Estimating Expected Losses and Liquidity Discounts Implicit in Debt Prices

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November 10, 2001
Revised June 20, 2002

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Abstract

This paper provides an empirical implementation of a reduced form credit risk model that incorporates both liquidity risk and correlated defaults. Liquidity risk is modeled as a convenience yield and default correlation is modeled via an intensity process that depends on market factors. Various different liquidity risk and intensity process models are investigated. Firstly, the evidence supports a non-zero liquidity premium that is firm specific, reflecting idiosyncratic and not systematic risk. Secondly, the credit risk model with correlated defaults fits the data quite well with an average R^2 of .87 and a pricing error of only 1.1 percent.

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1. Introduction

Given the recent exponential growth in the credit derivatives market [see Risk (2000)] and the regulatory induced need to account for credit risk in the determination of equity capital [see Jarrow and Turnbull (2000b)], credit risk modeling has become a topic of current and paramount interest. Although credit risk pricing theory has exploded [see Jarrow (1998) for a review], the empirical estimation of these models has lagged behind [see Duffie and Singleton (1997), Madan and Unal (1998), Duffee (1999) and Duffie, Pedersen, Singleton (2000)]. To help rectify this imbalance, this paper provides a comprehensive empirical implementation of a reduced-form credit risk model that includes both liquidity risk and correlated defaults. The reduced-form credit risk model implemented is that contained in Jarrow (2001) where a liquidity discount is modeled as a convenience yield and correlated defaults arise due to the fact that a firm's default intensities depend on common macro-factors.

The data used for this investigation is the University of Houston's Fixed Income Database consisting of monthly bid prices taken from Lehman Brothers over May 1991 – March 1997. Twenty different firms' debt issues are investigated where the firms are chosen to stratify various industry groupings.

Five different liquidity premium models were investigated differing in their dependence on various market-wide variables including the spot interest rate, the return on an equity market index, and the equity market index's volatility. These variables were chosen to capture systematic market risks related to interest rates, equities, and the market's volatility. Similarly, the intensity process was allowed to be dependent on the spot rate of interest and the cumulative return on an equity market index.

Overall, the evidence supports the model quite well. First, the best performing liquidity premium model appears to be firm specific and not dependent on market-wide variables. This result is consistent with liquidity risk reflecting only firm specific/ idiosyncratic and not systematic risk. Second, the best fitting reduced form credit risk model fits the data quite well with stationary estimated parameters, an average R^2 of .87, and an average percentage pricing error of only .011.

The previous literature estimating reduced form credit risk models include Duffie and Singleton (1997), Madan and Unal (1998), Duffee (1999), and Duffie, Pedersen, Singleton (2000). Duffie and Singleton (1997) estimate swap spreads, Madan and Unal (1998) estimate yields on thrift institution certificates of deposit, and Duffie, Pedersen, Singleton (2000) estimate credit and liquidity spreads for Russian debt. Duffee's (1999) paper is closest to our approach. Using the same bond data, he estimates a reduced form credit risk model where both the default intensity and the default free term structure follow a square root process. The default intensity also depends on the spot rate of interest, so his model captures correlated defaults. Our paper differs from Duffee (1999) in three ways: (i) we use Guassian processes for the default intensity and the default free term structure, (ii) our default intensity has an additional factor – it also depends on the cumulative excess return per unit of risk on an equity market index, and (iii) we explicitly model liquidity risk. Our observation period and firm sample also significantly differ from that in Duffee (1999).

An outline of this paper is as follows. Section 2 introduces both the notation and the reduced form credit risk model. Section 3 provides a description of the data. The parameter estimation is performed in section 4. Section 5 tests the time series stationarity of the parameter estimates, section 6 provides an analysis of the expected loss parameters, while section 7 studies the relative performance of the five different liquidity discount models. Section 8 discusses the absolute performance of the credit risk model studied, while section 9 concludes the paper.

2. The Model Structure

This section introduces the notation and briefly summarizes the reduced form credit risk model contained in Jarrow (2001). Trading can take place anytime during the interval $[0,\overline{T}]$. Let $\{(\Omega, F_{\overline{T}}, P),(F_t : t \in [0,\overline{T}]\}$ be a filtered probability space satisfying the usual conditions.[1] This filtered probability space represents the underlying randomness and information generated in the economy. Traded are default-free zero-coupon bonds and risky (defaultable) zero-coupon bonds of all maturities. Markets are assumed to be frictionless with no arbitrage opportunities, but they can be incomplete with illiquidities present.

Let $p(t,T)$ represent the time t price of a default-free dollar paid at time T where $0 \le t \le T \le \overline{T}$. The instantaneous forward rate at time t for date T is defined by $f(t,T) = -\partial \log p(t,T) / \partial T$. The spot rate of interest is given by $r(t) = f(t,t)$.

Consider a firm issuing risky debt. Let $v(t,T)$ represent the time t price of a promised dollar to be paid by this firm at time T where $0 \le t \le T \le \overline{T}$. The debt is risky because if the firm defaults prior to time T, then the promised dollar may not be paid. Let the random variable τ represent the first time that this firm defaults ($\tau > \overline{T}$ is possible if the firm does not default). Then,

$$N(t) = 1_{\{\tau \le t\}} = \begin{cases} 1 & if \quad \tau \le t \\ 0 & otherwise \end{cases} \tag{1}$$

[1] See Protter (1990, page 3) for a discussion of the usual conditions.

denotes the point process indicating whether or not default has occurred prior to time t. We assume that this point process has an intensity $\lambda(t)$ with respect to the given filtration.[2] The time t intensity process, $\lambda(t)\Delta$, gives the approximate probability of default for this firm over the time interval $[t, t+\Delta]$.[3]

If default occurs, we let the zero-coupon bond receive a *fractional recovery* of $\delta(\tau)v(\tau-,T)$ dollars where $0 \leq \delta(\tau)$ and $\tau-$ represents an instant before default.

Under the assumption of no arbitrage, standard arbitrage pricing theory implies that there exists a probability Q equivalent to P such that [4]

$$p(t,T) = E_t\left(e^{-\int_t^T r(u)du}\right) \tag{2}$$

and

$$v(t,T) = 1_{(t<\tau)} E_t\left(\delta(\tau)v(\tau-,T)e^{-\int_t^\tau r(u)du} 1_{(t<\tau\leq T)} + 1\, e^{-\int_t^T r(u)du} 1_{(T<\tau)}\right) \tag{3}$$

where $E_t(.)$ is conditional expectation with respect to Q at time t.

The risky debt value is composed of two parts. The first part is the present value of the promised payment in default. The second part is the present value of the promised payment if default does not occur. Duffie and Singleton (1999) show that expression (3) can be alternatively written as (3a):

$$v(t,T) = 1_{(t<\tau)} E_t\left(e^{-\int_t^\tau [r(u)+(1-\delta(u))\lambda(u)]du}\right). \tag{3a}$$

This expression shows that the risky zero-coupon's value can alternatively be computed by taking the discounted expectation of the promised dollar, discounting at a rate augmented by the expected loss $(1-\delta(u))\lambda(u)$ per unit time. As pointed out by Duffie and Singleton (1999), it is important to emphasize that expression (3a) enables only the estimation of the expected loss and not its separate components.

In this empirical investigation, almost all of the U.S. government debt and all the corporate debt studied are coupon bearing. Consequently, we need to price coupon-bearing bonds. First, for the U.S. government debt, a coupon bond is defined to pay coupons of C_j

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[2] See Jeanblanc and Rutkowski (2000) for conditions under which such an intensity process exists.
[3] The intensity process is defined under the risk neutral probability. This statement will become clear below.
[4] See Jarrow and Turnbull (1995). No arbitrage guarantees the existence, but not the uniqueness of the probability Q. Without any additional hypotheses on the economy, the uniqueness of Q is equivalent to markets being complete, see Battig and Jarrow (1999). In incomplete markets, equilibrium (additional hypotheses) guarantees the uniqueness of Q. The uniqueness of Q is essential for estimation.

dollars at times t_j for $j = 1, ...,n$ where $t_n = T$ is the maturity date. The last coupon at the maturity date is assumed to include the principal repayment. Let the time *t* price of this default free coupon bond be denoted by *B(t,T)*. Standard no arbitrage arguments give the price of the default free coupon bond as a portfolio of default free zero-coupon bonds, i.e.

$$B(t,T) = \sum_{j=1}^{n} C_{t_j} p(t,t_j) \ . \tag{4}$$

Next, consider a risky coupon-bearing bond. Using similar notation, except for the bond's price which will be denoted by a script "$\mathcal{B}$", the risky coupon bond is defined to pay coupons of C_j dollars at times t_j for $j = 1, ...,n$ where $t_n = T$ is the maturity date. The coupon bond is risky because if the firm defaults prior to the maturity date, the remaining coupons (and principal) may not be paid in full. In default, we assume that the coupon bond is worth the fractional recovery amount of $\delta(\tau)\mathcal{B}(\tau-,T)$. Other recovery rate assumptions are possible [see Jarrow and Turnbull (2000a)].

Under this recovery rate structure, the value of a risky coupon-bearing bond at time *t*, denoted by $\mathcal{B}(t,T)$, is equivalent to the cost of the following portfolio of risky zero-coupon bonds:

$$\mathcal{B}(t,T) = \sum_{j=1}^{n} C_{t_j} v(t,t_j) \ . \tag{5}$$

The coupon bond prices in both expressions (4) and (5) are for bonds trading in perfectly liquid markets. Although this is a reasonable approximation for U.S. government debt, it is not so for U.S. corporate debt. Thus, we need to introduce an adjustment for liquidity risk in the pricing of corporate debt.

Let $\mathcal{B}_I(t,T)$ denote the price of an otherwise identical risky coupon bond trading in an illiquid market. The subscript "*I*" indicates the bond's price in an illiquid market. In an illiquid and incomplete market, Jarrow (2001) shows that there exists a stochastic process[5] $\gamma(t,T)$ such that

$$\mathcal{B}_I(t,T) = e^{-\gamma(t,T)}\mathcal{B}(t,T) \ . \tag{6}$$

The argument is that when there are shortages, the risky bond cannot be shorted,[6] and hence $\mathcal{B}_I(t,T) > \mathcal{B}(t,T)$ is possible. The reverse case occurs when there is an oversupply. The process

[5] The process $\gamma(\omega,t,T)$ for $\omega \in \Omega$ is adapted to the filtration (F_t).

[6] The bond cannot be shorted because to short, one has to first borrow the bond. The bond shortage makes this precondition impossible to satisfy. Repurchase agreements are often used to short both government and corporate debt.

$\gamma(t,T)$ has the interpretation of being a convenience yield for holding the risky debt. When there is a shortage and one cannot readily buy the risky bond, then $-\gamma(t,T) \geq 0$. When there is a glut and one cannot readily sell the risky bond, then $-\gamma(t,T) \leq 0$. In this context, liquidity risk is analogous to a convenience yield from holding an illiquid bond in one's portfolio. The convenience yield is sometimes positive or negative, depending upon market conditions.

To obtain an empirical formulation of the above model, more structure needs to be imposed on the stochastic nature of the economy. Following Jarrow (2001) we consider an economy that is Markov in two state variables: the spot rate of interest and the cumulative excess return per unit of risk on an equity market index. We next introduce the stochastic evolution of these two state variables.

For the spot rate of interest, we use a single factor model with deterministic volatilities, sometimes called the extended Vasicek model, i.e.

(Spot Rate Evolution)

$$dr(t) = a_r\left[\bar{r}(t) - r(t)\right]dt + \sigma_r dW(t) \tag{7}$$

where $a_r \neq 0$, $\sigma_r > 0$ are constants, $\bar{r}(t)$ is a deterministic function of t chosen to match the initial zero-coupon bond price curve,[7] and $W(t)$ is a standard Brownian motion under Q initialized at $W(0) = 0$. The evolution of the spot rate is given under the risk neutral probability Q.

The second state variable is related to an equity market index, denoted by $M(t)$. The evolution for the equity market index is assumed to satisfy a geometric Brownian motion with drift $r(t)$ and volatility σ_m. The correlation coefficient between the return on the market index and changes in the spot rate is denoted by φ.

