (n/N)$
Mumbai	$H/H_0 = 0.1053 + 0.5698 (n/N)$
Kolkata	$H/H_0 = 0.0518 + 0.6443 (n/N)$
Pune	$H/H_0 = 0.0937 + 0.6315 (n/N)$

Regression line for New Delhi (1986–2000)

$$H/H_0 = 0.131 + 0.6043n/N$$

$$R^2 = 0.6539$$

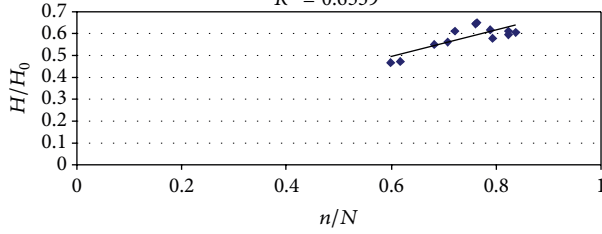


FIGURE 1

Comparison between measured and predicted solar radiation in New Delhi

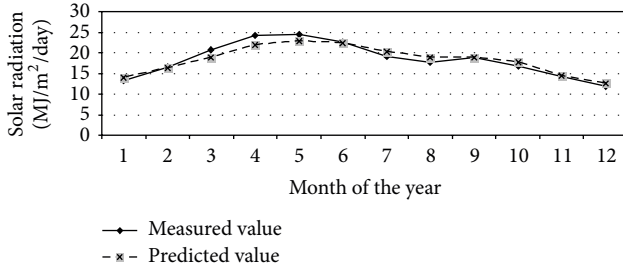


FIGURE 2

It is validated by evaluating the annual solar radiation for the same meteorological stations and comparing them with measured values of solar radiations for these stations. Table 4 shows estimates of annual global solar radiation for selected meteorological stations in India.

Observing Table 4 we find a good agreement between measured and predicted values. Plotting them on the same graph we obtained Figure 15 which shows the same.

2.2.3. *Correlation between Sunshine Hours and Global Solar Radiation in India.* Using the average of the correlation obtained in Table 4, we get the Angstrom-Prescott model for India as

$$\frac{H}{H_0} = 0.1382 + 0.5564 \frac{n}{N}. \quad (6)$$

2.2.4. *Comparison Techniques.* Differences between estimated and measured values of solar radiation are determined

Regression line for Jodhpur (1986–2000)

$$H/H_0 = 0.1257 + 0.6002n/N$$

$$R^2 = 0.6492$$

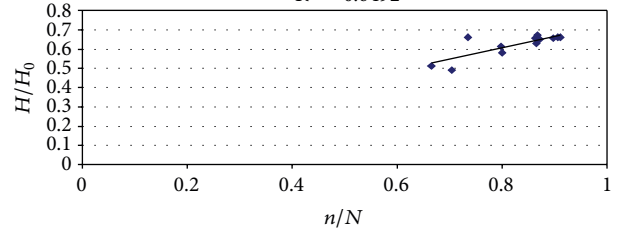


FIGURE 3

Comparison between measured and predicted values in Jodhpur

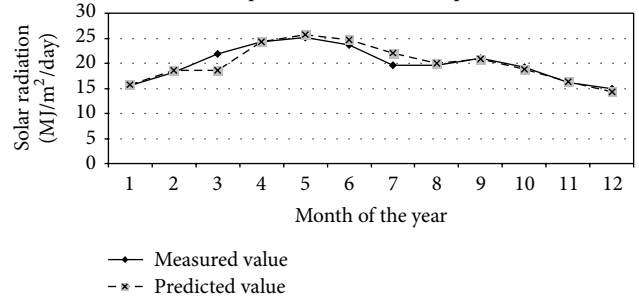


FIGURE 4

by the Mean Bias Error (MBE), the Root Mean Square Error (RMSE), and the Mean Percentage Error (MPE), given by the following respective expressions:

$$MBE = \frac{\sum (H_{cal} - H_{meas})}{n},$$

$$RMSE = \sqrt{\frac{\sum (H_{cal} - H_{meas})^2}{n}}, \quad (7)$$

$$MPE = \frac{1}{n} \sum \frac{H_{cal} - H_{meas}}{H_{meas}} \times 100,$$

where n is the number of data pairs.

