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ECONOMIC THEORY, APPLICATIONS AND ISSUES

Working Paper No. 78

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Parametric Framework: Application to
State-Level Agricultural Data for the
United States**

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Estimating Input-Mix Efficiency in a Parametric Framework: Application to State-Level Agricultural Data for the United States

ABSTRACT

This paper contributes to the productivity literature by demonstrating novel econometric methods to estimate input-mix efficiency (IME) in a parametric framework. Input-mix efficiency is defined as the potential improvement in productivity with change in input mix. Any change in input-mix (e.g., land to labor ratio) will result in change in productivity. We minimize a nonlinear input-aggregator function (e.g., Constant Elasticity of Substitution) to derive an expression for input-mix efficiency. We estimate a Bayesian stochastic frontier for obtaining mix efficiency using US state-level agricultural data for the period 1960 – 2004. We note significant variation in input-mix efficiency across the states and regions, attributable to diverse topographic, geographic and infrastructure conditions. Furthermore, comparisons of allocative and mix efficiencies provide insightful policy implications. For example, the production incentives such as taxes and subsidies could help farmers in adjusting their input mix in response to changes in input prices, which can affect the US agricultural productivity significantly. We provide a simple way of estimating mix efficiency in an aggregate-input, aggregate-output framework. This framework can be extended by i) using flexible functional forms; ii) introducing various time- and region-varying input aggregators; and iii) defining more sophisticated weights for input aggregators.

Key words: Mix efficiency, aggregator function, Bayesian stochastic frontier, productivity

JEL Codes: D21, D24, C43

Estimating Input-Mix Efficiency in a Parametric Framework: Application to State-Level Agricultural Data for the United States

1 Introduction

Identification of recognizable sources of productivity change plays an important role in policy development for various industries. Consistent and reliable productivity components help policy makers to assess whether more benefit could be reaped through expenditure on research and development for technical progress, or through achieving scope economies (O'Donnell, 2012a). Productivity can be changed through policy instruments, such as taxes and subsidies, by altering input-output combinations (i.e., scope economies). As noted by Hsieh and Klenow (2009, p. 1404), incorrect use of input or output mixes can ultimately reduce the level of productivity. It is well accepted that increased productivity is important for a society's long term economic welfare. O'Donnell (2012b) proposed a comprehensive decomposition of productivity measures using an aggregate-input, aggregate-output framework. Within this framework, he decomposed productivity change into technical-efficiency change, scale-efficiency change, mix-efficiency change and other measures of efficiency change (e.g., scale-mix efficiency).

Mix efficiency (a relatively new concept) is defined as the potential improvement in productivity when input or output mixes are altered. Any change in output mix (e.g., balance of crops and livestock) or input mix (e.g., land-to-labor ratio) results in a change in productivity. Mix efficiency is similar to allocative efficiency, but it differs in terms of its economic interpretation. Both measures are derived by solving a (cost or aggregate input) minimization problem. An improvement in mix efficiency increases productivity, which in turn contributes to the improved wellbeing of the people. On the other hand, improved allocative efficiency results in increased profit or reduced cost for a firm, which increases that firm's prosperity. The benefits of increased productivity far exceed the benefits of increased profit in enhancing social welfare. In this context, mix efficiency seems to be more important than allocative efficiency. The concept of mix efficiency used here is based on aggregate input quantities (a purely productivity-based concept) whereas allocative efficiency is a distinct value-based concept.

This paper contributes to the literature on productivity by developing an analytical method to measure mix efficiency in a parametric framework. The measurement of mix efficiency has been proposed using nonparametric data envelopment analysis (see O'Donnell, 2010, 2012a; Rahman and Salim, 2013; Mughera et al., 2016). The paper develops a parametric econometric measure for input-oriented mix efficiency using linear and nonlinear aggregator functions. An important advantage of this econometric approach is that it is relatively easy to impose regularity (curvature conditions) on the production function and to undertake statistical inference about mix efficiencies. Our expression for input-mix efficiency is derived along the lines of the work of Schmidt and Lovell (1979).

The aim of the paper is to minimize both linear and nonlinear aggregator functions to derive an expression for input-oriented mix efficiency. We choose the constant elasticity-of-substitution (CES) aggregator function for our input-minimization problem. While economists have used value-based CES aggregators to construct aggregate utility and demand functions and aggregate production functions, these aggregators have hardly been used as input aggregators (e.g., see Shapiro and Wilcox, 1997). The flexibility of the CES aggregator function leads to different linear and nonlinear aggregators that can be used to construct various multiplicatively complete productivity indexes (for details, see O'Donnell, 2012b). The CES aggregator is a generalized form which encompasses various other forms of aggregators (e.g., Lowe, Cobb-Douglas) as special cases (see Arrow et al., 1961, p. 230). The CES aggregator also leads to several corresponding productivity indexes such as the Geometric Young, the Lloyd-Moulton or the Färe-Primont indexes (de Haan et al., 2010; Lent and Dorfman, 2009).

The results of mix efficiency are illustrated by applying our methodology to agricultural data for 48 states of the United States over a 45-year period, 1960-2004. Using a Bayesian stochastic frontier, we estimate the highest posterior densities (HPD) and their respective confidence intervals (to draw statistical inferences) for state and regional mix efficiencies. Empirical findings indicate that large variations in input mix across states or regions are associated with substantial differences in mix efficiency. For instance, the mix efficiencies of states in the Mountain Region differ considerably from those of states in the Corn Belt. These findings have several implications for agricultural productivity growth in the US, as estimation of mix efficiency can help identify likely change in net returns resulting from varying input mixes.

The paper is organized as follows: Section 2 describes the different input-aggregator functions that can be used to derive input-mix efficiency in an aggregate-input framework. The distinction between allocative and mix efficiencies is set out in Section 3. Analytical methods for deriving input-mix efficiency are discussed in Section 4. Section 5 provides an empirical application of mix efficiency using US state level agricultural data. In this example, we estimate the Bayesian production function to compute input-oriented mix and allocative efficiencies. Finally, Section 6 provides concluding comments.

2 Technology-Based Aggregator Functions

Computation and decomposition of productivity in an aggregate framework requires choosing particular input and output aggregators. The class of input and output aggregators includes both linear and nonlinear functions. O'Donnell (2012b) provides a detailed discussion of various price - and technology-based input- and output-aggregator functions and their respective properties. To conserve space, we only illustrate technology-based input-aggregator functions.

Let $q_t \in \mathcal{Y}_+^M$ and $q_s \in \mathcal{Y}_+^M$ be output vectors in the current and reference periods, and let $x_t \in \mathcal{X}_+^K$ be an input vector in the current period. An input-aggregator function can be expressed as $X_t = X(x_t)$, where $X(\cdot)$ is a non-negative, nondecreasing and linearly homogenous function. Technology-based input-aggregator functions can be represented by input distance functions $D_I(q, x_t) = \max\{\lambda: x_t/\lambda \in L(q)\}$, with $L(q)$ being the set of input vectors feasible for output vector q . Four input-aggregator functions based on input distance functions are

$$X(x_t) = D_I^t(q_t, x_t) \quad (1)$$

$$X(x_t) = D_I^s(q_s, x_t) \quad (2)$$

$$X(x_t) = D_I^r(q_r, x_t) \quad (3)$$

$$X(x_t) = \left[D_I^t(q_t, x_t) D_I^s(q_s, x_t) \right]^{1/2} \quad (4)$$

Input-aggregator function (1) uses current-period output and technology to aggregate the elements of x_t ; input-aggregator function (2) uses reference-period output and technology; input-aggregator function (3) uses an arbitrary-period output and technology, where $r \neq s, t$; and input-aggregator function (4) is the geometric mean of (1) and (2). Adapting the work of Malmquist (1953), in a consumer context, to a producer context, Caves et al. (1982), Bjurek (1996), and Färe and Primont (1995) used these input-aggregator functions and analogous output-aggregator functions, based on output distance functions, $D_O(x, q_t) = \min\{\mu:$

$q_t/\mu \in P(x)$, where $P(x)$ is the set of output vectors feasible with input vector x , to construct different input- and output-quantity indexes, from which they constructed different productivity indexes (see O'Donnell, 2014, p. 190).

Other technology-based input-aggregator functions, not based on input distance functions, are available, three of which are:

$$X(x_t) = \left[\sum_{k=1}^K \delta_k x_{kt}^\theta \right]^{1/\theta} \quad (5)$$

$$X(x_t) = \prod_{k=1}^K x_{kt}^{\delta_k} \quad (6)$$

$$X(x_t) = \sum_{k=1}^K \delta_k x_{kt} \cdot \quad (7)$$

The CES input-aggregator function (5) forms the basis for a CES input-quantity index that shares a structure with the Lloyd–Moulton consumer price index (Lloyd, 1975; Moulton, 1996), which approximates a superlative price index without requiring current-period data (Shapiro and Wilcox, 1997; de Haan et al., 2010). The CES input-aggregator function approaches the Cobb-Douglas input-aggregator function (6), which forms the basis for the geometric Young input-quantity index, as $\theta \rightarrow 0$, and collapses to the linear input-aggregator function (7) if $\theta=1$. Each satisfies the requisite regularity conditions under parametric restrictions. These aggregators can be used to derive various measures of productivity. However, we confine our attention to measure the input-mix efficiency component.