(Market Index Evolution)

$$dM(t) = M(t)\big(r(t)dt + \sigma_m dZ(t)\big) \tag{8}$$

where σ_m is constant, and $Z(t)$ is a standard Brownian motion under Q initialized at $Z(0) = 0$ correlated with $W(t)$ as $dZ(t)dW(t) = \varphi \; dt$ with φ a constant.

Our second state variable is $Z(t)$. We see here that $Z(t)$ is a measure of the cumulative excess return per unit of risk (above the spot rate of interest) on the equity market index.

Given the evolutions of the state variables, we next need to specify their relationship to the bankruptcy parameters, the recovery rate and the liquidity discount. This is the task to which we now turn. First, for the default parameters, we assume that:

(Expected Loss: A Function of the Spot Rate and the Market Index)

[7] In particular, $\bar{r}(t) = f(0,t) + \left(\partial f(0,t)/\partial t + \sigma_r^2(1 - e^{-2a_r t})/2a_r\right)\Big/a_r$ for $a_r \neq 0$.

$$(1-\delta(t))\lambda(t) \equiv max\{a_0 + a_1 r(t) + a_2 Z(t),\ 0\} \quad where$$
$$a_0 \geq 0 \quad and \quad a_1, a_2 \quad are \quad constants. \tag{9}$$

In this formulation, the expected loss per unit time (i.e., the (pseudo) probability of default per unit of time multiplied by one minus the recovery rate) is assumed to be a linear function of the state variables *r(t)* and *Z(t)* as long as this linear combination is non-negative, zero otherwise. Note that in this formulation of the expected loss process, the recovery rate is allowed to be a stochastic.

For analytic tractability in the empirical implementation, we drop the maximum operator in expression (9). In this case, as the recovery rate is non-negative, this implies that negative default rates ($\lambda(t)$ < 0) are possible. If the likelihood of ($\lambda(t)$ < 0) is small, this simplification should provide a reasonable approximation to expression (9). Unfortunately, when the intensity process is negative, the default distribution is no longer a proper probability distribution [see Bremaud (1981)]. Nonetheless, given the tractability of the subsequent expressions, and the difficulty of the numerical inversion without a closed form solution, we empirically investigate the validity of this linear approximation.

Given these expressions, it is shown Jarrow (2001) that the default free zero-coupon bond and the risky zero-coupon bond's price can be rewritten as:

$$p(t,T) = e^{-\mu_1(t,T)+\sigma_1^2(t,T)/2} \tag{10}$$

and

$$v(t,T) = 1_{(t<\tau)}\, p(t,T)e^{-a_0(T-t)-a_1\mu_1(t,T)+\left(2a_1+a_1^2\right)\sigma_1^2(t,T)/2} \bullet$$
$$e^{-a_2Z(t)(T-t)+(1+a_1)a_2\varphi\eta(t,T)+[T-t]^3 a_2^2/6} \tag{11}$$

where

$$\mu_1(t,T) = \int_t^T f(t,u)du + \int_t^T b(u,T)^2 du\Big/2, \quad \sigma_1^2(t,T) = \int_t^T b(u,T)^2 du,$$
$$b(u,t) = \sigma_r\left(1-e^{-a_r(t-u)}\right)\Big/a_r, \quad and \tag{12}$$
$$\eta(t,T) = -(\sigma_r/a_r^3)[1-e^{-a_r(T-t)}] + (\sigma_r/a_r^2)e^{-a_r(T-t)}(T-t) + (\sigma_r/2a_r)[T-t]^2.$$

A direct substitution of these zero-coupon bond price formulae into the coupon bond price expressions (4) and (5) gives the analytical expressions used in this empirical investigation, with one exception. To complete the empirical specification of the risky debt model, we need to specify an explicit functional form for the liquidity premium.

To empirically separate the estimates of the liquidity premium $\gamma(t,T)$ from the expected loss $(1-\delta(t))\lambda(t)$, the time to maturity behavior of the liquidity premium and the expected loss needs to be utilized. First note that if the firm is not in default at time *t*, then as $T \to t$, all the default related terms in the exponent of the risky zero-coupon bond's price in expression (11) approach zero. This follows because the probability of default by the risky firm goes to zero as $T \to t$, so that $v(t,T) \to 1$. Hence, the expected loss component in the risky-zero coupon bond's price is proportional to time to maturity.

In contrast, the liquidity premium's time to maturity behavior is, in general, not proportional to time to maturity. Indeed, liquidity risk is usually thought of as being determined by factors that are independent of the maturity of the bond, including the size of the bond issue, market sentiment concerning its re-trade value, and the size of institutional holdings. If these beliefs are valid, then the liquidity premium contains a fixed component that is not proportional to time to maturity. To the extent that the liquidity premium contains only this fixed component, the subsequent methodology enables us to empirically separate the liquidity premium from the expected loss. To the extent that this is not true, any time to maturity component of the liquidity premium will be confounded into our estimate of the expected loss.

Based on this discussion, as a joint hypothesis to the empirical methodology, we assume that the liquidity premium is independent of the debt's time to maturity:

(Liquidity Discount)

$$\gamma(t,T) = \gamma_0 + \gamma_1 \sum_{j=t-4}^{t} r(j)/5 + \gamma_2 \sigma_m^2(t) + \gamma_3 \sum_{j=t-4}^{t} \left(\frac{M(j) - M(j-1)}{M(j-1)} \right) / 5 \qquad (13)$$

where $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are constants. [8]

First, the right side of expression (13) is independent of the time to maturity *(T-t)*. Secondly, the liquidity discount is assumed to be an affine function of three market-wide variables: the 5-day average spot rate, the volatility of an equity market index, and the 5-day average return on the equity market index. These variables were chosen to capture systematic market risks related to interest rates, equities, and the market's volatility. Although other firm specific variables correlated with debt market liquidity could have been included like the bid/ask spread, volume traded, or volume outstanding, unfortunately, none of this information was available in our bond database. Given this omission, however, the reader should be aware that the liquidity premium estimates obtained might incorporate residual model error. This limited formulation, however, does enable us to investigate whether liquidity risk is either firm specific/idiosyncratic or systematic by testing whether ($\gamma_1 = \gamma_2 = \gamma_3 = 0$).

Substitution of expression (13) into the risky coupon-bond price formula (6) completes the empirical specification of the reduced form credit risk model. As seen, analytic formulas are

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[8] This implies that $\gamma(t,T) \equiv \gamma(t)$ so that

$$\mathcal{B}_l(t,T) = e^{-\gamma(t)} \mathcal{B}(t,T) = \sum_{j=1}^{n} C_{t_j} e^{-\gamma(t)} v(t,t_j) = \sum_{j=1}^{n} C_{t_j} v_l(t,t_j) \text{ where } v_l(t,T) \equiv e^{-\gamma(t)} v(t,T).$$

available for both the default free and risky debt issues. These analytic formulae are the basis for the empirical estimation procedure described in the next sections.

3. Description of the Data

The data used for this investigation is the University of Houston's Fixed Income Database. This data consists of monthly bid prices for various fixed income securities, including U.S. Treasuries and U.S. corporate debt. The bid prices are taken from Lehman Brothers trading sheets on the last calendar day in each month. For each security included, various identifying information is also provided including embedded options, seniority status, and whether the bid price is transaction based or matrix priced, see Warga (1999) for additional details.

The time period covered in this study is May 1991 – March 1997. The University of Houston Fixed Income Data terminates after March 1997 and no further updates are available. For the U.S. Treasury securities, all outstanding bills, notes and bonds are included in this data and, therefore, included in this study. Being such a large database (containing over 2 million entries), the potential for data errors is quite large. Indeed, a careful examination of the data confirmed this suspicion. Hence, we filtered the data to remove obvious data errors. We excluded Treasury bonds with matrix prices and inconsistent or suspicious issue/dated/maturity dates and coupons. Lastly, using a median yield filter of 2.5%, we also removed U.S. Treasury debt listings whose yields exceeded the median yield by this percent. After filtering, there are approximately 29,100 U.S. Treasury prices left in the sample set.

For the corporate bond price data, we first excluded all debt issues that contained embedded options (call provisions, extendible bonds, convertible bonds, etc.) and that were matrix priced. Matrix prices are linear interpolations of bid prices for other traded issues. These prices are not good approximations to traded prices and therefore omitted from the analysis. These two filters left only bid prices on straight coupon bearing bonds.

From these debt issues, we selected twenty different firms chosen to stratify various industry groupings: financial, food and beverages, petroleum, airlines, utilities, department stores, and technology. Within each industry, the firms were chosen to ensure that at least three debt issues were available sometime during the sample period. Only debt classified as senior, senior debentures, and senior notes are included in the subsequent investigation.

The twenty firms included in this study are provided in Table 1. Their industry association, and the starting and ending date for each of the bond price observations are noted. For each firm, on any particular day in the observation period, a bid price may be missing from the data. For this reason, different firms can have different starting dates and different numbers of bond issues at specific dates in the observation period. The number of distinct bonds available on the first date in the estimation period is also provided. For example, AMR Corporation has only two senior debt issues outstanding on this date, while Merrill Lynch & Co. has fourteen. The Moodies and S&P's ratings for each company's debt issues at the start of our sample period (May 24, 1991) are also included. These ratings did not change over our sample period. As seen, our sample consists of only investment grade debt. Using S&P's ratings, the debt ranges from AAA for Shell Oil Company's to BBB for Union Oil of California.

For the equity market index, we used the S&P 500 index with daily observations obtained from CRSP. For parameter estimation of the state variable processes a daily spot rate is needed. Since the fixed income data provides only monthly observations, we use daily observations of the 3 month T-bill yield available from CRSP as well.

4. Estimation of the State Variable Process Parameters

To implement the estimation of the default and liquidity discount parameters, we first need to estimate the parameters for the state variable processes (*r(t),Z(t)*).

a. Spot Rate Process Parameter Estimation

The inputs to the spot rate process evolution are the forward rate curves over an extended observation period (*f(t,T)* for all months $t \in$ *January 1975 – March 1997*) and the spot rate parameters (a_r, σ_r). We discuss the estimation of these inputs in this section.

For the estimation of the forward rate curves, a two-step procedure is utilized. First, for a given time *t*, the discount bond prices (*p(t,T)* for various *T*) are estimated by solving the following minimization problem:

$$\text{choose} \quad (p(t,T) \text{ for all relevant } T \le max\{T_i : i \in I_t\})$$

$$\text{to minimize} \sum_{i \in I_t} \left[B_i(t,T_i) - B_i(t,T_i)^{bid} \right]^2 \tag{14}$$

where I_t is an index set containing the various U.S. Treasury bonds, notes and bills available at time *t*, $B_i(t,T_i)$ is the model price (expression (4)) for the i^{th} bond with maturity T_i as a function of (*p(t,T)*), and $B_i(t,T_i)^{bid}$ is the market bid price for the i^{th} bond with maturity T_i.

The discount bond prices' maturity dates T coincide with the maturities of the Treasury bills, and the coupon payment and principal repayment dates for the Treasury notes and bonds.

Step 2 is to fit a continuous forward rate curve to the estimated zero-coupon bond prices (*p(t,T)* for all $T \le max\{T_i : i \in I\}$). We use the maximum smoothness forward rate curve as developed by Adams and van Deventer (1994) and refined by Janosi and Jarrow (2002). Briefly, we choose the unique piecewise, 4th degree polynomial with the left and right end points left "dangling" that minimizes $\int_t^{max\{T_i : i \in I_t\}} \left| \partial^2 f(t,s) / \partial s^2 \right| ds$.

For the spot rate parameters (a_r, σ_r) estimation, the procedure follows that used in Janosi, Jarrow, Zullo (1999). However, the procedure is extended to include rolling estimation of the parameters using only information available at the time of the estimation. This rolling procedure makes the parameter estimates (a_{rt}, σ_{rt}) dependent on time *t* as well.

