

# Estimating Recreation Preferences Using Hedonic Travel Cost and Random Utility Models

## LINWOOD PENDLETON<sup>1</sup> and ROBERT MENDELSOHN<sup>2</sup>

<sup>1</sup>The University of Southern California, Los Angeles, CA 90089-0253, USA; <sup>2</sup>Yale School of Forestry and Environmental Studies, 360 Prospect Street, New Haven, CT 06511, USA

Accepted 3 September 1999

**Abstract.** Over the last decade, several authors have questioned the validity of the hedonic travel cost model, arguing instead that the random utility model is a superior method for valuing recreational site attributes. This paper demonstrates that the two methods emanate from a similar utility theoretic framework; yet in practice these methods differ in the assumptions made in their application. Constraining the underlying utility functions to be consistent, both models are applied to the valuation of recreational site attributes in the Southeastern United States. The way in which each method estimates preferences for site attributes is shown to depend critically on the method and the functional form of the underlying utility function.

Key words: hedonic travel cost, RUM, recreation demand

JEL classification: C25, Q23, Q26

#### 1. Introduction

Micro-economic theory began as an attempt to describe, predict, and value the demand and supply of consumption goods. Quality was largely ignored in initial theoretical treatises; goods were assumed to be homogeneous. Over the last two decades, however, economists have started to address quality within the theory of demand. Environmental economists have extended the theory further to value the quality of recreational sites – an important component of land management. Two distinct approaches for incorporating quality into recreational analyses have emerged: the hedonic travel cost method (HTC) and the discrete choice random utility methods (RUM). The hedonic method views site attributes as though they were individual goods which are bundled together in a single purchase. The random utility model treats quality as an index which is estimated by examining a discrete choice of alternative sites facing a consumer.

Because the mathematical derivations for the hedonic (Rosen 1974) and random utility models (McFadden 1978) are quite different, many practitioners do not recognize that both models are based on a common utility theoretic foundation. In the first section of this paper, we show how the hedonic and random utility methods are consistent with the same utility framework.

Curiously, practitioners of the two methods have often made different *a priori* assumptions about utility when applying the methods. Many studies using the RUM method have assumed linear utility functions (see Morey et al. 1993, for an exception) while studies using the hedonic method frequently rely on quadratic utility functions (e.g. Brown and Mendelsohn 1984; Englin and Mendelsohn 1991; Pendleton et al. 1998b). The choice of functional form imposes important restrictions on the way the researcher believes that consumers value site quality. In Section 2 of the paper, we examine linear and quadratic functional forms for utility and show explicitly how these functional forms effect preferences.

Although both methods are based on the same utility theoretic framework, assumptions made in the econometric estimation of the models differ significantly. Each method makes very different assuptions about (a) the nature of the error terms in consumer decisions, (b) the smoothness of available attributes, and (c) the consumers' choice sets. These econometric assumptions can significantly influence the way the models estimate consumer preferences for site attributes. Since there may be little theoretical justification for certain underlying assumptions, it is important to consider how different assumptions influence the econometric performance of the models. Of course, one could compare results. In Section 4 we estimate consumer preferences for wilderness attributes in the Southeastern United States. The two models also differ greatly in the assumptions in the calculation of welfare change. We leave a discussion of this matter for another paper.

## 2. The Utility Framework

It is well known that the quantity of goods purchased are arguments in the utility function of consumers. Although common utility theory glosses over quality, it is equally plausible that quality also is an argument in the utility function of consumers. Lancaster (1966) provides a rigorous framework for the role of characteristics as arguments in the utility function. Following Lancaster's characteristicsbased utility model, applied economists now value quality in terms of a good's attributes. Two approaches have emerged to model the role of characteristics in consumer utility and choice. The hedonic approach estimates the demand for attributes after first uncovering the implicit prices of attributes. The random utility approach treats quality as an attribute-based index to be attached to goods. In both cases, the techniques attempt to place values on these attributes by observing how consumers choose from amongst the packages of available goods. In this section, we demonstrate that the underlying theoretical foundation for both methods is the same utility maximization subject to budget constraints. We argue that the theoretical foundations of both approaches are the same and consequently cannot be used to argue for one rather than the other approach.

HEDONIC AND RANDOM UTILITY MODELS

#### 2.1. The hedonic travel cost method

The theoretic derivation of the demand for goods from utility maximization subject to a budget constraint is a well established part of basic micro-economic theory. Without loss of generality, we extend this derivation to include quality. We begin by considering a set of Hicksian demand functions for a vector of site attributes (qualities),  $\mathbf{Z}$ , described by a vector of attribute prices,  $\mathbf{P}$ , utility *u*, and an estimation error term,  $\phi$ .

$$Z = h(\mathbf{P}, u, \phi). \tag{1}$$

In the case of recreation demand, the price is not a market price, but an implicit price. This implicit price is found by estimating the hedonic price function. The hedonic price function is the empirical estimation of the hedonic price frontier across visited sites. The cost of accessing any site on the frontier is a function of the attributes of that site. Formally, the hedonic price function<sup>1</sup> is

$$C(\text{site } j) = \text{fn}(\mathbf{Z}_j) \tag{2}$$

and the vector of implicit prices for the site attributes is given by the gradient of (2)

$$\mathbf{P} = \mathrm{d}C/\mathrm{d}\mathbf{Z}.\tag{3}$$

If the demand functions of (1) meet the Slutsky Criteria, we can find a set of inverse demand functions:

$$\mathbf{P} = h^{-1}(\mathbf{Z}, u, \phi). \tag{4}$$

Here **P** reflects the marginal value that the consumer would pay for an incremental unit of quality. We derive the consumer surplus associated with the consumption of  $\mathbf{Z}^*$  by taking a line integral of (4) from  $\mathbf{Z} = 0$  to  $\mathbf{Z} = \mathbf{Z}^*$ :

$$CS = \int_0^{\mathbf{Z}^*} h^{-1}(\mathbf{Z}, u, \phi) d\mathbf{Z} - C(\mathbf{Z}^*) = \int_0^{\mathbf{Z}^*} h^{-1}(\mathbf{Z}, u) d\mathbf{Z} - C(\mathbf{Z}^*) + g(\phi).$$
(5)

