ESTIMATING SHORT-TERM SOLAR VARIATIONS BY A

SIMPLE ENVELOPE MATCHING TECHNIQUE

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ABSTRACT

A simple matching technique is explained which allows the computation of the response of the solar surface to perturbations which occur at any depth within the convective envelope of the Sun. This technique was applied to a perturbation of the convective efficiency (α-mechanism), and of the non-gas component of the pressure (β-mechanism) in different regions of the convection zone. The results indicate that either perturbation affects the solar luminosity. However, the α-mechanism has little effect in the solar radius, regardless of the location of the perturbed region, whereas the β-mechanism produces radius changes that become quite large if the location of the perturbed region is deep within the solar convection zone.

Changing the mixing length ratio used in computing the solar convective envelope is one mechanism capable of producing a fluctuation of the solar luminosity (refs. 1,2). However, there are other possible mechanisms which can also produce solar variations. To facilitate the investigation of the effects of as many possible mechanisms as possible in an economical way, we shall describe here a simple matching technique for quick estimation of these effects. This technique cannot give the time evolution of the fluctuations, and the action of the perturbing mechanism must be confined to be above the bottom of the convection zone. However, this technique can give a rather accurate estimate of the amplitudes of the solar variations with very little computational effort. We shall first describe the mechanics of the technique, then its justification, and finally some applications.

The procedure to follow is very simple. First, one calculates the envelope structure of an unperturbed solar model using some standard stellar envelope code (cf. ref. 3). Then the perturbing mechanism must be implemented in the evelope code, and a new envelope matched to the old envelope at a mass point near the base, but within, the convection zone. By "matching", we mean that the luminosity and the outer radius of the new envelope are changed until its pressure, temperature, and radius at the matching mass point become the same as those of the old envelope. After the fitting parameters are determined, the differences between the new and old envelopes can be interpreted to be the variation of the solar luminosity and the spontaneous change in the solar radius.

To justify this proceedure, we need to answer the following questions:

1. There are three variables: pressure, temperature, and radius, to be matched but there are only two fitting parameters - luminosity, and outer radius. Is it possible to match all three variables consistently?

2. The matching point at which the matching is done is not a priori fixed. Are the results of matching insensitive to this point?

3. The envelope integrations need to assume the luminosity to be uniform above the matching point. Is this procedure valid?

4. The variation of the pressure, temperature, and radius at the matching point are taken to be zero. Is this a valid approximation? For how long?

To answer questions 1 and 2, let us examine Table 1. This table shows the results of perturbing the solar envelope by increasing the mixing length ratio α by +0.1% from an initial value of 1.5. The first row at a given mass point (M_r) gives the results obtained by matching the pressure and the temperature at M_r, and the second row gives the results obtained by matching the radius and the temperature at M_r. As one can see, the variations in luminosity are identical, and the variations in the solar radius all agree to within 15%. The ratios between $\delta \log R$ and $\delta \log L$ are all about 6×10^{-4}, in very good agreement with time-dependent calculations (refs. 4 and 5). Therefore, matching either pressure-temperature or radius-temperature give consistent results. Furthermore, this table shows that the results are insensitive to the exact location of the matching point.

For the luminosity L to be uniform in a perturbed envelope, it is necessary that the time scale for the relaxation of a non-uniform distribution of the luminosity be very short. To estimate this time scale, we derived a diffusive type equation for a non-uniform distribution of luminosity ΔL in the almost-adiabatic convective region (using mixing-length theory of convection)

$$\frac{\partial}{\partial t} \Delta L = \frac{\epsilon}{\tau} \frac{\partial}{\partial \ln p} \left(\frac{1}{\epsilon} \frac{\partial}{\partial \ln p} \Delta L \right), \tag{1}$$

where $\epsilon = 4\pi \rho C_p Tr^2 H$, $\tau = (4/3 \alpha)(H/V)$. H is the pressure scale height, V is the convective velocity, and all other symbols have their standard meaning. The important time scales here is τ which is close to the eddy-turn-over time scale of the convective turbulence. Near the bottom of the convection zone, this time scale is of the order of half a month. Therefore, convection can smooth the luminosity distribution in very short time. This smoothing time scale constitutes the lower limiting time scale for the applicability of our approach.

The upper limiting time scale for the validity of this approach can be estimated as follows. An extra energy leaving the solar surface $\delta L \Delta t$ is approximately given by half of the gravitational energy released by a collapse of a layer of mass ΔM by an amount δr in radius

$$\delta L \Delta t \sim \frac{1}{2} \left[\frac{GM_o^2}{R_o} \right] \left(\frac{\delta r}{r} \right) \left(\frac{\Delta M}{M_o} \right). \tag{2}$$

This provides an estimate for the growth of the collapse δr in time. If we

want the estimate of the variation in the outer radius δR to be not completely washed away by the collapse δr, then we need δr to be less than δR. This implies a restriction on the time period given by the inequality

$$\Delta t < 2 \times 10^5 \text{ W (yrs)} \tag{3}$$

where $W = \delta \log R / \delta \log L$. In the case for perturbing α, $W = 6 \times 10^{-4}$; therefore Δt must be less than 120 yrs.

