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Estimating the Effects of the Minimum Wage in a Developing Country: A Density Discontinuity Design Approach

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Abstract

This paper proposes a new framework to identify the effects of the minimum wage on the joint distribution of sector and wages in a developing country. I show that under reasonable assumptions, cross-sectional data on the worker's wage and sector can identify the joint distribution of the latent counterparts of these variables; that is, the sector status and wage that would prevail in the absence of the minimum wage. I apply the method in the "PNAD", a nationwide representative Brazilian cross-sectional dataset for the years 2001 to 2009. The results indicate that the size of the informal sector is increased by around 39% compared to what would prevail in the absence of the minimum wage, an effect attributable to (i) unemployment effects of the minimum wage on the formal sector, (ii) movements of workers from the formal to the informal sector as a response to the policy.

JEL No. J60, J30, J31

Keywords: Minimum wage, informality, unemployment, density discontinuity design, wage inequality, labor tax revenues, formal sector

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1. INTRODUCTION

Despite its widespread use, controversy persists regarding the economic impact of the minimum wage. A simple one-sector competitive market model predicts that a minimum wage will generate unemployment when the minimum exceeds the market-clearing wage. However, if the employer has market power, then a minimum wage can lead to an increase in wages and employment. In an economy with a large informal sector, where some employers do not comply with the minimum wage policy, the minimum wage might not generate unemployment effects even in the absence of market power on the part of the employer. This will hold as long as the workers can freely migrate from one sector to the other and the informal sector is sufficiently large to accommodate such movements.

These conflicting theoretical predictions provide a strong motivation for empirical studies on the effects of minimum wage policies. In this paper, I develop a Dual-economy model based on Meyer and Wise (1983) to assess the impacts of the minimum wage on (a) unemployment, (b) average wages, (c) wage inequality, (d) sector mobility, (e) the size of the informal sector, and (f) labor tax revenues. I model the joint distribution of wages and sectors (latent and observed), as opposed to the marginal distribution of wages, as in Meyer and Wise (1983). A model for the joint distribution of sector and wages allows me to infer the size of the formal sector that would prevail in the absence of the minimum wage and compute the proportion of workers who move to the informal sector in response to the policy. I provide the conditions for identifying the Dual-economy model parameters and the latent joint distribution of sector and wages, that is, the distribution that would prevail in the absence of the minimum wage policy. My identification strategy relies on the discontinuity in wage density at the minimum wage and the differences in the response to the minimum wage between the formal and informal sectors.

This paper's contributions to the literature are the following: (i) I document key empirical facts concerning the relationship between formal and informal wage distributions that have been overlooked in previous research, namely, the similarity between these distributions conditional on values above the minimum wage; (ii) I provide a novel identification strategy that combines a non-parametric density discontinuity design with a parametric model for the conditional probability of sector given the wage. In particular, I show that under reasonable conditions, the parameters that describe the effects of the minimum wage and the underlying latent joint distribution of sector and wages are identified using only cross-sectional data. (iii) I estimate a sector mobility parameter, the probability that a worker in the formal sector moves to the informal sector in response to the minimum wage; (iv) I demonstrate how to test some of the assumptions invoked to identify the parameters of the model and I show that these assumptions hold in the empirical application; (v) I estimate the effect of the minimum wage on the joint distribution of sector and wages and estimate the effect of the minimum wage on labor tax revenues. To the best of my knowledge, this is the first paper that attempts to identify both the latent share of the formal sector and the effects of the minimum wage on labor tax revenues.

The identification problem studied in this paper includes minimum wage policies as a special case. In general, any policy that introduces a boundary type of restriction can potentially be analyzed under this framework. Examples of such policies are price floors, price ceilings, and age restrictions (such as the restrictions for alcohol and tobacco consumption). These policies will typically affect the size of the market, the distribution of quality of the good, taxes, and induce some transactions to occur "off the table" (and by so increase the size of a "black" market). This paper provides a set of assumptions and data requirements under which it is possible to identify the effects of such policies on these outcomes.

The model is estimated using the years 2001 to 2009 from "*Pesquisa Nacional por Amostra de Domicílios*" (PNAD), a dataset comprising repeated cross sections of an annual household survey that is representative of the Brazilian population. I find that the probability of a formal worker switching to the informal sector as a result of the policy is small – approximately 12%. The combined effect of unemployment and transitions to the informal sector generated by the introduction of the minimum wage leads to an 9% decrease in the size of the formal sector relative to the counterfactual state defined by the absence of the minimum wage. This associated growth in the size of the informal sector as a result of the policy is 39% - an effect attributable to the fact that the latent formal sector is four times larger than the informal sector of the economy. Unemployment effects of the minimum wage are, as expected, highly correlated with the real value of the minimum wage. Moreover, the minimum wage strongly affects average wages (promoting an increase of approximately 16%), wage inequality (an approximately -19% effect on the standard deviation of log wages and a -24% impact on the Gini Index), and labor tax revenues (-6%).

2. BACKGROUND

In Brazil, all workers are required to carry a government document called a "*Carteira de Trabalho*", or worker's card. This document, introduced in 1932, serves as proof of the worker's legal employment status. If a worker is formally employed in the Brazilian labor market, then his contract is signed by the employer on a page of the worker's card. This labor contract implies that the worker's employment is in compliance with labor taxes and labor regulations such as the minimum wage. Formal employment gives the worker access to benefits that include unemployment insurance and severance payments.

Not all labor contracts are signed by the employer and included in the worker's card. When an employer and a worker agree to a labor contract but decide not to formally sign it and include it in the worker's card, the worker's employment is called informal. Reasons for the existence of informal contracts include the evasion of labor regulations, such as the payment of labor taxes, compliance with the minimum wage, job safety standards, and restrictions on hours worked per week.[1] This definition of

[1]Firms face a trade-off between the costs of complying with the regulation and the probability/magnitude of punishment. The firms' decision to hire formal versus informal workers was investigated in Almeida and Carneiro (2012), and Mattos and Ogura (2009).

informality is tightly related to compliance with the minimum wage. However, these concepts are not equivalent. A worker with a wage below the minimum wage level is surely an informal worker. However, a worker whose wage is above the minimum wage may be formal or informal depending on whether his contract is signed by the employer.[2] The proportion of private sector workers between the ages of 19 and 59 who are employed in the formal sector is .74. In other words, more than one quarter of private sector workers do not have a signed contract included in their worker's card.

The minimum wage in Brazil has been set at the federal level since 1984. In theory, all jobs are covered, meaning that the (same) minimum wage level should apply to every worker. In practice, coverage only extends to workers with a contract written on the worker's card (Lemos, 2009). A unified minimum wage set at the federal level with full coverage complicates the task of finding an appropriate control group. This is because cross-border differences-in-differences analysis, such as that in Card and Krueger (1994), is ruled out as a practical option, as the same level prevails in all states. Another feature of the minimum wage changes in Brazil is that since 2005, they have been linked to inflation and GDP growth, which poses further challenges to the use of time-series variation to estimate the effects of the minimum wage. Under these conditions, it is more difficult to disentangle the effects of the minimum wage from other sources of changes in the wage distribution that are linked to changes in economic activity.

Despite these challenges, it is nevertheless possible to identify the effect of the minimum wage using only cross-sectional data on sector and wages. This paper describes a set of *a priori* restrictions – on the joint distribution of sector and wage, and on the effects of the policy – that allows for identification of the effects of the minimum wage using only cross-sectional data on sector and wages. This research design is well suited to analyze markets characterized by the absence of cross-sectional or time series variation on the policy level that can be used for identification, such as the case

[2] As we will discuss in greater detail below, approximately 20% of the workers whose wages are above the minimum are informal workers.

of the minimum wage in the Brazilian labor market.[3]

The points of departure for this paper are the works of Meyer and Wise (1983) and Doyle (2006). These papers show how to identify the effects of the minimum wage on the distribution of wages. I extend their model to a two-sector, or dual-economy, setting with sector mobility. This allows wages in both sectors to be affected, but in different ways, by the minimum wage. It also allows me to capture the effects of the minimum wage on the size of the formal sector and other related outcomes, such as labor tax revenues.

The dual-economy extension I develop presents new challenges for identification. This is because the techniques presented in Doyle (2006) are not sufficient to recover the sector-specific parameters of the model in this general version. The reason is that applying Doyle's strategy to the aggregate economy only recovers a weighted average of the parameters, which will be uninformative for most of the outcomes of interest. Applying his method to each sector separately is not feasible, as workers have moved from one sector to the other as a result of the policy. Thus, one of the contributions of this paper is demonstrating how to identify the effects of the minimum wage in this dual-economy setting.

In the next sections, I briefly describe the models of Meyer and Wise (1983) and Doyle (2006) to highlight the similarities and differences between their papers and the approach followed here.

3. MODEL

The effect of the minimum wage on a worker's wage is the difference between his wage under the policy and the wage he would receive in its absence. The fundamental problem of causal evaluation is that this difference is conceptually well defined but

[3]The assumptions made to address the question of interest will not be in terms of agents' preferences, technology or equilibrium mechanisms; rather, they will be in terms of the relationship between latent and observed variables. In this sense, the identification is semi-structural. It is structural in the sense that it relies on assumptions concerning the effects of the policy and semi-structural in the sense that those assumptions can be satisfied by a wide set of different fully specified structural models.

never observed in the data. This is true because we can at most observe the wages for each worker in one of the two possible states of the world. However, it is nevertheless helpful to consider these objects. Thus, let worker i be characterized by an observed wage $W_i(1)$ and a corresponding latent wage $W_i(0)$, which is defined as the wage that the worker would receive in the absence of the minimum wage. I will denote the minimum wage level by m. I will denote by $F_0(w)$ ($f_0(w)$) the CDF (pdf) of latent wages. Similarly, denote by $F(w)$ ($f(w)$) the CDF (pdf) of observed wages. To keep the model as simple as possible, assume that these workers come from a population with similar observable characteristics, and hence, we do not need to control for these characteristics. In the absence of the minimum wage, every worker i in this population obtains a draw $W_i(0)$ from the distribution F_0, which I will refer to as the underlying wage distribution or the distribution of "market" wages. Although workers are intrinsically homogeneous *ex-ante*, meaning that they draw their wages from the same distribution, they will have different wages *ex-post*.

In the presence of the minimum wage policy, the worker will receive a draw $W_i(1)$ from the distribution F, which I will refer to as the distribution of observed wages.[4]

To make the problem of identifying the effects of the minimum wage tractable, I follow Meyer and Wise by imposing a set of *a priori* restrictions on the distribution of the latent variables and on the effects of the policy.[5] As we will see, these restrictions allow me to identify the effects of the policy without relying on exogenous policy variations or time-series data.

[4]The most flexible way to model the effects of the minimum wage in the wage distribution is to assume that each worker can potentially be affected by the policy. If we consider wages in terms of a discrete variable, the effects of the minimum wage on the distribution of wages can be completely characterized by a matrix of transitions that govern the probability that a worker at any point w of the latent wage distribution will end up at any point w' in the observed wage distribution. That is, a completely general (and, by construction, correctly specified) model for the effects of the minimum wage on the wage distribution is a transition matrix in which every entry is given by $Pr[W(1) = w'|W(0) = w]$.

[5]I discuss in Section 6.4 and in Appendix D how to indirectly test the validity of these *a priori* restrictions.

3.1. *The Meyer and Wise Approach*

This section describes the assumptions and estimation strategy used by Meyer and Wise (1983). Assume that the econometrician observes a random sample of observed wages $\{W_i(1)\}$ of size N from a population of interest.[6] Let the following hold:

Assumption MW1 *The latent wage is log-normally distributed. That is,* $\log(W(0)) \sim N(\mu, \sigma^2)$.

Assumption MW2 *There are no spillovers from the minimum wage. This means that* $W(1) = W(0)$ *when* $W(0) > m$.

Assumption MW3 *If* $W(0) < m$*, then with probability* π_m*, the worker receives the minimum wage* ($W(1) = m$)*. With probability* π_d*,* ($W(1) = W(0)$)*, the worker's wage is the same as the latent wage (non-compliance). With the complementary probability* $\pi_u = 1 - \pi_m - \pi_d$*, the worker becomes unemployed* ($W(1) = \cdot$).[7]

The probabilities (π_m, π_d, π_u) represent the likelihood of receiving the minimum wage, non-compliance and unemployment. These parameters arise so naturally in the context of the minimum wage that it is occasionally difficult to recognize how they restrict the model in any way. They seem to resemble a list of all possible outcomes. This is not the case, however. The restrictions imposed by defining these probabilities are as follows: (i) $Pr[W(1) > m|W(0) < m] = 0$, that is, no worker whose market wage is below the minimum wage will receive a wage greater than the minimum wage when the policy is introduced; (ii) the probabilities π_d, π_m and π_u are not a function of the worker's latent wage, such as, for example, a function of how far they are from

[6]Note that this is a non-standard policy evaluation problem in which all individuals are treated ($W_i = W_i(1)$), meaning that the (same) minimum wage level holds for everyone in the population. The absence of a control group forces the use of a model to identify the effects of the policy.

[7]Strictly speaking, the appropriate expression should be "non-employment". I will refer to this effect as the "unemployment" effect of the minimum wage. Throughout the paper I will use non-employment and unemployment interchangeably, given that the model cannot distinguish these effects.

the minimum wage; and (iii) workers who do not comply with the policy retain the same wage, that is, $W_i(1) = W_i(0)$.[8,9]

The goal of the exercise is to recover the parameters of the underlying latent distribution of market wages (μ, σ) and the parameters (π_d, π_m, π_u) that govern how the minimum wage affects the economy. The key contribution of Meyer and Wise is to show that those parameters are identified using data on observed wages $(W_i(1))$. Perhaps surprisingly, one need not have any variation in the policy to recover its effects. To observe how this is achieved, define the log-likelihood of observing $W_i(1) = w$ as: $\log L(W_i = w|\mu, \sigma, \pi) = \mathbb{1}\{w < m\} \log \frac{\pi_d f_0(w)}{c} + \mathbb{1}\{w = m\} \log \frac{\pi_m F_0(m)}{c} + \mathbb{1}\{w > m\} \log \frac{f_0(m)}{c}$, where $\mathbb{1}\{A\}$ is the indicator function of the event A and $c \equiv 1 - \pi_u F_0(m)$ is a rescaling factor that ensures that the observed density of wages integrates to one. The parameter c can be interpreted as the ratio of employment before and after the introduction of the policy. Meyer and Wise use maximum-likelihood to estimate the parameters of the model. An intuitive way to think about the identification is to recognize that the model allows us to use the information on wages above the minimum wage level to predict the shape of the wage distribution in the absence of the policy.[10]

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[8]At first glance, the restriction (i) may not appear problematic, as it is difficult to imagine why someone would comply with the policy by increasing a worker's wage to a value *greater* than m. This is not impossible, however. An example of a model that is excluded by this assumption is that of Teulings (2000).

[9]Restriction (ii) can be relaxed in certain ways, for example, by making the probabilities (π_d, π_m, π_u) low-order polynomials of the worker's latent wage. Restriction (iii) can be relaxed without affecting the identification strategy by making the worker draw from the distribution of market wages conditional on values below the minimum wage. Changes to the average wages of those who do not comply with the policy also can be incorporated. However, this change requires some modifications in the identification strategy.

[10]A closer inspection on the likelihood function shows that Meyer and Wise's approach nests "standard measurement", truncation and censoring of the wages below the threshold m. If π_u is equal to one, the likelihood function of Meyer and Wise's model is the same as the likelihood of a truncation model. If π_m is equal to one, the likelihood of the model is the same as the likelihood of a Tobit model (censoring). If the probability of non-compliance π_d is equal to one, the likelihood becomes the standard likelihood of a normal distribution, with no censoring or truncation. Moreover, the probabilities of "measurement", truncation and censoring have a direct economic interpretation as different responses of the economy to the minimum wage policy.

3.2. *Doyle's Approach*

A limitation of Meyer and Wise's (1983) approach is that it relies on a parametric assumption concerning the latent wage distribution.[11] The contribution of Doyle (2006) is to show that this is not actually necessary for identification when one is only willing to assume continuity in the distribution of latent wages. The key idea behind this strategy is that the continuity of the distribution of latent wages implies that the ratio of the density of observed wages just above and below the minimum wage identifies π_d, the likelihood of non-compliance with the policy.[12] In this section, I discuss the identification of the minimum wage effects under the model proposed by Doyle (2006). In the following discussion, I will maintain assumptions MW2 (no spillovers) and MW3 (minimum wage effects) from Meyer and Wise (1983). Again, assume that the econometrician only observes wages in the presence of the policy; that is, a random sample of size N from the distribution of $\{W_i(1)\}$ is available.

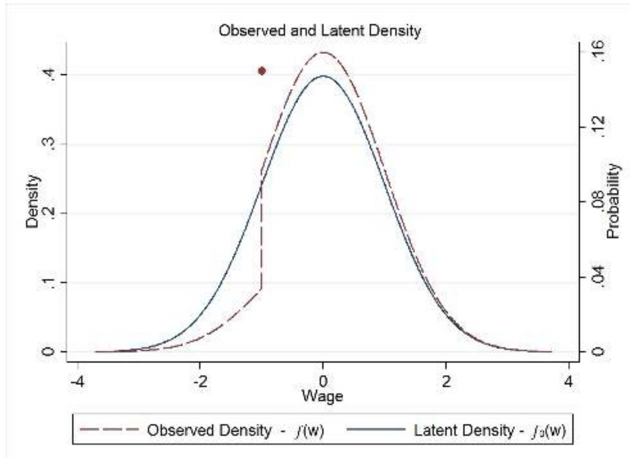
Assumption D1 *The density of latent wages is continuous at m. That is, $\lim_{w \to m^+} f_0(w) = \lim_{w \to m^-} f_0(w)$.*

As discussed in Doyle (2006), this assumption exploits the fact that the distribution of worker productivity is likely to be smooth, but the observed density of wages has a jump around the minimum wage. This jump can provide exactly the information necessary to trace back the effects of the policy on the outcomes of interest. Under assumptions MW2 and MW3, there is a relationship between the latent and observed distribution of wages. This relationship is given by: $f(w) = \mathbb{I}\{w < m\}\frac{\pi_d f_0(w)}{c} + \mathbb{I}\{w = m\}\frac{\pi_m F_0(w)}{c} + \mathbb{I}\{w > m\}\frac{f_0(w)}{c}$, where $c = 1 - \pi_u F_0(m)$, as before. Figure 1 provides a graphical example of the relationship between the observed and the latent densities. Taking the ratio of the density of observed wages just below and above the minimum wage, that is, considering the lateral limits of the density at

[11] The sensitivity of the estimates with respect to the parametric assumptions was studied in Dickens, Machin and Manning (1998).