Generally, practitioners take the expectation of  $g(\phi)$  to be zero, but the exact structure of the error term in (5) depends on the nature of the error (e.g. omitted variables or measurement error; see Bockstael and Strand 1987). Note that the definition of (5) allows for nonlinearity in the price schedule of **Z**. Since  $h(\mathbf{Z}, u, \phi)$  is a Hicksian demand, the consumer surplus measure in (5) is an exact measure of the welfare associated with  $\mathbf{Z}^*$ ; it is also a money metric utility function,  $U^m$ ,

$$U^{m} \cdot = \int_{0}^{\mathbf{Z}^{*}} \mathbf{P} dZ = \int_{0}^{\mathbf{Z}^{*}} h^{-1}(\mathbf{Z}, u) d\mathbf{Z} - C(\mathbf{Z}^{*}) + g(\phi).$$
(6)

One criticism of the hedonic method is that it estimates Marshallian demand, not Hicksian demand. Using a Marshallian demand function for (1) yields an inexact measure of consumer welfare in (5). However, Hausman (1981) shows that an exact welfare measure can be recovered directly from the Marshallian demand function. Alternatively, when the assumed utility function is linear in income, the Marshallian demand is identical to the Hicksian compensated demand and consumer surplus is an exact welfare measure. Finally, in most circumstances which pertain to recreation, policy measures affect only a small fraction of user's potential incomes. Consequently, it is reasonable to assume that the Marshallian measure is a good approximation of true welfare (see Randall and Stoll 1980; Willig 1976).

#### 2.2. The random utility method

The random utility method envisions that site qualities form an index to be associated with each good (a visit to a site). The method models how a representative consumer chooses from a set of discrete sites each of which embodies a vector of attributes (qualities). Following McFadden (1978), the representative consumer chooses a site to maximize their conditional utility (conditioned upon making a visit):

# $U(\mathbf{Z}_J, X) + \varepsilon_j$

where  $\varepsilon_j$  is known to the consumer, but appears random to the researcher. The conditional utility of the RUM may have the same functional form as  $U^m$  from the hedonic approach. The random utility function consists of a deterministic core,  $U(\mathbf{Z}_j, X)$ , and a random component,  $\varepsilon_j$ . This random utility is a function of the attributes,  $\mathbf{Z}_j$ , of the site chosen, *j*, and all the remaining goods, *X*, that can be consumed. *H* is the price of other goods *X*,  $\varepsilon$  is a random variable and *Y* is income. The probability that any site is chosen is the probability that the utility derived from that site is greater than that from all other sites. Formally,

Prob(choose site 
$$j$$
) = Prob[ $U(\mathbf{Z}_j, X) + \varepsilon_j$ ]  $\geq$  Prob[ $U(\mathbf{Z}_j, X) + \varepsilon_{\setminus j}$ ]  
such that  $Y = [HX + C(\mathbf{Z}_j)]$  (7)

This probability relationship lets the researcher estimate the parameters of the utility function to approximate the observed probability of site choice. This probability function and thus the estimation of the RUM depends critically on the distribution of the error term,  $\varepsilon$ . Unlike the hedonic approach, the random term is rarely assumed to have a normal distribution since this distribution makes econometric estimation cumbersome. Instead, practitioners usually assume a generalized extreme value distribution for the error term. While the deterministic component of the RUM method corresponds to the underlying utility function in the HTC, the models differ significantly in important econometric assumptions (e.g. error).

These differences can seriously effect the way in which each model values quality. In the discussion that follows, we focus solely on the ability of two models to estimate consumer preferences for site attributes.

# 3. Utility Functional Form

Although a small handful of researchers have conducted RUM and HTC studies on the same data sets, no study has yet made theoretically consistent comparisons. All of the empirical comparisons made to date have made different assumptions about the form of the utility function in the HTC and RUM models compared. In practice, both models usually are estimated in aggregate and for a representative consumer (see for instance Anderson et al. (1992) for a discussion of the representative consumer and random utility models and Bockstael (1997) for a discussion related specifically to recreation demand). Even though the actual utility functions of individual consumers may deviate from the representative consumer, it is important to compare the two models assuming at the very least that the utility functions of the representative consumers are consistent.

Typically, recreation studies assume that utility is linear in RUM models and quadratic in HTC models. These are *a priori* assumptions made by the researchers, not theoretical properties of each technique. Both Hanemann (1984) and Mendelsohn (1987) warn that the choice of linear in attributes utility functional forms can lead to severe structural restrictions on preferences for site quality. Quadratic utility functions are more flexible than linear functions, but also imply negative marginal values at high levels of attributes. Morey et al. (1993) depart from the standard linear in attributes utility function. While this utility function is not quadratic, it still allows for declining marginal returns for site attributes. In this section, we examine both linear and quadratic utility functions for both the RUM and HTC models.

#### 3.1. LINEAR UTILITY

Utility functions that are linear in both attributes and income (cost) are used commonly in applications of the RUM to recreational quality (e.g. Bockstael et al. 1987; Hausman et al. 1995; Kaoru et al. 1995; Parsons and Kealy 1992; Parsons and Needleman 1992). The standard deterministic core of the linear utility function is

$$U_{i} = \gamma \mathbf{Z}_{i} + X \text{ subject to } Y = HX + C(\mathbf{Z}_{i}),$$
(8)

where subscript *j* refers to site *j*,  $\mathbb{Z}_j$ , is the vector of quality attributes that describe site *j*,  $C(\mathbb{Z}_j)$  is the cost of accessing site *j* with attributes  $\mathbb{Z}_j$ , and *Y*, *H*, and *X* are as before. If we assume that utility (8) is linear in income (all other goods), and that

*H* is fixed and can be set arbitrarily to unity, then we can use the income constraint to substitute  $Y - C(\mathbf{Z}_i)$  for X giving us:

$$U_j = \gamma \mathbf{Z}_j + \lambda [Y - C(\mathbf{Z}_j)], \tag{9}$$

where  $\lambda$  can be interpreted as the (constant) marginal utility of income. Equation (9) forms the deterministic core of the RUM in which the conditional random utility derived from choosing site *j* is

$$v_j = U_j + \varepsilon_j \tag{10a}$$

$$v_j = \gamma \mathbf{Z}_j + \lambda [Y - C(\mathbf{Z}_j)] + \varepsilon_j, \tag{10b}$$

where  $\varepsilon_j$  is a random term. In standard applications of the RUM,  $\lambda$  cannot be estimated (*Y* does not vary over choices) and the estimated utility function is

$$v_{j} = \gamma \mathbf{Z}_{j} - \lambda C(\mathbf{Z}_{j}) + \varepsilon_{j}.$$
<sup>(11)</sup>

Note that (11) also is a conditional indirect utility function in price  $C(\mathbf{Z})$  and quality,  $\mathbf{Z}_{i}$ .