3 Allocative Efficiency versus Input-mix efficiency

Traditional input allocative efficiency (IAE) is a component of cost efficiency, a measure of the success with which a firm pursues the economic objective of minimising the cost of producing its chosen vector of outputs, given an input price vector, $w \in \mathcal{R}_{++}^K$. Cost efficiency $CE(q, w, x) = c(q, w)/w^T x \leq 1$, with actual cost $w^T x$ and minimum cost $c(q, w)$, the value of the solution to the problem $\min_x \{w^T x : x \in L(q)\}$. Cost efficiency decomposes into the product of technical efficiency, $TE(y, x) = w^T(x/D_I(q, x))/w^T x \leq 1$, and input allocative efficiency (IAE), $(q, w, x) = c(q, w)/w^T(x/D_I(q, x)) \leq 1$. At an allocatively efficient input vector all input price ratios are equal to the corresponding marginal rates of technical substitution.³

³ If $f(x)$ is not everywhere differentiable, then $[f_i(x)/f_j(x)]^+ \leq w_i/w_j \leq [f_i(x)/f_j(x)]^- \forall i, j$, as would be the case if technology were modelled with data envelopment analysis.

The economic consequences of allocative efficiency are well understood, motivating researchers to use different approaches to measure cost and revenue (or profit) allocative efficiency (Leibenstein, 1966; Coelli et al., 2005; Kumbhakar and Lovell, 2000). Misallocation of resources reduces the benefits for producers. If firms are unable to equate the marginal productivity of input factors to their respective prices, this may increase the cost of production, ultimately decreasing the benefit to society. A vast literature has debated the consequences of allocative inefficiencies for firms in different sectors including the manufacturing, agricultural and services sectors (Lau and Yotopoulos, 1971; Toda, 1976; Schmidt and Lovell, 1979, 1980; Greene, 1980; Kumbhakar, 1991, 1997 Kumbhakar and Tsionas, 2005; Brissimis et al., 2010). It is argued that the introduction of regulations and distortionary policies can affect the costs and revenues of firms, leading to inappropriate allocation of resources which reduces profit. However, mix efficiency indicates the extent to which a firm's productivity can be altered by changing input-output combinations.

Input-mix Efficiency

Following O'Donnell (2012b), we define the concept of technical and mix efficiency in aggregate quantity space and provide a graphical illustration. Consider that two firms, A and B, use aggregate inputs, $X_t^A = \delta_1 x_{1t}^A + \delta_2 x_{2t}^A$ and $X_t^B = \delta_1 x_{1t}^B + \delta_2 x_{2t}^B$, to produce an output vector represented by an isoquant, $I(q_t)$, where δ_1 and δ_2 have specific values (e.g., input shares). The dashed lines represented by X_t^A and X_t^B are called iso-aggregate inputs because the lines map all the points having specific aggregate input for each firm passing through a and b , that are technically inefficient. The curve passing through c and e shows all technically efficient inputs to produce an output vector (isoquant $I(q_t)$). While holding input mixes and the output vector fixed, the firm operating at point a can reduce its aggregate input to the point, c . This radial contraction is well known as the concept of technical efficiency (Farrell, 1957). Thus, the input-oriented technical efficiency of firm a can be defined as $ITE_t = \bar{X}_t^A / X_t^A$, where, \bar{X}_t^A is the minimum aggregate input quantity when both input mixes and the output vector are held fixed.⁴ On the other hand, input-mix efficiency occurs as a consequence of relaxing restrictions on input and output mixes. If input mixes are allowed to vary while holding the output vector fixed then the firm can further reduce its input aggregate (i.e., the minimum possible input aggregate given the same coefficients δ_1 and δ_2 , defined above). This occurs at point e , a

⁴ $\bar{X}_t = X_t / D_t(q_t, x_t)$.

further reduction in aggregate input by changing input mixes but holding the output vector constant, which is represented by \hat{X}_t .⁵ Thus, the input-mix efficiency of Firm A is defined as $IME_t = \hat{X}_t / \bar{X}_t^A$. O'Donnell (2012b, p. 261) also uses the term ‘pure’ input-mix efficiency because input mixes are allowed to change while holding the output vector fixed. Thus, input-mix efficiency measures the potential change in productivity when restrictions on input mixes are relaxed (as shown in panel b of Figure 1). An obvious consequence of mix efficiency (particularly IME) is that managers may avoid overuse of some inputs in response to substitution policies, which may increase the productivity of firms. However, the concept of mix efficiency has not been well-understood in the context of productivity measurement.

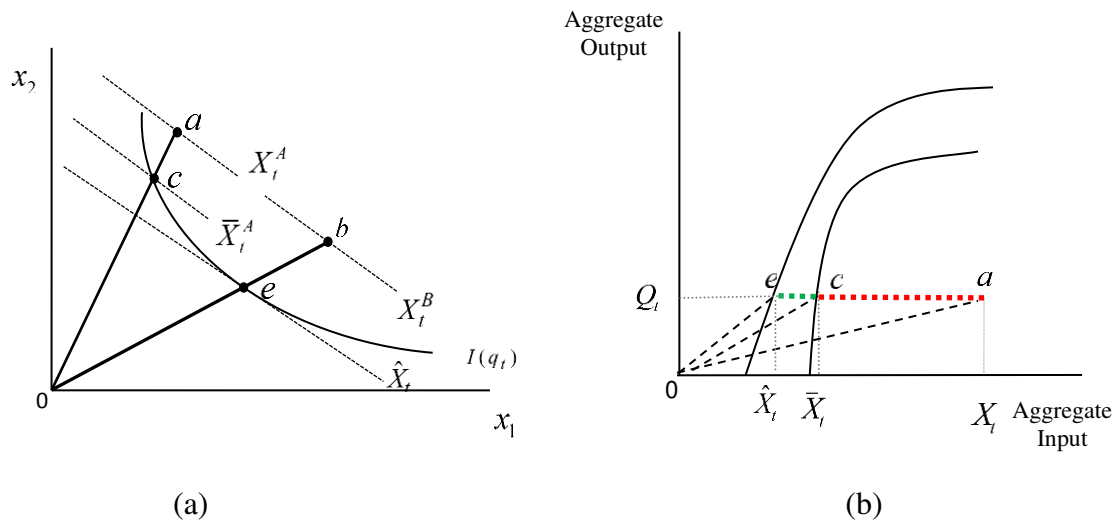


Figure 1: Relationship between Mix Efficiency and Productivity

Proposition 1. *For a given level of technical efficiency (arbitrarily chosen), any increase in input-mix efficiency will lead to an increase in productivity.*

Proof. This can be proved using Figure 1. Consider that Firm A and Firm B are technically efficient at points, c and e , respectively. Because both firms produce the same output (as can be seen geometrically), any further reduction in the input aggregate (towards the minimum possible point) by relaxing restrictions on input mixes leads to an increase in input-mix efficiency. In other words, any movement from c to e (i.e., change in input mixes) leads to an increase the input-mix efficiency; hence, productivity increases (i.e., $IME_t = \hat{X}_t / \bar{X}_t^A$). This

⁵ $\hat{x}_t = \operatorname{argmin}_{x>0} \{X(x_t) : (q_t, x_t) \in T_t\}$.

statement is valid for any level of technical efficiency, given the specific aggregator function used to construct input aggregates passing through a , b , c and e . An increase in IME can also be confirmed from Figure 1(b), where movement from point c to point e , allows the firm to vary input mixes, while holding the output vector fixed; hence, productivity increases. *QED*.

4 Analytical Model of Mix Efficiency

we consider a multi-output, multi-input technology. The production possibility set is presented as follows:

$$T_t = \{(x_t, q_t, z_t) \in \mathfrak{R}_+^K \times \mathfrak{R}_+^M \times \mathfrak{R}_+^J : g(q_t) \leq c(z_t)h(x_t)^r\} \quad (8)$$

where $q_t = (q_{i1}, \dots, q_{iM}) \in \mathfrak{R}_+^M$ is a vector of output quantities; $x_t = (x_{1t}, \dots, x_{Kt}) \in \mathfrak{R}_+^K$ is a vector of input quantities; $z_t = (z_{1t}, \dots, z_{Jt}) \in \mathfrak{R}_+^J$ is a vector of exogenous factors such as technical change and other production characteristics (e.g., geographic); r represents returns to scale; $g(\cdot): \mathfrak{R}_+^M \rightarrow \mathfrak{R}_+$ is an output-aggregator function, assumed to be non-negative, nondecreasing and linearly homogenous in outputs; $h(\cdot): \mathfrak{R}_+^K \rightarrow \mathfrak{R}_+$ is assumed to be monotonic (i.e., nondecreasing in inputs), quasi-concave, upper-semi continuous and linearly homogeneous; and $c(\cdot): \mathfrak{R}_+^J \rightarrow \mathfrak{R}_+$ is nondecreasing. Both $g(\cdot)$ and $h(\cdot)$ are separable in outputs and inputs (Chambers, 1988, p. 285; Chamber and Fare, 1993, pp. 197–198). Further, this technology characterization satisfies the regularity properties of inactivity, boundedness, free disposability of inputs and outputs, and essentiality. The alternative representation of the above technology is the input distance function as described by Shephard (1953, 1970). The input distance function shows how much the input vector can be scaled down while the output vector is held fixed.⁶ Given the technology in equation (8), the input distance function is described as

$$D_t(q_t, x_t, z_t) = \frac{c(z_t)^{\frac{1}{r}} h(x_t)}{g(q_t)^{\frac{1}{r}}} \geq 1. \quad (9)$$

The input distance function given in equation (9) is the characterization of the underlying production technology presented in equation (8) and satisfies certain properties (under the regularity conditions of the production possibility set discussed above). If the technology satisfies the above assumptions, then the input distance function is concave, nondecreasing and linearly homogenous in inputs (see Färe et al., 1985; Färe and Primont, 1995; Kumbhakar and Lovell, 2000).

⁶ Properties of input and output distance functions are given in footnote 5 of O'Donnell and Nguyen (2013).