[12] Given the similarity between this identification strategy and RD Designs, Doyle (2006) termed it a density discontinuity design.

FIGURE 1.— Doyle's Model

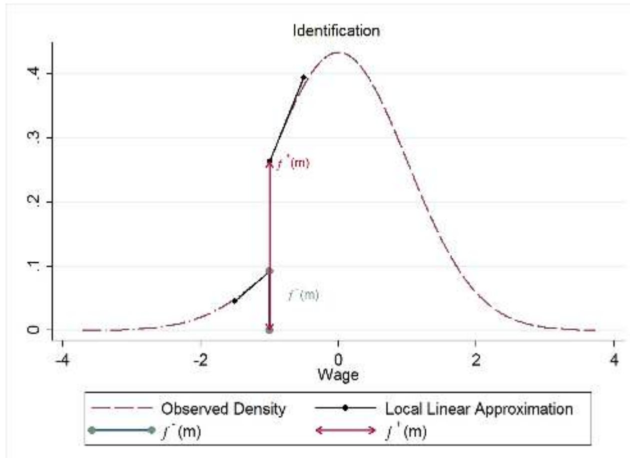


m, we have $\frac{\lim_{w\to m^-} f(w)}{\lim_{w\to m^+} f(w)} = \frac{\lim_{w\to m^-} \frac{\pi_d f_0(w)}{c}}{\lim_{w\to m^+} \frac{f_0(w)}{c}} = \pi_d$, where the last equality is obtained using assumption D1. Figure 2 graphically depicts the mechanics of the estimation of the non-compliance probability. To recover the remaining parameters, it is easy to see that by integrating the density of observed wages up to the minimum wage, we have $Pr[W(1) < m] = \frac{\pi_d F_0(m)}{c}$. Then, we have $\frac{Pr[W(1)=m]}{Pr[W(1)<m]} = \frac{\pi_m}{\pi_d}$.[13] Because the left-hand side of this equation is identified from the data and the right-hand side is a function of only one unknown, this implies that π_m is identified. This also implies that $\pi_u = 1 - \pi_d - \pi_m$ is identified. To recover the latent density of wages, one needs to recover $F_0(m)$. This is the case because the relationship between the observed and latent densities can be inverted once we know the rescaling factor c. To see this, note: $f_0(w) = \frac{f(w)\cdot c}{\pi_d}$ if $w < m$, and $f_0(w) = f(w) \cdot c$ if $w > m$.

One way to identify $F_0(m)$ is to use the fact that $F_0(m) = \frac{Pr[W(1)<m]}{Pr[W(1)<m]+\pi_d Pr[W(1)>m]}$, which follows from: $\frac{Pr[W(1)<m]}{Pr[W(1)<m]+\pi_d Pr[W(1)>m]} = \frac{\pi_d F_0(m)/c}{\pi_d F_0(m)/c+\pi_d(1-F_0(m))/c} = \frac{F_0(m)}{F_0(m)+1-F_0(m)} = F_0(m)$. This implies that the latent distribution of wages can be recovered under assumptions D1, MW1, and MW2. The discontinuity in the

[13]Interestingly, the identification of π_m also can be understood in terms of a discontinuity. The ratio π_m/π_d is identified by a discontinuity in the *distribution* of wages at the m. Thus, the identification of Doyle's model uses a discontinuity in the CDF of wages to identify π_m/π_d and a discontinuity in the density of wages to identify π_d.

FIGURE 2.— Doyle's Model: Identification



observed distribution around the minimum wage identifies the probability of non-compliance with the policy π_d.[14] This in turn allows us to recover π_m, $F_0(m)$ and, consequently, the entire latent wage distribution.[15]

3.3. *Minimum Wage Effects in a Dual Economy*

The Brazilian economy, similar to those of many other developing countries, is characterized by a large informal sector. In Brazil, an informal worker is defined as a worker whose worker's card does not include a signed labor contract. Informality is thought to arise in developing countries as a result of restrictive and costly labor laws. Note that once the worker's card is signed, the collection of labor taxes should follow and compliance with minimum wage and other labor standards has to be assured. A natural question that arises in this context is the following: What is the role of

[14]Each step after the identification of π_d from the limit of the ratio of densities relies on the assumption that this probability is not a function of the wage. This feature contrasts with the parametric model of Meyer and Wise. By restricting the set of latent wage distributions, more flexibility can be introduced in the functional form of the relationship between the latent wage $W(0)$ and the model parameter π_d. This is the case because the shape of the latent wage distribution can be recovered in the parametric setting using the information above the minimum wage. This allows us to identify not only a probability of non-compliance but also a function $\pi_d(w)$ that maps wages to non-compliance probabilities. This function need not be constant with respect to latent wages.

[15]Doyle's model can be identified under the assumption that π_d is a low-order polynomial of the latent wage. However, in this case, identification can only be achieved by using derivatives of the wage density at the minimum wage level.

the minimum wage in a economy with such a large informal sector? A large fraction of contracts outside the "umbrella" of the labor laws may be a consequence of the minimum wage, meaning that many workers (intentionally or not) have moved to the informal sector as a consequence of the minimum wage policy. However, in principle, it could also be the case that the observed proportion of workers in the informal sector is completely unrelated to the level of the minimum wage. Informality may instead depend on labor taxes and other forms of labor regulation (hours worked, job safety standards and so forth) that have to be met regardless of where the worker is located in the wage distribution. These two explanations have markedly different policy implications but are in principle equally plausible explanations for the observed size of the informal sector. One of the goals of this paper is to assess the relative importance of these explanations.

To do so, I generalize the models of Meyer and Wise (1983) and Doyle (2006) to the case of a dual economy. I model the joint distribution of wages and sectors (latent and observed), as opposed to the marginal distribution of wages. This allows me to infer the size of the formal sector that would prevail in the absence of the minimum wage and compute the proportion of workers who move to the informal sector in response to the policy.

Let worker i be characterized by a pair of wage ($W_i(1)$) and sector ($S_i(1)$), which is equal to one if he is employed in the formal sector and zero otherwise. Compliance with minimum wage legislation is perfect in the formal sector but not in the informal sector. This effectively means that the workers in the formal sector are not allowed to have wages below the minimum wage once the policy is introduced. If they remain employed in the presence of the policy, they must either move to an informal contract or comply with the policy by receiving a wage equal to m. In addition, for each worker, define a pair $(W_i(0), S_i(0))$ that denotes the counterfactual - or latent - wage and sector in the absence of the minimum wage. Finally, define $F_0(w)$ ($f_0(w)$) as the c.d.f (p.d.f) of $W(0)$ and $F(w)$ ($f(w)$) as the c.d.f (p.d.f) of observed wages ($W_i(1)$

or, in short notation, W_i). I will assume that the *latent* wage and sector distribution have the following characteristics:

ASSUMPTION 1 (Continuity) *The density of latent wages and its first derivative are continuous at m. That is,* $\lim_{w \to m^+} f_0(w) = \lim_{w \to m^-} f_0(w)$, *and* $\lim_{w \to m^+} f_0'(w) = \lim_{w \to m^-} f_0'(w)$.

Because this is a model of the joint distribution of sector and wages, we need to define another object, $Pr[S(0) = 1|W(0) = w]$:

ASSUMPTION 2 (Conditional probability of (latent) sector given the wage) *The conditional distribution of latent sector given the latent wage belongs to a parametric family* $\{\Lambda(w, \beta) : \beta \in B \subset \mathbb{R}^k\}$. *That is,* $Pr[S(0) = 1|W(0) = w] = \Lambda(w, \beta)$ *for some* $\beta_0 \in B$. *Moreover,* $Pr[\Lambda(W(0), \beta_0) \neq \Lambda(W(0), \beta')|W(0) > m] > 0$ *for all* $\beta' \neq \beta_0$.

With the conditional probability of latent sector (given the wage) and the marginal distribution of latent wages, we have completely specified the joint distribution of these variables.[16] The restrictive part of this assumption is that the conditional probability of the latent sector given latent wages can be described by a parametric model. The first part of the above assumption states that there is a parameter β for which the probability of the latent sector given the latent wage w is exactly equal to $\Lambda(w, \beta)$. The second part of the assumption ensures that there is only one parameter for which this condition holds. Both assumptions are standard in models with binary outcomes. For concreteness, assume that the parametric model is a logit.[17,18]

[16]This joint distribution could come, for example, from a Roy-type model of sector choice, in which workers would choose the sector that yields the highest utility. Another model would be one in which workers are assigned to firms that, based on labor taxes and probability of punishment, decide whether they will employ formal or informal workers.

[17]The logistic functional form is assumed only for clarity in the exposition. All identification results are preserved if the logistic functional form is replaced by another parametric form, such as a probit. Moreover, one can make the model flexible by adding higher-order polynomials of wages (squares and cubes) as regressors in the logit to better adjust the curve. As long as the degree k of the polynomial is fixed with respect to the sample size, that is, the model remains parametric, the identification results will hold.

[18]The reason for the need of a parametric model, as will become clear in the identification section,

Assumption 3 (No spillovers) *Workers whose latent wages would be above the minimum wage are not affected by the policy. That is,* $W(1) = W(0)$ *and* $S(1) = S(0)$ *when* $W(0) > m$*.*

This assumption is potentially strong. In the non-parametric framework, this assumption is also untestable. However, it is straightforward to see that the model is still identified under any *known* and invertible spillover function. Moreover, bounds can be computed for the parameters when positive spillovers are assumed to exist and the researcher has no prior information on their size.[19] Furthermore, spillovers can also be identified and estimated if one is willing to assume a parametric family for the latent wage distribution. In the empirical application I investigate the robustness of my results to the presence of limited spillovers.

To complete the model we need to define the minimum wage effects in the lower-tail of the wage distribution. As discussed by Meyer and Wise, workers in sectors operating in competitive markets whose wages would be below the minimum might become unemployed as a result of the minimum wage. If there is some bargaining involved in the wage determination, or if the employers hold market power, some workers will "cluster" at the minimum as a result of the policy. Finally, because compliance with the minimum is imperfect in some markets, workers might migrate from the formal to the informal sector to avoid unemployment. In terms of the model, this leads to the following assumption:

Assumption 4 (Minimum wage effects) *For wages below the minimum wage* ($W(0) < m$)*, we have the following: If* $S(0) = 0$*, then* $S(1) = S(0)$*. Moreover, with probability* $\pi_d^{(0)}$*, the wage continues to be observed* ($W(1) = W(0)$)*. With the complementary probability* $\pi_m^{(0)} = 1 - \pi_d^{(0)}$*, the worker earns the minimum wage*

is that this model induces censoring in the probabilities of working in the formal sector for wages below the minimum wage. This forces us to rely on extrapolation using values above the minimum to identify the share of formal workers for low wages. The need for extrapolation excludes non-parametric methods as an option.

[19] See Appendix E.5 for further discussion of this issue.

$(W(1) = m)$.[20] *If* $S(0) = 1$, *then with probability* $\pi_d^{(1)}$, *the wage continues to be observed* $(W(1) = W(0))$, *meaning that the worker successfully transits from the formal sector to the informal sector.*[21] *In this case, the observed sector will be* $S(1) = 0$, *being different from the latent sector. With probability* $\pi_m^{(1)}$, *the worker earns the minimum wage* $(W(1) = m, S(1) = 1)$. *With the complementary probability* $(\pi_u^{(1)} = 1 - \pi_d^{(1)} - \pi_m^{(1)})$, *the worker becomes unemployed* $(W(1) = \cdot, S(1) = \cdot)$.[22]

3.4. *Discussion*

The goal of the exercise is to recover the unknown parameters $\pi \equiv (\pi_d^{(1)}, \pi_m^{(1)}, \pi_u^{(1)}, \pi_d^{(0)}, \pi_m^{(0)})'$ and the joint distribution of latent sector and wages, that is, the joint density that would prevail in the absence of the minimum wage. By comparing this distribution with the observed distribution, I can evaluate the impact of the minimum wage on expected wages, wage inequality, employment and other labor market outcomes. By defining the latent sector and the sector-specific parameters, a broader range of implications of the minimum wage becomes assessable, such as changes in tax revenues and movements between sectors. In Sections 3.5 and 6.3, I will discuss in detail how the minimum wage affects these outcomes.

The assumptions used in this model are similar to the assumptions used previously in this literature. I maintain all assumptions from Doyle – or Meyer and Wise, if one

[20]The reason for allowing $\pi_m^{(0)}$ to be greater than zero, that is, to allow workers in the informal sector to cluster at the minimum wage, is for the model to account for the empirical fact that they seem to do so. The informal sector wage distribution presents a spike similar to the formal sector distribution at the minimum wage. The economic logic behind this regularity is under debate. One hypothesis is that the minimum wage acts as a signal to the agents of a fair price for unskilled labor, which might affect the way workers in the informal sector bargain with their employers. This feature seems to be related to the "self-enforcing" nature of the minimum wage.

[21]The assumption that the wage remains exactly the same when the worker moves to the informal sector, that is $(W(1) = W(0))$, substantially simplifies the exposition. The same results hold when this assumption is replaced with one in which the worker draws a new wage from $f_0(w|S(0) = 1, W(0) < m)$.

[22]To ease the exposition, I have assumed that $\pi_m^{(1)}$ and $\pi_u^{(1)}$ do not vary as a function of the latent wage. In this case in which they vary over the latent wages, the parameter recovered by assuming that they are constants is the expectation of the distribution of $\pi_m^{(1)}$ and $\pi_u^{(1)}$ over the distribution of wages below the minimum. This result holds only as long as $\pi_d^{(1)}$ remains constant as a function of the wage.

prefers a parametric specification for latent wages – and generalize their approach to address sector-specific responses. Assumption 4, the assumption that defines the sector-specific effects of the minimum wage, implies the assumptions used by Meyer and Wise (1983) and Doyle (2006) concerning the marginal distribution of wages. That is, the marginal distribution of wages (which is obtained after integrating out the sector-specific wage distributions) will resemble the density of wages that appears in Meyer and Wise (1983) and Doyle (2006).

Despite these similarities, there are numerous advantages of using a model for the joint distribution of sector and wages. This is especially true for developing countries, where the informal sector plays an important role in the economy. This model accommodates a variety of responses of the economy to the minimum wage policy. The model allows for the standard unemployment effect. The model allows the minimum wage to have a "supporting" effect on the lower tail of the wage distribution in such a way that the policy can affect average wages and wage inequality. The model allows wages in the informal sector to be affected by the introduction of the minimum wage – an effect captured by the parameter $\pi_m^{(0)}$. This model also allows workers to move to the informal sector as a response to the minimum wage – this event is captured by the parameter $\pi_d^{(1)}$. Combined, these unemployment and sector mobility effects allow the minimum wage to affect the relative size of the formal sector in the economy, which in turn can affect labor tax revenues.

A two-sector model helps to interpret the parameters identified in the previous work from Meyer and Wise (1983) and Doyle (2006). Meyer and Wise discuss the possible reasons that one would observe a non-zero density of wages below the minimum, such as uncovered jobs and non-compliance in covered sectors. Ultimately, however, Meyer and Wise's model identifies the aggregate likelihood of non-compliance (π_d). This parameter is the proportion of workers who, following the introduction of the minimum wage, do not ultimately respond to the policy. An application of the law of iterated expectations shows that the parameter estimated in their model is a weighted average of the sector-specific parameters, with weights given by the *latent* shares of the sectors in the economy. The parameter π_d does not identify whether workers earn sub-minimum wages because they would already be working in non-covered sectors regardless of the policy or because they moved there as a *response* to the policy. These two different stories are implied by different values of the sector-specific parameters. However, they can imply the exact same value for π_d. Moreover, any combination of the two is also equally likely when one estimates only the aggregate or "unconditional" parameter π_d. Thus, the sector mobility parameter $\pi_d^{(1)}$ and the latent size of the uncovered sector $Pr[S(0) = 1]$ are more economically meaningful than the aggregate parameters.

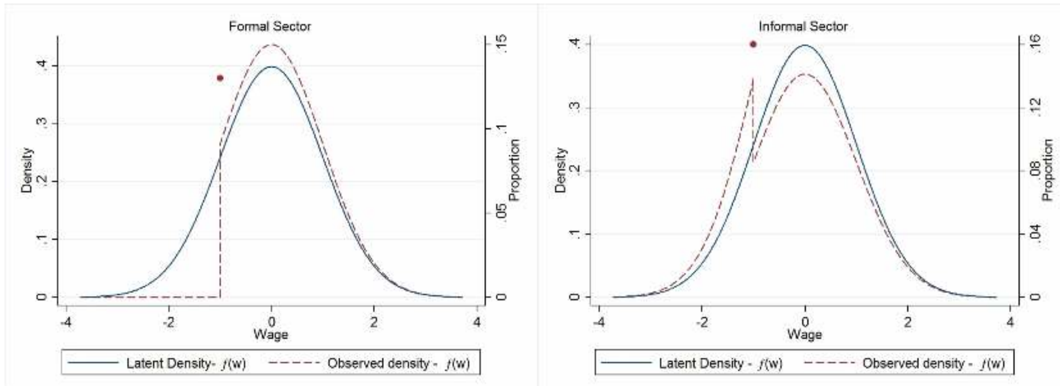
3.5. *Model Analysis*

In this section, I show that this model can capture a wide range of potential effects of the minimum wage policy. To do so, I discuss the model's implications for some objects of interest, such as the sector-specific wage densities and the conditional probability of formality given the wage.

Given assumptions 2 to 4 above, there is a relationship between the latent and observed unconditional wage distributions. It is given by: $f(w) = \mathbb{1}\{w < m\}\frac{\pi_d(w)f_0(w)}{c} + \mathbb{1}\{w = m\}\int^m \frac{\pi_m(w)f_0(w)}{c}dw + \mathbb{1}\{w > m\}\frac{f_0(w)}{c}$, where $c \equiv 1 - \int^m \pi_u^{(1)}\Lambda(w)f_0(w)dw$ is a rescaling factor that ensures both densities integrate to one. This parameter has the interpretation as the ratio of employment in the presence of the policy to that in the absence of the policy.