The deterministic portion of the linear utility function is not strictly "wellbehaved" in the sense that it is not strictly concave. The linearity of the utility function means that the marginal value of any attribute remains the same for all levels of quality (i.e. the marginal value is constant). Since the budget constraint is assumed to be linear, the marginal utility of income is "minus the marginal utility of cost". So, the inverse demand (marginal willingness to pay) function for this utility is constant and can be given by

marginal willingness to pay = 
$$\left(\frac{\partial U}{\partial \mathbf{Z}}\right) / \left(-\frac{\partial U}{\partial C}\right) = \frac{\gamma}{-\lambda},$$
 (12)

 $\partial U/\partial \mathbf{Z}$  is a column vector of marginal utilities and  $\gamma$  is a column vector of coefficients and  $(-\lambda)$  is the marginal utility of income. Using this functional form, the representative consumer places the same marginal value on attributes, regardless of the level of attributes.

Since the inverse demand function associated with the linear utility function is constant, the corresponding hedonic model must be constrained to have a constant hedonic price. To do this requires that we estimate a single linear hedonic price function for all markets (origins). The hedonic price function must be the same for all markets since, by assumption, the marginal utility of another unit is equal to a constant.

#### 3.2. QUADRATIC UTILITY

Many applications of the hedonic method to recreational quality implicitly assume a utility function that is quadratic in attributes (Brown and Mendelsohn 1984;

94

Englin and Mendelsohn 1991; Mendelsohn 1984; Pendleton et al. 1998b; Smith and Kaoru 1987; Smith et al. 1991). More sophisticated applications of the HTC assume quadratic utilities that also contain cross-price terms (e.g. Brown and Mendelsohn 1984; Englin and Mendelsohn 1991; Pendleton et al. 1998b). The functional form for the deterministic core of the quadratic utility function is:

$$U_j = 1/2(\mathbf{Z}_j - \alpha)'\beta^{-1}(\mathbf{Z}_j - \alpha) + \lambda X$$
, subject to  $Y = HX + C(\mathbf{Z}_j)$ , (13)

where **Z** is a vector of site attributes,  $\alpha$  is a vector of constants, and  $\beta$  is a matrix to be estimated. A well-behaved quadratic utility function requires that all elements of the vector  $\alpha$  are positive and that the matrix  $\beta$  is negative semi-definite. The cross-price terms allow attributes to act as substitutes or complements.

With the quadratic utility function, it is theoretically possible to have oversatiation if a consumer faces a cheap (nearby) and over-abundant supply of a specific attribute. For some economists, the potential for negative prices (decreasing utility with increasing attributes) is sufficient reason to reject a quadratic utility functional form (Freeman 1993). There are, however, two cases in which a quadratic utility function might be appropriate for the analysis of recreational quality. The first case is where the feasible consumption set is one in which all or most consumers have a utility that lies within the increasing range of the utility function. The second case is when consumers do not enjoy free disposal and may be forced to consume some attributes at a level that exceeds complete satiation.<sup>2</sup> For example, surfers may generally find that larger waves are better than smaller waves. Nevertheless, a surfer may live near a beach that has exceedingly large and dangerous waves. The suffer cannot sell off these large waves and may be observed to occasionally travel further (pay more) to go to a beach with smaller waves. The negative prices often found in applications of the HTC can reflect oversatiation. Results using the same data that follow in Section 3 and published in another paper (Pendleton et al. 1998b) show that for hiking in the Southeastern United States, negative implicit prices are associated with attribute levels that are significantly higher than attribute levels where prices are positive.

A system of inverse demand and demand functions result from the assumption of quadratic utility (see LaFrance 1985). The system of demand functions is

$$\mathbf{Z} = \alpha + \beta \mathbf{C}_z + \phi. \tag{14}$$

If the matrix  $\beta$  is symmetric and invertable (the Slutsky Conditions) then the corresponding inverse demand function can be found as

$$\mathbf{C}_{z} = -\beta^{-1}\alpha + \beta^{-1}\mathbf{Z} - \beta^{-1}\phi.$$
<sup>(15)</sup>

The hedonic method estimates the parameters of the quadratic utility function by first estimating a hedonic price function for each origin in which  $C(\mathbf{Z})$  is regressed upon  $\mathbf{Z}$ . Any functional form can be used in the regression.<sup>3</sup> Using these hedonic prices, the system of seemingly unrelated demand functions in (14) is estimated,

where **Z**,  $\alpha$ , **C**<sub>z</sub> are the same vectors as before,  $\phi$  is a vector of error terms, and  $\beta$  is a matrix.

The corresponding random utility function can be estimated after expanding the vector notation of (13). A simplified form of the expanded utility would follow:

$$U = [\beta_1^{\text{rum}} z_1 + 1/2\beta_2^{\text{rum}} z_1^2 + \ldots + \beta_{n-1}^{\text{rum}} z_n + 1/2\beta_n^{\text{rum}} z_n^2 + \beta_{n+1}^{\text{rum}} z_1 z_2] + \alpha^{\text{rum}} \ldots + \lambda (Y - C(\mathbf{z})) + \varepsilon,$$
(16)

where the coefficients,  $\alpha^{\text{rum}}$  and  $\beta^{\text{rum}}$  represent collected terms (i.e. the complex coefficients that result from the matrix multiplication in (13)). The income constraint is substituted in for all other goods X in (13). Unlike the hedonic estimation, there is no need to restrict cross-price terms since only one coefficient is estimated for each cross-attribute pairing. The constant,  $\alpha^{\text{rum}}$ , cannot be estimated using the RUM and is irrelevant for welfare and utility calculations. As with the linear utility function, the income term, Y, is dropped in the standard RUM estimation.

As before, the inverse demand functions are found by solving for

marginal willingness to pay = 
$$\left(\frac{\partial U}{\partial \mathbf{Z}}\right) / \left(-\frac{\partial U}{\partial C}\right)$$
. (17)

#### 4. An Empirical Comparison

Past comparisons between hedonic and RUM methods have made no attempt to make consistent assumptions about the underlying form of the utility function (e.g. Bockstael et al. 1987; Cropper et al. 1993). In this section we estimate both linear and quadratic utility functions for the HTC and RUM methods. Note that a linear RUM and a quadratic HTC are not theoretically comparable.

