

ESTIMATING THE MEAN OF A CORRELATED BINARY SEQUENCE  
WITH AN APPLICATION TO DISCRETE EVENT SIMULATION

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This paper discusses a procedure for interval estimation of the mean  $\theta$  of a correlated binary (0,1) sequence. The method assumes that the sequence is strictly stationary and that a particular string of  $m$  binary digits is a recurrent event in the sequence, where  $m \geq 1$  is unknown. Of the  $2^m$  choices for the possible recurrent events, the strings of all zeros and of all ones are examined.

For each  $m=1,2,\dots$  the sequence is demarcated by entrance to the recurrent event. The subsequences between the demarcation points thus form independent epochs by assumption. Classical techniques then yield variance estimates for the number of ones and zeros in the epoch as well as an estimate of the covariance of the ones and zeros. A quadratic equation in  $\theta$  is solved to obtain an interval estimate.

Each string of all ones or all zeros examined yields a  $1-\alpha$  confidence interval. The intervals are intersected to obtain shorter intervals with confidence greater than  $1-2\alpha$ . Since each  $m=1,2,\dots$  yields an interval, a conservative rule is developed to determine the

$m$  whose interval is finally used. This rule is based upon the empirical run lengths in the binary sequence.

The procedure is then applied to interval estimation of the fractile for the waiting time distribution in a simulation of the M/M/1 queue with activity level 0.9. For  $\theta=0.1$  and 0.5 the proposed method worked well. For  $\theta=0.9$  results showed some degradation. An error analysis led to a set of recommendations for keeping performance in practice close to the desired theoretical levels. An appendix describes algorithms for computing the critical quantities upon which the proposed method relies.