IME: An Aggregator Function with Homogenous Technology

In this case, we assume a homogenous production technology for our input-aggregator function minimization problem and derive the first-order conditions to obtain the minimum input aggregator. Consider the technology given in equation (8) as

$$g(q_t) \leq c(z_t)h(x_t)^r. \quad (10)$$

We solve the input-aggregator function minimization with a nonlinear aggregator function to obtain an expression for input-mix efficiency by choosing a CES nonlinear aggregator and a homogenous Cobb-Douglas production function by using an aggregate output, to obtain minimum input aggregates for a technically and mix-efficient firm. The CES aggregator function, as given in equation (5), is a non-negative, nondecreasing and linearly homogeneous function and subsumes many other aggregator functions (see Samuelson and Swamy, 1974, p. 574).

If a firm chooses an input vector which is technical efficient on the production frontier, then $D_t(q_t, x_t) = 1$. The aggregate input-minimization problem for the firm is:

$$\hat{X}_t = \min_x \{X(x_t) : g(q_t) = c(z_t)h(x_t)^r\}. \quad (11)$$

The returns-to-scale parameter r , can take any positive value (i.e., $r > 0$). For example, $r > 1$ implies that the production function exhibits increasing returns to scale (IRS), $r < 1$ shows decreasing returns to scale (DRS) and $r = 1$ exhibits constant returns to scale. The minimization problem discussed here is different from that discussed by Schmidt and Lovell (1979). We use a nonlinear aggregator to derive an expression for mix efficiency whereas conventional cost minimization or revenue (or profit) maximization is undertaken using linear aggregators, and they have market prices to characterize the aggregator; in contrast, we only have parameters to specify and estimate.

The Lagrangian for the input-aggregator minimization is defined as:

$$L = X(x_t) + \lambda [g(q_t) - c(z_t)h(x_t)^r]. \quad (12)$$

The first-order conditions are:

$$\frac{\partial L}{\partial x_k} = X_k(x_t) - \lambda r c(z_t) h(x_t)^{r-1} h_k(x_t) \leq 0 \quad \text{for } k = 1, \dots, K. \quad (13)$$

and

$$\frac{\partial L}{\partial \lambda} = g(q_t) - c(z_t)h(x_t)^r = 0 \quad (14)$$

where, $X_k(x_t) \equiv \partial X(x_t) / \partial x_{kt}$ and $h_k(x_t) \equiv \partial h(x_t) / \partial x_{kt}$. If the firm is technically efficient then taking the ratio of the first-order conditions of the k -th input and first input yields

$$\frac{X_k(x_t)}{X_1(x_t)} = \frac{h_k(x_t)}{h_1(x_t)} \quad \text{for } k = 2, \dots, K. \quad (15)$$

If a firm is technically and mix efficient, it will choose x that satisfies equation (15). However, if the firm makes errors in choosing the correct input mixes then mix inefficiency would prevail. In this situation, the first-order conditions fail to hold. Therefore,

$$\frac{X_k(x_t)}{X_1(x_t)} = \frac{h_k(x_t)}{h_1(x_t)} \exp(\tau_k) \quad \text{for } k = 2, \dots, K \quad (16)$$

where $\tau = 1, \tau_2, \dots, \tau_k$ is a vector of $K-1 \times 1$ input quantity-adjustment scalars, which becomes zero if the firm utilizes an input mix that minimizes the aggregator input function. The interpretation of τ is straightforward. The presence of optimization errors (i.e. $\tau \neq 0$) may increase or decrease the use of x_k (relative to x_1) depending on value of τ . We interpret these errors as mix inefficiency, whereas Schmidt and Lovell (1979) in their cost-minimization problem refer to it as allocative inefficiency. If $\tau > 0$, then the firm will underutilize the k -th input mix with respect to input 1 and $\tau < 1$ indicates that the firm will overutilize the k -th input mix with respect to input 1. The significant difference between our minimization problem and existing cost minimization or revenue-maximization problems is that we use the nonlinear aggregator function for optimization. If we use a linear aggregator function then the input-mix efficiency can be derived in a similar fashion to the derivation of Schmidt and Lovell (1979) cost allocative inefficiency. However, our expression has a fundamentally different interpretation because we do not use input prices to construct our input-aggregator function.

Proposition 2. *If the input-aggregator function is nonlinear (i.e., of CES form) and the technology is homogeneous (i.e., Cobb-Douglas), then the level of IME can be obtained by solving the input-aggregator minimization problem, which gives*

$$IME_t = \frac{\hat{X}_t}{\bar{X}_t} = \left[\frac{r}{\exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k\right) \left(\sum_{k=1}^K \beta_k \exp(-\tau_k)\right)} \right]^{\frac{1}{\theta}} \leq 1.$$

Proof. We use Cobb-Douglas production technology, homogenous of degree r , as follows

$$h(x_t)^r = \prod_{k=1}^K x_{kt}^{\beta_k} \quad (17)$$

By differentiating the input-aggregator function given in equation (5) and the production technology represented by equation (17) with respect to x_k , we obtain

$$X_k(x_t) \equiv \frac{\partial X(x_t)}{\partial x_k} = \delta_k x_{kt}^{\theta-1} \left[\sum_{k=1}^K \delta_k x_{kt}^\theta \right]^{\frac{1}{\theta}-1} \quad (18)$$

$$h_k(x_t) \equiv \frac{\partial h(x_t)}{\partial x_{kt}} = \beta_k x_{kt}^{-1} \prod_{k=1}^K x_{kt}^{\beta_k} \quad (19)$$

If the firm is mix inefficient then the first-order conditions become

$$\frac{\delta_k x_{kt}^{\theta-1}}{\delta_1 x_{1t}^{\theta-1}} \exp(\tau_k) = \frac{\beta_k x_{kt}^{-1}}{\beta_1 x_{1t}^{-1}} \quad (20)$$

The mix inefficient (but technically efficient) firm chooses x_t that solves the following:

$$x_{kt} = x_{1t} \left[\frac{\delta_1 \beta_k}{\delta_k \beta_1} \exp(-\tau_k) \right]^{\frac{1}{\theta}}. \quad (21)$$

After substituting equation (21) into equation (14) and combining with equation (17), we solve for x_{1t} and x_{kt}

$$x_{1t} = c(z_t)^{-\frac{1}{r}} g(q_t)^{\frac{1}{r}} \left[\prod_{k=1}^K \left(\frac{\delta_k}{\beta_k} \right)^{\frac{\beta_k}{r}} \frac{\beta_1}{\delta_{k1}} \exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k \right) \right]^{\frac{1}{\theta}} \quad (22)$$

$$x_{kt} = c(z_t)^{-\frac{1}{r}} g(q_t)^{\frac{1}{r}} \left[\prod_{k=1}^K \left(\frac{\delta_k}{\beta_k} \right)^{\frac{\beta_k}{r}} \frac{\beta_k}{\delta_k} \exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k \right) \exp(-\tau_k) \right]^{\frac{1}{\theta}}. \quad (23)$$

Combining equations (22) and (23) with equation (5), we find the input aggregate of a technically efficient firm

$$\bar{X}(x_t) = b_k^* \left[\exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k \right) \left(\sum_{k=1}^K \beta_k \exp(-\tau_k) \right) \right]^{\frac{1}{\theta}} g(q_t)^{\frac{1}{r}} \quad (24)$$

where, $b_k^* = \left[c(z_t)^{-1} \prod_{k=1}^K \left(\frac{\delta_k}{\beta_k} \right)^{\frac{\beta_k}{r}} \right]$.

If the firm is mix efficient then x_k corresponds to $\tau_k = 0$ for $k = 2, \dots, K$, and equation (24) becomes

$$x_k = b_k^* \left[\prod_{k=1}^K \left(\frac{\delta_k}{\beta_k} \right)^{\frac{\beta_k}{r}} \frac{\beta_k}{\delta_k} \right]^{\frac{1}{\theta}} g(q_t)^{\frac{1}{r}} \quad (25)$$

By substituting equation (25) into equation (5), we obtain the minimum possible aggregate input (holding the output vector fixed) as

$$\hat{X}(x_t) = b_k^* \left[\sum_{k=1}^K \beta_k \right]^{\frac{1}{\theta}} g(q_t)^{\frac{1}{r}}. \quad (26)$$

Finally, input-mix efficiency is given as the ratio of equation (26) to equation (24) and $\sum_{k=1}^K \beta_k = r$

$$IME_t = \frac{\hat{X}_t}{\bar{X}_t} = \left[\frac{r}{\exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k\right) \left(\sum_{k=1}^K \beta_k \exp(-\tau_k)\right)} \right]^{\frac{1}{\theta}} \leq 1. \quad (27)$$

QED.

This expression for mix efficiency can be used to describe many other special cases, some of which we consider in the following section. As discussed previously, the final expression of IME in equation (27) is a closed-form solution that is bounded, i.e., $0 < IME_t \leq 1$. (see Schmidt and Lovell, 1979; Kumbhakar, 1988).

Some Special Cases

As discussed earlier, the CES aggregator function encompasses many functional forms, so that we can obtain many other expressions of mix efficiency by restricting the value of θ .

For instance, if the input-aggregator function is linear as given equation (7), then the IME expression becomes

$$IME_t = \frac{\hat{X}_t}{\bar{X}_t} = \left[\frac{\left(\sum_{k=1}^K \beta_k\right)}{\exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k\right) \left(\sum_{k=1}^K \beta_k \exp(-\tau_k)\right)} \right] \leq 1. \quad (28)$$

The expression of IME in equation (28) is similar to that of Schmidt and Lovell (1979), which can also be obtained by minimizing a linear input aggregator given in equation (7), subject to the input distance function given by the equation (9). The IME defined in equation (28), can be derived using any linear input aggregator (e.g., Lowe).

Proposition 3. *If the parameter δ_k is substituted with input prices w_k , these expressions will exactly produce Schmidt and Lovell (1979) allocative efficiency estimates.*

See proof in Appendix A.