# 4.1. Data

Data were collected on 4778 visits to 46 trails in 20 different forest areas near the Smoky Mountains (see Pendleton et al. 1998b). Visitor data came from permits collected by the United States Forest Service (USFS) and an independent survey. We limit the data set to visitors from within 300 miles of the North Carolina and Tennessee border in order to focus the analysis on single purpose, day trips. The data were collected between 1992 and 1994. Trails were surveyed in wilderness areas, non-wilderness areas, the State Park system, and the Great Smoky Mountain National Park.

Important trail attributes were identified by interviewing hikers and reading popular trail guides. Standard ecological techniques were used to measure these attributes along each of the 46 trails in the study. The set of trail attributes includes "basal area" (a measure of the size of trees and tree density), "elevation" (the maximum elevation of each trail), "riparian" (percent of trail along a creek), and

"isolation" (measured as miles from the paved road to the trail head). Appendix A gives summary statistics for the trail attributes. In addition, the distance from each origin to a trailhead was calculated using the program ZIPFIP (USDA 1993). All distances are in one way miles.

#### 4.2. The methods

Both the RUM and HTC methods are estimated according to standard practice. We give a brief review of the estimation methods here.

#### 4.3. The hedonic cost function

We estimate the implicit price of trail attributes by regressing the total travel costs to sites visited,  $C(\mathbf{Z})$ , on levels of environmental attributes at these sites. Because the geographic configuration of sites differs for every origin, a different hedonic price function is estimated for each origin. Using OLS, we estimate the hedonic price function only for those sites actually visited by residents of a given origin. It is assumed that sites that are not visited are not on the hedonic price frontier (i.e. these sites are inferior). We assume that the hedonic price function is linear:

$$C(\mathbf{Z}) = c_0 + C_1(\text{basal}) + C_2(\text{elevation}) + C_3(\text{riparian area}) + C_4(\text{isolation}) + \psi.$$
(18)

where **Z** is a vector of quantities for the selected attributes (basal, elevation, riparian, isolation) and  $\psi$  is the estimation error. The coefficients,  $C_i$ , represent the implicit prices for the attributes. Because we run a different regression for each origin, a different vector of implicit prices,  $C_z$ , exists for each origin.

Some critics argue that the hedonic price function cannot be estimated since the cost of obtaining the recreational good is exogenous. As described earlier, consumers choose only the sites that lie along the hedonic price frontier. Arguea and Hsiao (1993) show that if attributes are independent, then consumers will make choices that are best represented by a linear in attributes price function. The linearity depends on the production function only to the degree that the attributes are independent. The linearity of this function is not dependent on the actual offer function and thus does not require any knowledge of the functional form of the offer function. To the degree that attributes are not independent in production, a linear in attributes hedonic price function may represent a mis-specification of the true hedonic price function. Mis-specification in the hedonic methods is not qualitatively different from mis-specification in other types of estimation (e.g. the RUM) and can be tested using standard techniques.

The coefficients of the hedonic cost function represent the implicit prices of attributes. These implicit prices represent the marginal value of any attribute. The linear in attributes utility function implies a constant marginal value for each attribute, regardless of the level of attributes consumed. Therefore, a single hedonic cost function also was estimated for all origins simultaneously. The coefficients of this "universal" linear hedonic cost function are consistent with the marginal values that would be derived from the linear utility function.

#### 4.4. The demand for site attributes

The second step in the hedonic travel cost analysis is to estimate the demand for site attributes based on the implicit prices faced by each visitor and the level of attributes chosen by each visitor. In this study, we estimate a system of demand functions that are linear in site attributes and socio-economic shift variables. Using data on all visitors, we estimate the following system of demand functions:

$$\mathbf{Z} = \alpha + \beta \mathbf{C}_{z} + \delta \mathbf{S} + \phi, \text{ or}$$
  
(basal area) =  $\alpha_{1} + \beta_{11}C_{\text{basal area}} + \beta_{12}C_{\text{elevation}} + \beta_{13}C_{\text{riparian}} + \beta_{14}C_{\text{isolation}} + \delta_{1}S$   
(elevation) =  $\alpha_{2} + \beta_{21}C_{\text{basal area}} + \beta_{22}C_{\text{elevation}} + \beta_{23}C_{\text{riparian}} + \beta_{24}C_{\text{isolation}} + \delta_{2}S$   
(riparian) =  $\alpha_{3} + \beta_{31}C_{\text{basal area}} + \beta_{32}C_{\text{elevation}} + \beta_{33}C_{\text{riparian}} + \beta_{34}C_{\text{isolation}} + \delta_{3}S$   
(isolation) =  $\alpha_{4} + \beta_{41}C_{\text{basal area}} + \beta_{42}C_{\text{elevation}} + \beta_{43}C_{\text{riparian}} + \beta_{44}C_{\text{isolation}} + \delta_{4}S$  (19)

where **Z** is a vector of quantities for the selected attributes (basal area, elevation, riparian, isolation),  $C_z$  is a vector of hedonic prices from the first stage regressions, **S** is a vector of socio-economic variables,  $\phi$  is a vector of estimation errors, and  $\alpha$ ,  $\beta$  and  $\delta$  are respectively a vector and two matrices of coefficients to be estimated. The socio-economic shift variables are characteristics of each origin and are derived from U.S. 1990 census data. Interestingly, we could not identify any socio-economic variables that significantly effected the demand for site attributes and so **S** was dropped from (19). Because the coefficient on income (an element of *S*) was not significantly different from zero, we conclude that the income elasticity of demand for forest attributes is zero and thus compensating variation, equivalent variation, and consumer surplus are equivalent.