Regarding the relationship between the sector-specific parameters and the aggregate ones, we have $\pi_d(w) \equiv \Lambda(w)\pi_d^{(1)} + (1 - \Lambda(w))\pi_d^{(0)}$, $\pi_m(w) \equiv \Lambda(w)\pi_m^{(1)} + (1 - \Lambda(w))\pi_m^{(0)}$, and $\pi_u(w) \equiv \Lambda(w)\pi_u^{(1)}$. The parameters $\pi_d(w)$, $\pi_m(w)$ and $\pi_u(w)$ are weighted averages of the sector-specific parameters with weights given by the relative shares of each sector in the *latent* distribution. They describe the unconditional probability of non-compliance, "clustering" at the minimum wage level and unemployment at a given

FIGURE 3.— Dual-economy Model: Latent and Observed Densities



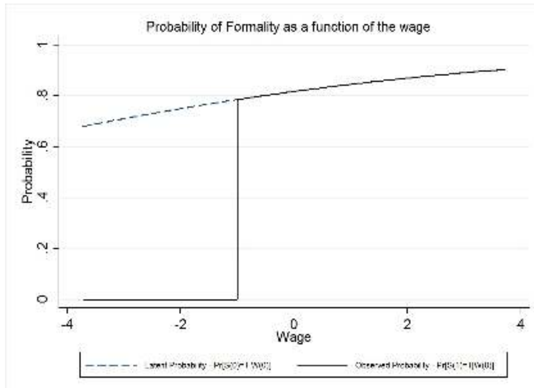
value of the wage. These are the parameters estimated in the previous approach employed by Meyer and Wise (1983) and Doyle (2006).[23]

Examining the sector-specific wage density, one can see that for the formal sector, we have: $f(w|S(1)=1) = \mathbb{I}\{w<m\}0 + \mathbb{I}\{w=m\}\frac{\pi_m^{(1)}F_0(m|S(0)=1)}{c^{(1)}} + \mathbb{I}\{w>m\}\frac{f_0(w|S(0)=1)}{c^{(1)}}$. For the informal sector, we have $f(w|S(1) = 0) = \mathbb{I}\{w<m\}\frac{\pi_d(w)f_0(w|S(0)=0)}{(1-\Lambda(w))c^{(0)}} + \mathbb{I}\{w=m\}\frac{\pi_m^{(0)}F_0(m|s(0)=0)}{c^{(0)}} + \mathbb{I}\{w>m\}\frac{f_0(w|S(0)=0)}{c^{(0)}}$, where I define $c^{(1)} \equiv 1 - F_0(w|S(0) = 1)(1-\pi_m^{(1)})$ and $c^{(0)} \equiv 1 + \pi_d^{(1)}\int^m \frac{\Lambda(w)}{1-\Lambda(w)} f_0(w|S(0)=0)dw$ so that both densities integrate to one. They have the interpretation of the ratio of employment observed in the sector to that in the absence of the policy. Figures 1 and 3 display the relationship between the latent and observed densities for the aggregate wage distribution, for the formal sector, and for the informal sector, respectively.

The dual-economy model preserves the same relationship between the latent and observed unconditional wage densities as in Meyer and Wise's model. However, the dual-economy model presents heterogeneity in the responses to the minimum wage across sectors. The formal sector wage density below the minimum wage vanishes, whereas in the informal sector, the density grows according to the inflow of workers from the formal sector. As a result, the density in the informal sector below the minimum wage can, for some values of the model parameters, present a discontinuity at

[23]Note that, here, they are allowed to be functions of w as long as the latent sizes of the sectors differ across wages and the model parameters differ across sectors.

FIGURE 4.— Dual-economy Model: Latent and Observed Conditional Probabilities



the minimum wage with the "inverse" shape relative to that observed in the aggregate wage distribution.

Regarding the conditional probability of working in the formal sector as a function of the wage, we have: $Pr[S(1) = 1|W(1) = w] = \mathbb{1}\{w < m\}0 + \mathbb{1}\{w = m\}\frac{\pi_m^{(1)} \int^m f_0(w)\Lambda(w)dw}{\int^m \pi_m(w)f_0(w)dw} + \mathbb{1}\{w > m\}\Lambda(w)$. Figure 4 graphically displays the relationship between the latent and the observed probabilities of formality with respect to the wage. The model offers a sharp prediction concerning the effect of the minimum wage on the conditional probability of the sector given the wage. It states that for values above the minimum wage, this probability is equal to the latent probability ($Pr[S(0) = 1|W(0) > m] = Pr[S(1) = 1|W(1) > m]$). It states that the probability of working in the formal sector given the wage will be zero for values below the minimum wage. At the minimum wage level, it should be a particular constant ($\frac{\pi_m^{(1)} \int^m \Lambda(w)f_0(w)dw}{\int^m \pi_m(w)f_0(w)dw}$), which is likely different from this function's left and right limits. This result follows from the fact that workers are not able to maintain wages below m in the formal sector and the assumption of the absence of spillovers above the minimum wage level.

It is helpful to understand the implications of the model using limiting cases for the parameter values. For example, if $\pi_d^{(1)}$ tends to zero, there is no mobility between sectors. At the other extreme, when $\pi_d^{(1)}$ tends to one, all affected workers in the formal sector manage to find a job in the informal sector, which also implies no employment effects from the minimum wage. In general, the employment effect will be given by $\pi_u^{(1)} F_0(m|S(0) = 1) Pr[S(0) = 1]$, which means that the employment loss will be higher when the mass of workers for whom the minimum wage "bites" is larger, and when the size of the formal sector is larger. In terms of market structures that could generate these values, $\pi_d^{(1)}$ tends to one if the economy can be described by a simple two-sector model with imperfect compliance with the minimum wage and costless sector mobility. $\pi_m^{(1)}$ tends to be higher if the economy primarily consists of employers with monopsonistic power in the labor market, and $\pi_u^{(1)}$ tends to be higher if the labor market operates close to perfect competition and mobility to the informal sector is limited.

4. IDENTIFICATION

It is not possible to directly use the techniques developed in Doyle (2006) in each sector separately, as I have introduced movements between them. To identify the model, a different approach must be used. Below, I state the main identification results of this paper, which concern the identification of (a) the latent *joint* distribution of sector and wages; that is, the distribution that would prevail in the absence of the minimum wage; (b) the vector of parameters π that governs how the minimum wage affects the economy; (c) the effects of the minimum wage on functionals of the distribution of sector and wages; and (d) the effects of the minimum wage on labor tax revenues. In what follows, assume that the econometrician observes a random sample of the pair $\{(W_i(1), S_i(1))\}$ of size N from a population of interest. I also assume the following easily verifiable technical conditions: the minimum wage m is set at a point with non-zero density, that is, $f_0(m) > 0$, $Pr[W(1) < m] > 0$, and $\Lambda'(m;\beta) \neq 0$.

LEMMA 4.1 (***Identification of sector-specific parameters***) Under Assumptions 1, 2, 3, and 4, π is identified. Proof: See Appendix A.

LEMMA 4.2 (***Identification of latent distributions***) Under Assumptions 1, 2, 3 and 4, the latent joint distribution of sector and wages is identified. Proof: See Appendix A.

Corollary 4.3 (***Identification of the minimum wage treatment effects***)

Under Assumptions 1, 2, 3 and 4, the effects of the minimum wage on functionals of the joint distribution of sector and wages are identified. Examples of such functionals are the effects of the minimum wage on average wages, on the standard deviation of wages, on quantiles of the wage distribution, on the size of the formal and informal sectors and on the average wages conditional on sectors.

Corollary 4.4 (***Identification of the minimum wage effects on labor tax revenues***)

Under Assumptions 1, 2, 3, and 4 and assuming no tax revenues from the informal sector, the effects of the minimum wage on labor tax revenues are identified. Identification of the effects of the minimum wage on labor tax revenues holds as long as the effects can be written as a functional of the latent and observed wage distributions and the model parameters π.[24]

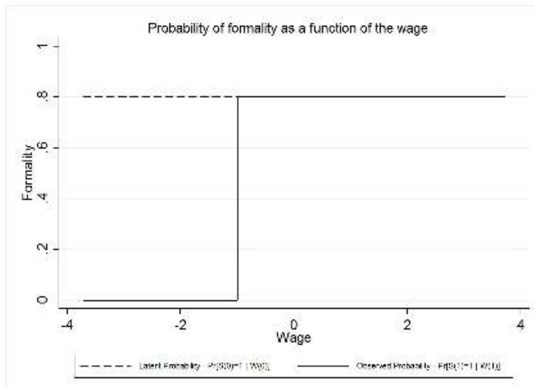
The key points that permit the identification are as follows: The shape of the relationship between sector and wages for values above the minimum wage is preserved in the presence of the policy. This allows us to obtain estimates of the latent share of the formal sector for values below the minimum wage level by extrapolating the curve we observe in the upper part of the wage distribution.[25] The identification of the latent wage density builds on the approach in Doyle (2006) in the sense that the probability

[24]See Section 6.3 for further discussion of this issue.

[25]The relationship between latent sector and wages can only be observed for values above m. If this function is specified non-parametrically, the latent share of formal workers for values below m would essentially be unidentified. However, by relying on the parametric functional form, I can extrapolate the relationship observed above m to predict the latent share of workers that would prevail below the minimum wage in the absence of the policy. This is achieved by estimating the parameters of the function $\Lambda(w)$ using wages above m and then using the estimated parameters for the prediction for *all* wages, both above and below m.

of non-compliance with the policy is identified using the ratio of the density of wages above and below the minimum wage level. To complete the identification, the sector-specific parameters are identified using the derivative of the wage density.[26]

Figure 5.— Dual Economy Model: Latent and Observed Conditional Probabilities under Independence



5. ESTIMATION

In this section, I discuss how to estimate the model parameters and latent distributions using non-parametric kernel methods. The non-parametric estimation strategy used here is local linear density estimators.

As in Doyle (2006), the model can also be estimated without assuming that the latent wage distribution belongs to a known parametric family. A crucial step in obtaining non-parametric estimates of the objects of interest, such as the model parameters and the counterfactual distributions, involves the estimation of a ratio of one-sided limits of the density at the minimum wage. The estimation of these quantities can be performed using non-parametric methods. Note that because the density is discontinuous around the minimum wage, only observations below the minimum are informative for $lim_{w \to m^-} f(w)$ (and similarly for the density above the minimum).

[26]If latent sector and wages are independent, one need not resort to the derivative of the wage density at m. In this case, identification of the sector-specific parameters can be achieved by examining the distribution of wages given the sector. See Appendix B for a detailed discussion of this issue.

This implies that the estimators of these quantities will behave as if the minimum wage were a boundary point of the density, which has implications in terms of bias and variance.

Therefore, it is advisable to use methods ensuring that the performance of the density estimator is satisfactory on points that are close to the support boundaries. I use local linear density estimators, which have the same order of bias at the boundary as at interior points of the distribution.[27] In Appendix C, I formally describe how to non-parametrically perform the density estimation.

For the remaining terms that need to be estimated, I will use the plug-in approach and replace the unknown objects in the identification section with their consistent estimators. Thus $\widehat{\pi}_d(m) \equiv \frac{\widehat{f}(m^-)}{\widehat{f}(m^+)}$, where $\widehat{f}(m^-)$ is the estimator of the density just below the minimum wage value using the local linear density estimator. In addition, for the estimator of $\pi_d'(m)$, we can define $\widehat{\pi}_d{}'(m) \equiv \left(\frac{\widehat{f}'(m^-)}{\widehat{f}'(m^+)} - \frac{\widehat{f}(m^-)}{\widehat{f}(m^+)}\right) \frac{\widehat{f}'(m^+)}{\widehat{f}(m^+)}$.

To complete the process of recovering the structural parameters π, one requires estimates of $\Lambda(m)$ and $\Lambda'(m)$. These objects are the latent share of the formal sector and the change in it at m. For that purpose, one needs to estimate β. Given Assumptions 2 and 3, the estimator can be defined as: $\widehat{\beta} \equiv \arg\min_\beta \sum_{i=1}^N (S_i - \Lambda(W_i;\beta))^2 \mathbb{I}\{W_i > m\}$.

Then, given that we estimated β, we can plug it into the function $\Lambda(.)$ to obtain an estimate of $\Lambda(m)$ and $\Lambda'(m)$. They will be given by $\widehat{\Lambda}(m) = \Lambda(m;\widehat{\beta})$ and $\widehat{\Lambda}'(m) = \Lambda'(m;\widehat{\beta})$. Using the estimate $\widehat{\Lambda}(m)$ of the latent share of the formal sector, we can define the plug-in estimator for the parameters $\pi_d^{(1)}$ and $\pi_d^{(0)}$: $\widehat{\pi}_d^{(0)} \equiv \widehat{\pi}_d(m) - \frac{\widehat{\Lambda}(m)}{\widehat{\Lambda}'(m)}\widehat{\pi}_d'(m)$, and $\widehat{\pi}_d^{(1)} \equiv [\widehat{\pi}_d(m) - (1 - \widehat{\Lambda}(m)) \cdot \widehat{\pi}_d^{(0)}] \cdot \widehat{\Lambda}(m)^{-1}$. To complete the estimation, we first need to estimate c before we recover the latent wage density: $\widehat{c} \equiv [\int^m \frac{\widehat{f}(u)}{\widehat{\pi}_d(u)} du + 1 - \widehat{F}(m)]^{-1}$. Then, the estimates of the latent wage

[27] This estimator builds on the idea of local linear conditional mean estimators. It begins by dividing the support of the density into a set of bins. Then, a "response variable" is defined as the bin counts of these disjoint intervals. After this process, one is left with a vector containing the "independent variable," which are the bin centers, and a corresponding "dependent variable," the bin counts. Finally, standard local polynomial smoothing estimates are applied to these constructed variables. See McCrary (2008) for a detailed discussion of this issue.

TABLE I

DESCRIPTIVE STATISTICS

Variable	Obs	Mean	Std. Dev.	Min	Max
Wage	579252	791.13	1233.28	1	350000
Female	579252	0.35	0.48	0	1
White	579252	0.56	0.50	0	1
Education	579252	9.88	3.87	1	17
Tenure	579252	4.24	5.65	0	53
Age	579252	32.97	10.08	19	59
Formal Sector	579252	0.74	0.44	0	1
Minimum Wage	579252	320.36	93.12	180	465

distribution can be defined, for $w \neq m$, as: $\widehat{f}_0(w) = \mathbb{I}\{w < m\}\frac{\widehat{f}(w)\widehat{c}}{\widehat{\pi}_d(w)} + \mathbb{I}\{w > m\}\frac{\widehat{f}(w)}{\widehat{c}}$.

The consistency of the $\widehat{\pi}$, $\widehat{\beta}$ and, consequently, $\widehat{\Lambda}(w)$ and $\widehat{f}(w)$ follows directly from the identification equations and the consistency of $\widehat{f}(w)$ and $\widehat{f'}(w)$. Closed-form expressions for the asymptotic variances can be derived. However, I will rely on resampling methods to estimate them in the empirical application.

6. EMPIRICAL APPLICATION: THE EFFECT OF THE MINIMUM WAGE IN BRAZIL

For my empirical application, I consider a stronger version of Assumption 2:

ASSUMPTION 5 *Independence:* $Pr[S(0) = 1|W(0) = w] = \Lambda \quad \forall \quad w.$

This assumption implies that *latent* sector and wages are independent. Figure 5 displays the relationship between the latent and observed conditional probabilities of formality with respect to the wages under this assumption. This assumption is testable. Below, I provide evidence that it is not violated in the context of the Brazilian labor market.

Independence greatly simplifies the identification and estimation, as can be seen in Appendix B. Independence (and the absence of spillovers) allows me to identify the latent share of the formal sector by examining the observed share of the formal sector for wages that are above the minimum wage level. Moreover, it implies that the aggregate minimum wage probabilities ($\pi_d(w)$,$\pi_m(w)$,$\pi_u(w)$) do not vary across wages even if the parameters differ across sectors. This is because the latent share of each sector becomes constant with respect to wages. This allows me to identify

TABLE II

Descriptive Statistics by Sector

	Formal Sector	Informal Sector	Difference
Wage	874.599	556.114	318.485***
	(1.716)	(3.755)	(3.661)
Female	0.359	0.340	0.019***
	(0.001)	(0.001)	(0.001)
White	0.583	0.482	0.101***
	(0.001)	(0.001)	(0.001)
Education	10.246	8.840	1.406***
	(0.006)	(0.010)	(0.011)
Tenure	4.639	3.134	1.505***
	(0.009)	(0.013)	(0.017)
Age	33.312	32.003	1.309***
	(0.015)	(0.026)	(0.030)
Minimum wage worker	0.103	0.151	-0.047***
	(0.000)	(0.001)	(0.001)
N	420,097	159,155	-

the model parameters by simply examining the discontinuity in the aggregate wage distribution at m and the sector-specific wage distributions, so I will not need to rely on estimating the first derivative of the wage distribution at the boundary point, m.

In Appendix B, I describe how to identify the model under this condition. The estimation strategy I use follows the same method as in the general form of the model. That is, I estimate the density of wages at the boundary using local linear density estimators and use a plug-in method for the remaining objects. Namely, once I estimate the lateral limits of the density of wages at m, I complete the estimation by replacing the objects in the identifying equations using their respective sample counterparts. In the next sections, I describe the data and discuss the results obtained when estimating this model for the Brazilian labor market.