5 An Econometric Model

The estimation of mix efficiency in an econometric framework requires estimation of the production technology and aggregator function. If technology is represented by multiple outputs as in equation (8), then the output distance function can be represented as

$$\ln D_o(q_t, x_t, z_t) = \ln g(q_t) - \ln c(z_t) - \ln h(x_t)^r - v_t \quad (29)$$

where $g(\cdot)$, $c(\cdot)$, and $h(\cdot)$ are as previously defined, and v_t is an error term taking into account statistical noise due to factors such as droughts or floods and other errors of approximation. The error terms for all t are assumed to be independent and identically distributed such that $v_t \sim iid N(0, \sigma_v^2)$.

In the multiple-output, multiple-input case, it is common practice to estimate an output or an input distance function (Atkinson et al., 2003; Coelli et al., 2005). The estimation of these input or output distance functions requires that one of inputs or outputs is considered a dependent (endogenous) variable, whereas the other (input or output) variables are treated as exogenous variables. However, it is likely that two or more inputs (in the case of the input distance function) and two or more outputs (in case of the output distance function) may be correlated with the statistical error terms. As a result, the estimates become biased because of the endogeneity issue. This endogeneity problem is usually remedied by applying two-stage least squares or the Generalized Method of Moments (see Atkinson and Primont, 2002). However, the choice of arbitrary moment conditions is disadvantageous if the instruments are not defined appropriately (O'Donnell, 2014, 2015).

In the case of a single output, the distance function defined by equation (29) can be expressed in conventional stochastic frontier form as

$$\ln Q_t = \ln c(z_t, \gamma) + \ln h(x_t, \beta)^r + v_t - u_t \quad (30)$$

where $Q_t = Q(q_t)$ is an aggregate output; $c(z_t, \gamma) = \exp(\alpha_0 + \alpha_1 t + \sum_{l=1}^L \gamma_l D_l)$; $h(x_t, \beta)^r = \prod_{k=1}^K x_{kt}^{\beta_k}$, where D_l represents regional dummies; t denotes the time period; $x_{kt} = (x_{1t}, x_{2t}, x_{3t}, x_{4t})$ represents the input variables, capital, land, labor and materials; and $u_t = -\ln D_o(x_t, q_t, z_t)$ is a one-sided error measuring the extent of firm technical inefficiency.

In this paper, we use the Cobb-Douglas approximation which has extensive application in agricultural productivity measurement (e.g., see Battese and Corra, 1977; Kalirajan, 1981,

1989; Battese and Coelli, 1988, 1992, 1995; Bravo-Ureta and Evenson, 1994; Timmer, 1971; O'Donnell (2012a, 2012b). Another popular approximation that has also been used widely in agricultural productivity is the translog production function (i.e., the second-order approximation of a linearly homogeneous production function) by Kumbhakar (1994), Darku et al., (2015), Moreira and Bravo-Ureta (2016), Reinhard et al. (1999), and Tsiaonas et al. (2016). However, these approximations cannot impose pointwise regularity (positivity, monotonicity, and curvature) restrictions unless all second-order coefficients collapse to zero, in which case the translog production reduces to the Cobb-Douglas production function (O'Donnell, 2013; Serletis and Feng, 2015). We use the Cobb-Douglas production function to estimate IME levels.

A log-linear Cobb-Douglas frontier with a single aggregate output is represented as follows:

$$\ln Q_t = \alpha_0 + \alpha_1 t + \sum_{l=1}^L \gamma_l D_l + \sum_{k=1}^K \beta_k \ln x_{kt} + v_t - u_t \quad (31)$$

where the parameters α_0 and α_1 represent constant terms and the rate of neutral technical change, respectively; whereas $\beta_k \geq 0$ and $\gamma = (1, \dots, \gamma_j)$ denotes a vector of unknown parameters such that $\sum_{k=1}^K \beta_k = r$. Because agricultural practices vary significantly in different regions of the US (e.g., Corn Belt vis-à-vis the Appalachians) due to geographical and climatic conditions (see Ball et al., 2010), it is important to account for regional differentials in mix efficiency in our analysis. The data show that the different regions in the US produce different agricultural outputs and use different input mixes. The variation in input mix may be partly due to variation in the production environment. For example, output from the Pacific region (e.g., fruit and vegetable crops) is different from output from the South East (e.g., livestock). Similarly, the use of inputs varies from region to region. One way to account for these differences is by introducing regional dummies into the model. The inclusion of regional dummy variables allows us to change the production environment across regions. We use the approach of Ball et al. (2010) to classify these regions. According to them, the Pacific region, which is considered one of the most productive regions, is treated as the base region.

We estimate the Bayesian stochastic production frontier by using pooled data of the US agricultural sector to obtain IME estimates. The main advantages of the Bayesian stochastic frontier estimation are that: a) one can draw exact inferences on efficiencies; b) it is easy to incorporate prior information and regularity restrictions; and c) the method provides a formal

treatment of parameters and model uncertainty through numerical integration methods for complex stochastic frontier models. Statistical inference on efficiency measures is essential for policy purposes. The Bayesian method allows estimation of the posterior distribution by assigning a subjective probability distribution to a parameter, using the available sample information. In this way, one can draw the highest posterior densities (HPD) of state-specific efficiency measures. Computation of exact standard errors enables us to draw inferences on whether the efficiencies of one state are statistically significantly different from those of another state.

In Bayesian stochastic frontier analysis, the distribution of inefficiency components is determined using a posterior simulator (e.g., the Gibbs sampler). Van den Broeck et al. (1994) provided the earliest estimation of a stochastic frontier function using Bayesian methods with cross-sectional data. Later, a series of papers described this method using both cross-sectional and panel data sets (see Koop et al., 1997; Osiewalski and Steel, 1998; and Griffin and Steel, 2007). More recently, O'Donnell (2014) has applied the Bayesian method to estimate US agricultural productivity.

The compact form of the model of equation (31) can be written as:

$$y = X\beta - v - u \quad (32)$$

where $y \equiv \ln Q_t$; X is a matrix of order $T \times (K + J + 2)$; $\beta = (\alpha_0, \alpha_1, \beta_1, \dots, \beta_K, \gamma_1, \dots, \gamma_J)'$ represents the vector of parameters to be estimated; $v = (v_1, \dots, v_T)$ is a vector of normal random errors representing the combined effect of measurement errors and errors occurring due to approximation of the functional form; and $u = (u_1, \dots, u_T)$ is the vector in non-negative inefficiency effects.

All elements in v are assumed to be identically and independently distributed with joint probability density function (pdf):

$$v \sim N(0_T, h^{-1}I_T) \quad (33)$$

where h is a precision variable (i.e., the inverse of the variance σ_v^2); 0_T represents a vector of zeros having dimension T ; and I_T denotes the identity matrix of order T . The pdf of v is given by $p(v|h) = f_N(v|0, h^{-1})$. The vector of random variables, u , is also independently distributed

and accounts for technical inefficiency, which is assumed to be exponentially distributed i.e., $u \sim \exp(\lambda)$.

To proceed with Bayesian estimation, we choose appropriate priors and distributions for the parameters of interest. To impose regularity conditions, in the case of the Cobb-Douglas production function, input elasticities are assumed to be non-negative. For instance, to incorporate regularity conditions into the estimation procedure, an informative prior, $p(\beta, h) \propto h \times I(\beta \in R)$, is considered, where $I(\beta)$ is an indicator function that is equal to 1 if the production function satisfies monotonicity, and 0 otherwise. In this analysis, the following priors are assumed to generate posterior densities:

$$p(\beta, h, u, \lambda) = p(\beta, h) p(u | \lambda) p(\lambda) \quad (34)$$

$$p(u | \lambda) = \prod_{t=1}^T p(u_t | \lambda) \quad (35)$$

$$p(\lambda) = -\ln(\zeta^*) \left\{ \frac{1}{\lambda} \ln(\zeta^*) \right\} \quad (36)$$

where ζ^* indicates the prior median technical inefficiency level, and also $\zeta^* \in (0,1)$. We choose the median technical efficiency of 0.90 (i.e., prior estimate).⁷ The posterior densities are not sensitive to the choice of priors.

The likelihood function is given by:

$$p(y | \beta, h, u) = \sqrt{\left(\frac{h}{2\pi}\right)^T} \exp\left[-\frac{h}{2}(y - X\beta + u)'(y - X\beta + u)\right]. \quad (37)$$

The combination of the prior with the likelihood function generates the posterior density $P(\theta)$. The conditional posterior pdfs can be derived by using the likelihood function of equation (8) and combining it with the priors defined by equations (5) through (7), which are described as:

$$p(\beta | h, \lambda, u, y) = f_N\left(\beta | \hat{\beta}, \frac{(X'X)^{-1}}{h}\right) \times I(\beta \in R) \quad (38)$$

$$p(h | \beta, \lambda, u, y) = f_G\left(h | T/2, \frac{1}{2}(y - X\beta + u)'(y - X\beta + u)\right) \quad (39)$$

$$p(\lambda^{-1} | \beta, h, u, y) = f_G\left(\lambda^{-1} | T+1, u' - \ln(\zeta^*)\right) \quad (40)$$

⁷ Studies using the same data set used the average efficiency equal to 0.90 as an informative prior. For instance, O'Donnell (2012) assumes $\zeta^* = 0.90$ using in his study for the same data set.

$$p(u_i | \beta, h, \lambda, y) = f_N(u_i | X' \beta - y - h^{-1} \lambda^{-1}, h^{-1}) \times I(u_i \geq 0) \quad (41)$$

where $\hat{\beta} = (X'X)^{-1}X'(y+u)$.

The pdfs given by equations (37) through (41) are simulated using the Gibbs sampling method, which involves the accept-reject Markov chain Monte Carlo (MCMC) algorithm. Further details can be found in Koop et al. (1997).