We have answered all the previous questions on the validity of the approach. Now, let us briefly describe some results of applying the present technique to a calculation of the variations induced by a perturbation of the magnetic pressure in the upper convection zone. Let β be defined as P_m/P_t, where P_m is the material pressure, and P_t is the total pressure, which in addition to P_m, includes the radiation pressure and some "effective" magnetic pressure. Treated as a fluid, the magnetic field actually behaves quite similar to the radiation in the sense that its polytropic index is also 3. A lowering of β would mean that the fraction of electromagnetic pressure is raised relative to the material pressure. Figure 1 shows the results obtained by suddenly lowering β by 0.1% in the region between the solar surface and M_r (the horizontal axis). The solid curve shows $\delta R/R_0$ and the broken curve shows $\delta L/L_0$ as a function of the depth of the perturbation (the matching point for all cases is at $M_r = 0.99$). One can see that as the perturbation goes deeper, the effects become more prominent. Especially, the perturbation in radius increases substantially when $\delta \beta$ occurs below $M_r = 1-10^{-4}$ M_0; the $\delta \log R/\delta \log L$ ratio becomes close to 0.1. However, we must stress that the present calculation is very preliminary, and it can only give an order of magnitude guide to the general response of the convective envelope to β perturbations.

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   2191, 1981.

TABLE 1. RESULTS FOR PERTURBING BY

$$\Delta\alpha + 0.1\% \text{ FROM } \alpha = 1.5$$

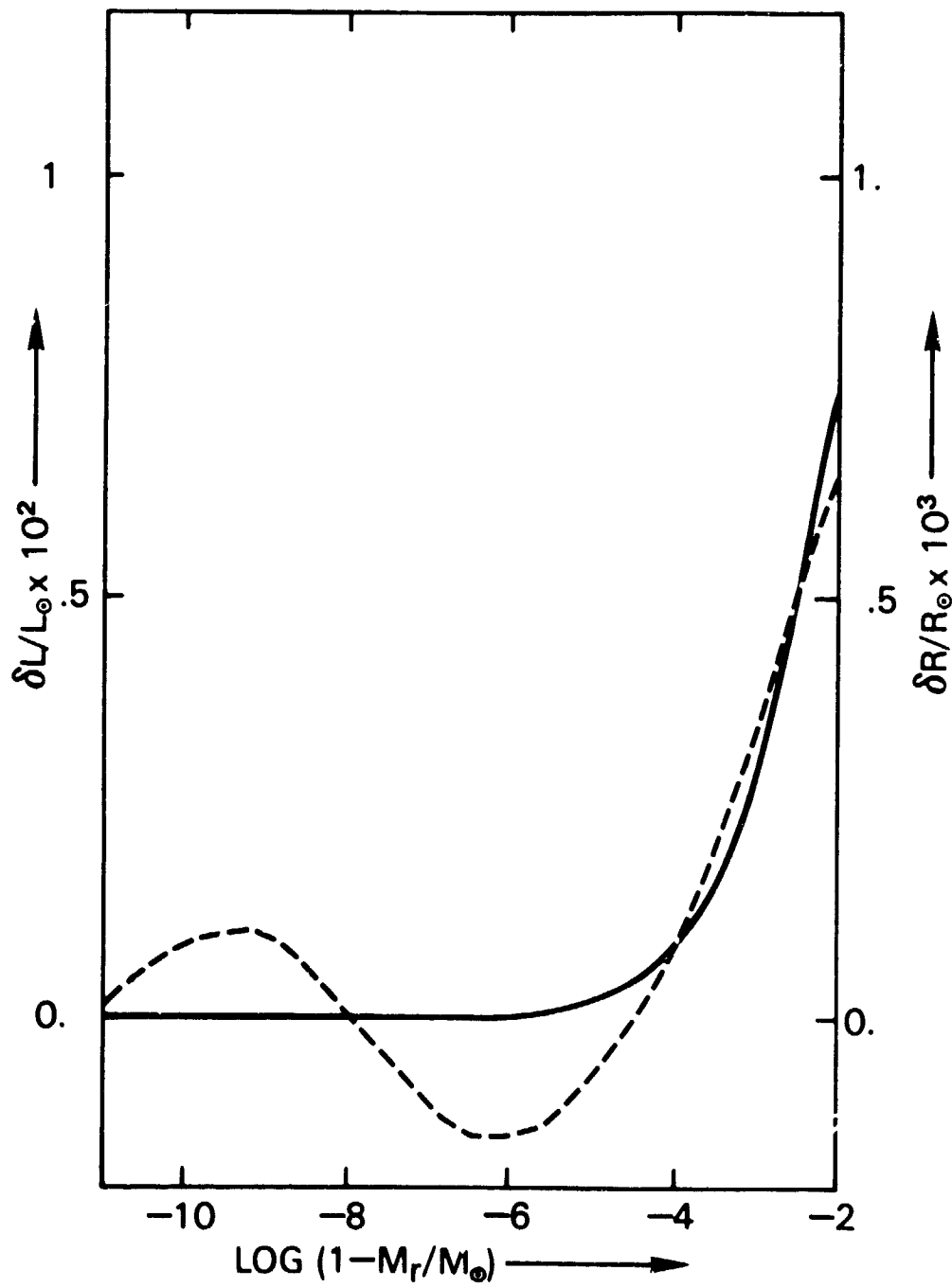| Matching Point M_r	$\delta \log (L)$	$\delta \log (R)$	$\dfrac{\delta \log (R)}{\delta \log (L)}$
.9850	.326(-3)	.189(-6)	.579(-3)
	.326(-3)	.193(-6)	.591(-3)
.9900	.326(-3)	.189(-6)	.580(-3)
	.326(-3)	.185(-6)	.566(-3)
.9990	.326(-3)	.182(-6)	.557(-3)
	.326(-3)	.170(-6)	.521(-3)
.9999	.326(-3)	.185(-6)	.567(-3)
	.326(-3)	.165(-6)	.507(-3)

Figure 1. Variations induced by a sudden decrease in β (0.1%) in the region above M_r. Solid line shows δR, and broken line δL.