6.1. *Data and Descriptive Statistics*

To evaluate the effects of the minimum wage on labor market outcomes, I used data for the period from 2001 to 2009 from the PNAD dataset. These data have been collected by the IBGE – which is a Portuguese acronym for "Brazilian Institute of Geography and Statistics" – since 1967 and contain information on income, education, labor force participation, migration, health and other socioeconomic characteristics of the Brazilian population. Workers who do not report wages, those who work in

TABLE III

DESCRIPTIVE STATISTICS: CONDITIONAL PROBABILITIES

	$Pr[S=1]$	$Pr[S=0]$	$Pr[W=m]$	$Pr[W<m]$
Unconditional	0.738	0.262	0.116	0.073
Conditional on sector				
Formal	1.000	0.000	0.103	0.006
Informal	0.000	1.000	0.151	0.265
Conditional on wage				
$W<m$	0.056	0.944	0.000	1.000
$W=m$	0.659	0.341	1.000	0.000
$W>m$	0.811	0.189	0.000	0.000
Conditional on gender				
Male	0.732	0.268	0.099	0.066
Female	0.748	0.252	0.147	0.088
Conditional on race				
White	0.773	0.227	0.082	0.047
Non-white	0.694	0.306	0.157	0.107
Conditional on education				
Less than 5 years	0.576	0.424	0.180	0.179
Less than 12 years	0.672	0.328	0.143	0.108
More than 12 years	0.797	0.203	0.041	0.022
Conditional on region				
South	0.803	0.197	0.062	0.040
Southeast	0.778	0.222	0.079	0.041
Center-West	0.706	0.294	0.129	0.050
North	0.623	0.377	0.182	0.106
Northeast	0.608	0.392	0.246	0.198

Note: $N = 579{,}252$

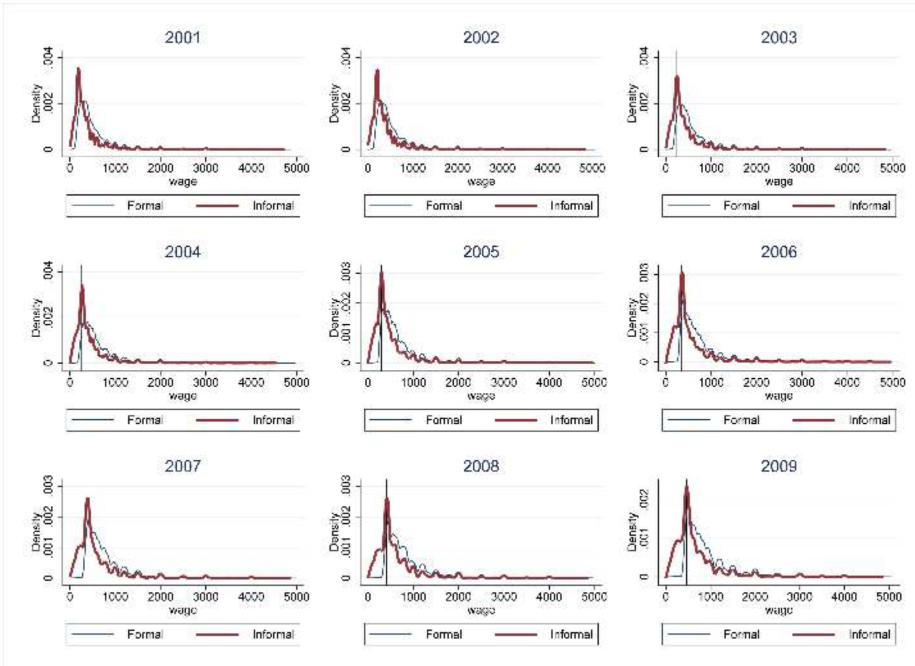
the public sector and workers who are older than 60 years of age or younger than 18 years of age were removed from the sample. The PNAD dataset includes information on the worker's labor contract status, which was used to define formality.

The variable of interest – the wage – is measured at the monthly level, which is the most natural unit in the Brazilian institutional context. A feature of the Brazilian labor market is that wages are typically specified at the monthly level, the same unit of measure as the minimum wage. The labor contract also establishes the number of hours of work per day (typically 6 or 8 hours).[28] I will treat the wage reported in the survey as the contracted wage, so no adjustment for hours need to be performed. As a result, wages below the minimum wage are not, in principle, a result of a "division bias".

The empirical strategy will assume also that the wage is measured without error. This is unquestionably a strong assumption. The observed wage distribution presents

[28] At the end of the month the worker will receive a payment "pro rata" based on the actual number of days he or she worked. This payment will present some small variation across months due to reasons such as holidays, absences, overtime pay and the like.

FIGURE 6.— Wage Densities



Note: Local linear density estimates using Silverman's rule of thumb bandwidth.

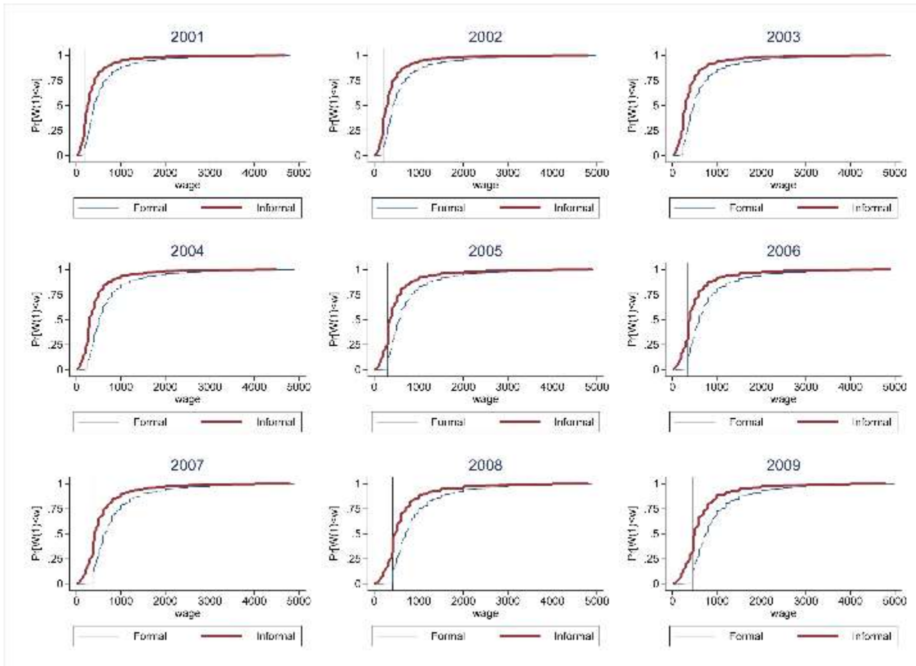
heaping at round numbers. I will show that the estimates of the parameters of the model are fairly robust to the presence of heaping by using different values of the bandwidth in the density estimation.

As mentioned above, all workers in Brazil carry an official document called "Carteira de Trabalho" (worker's card). This document is signed by the employers in the formal act of hiring. The lack of a formal signed labor contract means that the employer is not forced to collect labor taxes or to comply with the minimum wage and other types of regulation. The Brazilian economy is known to be characterized by a large informal sector. Tables I, II, and III illustrate this fact and describe the main features of the data.[29]

Figure 7 displays the empirical CDFs of the formal and informal sectors. A few interesting facts can be noted: The empirical cumulative distribution of wages seems to have a spike at the minimum wage level in *both* sectors, and virtually no worker

[29] All estimates are computed using survey weights.

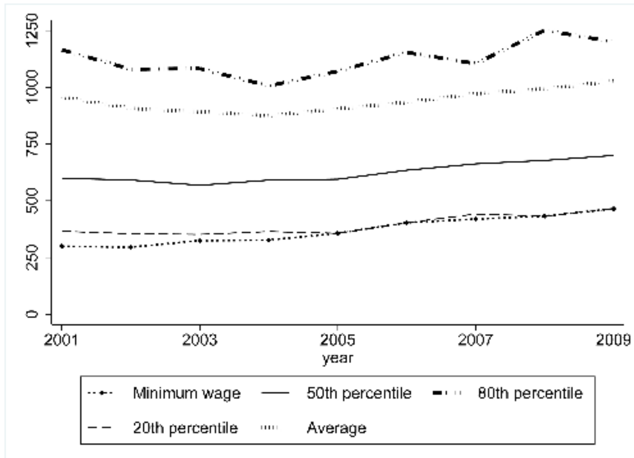
FIGURE 7.— Empirical CDFs



in the formal sector receives wages below m. The same pattern appears on Figure 6, where I display the estimates of the wage density of the formal and informal sectors. Thus, informality is closely related to sub-minimum wages. However, these concepts are not equivalent, as a sizable fraction of informal workers earn wages above the minimum wage level.

Table II shows that workers in the informal sector earn on average approximately 36% less than workers in the formal sector. In addition, in terms of the observable characteristics, workers in the informal sector are more likely to be male, non-white, less educated and young. Considering the likelihood of earning minimum and sub-minimum wages, Table III shows the heterogeneity of these probabilities across population subgroups. For example, white workers are 48% less likely to earn the minimum wage than are non-white workers. Workers with less than 5 years of education have an approximately 18% likelihood of earning the minimum wage, whereas the corresponding likelihood is only 4% for workers with more than 12 years of education. Regarding

FIGURE 8.— Real Wages and Minimum Wage Evolution



the geographic variation, workers in the South Region have a 6% probability of earning the minimum wage, whereas workers in the Northeast have an approximately 25% probability of earning the minimum wage. A similar heterogeneity pattern appears when we consider the probability of earning sub-minimum wages.

Table III shows that formality presents considerable heterogeneity across observable characteristics. It shows that the probability of formality is close to zero for workers with wages below the minimum wage, as predicted by the dual-economy model. Also, it shows that the probability of working in the formal sector is lower for low education groups, non-white, and in the North and Northeast regions.

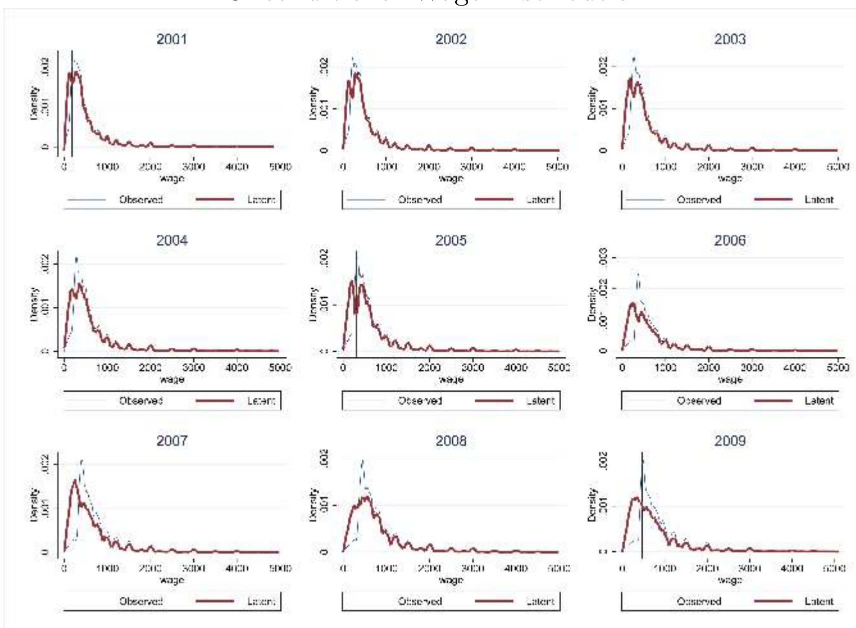
The history of the minimum wage in Brazil began during the Getulio Vargas government, on May 1st, 1940. Initially, the minimum wage varied across regions to accommodate differences in price levels across the country. Subsequently, in 1984, regional minimum wages were unified into a single wage at the national level.[30] The periodicity of changes in the minimum wage has been annual since the economy stabilized in 1994 (Lemos, 2009). Figure 8 depicts the evolution of average wages, minimum wage, and different quantiles of the wage distribution over the last decade.[31]

Regarding Figure 8, the challenge of relying on time-series variation to identify the

[30]The Constitution of 1988 prohibited the use of the minimum wage as a reference for wage bargaining for other categories of workers and contracts. The aim of this prohibition was to reduce

Figure 9.— Kernel Density Estimates

Unconditional Wage Distribution



effects of the minimum wage becomes clear, as there is nearly as much evidence in favor of the minimum wage affecting the 20th percentile as there is of it affecting the 80th percentile.[32] The correlation between minimum wage changes and changes in such high percentiles of the wage distribution is likely a reflection of the pro-cyclical nature of changes to the minimum wage. Given this, effects of the minimum wage on other objects such as average wages or lower quantiles that are based on time-series variation should also be interpreted with caution.

6.2. *Main Results*

In this section, I will discuss the results obtained after estimating the model for the Brazilian labor market. The model is estimated (separately) for the years 2001 to

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the over-indexation of the economy, which was thought to be fueling inflation.

[31]Real wages displayed in Figure 8 were computed using the IPCA, a Portuguese acronym for "Nation-wide consumer price index". IPCA is the consumer price index used by the Central Bank in its inflation target system.

[32]A similar point was made by Lee (1999) when analyzing U.S. data.

TABLE IV

MODEL PARAMETER ESTIMATES BY YEAR

	2001	2002	2003	2004	2005	2006	2007	2008	2009
π_d	0.202***	0.217***	0.206***	0.232***	0.180***	0.157***	0.113***	0.192***	0.121***
	(0.006)	(0.009)	(0.005)	(0.006)	(0.006)	(0.004)	(0.002)	(0.005)	(0.003)
π_m	0.256***	0.356***	0.289***	0.293***	0.349***	0.262***	0.176***	0.304***	0.208***
	(0.007)	(0.015)	(0.007)	(0.008)	(0.011)	(0.006)	(0.004)	(0.007)	(0.005)
$F_0(m)$	0.253***	0.260***	0.311***	0.291***	0.328***	0.400***	0.446***	0.345***	0.434***
	(0.005)	(0.008)	(0.004)	(0.005)	(0.007)	(0.005)	(0.004)	(0.004)	(0.005)
$\pi_d^{(1)}$	0.106***	0.222***	0.139***	0.179***	0.165***	0.077***	0.000	0.131***	0.023***
	(0.013)	(0.021)	(0.010)	(0.012)	(0.012)	(0.007)	(0.000)	(0.008)	(0.005)
$\pi_m^{(1)}$	0.191***	0.231***	0.223***	0.226***	0.258***	0.213***	0.137***	0.264***	0.186***
	(0.006)	(0.011)	(0.006)	(0.006)	(0.009)	(0.005)	(0.007)	(0.006)	(0.004)
$\pi_d^{(0)}$	0.525***	0.199***	0.461***	0.443***	0.244***	0.516***	0.640***	0.494***	0.669***
	(0.018)	(0.038)	(0.017)	(0.019)	(0.026)	(0.013)	(0.015)	(0.016)	(0.011)
$\pi_m^{(0)}$	0.475***	0.801***	0.539***	0.557***	0.756***	0.484***	0.360***	0.506***	0.331***
	(0.018)	(0.038)	(0.017)	(0.019)	(0.026)	(0.013)	(0.015)	(0.016)	(0.011)
π_u	0.543***	0.427***	0.506***	0.475***	0.471***	0.581***	0.711***	0.504***	0.670***
	(0.012)	(0.024)	(0.011)	(0.013)	(0.016)	(0.010)	(0.005)	(0.010)	(0.007)
A	0.772***	0.781***	0.792***	0.798***	0.816***	0.819***	0.823***	0.832***	0.847***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
m	180	200	240	260	300	350	380	415	465
N	55,487	58,223	58,265	62,576	65,745	68,193	68,315	71,051	71,397

Note: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Standard errors (in parentheses) computed by 100 bootstrap replications.

2009. As discussed in the estimation section, all objects in the model can be estimated by replacing the population object with its sample analog. The only exception to this is the density of wages at the boundary. To estimate this object, I use a local linear kernel estimator with a normal kernel and a bandwidth equal to eight times Silverman's rule of thumb, which has been shown to be mean squared error optimal in Monte Carlo simulations. In the robustness section, I show that the estimates are not sensitive to this choice by using McCrary's automatic bandwidth selection rule.

Figure 9 shows a plot of the observed density of wages and its latent counterpart. We can see that, as a consequence of sizable unemployment effects, the observed density above the minimum wage is higher than the latent density. Due to both truncation at the minimum and unemployment, the observed density below the minimum wage is smaller than the latent density. The estimates of the model parameters used to construct this latent density are shown in Table IV.

In examining the point estimates and standard errors in Table IV, we see sizable estimates of the unemployment effects of the minimum wage. This result is comparable to the estimates of π_u obtained in other applications of this approach. Doyle, for example, found that approximately 60% of young workers who would earn below the

TABLE V

DISTRIBUTIONAL EFFECTS OF THE MINIMUM WAGE

	2001	2002	2003	2004	2005	2006	2007	2008	2009
E[log(w)]									
Observed	5.994***	6.054***	6.152***	6.220***	6.322***	6.400***	6.489***	6.578***	6.658***
	(0.004)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Latent	5.793***	5.874***	5.926***	6.020***	6.090***	6.093***	6.097***	6.363***	6.310***
	(0.009)	(0.014)	(0.008)	(0.009)	(0.012)	(0.010)	(0.010)	(0.008)	(0.008)
Minimum wage effect	0.201***	0.180***	0.225***	0.200***	0.231***	0.307***	0.392***	0.215***	0.348***
	(0.008)	(0.012)	(0.007)	(0.008)	(0.011)	(0.009)	(0.009)	(0.007)	(0.007)
Sd[log(w)]									
Observed	0.769***	0.774***	0.753***	0.737***	0.719***	0.699***	0.693***	0.684***	0.661***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.004)	(0.003)	(0.003)
Latent	0.916***	0.929***	0.916***	0.885***	0.901***	0.881***	0.897***	0.846***	0.851***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.007)	(0.006)	(0.009)	(0.006)	(0.006)
Minimum wage effect	-0.147***	-0.155***	-0.163***	-0.149***	-0.182***	-0.182***	-0.204***	-0.162***	-0.190***
	(0.004)	(0.005)	(0.003)	(0.004)	(0.005)	(0.004)	(0.006)	(0.004)	(0.004)
q^{80} *(log(w))*-q^{20} *(log(w))*									
Observed	1.157***	1.112***	1.124***	1.015***	1.099***	1.050***	0.916***	1.062***	0.948***
	(0.003)	(0.020)	(0.012)	(0.016)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)
Latent	1.419***	1.476***	1.267***	1.386***	1.386***	1.386***	1.447***	1.204***	1.204***
	(0.022)	(0.045)	(0.019)	(0.000)	(0.012)	(0.021)	(0.015)	(0.010)	(0.012)
Minimum wage effect	-0.262***	-0.364***	-0.143***	-0.372***	-0.288***	-0.336***	-0.531***	-0.142***	-0.256***
	(0.022)	(0.052)	(0.022)	(0.016)	(0.012)	(0.021)	(0.015)	(0.010)	(0.012)
Gini									
Observed	0.069***	0.069***	0.065***	0.063***	0.061***	0.058***	0.056***	0.055***	0.052***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Latent	0.087***	0.087***	0.085***	0.081***	0.081***	0.080***	0.080***	0.072***	0.074***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
Minimum wage effect	-0.018***	-0.018***	-0.019***	-0.017***	-0.021***	-0.022***	-0.024***	-0.017***	-0.022***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
N	55,487	58,223	58,265	62,576	65,745	68,193	68,315	71,051	71,397

Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

minimum became unemployed. A possible explanation for this regularity is that the reduced-form and panel data approach, such as the work of Card and Krueger (1994), estimates the effects of *marginal* changes in the minimum wage, whereas I estimate the effect of the minimum wage when compared to the counterfactual scenario defined as the absence of it. High unemployment probabilities can generate a small marginal effect of the minimum wage, depending on the size of the density around the minimum wage and the magnitude of the change in the minimum wage. [33]

The evidence from Table IV also suggests that sector mobility is limited. The estimates of the sector-mobility parameter ($\pi_d^{(1)}$) are approximately 12%, with a maximum of 22%. I discuss in greater detail the implications of this result in Section 6.3.