To illustrate the proposed methods of mix efficiency (and also allocative efficiency), US agricultural data are used for the econometric estimation of mix efficiency. The state-level US agricultural data were compiled by the Economic Research Service of the United States Department of Agriculture (ERS-USDA). The data consist of an aggregate output and four inputs for 48 states for 45 years from 1960 to 2004, which are pooled to obtain 2160 observations.⁸ The input data cover capital, land, labor and materials. The capital input comprises equipment and buildings, whereas labor includes both hired labor and the self-employed (see Ball et al., 2004). Material inputs include fertilizers, other chemicals and energy components.⁹ We use the aggregate output that was constructed by ERS-USDA.¹⁰

6 Empirical Example

To obtain a parsimonious model, we estimated several specifications of the Cobb-Douglas production and the final model includes time interaction with the materials input (i.e., $x_{4t} \times t$).¹¹ We only report the maximum likelihood estimates of the restricted production function (without regional dummies) and Bayesian ML estimates of the Cobb-Douglas production function (that includes regional dummies) in Table 1. Further, a joint hypothesis test is conducted to see if inclusion of regional dummy variables affects estimates of the agricultural production function. The value of the likelihood ratio test statistic (450.14) is highly significant at the 1% level of significance ($\chi^2_{9(0.01)} = 3.17$), indicating that different geographic and climatic environments profoundly impact agricultural production. Panel (a) of Figure 1 presents the

⁸ More details on the construction of the output and input variables can be found in Ball et al. (2004). These details are also available on ERS-USDA website: <http://www.ers.usda.gov/data>.

⁹ All inputs are adjusted for quality using hedonic prices. A detailed methodology for the quality adjustment of inputs can be found in Kellogg et al. (2002).

¹⁰ Output quantity indexes have been constructed using the methods of Elteto and Koves (1964) and Szulc (1964), known as the EKS indexes (see Ball et al., 2004).

¹¹ This specification has been chosen based on Akaike Information Criteria (AIC), which gives the minimum value of likelihood function. We also note that the materials input variable increased consistently throughout the study period.

convergence plots for all elements of these estimates, indicating that MCMC is stationary for all reported parameters. These plots are based on 500,000 draws after burning in the first 5000 draws. The marginal posterior densities of all estimated coefficients are also shown in panel (b) of Figure 2.

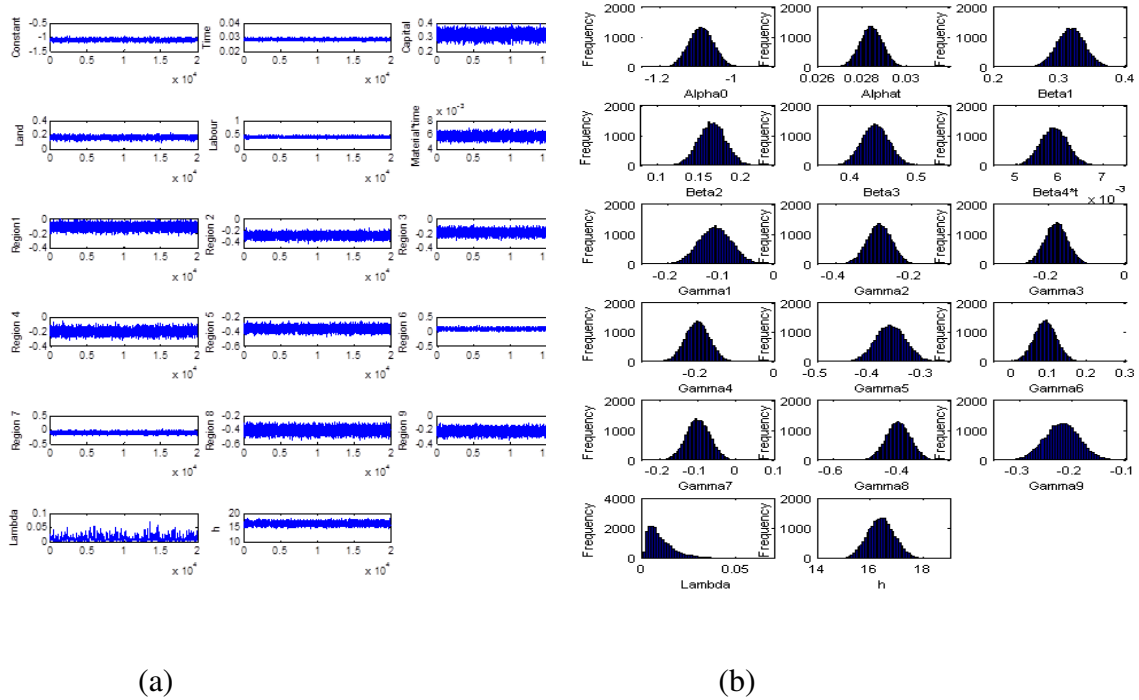


Figure 2: MCMC Convergence Plots and Posterior Densities of Parameter Estimates

Results indicate that all input coefficients are positively and statistically significant at the 1% level. For instance, the output elasticity of capital is 0.32, which indicates that a 1% increase in capital input contributes about 0.32% to total agricultural output. However, the highest posterior densities (HPD) interval indicates that the elasticity coefficient varies between 0.271 and 0.362 with 95% probability. For instance, the 95% HPD for the dummy variable coefficient for the Southeast region indicates that the average increase in agricultural production in this region lies between 2.1% and 15.1%, whereas the Mountain region experienced an average decrease in production by 22.1% with 95% HPD [-0.296 and -0.149]. The neutral part of technical change (α_t) shows an average annual increase in agricultural production of about 3.0%.¹²

¹² However, the geometric mean of technical change (computed by using the expression, $\Delta\text{Tech} = \exp[(\alpha_t + \beta_{4t} \ln x_{4t})(t-1)]$), shows an annual increase of 1.73%.

Table 1: Estimates of Cobb-Douglas Stochastic Production Frontier for US Agricultural Sector

| Variables | Coefficients | Maximum Likelihood Estimation | | Bayesian Maximum Likelihood Estimation | | |
|-------------------------|--------------------|-------------------------------|-------------------|--|---------|--------|
| | | Estimates | Standard Errors | Estimates | 2.50% | 97.50% |
| Capital | β_1 | 0.250 | 0.001 | 0.316 | 0.271 | 0.362 |
| Land | β_2 | 0.128 | 0.016 | 0.165 | 0.128 | 0.200 |
| Labour | β_3 | 0.495 | 0.020 | 0.437 | 0.392 | 0.483 |
| Materials | $\beta_4 \times t$ | 0.007 | 3.0×10^3 | 0.006 | 0.005 | 0.007 |
| Northeast | γ_1 | --- | --- | -0.112 | -0.178 | -0.041 |
| Great Lakes | γ_2 | --- | --- | -0.288 | -0.362 | -0.219 |
| Corn Belt | γ_3 | --- | --- | -0.186 | -0.251 | -0.119 |
| Northern Plains | γ_4 | --- | --- | -0.203 | -0.273 | -0.133 |
| Appalachian | γ_5 | --- | --- | -0.363 | -0.425 | -0.300 |
| Southeast | γ_6 | --- | --- | 0.086 | 0.021 | 0.151 |
| Delta | γ_7 | --- | --- | -0.101 | -0.171 | -0.033 |
| Southern Plains | γ_8 | --- | --- | -0.408 | -0.488 | -0.327 |
| Mountain | γ_9 | --- | --- | -0.221 | -0.296 | -0.149 |
| Time | α_t | 0.029 | 0.001 | 0.028 | 0.027 | 0.030 |
| Constant | α_0 | -1.296 | 0.031 | -1.093 | -1.163 | -1.022 |
| Log Likelihood Function | | -265.460 | --- | | -40.400 | --- |
| | σ^2 | 0.075 | --- | 0.004 | 0.058 | 0.065 |
| | λ | 0.007 | --- | 0.008 | 0.002 | 0.032 |
| Sample Size | 2160 | --- | --- | --- | --- | --- |

6.1 Estimates of Input-mix Efficiency (IME) Levels

A central theme of this paper is the estimation of the mix efficiency levels of states/regions within an econometric framework, which has not been attempted before. The methodology we propose for estimation of input-mix efficiency is obtained by minimizing aggregator functions. A Bayesian stochastic production frontier is used to demonstrate IME estimation. As mentioned earlier, Bayesian estimation enables us to draw finite-sample statistical inferences about unknown parameters. The HPD intervals are useful, particularly for efficiency estimates where posteriors are not symmetric. In that situation, HPD intervals are more useful than point and interval estimates obtained from classical stochastic frontier estimation. We use both linear and nonlinear (constant elasticity of substitution) input aggregators to obtain expressions for mix efficiency.

We compute measures of allocative efficiency obtained from the same formulae by substituting input prices in equation (28).¹³ Allocative efficiency estimates can be obtained by using both linear and nonlinear aggregators, but, for illustrative purposes, we only obtain allocative efficiency estimates using (conventional) linear price aggregators. We illustrate mix and allocative efficiency estimates at state and regional levels for the period 1960–2004.

6.2 Mix Versus Allocative Efficiency Estimates: Linear Aggregator

The proposed econometric estimation of mix efficiency requires estimates of the parameters of the aggregator function and production technology. We obtain the estimates of the production technology by estimating a Bayesian stochastic production frontier. Then, to compute mix efficiency for the linear aggregator based on equation (28)), we use the Lowe aggregator, which requires average input prices to construct those input aggregates (O’Donnell, 2012a). We then combine these aggregates with the estimates of the technology parameters to obtain input-mix efficiency.