Values of regression constants, the correlation coefficients “ R ,” and the values of the MBE, RMSE, and MPE for the seven meteorological stations are summarized in Table 5.

2.3. *Correlation between Parameter of the Angstrom-Prescott Model and Sunshine Ratio n/N .* Since the number of meteorological stations is not adequate in India, even the places where these stations are available, many of them have no arrangement for measuring solar radiation. But comparatively it is easier to measure actual sunshine duration for different places, and therefore this data is more widely available across India. So if we have correlation that uses only sunshine hour data for calculating parameter of the Angstrom-Prescott model, then it will be possible to estimate solar radiation even for those places where solar radiation data is not available.

TABLE 4: Estimates of annual global solar radiation for selected meteorological stations in India.

Location	Average annual n/N	a	b	$a + b$	Annual H_{meas} $MJ m^{-2} day^{-1}$	Annual H_{cal} $MJ m^{-2} day^{-1}$
Delhi	0.74	0.1310	0.6043	0.7353	18.25	18.44
Jodhpur	0.82	0.1257	0.6002	0.7259	19.97	20.09
Jaipur	0.79	0.3359	0.3328	0.6687	19.42	19.62
Ahmedabad	0.81	0.1212	0.5822	0.7034	19.30	19.56
Mumbai	0.76	0.1053	0.5698	0.5751	18.25	18.40
Kolkata	0.70	0.0518	0.6443	0.6951	16.17	16.48
Pune	0.76	0.0963	0.6315	0.7252	19.51	19.80
Average		0.1382	0.5664	0.7046		

TABLE 5: Estimated values of Angstrom-Prescott parameters “ a ,” “ b ,” the correlation coefficient “ R ,” and the values of the MBE, RMSE, and MPE for the seven meteorological stations.

Location	Correlation coefficient R	Estimated Angstrom-Prescott parameters			MBE	RMSE	MPE %
		a	b	$a + b$			
Delhi	0.81	0.1310	0.6043	0.7353	-0.105	1.1243	0.3408
Jodhpur	0.81	0.1257	0.6002	0.7259	0.0758	1.2166	0.3650
Jaipur	0.51	0.3359	0.3328	0.6687	-0.0583	2.1703	1.1058
Ahmedabad	0.88	0.1212	0.5822	0.7034	-0.0517	1.4768	0.7692
Mumbai	0.91	0.1053	0.5698	0.5751	-0.0517	1.4531	0.7100
Kolkata	0.84	0.0518	0.6443	0.6951	0.0242	1.3019	0.0025
Pune	0.96	0.0937	0.6315	0.7252	0.0092	0.7528	0.2058

Regression line for Jaipur (1986–2000)

$$H/H_0 = 0.3359 + 0.3328n/N$$

$$R^2 = 0.2566$$

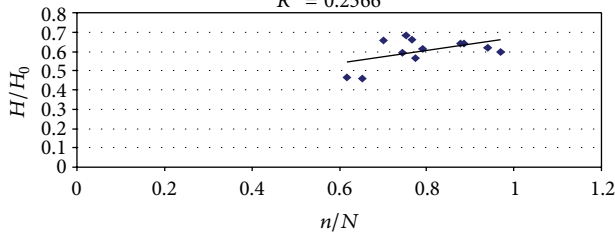


FIGURE 5

Regression line for Ahmedabad (1986–2000)

$$H/H_0 = 0.1212 + 0.5822n/N$$

$$R^2 = 0.7767$$

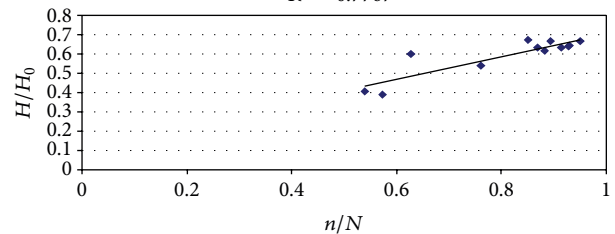
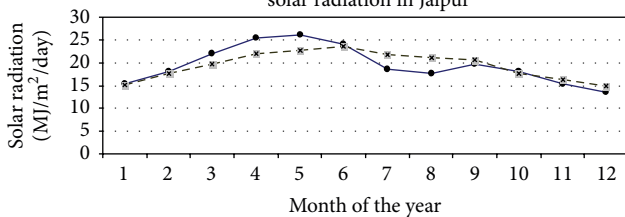