The prices from the first stage and the quantities of site attributes chosen by hikers allow us to estimate the demand functions of (19). Because hikers from different origins face different prices, we treat each origin as a separate market. The existence of multiple markets allows the estimation to be specified and avoids the pitfalls common to single market hedonic applications (see Mendelsohn 1985). We estimate (19) using a generalized least squares, seemingly unrelated regression procedure. We constrain the cross-prices of  $\beta$  to be symmetric in order to ensure that welfare measures are path independent. We also derive the inverse demand

functions for the HTC based on the quadratic utility function. Following from (19), we find the marginal benefit (willingness to pay function) as

$$MB = \beta^{-1}(\mathbf{Z} - \alpha). \tag{20}$$

# 4.5. The random utility models

We estimate the RUM models using standard non-nested multinomial logit methods. We choose not to nest since there is no reason to believe that sites can be grouped to account for possible correlations in the error term. All sites are part of a contiguous expanse of forest. All trails are included in the choice sets of individuals. We estimate a linear in attributes conditional random utility function

$$v_{j} = \beta^{\text{linear rum}} \mathbf{Z}_{j} + \lambda^{\text{linear rum}} [C(\mathbf{Z}_{j})] + \varepsilon_{j}, \text{ or}$$

$$v_{j} = \beta^{\text{linear rum}}(\text{basal area}) + \beta_{2}^{\text{linear rum}}(\text{elevation}) + \beta_{3}^{\text{linear rum}}(\text{riparian}) + \beta_{4}^{\text{linear rum}}(\text{isolation}), \qquad (21)$$

where  $Z_j$  is defined as before (i.e.  $Z_j = \{\text{basal area, elevation, riparian, isolation}\}$ ). The inverse demand function associated with the linear in attributes RUM is simply

$$MB(\mathbf{Z}) = (\mathrm{d}v/\mathrm{d}\mathbf{Z})/-\lambda^{\mathrm{linear\ rum}} = \beta^{\mathrm{linear\ rum}}/-\lambda^{\mathrm{linear\ rum}}.$$
 (22)

We also estimate a quadratic in attributes random utility function

$$v_{j} = \beta_{1}(\text{basal area}) + \beta_{2}(\text{basal area})^{2} + \beta_{3}(\text{elevation}) + \beta_{4}(\text{elevation})^{2} + \beta_{5}(\text{riparian}) + \beta_{6}(\text{riparian})^{2} + \beta_{7}(\text{isolation}) + \beta_{8}(\text{isolation})^{2} + \beta_{9}(\text{basal area * riparian}) + \beta_{10}(\text{isolation * riparian}) + \beta_{11}(\text{elevation * riparian}) + \beta_{12}(\text{elevation * basal area}) + \beta_{13}(\text{elevation * isolation}) + \beta_{14}(\text{basal area * isolation}) + \varepsilon_{j}$$
(23)

where all of the coefficients, of course, refer only to the quadratic specification. As before, we find the inverse demand function as

$$MB(\mathbf{Z}) = (\mathrm{d}v/\mathrm{d}\mathbf{Z})/-\lambda^{\text{linear rum}}.$$
(24)

#### 4.6. Econometric results

The results of the linear utility estimations are given in Table Ia; the coefficients of the derived inverse demand function are given in Table Ib. The linear utility parameters for the HTC model suggest that basal area and elevation are both goods whereas isolation is an economic bad and riparian is not relevant. (Note that a single linear in attributes hedonic price function for all origins is an inappropriate

HTC results $C(\mathbf{z}) =$ (t-statistics) Observations = 4778	Constant 66.2 (3.90)	Basal area 0.199 (1.57) Corrected $r^2 =$	Elevation $5.81 \times 10^{-3}$ (3.27) 0.0201	Riparian -1.96 (-0.216)	
RUM results $v(\mathbf{z}, C) =$ ( <i>t</i> -statistics) Observations = 4778	Basal area 2.57 × $10^{-2}$ (21.6)	Elevation $-4.99 \times 10^{-5}$ (-25.6) Percent sites co predicted = 31.4	(-6.64) prrectly	0.103 (24.0)	Travel cost $-2.97 \times 10^{-2}$ (-46.0) hood = -13197

Table Ia. The estimated parameters of the HTC and RUM: Linear utility.

Table Ib. The parameters of the inverse demand functions: Uniform linear utility.

	C <sub>basal area</sub>	Celevation	Criparian	Cisolation
HTC	0.199	$5.81 \times 10^{-3}$	-1.96	-2.12
RUM	0.866	$-1.68 \times 10^{-2}$	-17.3	3.45
Wald test	24.3	0.335	2.86	239

application of the HTC. We include this estimation solely for comparison.) The results from the linear RUM analysis suggest that both elevation and riparian are undesirable whereas basal area and isolation are good. Although the basal area and isolation results are consistent with prior expectations, the remaining results from the RUM analysis seem inconsistent with the description of trail attributes in hiking books.

The results of the quadratic utility estimations are given in Table IIa; the coefficients of the derived inverse demand functions are given in Table IIb. In general, both models perform better under the assumption of quadratic utility. More coefficients are significant and have the expected sign and the models explain a greater fraction of the observed behavior. The quadratic utility parameters for the HTC model imply negative own price elasticities (downward sloping demand functions) for all four attributes. The cross price elasticities between basal area and both riparian and isolation are positive, implying these attributes are substitutes. Elevation also has a positive cross price elasticity with respect to isolation. The quadratic utility parameters for the RUM model yield similar results. All of the linear terms in the quadratic RUM are of the expected sign, but only the squared terms for basal area and riparian have the expected sign. All interaction terms between attributes suggest that the attributes are substitutes. The RUM model suggests that the more isolation and the more elevation, the better the site becomes at an increasing rate.

*Table IIa.* The estimated parameters of the HTC and RUM: Quadratic utility (*t*-statistics in parentheses).

HTC		Basal area	Elevation	Riparian	Isolation	
	Constant	79.2	2990	0.284	5.73	
		(215)	(143)	(65.6)	(100)	
	C <sub>basal area</sub>	-7.18	22.8	$0.502 \times 10^{-1}$	0.652	
		(-11.8)	(0.689)	(8.95)	(16.1)	
	Celevation	22.8	-8610	$-0.154 \times 10^{-1}$	22.7	
		(0.689)	(-3.12)	(-0.049)	(8.56)	
	Criparian	$0.502 \times 10^{-1}$	$-0.154\times10^{-1}$	$-0.434\times10^{-3}$	$-0.911\times10^{-3}$	
	•	(8.95)	(-0.049)	(-5.13)	(-1.83)	
	Cisolation	0.652	22.7	$-0.911\times10^{-3}$	-0.43	
		(16.1)	(8.56)	(-1.83)	(-60.8)	
	Observations	4778				
	Corrected, $r^2$	0.135	0.054	0.242	0.505	
	Wald test on l	inear restrictions	= 264			
RUM	v =	Basal area	Elevation	Riparian	Isolation	Travel cost
	Coefficient	0.567	$2.43 \times 10^{-3}$	42.0	1.62	$-2.94 \times 10^{-2}$
		(28.0)	(7.93)	(31.1)	(25.3)	(-43.5)
		(Basal area) <sup>2</sup>	(Elevation) <sup>2</sup>	(Riparian) <sup>2</sup>	(Isolation) <sup>2</sup>	Basal area*
						riparian
	Coefficient	$-1.95 \times 10^{-3}$	$2.08 \times 10^{-7}$	-13.5	$1.69 \times 10^{-2}$	-0.346
		(-19.6)	(7.63)	(-27.4)	(9.30)	(-31.1)
		Isolation*	Elevation*	Elevation*	Elevation*	Basal area*
		riparian	riparian	basal	isolation	isolation
	Coefficient	-0.882	$-1.31 \times 10^{-3}$	$-3.55 \times 10^{-5}$	$-1.75 \times 10^{-4}$	$-1.34 \times 10^{-2}$
		(-29.1)	(-10.4)	(-18.7)	(-27.2)	(-23.2)
	Observations $= 4778$		Percent sites corr	rectly predicted =	31.94	
	Observations $= 4778$		Percent sites con	rectly predicted =	31.94	