The procedure is based on an explicit formula for the variance of the default-free zero-coupon bond prices derived using expression (7). For $\Delta = 1/12$ (a month), the expression is:

$$var_t [log(P(t+\Delta,T) / P(t,T)) - r(t)\Delta] = \left(\sigma_{rt}^2 \left(e^{-a_{rt}(T-t)} - 1 \right)^2 / a_{rt}^2 \right) \Delta . \qquad (15)$$

First we fix a time to maturity $T-t \in$ {3 months, 6 months, 1 year, 5 years, 10 years, the longest time to maturity of an outstanding Treasury bond closest to 30 years}. Then, we fix a current date $t \in$ {May 1991 – March 1997}. Going backwards in time 60 months (5 years), we compute the sample variance, denoted s_{tT}, using the smoothed forward rate curves previously generated. Note that the sample variance depends on both the date of estimation and the bond's maturity.

Then, for each date $t \in$ *{May 1991 – March 1997}*, to estimate the parameters (σ_{rt}, a_{rt}) we run a nonlinear regression

$$s_{tT} = \left(\sigma_{rt}^2 \left(e^{-a_{rt}(T-t)} - 1 \right)^2 / a_{rt}^2 \right) \Delta + e_{tT} \qquad (16)$$

across the bond time to maturities $T-t \in$ *{ ¼, 1/2, 1, 5, 10, longest time to maturity closest to 30}* where e_{tT} is the error term.

The parameter estimates are:

	Min	Mean	Max	StdDev
a_{rt}	0.0109	0.0282	0.0428	0.0101
σ_{rt}	0.0100	0.0109	0.0117	0.0004

The R^2 for each of these monthly non-linear regressions (not reported) exceeded .99 in all cases. The spot rate volatility (σ_{rt}) is nearly constant over this period. In contrast, the mean reverting parameter (a_{rt}) appears to be more volatile.

To test for the time series stability of these parameter estimates, a unit root test was performed.[9] For the volatility σ_{rt}, the test rejects a unit root, implying the time series is stationary. In contrast, one cannot reject a unit root for the mean reverting parameter a_{rt}.

b. Market Index Parameter Estimation

Using the daily S&P 500 index price data and the 3-month T-bill spot rate data, we need to estimate the parameters of the market index process (σ_m, φ) as given in expression (8) and the cumulative excess return on the market index as given by *Z(t)* in expression (8). This section discusses this estimation.

This estimation of the parameters (σ_m, φ) is based on daily data ($\Delta = 1/365$). As before, the procedure involves a rolling estimation using only information available at the time of the estimation. This procedure implies that the parameter estimates depend on time *t* as well, denoted by (σ_{mt}, φ_t).

For a given date $t \in$ *{May 24,1990 – March 31,1997}*, we go back in time 365 business days and estimate the time dependent sample variance and correlation coefficients (σ_{mt}, φ_t) using the sample moments, i.e.

$$\sigma_{mt}^2 = var_t\left(\frac{M(t)-M(t-\Delta)}{M(t-\Delta)}\right)\frac{1}{\Delta} \quad \text{and} \quad \varphi_t = corr_t\left(\frac{M(t)-M(t-\Delta)}{M(t-\Delta)}, r(t)-r(t-\Delta)\right). \quad (17)$$

The parameter estimates are:

	Min	Mean	Max	StdDev
σ_{mt}	0.0982	0.1261	0.1897	0.0270
φ_t	-0.2706	-0.0990	0.1262	0.1142

[9] A unit root test is performed to check for the stationarity of the time-series parameter estimates. Stationarity means that the time series fluctuates around a fixed mean. Hence, this is basically a random walk test. We run the following augmented Dickey-Fuller test with a time trend,

$$\Delta y_t = \mu + \rho y_{t-1} + \beta t + \sum_{j=1}^{p-1} c_j \Delta y_{t-j} + \varepsilon_t$$

Since there was no time trend and no augmented terms, we reduced the equation to the Dickey-Fuller test with no trend and augmented terms:

$$\Delta y_t = \mu + \rho y_{t-1} + \varepsilon_t.$$

Therefore, we only report the DF test statistics for the stationarity of the parameters. The test statistic for the DF test is given as the t-statistic of the ρ coefficient in the regression:

$$\Delta y_t = \mu + \rho y_{t-1} + \varepsilon_t.$$

The null hypothesis is $H_0: \rho = 0$. If we accept the null hypothesis, we have unit root. Otherwise, we accept the stationarity of the parameters [see Greene (1993) for more discussion]. The unit root test statistics are: σ_r (– 2.6348) and a_r (– 1.1632).

The market volatility is relatively constant between .1 and .2 over this observation period. The correlation coefficient appears to be more variable. As before, to test for the stability of the parameters a unit root test was performed. The results show that a unit root can be rejected at the 90 percent confidence level for the market volatility but not for the correlation coefficient.[10]

Given the parameter estimates for the market volatility (σ_{mt}) and the daily 3-month Treasury bill yields, the *Z(t)* process is computed using a discretized approximation to expression (8), starting the series on May 24, 1991.

$$Z(t) = Z(t-\Delta) + [\log M(t)/M(t-\Delta) - r(t-\Delta)\Delta + (1/2)\sigma^2_{m(t-\Delta)}\Delta] / \sigma_{m(t-\Delta)}\sqrt{\Delta}$$
$$for\ t \geq \Delta\ and\ Z(0) = 0.$$

Finally, we computed the market-wide risk variables for the liquidity discount process (expression (13)). These include the 5-business day average spot rate, the 5-business day average return on the S&P 500 index, and for consistency, a 5-business day rolling estimate of the volatility for the S&P 500 index. The 5-day rolling estimate of the volatility for the S&P 500 index differs from the market volatility (σ_{mt}) estimate generated previously only in the number of the past observations used. The previous estimate used 365 past observations, while the current estimate only uses 5.

c. Default and Liquidity Discount Parameter Estimation

Given the state variables (*r(t),Z(t)*) parameters as estimated in the previous sections, this section presents the default and liquidity discount parameter estimation. The default parameters are the expected losses per unit time from the intensity process (expression (10)): a_0, a_1, a_2; and the liquidity discount parameters from expression (13): $\gamma_0, \gamma_1, \gamma_2, \gamma_3$. These parameters are constants. However, since we utilize a rolling estimation procedure at each date *t* (the details of which are discussed below), the parameter estimates will depend on *t* as well, denoted by $(a_{0t}, a_{1t}, a_{2t}, \gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t})$.

For the estimation of the default and liquidity discount parameters, a non-linear regression procedure is implemented using both cross-sectional and past time series observations of bond prices. Table 1 contains the number of bonds available on the first date in the estimation period. At each time *t*, only a few bonds of any single firm with a particular seniority status trade (and have bid prices). For example, Fleet Financial Group only has three outstanding senior bonds with no embedded options on the first date in the observation period. This is the cross-sectional price data at time *t*. These are too few observations to estimate the seven different default and liquidity parameters. In order to augment these observations we use the past seven

[10] The unit root test statistics are: σ_m (– 3.9407) and φ (– 1.3479). See footnote 9 for a more detailed explanation of the unit root test.

months of bond prices as well. This is the time series data. As before, only information available at time *t* is used in the estimation procedure. Augmenting the data in this way increases the sample size significantly, for example for Fleet Financial Group, using the past seven months of data increases the sample size from 3 to 16 observations.

For a particular firm at time *t*, let I_t be the index set containing the bid prices of the firm's debt issues over the current month and the past seven months[11]. The twenty company debt issues involved in the estimation are given in Table 1.

The time *t* default intensity and liquidity discount parameters are estimated by solving the following minimization problem[12]:

$$\text{choose } (a_{0t}, a_{1t}, a_{2t}, \gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t}) \quad \text{to minimize } \sum_{i \in I_t} \left[\mathcal{B}_{li}(t,T_i) - \mathcal{B}_{li}(t,T_i)^{bid} \right]^2 \qquad (18)$$

subject to the constraint that $a_{0t} \geq 0$,

where $\mathcal{B}_{li}(t,T_i)$ is the model price (expression (6)) for the i^{th} bond with maturity T_i as a function of $(a_{0t}, a_{1t}, a_{2t}, \gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t})$ and $\mathcal{B}_{li}(t,T_i)^{bid}$ is the market bid price for the i^{th} bond with maturity T_i.

The non-negativity constraint for a_{0t} is included in order to keep the intensity process positive in the case when both a_{1t}, a_{2t} are zeros.

As noted, our default and liquidity parameter estimation involves a two-step procedure. The first step computes the state variable parameter estimates using their sample moments. The second step uses these parameter estimates in the non-linear regression (18). This second step introduces additional sampling error into the estimation procedure. An alternative approach would have been to use a standard GMM procedure, estimating all of the parameters (including the state variable parameters) in a single step. We choose not to use the GMM procedure for two reasons. One, GMM is only asymptotically consistent, and in our situation, we do not know its small sample properties. Two, our two-step procedure is also asymptotically consistent (under certain error structures for the parameters), but simpler to implement.

Five different models for the liquidity discount are estimated. The models differ with respect to the number of independent variables included in the liquidity discount. Model 1 has all the liquidity parameters set equal to zero: $\gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 0$. This is the base case with no liquidity discount. Model 2 is the test for liquidity risk being idiosyncratic or systematic:

[11] The first estimation is for December 1991. The data starts 8 months earlier in May 1991.
[12] Matlab's non-linear regression procedure is used to do this minimization. All parameter estimates are initialized at zero for the numerical procedure.

$\gamma_1 = \gamma_2 = \gamma_3 = 0$. Model 3 has $\gamma_2 = \gamma_3 = 0$, model 4 has $\gamma_3 = 0$, and model 5 includes all of the liquidity discount parameters. These five models are nested and a relative comparison of model performance is subsequently provided.

For example, Xerox's (symbol xrx) parameters are estimated each month from May 1991 – March 1997 for a total of 64 regressions, giving 64 time series observations of ($a_{0t}, a_{1t}, a_{2t}, \gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t}$). For each month in the observation period, on average, 39 bonds were used in the time *t* non-linear regression. Graphs of the time series parameters for Xerox are contained in Figures 1a and 1b. Figure 1b contains Xerox's expected loss per unit time parameter estimates (a_{0t}, a_{1t}, a_{2t}). As depicted, Xerox's expected loss appears to be declining over the observation period for all five liquidity discount models estimated. As suggested, Xerox's default risk is declining. In contrast, its credit rating is unchanged (see Table 1).

Figure 1a contains Xerox's liquidity discount factor ($exp(-\gamma(t,T))$) for the five different models using expression (13) and the parameter estimates ($\gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t}$). For the first half of the observation period, for models 2 – 5, Xerox's debt appears to have traded at a premium (greater than one), while over the last half of the observation period it traded at a discount. A premium implies that Xerox's bonds were in excess supply, while a discount implies that Xerox's bonds were in shortage (relative to a liquid market).

To summarize the time series estimates across all models and across all times, Table 2 provides the average values for the point estimates of the liquidity discount and the expected loss parameters. The average number of bonds used in each monthly regression, the average R^2, and the number of monthly regressions are also included. For each firm, on any particular day in the observation period, a bid price may be missing from the data. For this reason, different firms can have different starting dates and different numbers of bond issues at different dates in the observation period. Table 1 provides the estimation periods for the different companies' debt issues.

The values in Table 2 are averages over the number of days in the observation period (May 1991 – March 1997) for which the non-linear regression estimates of the parameters are computed.[13] Table 3 provides the t-scores[14] for the averages of the parameter estimates as contained in Table 2 as well as the average P-scores for the coefficients (across the number of regressions). The P-score is the probability of rejecting the null hypothesis (that the coefficient is zero), when it is true. Summary statistics for various F-tests are also provided. The first F-test has

[13] This is not to be confused with the number of observations used in the time *t* regression for a particular firm. At the time *t* regression, we use all bond prices for issues of a particular seniority over the past eight months.

[14] The t-score is adjusted to reflect the fact that the regressions contain overlapping time intervals. The justification for the t-score adjustment is contained in the appendix.

as its null hypothesis ($a_0 = a_1 = a_2 = 0$). Given are the average P-scores of the F-tests (across the number of regressions). The remaining F-tests have as their null hypothesis the liquidity premium models 1 – 5 (i.e., $\gamma_j = 0$ for all *j* where $j < i$ for model *i*). Model 2 is a test for liquidity risk being idiosyncratic versus systematic. The average P-scores of these F-tests (across the regressions) are provided. The next sections discuss these statistics and various tests for the relative performance of the various models.