FIGURE 7

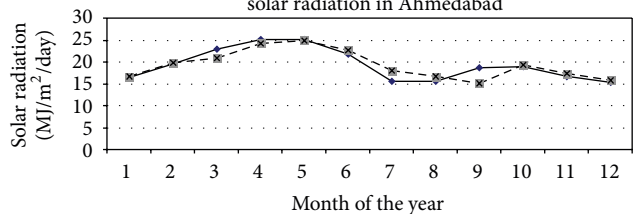
Comparison between measured and predicted solar radiation in Jaipur



—●— Measured value
- * - Predicted value

FIGURE 6

Comparison between measured and predicted solar radiation in Ahmedabad



—●— Measured value
- * - Predicted value

FIGURE 8

Regression line for Mumbai (1986–2000)
 $H/H_0 = 0.1053 + 0.5698n/N$
 $R^2 = 0.8352$

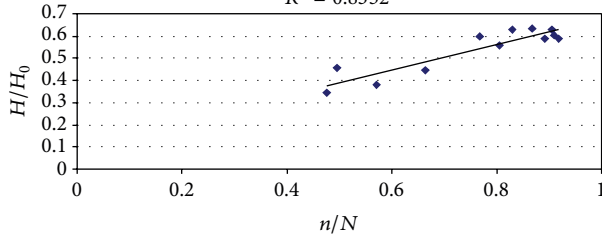


FIGURE 9

Regression line for Pune (1986–2000)
 $H/H_0 = 0.0937 + 0.6315n/N$
 $R^2 = 0.9258$

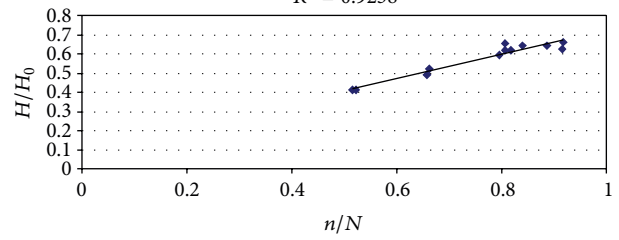


FIGURE 13

Comparison between measured and predicted solar radiation in Mumbai

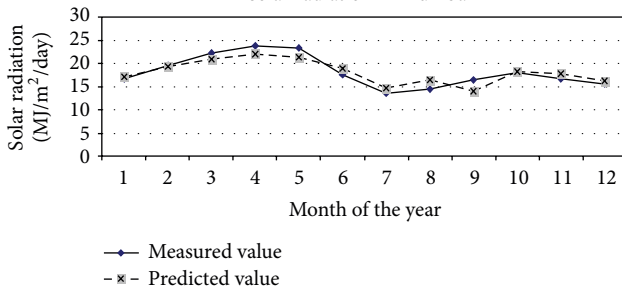


FIGURE 10

Comparison between measured and predicted solar radiation in Pune

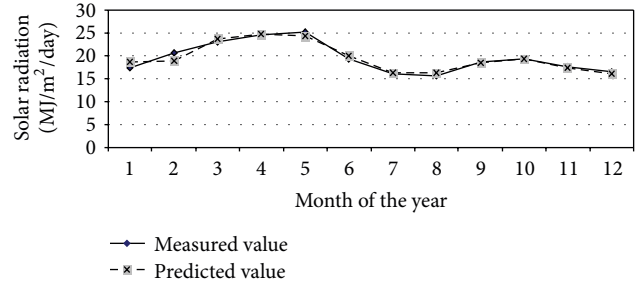


FIGURE 14

Regression line for Kolkata (1986–2000)
 $H/H_0 = 0.0518 + 0.6443n/N$
 $R^2 = 0.713$