Seventeen of the 20 common coefficients between the inverse demand functions derived from the HTC and RUM models are of the same sign, but only 11 were statistically similar by a Wald test.<sup>4</sup> All of the coefficients, however, differ by at least one order of magnitude, even those that passed the Wald test. All but one of the coefficients which were significantly different between the RUM and HTC models involved riparian.

# 4.7. Welfare and error

That coefficients in Table IIb can appear to be so dissimilar, yet still pass a Wald test, highlights the complex role of error in these estimations. Both models contain estimation error that may have its roots in measurement error or a specification

		C <sub>basal area</sub>	Celevation	Criparian	Cisolation
Constant	HTC	850	6.19	96700	1430
	RUM	19.3	$8.26 \times 10^{-2}$	1430	55.0
	Wald test	>1000	$\textbf{4.22}\times\textbf{10}^{-3}$	>1000	>1000
$\beta_{\text{basal area}}$	HTC	-3.53	$-\textbf{2.39}\times\textbf{10}^{-2}$	-399	-5.77
	RUM	-0.133	$-1.21  imes 10^{-3}$	-11.8	-0.454
	Wald test	0.007	$\textbf{2.50}\times\textbf{10}^{-12}$	>1000	$1.51 \times 10^{-2}$
$\beta_{\text{elevation}}$	HTC	$-2.39 \times 10^{-2}$	$-2.96  imes 10^{-4}$	-2.68	$-4.62  imes 10^{-2}$
	RUM	$-1.21 \times 10^{-3}$	$1.41 \times 10^{-5}$	$-4.46 \times 10^{-2}$	$-5.96 \times 10^{-3}$
	Wald test	$\textbf{2.50}\times\textbf{10}^{-12}$	$\textbf{3.34} \times \textbf{10}^{-19}$	$1.36 \times 10^{-4}$	$\textbf{1.14}\times\textbf{10}^{-10}$
$\beta_{riparian}$	HTC	-399	-2.68	-47500	-647
	RUM	-11.8	$-4.46  imes 10^{-2}$	-920	-30.0
	Wald test	>1000	$\textbf{1.36}\times\textbf{10}^{-4}$	>1000	>1000
$\beta_{\rm isolation}$					
	HTC	-5.77	$-4.62  imes 10^{-2}$	-647	-12.2
	RUM	-0.454	$-5.96 \times 10^{-3}$	-30.0	1.15
	Wald test	$\textbf{1.51}\times\textbf{10}^{-2}$	$\textbf{1.14} \times \textbf{10}^{-10}$	>1000	2.91

*Table IIb.* The parameters of the inverse demand functions: Quadratic utility (bold face indicates coefficients are not different at the 5% significance level).

error (e.g. omitted variable error). Obviously, the estimated parameters themselves are estimated with error.

Because welfare calculations, especially for non-marginal changes in attributes, sometimes require complex mathematical manipulations of the estimated coefficients, it is often extremely difficult to calculate the standard error (or variance) of welfare estimates. Welfare measures are rarely given with standard errors in the literature, especially for non-marginal welfare changes. From Bockstael et al. (1991), the expected welfare measures from the RUM are derived as

$$E[\Delta CS] = \frac{1}{\lambda} \left\{ \ln \left[ \sum_{j} \exp\{U(\mathbf{Z}_{j}^{1})\} \right] - \ln \left[ \sum_{j} \exp\{U(\mathbf{Z}_{j}^{0})\} \right] \right\}$$
(25)

It is important to note that the expectation of the compensating variation is an expectation form the perspective of the researcher only, since the researcher cannot predict perfectly how consumers will chose among attribute bundles (Freeman 1993, p. 471). The probability in this case is in the estimated utility itself and not in the arguments of the utility function (as in the Von Neumann utility function). Calculation of welfare standard errors in the RUM requires that we find the variance of the natural logarthim of the sum of exponentials of the utilities for each bundle, itself a function of estimated coefficients, and divide by the marginal utility of income, also estimated with error. The exact definition of the choice set also

effects welfare estimates through the summation operation over the *j* choices. In the end, calculation of a standard error for expected welfare change is exceedingly difficult and rarely offered in the literature.

The calculation of welfare and standard errors in the HTC is no less difficult, requiring that we integrate the inverse of the estimated demand function and find the error distribution of the integrand.

$$\Delta \text{Consumer Surplus} = \int_{z_j^0}^{z_j^1} h^{h-1}(z, u) dz - C(z_j^1) + C(z_i^0) + g(\phi^1) - g(\phi^0).$$
(26)

(See Bockstael and Strand (1987) for an exposition on the complexities of incorporating error in surplus estimates from simple demand functions.) The problems in the two stage HTC are confounded by the fact that the second stage is estimated on prices that are themselves estimated with error.

#### 4.8. Comparing the welfare calculations

As illustrated above, the two methods differ fundamentally in the way in which welfare is calculated. Welfare estimates in the RUM are expected measures that depend importantly on how the choice set is defined and the distribution of the random error in the model. Despite Pudney's observation that the utility function of the RUM is ordinal and not cardinal (see for instance, Pudney 1989, p. 111), researchers continue to estimate the RUM and calculate expected welfare change as given above. While the welfare calculations of the HTC are cardinally comparable, the HTC suffers from the restriction that it can estimate welfare only for those who choose the same bundle no matter the level of attributes.