5. Analysis of the Time Series Properties of the Parameters

Under the assumed model structure, the default and liquidity premium parameters ($a_{0t}, a_{1t}, a_{2t}, \gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \gamma_{3t}$) should be constant across time. Given measurement error in the input data (bond prices and the state variable parameters) and its effect on the debt parameter estimates, we test the hypothesis that the time series variation in these parameters is solely due to random (white) noise. Alternatively stated, we test to see if the parameter estimates follow a random walk around a given mean. A unit root test is used in this regard.[15]

Table 4 contains a summary of the unit root test statistics across model types. For model 1, no liquidity premium, around 50 percent of the different firm's default parameters accept the null hypothesis of a unit root, rejecting the time series stationarity of the parameters. Firms with at least two thirds of the default parameters accepting a unit root include ten out of eighteen companies (financials: spc, bt; food and beverages: cce; airlines: amr; utilities: txu; petroleum: mob; department stores: dh; technology: ek, txn, ibm). Model 1's estimated parameters appear to have a stationarity problem.

The liquidity premium corrects this non-stationarity. Indeed, inclusion of the liquidity premium significantly improves the stationarity of the default parameter estimates. For models 2 – 5, a majority of the default parameter coefficients reject a unit root. The more complex the model, the more unit root rejections occur. The best performing model in this regard is model 5. For model 5, for almost all companies, the liquidity premium and default parameters reject the hypothesis of a unit root. Although the unit root test is a weak test for stationarity, these rejections are consistent with the validity of the pricing model and the necessity of including a liquidity premium.

6. Analysis of the Expected Loss

As previously mentioned, the average expected loss parameters are contained in Table 2 with t-scores and average values for the P-scores provided in Table 3. The firms' estimates are presented in industry groupings for easy comparison.

First to be noticed in Table 2 is that the fit of the non-linear regressions are quite high for all firms and all models with an R^2 of .72 or higher, with one exception. The exception is Fleet

Financial Group (flt) with an R^2 between .41 - .66, depending upon the model selected. These high R^2's are obtained in spite of the fact that the number of bonds used in each regression is quite small – between 16 and 132 for all firms. The low R^2 for Fleet Financial Group can be explained by the fact that it had only an average of 16 bond observations for each regression.

Second, it is interesting to examine the signs of the coefficients for the expected loss parameters. The signs of a_1 and a_2 indicate the sensitivity of the firm's default likelihood to changes in the spot rate and the equity market index, respectively. For example, for Security Pacific Corp (spc), for all five models considered, we see that $a_1 > 0$ and $a_2 < 0$. This implies that as spot rates increase, the firm's default risk increases; and as the equity market index increases, the firm's default risk declines. These signs are consistent with simple economic intuition.

This economic intuition is based on the effect of higher interest rates on the firm's ability to service its short-term floating rate debt. As spot rates rise, given fixed operating income flows, debt servicing costs increase, thereby making it more likely for the firm to default. This intuition, "at first blush", appears to be inconsistent with the structural approach to risky debt valuation. For example, in Merton (1974; p. 457, equ.(26)), we see that as spot rates increase, the credit spread declines (implying default risk declines). The reason for this difference is easily explained. In Merton's structural model, the firm has only the equivalent of long-term debt on its balance sheet (a single fixed maturity discount bond). The previous economic intuition is formulated for firms with more complex liability structures that contain a significant amount of short-term floating rate debt. Consequently, it is possible for different firms to exhibit different interest rate sensitivity to default based on the relative importance of floating rate versus fixed rate borrowings in their capital structure.

The signs of these coefficients appear to be stable across time for any particular company's debt, but they differ across industries and they sometimes differ across companies within an industry. An example of different signs within an industry is for the department stores grouping, where Sears Roebuck and Company (s) and Wal-Mart Stores, Inc. (wmt) have contrasting signs for both the interest rate and market index variables. These differences reflect different capital structures (e.g. relative dependence on floating rate versus fixed rate debt) and different customer pools (customer income correlation with the market index variables).

Glancing now at Table 3, we discuss the statistical significance of these point estimates. First, we investigate the joint significance of all three of the default parameters (a_0, a_1, a_2). The F-test for model 1 provides the appropriate test. As seen, for 19 out of 20 companies, the average P-score is less than 5 percent, accepting the joint statistical significance of the three parameters

[15] See footnote 9 for a more detailed explanation of the unit root test.

(a_0, a_1, a_2). The exception is Fleet Financial Group (flt) with an average P-score of .3362. Fleet Financial Group has the smallest sample size – number of bonds (16 observations).

We next investigate the individual t-scores, given the joint statistical significance of the credit risk model. With respect to the constant in the expected loss function, a_0, its statistical significance varies across model types. For model 1, no liquidity premium, it is significant for 10 out of the 20 companies. For model 2 it is significant for 18 out of 20 companies (the exceptions are txu and txn). For models 3 – 5, a_0 is never significant. The absence of individual significance in models 3 – 5 is due to both the increased number of parameters to estimate and the increased multi-collinearity of the independent variables. Although the multi-collinearity increases the standard error of the estimates, the estimates remain unbiased. The average P-scores across the individual regressions confirm the above conclusions.

With respect to the spot rate coefficient in the expected loss, a_1, the significance of its t-scores also varies across model types. For model 1, no liquidity premium, it is significant in only 8 out of 20 cases. For model 2, it is significant in 18 out of 20 cases. The exceptions are Carolina Power and Light (cpl) and Dayton Hudson (dh). But, in both of these cases, the average P-scores are less than 5 percent, indicating significance in this alternative test. This is strong evidence that the expected loss depends on the spot rate of interest. In contrast, for the more complex liquidity discount models 3 – 5, a_1 is never significant. Again, this is due to the increased number of parameters to estimate and the increased multi-collinearity of the independent variables.

Finally, with respect to the market index coefficient in the expected loss, a_2, only 5 out of the 20 firms are significant for model 1, and none are significant for models 2 – 5. Given the other two expected loss coefficients, it appears that the expected loss does not depend on the market index. The only near exception to this statement is for Merrill Lynch (mer) in the case of model 3. Here the t-score is nearly significant (-1.4089) and the average P-score is low (.1719). This could be due to chance, but it also is consistent with the conjecture that an industry specific index should be included, rather than a market index. For example, for the petroleum industry grouping, oil prices may have been a better index choice; and for the utilities industry, electricity prices may have been a better choice. This is consistent with the weak evidence available from Merrill Lynch because the market index is probably highly correlated with an industry index for investment banking. This conjecture, however, awaits subsequent research.

The impact of these different parameter estimates on the one-year default probabilities for each firm across model types can be gleaned from column 2 in Table 5. Column 2 in Table 5 provides the average one-year default probabilities (computed under the risk neutral measure) across the different regressions *assuming a constant recovery rate of 0.5.* Except for Fleet Financial Group, due to its small sample size, the one-year default probabilities do not appear to differ significantly across the liquidity premium models 2 – 4. For each firm, the biggest difference occurs between model 1 and models 2 – 4, i.e. no liquidity discount versus a liquidity discount. As seen, the inclusion of a liquidity discount appears to have a significant impact on the estimated probability of default. The necessity (or lack thereof) of a liquidity discount is addressed in the next section.

7. Analysis of the Liquidity Discount Model

This section studies the relative performance of the five liquidity discount models. First, for each firm and for each model type, Table 3 contains the average P-scores for the F-statistic testing the joint nullity of the parameters ($\gamma_0, \gamma_1, \gamma_2, \gamma_3$). Model 1 tests for the inclusion of a liquidity discount. Model 2 tests whether liquidity risk is idiosyncratic or systematic. Models 3 - 5 test for the sensitivity of liquidity risk to interest rate, equity market, and equity market volatility risk, respectively.

These F-tests confirm the necessity of including a liquidity discount. For model 2, the average P-score is less than 30 percent for 12 out of 20 cases. For models 2 – 5 the average P-score is less than 50 percent for all companies except five (flt, luv, cpl, txu, wmt). This is strong evidence consistent with the importance of including a liquidity discount into the credit risky model structure.

The t-scores and the average P-scores for the liquidity coefficients, across regressions, are also contained in Table 3. This simple t-test checks for the significance of each coefficient, given that the other coefficients are included in the regression. For models 2 – 5, almost all of the coefficients are significantly different from zero. This is true using either the t-score or average P-score statistics. This evidence confirms the F-test analysis previously discussed and the importance of including a liquidity discount in the credit risk model.

Three additional statistical analyzes were performed to investigate relative model performance. The first was the unit root test for parameter stability discussed in section 5 above. As noted there, the liquidity coefficients' time series properties are inconsistent with a unit root. The best performing model on this metric is model 5. However, this is a weak test. Hence, two additional tests were performed. For each firm and for each model's regression, both a root mean squared error statistic (RMSE) and a generalized cross validation statistic (GCV) are computed. The RMSE statistic measures the "average" pricing error between the model and the market bid. It is an in-sample goodness of fit measure. As with all in-sample goodness of fit measures, a potential problem with RMSE is that it may provide a biased picture of the quality of model performance due to a model over-fitting the noise in the data. With in-sample estimation, usually the more parameters utilized, the better the fit. To avoid this problem, we provide an out-of-sample test. The second GCV test statistic is an out-of-sample goodness of fit measure that is predictive in nature.[16] The lower the GCV statistic, the better the out-of-sample model fit.

The average RMSE and GCV statistics for each firm and model are contained in Table 5. As indicated, the RMSE is lower for models 2 – 5 than for model 1. This is as expected, however, because the RMSE is an in-sample statistic and models 2 – 5 have more parameters. More importantly, the out-of-sample GCV statistic is lower for model 2 than it is for model 1. This again confirms the importance of including a liquidity discount into the model structure.

In summary, the best performing model based on either RMSE or the GCV statistic is model 2. This evidence is consistent with liquidity risk being idiosyncratic and not systematic risk. The importance of additional company and industry specific variables is an interesting topic for future research.

The impact of these different liquidity discount models on the aggregate estimate of the liquidity discount for each firm is contained in column 7 of Table 5. The biggest impact occurs between model 1 (no liquidity discount) and models 2 – 5. Across the different liquidity discount models 2 – 5, the differential impact on the estimate of the liquidity discount does not appear to be that significant.

8. Absolute Performance of the Credit Risk Model

The above analysis was for the relative performance of models 1 – 5. The absolute performance of the models is much more difficult to ascertain. Conceptually, this is because the default process' parameters are unobservable. The default parameters are from a distribution whose realization is a binary variable – default, no default. And, for most firms (in fact, for all firms in our sample), the default realization has not occurred. It is possible to compare the

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[16] To understand this out-of-sample GCV statistic, we first consider the CV method in terms of its forecasting ability. Assuming that the random errors have zero mean, the true regression curve g has the property that, if an observation Y is taken at a point t, the value g(t) is the best predictor of Y in terms of MSE. Thus a good choice of estimator $\hat{g}(t)$ would be one that gives a small value of $\left(Y - \hat{g}(t)\right)^2$ for a new observation Y at point t. Therefore, we can write the CV as: $CV = \sum_{i=1}^{n}\left(Y_i - \hat{g}^{(-i)}(t_i)\right)^2 / n$ where $\hat{g}^{(-i)}(t_i)$ is the slope estimate without using the ith observation. Since we have cross-sectional and time series data, CV is computationally intensive. Indeed, we need to do the NNLS estimation nxn times. To reduce this computation to only n times we use the GCV statistic: $GCV = \sum_{i=1}^{n}\left(Y_i - \hat{g}(t_i)\right)^2 \Big/ n\left(1 - \frac{1}{n}tr(A)\right)^2$ where $\sum_{i=1}^{n}\left(Y_i - \hat{g}(t_i)\right)^2 / n$ is the SSE of the cross-section regression. A is equal to $X\left(X'X\right)^{-1}X'$, where X is the Jacobian at each date. The smaller the GCV statistic, the better the model in terms of its prediction power [for more detail see Wahba (1985)].

implied default parameters with historical based default frequencies for “like” firms. This is, in fact, the topic for a companion paper [see Chava and Jarrow (2000)].