Table 2 presents state-level comparisons of mix efficiencies for the linear input aggregators, for the years 1960-2004. The posterior means and the respective confidence intervals of mix efficiency are based on equation (28). Estimates of state-level mix efficiency are presented in columns 2–4. All mix-efficiency estimates are bounded between 0 and 1 for individual states over the entire period. The average mix-efficiency score for the entire sample is 0.85; however, there is large variation in input-mix efficiencies (0.46–0.99) across the different states. We also observe substantial differences in mix efficiencies across the regions, which also vary over time. For example, for the US as a whole, the average mix efficiency increased from 0.77 in the year 1960 to 0.85 in 2004. However, the changes over this time period were markedly different across the states and regions. We highlight Bayesian point estimates and 95% HPD intervals of mix efficiency for a few selected states (i.e., 2.5%, mean, 97.5%). For example, Iowa exhibited the highest level of mix efficiency [0.932, 0.953, 0.971] for the entire study period. States with notably lower mix efficiencies include New Mexico [0.499, 0.541, 0.585], and Wyoming [0.552, 0.594, 0.636]. It is notable that the most mix-efficient states are located in the Corn Belt, whereas the least mix-efficient states are in the Mountain region. These differences in mix efficiencies across states may partly be due to selection of different input

¹³ I am grateful to Eldon Ball and Knox Lovell for providing me a series of input prices which have been used to compute allocative efficiencies in the US agricultural sector.

mixes depending on economic factors such as input prices, soil productivity, crop yield and relevant infrastructure, as well as geographic and climatic factors. For instance, farmers in different states allocate different combinations of input resources, based on economic factors (i.e., farmers in one state may be using more capital per acre compared with other states), which may affect mix efficiency (or productivity). As discussed earlier, mix efficiency is a productivity concept, which varies with change in input mix. For example, the highly mix-inefficient states may be selecting cost-effective input mixes (i.e., they are allocatively efficient) rather than choosing an input mix that gives the maximum productivity. By using input prices in the estimator of equation (28), we can obtain the allocative efficiency levels, as proposed by Schmidt and Lovell (1979). We substitute input coefficients (δ_k) with respective input prices (w_k) in equation (28) to obtain allocative efficiency estimates. These estimates and their respective HPDs are presented in columns 5–7 of Table 2. It appears from the table that state-level estimates of allocative efficiency vary widely. There are substantial differences in allocative efficiencies across states which change over time. At the beginning of the period, significant allocative inefficiencies were prevalent in the US agricultural sector. These have improved over time. Differences remained widespread across states (from 31% to 88%), and over time (from 47.2% to 85.7%). Whereas most of the highly mix-efficient states show higher allocative efficiencies, their rankings for allocative efficiency differs markedly from the rankings observed for mix efficiency. Looking at the allocative efficiencies of selected states, we note that Iowa ranked 18th, with mean values for allocative efficiency of [0.79, 0.82, 0.85], whereas the values for New Mexico with [0.28, 0.31, 0.35] and Wyoming with [0.29, 0.32, 0.36] were the lowest of all states. These differences might be partly due to economic factors. For example, if prices changed for any reason (e.g., tax changes) then farmers who were unable to adjust their input mixes quickly would become allocatively inefficient.

Figure 3 portrays comparisons of regional mix and allocative efficiencies. The posterior densities of mix and allocative efficiencies show wide variations within as well as across the regions. It is noticed that the Corn Belt region ranks highest, with an average mix efficiency of 0.935, followed by the Great Lakes with an efficiency score of 0.919. The Mountain region shows the lowest mix efficiency of all regions in our estimates. This region includes New Mexico, Arizona, Nevada and Wyoming; they are located in southwest and northwest regions, which have diverse topography and climate, ranging from the Rocky Mountains to the deserts. There may be other factors (e.g., production environment) affecting the value of land in these

states, including location, soil quality, topography and geographic and economic factors. Looking at geographic differences and availability of resources such as land, we note that Alabama and Florida are in the Southeast region, while Iowa is in the Corn Belt; both these regions are more productive than the Mountain region where Wyoming is located.

Table 2: State-Level Posterior Means and 95% HPD Intervals for IME and IAE Efficiencies

| State | IME | | | AE | | |
|----------------|-------|-------|-------|-------|-------|-------|
| | 2.5% | Mean | 97.5% | 2.5% | Mean | 97.5% |
| Alabama | 0.883 | 0.915 | 0.943 | 0.701 | 0.740 | 0.776 |
| Arkansas | 0.879 | 0.911 | 0.940 | 0.740 | 0.777 | 0.812 |
| Arizona | 0.564 | 0.609 | 0.655 | 0.371 | 0.411 | 0.453 |
| California | 0.795 | 0.838 | 0.879 | 0.787 | 0.820 | 0.851 |
| Colorado | 0.840 | 0.876 | 0.909 | 0.604 | 0.645 | 0.685 |
| Connecticut | 0.814 | 0.849 | 0.881 | 0.810 | 0.845 | 0.877 |
| Delaware | 0.923 | 0.947 | 0.967 | 0.713 | 0.744 | 0.773 |
| Florida | 0.776 | 0.820 | 0.863 | 0.770 | 0.805 | 0.839 |
| Georgia | 0.895 | 0.925 | 0.952 | 0.725 | 0.763 | 0.800 |
| Iowa | 0.932 | 0.953 | 0.971 | 0.792 | 0.824 | 0.853 |
| Idaho | 0.873 | 0.905 | 0.935 | 0.758 | 0.797 | 0.833 |
| Illinois | 0.931 | 0.951 | 0.968 | 0.754 | 0.792 | 0.828 |
| Indiana | 0.923 | 0.945 | 0.964 | 0.787 | 0.823 | 0.856 |
| Kansas | 0.872 | 0.904 | 0.933 | 0.675 | 0.715 | 0.753 |
| Kentucky | 0.855 | 0.887 | 0.917 | 0.810 | 0.846 | 0.880 |
| Louisiana | 0.885 | 0.914 | 0.941 | 0.799 | 0.835 | 0.868 |
| Massachusetts | 0.774 | 0.812 | 0.848 | 0.786 | 0.824 | 0.859 |
| Maryland | 0.912 | 0.939 | 0.961 | 0.795 | 0.828 | 0.859 |
| Maine | 0.835 | 0.869 | 0.902 | 0.771 | 0.805 | 0.836 |
| Michigan | 0.892 | 0.919 | 0.943 | 0.838 | 0.871 | 0.901 |
| Minnesota | 0.911 | 0.936 | 0.959 | 0.817 | 0.846 | 0.873 |
| Missouri | 0.863 | 0.895 | 0.924 | 0.783 | 0.819 | 0.853 |
| Mississippi | 0.874 | 0.906 | 0.935 | 0.724 | 0.766 | 0.806 |
| Montana | 0.638 | 0.679 | 0.719 | 0.423 | 0.464 | 0.506 |
| North Carolina | 0.852 | 0.886 | 0.917 | 0.813 | 0.848 | 0.881 |
| North Dakota | 0.820 | 0.853 | 0.885 | 0.604 | 0.647 | 0.689 |
| Nebraska | 0.874 | 0.907 | 0.936 | 0.682 | 0.723 | 0.763 |
| New Hampshire | 0.817 | 0.851 | 0.883 | 0.770 | 0.807 | 0.843 |
| New Jersey | 0.818 | 0.852 | 0.885 | 0.804 | 0.840 | 0.873 |
| New Mexico | 0.499 | 0.541 | 0.585 | 0.276 | 0.311 | 0.349 |
| Nevada | 0.619 | 0.661 | 0.702 | 0.397 | 0.437 | 0.479 |
| New York | 0.864 | 0.895 | 0.923 | 0.816 | 0.845 | 0.873 |
| Ohio | 0.898 | 0.923 | 0.945 | 0.822 | 0.857 | 0.888 |
| Oklahoma | 0.815 | 0.853 | 0.888 | 0.662 | 0.705 | 0.747 |
| Oregon | 0.837 | 0.872 | 0.904 | 0.780 | 0.819 | 0.856 |
| Pennsylvania | 0.845 | 0.878 | 0.909 | 0.825 | 0.855 | 0.883 |
| Rhode Island | 0.812 | 0.846 | 0.878 | 0.816 | 0.849 | 0.880 |
| South Carolina | 0.856 | 0.889 | 0.919 | 0.766 | 0.806 | 0.844 |
| South Dakota | 0.816 | 0.851 | 0.885 | 0.571 | 0.613 | 0.656 |
| Tennessee | 0.859 | 0.891 | 0.920 | 0.803 | 0.840 | 0.875 |
| Texas | 0.753 | 0.794 | 0.832 | 0.553 | 0.596 | 0.639 |
| Utah | 0.789 | 0.828 | 0.865 | 0.593 | 0.637 | 0.682 |
| Virginia | 0.873 | 0.903 | 0.931 | 0.806 | 0.842 | 0.876 |
| Vermont | 0.853 | 0.885 | 0.915 | 0.761 | 0.798 | 0.832 |
| Washington | 0.874 | 0.905 | 0.933 | 0.852 | 0.882 | 0.910 |
| Wisconsin | 0.867 | 0.898 | 0.926 | 0.823 | 0.851 | 0.876 |
| West Virginia | 0.823 | 0.857 | 0.889 | 0.760 | 0.801 | 0.841 |
| Wyoming | 0.552 | 0.594 | 0.636 | 0.289 | 0.323 | 0.360 |