While a complete discussion of the welfare differences in the RUM and HTC is beyond the scope of this paper, we can make simplifying assumptions to show how the empirical results of the two models would effect changes in the underlying level of utility of the representative consumer, irrespective of the treatment of error.

We compare the welfare implications of the way in which the two models estimate preferences by calculating the value<sup>5</sup> of strict utility changes that result from a 10% decline from the mean for each attribute, separately. The magnitude of welfare changes in the HTC assuming a linear utility function (note this is not a valid application for the HTC and is included only for comparison) is given by

Welfare Change for 
$$\Delta Z_i = \Delta Z_i \beta_i$$
. (27)

The deterministic utility change for the representative consumer using the linear in attributes RUM is

Welfare Change for 
$$\Delta Z_i = \Delta Z_i \beta_i / (-\lambda)$$
. (28)

Welfare change calculations under the assumption of a utility function that is quadratic in attributes are necessarily more complex. Comparing only a deterministic

	Attribute			
Method	Basal area	Maximum	Riparian elevation	Isolation
Linear utility:				
RUM	5.64	5.59	-0.59	1.54
HTC	1.30	1.93	-0.067	-0.94
Quadratic utility:				
RUM	-6.41	-2.27	-2.69	-0.025
HTC	-142.42	-17.56	-71.01	-11.54

*Table III.* Representative welfare measures for changes in individual attributes (10% decline from mean site attribute levels).

change in utility, we find that the value for a change in utility in the random utility model is

Welfare Change for 
$$\Delta Z_i = \Delta U (Z_i^1 - Z_i^0) / (-\lambda),$$
 (29)

which is simply the compensating variation for a change in attributes.

Similarly, from Englin and Mendelsohn (1991) and LaFrance (1985) we can use the compensating variation for a change in attributes to find the deterministic utility change in the HTC as

$$CV = \cdot 5(\mathbf{Z}^1 - \alpha)'\beta^{-1}(\mathbf{Z}^1 - \alpha) - \cdot 5(\mathbf{Z}^0 - \alpha)'\beta^{-1}(\mathbf{Z}^0 - \alpha).$$
(30)

Table III gives the results for the deterministic welfare measures using both the RUM and HTC methods under assumptions of linear and quadratic utility. While the magnitudes differ considerably, the signs are very similar between the models.

#### 5. Conclusion: Lessons Learnt

# 5.1. BOTH THE RUM AND HTC HAVE THE SAME UTILITY THEORETIC FOUNDATIONS

We have shown that the random utility and hedonic models both follow from the same basic premise of utility maximization over a choice of sites that differ in attributes and costs. Contrary to the claims of Bockstael et al. (1987), the hedonic methods cannot be dismissed simply because the theory is wrong or the approach to modeling the demand for attributes is inconsistent with behavior. Choices between the two models must boil down to econometric considerations.

# 5.2. THE TWO MODELS MAKE VERY DIFFERENT ASSUMPTIONS ABOUT THE ROLE OF ERROR IN THE CHOICE OF SITE QUALITY

In the hedonic framework, consumers have different preferences for site qualities due to neoclassical assumptions about demand (e.g. declining marginal returns to consumption and the presence of individual demand shift parameters). Representative consumers from the same origin may still choose different sites provided that representative consumers can be disaggregated based on personal characteristics. Consumers with similar personal characteristics choose different levels of site attributes only if they face different (marginal) costs of acquiring these attributes. In short, differentiation in site choice can be explained by deterministic mechanisms. In the hedonic method, the apparent randomness of site choice is considered a result of three things: (1) random behavior on the part of the consumer, (2) measurement error, and (3) misspecification of the demand model.

In the standard application of the random utility method, the choice of different sites depends largely on a randomness in consumer tastes. Even if the linear random utility of the representative consumer results from the aggregation of non-linear individual random utility functions, it is not possible to recover these individual utility functions from the estimation. The fact that income routinely is dropped from the indirect utility function in RUM applications further places the burden of choice variations on the random term.

Ultimately the dissimilar assumptions about error in the two models are likely to influence the way each model values site attributes. As long as generalized least squares methods are used to estimate the demand functions of the HTC, the model does not require any restrictive assumptions about the distribution of the error term. The RUM, however, requires strong assumptions about the distribution of error. Most commonly, the application of the RUM assumes an extreme value distribution for the error term. As Pudney (1989, p. 117) points out, however, the choice of the extreme value distribution should be considered only "as an arbitrary device for generating convenient logistic forms" for estimation.

#### 5.3. THE HEDONIC MODEL SUFFERS FROM A TWO-STEP APPROACH

To date, the greatest perceived limitation of the hedonic travel cost model comes in the two-stage nature of the demand estimation. We argue in this paper and others (Pendleton et al. 1998a, 1998b) that there are many reasons to believe that the hedonic price function (the first stage) can be estimated with precision. Nevertheless, the two-stage estimation of first hedonic prices and then demand functions causes a significant loss of econometric information not found in the RUM. If the HTC is to be used to value non-marginal changes in attributes, then the development of a joint estimation function for prices and demand must be used. 5.4. The role of functional form in the random utility models is important

We hope that we have shown that the choice of functional form for the RUM should not be taken lightly. Specifically, the linear RUM imposes strong assumptions about the way in which the (representative) consumer values larger and larger levels of attributes. Linear in attributes indirect utility functions may be particularly restrictive, and even inadequate, in cases where the models are estimating utility (and welfare change) over a large range of attribute levels. Our results show that more parameter estimates for the RUM are of the expected sign under the assumption of quadratic rather than linear utility. More attention needs to be given to the functional form of the indirect utility function in the RUM and the assumptions implicit in the chosen functional form should be made explicit. When a linear budget constraint is assumed (i.e. constant marginal utility of income), authors should be clear that their expected welfare measures are Marshallian as well as Hicksian.

## Acknowledgement

This research was funded by the U.S. Department of Energy with data collection support from the U.S. Forest Service. Special thanks to John Houghton and Tom Holmes for their support.