Nonetheless, the absolute performance of the reduced form credit risk model can be partially gauged by examining the time series stability of the estimated parameters, the R^2 of the regression model, and the percentage pricing error (RMSE/average bond price). The time series stability of the estimated parameters was discussed in section 5 above. In summary, that evidence supports the necessity of including a liquidity discount into the debt-pricing model. The R^2 statistic, as mentioned previously, is quite high for all model structures – often greater than .85 (see Table 5). This indicates the ability of the model to explain a large percentage of the variation in the bond price data. The average R^2 for the “best” performing model 2 is .87.

The average percentage pricing error across firms and model types is quite low. As seen in Table 5, the average percentage pricing error fluctuates around 1 percent of the bond’s bid price, and is often much less. The overall average percentage pricing error for the “best” performing model 2 is 1.1 percent. This is a small pricing error despite the facts that: (i) only a small number of bonds were used in the estimation, (ii) the estimates are based on monthly observations (not daily or weekly), (iii) a rolling estimation procedure is employed, and (iv) the term structure and credit risk models implemented are quite simple.

9. Conclusion

This paper provides an empirical investigation of a reduced form credit risk model that includes both liquidity risk and correlated defaults. The estimation is for twenty different firms’ debt issues using monthly bond prices over a six-year time period from May 1991 – March 1997. Five different liquidity discount models are investigated.

Based on various statistical measures, both in- and out- of sample, the evidence supports the importance of including a liquidity discount into a credit risk model to capture liquidity risk. The inclusion of a liquidity discount increases the stability of the estimated parameters, it reduces the credit risk model’s average pricing error, and it significantly impacts the one-year default probability estimation.

Three conclusions can be drawn with respect to the specific debt-pricing model estimated. First, the model fits the data quite well. Second, the expected loss appears to depend on the spot rate of interest, but not a market index. This captures the integration of market and credit risk. Third, liquidity risk appears to be idiosyncratic and not systematic risk. The importance of an industry effect in both the default intensity and the liquidity discount is an open question.

Future research is also needed to compare the implied default probabilities estimated using the above model to both: historical default frequencies and default probabilities implicit in credit derivative prices.

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Appendix: Determination of the t-scores in Table 3.

Let x_i represent the coefficient from the i^{th} regression for $i = 1, ..., m$ where m = the number of regressions.

Table 2 contains $\overline{x} = \sum_{i=1}^{m} x_i / m$. We want to compute the standard error for $\overline{x}$.

Let $E(x_i) = \mu$ and $Var(x_i) = \sigma_x^2$ for all i.

The x_i are identically distributed, but not independent. The regression coefficients x_i are correlated because the i^{th} regression uses the past n months of bond price observations, so the i^{th} and $(i+1)^{st}$ regression overlap $(n-2)$ months of data.

So, $E(\overline{x}) = E(\sum_{i=1}^{m} x_i / m) = \mu$. We need to determine $Var(\overline{x})$.

As an approximation in distribution (to the regression estimation), let $x_i = \sum_{j=i}^{n+i-1} y_j / n$ where y_j are independent and identically distributed with $E(y_j) = \mu$ and $Var(y_j) = \sigma^2$ for all i and n = the number of months in the regression ($n = 8$ for the estimation). This implies that x_i and x_{i+1} are overlapping $n-2$ observations. Note that:

$$\sum_{i=1}^{m} x_i / m = \sum_{i=1}^{m} \sum_{j=i}^{n+i-1} y_j / nm = \begin{cases} \sum_{j=1}^{n-1} j y_j / nm + \sum_{j=n}^{m} y_j / m + \sum_{j=m+1}^{n+m-1} (n+m-j) y_j / nm & if \quad m \geq n \\ \sum_{j=1}^{m-1} j y_j / nm + \sum_{j=m}^{n} y_j / n + \sum_{j=n+1}^{n+m-1} (n+m-j) y_j / nm & if \quad m < n \end{cases}$$

$$= \begin{cases} \sum_{j=1}^{n-1} j y_j / nm + \sum_{j=n}^{m} y_j / m + \sum_{j=1}^{n-1} j y_{n+m-j} / nm & if \quad m \geq n \\ \sum_{j=1}^{m-1} j y_j / nm + \sum_{j=m}^{n} y_j / n + \sum_{j=1}^{m-1} j y_{n+m-j} / nm & if \quad m < n \end{cases}$$

Then,

$$var(\sum_{i=1}^{m} x_i / m) = \begin{cases} var(\sum_{j=1}^{n-1} j y_j / nm + \sum_{j=n}^{m} y_j / m + \sum_{j=1}^{n-1} j y_{n+m-j} / nm) & if \quad m \geq n \\ var(\sum_{j=1}^{m-1} j y_j / nm + \sum_{j=m}^{n} y_j / n + \sum_{j=1}^{m-1} j y_{n+m-j} / nm) & if \quad m < n \end{cases}$$

$$= \begin{cases} \frac{\sigma^2}{m^2} [2 \sum_{j=1}^{n-1} j^2 / n^2 + (m-n+1)] & if \quad m \geq n \\ \frac{\sigma^2}{m^2} [2 \sum_{j=1}^{n-1} j^2 / n^2 + \frac{m^2}{n^2} (n-m+1)] & if \quad m < n \end{cases}$$

Using $\sum_{j=1}^{k} j^2 = k(k+1)(2k+1)/6$, we get:

$$var(\bar{x}) = \begin{cases} \sigma^2 [\frac{1}{m} + \frac{(1-n^2)}{3nm^2}] & if \quad m \geq n \\ \sigma^2 [\frac{1}{n} + \frac{(1-m^2)}{3n^2 m}] & if \quad m < n \end{cases}.$$

But,

$$\sigma_x^2 = var(x_i) = var(\sum_{j=i}^{n+i-1} y_j / n) = \sigma^2 / n.$$

So, $\sigma^2 = n\sigma_x^2$. Thus, $\sqrt{var}(\bar{x}) = \begin{cases} \sigma_x \sqrt{\frac{n}{m} + \frac{(1-n^2)}{3m^2}} & if \quad m \geq n \\ \sigma_x \sqrt{1 + \frac{(1-m^2)}{3nm}} & if \quad m < n \end{cases}$

A consistent estimator for the standard error of $\bar{x}$ is:

$$stderror(\bar{x}) = \begin{cases} \frac{\sum_{i=1}^{m} \hat{\sigma}_{x_i}}{m} \sqrt{\frac{n}{m} + \frac{(1-n^2)}{3m^2}} & if \quad m \geq n \\ \frac{\sum_{i=1}^{m} \hat{\sigma}_{x_i}}{m} \sqrt{1 + \frac{(1-m^2)}{3nm}} & if \quad m < n \end{cases}$$

where $\hat{\sigma}_{x_i}$ is the standard error of the coefficient x_i from the i^{th} regression, and $\frac{\sum_{i=1}^{m} \hat{\sigma}_{x_i}}{m}$ = average standard errors of the coefficients x_i from the *m* regressions.

The t-score is: $\frac{\bar{x}}{stderror(\bar{x})}$. Under the null hypothesis that $\mu = 0$, this is asymptotically normal with mean 0 and standard derivation 1. The t-scores are reported in Table 3.

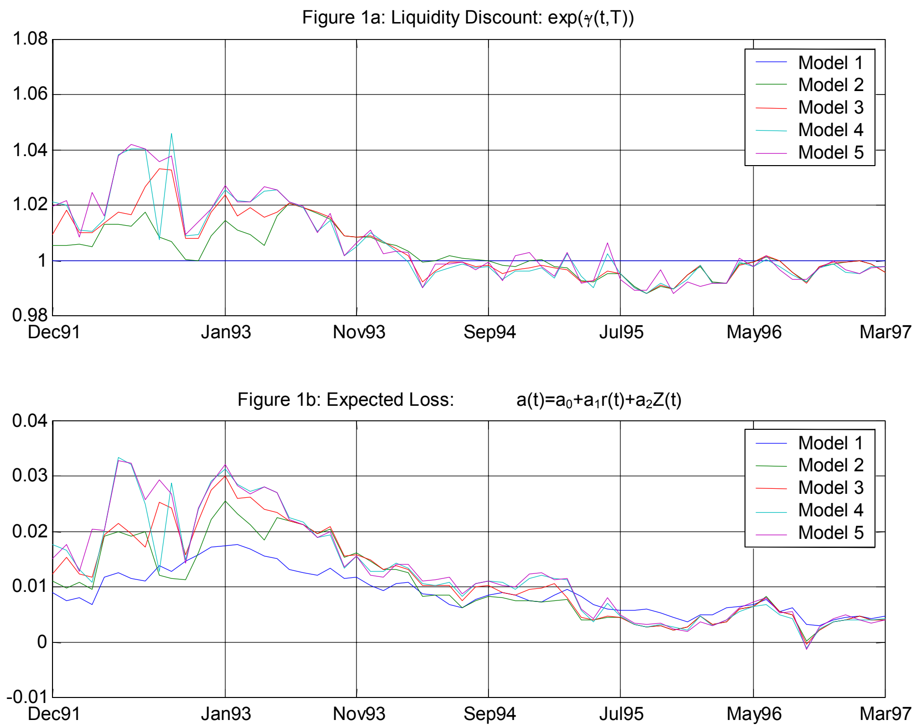

Figure 1: Time Series Estimates of Xerox's Liquidity Discount and Expected Loss (per unit time) from December 1991 to March 1997.

The liquidity discount is $\gamma(t,T) = \gamma_0 + \gamma_1 \sum_{j=t-4}^{t} r(j)/5 + \gamma_2\, \sigma_m^2(t) + \gamma_3 \sum_{j=t-4}^{t} \left(\frac{M(j) - M(j-1)}{M(j-1)} \right) / 5$

where $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are constants, *r(t)* is the 3-month Treasury yield at time *t*, $\sigma_m^2(t)$ is the 5-business day volatility of the return on the S&P 500 index, and *M(t)* is the value of the S&P 500 index at time *t*. The expected loss is $a(t) = a_0 + a_1 r(t) + a_2 Z(t)$ where a_0, a_1, a_2 are constants and *Z(t)* is the time *t* cumulative excess return per unit of risk on the S&P 500 index. The parameters $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ and a_0, a_1, a_2 are estimated implicitly from the market price of Xerox's debt over the current and previous seven months. The time *t* estimation uses only information available at time *t*. Given are monthly observations.

	Ticker Symbol	SIC Code	First Date used in the Estimation	Last Date used in the Estimation	Number of Bonds	Moodies	S&P
Financials							
SECURITY PACIFIC CORP	spc	6021	12/31/1991	07/31/1994	7	A3	A
FLEET FINANCIAL GROUP	flt	6021	12/31/1991	10/31/1996	3	Baa2	BBB+
BANKERS TRUST NY	bt	6022	01/31/1994	04/30/1994	3	A1	AA
MERRILL LYNCH & CO	mer	6211	12/31/1991	03/31/1997	14	A2	A
Food & Beverages							
PEPSICO INC	pep	2086	12/31/1991	03/31/1997	8	A1	A
COCA - COLA ENTERPRISES INC	cce	2086	12/31/1991	06/30/1994	3	A2	AA-
Airlines							
AMR CORPORATION	amr	4512	02/29/1992	08/31/1994	2	Baa1	BBB+
SOUTHWEST AIRLINES CO	luv	4512	05/31/1992	03/31/1997	3	Baa1	A-
Utilities							
CAROLINA POWER + LIGHT	cpl	4911	08/31/1992	01/31/1993	3	A2	A
TEXAS UTILITIES ELE CO	txu	4911	04/30/1994	03/31/1997	4	Baa2	BBB
Petroleum							
MOBIL CORP	mob	2911	12/31/1991	02/29/1996	3	Aa2	AA
UNION OIL OF CALIFORNIA	ucl	2911	12/31/1991	03/31/1997	6	Baa1	BBB
SHELL OIL CO	suo	2911	03/31/1992	02/28/1995	5	Aaa	AAA
Department Stores							
SEARS ROEBUCK + CO	s	5311	12/31/1991	08/31/1996	7	A2	A
DAYTON HUDSON CORP	dh	5311	04/30/1993	03/31/1997	2	A3	A
WAL-MART STORES, INC	wmt	5331	12/31/1991	03/31/1997	3	Aa3	AA
Technology							
EASTMAN KODAK COMPANY	ek	3861	01/31/1992	09/30/1994	3	A2	A-
XEROX CORP	xrx	3861	12/31/1991	03/31/1997	4	A2	A
TEXAS INSTRUMENTS	txn	3674	10/31/1992	03/31/1997	3	A3	A
INTL BUSINESS MACHINES	ibm	3570	01/31/1994	03/31/1997	3	A1	AA-

Table 1: Details of the Firms Included in the Empirical Investigation.