As seen in Figure 8, the period of 2001 to 2009 is characterized by an increase in the real value of the minimum wage. We should expect the estimates of the mass of

[33]Jales (2015) discusses the predictions of this Dual-economy model for marginal changes in the minimum wage under a parametric assumption for the latent distribution of wages.

TABLE VI

The Geographic Heterogeneity of Minimum Wage Effects

	Southeast	Northeast
π_d	0.142***	0.328***
	(0.007)	(0.011)
π_m	0.204***	0.345***
	(0.009)	(0.012)
$F_0(m)$	0.267***	0.532***
	(0.006)	(0.007)
$\pi_d^{(1)}$	0.044***	0.264***
	(0.013)	(0.020)
$\pi_m^{(1)}$	0.146***	0.306***
	(0.007)	(0.012)
$\pi_d^{(0)}$	0.552***	0.531***
	(0.025)	(0.021)
$\pi_m^{(0)}$	0.448***	0.469***
	(0.025)	(0.021)
π_u	0.654***	0.327***
	(0.015)	(0.021)
Λ	0.808***	0.760***
	(0.002)	(0.004)
m	260	260
N	22158	14982

Note: Data from [illegible] PNAD. *** p<0.01, ** p<0.05, * p<0.1. Standard errors (in parentheses) computed by [illegible] bootstrap replications

affected workers, $F_0(m)$, to reflect this feature of the data. Regarding Table IV, we observe a close relationship between the minimum wage level and estimates of $F_0(m)$. The correlation coefficient with the minimum wage level is approximately 0.90.[34]

Table V shows how the minimum wage affects the shape of the (log-) wage distribution. Here, I compute the effects of the minimum wage on the usual measures of wage inequality, such as the standard deviation of log wages and the Gini coefficient. The estimates show that the minimum wage has a positive impact on average wages (conditional on employment). The maximum difference is .39 log points in 2007, and the minimum is .18 in 2002. The minimum wage also reduces wage inequality, as measured by differences in quantiles, the standard deviation, or the Gini coefficient. These estimates indicate the trade-off faced by policy makers when choosing the minimum wage level. On the one hand, there is a gain in terms of reducing wage inequality and increasing average wages. On the other hand, workers tend to have more difficulty finding jobs.

The Brazilian economy is characterized by considerable geographic variation in the size of the formal sector, as shown in Table III. The size of the formal sector in the

[34]The results also suggests a high correlation over time between unemployment estimates (π_u) and the minimum wage level.

Southeast region is approximately 0.78, whereas in the Northeast region the size of the formal sector is approximately 0.61. Table VI shows the model parameter estimates separately for the Southeast and Northeast regions. In the Northeast region the minimum wage "bites" at a much higher point in the wage distribution when compared to the Southeast. The latent size of the formal sector in the Southeast is 0.81. In the Northeast region, the latent size is 0.76. These regions also differ in their responses to the (same) minimum wage policy. In the Southeast, we observe a high probability of unemployment (0.65). We also observe a low estimate of the sector mobility parameter (0.04). In the Northeast, we observe a lower probability of unemployment (0.33), higher probability of non-compliance (0.33), and higher probability of moving to the informal sector (.26).[35]

Regarding the informal sector parameters, $\pi_d^{(0)}$ and $\pi_m^{(0)}$, I do not reject the null hypothesis that the coefficients are the same across regions. This suggest that the differences we observe in the joint distribution of sector and wages across these regions come from differences in their latent distributions and differences in the *formal* sector's response to the minimum wage. A decomposition exercise based on the estimates from Table VI show that approximately 72% of the differences in the observed size of the formal sector between the Northeast and the Southeast are a result of the minimum wage. The remaining 28% of the differences in the size of the formal sector across these regions are due to other economic factors that cause the Southeast to have a larger size of the formal sector beyond their differences in the minimum wage effects. This exercise indicates that the minimum wage affects a substantially larger proportion of workers in the Northeast economy, thereby inducing a larger inflow of workers to the informal sector in that region.

[35] The region where the *latent* size of the formal sector is higher also presented a higher likelihood of sector mobility. This result may also suggest that formal and informal sectors operate in most cases in distinct labor markets, in the sense that they are located in different geographic regions or different industries. This could be one explanation for the small estimates of the likelihood of sector mobility found in the aggregate economy.

TABLE VII

Minimum Wage Effects on Labor Tax Revenues

	2001	2002	2003	2004	2005	2006	2007	2008	2009
R	0.966***	0.969***	0.955***	0.956***	0.954***	0.922***	0.893***	0.939***	0.897***
	(0.001)	(0.002)	(0.001)	(0.001)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
Pr[S(1)=1]/Pr[S(0)=1]	0.929***	0.908***	0.909***	0.904***	0.898***	0.897***	0.915***	0.909***	0.916***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
c	0.863***	0.889***	0.843***	0.862***	0.845***	0.767***	0.683***	0.826***	0.709***
	(0.006)	(0.008)	(0.005)	(0.006)	(0.010)	(0.007)	(0.006)	(0.006)	(0.006)
E[W(1) | S(1) = 1]/E[W(0) | S(0) = 1]	1.205***	1.201***	1.246***	1.227***	1.257***	1.340***	1.429***	1.251***	1.380***
	(0.007)	(0.009)	(0.006)	(0.007)	(0.011)	(0.008)	(0.009)	(0.006)	(0.009)
N	55,487	58,223	58.265	62,576	65,745	68,193	68.315	71,051	71,397

Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

6.3. *Tax Revenues and the Size of the Informal Sector*

A comparison of Tables I and IV shows that the minimum wage reduces the share of the formal sector in the economy. This occurs through two different but related channels: First, the minimum wage reduces the size of the formal sector as long as the unemployment effects are greater than zero, as has been found in Brazil. Second, the minimum wage increases the size of the informal sector through sector movements that are driven by the policy itself. Overall, the share of the formal sector in the Brazilian economy is reduced by approximately 9% as a result of the minimum wage policy.[36]

For this reason, the minimum wage indirectly affects the government budget.[37] The minimum wage affects the shape of wage distribution, the relative size of the formal sector and the likelihood of employment. Each of these effects has the potential to alter tax revenues.

The goal of this section is to derive an estimate of these effects. I consider the effects on revenues from the INSS tax, which is the Brazilian labor tax. The INSS is collected to fund the social insurance system in Brazil, and the rate is 20% for companies included in the regular system of taxation and 12% for small companies

[36] My estimates imply that the mass of workers at and below the minimum wage level is inconsistent with absence of disemployment effects under smooth non-compliance probabilities and a continuous latent distribution of wages. The "missing" mass of workers at or below the minimum wage level is attributed in the model to unemployment effects of the policy. Similarly, high sector-mobility probabilities ($\pi_d^{(1)}$) are inconsistent with my estimate of the latent share of the formal sector and the density of low-wages in the informal sector. That is, we do not observe enough small wages in the informal sector to justify larger sector mobility parameter estimates.

[37] Here, I use the term "indirectly" because the minimum wage affects the government's budget through the spending channel. This is due to the indexation of pensions to the minimum wage.

that opt for the "simplified" system. To estimate the effects, I will rely on the following assumption:

ASSUMPTION 6 *No Tax Revenues in the Informal Sector*

Let $T(1)$ represent the tax revenues in the formal sector under the imposition of a minimum wage and $T(0)$ in its absence.[38] By definition, we have $T(1) \equiv \sum_{i=1}^{N} \tau(W_i(1))W_i(1)S_i(1)$, and $T(0) \equiv \sum_{i=1}^{N} \tau(W_i(0))W_i(0)S_i(0)$. The object of interest is the ratio between these two quantities. After some algebra, we have $R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1)=1]}{Pr[S(0)=1]} \cdot c \cdot \frac{E[\tau(W(1))W(1)|S(1)=1]}{E[\tau(W(0))W(0)|S(0)=1]}$. This expression is further simplified in the Brazilian case, where labor taxes are a constant fraction of wages. In this case, R is given by: $R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1)=1]}{Pr[S(0)=1]} \cdot c \cdot \frac{E[W(1)|S(1)=1]}{E[W(0)|S(0)=1]}$. Thus, the effects on tax revenues can be decomposed into three components: compression of the formal sector, employment effects, and change in expected wages in the formal sector.[39] This equation shows that the tax effect of the minimum wage will depend on the relative magnitude of these effects.[40]

I compute the tax effects of the minimum wage using a plug-in approach for the components of the equation above based on the model parameter estimates from Table IV. Table VII displays the estimated effects. The minimum wage policy seems to generate sizable unemployment effects and to reduce the size of the formal sector. These effects are large enough to compensate for the increase in expected wages. Therefore, the minimum wage reduces the mass of wages in the formal sector, with a corresponding decline in labor tax revenues. The estimates range from 3% in 2001 to 11% in 2007.[41]

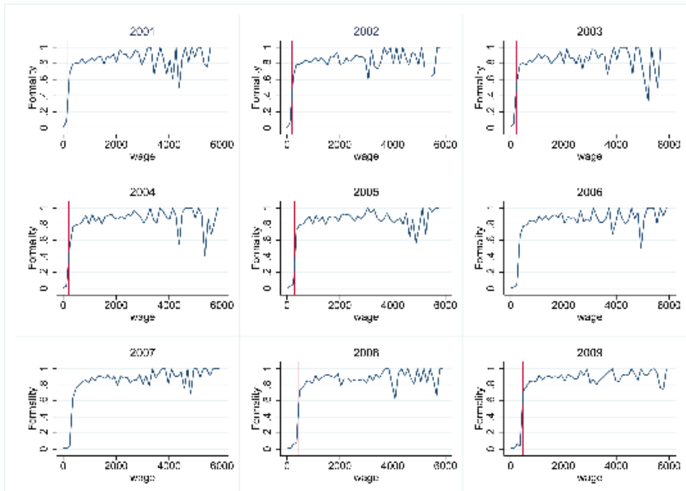
[38]Note that I abuse notation here and use N to refer to the size of the population, not the size of the sample.

[39]The expression for R, the effect of the minimum wage on labor tax revenues, relies exclusively on Assumption 6. It does not rely on the particular assumptions I used for the dual economy-model.

[40]Note that the parameter R also answers a related question: Is the mass of wages, the sum of the wages of all workers in the formal sector, higher under the minimum wage or in its absence? Because the tax rate τ is a constant function of the wages, the effects on tax revenues are proportional to the effects on the mass of wages.

[41]As a sensitivity test, fixing all other parameters, the employment effect of the minimum wage

FIGURE 10.— Formality vs. Wages



Note: Conditional probabilities estimates based on a local-constant estimator using an Epanechnikov kernel and the standard "rule of thumb" bandwidth.

6.4. *Testing the Underlying Assumptions and Robustness Checks*

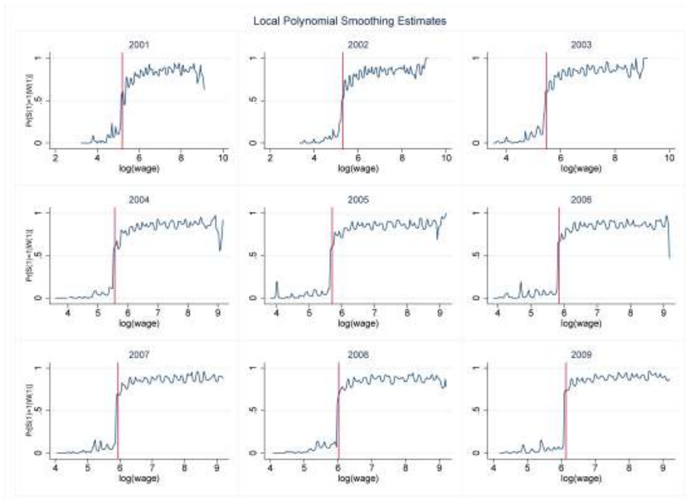
This research design allows me to indirectly test some of model assumptions.[42] First, I will indirectly test Assumption 5, the independence between latent sector and wage. This assumption is testable in different ways. One way to test it is to consider the proportion of workers in each sector as a function of the wage. If the assumption holds, this proportion should not vary with the wage for wage values that are above the minimum.[43] A naive regression of formality on wages should mechanically detect a negative relationship because no worker in the formal sector can earn below the minimum wage. However, after restricting our attention to wage values above the minimum, the relationship should disappear. Another related way to test the assumption is to examine the CDF conditional on wages above the minimum. If the model is correct, differences in the observed wage distributions across sectors at values above

in 2009 needs to be approximately 28% smaller than my estimates for the minimum wage to have no effect on labor taxes. Similarly, the minimum wage effect on average wages needs to be underestimated by at least 29.6% for the minimum wage to have no impact on labor tax revenues. This suggests that the model needs to be severely misspecified for the estimates of the *direction* of the effect to be wrong.

[42] I discuss the theory behind the tests performed in this section in Appendix D.

[43] See Figure 5.

Figure 11.— Formality vs. Log-wages



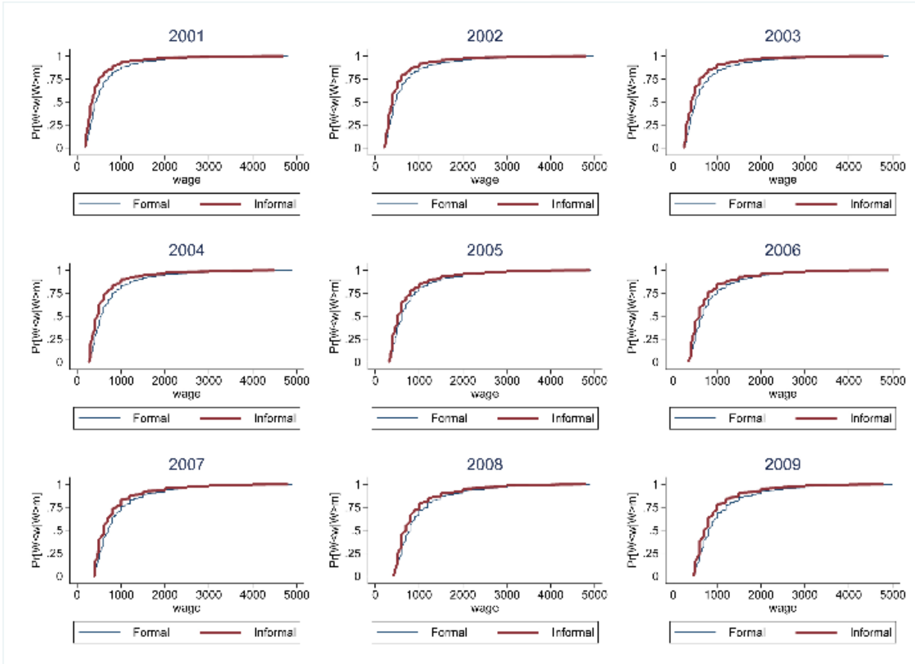
Note: Conditional probability estimates based on a local-constant estimator using an Epanechnikov kernel. Bandwidth: 0.03.

the minimum will only be due to rescaling induced by unemployment and sector mobility. Thus, by conditioning on values above the minimum, the effects of rescaling and sector movements should disappear, and the densities should be approximately the same.

Figures 10, 11, and 12 provide visual evidence of the accuracy of this assumption within the Brazilian context. Above the minimum wage level, the proportion of workers in the formal sector of the economy does not seem to systematically vary with the wage. Figure 11 shows that this is also true when we inspect the relationship between formality and log-wages.[44] This evidence supports the assumption that the underlying latent density of wages should be the same between sectors. The plots of the empirical CDFs in Figure 12 across formal and informal sectors point in the same direction: Workers in the formal and informal sectors apparently draw from similar distributions for wages above the minimum wage. This suggests that the differences between the overall distribution of wages occur as a result of the different ways in which the sectors respond to the minimum wage. Note, however, that the assump-

[44]Under Assumption 5 there should be no relationship between formality and *any* function of the wage above the minimum wage level.

Figure 12.— Empirical CDF by Sector above the Minimum Wage



tion required for identification is that the entire wage distribution be the same across sectors. The presence of the minimum wage prevents me from testing this condition for values below m. Thus, it is still possible that the latent wage distributions are indeed equal conditional on wages above the minimum wage, while this is not the case for values below it. This last part of the identifying assumption is untestable. The evidence that the wage distributions are similar for values above the minimum wage seems to indicate that they may also be so for values below m in the absence of the policy. However, this conclusion is subject to debate.

Table VIII shows the estimates of the elasticity of formality with respect to the wage based on a linear probability model, using different restrictions on the sample. The relationship between sector distribution and wages becomes substantially weaker after one conditions the regression to only consider wages above the minimum. Regarding the coefficient while conditioning on higher values, several estimates that are not different from zero were found.