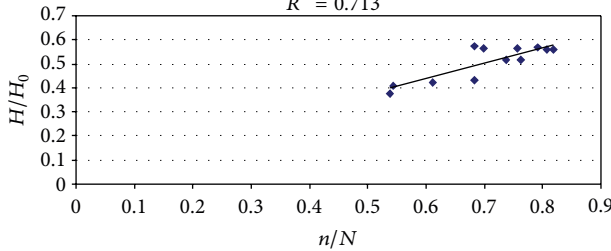


FIGURE 11

Comparison between measured and predicted solar radiation in Kolkata

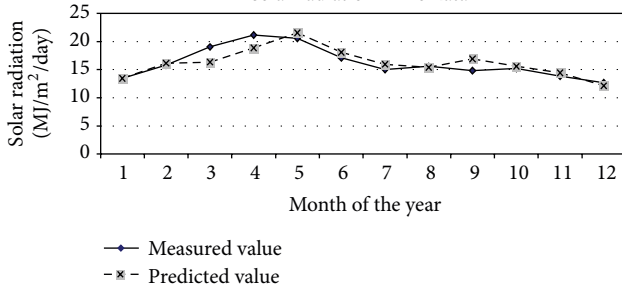


FIGURE 12

To obtain this correlation parabolic regression curve is fit between “a” versus “n/N” and “b” versus “n/N” data given in Table 4, and we obtained the following correlation equation for India (see Figures 16 and 17):

$$a = -17.222\left(\frac{n}{N}\right)^2 + 27.18\left(\frac{n}{N}\right) - 10.533, \tag{8}$$

$$b = 18.676\left(\frac{n}{N}\right)^2 - 29.395\left(\frac{n}{N}\right) + 12.098.$$

The importance of this correlation lies in the fact that using this correlation solar radiation can be estimated for every location in India, even at the places where we do not have a system to measure solar radiation. Thus, coefficients of the Angstrom-PreScott model for India can be estimated using (8), even if only sunshine data is available.

3. Results and Discussion

Results of data analysis show that maximum solar radiation is observed in May and minimum solar radiation is observed in December.

The sum of the regression coefficients (a + b) as shown in Table 4 is interpreted as the transmissivity of the atmosphere for global solar radiation under perfectly clear sky conditions [20]. A clear day means n/N = 1, and then by (2)

$$\frac{H}{H_0} = a + b. \tag{9}$$

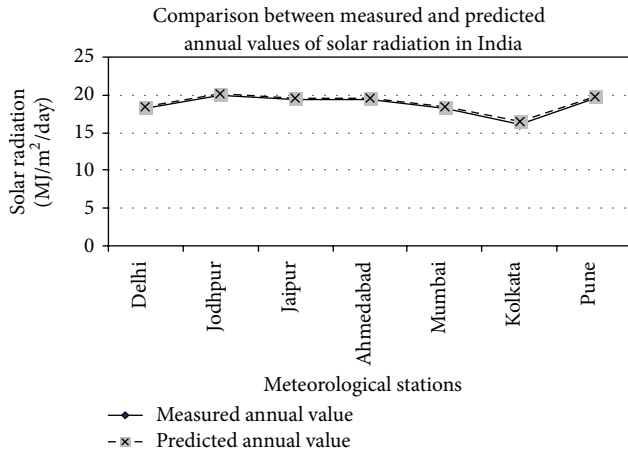


FIGURE 15

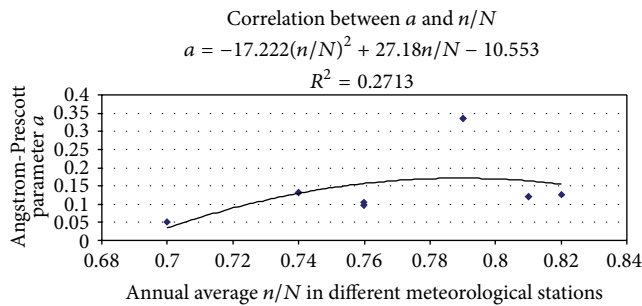


FIGURE 16

Also for a completely overcast day, $n/N = 0$, which reduces (2) to the form

$$\frac{H}{H_0} = a. \quad (10)$$

Thus parameter “ a ” can be interpreted as the transmissivity of an overcast atmosphere, which essentially accounts for the diffuse component.