Ticker Symbol is the firm's ticker symbol. SIC is the Standard Industry Code. Number of Bonds is the number of the firm's different senior debt issues outstanding on the first date used in the estimation. Moodies refers to Moodies' debt rating for the company's senior debt on the first date used in the estimation. S&P refers to S&P's debt rating for the company's debt on the first date used in the estimation.

FINANCIALS

1- SECURITY PACIFIC CORP

spc	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0012	0.1934	-0.0021	47	0.812	32
Model 2	-0.0065				0.007	0.1196	-0.0008	47	0.8366	32
Model 3	-0.0084	0.0411			0.0069	0.1197	-0.0003	47	0.8385	32
Model 4	-0.0078	-0.0046	0.1056		0.0035	0.1985	-0.0029	47	0.8499	32
Model 5	-0.0034	-0.0992	0.0383	0.3476	0.0033	0.2056	-0.0033	47	0.8557	32

2- FLEET FINANCIAL GROUP

flt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0085	0.0054	0.0001	16	0.4143	57
Model 2	-0.0023				0.0157	-0.1195	-0.0006	16	0.4501	57
Model 3	-0.0075	0.254			0.0219	-0.2078	-0.0015	16	0.5031	57
Model 4	-0.0094	0.5403	0.1016		0.0327	-0.46	-0.0043	16	0.5803	57
Model 5	-0.0074	0.3125	0.2018	-0.3375	0.0336	-0.45	-0.0066	16	0.6582	57

3- BANKERS TRUST NY

bt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0014	0.018	0	24	0.9504	4
Model 2	-0.0031				0.0064	-0.0508	-0.0003	24	0.9576	4
Model 3	0.0028	-0.1824			0.0056	-0.028	-0.0006	24	0.9583	4
Model 4	0.0095	-0.3909	0.0141		0.0042	0.0091	-0.0009	24	0.9589	4
Model 5	0.013	-0.4817	0.01	-0.662	0.0034	-0.0181	0.001	24	0.9613	4

4-MERRILL LYNCH & CO

mer	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0083	0.0154	-0.0007	132	0.8918	64
Model 2	-0.0068				0.0156	-0.0639	-0.0017	132	0.9	64
Model 3	-0.017	0.2331			0.0168	-0.0809	-0.0013	132	0.9024	64
Model 4	-0.0251	0.4294	0.023		0.0176	-0.0864	-0.0017	132	0.9044	64
Model 5	-0.0216	0.3445	-0.0383	0.4829	0.0181	-0.0873	-0.0015	132	0.907	64

FOOD & BEVERAGES

5-PEPSICO INC

pep	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0026	0.0629	0	57	0.8554	64
Model 2	-0.0061				0.0063	0.0255	-0.0002	57	0.8616	64
Model 3	0.006	-0.2528			0.0064	0.0331	-0.0004	57	0.8635	64
Model 4	0.0199	-0.5636	0.0432		0.006	0.0544	-0.0007	57	0.8667	64
Model 5	0.0032	-0.2422	-0.0049	-0.2383	0.0071	0.026	-0.0005	57	0.8682	64

6-COCA-COLA ENTERPRISES INC

cce	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.003	0.0494	0.0003	24	0.8142	31
Model 2	-0.0096				0.011	-0.045	0.001	24	0.8783	31
Model 3	-0.0111	0.0064			0.0118	-0.0593	0.0018	24	0.8812	31
Model 4	-0.0191	0.2505	0.0472		0.0114	-0.0465	0.001	24	0.8884	31
Model 5	-0.0107	0.057	-0.1067	0.1239	0.0107	-0.0337	0.0012	24	0.8926	31

AIRLINES

7-AMR CORPORATION

Amr	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0134	0.064	0.0011	26	0.9216	31
Model 2	-0.0226				0.0283	-0.0863	0.0001	26	0.9346	31
Model 3	-0.0151	-0.1824			0.0259	-0.0546	0	26	0.9378	31
Model 4	-0.0172	-0.1521	-0.2317		0.0246	-0.0394	0.0018	26	0.9484	31
Model 5	-0.0157	-0.0927	-0.2293	-1.3426	0.0252	-0.0579	0.0026	26	0.9548	31

8-SOUTHWEST AIRLINES CO

luv	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0184	-0.0986	-0.0035	27	0.8432	59
Model 2	0.0071				0.0142	-0.0617	-0.0027	27	0.8513	59
Model 3	0.0108	-0.0417			0.0146	-0.0617	-0.0027	27	0.8539	59
Model 4	0.0168	-0.2548	-0.0536		0.0218	-0.13	-0.0044	27	0.8621	59
Model 5	0.009	-0.2063	-0.1335	-0.3788	0.0238	-0.1489	-0.0043	27	0.8718	59

UTILITIES

9-CAROLINE POWER + LIGHT

cpl	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0105	-0.0006	-0.018	24	0.8559	6
Model 2	-0.0042				0.0146	-0.0305	-0.0181	24	0.8598	6
Model 3	-0.0091	0.1352			0.0165	-0.0688	-0.0178	24	0.8605	6
Model 4	-0.0118	0.3229	-0.4386		0.0191	-0.0898	-0.0194	24	0.8686	6
Model 5	-0.022	0.6041	-0.4699	0.1241	0.0214	-0.1067	-0.0219	24	0.8727	6

10-TEXAS UTILITIES ELE CO

txu	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0108	-0.0215	-0.0022	29	0.8329	36
Model 2	0.0054				0.0057	0.0264	-0.0015	29	0.837	36
Model 3	0.0201	-0.2835			0.004	0.0499	-0.0015	29	0.8395	36
Model 4	0.0349	-0.5343	-0.2725		0.0062	0.0305	-0.0018	29	0.85	36
Model 5	0.0161	-0.1118	-0.1959	-1.7487	0.0024	0.0611	-0.0008	29	0.8591	36

PETROLEUM

11-MOBIL CORP

mob	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0038	0.0114	-0.0005	31	0.9787	51
Model 2	-0.0028				0.0067	-0.0201	-0.0009	31	0.9824	51
Model 3	0.0003	-0.0958			0.0068	-0.0207	-0.001	31	0.9829	51
Model 4	0.0036	-0.2213	0.0071		0.0076	-0.0321	-0.0012	31	0.9835	51
Model 5	0.0005	-0.1364	0.0257	-0.1457	0.0077	-0.0347	-0.001	31	0.984	51

12-UNION OIL OF CALIFORNIA

ucl	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0109	-0.0153	-0.0013	31	0.9219	64
Model 2	-0.0024				0.013	-0.0385	-0.0016	31	0.9255	64
Model 3	-0.0226	0.4015			0.0155	-0.0778	-0.0018	31	0.927	64
Model 4	-0.0387	0.7875	-0.0232		0.0136	-0.062	-0.0017	31	0.9298	64
Model 5	-0.0289	0.5747	-0.0698	0.596	0.0139	-0.0597	-0.0016	31	0.9312	64

13-SHELL OIL CO

suo	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0008	0.0721	0.0003	36	0.8309	36
Model 2	-0.0097				0.0103	-0.0386	-0.0004	36	0.8556	36
Model 3	-0.0113	0.0286			0.0108	-0.0437	-0.0005	36	0.8575	36
Model 4	-0.0046	-0.1822	-0.1238		0.013	-0.0703	-0.0005	36	0.8618	36
Model 5	-0.0084	-0.0672	-0.0622	0.1001	0.0144	-0.0808	-0.0014	36	0.8638	36

DEPARTMENT STORES

14-SEARS ROEBUCK + CO

s	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0069	0.0833	0.0009	42	0.7245	57
Model 2	-0.0063				0.0108	0.0455	0.0007	42	0.7338	57
Model 3	0.0029	-0.2729			0.0095	0.0654	0.0006	42	0.7379	57
Model 4	0.0119	-0.5485	-0.2141		0.0122	0.0243	0.0012	42	0.7474	57
Model 5	0.0307	-0.927	-0.1149	0.7994	0.0122	0.0799	-0.0005	42	0.7641	57

15-DAYTON HUDSON CORP

dh	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0024	0.0889	0.001	20	0.8859	48
Model 2	-0.051				0.0138	0.0043	0.0005	20	0.9219	48
Model 3	-0.0372	-0.3116			0.0129	0.0174	0.0006	20	0.9229	48
Model 4	0.0237	-1.7798	-0.0709		0.0118	0.0393	0.0004	20	0.927	48
Model 5	-0.0136	-0.9273	-0.1076	-1.8731	0.0123	0.0161	0.0011	20	0.9322	48

16-WAL-MART STORES, INC

wmt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0098	-0.0639	-0.0016	58	0.9415	64
Model 2	0.0042				0.0063	-0.0282	-0.0012	58	0.943	64
Model 3	0.0079	-0.1257			0.0059	-0.0236	-0.0011	58	0.9438	64
Model 4	0.0157	-0.3443	-0.0186		0.0058	-0.0228	-0.0011	58	0.945	64
Model 5	0.0082	-0.1769	-0.0285	-0.4038	0.0064	-0.0357	-0.0008	58	0.946	64

TECHNOLOGY

17-EASTMAN KODAK COMPANY

ek	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.003	0.0984	-0.0005	29	0.9257	33
Model 2	-0.0089				0.0102	0.0235	-0.0012	29	0.9379	33
Model 3	-0.0177	0.3019			0.0102	0.0247	-0.0013	29	0.9394	33
Model 4	-0.0247	0.541	-0.0249		0.0092	0.04	-0.0012	29	0.9421	33
Model 5	-0.022	0.482	-0.0653	-0.083	0.0092	0.039	-0.0011	29	0.9429	33

18-XEROX CORP

xrx	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0151	-0.1022	-0.0017	39	0.921	64
Model 2	-0.0024				0.0157	-0.1123	-0.0004	39	0.9235	64
Model 3	-0.0224	0.4989			0.0189	-0.1562	0	39	0.926	64
Model 4	-0.0215	0.4888	-0.1402		0.0214	-0.1788	0	39	0.9286	64
Model 5	-0.027	0.5427	-0.1375	0.5588	0.0229	-0.1936	-0.0004	39	0.9308	64

19-TEXAS INSTRUMENTS

txn	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0094	-0.0164	-0.0007	24	0.8947	54
Model 2	0.0049				0.0066	0.0114	-0.0004	24	0.9142	54
Model 3	0.0135	-0.1859			0.0057	0.0232	-0.0004	24	0.9153	54
Model 4	0.0348	-0.7707	-0.1243		0.0045	0.0367	0	24	0.9211	54
Model 5	0.0251	-0.5054	-0.0549	-0.6919	0.0045	0.0347	0.0003	24	0.8947	54

20-INTL BUSINESS MACHINES

ibm	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	R^2	Number of Reg
Model 1					0.0049	0.0188	-0.0001	24	0.8942	39
Model 2	-0.016				0.0161	-0.0997	-0.001	24	0.9134	39
Model 3	-0.0086	-0.1215			0.0154	-0.0918	-0.0009	24	0.9149	39
Model 4	-0.0352	0.484	-0.0148		0.0196	-0.1394	-0.0012	24	0.9231	39
Model 5	-0.0305	0.3662	-0.0097	0.2081	0.0204	-0.1442	-0.0014	24	0.9281	39

Table 2: Averages of the Parameter Estimates from the Non-linear Debt Regression.

Table 2 contains the average parameter estimates $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, a_0, a_1, a_2)$, across the months in Table 1, from the non-linear debt regressions. They are presented for each company and for each model type, separated by industries. Model 1 has no liquidity discount. Model 2 includes only the first liquidity discount parameter γ_0, Model 3 contains the first two liquidity discount parameters, and so forth. The number of bonds corresponds to the average number of bonds used in each of the monthly regressions. The average R^2 is given. The Number of Reg refers to the number of distinct regressions performed over the observation period given in Table 1.