TABLE VIII

FORMALITY VS. WAGES - LINEAR REGRESSION ESTIMATES

	2001	2002	2003	2004	2005	2006	2007	2008	2009
$W > 0$	1.93***	2.15***	2.77***	2.43***	2.73***	2.63***	2.76***	2.51***	0.72
	(0.14)	(0.14)	(0.13)	(0.34)	(0.15)	(0.62)	(0.15)	(0.48)	(0.59)
$W > m$	0.87***	0.71***	0.99***	0.79***	0.54***	0.61***	0.73***	0.51***	0.09
	(0.08)	(0.08)	(0.07)	(0.12)	(0.08)	(0.15)	(0.08)	(0.11)	(0.16)
$W > 1.5m$	0.40***	0.26***	0.42***	0.38***	0.12*	0.15**	0.17**	0.02	-0.03
	(0.06)	(0.07)	(0.06)	(0.07)	(0.07)	(0.06)	(0.07)	(0.06)	(0.08)
$W > 2m$	0.15**	0.07	0.34***	0.08	-0.07	0.07	0.08	-0.05	-0.08*
	(0.06)	(0.06)	(0.06)	(0.05)	(0.07)	(0.06)	(0.07)	(0.07)	(0.04)

Note: Estimates of the (100 times the) elasticity of formality with respect to the wage at the minimum wage level, using different sample restrictions. Standard errors in parentheses.

Another maintained assumption of the model is that the latent wage density is continuous around the minimum. If the wage density is continuous, then my estimates should not reveal any effect for values other than the minimum wage.

Table IX displays the estimates of the ratio of the left and right limits of the wage density for values different than the minimum wage pooling data from all years (normalizing the minimum wage to zero). I display the point estimates for using two values for the bandwidth: the same bandwidth as in Table IV and a bandwidth half of its size. I perform the placebo test at 20 different points, from R$200 to R$300 above the minimum wage. The point estimates tend to be around one, which should be the case in the absence of a discontinuity. Using the baseline bandwidth, the estimates range from 0.88 to 1.17.

However, the null hypothesis of no gap is still rejected at several points. Discontinuities of such small magnitude are likely a result of "heaping" at round numbers. If a discontinuity of similar size is present in the latent wage density, then the magnitude of the bias on the estimates of the model structural parameters would be of negligible economic significance. All the qualitative implications of the model parameter estimates based on the continuity assumption would remain valid.[45]

[45] I discuss in Appendix E.3 the consequences of estimating the model incorrectly assuming continuity for the latent wage distribution. The estimators of the probabilities of non-compliance and "clustering" at the minimum wage will be inconsistent if the latent wage distribution is discontinuous at the minimum wage level. The ratio between the true structural parameters and the (probability limit of the) estimators will be given by the magnitude of the discontinuity in the *latent* density at the minimum wage level. For example, adjusting the estimates for a discontinuity of 0.92 in the latent density increases the estimate of π_d for the year 2001 from 0.20 to 0.22. Similarly, π_m increases

TABLE IX

Placebo Tests: Discontinuity Estimates using Values Other than the Minimum Wage

	Wage	10	20	30	40	50	60	70	80	90	100
Baseline	200	1.007***	0.929***	0.993***	1.111***	1.130***	1.173***	1.097***	0.920***	0.961***	0.882***
		(0.008)	(0.008)	(0.008)	(0.009)	(0.009)	(0.009)	(0.007)	(0.007)	(0.007)	(0.007)
Half		1.039***	0.876***	1.003***	1.191***	1.178***	1.161***	0.969***	0.654***	0.742***	0.579***
		(0.011)	(0.011)	(0.010)	(0.013)	(0.015)	(0.014)	(0.010)	(0.008)	(0.009)	(0.010)
Baseline	300	1.029***	0.907***	0.894***	1.076***	1.084***	1.104***	1.046***	0.899***	1.059***	0.995***
		(0.008)	(0.008)	(0.008)	(0.009)	(0.009)	(0.009)	(0.010)	(0.009)	(0.011)	(0.010)
Half		0.901***	0.698***	0.737***	1.081***	1.101***	1.174***	1.031***	0.760***	1.023***	0.865***
		(0.013)	(0.011)	(0.010)	(0.012)	(0.013)	(0.013)	(0.012)	(0.010)	(0.013)	(0.013)

Note: Standard errors in parentheses.

As a robustness check, I investigate the sensitivity of my estimates to the choice of bandwidth and the presence of spillovers. A key parameter of the model, π_d, is identified by the ratio of the wage density above and below the minimum wage. In the baseline specification, the estimation was performed using local linear density estimators with the bandwidth equal to eight times Silverman's rule of thumb. I estimate π_d using the automatic bandwidth selection procedure proposed by McCrary (2008). The estimates of π_d, available through request, range from 11 to 16%, whereas with the baseline bandwidth they range from 11 to 23%. Thus, I tend to find slightly smaller estimates of the likelihood of non-compliance using the automatic bandwidth selection procedure. The qualitative implications of the results, however, remain similar.

In Appendix E.5, I discuss the identification of the effect of the minimum wage on the size of the formal sector under the presence of spillovers. Identification of the latent size of the formal sector can be achieved by assuming that spillovers vanish at a point higher up in the wage distribution. My spillover-robust estimates of the impact of the minimum wage on the size of the formal sector are approximately -14%. Thus, these estimates are higher than the baseline estimates from Table IV that are obtained under the assumption of absence of spillovers. This suggests that the -9% effect from the baseline estimate *underestimates* of the true effect of the minimum wage on the size of the formal sector if Assumption 3 is violated.

from 0.26 to 0.28.

7. CONCLUSION

This paper develops a dual-economy model to analyze the effects of the minimum wage in a country with a large informal sector. I discuss the conditions under which the effects of the policy are identified using only cross-sectional data on wages and sector (defined by formality status) and the same level of the policy is applied to all workers. I show that the discontinuity of the wage density at the minimum wage level identifies the probability of non-compliance with the policy, and the latent relationship between sector and wages can be recovered using data on wages and sector above the minimum wage. I then show that the latent *joint* distribution of sector and wages can be identified based solely on data on sector and wages. This result allows me to estimate the impact of the policy on a broad range of labor market outcomes, such as expected wages, unemployment, wage inequality, the size of the formal sector, and labor tax revenues.

The main results are that the minimum wage significantly alters the shape of the lower part of the wage distribution and thereby reduces wage inequality. My estimates show that expected wages increase by approximately 16% and the Gini coefficient decreases by approximately 24%. However, the minimum wage policy generates sizable unemployment effects and a reduction in the size of the formal sector of the economy. My estimates imply a decrease of approximately 9% in the size of the formal sector. This result is due to both unemployment effects on the formal sector and movements of workers from the formal sector to the informal sector as a consequence of the policy. My estimates also indicate that the latent size of the formal sector is approximately four times larger than the informal sector. Thus, small movements from the formal to the informal sector still induce a sizable change in the relative size of the informal sector. My estimates show that the minimum wage increases the size of the informal sector by approximately 39%. Together, these effects imply a reduction in the tax revenues collected by the government to support the social welfare system of approximately 6%.

The research design based on the sharp contrast in the effects of the minimum wage between workers on each side of the minimum wage value allows for indirect tests of the underlying identification assumptions of the model. The graphical and statistical evidence supports the maintained assumptions. The robustness checks performed produced similar results to those of the baseline estimator.

There are, however, several limitations of this strategy. A fully structural model of workers and firms behavior is not specified. Thus, this approach does not recover deep parameters of the economy such as the elasticity of labor demand.[46] An extended version of the dual-economy model presented in this paper that fully incorporate optimizing behavior from the workers' side, such as a Roy-model of sector choice, is the object of ongoing research.

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ESTIMATING THE EFFECTS OF THE MINIMUM WAGE IN A DEVELOPING COUNTRY: A DENSITY DISCONTINUITY DESIGN APPROACH, SUPPLEMENTAL MATERIAL

APPENDIX A: IDENTIFICATION

This section I prove identification of the parameters of the model and the joint distribution of latent sector and wages under the assumptions of the Dual-Economy model. In what follows, assume that the econometrician observes a random sample of the pair $\{(W_i(1), S_i(1))\}$ of size N from a population of interest. I also assume the following easily verifiable technical conditions: the minimum wage m is set at a point with non-zero density, that is, $f_0(m) > 0$, $Pr[W(1) < m] > 0$, and $\Lambda'(m;\beta) \neq 0$.

Lemma A.1 (***Identification of sector-specific parameters***) Under Assumptions 1, 2, 3, and 4, π is identified.

Lemma A.2 (***Identification of latent distributions***) Under Assumptions 1, 2, 3 and 4, the latent joint distribution of sector and wages is identified.

Proof: Given Assumptions 3 and 4, the relationship between the observed density and the latent one can be written as:

$$(1) \qquad f(w) = \begin{cases} \frac{\pi_d(w) f_0(w)}{c} & \text{if} \quad w < m \\ \int^m \frac{\pi_m(w) f_0(w)}{c} dw & \text{if} \quad w = m \\ \frac{f_0(w)}{c} & \text{if} \quad w > m. \end{cases}$$

Given Assumptions 2, 3 and 4, the latent share of the formal sector $\Lambda(w(0))$ is identified using the information above the minimum wage. This is true because

$Pr[S(0)=1|W(0)=w]=Pr[S(1)=1|W(1)=w]$ when $w>m$. Then, we have:

$$\beta_0 = \arg\min_{\beta} \int_m^{\infty} (Pr[S(1)=1|W(1)=u] - \Lambda(u;\beta))^2 f(u|W(1)>m)du.$$

Furthermore, we have that $\Lambda(w;\beta_0) = Pr[S(0)=1|W(0)=w]$ for all w.[1] Given Assumptions 1, 3 and 4, we have:

$$\pi_d(m) = lim_{\epsilon\to 0^+} \frac{f(m-\epsilon)}{f(m+\epsilon)}.$$

Moreover, regarding the derivative of the wage density, we have:

$$f'(w) = \begin{cases} \frac{\pi_d'(w) f_0(w)}{c} + \frac{\pi_d(w) f_0'(w)}{c} & \text{if} \quad w<m \\ \frac{f_0'(w)}{c} & \text{if} \quad w>m. \end{cases} \tag{2}$$

Then, it can be shown that:

$$\pi_d'(m) = lim_{\epsilon\to 0^+} \left(\frac{f'(m-\epsilon)}{f'(m+\epsilon)} - \frac{f(m-\epsilon)}{f(m+\epsilon)} \right) \cdot \frac{f'(m+\epsilon)}{f(m+\epsilon)}.$$

Because the RHS of this equation contains only objects of the observed wage distribution, this implies that $\pi_d'(m)$ is identified. Given that the function $\Lambda(m)$ is identified, we have:

$$\pi_d^{(0)} = \pi_d(m) - \frac{\Lambda(m)}{\Lambda'(m)} \cdot \pi_d'(m)$$

$$\pi_d^{(1)} = [\pi_d(m) - (1-\Lambda(m)) \cdot \pi_d^{(0)}] \cdot \Lambda(m)^{-1}.$$

This can be shown using the equation below and its derivative with respect to the

[1]Note the importance of *all* w in this sentence. This means that once we recover β_0, we can forecast $Pr[S(0)=1|W(0)=w]$ for values of w that are below the minimum wage level. It should be clear here why non-parametric estimation of the conditional probability of sector given the wage is not an option. By assuming a parametric form, I can use the parameters to predict the latent probability of sector given the wage for values at which, in the data, this probability is equal to zero due to the minimum wage policy.

wage:

$$\pi_d(w) = \pi_d^{(1)}\Lambda(w) + \pi_d^{(0)}(1 - \Lambda(w)).$$

Given that all terms of the equation above are identified, we have that the function $\pi_d(w)$ is identified. Inverting the relationship between the observed and latent wage densities, we have:

$$f_0(w) = \begin{cases} \frac{f(w)\cdot c}{\pi_d(w)} & \text{if} \quad w < m \\ f(w)\cdot c & \text{if} \quad w > m. \end{cases} \tag{3}$$

Which implies:

$$c = [\int^m \frac{f(w)}{\pi_d(w)} dw + 1 - F(m)]^{-1}.$$

Because the function $\pi_d(w)$ is already identified and $F(m)$ is simply the fraction of workers in the observed wage distribution who earn less than or equal to the minimum wage, c is identified. This implies the identification of the entire latent wage distribution $f_0(w)$. Using the latent wage density and the function $\Lambda(w)$ allows the identification of the latent densities of the formal and informal sectors and, finally, the remaining parameters $\pi_m^{(1)}$ and $\pi_u^{(1)}$.

$$\begin{aligned}
f(W(0) = w|S(0) = 1) &= \frac{Pr[S(0)=1|W(0)=w]\cdot f_0(w)}{Pr[S(0)=1]} &&= \frac{\Lambda(w)\cdot f_0(w)}{\int \Lambda(u) f_0(u) du} \\
f(W(0)|S(0) = 0) &= \frac{Pr[S(0)=0|W(0)=w]\cdot f_0(w)}{Pr[S(0)=0]} &&= \frac{(1-\Lambda(w))\cdot f_0(w)}{\int (1-\Lambda(u)) f_0(u) du} \\
\pi_m^{(1)} &= \frac{Pr[W(1)=m|S(1)=1]}{1-Pr[W(1)=m|S(1)=1)]} \cdot \frac{1-F_0(m|S(0)=1)}{F_0(m|S(0)=1)} \\
\pi_u^{(1)} &= 1 - \pi_d^{(1)} - \pi_m^{(1)} \\
\pi_m^{(0)} &= 1 - \pi_d^{(0)}.
\end{aligned}$$

Q.E.D.

It is important to note that the identification result holds if one assumes that $\pi_m^{(1)}$ and $\pi_u^{(1)}$ are non-specified functions of the latent wage, as long as $\pi_d^{(1)}$ remains constant. In this scenario, the parameters recovered above are expectations - $E(\pi_d^1)$ and $E(\pi_u^{(1)})$ - over the distribution of workers whose latent wages are below the minimum wage. Formally, the parameters identified are $\pi_m^{(1)} = Pr[W(1) = m|S(0) = 1, W(0) < m]$ and $\pi_u^{(1)} = Pr[W(1) = .|S(0) = 1, W(0) < m]$. Under the maintained assumptions, this probability is the same for all workers regardless of their latent wage. In the case in which workers are heterogeneous in the probability of becoming unemployed, or receiving the minimum wage, with respect to their latent wages, the model recovers the natural extension of this parameter in the presence of such heterogeneity. That is, it recovers the average effect for the population of affected workers. Interestingly, this does not imply that the latent wage distributions obtained under the assumption of constant probabilities will be inconsistent. The assumption of constant probabilities is maintained only to simplify the exposition.[2]

Further, it should be stressed that this proof does not require the wage distribution to peak above the minimum wage. In fact, one can identify the effects of the minimum wage regardless of where in the latent wage distribution the minimum wage happens to be set, as long as the density of wages is greater than zero at the minimum wage, $\pi_d^{(1)}$ and $\pi_d^{(0)}$ are constants and either one of them is greater than zero.

COROLLARY A.3 (***Identification of the minimum wage treatment effects***) Under Assumptions 1, 2, 3 and 4, the effects of the minimum wage on functionals of the joint distribution of sector and wages are identified.

PROOF: The identification of treatment effect parameters follows directly from the identification of the joint distribution of observed and latent sector and wages from i.i.d data on $\{(W_i(1), S_i(1))\}$. *Q.E.D.*

[2]This will hold as long as the part regarding the probability of non-compliance is correctly specified with respect to latent wages – for example, if it is constant. See more on this in Appendix 3.

APPENDIX B: IDENTIFICATION UNDER INDEPENDENCE BETWEEN SECTOR AND WAGES

In this section, I discuss the identification given the independence between (latent) sector and wages, that is, $Pr[S(0) = 1|W(0) = w] = \Lambda \;\; \forall \;\; w$. I maintain Assumptions 1 (continuity), 3 (no spillovers) and 4 (minimum wage effects). Given those assumptions, the aggregate wage density will be given by:

$$f(w) = \begin{cases} \frac{\pi_d f_0(w)}{c} & \text{if} \quad w < m \\ \frac{\pi_m F_0(m)}{c} & \text{if} \quad w = m \\ \frac{f_0(w)}{c} & \text{if} \quad w > m. \end{cases} \tag{4}$$

This is exactly the one-sector version of this model, as proposed by Doyle (2006). This means that at least the aggregate parameters π_d, π_m and π_u are identified as:

$$\pi_d = lim_{\epsilon \to 0} \frac{f(m-\epsilon)}{f(m+\epsilon)}.$$

To identify π_m, one simply needs to verify that:

$$\pi_m = \pi_d \cdot \frac{Pr[W(1) = m]}{Pr[W(1) < m]}.$$

Given π_d, $F_0(m)$ can identified by: [3]

$$F_0(m) = \frac{Pr[W(1) < m]}{\pi_d Pr[W(1) > m] + Pr[W(1) < m]}.$$

[3]See the section on the identification of Doyle's model.

The relationship between the aggregate data parameters π_d and π_m and the sector-specific model parameters can be derived as:

$$\pi_d = \Lambda\pi_d^{(1)} + (1-\Lambda)\pi_d^{(0)}$$

$$\pi_m = \Lambda\pi_m^{(1)} + (1-\Lambda)\pi_m^{(0)}$$

$$\pi_u = \Lambda\pi_u^{(1)}$$

$$\pi_d^{(1)} + \pi_m^{(1)} + \pi_u^{(1)} = 1$$

$$\pi_d^{(0)} + \pi_m^{(0)} = 1.$$

Having recovered the aggregate parameters, the goal is solve for the sector-specific parameters. To do so, one first needs to identify Λ. Note:

$$\Lambda \equiv Pr[S(0)=1] = Pr[S(0)=1|W(0)>m] = Pr[S(1)=1|W(1)>m],$$

where the first equality holds because of the independence between latent sector and wages, and the second holds due to the lack of spillovers on sector probabilities. Interestingly, the identification of the latent size of the formal sector does not rely on anything but independence, the lack of spillovers, and the assumption that $Pr[W(1)>m|W(0)<m]=0$.[4] This means that we can correctly identify the size of the formal sector even if we mis-specify the continuity of the latent distribution of wages or the way in which that the minimum wage affects the lower tail of the wage distribution. Note that, given the aggregate data parameters and Λ, this is a system of five equations and five unknowns. Unfortunately, the system is rank deficient, and hence, an additional equation needs to be added to recover the sector-specific parameters. Relying on the identification of Λ , $\pi_u^{(1)}$ is identified by:

$$\pi_u^{(1)} = \frac{\pi_u}{\Lambda} = \frac{1-\pi_d-\pi_m}{\Lambda}.$$

[4] $Pr[W(1)>m|W(0)<m]=0$ is implied by Assumption 4. When Assumption 4 does not hold, the identification strategy described above will be valid if $Pr[W(1)>m|W(0)<m]=0$. An example of this situation is when the probability of non-compliance is a function of the worker's latent wage. This would invalidate Assumption 4 while preserving the condition $Pr[W(1)>m|W(0)<m]=0$.