It is obvious from Table 5 that neither “ a ” nor “ b ” varies with latitude or altitude in any systematic manner. However, the values of the sum of the regression constants $a + b$, which represent the maximum Clearness Index for $n/N = 1$, are found to be almost equal for various meteorological stations.

The values of the regression constants a and b are in close agreement with those given for 101 locations in the paper of Martínez-Lozano et al. [21], ranging from 0.06 to 0.44 for “ a ” and from 0.19 to 0.87 for “ b ”.

It can be easily observed from the Table 5 that the values of correlation coefficients for different meteorological stations are more than 0.80 (except for Jaipur). It means a good fit between the Clearness Index H/H_0 and the relative possible number of sunshine hours n/N .

Also, analysing Table 4 it is found that there is a good agreement between the measured and calculated values of global radiation for the different meteorological station. It is further established by very low values of MBE, RMSE, and MPE as shown in Table 5. The percentage error is less than

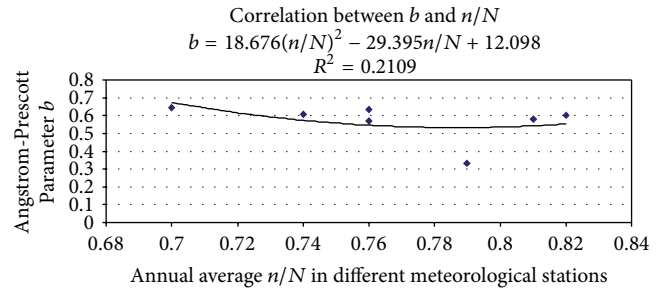


FIGURE 17

$\pm 2\%$ at any of the meteorological stations. This suggests that the Angstrom-Preccott correlations model is a good model to estimate the solar radiation in India.

Table 3 gives the summary of the first-order Angstrom-Preccott correlations model that has been developed for use in estimating values of H at each of the respective seven meteorological stations.

Also based on these results summarized in Tables 3, 4, and 5 a linear regression model for estimating solar radiation in India is developed, which comes out to be

$$\frac{H}{H_0} = 0.1382 + 0.5564 \frac{n}{N}. \quad (11)$$

Also, a new parabolic correlation has been developed so that the coefficient of the Angstrom-Preccott models can be estimated even if only sunshine hour data is available;

$$\begin{aligned} a &= -17.222 \left(\frac{n}{N} \right)^2 + 27.18 \left(\frac{n}{N} \right) - 10.533, \\ b &= 18.676 \left(\frac{n}{N} \right)^2 - 29.395 \left(\frac{n}{N} \right) + 12.098. \end{aligned} \quad (12)$$

4. Conclusion

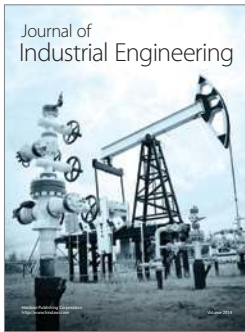
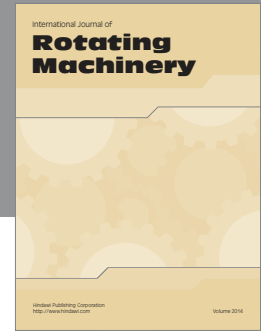
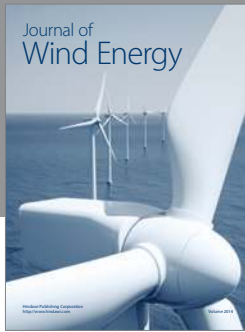
Energy is the primary requirement for development of any country. The world is heading towards energy crisis as fossil fuel reserves are depleting rapidly and demand for energy is increasing exponentially. Therefore, we need to exploit the widely available nonconventional energy resource “The Sun.” Designing a solar system is crucial for fulfilling the energy requirement of the people. In this direction the knowledge of the amount of irradiance reaching any point on the Earth’s surface is very essential.

The Angstrom-Preccott models that are developed in this study (see Table 3, (6), and (8)) may help the Indian Government to develop realistic energy policies and programmes based on sound scientific knowledge.

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