FINANCIALS

1- SECURITY PACIFIC CORP

spc	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.2819	2.7639^{+}	-0.6505	47	0.0002	32
					0.4254	0.0701	0.1701			
Model 2	-2.8164^{+}				4.1070^{+}	23.5748^{+}	-0.0035	47	0.1124	32
	0.0448				0.2215	0.0045	0.4893			
Model 3	-0.0826	22.7643^{+}			0.7333	0.1660	0.0043	47	0.1409	32
	0.4632	0.0049			0.3753	0.2897	0.3691			
Model 4	-0.9345	-0.4333	12.1760^{+}		0.4156	0.4182	-0.0169	47	0.1041	32
	0.1823	0.2218	0.0000		0.4204	0.2976	0.4886			
Model 5	-0.0118	-17.8704^{+}	0.6163	171.2255^{+}	0.2841	0.4279	-0.0210	47	0.0751^{+}	32
	0.4864	0.0167	0.1043	0.0000	0.4232	0.3185	0.4875			

2- FLEET FINANCIAL GROUP

flt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.6711	0.3265	-0.2349	16	0.3362	57
					0.3333	0.2363	0.1625			
Model 2	-2.9728^{+}				3.2078^{+}	-2.6361^{+}	0.0063	16	0.4753	57
	0.0660				0.1705	0.0525	0.4841			
Model 3	0.0466	-42.1781^{+}			0.5309	-0.0869	-0.0014	16	0.5717	57
	0.4257	0.0072			0.3407	0.3625	0.3374			
Model 4	2.0675^{+}	-2.3262^{+}	41.4808^{+}		0.7369	-0.2621	-0.0315	16	0.5585	57
	0.0934	0.0880	0.0006		0.3163	0.3362	0.4723			
Model 5	-0.0178	-4.8128^{+}	0.9689	-264.818^{+}	0.6068	-0.2036	-0.0268	16	0.5963	57
	0.4609	0.0155	0.1815	0.0006	0.3416	0.3224	0.4664			

3- BANKERS TRUST NY

bt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.1358	0.0550	-0.0240	24	0.0070	4
					0.4465	0.3707	0.3948			
Model 2	-0.9243				4.4063^{+}	-5.5819^{+}	-0.0024	24	0.0947	4
	0.1781				0.1250	0.0026	0.4986			
Model 3	0.0124	-55.5685^{+}			0.3330	-0.0825	-0.0541	24	0.2150	4
	0.4944	0.0000			0.3726	0.4556	0.4716			
Model 4	0.7816	-1.7180^{+}	3.3353^{+}		0.2418	-0.0214	-0.0064	24	0.3556	4
	0.2605	0.1478	0.0004		0.4072	0.4537	0.4971			
Model 5	0.0242	-39.8571^{+}	0.0222	-173.768^{+}	0.1876	-0.0409	0.0045	24	0.3540	4
	0.4903	0.0000	0.4097	0.0000	0.4272	0.4699	0.4975			

4-MERRILL LYNCH & CO

mer	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					8.1734^{+}	0.3176^{+}	-4.1362^{+}	132	0.0000	64
					0.0907	0.0848	0.0370			
Model 2	-16.1905^{+}				16.9420^{+}	-51.7314^{+}	-0.1162	132	0.1769	64
	0.0105				0.0370	0.0130	0.4191			
Model 3	-0.6937	493.1328^{+}			2.2514^{+}	-0.4761	-1.4089	132	0.2475	64
	0.2778	0.0002			0.1293	0.3090	0.1719			
Model 4	-13.8636^{+}	17.9381^{+}	21.1521^{+}		2.1107^{+}	-0.4735	-0.0284	132	0.2454	64
	0.0514	0.0777	0.0009		0.1437	0.2976	0.4751			
Model 5	-0.0949	194.8086^{+}	-1.6719^{+}	1193.061^{+}	19.7803^{+}	-0.4329	-0.0334	132	0.2458	64
	0.4619	0.01461	0.0647	0.0000	0.1557	0.3161	0.4748			

FOOD & BEVERAGES

5-PEPSICO INC

pep	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.2749	2.2308^{+}	0.2828	57	0.0001	64
					0.4294	0.0712	0.1724			
Model 2	-7.2623^{+}				3.6325^{+}	10.3711^{+}	-0.0041	57	0.2893	64
	0.0260				0.1568	0.0100	0.4838			
Model 3	0.0115	-191.0761^{+}			0.3657	0.2069	-0.0775	57	0.3477	64
	0.4501	0.0003			0.3755	0.3807	0.3547			
Model 4	2.0088^{+}	-5.9499^{+}	63.3143^{+}		0.2374	0.2693	-0.0017	57	0.3519	64
	0.1166	0.0888	0.0000		0.4196	0.3693	0.4899			
Model 5	0.0042	-53.3278^{+}	1.1639	-217.142^{+}	0.2549	0.1950	0.0011	57	0.3823	64
	0.4832	0.0170	0.1482	0.0000	0.4174	0.3878	0.4899			

6-COCA-COLA ENTERPRISES INC

cce	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					1.3866	0.8110	-0.6902	24	0.0161	31
					0.3260	0.0815	0.1385			
Model 2	-7.8061^{+}				4.9461^{+}	-11.2644^{+}	0.0201	24	0.1309	31
	0.0222				0.0225	0.0433	0.4644			
Model 3	-0.1935	-8.1359^{+}			0.8045	-0.2558	0.3406	24	0.1616	31
	0.4205	0.0107			0.2795	0.4236	0.2459			
Model 4	-4.2193^{+}	3.6938^{+}	21.1966^{+}		0.7362	-0.2211	0.0064	24	0.2094	31
	0.0386	0.0703	0.0151		0.3086	0.4206	0.4869			
Model 5	-0.0318	21.2825^{+}	-1.3620	-25.8086^{+}	0.6432	-0.1745	0.0081	24	0.2427	31
	0.4784	0.0000	0.1006	0.0000	0.3263	0.4282	0.4884			

AIRLINES

7-AMR CORPORATION

Amr	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					5.7981	1.2952	0.9313	26	0.0173	31
					0.0330^{+}	0.1603	0.1477			
Model 2	-17.7134^{+}				5.0806^{+}	-16.3786^{+}	-0.0089	26	0.3054	31
	0.0134				0.0306	0.0053	0.4752			
Model 3	-0.1897	-168.585^{+}			1.0632	-0.1146	-0.1466	26	0.3957	31
	0.3911	0.0000			0.2063	0.4043	0.2490			
Model 4	-1.6442^{+}	-1.1826	-295.966^{+}		1.1666	-0.1536	0.0046	26	0.2729	31
	0.0502	0.0592	0.0000		0.2594	0.3400	0.4911			
Model 5	0.0003	-49.6778^{+}	-2.3903^{+}	-1431.08^{+}	1.0377	-0.1159	0.0119	26	0.2069	31
	0.4584	0.0101	0.0146	0.0000	0.2608	0.3616	0.4910			

8-SOUTHWEST AIRLINES CO

luv	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					6.9117^{+}	-2.7826^{+}	-3.9155^{+}	27	0.0044	59
					0.0664	0.0398	0.0540			
Model 2	5.0020^{+}				3.7845^{+}	-13.2640^{+}	-0.0479	27	0.6245	59
	0.0505				0.1339	0.0000	0.4773			
Model 3	0.1609	-181.142^{+}			0.5043	-0.0916	-0.3751	27	0.6337	59
	0.3603	0.0016			0.3324	0.4208	0.3427			
Model 4	4.7397^{+}	-5.0962^{+}	-99.4887^{+}		0.6315	-0.1701	-0.0280	27	0.5360	59
	0.0589	0.0436	0.0003		0.3004	0.3891	0.4876			
Model 5	0.0207	-48.2290^{+}	-1.2779	-578.965^{+}	0.6730	-0.1965	-0.0267	27	0.4910	59
	0.4599	0.0113	0.0342	0.0000	0.3004	0.3822	0.4874			

UTILITIES

9-CAROLINE POWER + LIGHT

cpl	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.7559	-0.0622	-1.4007	24	0.0070	6
					0.2540	0.2939	0.0992			
Model 2	-0.3761				2.2033^{+}	-1.4132	-0.0761	24	0.5641	6
	0.3572				0.1168	0.0525	0.4697			
Model 3	-0.0189	8.8490^{+}			0.3872	-0.0587	-0.7423	24	0.7487	6
	0.4919	0.0594			0.3538	0.4452	0.2500			
Model 4	-0.4438	0.6867	-34.3115^{+}		0.4478	-0.0676	-0.0468	24	0.6528	6
	0.3487	0.3112	0.0337		0.3319	0.4341	0.4813			
Model 5	-0.0152	20.1177^{+}	-1.0647	-13.7776^{+}	0.4572	-0.0569	-0.0491	24	0.6955	6
	0.4939	0.0028	0.1694	0.0000	0.3287	0.4453	0.4804			

10-TEXAS UTILITIES ELE CO

txu	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					3.1579^{+}	-0.8421	-3.0611^{+}	29	0.0031	36
					0.1335	0.1544	0.1217			
Model 2	5.6394^{+}				1.5927	4.7999^{+}	-0.0280	29	0.5150	36
	0.0354				0.1875	0.0434	0.4867			
Model 3	0.1860	-287.861^{+}			0.0949	0.1011	-0.1569	29	0.6128	36
	0.4267	0.0057			0.4626	0.4510	0.4166			
Model 4	2.5856^{+}	-3.6714^{+}	-199.786^{+}		0.0956	0.0882	-0.0149	29	0.5015	36
	0.0777	0.0832	0.0000		0.4627	0.4372	0.4934			
Model 5	0.0332	-8.3863^{+}	-1.6014	-1339.96^{+}	0.0233	0.1099	-0.0059	29	0.4466	36
	0.4729	0.0117	0.1583	0.0000	0.4908	0.4520	0.4965			

PETROLEUM

11-MOBIL CORP

mob	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					4.1398^{+}	0.5739	-0.6277	31	0.0009	51
					0.2184	0.0327	0.1417			
Model 2	-13.74^{+}				8.6393^{+}	-18.98^{+}	-0.0453	31	0.2297	51
	0.0221				0.0211	0.0232	0.4783			
Model 3	0.0144	-151.1^{+}			1.1888	-0.1405	-0.4414	31	0.2513	51
	0.3389	0.0063			0.1893	0.3904	0.3152			
Model 4	1.3997	-7.853^{+}	-13.24^{+}		1.3660	-0.2807	-0.0370	31	0.2480	51
	0.0402	0.0242	0.0003		0.2240	0.3361	0.4835			
Model 5	-0.0400	-36.91^{+}	1.1006	-397.9^{+}	1.3519	-0.3080	-0.0282	31	0.2808	51
	0.4382	0.0000	0.1106	0.0084	0.2258	0.3467	0.4844			

12-UNION OIL OF CALIFORNIA

ucl	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					2.7483^{+}	0.2021	-1.4434	31	0.0034	64
					0.1729	0.1026	0.0947			
Model 2	-2.821^{+}				4.9479^{+}	-12.06^{+}	-0.0343	31	0.4866	64
	0.0219				0.1369	0.0293	0.4763			
Model 3	-0.3393	459.71^{+}			1.0650	-0.3185	-0.3399	31	0.5034	64
	0.3619	0.0000			0.2884	0.3474	0.3005			
Model 4	-8.225^{+}	11.537^{+}	-48.24^{+}		1.0700	-0.3444	-0.0188	31	0.4501	64
	0.0889	0.0962	0.0000		0.3257	0.3442	0.4866			
Model 5	-0.0714	122.63^{+}	-2.235^{+}	523.49^{+}	0.9765	-0.3127	-0.0163	31	0.4858	64
	0.4494	0.0108	0.0520	0.0000	0.3291	0.3531	0.4871			