To recover $\pi_m^{(1)}$, it is necessary to consider the density of the formal sector:

$$f(w|S(1)=1) = \begin{cases} 0 & \text{if} \quad w < m \\ \frac{\pi_m^{(1)} F_0(m)}{c^{(1)}} & \text{if} \quad w = m \\ \frac{f_0(w)}{c^{(1)}} & \text{if} \quad w > m, \end{cases} \tag{5}$$

where $c^{(1)} = 1 - F_0(m)(1 - \pi_m^{(1)})$ is a scaling factor such that the two densities integrate to one. The key feature of the formal sector that allows for the identification of $\pi_m^{(1)}$ is that because the density is zero below the minimum wage, the scaling factor on the denominator is a function of only one unknown parameter (note that $F_0(m)$ is already identified). Finally, using:

$$Pr[W(1) = m|S(1) = 1] = \pi_m^{(1)} F_0(m)/c^{(1)},$$

it is possible to show that:

$$\pi_m^{(1)} = \frac{Pr[W(1) = m|S(1) = 1]}{1 - Pr[W(1) = m|S(1) = 1]} \cdot \frac{1 - F_0(m)}{F_0(m)}.$$

The RHS of this equation consists only of quantities that are already identified. Given that $\pi_m^{(1)}$ is identified based on the expression above, we can now return to the system and recover all the other parameters:

$$\pi_m^{(0)} = \frac{\pi_m - \Lambda \pi_m^{(1)}}{1 - \Lambda}.$$

Thus:

$$\pi_d^{(0)} = 1 - \pi_m^{(0)}.$$

Finally:

$$\pi_d^{(1)} = 1 - \pi_m^{(1)} - \pi_u^{(1)}.$$

The latent wage density can be recovered in the same way as in the baseline model, that is, by inverting the relationship and using the fact that c and π_d were already identified:

$$f_0(w) = \begin{cases} \frac{f(w) \cdot c}{\pi_d} & \text{if} \quad w < m \\ f(w) \cdot c & \text{if} \quad w > m. \end{cases} \tag{6}$$

This implies that we have identified the latent distribution of wages $f_0(w)$, the latent size of the formal sector Λ and the parameters π that govern how the minimum wage affects the economy.

Note that estimation in this context is considerably easier than in the baseline model. This is the case because it is not necessary to estimate the derivative of the density of wages at m to solve for the sector-specific parameters. All objects in the identifying equations – except by the lateral limit of the density of wages at m – can be estimated by replacing the population object with its respective sample counterpart. I used this plug-in method to estimate the parameters of the model in the empirical application.

APPENDIX C: ESTIMATION

C.1. *Local Linear Density Estimation*

In this section, I describe the local linear approach to density estimation. A standard approach to non-parametrically estimate densities at boundary points is to use a local linear density estimator. This estimator builds on the idea of local linear conditional mean estimators. It begins by dividing the support of the density into a set of bins. Thereafter, a "response variable" is defined as the bin counts of these disjoint intervals. After this process, one is left with a vector containing the "independent variable," which is the bin center, and a corresponding "dependent variable," the bin counts. Finally, standard local polynomial smoothing estimates are applied to these constructed variables. The discussion and notation here will follow the approach advocated by McCrary (2008) in the context of testing for manipulation in RD designs. To begin, define $g(w_i)$ as the discretized version of the wage support for a bin size equal to b.

$$g(w) = \begin{cases} \lfloor \frac{w-m}{b} \rfloor b + \frac{b}{2} + m & \text{if} \quad w \neq m \\ m & \text{if} \quad w = m, \end{cases}$$

where $\lfloor a \rfloor$ is the greatest integer in a.[5] Clearly, it holds that $g(w) \in \chi \equiv \{..., m - 5\frac{b}{2}, m - 3\frac{b}{2}, m - \frac{b}{2}, m, m + \frac{b}{2}, m + 3\frac{b}{2}, m + 5\frac{b}{2}, ...\}$. I will call the jth element of this set X_j. [6] Define the normalized cell size for the jth bin $Y_j = \frac{1}{Nb} \sum_{i=1}^{N} \mathbb{1}\{g(W_i) = X_j\}$. Let $K(.)$ be a symmetric kernel function satisfying the usual properties and let h be a bandwidth satisfying the conditions $h \to 0$, $nh \to \infty$, $(nh)^{1/2}h^2 \to 0$ and $b/h \to 0$. Then, the local linear estimator of the density and its derivative are defined, for $w \neq m$, as:

$$\begin{bmatrix} \widehat{f}(w) \\ \widehat{f}'(w) \end{bmatrix} = \arg\min_{(a_0,a_1)'} \sum_{j}^{J} (Y_j - a_0 - a_1(X_j - w))^2 K(\tfrac{w_j - w}{h})(\mathbb{1}\{X_j > m\}\mathbb{1}\{w > m\} + \mathbb{1}\{X_j < m\}\mathbb{1}\{w < m\}).$$

APPENDIX D: TESTING

This research design allows us to perform partial tests of the validity of the model's assumptions. This section I describe how these tests can be performed and their limitations.

Assumption 1, the continuity of the latent wage distribution, can be verified by

[5]As discussed by McCrary, the greatest integer in a is the unique integer Q such that $Q < a < Q+1$ (round to the left). In software, this is known as the "floor" function.

[6]As discussed in McCrary (2008), the endpoints X_1 and X_j may always be chosen arbitrarily small (large) such that all points in the support of the distribution of wages are inside one of the bins.

visual inspection of the histogram and the kernel density estimates using different values for the bandwidth. Formally, this condition can be tested by performing a placebo test, that is, by checking whether there are differences between the left and right limits of the density estimates at wage points other than the minimum wage.

Assumption 2 can be tested by comparing the fit of the parametric model with non-parametric smoothing estimates. If $\Lambda(w;\beta)$ is correctly specified, for the true value of the parameters β_0, we have:

$$\int_m^\infty \big(Pr[S(0)=1|W(0)=u]-\Lambda(u;\beta_0)\big)^2 f_0(u)du = 0.$$

While this equation is in terms of latent variables, we can restate it using observables by relying on Assumption 3. Thus, we have:

$$I \equiv \int_m^\infty \big(Pr[S(1)=1|W(1)=u]-\Lambda(u;\beta_0)\big)^2 \mathbb{1}\{u>m\} f(u)du = 0,$$

where $\beta_0 \equiv \arg\min_\beta E[(S_i(1)-\Lambda(W_i(1);\beta))^2\mathbb{1}\{W_i(1)>m\}]$. This condition is in terms of quantities we can observe. Correctness of the specification of the model for $\Lambda(W(0),\beta)$ implies that $I=0$. This is a integrated mean squared error type of condition that can be used for specification testing (see Pagan and Ullah (1999)). The idea behind it is to compare the fit of a parametric model with the fit of a non-parametric model. This type of comparison can be used to identify the proper functional form for the sector-wage relationship. This is relevant because part of the identification relies on extrapolating this conditional mean function to values below the minimum wage.[7] It should be noted, however, that this is, at best, a partial test of

[7] There are also parametric versions of these tests. For example, testing Assumption 2 in a parametric setting can be achieved by increasing the order of the polynomial of the wage and testing the restriction that the higher order terms are equal to zero. In the simplest case in which one has a linear logit of the sector given the wage, the correctness of the specification can be tested by estimating a model in which the square of the wage is added as a regressor and assessing whether the coefficient associated with the squared term is different from zero.

the assumption. There are some deviations from the null for which this test does not have the power to reject. To make this point clear, observe the following condition:

$$\int_{-\infty}^{\infty} \left(Pr[S(0) = 1|W(0) = u] - \Lambda(u; \beta_0)\right)^2 f_0(u)du = 0.$$

This condition is equivalent to the correctness of the specification of the parametric model for the conditional probability of the latent sector given the wage. The crucial difference between this condition and that used in the test above is that it can detect when the model is incorrectly specified for values below the minimum wage. Unfortunately, it is not possible to create a feasible version of this condition, as once we move from latent to observed wages, all information on the conditional probability of latent sector given the wages is lost for values below the minimum. In sum, it is conceivable that the parametric functional form holds for values above the minimum wage but fails to hold for values below it. This part of the assumption remains untestable.

It is also possible to test Assumption 4. In Assumption 4, the probabilities that capture the effects of the minimum wage are defined. A restriction imposed by that assumption is that the probabilities of non-compliance ($\pi_d^{(1)}$ and $\pi_d^{(0)}$) are invariant across workers with different latent wages in the same sector.[8] This is a restrictive assumption, as workers whose latent wage is close to the minimum wage level could be more likely to comply with the policy than workers whose latent wage is far from the minimum. To see why this assumption is testable, one must first examine the second derivative of the observed wage density:

$$(7) \qquad f''(w) = \begin{cases} \frac{\pi_n''(w)f(w)}{c} + 2\frac{\pi_n'(w)f'(w)}{c} + \frac{\pi_d(w)f''(w)}{c} & \text{if} \quad w < m \\ \frac{f''(w)}{c} & \text{if} \quad w > m. \end{cases}$$

[8] One can see that in aggregate, the likelihood of non-compliance $\pi_d(w)$ will be a function of latent wages due to changes in the composition of each sector as we move along different wages.

If the continuity assumption on the latent wage distribution is strengthened up to the second derivative, that is, if $\lim_{w\to m^+} f_0''(w) = \lim_{w\to m^-} f_0''(w)$, then we have:

$$lim_{\epsilon\to 0^+}\Big(cf''(m+\epsilon) - \frac{cf''(m-\epsilon) - \pi_d''(m)f_0(m) - 2\pi_d'(m)f_0'(m)}{\pi_d(m)}\Big) = 0.$$

Intuitively, we can test Assumption 4 because by examining the second derivative, we have added another equation while the number of parameters remained the same. This provides us the overidentification condition that allows us to test the model.[9,10,11]

APPENDIX E: ROBUSTNESS

This section demonstrates how generalizable the inferences based on the model are when the model is misspecified or when we fail to obtain data on other determinants of the joint distribution of sector and wages. I claim that (i) the model is still correctly specified if the unobserved heterogeneity affects either the model parameters or the latent wage distribution, but not both, (ii) the model correctly identifies the desired features of the data when the model parameters are allowed to vary across latent wages and individuals under some restrictions on the unobserved heterogeneity, (iii) the odds ratio of clustering at the minimum wage (π_m) versus non-compliance (π_d), and the

[9]This is easier to see in the case in which one assumes a linear probability model for $Pr[S(0) = 1|W(0)]$. In this scenario, it is possible to find a closed-form solution for the model parameters using either the first or the second derivative of the wage density. These different ways of identifying the parameters must yield the same result if the model is correctly specified. However they do not coincide if the model is misspecified, that is, when the probabilities of non-compliance are functions of latent wages.

[10]If one is willing to impose further smoothing conditions on the latent wage distribution, it is possible to identify the model by imposing flexible conditions on the relationship between the parameters and the wages. For example, if one believes that $(\pi_d^{(1)}, \pi_d^{(0)})$ is appropriately described by a quadratic (cubic) function, then one needs to go up to the third (fourth) derivative of the wage density to estimate the model parameters.

[11]This condition is easier to test in the parametric version of the model. To do so, one simply needs to estimate a version of the model in which the probabilities of non-compliance $\pi_d^{(1)}$ and $\pi_d(0)$ are allowed to be a low-order polynomial of the latent wages and compute a likelihood ratio test that uses the baseline version of the model as a comparison. A rejection of the null indicates that the more general version is a better description of the economy, that is, the probabilities of non-compliance are indeed functions of latent wages.

latent share of the formal sector are correctly identified even when unemployment effects cannot be, and (iv) the aggregate parameters π_d, π_m and π_u are correctly identified even when Assumption 2 does not hold or when unemployment effects are also present in the informal sector.

To show (i), I reformulate the model and allow its parameters or distributions to be functions of potentially unobservable worker characteristics. I show that under some conditions, the assumptions I require for the baseline model to hold will still be valid. (ii) I reformulate the model under a random coefficients framework. I show that under reasonable conditions for the heterogeneity of the parameters across individuals, the estimands based on the baseline model identify the expectation of the distribution of parameters over the set of workers affected by the minimum wage. To show (iii), I prove that a lack of continuity implies the inconsistency of some, but not all, of the parameters of interest in the model. To show (iv), I recall that identification of Doyle's aggregate parameters does not rely on all four assumptions that I use to identify the baseline model.

These results reveal an important feature of the baseline model. It is easy and feasible to infer the direction that the parameter estimates will go when some of the model's assumptions are violated. Moreover, as the identification is achieved using "separable" pieces – a model for the conditional distribution of sector given the wages, continuity of latent wage distribution to identify π_d, and so forth – some of the results will still hold when the model is partially misspecified. Taken together, these features should increase the credibility of the results when there are some concerns with the correctness of the model specification. Some pieces of information based on this approach can be useful even in the worst case scenario in which the model is guaranteed to be inconsistent for some parameters.

E.1. *Role of Covariates and Unobserved Heterogeneity*

By exploring the different effects of the minimum wage across sectors and the discontinuity of the density of wages around the minimum, one can estimate how the economy responds to this policy. This approach has some similarities to the quasi-experimental Regression Discontinuity Designs. Because one of the main advantages of Regression Discontinuity Designs is to provide a way to avoid most of the endogeneity concerns associated with using observational data to infer causality, it is useful to discuss the extent to which these advantages are also present in this method.

Assume that there is a random variable Z – say, for example, age – that is known to affect individual labor market conditions. One example is when workers with different values of Z draw from different latent wage distributions. Another way that Z can affect a worker's labor market conditions is through the model parameters. For example, after the introduction of the minimum wage, younger workers might be more likely to move into the informal sector than older workers, which, in the model, would be represented by a higher $\pi_d^{(1)}$. In these cases, is it necessary to estimate the model conditional on Z for the inferences to be valid?

In the following discussion, I will always assume continuity of the Z-specific latent wage distribution, an absence of spillovers and a covariate-specific version of the assumption that describes the minimum wage effects. I will also assume the following:

Assumption 1 *Conditional probability of latent sector given the wage:*

$$Pr[S(0) = 1|W(0) = w] \equiv \int Pr[S(0) = 1|W(0) = w, Z = z]f(z|w)dz = \Lambda(w; \beta).$$

This assumption simply means that whatever the model for the conditional probability of the sector given the wage and Z is, this model can be aggregated to a unconditional one with parameters β.[12] Two sufficient conditions for the inferences

[12]In general, this model will be more complex than the covariate-specific one. A simple, sufficient,

based on the unconditional wage distribution to be valid in the presence of covariates are the following:

Case 1:

Assumption 2 *Equality of parameters:* $\pi(z) = \pi \ \ \forall z$.

When the effect of Z occurs through changes in the latent joint distribution of sector and wages but not through differential responses to the minimum wage, then Z can be safely ignored when making inferences with regard to the unconditional distribution. The reason for this result is simple. The assumptions above imply that all assumptions of the model for the aggregated data remain valid.

Case 2:

Assumption 3 *Equality of latent distributions:* $W(0)_{|Z} {\sim} F \ \ \forall z$.

This assumption means that $Pr[W(0) < w|Z = z] = Pr[W(0) < w|Z = z']$ for all (z, z') and all w. By restricting the latent wage distribution to be the same for all values of z, inference based on the unconditional distribution ignoring the covariate will be valid when parameters are allowed to vary over Z. The parameters π recovered from the aggregate data will be weighted averages of the covariate-specific ones, with correct weights to reflect the share of each group of values of Z in the population. These, of course, are much stronger conditions than those in Case 1, as the role of covariates is severely limited when they are only allowed to determine wages through the differences in minimum wage effects.

When both the latent wage distribution and the parameters are allowed to vary over Z, the estimate of π_d can be interpreted as a local effect, as it recovers the likelihood of non-compliance for those with latent wages around the minimum wage.[13]

but clearly not necessary, condition to guarantee that such a model will exist is when strengthened to $Pr[S(0) = 1|W(0) = w, Z = z] = \Lambda(w, \beta)$, that is, covariates only enter the conditional probability of sector given the wage though their effects on wages.

[13]Preliminary results from simulations show that an unreasonably large degree of heterogeneity in both the latent distributions and the model parameters is necessary for the inference based on unconditional distribution to show sizable distortions.

The relevance of these results is quite small if the wage determinants are observable, as the model can be easily estimated conditional on these variables. If the estimation is performed while conditioning on the covariates, one need not be concerned with the cases above, meaning that the model parameters and latent joint distributions can be different for different values of Z. However, the situation differs when not all wage determinants are observable. Failure to observe wage determinants is a major source of bias in inferences based on regression models. In this design, this is not the case, as long as the model parameters remain constant over the distribution of the variables that are ignored, which seems to be a much easier condition to satisfy than the zero correlation usually assumed in regression models. In this sense, this research design resembles most of the characteristics of Regression Discontinuity Designs, overcoming the difficulties in assessing causal effects based on observational data due to endogeneity concerns. The reason for this is that the identification does not rely on variation in the minimum wage to assess the policy's impact. Instead, identification relies on the sharp contrast between the effect of the minimum wage across individuals whose wages would fall on each side of it. Thus, concerns with omitted variable biases should be much more limited.

E.2. *Random Coefficients*

In the model, a worker is characterized by a pair (W_i, S_i) of observed sector and wage, a vector $(W_i(0), S_i(0), \zeta_i)$, and a vector $(\pi(\zeta) \equiv (\pi_n^{(1)}(\zeta), \pi_m^{(1)}(\zeta), \pi_u^{(1)}(\zeta), \pi_n^{(0)}(\zeta), \pi_m^{(0)}(\zeta))$, which is now ζ-specific. One way to interpret this is that we are treating ζ as the worker's unobserved type. For now, I will not assume anything regarding the relationship between the worker's type and his latent wages. Of course, it still holds that $\pi_m^{(0}(\zeta) + \pi_n^{(0)}(\zeta) = 1$ for all ζ and similarly for the formal sector parameters. This means, in addition to the worker's latent sector and wages, he receives a draw for the model parameters. Here, I also allow this draw to be a function of the latent wage. Thus, for example, workers with higher latent wages can have a higher probability of receiving the minimum wage versus becoming unemployed. This extension captures the idea that (i) minimum wage effects might vary across dimensions of worker's characteristics that are unobservable to the researcher and (ii) minimum wage effects can, and likely will, vary across workers with respect to the distance of their latent productivities from the minimum wage level. The rest of the model remains the same, meaning that I will retain continuity and the absence of spillovers. I will also assume independence between latent sector and wages for simplicity in the rest of this discussion.