#### Notes

- 1. In this section, we assume that the true functional form of the hedonic price function is known. We address questions of estimation in later sections.
- 2. This idea can be attributed first to Jeffrey Englin's doctoral dissertation: 'Backcountry hiking and optimal forest management' (p. 57), University of Washington, Seattle, 1986.
- 3. Arguea and Hsiao (1993) offer reasons why a linear in attributes utility function may be justified when attributes can be combined independently.
- 4. Note that standard errors cannot be calculated for the inverse demand function recovered from the two-stage HTC. Wald tests for the models based on a quadratic utility function assume standards errors only for the RUM estimates.
- 5. Values are given in terms of one-way miles. One-way miles can be converted to dollars by multiplying by two and then by a conversion term (e.g. the AAA value is \$0.25/mile; see AAA 1992).

#### 106

Attribute	Description	Sample mean (standard deviation)
Basal area	Square feet of trees/acre	65.1 (21.0)
Riparian	% of trail along riparian	34.4 (34.0)
Elevation	Maximum elevation of trail	3330 (1090)
Isolation	Miles from paved road to trailhead	4.45 (4.65)

**Appendix A: Summary Statistics** 

#### References

- AAA (1992), 'Your Driving Costs', American Automobile Association Public Affairs Publications. Heathrow, FL.
- Anderson, S., A. de Palma and J. Thisse (1992), *Discrete Choice Theory of Product Differentiation*. Cambridge, MA: The MIT Press.
- Arguea, N. M. and C. Hsiao (1993), 'Econometric Issues of Estimating Hedonic Price Functions', *Journal of Econometrics* 56, 243–267.
- Bockstael, N. (1997), 'Travel Cost Models', in D. Bromley, ed., *Handbook of Environmental Economics*. Cambridge, MA: Blackwell Handbook in Economics.
- Bockstael, N. and I. Strand (1987), 'The Effect of Common Sources of Regression Error on Benefit Estimates', *Land Economics* **63**(1), 11–20.
- Bockstael, N., W. M. Hanemann and C. Kling (1987), 'Estimating the Value of Water Quality Improvements in a Recreational Demand Framework', *Water Resources Research* 23, 951–960.
- Bockstael, N., K. McConnell and I. Strand (1991), 'Recreation', in J. Braden and C. Kolstad, eds., *Measuring the Demand for Environmental Quality*. North-Holland: Amsterdam.
- Brown, G. and R. Mendelsohn (1984), 'The Hedonic Travel Cost Method', *Review of Economics and Statistics* **66**, 427–433.
- Cropper, M., L. Deck, N. Kishor and K. McConnell (1993), 'Valuing Product Attributes Using Single Market Data: A Comparison of Hedonic and Discrete Choice Approaches', *Review of Economics* and Statistics **75**(2), 225–232.
- Englin, J. and R. Mendelsohn (1991), 'A Hedonic Travel Cost Analysis for Valuation of Multiple Components of Site Quality: The Recreation Value of Forest Management', *Journal of Environmental Economics and Management* 21, 275–290.
- Freeman, M. (1993), *The Measurement of Environmental and Resource Values: Theory and Methods*. Resources for the Future.
- Hanemann, M. (1984), 'Discrete-continuous Models of Consumer Demand', *Econometrica* 52, 541– 561.
- Hausman, J. (1981), 'Exact Consumer's Surplus and Deadweight Loss', *The American Economic Review* 71(4), 662–676.
- Hausman, J., G. Leonard and D. McFadden (1995), 'A Utility-Consistent, Combined Discrete Choice and Count Data Model: Assessing Recreational Use Losses Due to Natural Resource Damage', *Journal of Public Economics* 56, 1–30.
- Kaoru, Y, V. Smith and J. Liu (1995), 'Using Random utility Models to Estimate the Recreational Value of Estuarine Resources', *American Journal of Agricultural Economics* **77**, 141–151.
- LaFrance, J. T. (1985), 'Linear Demand Functions in Theory and Practice', Journal of Economic Theory 37, 137–166.
- Lancaster, K. (1966), 'A New Approach to Consumer Theory', *Journal of Political Economy* **74**(2), 132–157.

- McFadden, D. (1978), 'Modeling the Choice of Residential Location', in A. Karlquist, L. Lundquist, F. Snickers and J. Weibull, eds., *Spatial Interaction Theory and Planning Models*, pp. 75–96. North-Holland: Amsterdam.
- Mendelsohn, R. (1984), 'An Application of the Hedonic Travel Cost Framework for Recreation Modeling to the Value of Deer', *Advances in Applied Microeconomics* **3**, 89–101.
- Mendelsohn, R. (1985), 'Identifying Structural Equations With Single Market Data', *Review of Economics and Statistics* 67, 525–529.
- Mendelsohn, R. (1987), 'A Review of Identification of Hedonic Supply and Demand Functions', *Growth and Change* 18(1), 82–92.
- Morey, E., R. Rowe and M. Watson (1993), 'A Repeated Nested Logit Model of Atlantic Salmon Fishing', American Journal of Agricultural Economics 75, 578–592.
- Parsons, G. and M. Kealy (1992), 'Randomly Drawn Opportunity Sets in a Random Utility Model of Lake Recreation', *Land Economics* 68(1), 93–106.
- Parsons, G. and M. Needleman (1992), 'Site Aggregation in a Random Utility Model of Recreation', Land Economics 68(4), 418–433.
- Pendleton, L. and R. Mendelsohn (1998a), 'Estimating the Economic Impact of Climate Change on The Freshwater Sportsfisheries of the Northeastern United States', *Land Economics 1998* 74(4), 483–497.
- Pendleton, L., B. Sohngen, R. Mendelsohn and T. Holmes (1998b), 'Measuring Environmental Quality in the Southern Appalachian Mountains', *Forest Science 1998* 44(4), 603–609.
- Pudney, S. (1989), Modelling Individual Choice. Oxford: Blackwell.
- Randall, A. and J. Stoll (1980). 'Consumer's Surplus in Commodity Space', American Economic Review 70(3), 449–455.
- Rosen, S. (1994), 'Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition', *Journal of Political Economy* 82, 34–55.
- Smith, V. K. and Y. Kaoru (1987), 'The Hedonic Travel Cost Method: A View From the Trenches', Land Economics 63, 779–792.
- Smith, V., R. Palmquist and P. Jakus (1991), 'Combining Farrell Frontier and Hedonic Travel Cost Models for Valuing Estuarine Quality', *Review of Economics and Statistics* 73(4), 694–699.
- USDA (1993). ZIPFIP: A ZIP and FIPS Database. Created by D. Hellerstein, D. Woo, D. McCollum, and D. Donnelly.
- Willig, R. (1976), 'Consumer's Surplus Without Apology', *The American Economic Review* **66**(4), 589–597.