13-SHELL OIL CO

suo	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.1369	1.4713	0.1705	36	0.0004	36
					0.4520	0.1115	0.1416			
Model 2	-6.415^{+}				4.9529^{+}	-10.48^{+}	0.0038	36	0.1563	36
	0.0213				0.0338	0.0616	0.4838			
Model 3	-0.1550	2.5431^{+}			0.8302	-0.1863	0.0650	36	0.2239	36
	0.4263	0.0000			0.2609	0.4031	0.3316			
Model 4	-0.3532	-3.344^{+}	-70.64^{+}		0.9976	-0.3071	-0.0062	36	0.2557	36
	0.0616	0.0776	0.0173		0.2641	0.3684	0.4902			
Model 5	-0.0172	-14.66^{+}	-0.4279	2.2451^{+}	0.9377	-0.2704	-0.0160	36	0.3123	36
	0.4765	0.0151	0.0847	0.0000	0.2547	0.3694	0.4887			

DEPARTMENT STORES

14-SEARS ROEBUCK + CO

s	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					1.3235	1.6674^{+}	0.6127	42	0.0004	57
					0.2311	0.1404	0.1129			
Model 2	-4.816^{+}				3.6028^{+}	7.1801^{+}	0.0345	42	0.4363	57
	0.0820				0.1500	0.0553	0.4645			
Model 3	0.0469	-154.6^{+}			0.4636	0.1066	0.1999	42	0.4946	57
	0.4351	0.0078			0.3584	0.3929	0.2974			
Model 4	2.2220^{+}	-6.796^{+}	-117.6^{+}		0.6184	-0.0089	0.0119	42	0.4445	57
	0.1358	0.1243	0.0000		0.3588	0.3510	0.4848			
Model 5	0.0872	-123.1^{+}	-1.860^{+}	276.31^{+}	0.5578	0.0545	-0.0093	42	0.3375	57
	0.4454	0.0099	0.1366	0.0000	0.3566	0.3580	0.4805			

15-DAYTON HUDSON CORP

dh	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					0.5250	1.1928	0.8991	20	0.0423	48
					0.4189	0.1272	0.1494			
Model 2	-44.83^{+}				1.7267^{+}	-1.1274	0.0032	20	0.0912	48
	0.0000				0.2341	0.0091	0.4915			
Model 3	-0.3282	-246.0^{+}			0.2030	0.0430	0.0764	20	0.1869	48
	0.3738	0.0000			0.4221	0.4597	0.4209			
Model 4	6.9778^{+}	-19.17^{+}	-57.59^{+}		0.1708	0.0528	0.0014	20	0.2439	48
	0.1076	0.0890	0.0000		0.4349	0.4619	0.4973			
Model 5	-0.0132	-130.1^{+}	-1.2059	-1597.8^{+}	0.1591	0.0404	0.0053	20	0.2591	48
	0.4818	0.0027	0.0780	0.0000	0.4392	0.4585	0.4971			

16-WAL-MART STORES, INC

wmt	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					6.2730^{+}	-3.089^{+}	-2.443^{+}	58	0.0013	64
					0.0431	0.0668	0.0482			
Model 2	6.5468^{+}				2.5800^{+}	-10.95^{+}	-0.0317	58	0.6031	64
	0.0455				0.0963	0.0151	0.4833			
Model 3	0.1441	-108.5^{+}			0.3646	-0.0695	-0.2371	58	0.6788	64
	0.4111	0.0002			0.3699	0.4450	0.3638			
Model 4	5.1426^{+}	-8.805^{+}	-58.55^{+}		0.3793	-0.1032	-0.0080	58	0.6017	64
	0.0467	0.0826	0.0000		0.3768	0.4309	0.4949			
Model 5	0.0077	-34.19^{+}	-0.7960	-542.183^{+}	0.3664	-0.1143	-0.0051	58	0.6193	64
	0.4701	0.0107	0.0480	0.0000	0.3776	0.4351	0.4957			

TECHNOLOGY

17-EASTMAN KODAK COMPANY

ek	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					1.3771	3.7408^{+}	-1.0150	29	0.0023	33
					0.2553	0.0661	0.1370			
Model 2	-12.00^{+}				4.5982^{+}	5.6228^{+}	-0.0353	29	0.1432	33
	0.0001				0.0565	0.0026	0.4758			
Model 3	-0.4943	461.49^{+}			0.8045	0.0248	-0.4289	29	0.2158	33
	0.3495	0.0017			0.2764	0.4316	0.2565			
Model 4	-8.196^{+}	15.403^{+}	-72.37^{+}		0.7044	0.0492	-0.0108	29	0.2479	33
	0.0843	0.0677	0.0000		0.3226	0.4171	0.4919			
Model 5	-0.0350	186.67^{+}	-1.746^{+}	-68.3966^{+}	0.5711	0.0472	-0.0069	29	0.3489	33
	0.4806	0.0094	0.0209	0.0000	0.3336	0.4269	0.4937			

18-XEROX CORP

xrx	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					6.9969^{+}	-3.615^{+}	-1.768^{+}	39	0.0003	64
					0.0645	0.0770	0.1017			
Model 2	-1.823^{+}				7.6634^{+}	-43.89^{+}	-0.0175	39	0.5057	64
	0.0744				0.0959	0.0105	0.4616			
Model 3	-0.4643	500.22^{+}			1.2901	-0.4918	-0.2347	39	0.4381	64
	0.3299	0.0006			0.1983	0.3202	0.2911			
Model 4	-4.213^{+}	8.8823^{+}	-119.8^{+}		1.3794	-0.5756	0.0015	39	0.4192	64
	0.0784	0.0635	0.0003		0.2047	0.3089	0.4892			
Model 5	-0.0574	100.16^{+}	-4.227^{+}	649.613^{+}	1.3643	-0.5659	-0.0046	39	0.4322	64
	0.4641	0.0009	0.0562	0.0000	0.2035	0.3135	0.4879			

19-TEXAS INSTRUMENTS

txn	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					4.8313^{+}	-1.1123	-1.6203	24	0.0070	54
					0.2245	0.0549	0.1055			
Model 2	1.6465^{+}				1.5178	3.4678^{+}	-0.0210	24	0.2911	54
	0.0075				0.2652	0.0079	0.4833			
Model 3	0.2326	-290.84^{+}			0.2001	0.1176	-0.2104	24	0.3984	54
	0.3668	0.0090			0.4243	0.4243	0.3762			
Model 4	6.6159^{+}	-13.281^{+}	-175.9^{+}		0.1310	0.1390	-0.0052	24	0.3697	54
	0.0732	0.0977	0.0000		0.4512	0.4140	0.4947			
Model 5	0.0623	-109.99^{+}	-0.9385	-681.103^{+}	0.1089	0.1262	-0.0010	24	0.4350	54
	0.4577	0.0120	0.0740	0.0000	0.4594	0.4196	0.4956			

20-INTL BUSINESS MACHINES

ibm	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2	Number of Bonds	F-test (gamma)	Number of Reg
Model 1					2.3048^{+}	0.4984	-0.1800	24	0.0070	39
					0.1920	0.1489	0.2187			
Model 2	-30.72^{+}				4.2548^{+}	-24.18^{+}	-0.0237	24	0.2624	39
	0.0033				0.0647	0.0046	0.4898			
Model 3	-0.1740	-162.97^{+}			0.4147	-0.0593	-0.1913	24	0.3137	39
	0.3715	0.0000			0.3503	0.4192	0.4211			
Model 4	-8.451^{+}	9.2233^{+}	86.762^{+}		0.5169	-0.1197	-0.0091	24	0.3152	39
	0.0473	0.0589	0.0000		0.3345	0.4025	0.4956			
Model 5	-0.0596	78.0873^{+}	0.7851^{+}	150.454^{+}	0.5224	-0.1155	-0.0121	24	0.3242	39
	0.4567	0.0207	0.0733	0.0000	0.3323	0.4047	0.4948			

Table 3: T-Scores and Average P-values for the Estimated Parameters from the Non-linear Debt Regression

In each cell under the columns $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, a_0, a_1, a_2)$ the first number is the t-score for the corresponding average parameter estimate in Table 1. This t-score is adjusted for the fact that the regressions contain overlapping time intervals. The adjustment to the average standard error is:

$$stderror = \frac{\sum_{i=1}^{m} \hat{\sigma}_i}{m} \sqrt{\frac{8}{m} + \frac{-63}{3m^2}} \quad if \quad m \geq 8 \quad and \quad \frac{\sum_{i=1}^{m} \hat{\sigma}_i}{m} \sqrt{1 + \frac{(1 - m^2)}{24m}} \quad if \quad m < 8$$

where $\hat{\sigma}_i$ is the standard error of the relevant coefficient from the i^{th} regression and *m* is the number of regressions. The second entry is the average P-score obtained from the t-tests of the individual regression coefficients. The P-score from an individual t-test corresponds to the probability of rejecting the null hypothesis that the coefficient is zero when it is true.

The Number of Bonds corresponds to the average number of bonds used in each of the monthly regressions.
The Number of Reg refers to the number of distinct regressions performed over the observation period given in Table 1.
The F-test column contains the average P-score where the P-scores are obtained from the F-tests of the individual regressions. The P-score from an individual F-test corresponds to the probability of rejecting the null hypothesis when it is true. The first row corresponds to the null hypothesis $a_0 = a_1 = a_2 = 0$. The second through fifth rows correspond to the null hypothesis: (i) $\gamma_0 = 0$, (ii) $\gamma_0 = \gamma_1 = 0$, (iii) $\gamma_0 = \gamma_1 = \gamma_2 = 0$, and (iv) $\gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 0$, respectively.

$^{+}$ Significant at 90% level.

	γ_0	γ_1	γ_2	γ_3	a_0	a_1	a_2
Model 1					8/18	9/18	11/18
Model 2	8/18				11/18$^+$	13/18	11/18
Model 3	12/18	16/18			11/18$^+$	12/18	10/18
Model 4	15/18	17/18	11/18		10/18	13/18	12/18
Model 5	17/18$^+$	18/18$^+$	15/18$^+$	15/18$^+$	11/18$^+$	15/18$^+$	15/18$^+$

$^+$ denotes best value.

Table 4: Unit Root Tests Summary

The modified Dickey-Fuller (DF) test statistic is given as the t-statistic of the ρ coefficient in the linear regression: $\Delta y_t = \mu + \rho y_{t-1} + \varepsilon_t$ where y_t represents the time t value of each parameter, $\Delta y_t = y_t - y_{t-1}$, and ε_t is an error term. The null hypothesis for a unit root is $\rho = 0$. In the table, the entries under the $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, a_0, a_1, a_2)$ columns correspond to the number of companies for the relevant coefficient where the null hypothesis of a unit root is rejected at the 90 percent level. There are 18 total companies – tests for a unit root.

Firm Averages	Number of Bonds	R^2	Number of Reg	GCV	RMSE	RMSE/ Price	$e^{-\gamma}$	λ	1ydf
Model 1	37	0.8555	44.7	1.7655	1.1770	0.0115	1.0000	0.0133	0.0136
Model 2	37	0.8709	44.7	1.6659	1.1246	0.0110	1.0072	0.0185	0.0174
Model 3	37	0.8753	44.7	1.8080	1.1367	0.0111	1.0074	0.0187	0.0175
Model 4	37	0.8844	44.7	1.9313	1.1277	0.0111	1.0079	0.0189	0.0174
Model 5	37	0.8910	44.7	2.3073	1.1298	0.0111	1.0082	0.0192	0.0174

Table 5: Summary Statistics for Model Performance

The Number of bonds corresponds to the average number of bonds used in each of the monthly regressions. The R^2 is the average value across all the regressions.
The Number of Reg refers to the number of distinct regressions performed over the observation period.
Given are the average Generalized Cross Validation statistics (GCV) and the average Root Mean Squared Error (RMSE) where the averages are taken across all the months in Table 1 from the non-linear debt regressions.
RMSE/ Price is the average Root Mean Squared Error (RMSE) from Table 4 divided by the average bond price. It is a measure of the percentage pricing error.
$exp(-\gamma)$ is the average liquidity discount determined using the estimated liquidity discount parameters underlying Table 2.
λ is the default intensity assuming a constant recovery rate of .5, based on the estimated default parameters underlying Table 2.
1 yr dfp is the 1-year default probability for the various models, based on the estimated default parameters underlying Table 2, and using a constant recovery rate of .5.