This extension adds a great degree of flexibility to the model. It relaxes Assumption 4 in two ways. It allows different workers with similar wages to have different minimum wage response probabilities in an unknown and unspecified way. It also allows workers in the formal sector to have different probabilities of becoming unemployed ($\pi_u^{(1)}$) versus truncating at the minimum wage ($\pi_m^{(1)}$)) for different values of the latent wage. Importantly, this can be achieved without relying on any specified functional form; that is, it is not assumed that these probabilities vary over latent wages in any parametric, continuous or known way.

To analyze the model, we now need to define some new objects. Let:

$$E(\pi_m^{(1)}(\zeta|w)) = \int \pi_m^{(1)}(u) f_{\zeta|w}(u) du.$$

This expression defines the "average probability of truncation at the minimum wage for a formal sector worker with latent wage equal to w" as the integral of this probability for each worker's unobserved type weighted by the proportion of each type for that wage value. We can analogously define similar objects for the other probabilities.

Now, under this new set of assumptions, the relationship between latent and observed densities will be given by:

$$f(w) = \begin{cases} \frac{E(\pi_d(\zeta|W(0)=w)f_0(w)}{c} & \text{if} \quad w < m \\ \frac{\int^m E(\pi_m(\zeta)|W(0)=u)f_0(u)du}{c} & \text{if} \quad w = m \\ \frac{f_0(w)}{c} & \text{if} \quad w > m. \end{cases}$$

Now, let us consider the behavior of the estimands defined for the baseline model used under this, more general, version.

$$\pi_d \equiv lim_{\epsilon \to 0} \frac{f(m-\epsilon)}{f(m+\epsilon)}.$$

It is easy to see that:

$$lim_{\epsilon \to 0} \frac{f(m-\epsilon)}{f(m+\epsilon)} = \Lambda E(\pi_d^{(1)}(\zeta|W(0) = m, S(0) = 1) + (1-\Lambda) E(\pi_d^{(0)}(\zeta)|W(0) = m, S(0) = 0).$$

Now, it is also easy to see that this estimand will converge to the number that we need if $E(\pi_d^{(1)}(\zeta|W(0) = w, S(0) = 1) = E(\pi_d^{(1)}(\zeta|W = w', S(0) = 1)$ and $E(\pi_d^{(0)}(\zeta)|W(0) = w, S(0) = 0) = E(\pi_d^{(0)}(\zeta)|W(0) = w', S(0) = 0)$. This means that the only restriction on the relationship between the types and latent wages is that the expectation of the non-compliance probabilities (taken with respect to the type distribution) is not a function of the wage. [14]

Assuming that this condition holds, we have that our baseline estimand $lim_{\epsilon \to 0} \frac{f(m-\epsilon)}{f(m+\epsilon)}$ identifies the expected value of π_d over the population of affected individuals. That is:

[14] This does not mean that the model is unidentified if this condition fails to hold. It means that in this case, we would need to rely on the derivatives of the wage density to identify the slope of the relationship between expected minimum wage probabilities and latent wages. This can be achieved in the same way as discussed in the testing section.

$$lim_{\epsilon \to 0} \frac{f(m-\epsilon)}{f(m+\epsilon)} = Pr[W(1) = W(0)|W(0) < m].$$

Regarding the estimand of π_m:

$$\pi_m \equiv \pi_d \frac{Pr[W_i = m]}{Pr[W_i < m]}.$$

It can be shown that:

$$\pi_d \frac{Pr[W_i = m]}{Pr[W_i < m]} = \frac{\int^m E(\pi_m(\zeta)|W(0) = u) f_0(u) du}{\int^m f_0(u) du} = Pr[W(1) = m|W(0) < m],$$

which means that π_m converges to the expectation of the parameter over the population of affected workers. The intuition for this result is that the estimand of π_m comes from the point mass at the minimum wage level, which is obtained by integrating the probability of "clustering" at the minimum wage level for all workers whose latent wages are below the minimum wage level. Thus, irrespective of what functional form exists between the latent wage and the probability of receiving the minimum wage, this form simply reveals itself in the data in the form of the proportion of workers at the minimum wage level. The mass of wages at the minimum wage level has already "integrated out" the unobserved heterogeneity. This allows us to consider estimating $Pr[W(1) = m|W(0) < m]$ without completely describing the shape of $\pi_m^{(1)}(\zeta)$ as a function of $W(0)$. The term $Pr[W(1) = m|W(0) < m, S(0) = 1]$ coincides with the parameter $\pi_m^{(1)}$ as defined in the baseline model when $\pi_m^{(1)}$ is not a function of the latent wage. When $\pi_m^{(1)}$ is indeed a function of the latent wage (through unobserved types, for example), we can bypass the task of modeling this function and directly identify the aggregate component $Pr[W(1) = m|W(0) < m, S(0) = 1]$. Similar calculations show that the same is the case with respect to the estimand of unemployment:

$$1 - \pi_d - \pi_m = Pr[W(1) = .|W(0) < m].$$

Finally, it can be shown that the estimates for the implied treatment effects and latent densities will also converge to the correct values. This is a somewhat remarkable result, as it can be tempting to say that the way in which one understands the relationship between the heterogeneity in parameters and latent wages will necessarily determine the estimated latent densities that, in turn, will drive the results for the treatment effects. This result shows that this intuition is incorrect. As long as the part that concerns the likelihood of non-compliance $(\pi_d^{(0)}, \pi_d^{(1)})$ is reasonably specified, which can be achieved in a very flexible way by utilizing higher order derivatives of the wage density, all the results will hold. This result will hold even if unemployment or truncation at the minimum wage happen to have a unknown pattern that varies across individuals, unobservable characteristics, or latent wages. [15]

E.3. *Lack of Continuity*

In the following discussion, I will assume independence between latent sector and wages. Now suppose that π_d is not identified. This could be the case for two reasons. The first case is when latent wage distribution is not continuous. In this case, the estimate of π_d actually identifies $\pi_d \kappa$, where κ is the (unknown) size of the discontinuity of the latent wage around the minimum wage. It is clear that as long as $\kappa = 1$, the estimate of π_d will be consistent. The second case is when spillovers are misspecified. For example, if one incorrectly assumes that spillovers are absent, when in fact they are present and reduce the density of wages just above the minimum wage,[16]

[15] Of course, the marginal effects estimates will break down, as one needs to know not only the average probability of truncating at the minimum wage and unemployment but, more important, also the marginal probability to identify the effect of changes in the minimum wage level. The marginal probabilities will only be recovered if they either coincide with the average probability, as in the baseline version of the model, or if they have a known or estimable functional form.

[16] This will be the case if one assumes that spillovers are weakly positive.

π_d is misspecified because the density of wages observed just above the minimum wage is not the correct quantity to scale the density below to measure the extent of non-compliance.

However, by using this identification strategy, one can compute bounds for the extent of non-compliance. For example, if spillovers are assumed to be weakly positive, meaning that workers above the minimum do not suffer wage cuts following the policy, then π_d estimated when ignoring spillovers represent an upper bound of the likelihood of non-compliance. This will also provide an upper bound for π_m and a corresponding lower bound for π_u. Importantly, this means that the sizable unemployment effects found in the application cannot be explained by having a misspecified model for spillovers, as unemployment effects will necessarily be magnified in the presence of spillovers.

Interestingly, some features of the minimum wage effects can still be correctly identified in this scenario. It is straightforward to see that the odds ratio of truncation versus non-compliance will still be correctly identified regardless of the lack of continuity in the latent wage distribution or misspecification of spillovers. Moreover, those quantities will be meaningful even if the correct specification for the minimum wage effects would need a more flexible form for the parameters – by making them vary across individuals or latent wages, for example. In general, the statistic $\frac{Pr[W(1)=m]}{Pr[W(1)<m]}$ will always identify $\frac{Pr[W(1)=m|W(0)<m]}{Pr[W(1)<m|W(0)<m]}$, which is the ratio of the expected likelihood of truncating at the minimum wage versus not complying with the minimum wage for those directly affected by the policy.[17]. Interestingly, note that the same does not hold for the ratio of unemployment to either non-compliance or truncation.

If the latent wage distribution present a discontinuity at the minimum wage level, then this identification strategy will identify $\pi_d \kappa$. That is, the probability of non-compliance will be scaled by the discontinuity in the latent wage density at m. Thus,

[17]The conditions needed for this result are weak, namely, the lack of point mass at the minimum wage level in the latent wage distribution and a "no-crossing condition" that rules out workers higher up in the wage distribution receiving the minimum wage or less in the presence of the policy

if the discontinuity is of moderate size, the implied change in the estimated of π_d will be of a small order.[18] Thus, sizable distortions will be present only if the latent wage distribution presents large discontinuities at the minimum wage level.

It should also be stressed that under independence between sector and wages, the latent share of the formal sector – which is perhaps one of the most relevant parameters of the model – is still identified regardless of misspecifying how the minimum wage affects the lower part of the wage distribution or a lack of continuity in the latent wage distribution.[19]

E.4. *Aggregate Parameters*

Doyle (2006) and Meyer and Wise (1983) define what I call "aggregated data" probabilities π_d, π_m and π_u. I call them aggregated because they are a weighted average of the corresponding sector-specific likelihood of non-compliance, truncating and becoming unemployed. Because their goal is to compute aggregate data parameters, they do not need to have a correctly specified form for the conditional probability of the sector given the wage.

The identification of the simplified version of the model here uses Doyle's estimate as a first step. Then, the weights of the sector-specific probabilities are estimated, and finally, one can solve for the sector-specific parameters. This is a worthwhile exercise because, as I have shown above, a broader set of counterfactuals, such as labor tax and the size of the formal sector, analyses can be performed with sector-specific parameters. Moreover, $\pi_d^{(1)}$ is a parameter with more economic meaning than π_d itself.

Importantly, misspecification of either sector-specific assumptions or the form of the joint distribution of sector and wages has different consequences for the aggre-

[18] For example, if the left limit of the latent density of wages is 10% smaller than the right limit and π_d is equal to 0.2, then the estimated probability limit of the estimator of π_d based on this identification strategy will be 0.18, only 2 percentage points smaller than the true value.

[19] The only additional assumption needed for that identifying this parameter is a lack of spillovers on sector probabilities. See the section on identification of the restricted version of the model.

gate parameters when compared to the sector-specific parameters. Two cases can illustrate this: If either (a) unemployment effects on the informal sector are present, which is ruled out by Assumption 4, or (b) the model for the conditional probability of the sector given the wages is incorrectly specified, then the sector-specific parameter estimators will be inconsistent. However, the aggregate ones will not be. It is straightforward to see this because neither Doyle or Meyer and Wise use this information.

The results concerning the misspecification of the minimum wage effects, the joint distribution of latent sector and wages, spillovers or unknown heterogeneity of parameters point in the same direction. They show that the quantities obtained by the identification of the baseline model remain informative when some of the model's assumptions are incorrectly specified.

E.5. *Robust Estimates of the Effects of the Minimum Wage on the Size of the Informal Sector*

This paper develops a model that allows one to estimate the effects of the minimum wage on a broad range of policy-relevant outcomes. Importantly, the model captures a channel through which workers move from the formal sector to the informal sector in response to the minimum wage policy. The effects of the minimum wage on the size of the informal sector have important policy implications. This parameter is key to understand the effects of the minimum wage on the government budget, for example.

Under the assumptions of the model, this parameter, the effect of the minimum wage on the size of the informal sector, can be consistently estimated. This section discuss the extent to which those estimates are robust to deviations from these assumptions, in particular the absence of spillovers. This will be achieved using Assumption 5, the independence between latent sector and wages. The object of interest is $\frac{Pr[S(1)=1]}{Pr[S(0)=1]}$, that is, the ratio of the size of the formal sector in the presence of the minimum wage versus its size in the absence of the minimum wage. The numerator of this fraction can be directly estimated from the data. The counterfactual object is the latent size of the informal sector. Under independence between latent sector and wages, we have:

$$Pr[S(0) = 1] = Pr[S(0) = 1|W(0) > m] = Pr[S(1) = 1|W(1) > m].$$

This expression uses the size of the formal sector above the minimum wage as the estimate of the latent size of the formal sector in the absence of the policy. Interestingly, this result does not rely on the continuity of the latent wage distribution or the correctness of the specification of the minimum wage effects on the bottom part of the wage distribution. It relies on the independence, the lack of spillovers assumptions, and $Pr[W(1) > m|W(0) < m] = 0$. To evaluate the robustness of this estimate to departures from the absence of spillovers, one simply needs to specify a limit at which the spillovers should vanish. In the most extreme version of this assumption, the effects of the minimum wage vanish at the minimum wage level. However, one can specify that the minimum wage effects vanish at twice, or in general, k-times the minimum wage level. This lead to the following identification equation:

$$Pr[S(0) = 1] = Pr[S(0) = 1|W(0) > km] = Pr[S(1) = 1|W(1) > km],$$

where k is a number greater than or equal to one. The first equality follows from independence between sector and wages, whereas the second follows from the absence of spillovers at points higher than km.[20] Table I reports the effects of the minimum wage on the size of the formal sector based on different assumptions concerning where spillovers should vanish. The baseline estimates are approximately 10%. The estimates robust to spillovers find an effect of around 12 to 16%. The point estimates are significantly different. The qualitative conclusions, however, remain similar. The minimum

[20]It is interesting to note that one can also add spillovers on wages above this threshold. The only restriction that needs to be imposed for this identification to be effective is the absence of spillovers on sector probabilities. That is, workers do not move across in response to the minimum wage if their latent wages are above km and $Pr[W(1) > km|W(0) < km] = 0$.

TABLE I

Robust Estimates of the Effects of the Minimum Wage on the Size of the Formal Sector

	W>2m	W>3m
2001	0.867***	0.853***
	(0.002)	(0.003)
2002	0.853***	0.844***
	(0.003)	(0.004)
2003	0.865***	0.849***
	(0.003)	(0.004)
2004	0.847***	0.845***
	(0.003)	(0.003)
2005	0.860***	0.866***
	(0.003)	(0.004)
2006	0.858***	0.850***
	(0.003)	(0.004)
2007	0.879***	0.872***
	(0.002)	(0.004)
2008	0.873***	0.873***
	(0.002)	(0.003)
2009	0.883***	0.878***
	(0.002)	(0.003)

wage has a sizable impact on the size of the formal sector. This section shows that those effects should be further magnified if spillovers are indeed present. These results are based on the minimal assumptions of independence and lack of spillovers in the upper part of the wage distribution. They are robust to limited spillovers, a lack of continuity in the latent wage distribution and misspecification of the minimum wage effects on the lower part of the wage distribution.

TABLE II

LABOR TAX EFFECTS UNDER A "NO UNEMPLOYMENT" ASSUMPTION

	2001	2002	2003	2004	2005	2006	2007	2008	2009
R	1.093***	1.077***	1.081***	1.059***	1.057***	1.052***	1.052***	1.037***	1.039***
	(0.006)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.007)
Pr(S(1)=1)/Pr(S(0)=1)	0.912***	0.892***	0.895***	0.891***	0.884***	0.884***	0.899***	0.889***	0.902***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
E(W(1) | S(1) = 1)/E(W(0) | S(0) = 1)	1.198***	1.208***	1.208***	1.189***	1.195***	1.191***	1.170***	1.167***	1.152***
	(0.006)	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.008)
N	71,397	71,051	68,319	68,196	65,755	62,587	58,269	58,241	55,502

Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors computed by 100 bootstrap replications.

APPENDIX F: TAX EFFECTS OF THE MINIMUM WAGE UNDER ALTERNATIVE ASSUMPTIONS

To provide an idea of the importance of the unemployment effects on the matter at hand, I will also compute the effects of the minimum wage on taxes based on a different model. In this version, I will force the unemployment effects to be equal to zero. By doing so, I no longer need to assume the continuity of the latent wage distribution. Formally, the model operates as follows. I will retain Assumptions 6 (independence) and 3 (no spillovers). Assumption 4 will be modified to force $\pi_u = 0$:

ASSUMPTION 4 *No Unemployment Effects*

Under the minimum wage, a fraction π_d of workers will earn the same wage as in the latent wage distribution. The remaining fraction will earn the minimum wage. These fractions can be sector-specific as in the baseline model. Note that there is no Assumption 1 (continuity) in this case. Under these assumptions, the observed wage density will relate to the latent density by the following equation:

$$f(w) = \begin{cases} \pi_d f_0(w) & \text{if} \quad w < m \\ (1-\pi_d)F_0(m) & \text{if} \quad w = m \\ f_0(w) & \text{if} \quad w > m, \end{cases}$$

where $f_0(w)$ is the latent wage distribution based on this different set of assumptions. In this case, we only need to estimate π_d. One way to do so is by recognizing that in this case:

$$\pi_d = \frac{Pr[W < m]}{Pr[W < m] + Pr[W = m]}.$$

Therefore, a consistent estimator can be constructed by plugging in the maximum likelihood estimator of the respective quantity. With an estimate of π_d, the latent wage density can be easily estimated by properly reweighting the observed wage density. Then, the tax effects of the minimum wage can be computed under the "no unemployment" assumption, given by:

$$R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1) = 1]}{Pr[S(0) = 1]} \cdot \frac{E(\tau(W(1))W(1)|S(1) = 1)}{E(\tau(W(0))W(0)|S(0) = 1)}.$$

This is exactly the same expression as before without the unemployment component c. Importantly, the expected wages under the latent distribution also change, as the estimate of the latent distribution is different under this different set of assumptions. Table II reports the estimates of R for the years from 2001 to 2009. The estimates under the assumption of no-unemployment indicate that the minimum wage has a sufficiently strong effect on average wages to compensate for the reduction in the share of the formal sector due to sector transition. Moreover, note that the for the same data, the implied effect of the minimum wage on the average wages of those employed is, as expected, smaller when one assumes the absence of unemployment effects.