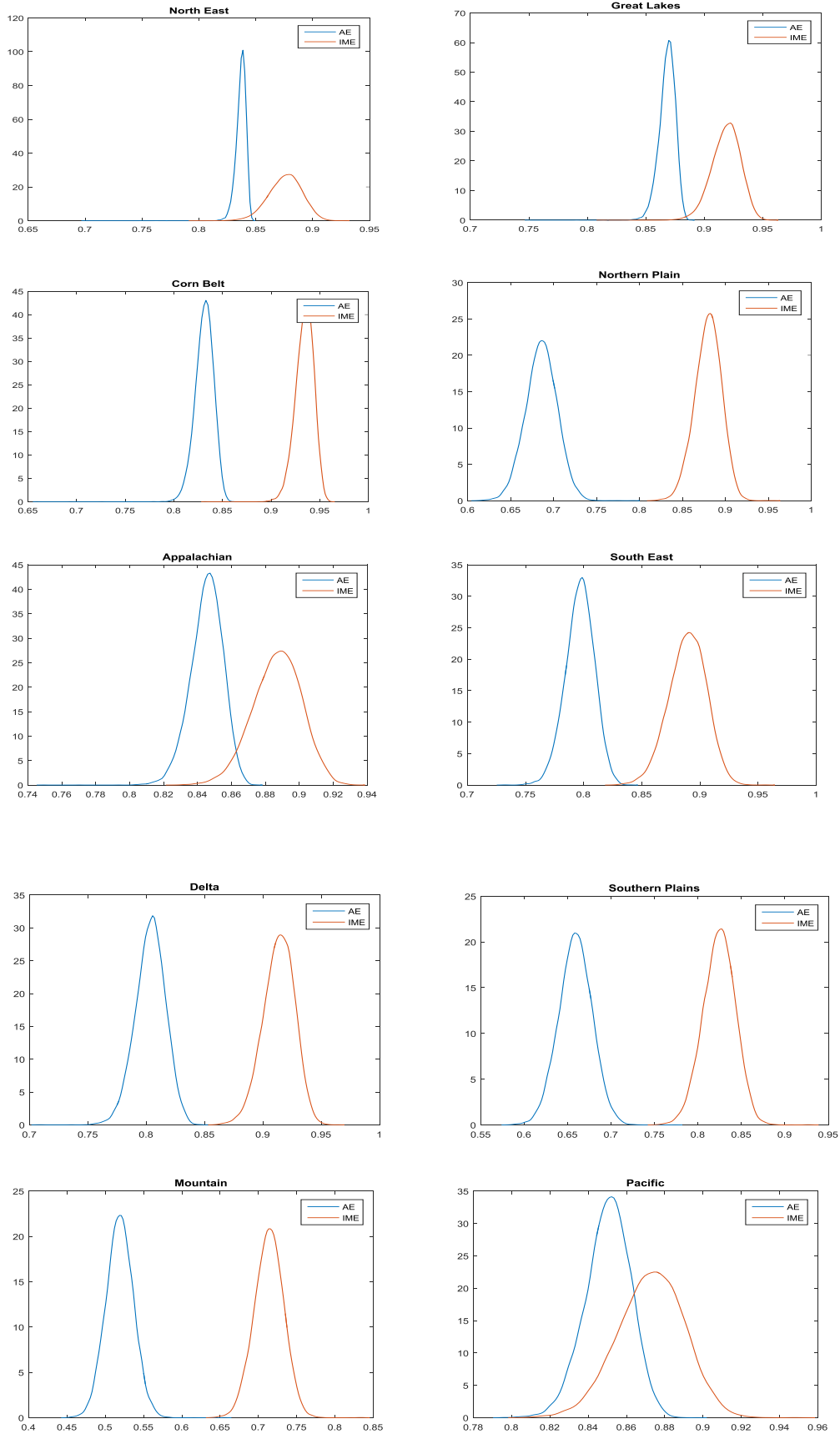


Figure 3: Posterior Densities of Regional Mix and Allocative Efficiencies

To make an indirect comparison of mix and allocative efficiencies among different states, we compute transitive Lowe indexes for three selected states, Alabama, Florida and Wyoming, for the period 1960-2004, with reference to the values of Alabama in 1960 (i.e., Alabama 1960 = 1). Figure 4 shows the Lowe indexes of mix and allocative efficiency changes in these states for this period. Mix efficiencies in Alabama remained higher than those of both Florida and Wyoming over the entire period. For example, in 2004, the change in mix efficiency in Alabama, compared with 1960, was 17.2% higher than that of Florida¹⁴, whereas the change in mix efficiency in Florida was 56.8% higher than that of Wyoming¹⁵. The transitivity axiom also allows us to compare Alabama with Wyoming indirectly via Florida. It implies that, in 2004, the mix efficiency in Alabama was 83.7% greater than in Wyoming.¹⁶ Similarly, we can compare the allocative efficiency of Alabama with that of Wyoming using this transitivity axiom.

To further illustrate state-level mix and allocative efficiencies, we calculate some input ratios for selected states. We find that, in 2004, the capital-to-land ratio in Iowa was nine times higher than in Wyoming ($1.46/0.16 = 8.98$), whereas the land-to-materials ratio was markedly higher in Wyoming than in Iowa ($9.39/0.96 = 9.78$). Similarly, the capital-to-land ratio in Alabama was almost eight times that in Wyoming. On the other hand, the land-to-materials ratio in Alabama was one-ninth that of Wyoming. A similar result is found for Florida where the capital-to-land ratio was much higher than that of Wyoming but the capital-to-materials ratio was quite low relative to that of Wyoming. It appears that large variation in input mix across states or regions is associated with substantial differences in mix efficiency. This may, in part, reflect variation in input prices and other economic incentives across states. Different prices of inputs may drive farmers to choose more land in states where land prices are low as compared with states where land prices are high.¹⁷

Although there has been a marked improvement in resource allocation in the US agricultural sector during 1960-2004, the posterior means and 95% HPD intervals of these estimates

¹⁴ $\Delta IME_{AL}^{2004} / \Delta IME_{FL}^{2004} = (IME_{AL}^{2004} / IME_{AL}^{1960}) / (IME_{FL}^{2004} / IME_{AL}^{1960}) = 1.225 / 1.077 = 1.172$

¹⁵ $\Delta IME_{FL}^{2004} / \Delta IME_{WY}^{2004} = (IME_{FL}^{2004} / IME_{AL}^{1960}) / (IME_{WY}^{2004} / IME_{AL}^{1960}) = (1.071 / 0.683) = 1.568$

¹⁶ $\Delta IME_{AL}^{2004} / \Delta IME_{WY}^{2004} = (\Delta IME_{AL} / \Delta IME_{FL}) \times (\Delta IME_{FL} / \Delta IME_{WY}) = 1.178 \times 1.568 = 1.837$

¹⁷ National Agricultural Statistics Service United States Department of Agriculture (NASS-USDA) also publishes cropland prices by region and states which differ from real estate prices. For example, cropland prices (per acre) for Alabama, Florida, Iowa and Wyoming in 2004 were \$1800, \$3900, \$2320 and \$1010 respectively.

confirm the substantial differences across states. For instance, Alabama has been ranked 10th in terms of mix efficiency but 35th in terms of allocative efficiency. Iowa and Illinois have been ranked 1st and 2nd in terms of mix efficiency but ranked 18th and 30th, respectively, in allocative efficiency.

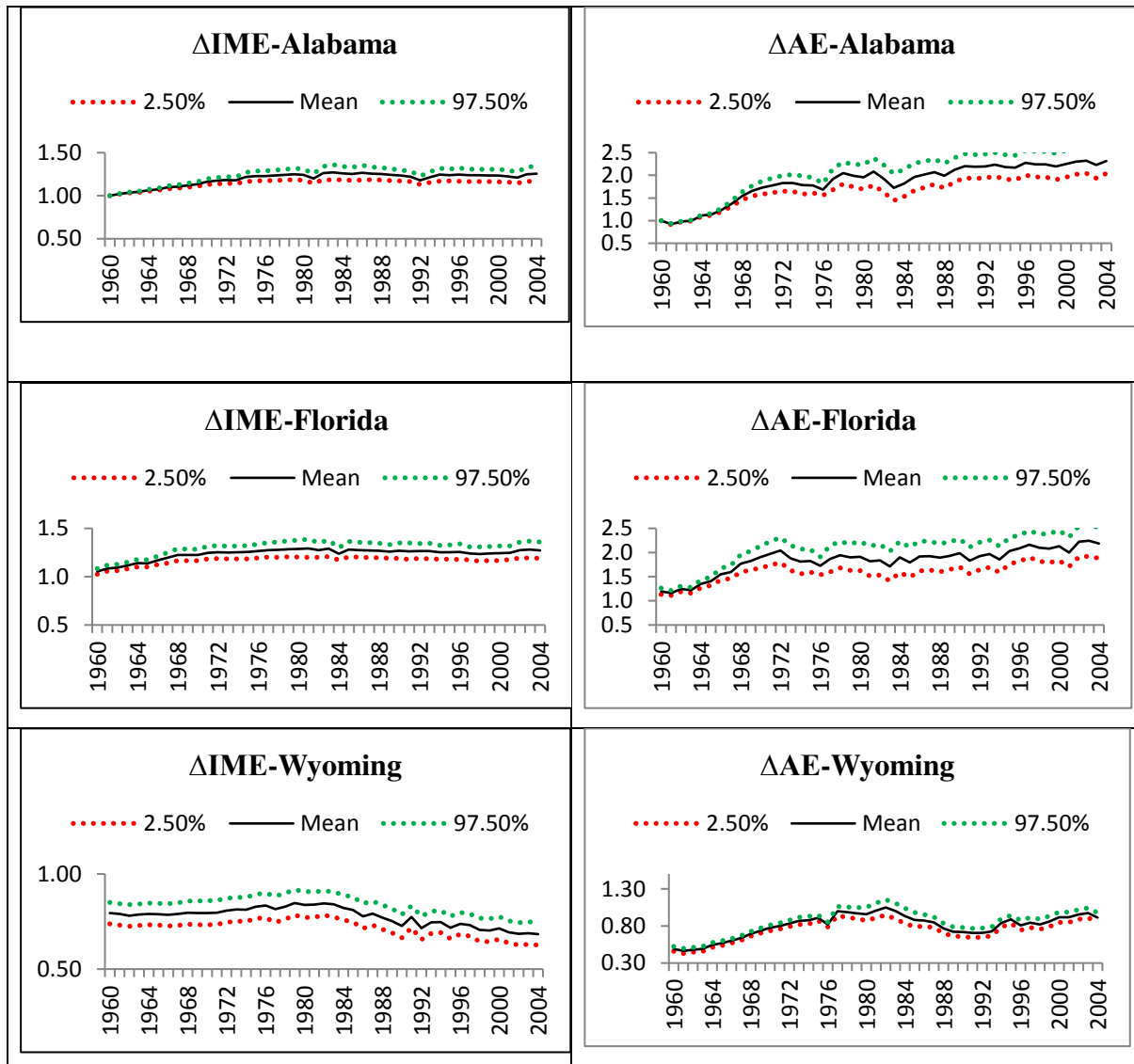


Figure 4: Mix Versus Allocative Efficiencies (Alabama, Florida, Wyoming)

6.3 Mix Efficiency Estimates: Nonlinear Aggregator

To compute mix efficiency using the nonlinear aggregator, we require values of δ and θ in addition to values for technology parameters. If the technology is represented by a CES production function, then we can obtain parameter values by estimating the production function. However, in the case of nonlinear aggregator function, different economically feasible values of these parameters can be used. For this analysis, we use average input shares (i.e., δ) to construct input aggregates along with estimates for technology parameters. We choose different arbitrary values of the substitution parameter θ to compute mix efficiency.¹⁸ Average mix efficiencies reported in this section vary with the changing values of θ . The use of different values of θ permits us to test the sensitivity of the mix efficiency with the possibility of substitution of inputs. State-level average input-mix efficiencies, based on the different values of the substitution parameter (i.e., $\theta = 0.5, 0.7$ and 1.2), are presented in Figure 3. Our results for mix efficiency should coincide with the results of the linear aggregator; however, these differ from the above-stated estimates because here we use input shares as weights of the aggregator function instead of average prices (as used previously). Because the expression of mix efficiency given in equation (27) with $\theta = 1$ collapses to the mix efficiency expression given in equation (28), it should produce identical results for the value of $\theta = 1$. But because we use different weights of the aggregate input (i.e., average input prices rather than input shares), these estimates differ slightly in magnitude. However, the ranking of states and regions is not affected at all. We note that the state-level mix efficiencies change monotonically with the changing value of θ (as presented in Figure 5). However, the ranking of the states does not change with different values of θ . Similarly, Figure 4 presents estimates of regional mix efficiencies based on different values of θ .

¹⁸ Because input-mix efficiency is monotonically increasing (decreasing) with the increasing (decreasing) values of θ , therefore, choosing different values of the substitution parameter only changes the efficiency score without changing the ranking of states.

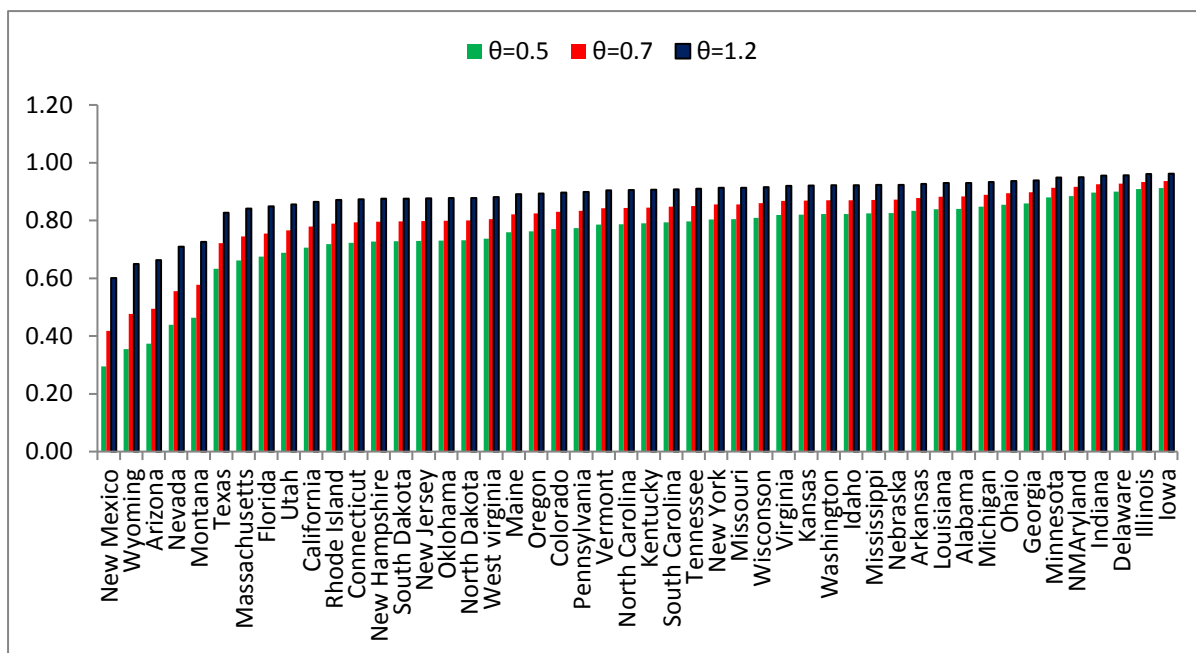


Figure 5: Regional Mix Efficiencies - CES Aggregator (1960–2004)

7 Conclusions

Effective policymaking requires identification of the main sources of productivity change. It is always helpful to identify whether productivity can be improved either by shifting the production frontier (i.e., technical change) or by changing input mixes (i.e., mix efficiency). Whereas conventional measures of efficiency such as technical and allocative efficiency have been in use for many years, the concept of mix efficiency in productivity measurement is a relatively new. This paper contributes to the efficiency and productivity literature by deriving an expression for input-oriented mix efficiency (by minimizing the input-aggregator function subject to homogenous Cobb-Douglas technology) in an econometric framework. A constant elasticity of substitution (CES) input aggregator has been used to derive these expressions. The empirical application confirms that econometric estimation of mix efficiency is feasible. The application of Bayesian econometric methods of estimating mix efficiency has the major advantage of drawing precise statistical inference, which is difficult when using nonparametric methods.

The empirical findings, based on the Bayesian stochastic frontier model, show a large variation in mix efficiencies across different states and regions. Allocative efficiency estimates were also obtained for all 48 states for the entire period, 1960–2004, and compared with the mix-efficiency estimates. However, the regions differ substantially in ranking when allocative

efficiencies are compared with mix efficiencies. We found a considerable improvement in resource allocation in almost every region for the period under consideration. However, the regions differ substantially in ranking when allocative efficiencies are compared with mix efficiencies. For example, Iowa and Illinois remained the highest mix-efficient states but their ranking changed to 18th and 30th in terms of allocative efficiency. It is noticed that states located in the Mountain region showed a declining trend in mix efficiency but an increasing trend in allocative efficiency during these years. For example, Wyoming showed a significant increase in allocative efficiency but a decrease in its mix efficiency. These variations in mix and allocative efficiencies provide a useful summary of their contribution to agricultural productivity and economic welfare. Appropriate allocation of input mixes in response to changes in input prices and the varying production environment has improved mix efficiency across different regions. These findings have many implications for future policy making. For instance, the introduction of production incentives, such as taxes and subsidies, will encourage farmers to adjust their input mixes (e.g., capital and labor) in response to changing input prices. This adjustment could significantly influence the productivity potential of the US agricultural sector.

We propose a simple method to derive an expression for measuring input-mix efficiency levels. However, this study can be extended in various directions. First, it would be useful to introduce the time-varying or region-varying input-aggregator functions, to account for the use of different inputs for the different regions. It would also be interesting to investigate the relative importance of input variability over time, particularly, due changes related to input-specific (e.g., seed) improvements in technology. Second, we use both average input prices and shares to construct linear and nonlinear input aggregates; these could be replaced with other types of aggregators, as discussed in Samuelson and Swamy (1974). Last, we used the Cobb-Douglas production technology, but this could be replaced by more flexible functional forms such as translog.

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9. Appendix

Proposition 3

Proof. Given the input-mix efficiency from equation (28)

$$IME_t = \left[\frac{\left(\sum_{k=1}^K \beta_k \right)}{\exp\left(\sum_{k=2}^K \frac{\beta_k}{r} \tau_k \right) \left(\sum_{k=1}^K \beta_k \exp(-\tau_k) \right)} \right] \leq 1.$$

where, $r = \sum \beta_k$, and $\exp(-\tau_k) = \left(\frac{x_k \beta_l \delta_k}{x_l \beta_k \delta_l} \right) \Rightarrow \tau_k = \ln \left(\frac{x_l \beta_k \delta_l}{x_k \beta_l \delta_k} \right)$.

Let
$$E_1 = \exp \left(\sum_k \frac{\beta_k}{r} \tau_k \right) \tag{42}$$

$$E_2 = \sum_{k=1}^K \beta_k \exp(-\tau_k) \tag{43}$$

Now by simply plugging $\delta_k = w_k$ into equation (42), we have

$$E_1 = \exp \left[\frac{1}{r} \beta_2 \ln \left(\frac{x_1 \beta_2 w_1}{x_2 \beta_1 w_2} \right) + \dots + \beta_k \ln \left(\frac{x_1 \beta_k w_1}{x_k \beta_1 w_k} \right) \right]. \tag{44}$$

Similarly, substituting $\delta_k = w_k$ into equation (59) gives,

$$E_2 = \beta_1 \left(\frac{x_1 \beta_1 w_1}{x_1 \beta_1 w_1} \right) + \dots + \beta_k \left(\frac{x_k \beta_1 w_k}{x_1 \beta_k w_1} \right). \tag{45}$$

Where,

$$E = E_1 \times E_2 \tag{46}$$

Now plugging Equations (50), and $r = \sum \beta_k$ back into equation (28) will produce input allocative efficiency estimates as given in Schmidt and Lovell (1979).

Hence,

$$IAE = r / E. \tag{47}$$

QED.

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