

Estimating the volume of tephra deposits: A new simple strategy

BONADONNA, Costanza, COSTA, A.

Abstract

Volume determination of tephra deposits is necessary to the characterization of active volcanoes, with obvious implications for environmental and climatic impact, estimation of magma-production rate, long-term hazard assessments and forecasting of future eruptions. Several methods have been proposed that mainly include the integrations of various deposit-thinning relationships and the inversion of field observations using computational models. Regardless of their strong dependence on tephra-deposit exposure, empirical integrations of deposit-thinning trends still represent the most widely adopted strategy due to their practical and fast application. The choice of best-fitting trends (e.g., exponential and power-law thinning on semi-log plots) has been subject of lively debate because they are all characterized by various advantages and disadvantages. We propose a new empirical method that is based on the Weibull distribution and shows a better agreement with observed data reconciling the debate on the use of the exponential versus power-law methods. Nonetheless, we also show that all empirical methods used to derive [...]

Reference

BONADONNA, Costanza, COSTA, A. Estimating the volume of tephra deposits: A new simple strategy. *Geology*, 2012, vol. 40, no. 5, p. 415-418

DOI : 10.1130/G32769.1

Available at:

<http://archive-ouverte.unige.ch/unige:19461>

Disclaimer: layout of this document may differ from the published version.



UNIVERSITÉ
DE GENÈVE

Geology

Estimating the volume of tephra deposits: A new simple strategy

C. Bonadonna and A. Costa

Geology published online 19 March 2012;
doi: 10.1130/G32769.1

Email alerting services click www.gsapubs.org/cgi/alerts to receive free e-mail alerts when new articles cite this article

Subscribe click www.gsapubs.org/subscriptions/ to subscribe to *Geology*

Permission request click <http://www.geosociety.org/pubs/copyrt.htm#gsa> to contact GSA

Copyright not claimed on content prepared wholly by U.S. government employees within scope of their employment. Individual scientists are hereby granted permission, without fees or further requests to GSA, to use a single figure, a single table, and/or a brief paragraph of text in subsequent works and to make unlimited copies of items in GSA's journals for noncommercial use in classrooms to further education and science. This file may not be posted to any Web site, but authors may post the abstracts only of their articles on their own or their organization's Web site providing the posting includes a reference to the article's full citation. GSA provides this and other forums for the presentation of diverse opinions and positions by scientists worldwide, regardless of their race, citizenship, gender, religion, or political viewpoint. Opinions presented in this publication do not reflect official positions of the Society.

Notes

Advance online articles have been peer reviewed and accepted for publication but have not yet appeared in the paper journal (edited, typeset versions may be posted when available prior to final publication). Advance online articles are citable and establish publication priority; they are indexed by GeoRef from initial publication. Citations to Advance online articles must include the digital object identifier (DOIs) and date of initial publication.

Estimating the volume of tephra deposits: A new simple strategy

C. Bonadonna¹ and A. Costa^{2,3}

¹University of Geneva, 13 rue des Maraîchers, 1205 Geneva, Switzerland

²University of Reading, P.O. Box 217, Reading, Berkshire RG6 6AH, UK

³Istituto Nazionale di Geofisica e Vulcanologia, Via Diocleziano 328, 80125 Naples, Italy

ABSTRACT

Volume determination of tephra deposits is necessary for the assessment of the dynamics and hazards of explosive volcanoes. Several methods have been proposed during the past 40 years that include the analysis of crystal concentration of large pumices, integrations of various thinning relationships, and the inversion of field observations using analytical and computational models. Regardless of their strong dependence on tephra-deposit exposure and distribution of isomass/isopach contours, empirical integrations of deposit thinning trends still represent the most widely adopted strategy due to their practical and fast application. The most recent methods involve the best fitting of thinning data using various exponential segments or a power-law curve on semilog plots of thickness (or mass/area) versus square root of isopach area. The exponential method is mainly sensitive to the number and the choice of straight segments, whereas the power-law method can better reproduce the natural thinning of tephra deposits but is strongly sensitive to the proximal or distal extreme of integration. We analyze a large data set of tephra deposits and propose a new empirical method for the determination of tephra-deposit volumes that is based on the integration of the Weibull function. The new method shows a better agreement with observed data, reconciling the debate on the use of the exponential versus power-law method. In fact, the Weibull best fitting only depends on three free parameters, can well reproduce the gradual thinning of tephra deposits, and does not depend on the choice of arbitrary segments or of arbitrary extremes of integration.

INTRODUCTION

One of the main ways volcanologists categorize the volume and explosivity of the world's major volcanic eruptions is through the analysis of tephra distribution. This is because tephra deposits retain a large amount of important information related to the dynamics and physical parameters of the associated volcanic eruptions (e.g., Pyle, 1989; Walker, 1973). One of the most important parameters that can be derived from the analysis of tephra deposits is the erupted volume, which is essential for the assessment of the associated hazards (e.g., Volcanic Explosivity Index, VEI; Newhall and Self, 1982). Nonetheless, the calculation of erupted volume is complicated by (1) the nonuniversal relationship of the deposit thinning with distance from the vent, (2) the poor preservation and accessibility to significant parts of tephra deposits (limited outcrops and/or tephra dispersal often over large water bodies), and (3) the difficulty in extrapolating thickness decay patterns of the medial portion of deposit, which is typically well preserved, to both proximal and distal areas. Several empirical volume calculation methods have been proposed over the past 40 years, ranging from the analysis of crystal/glass ratio of large pumices to various integration methods of thickness-versus-distance-from-the-vent relations, such as the two segments on a log-log plot, the trapezoidal rule, and the recent methods based on the exponential and power-law thinning on a semilog plot (see Fierstein and Nathenson [1992], Bonadonna and Houghton [2005], and Gonzalez and De la Cruz

[2010] for a review). As an alternative, calculation of erupted mass has also been approached by solving an inverse problem for tephra loads through the use of both analytical and computational advection-diffusion models (e.g., Connor and Connor, 2006). These inversion strategies are very promising, but require larger computational resources and have intrinsic limitations mainly related to (1) the difficulty of describing deposition in the very proximal area (dominated by column dynamics) and medial to distal area when dominated by aggregation, and (2) the common assumption of a uniform wind profile. In contrast, the integration of the thickness-versus-distance relationship based on various assumptions is of easier and faster application, but each empirical method has proved specific limitations. In particular, the exponential integration has been a subject of lively debate, as it was suggested that, even though the empirical fit of one or two exponential segments displays a good agreement with some tephra deposits, the volume can be significantly underestimated when the distal data are missing (e.g., Fierstein and Nathenson, 1992, 1993; Pyle, 1989, 1995; Rose, 1993). In fact, more recent studies have shown that well-preserved tephra deposits do not follow a simple exponential decay because distal ash settles differently (e.g., Rose, 1993; Sparks et al., 1992). The power-law fit and the use of at least three exponential segments were introduced to better describe the thinning of well-preserved deposits (Bonadonna and Houghton, 2005). In addition, numerical models also indicate a much

more gradual deposit thinning than predicted by one or two exponential segments, especially for eruptions that produce large amounts of both volcanic ash and lapillus-size particles (Bonadonna et al., 1998; Sparks et al., 1992). However, three segments cannot always be identified, especially for poorly preserved deposits, and the power-law fit cannot be integrated between zero and infinity. This paper presents a new fast empirical method, the fit of deposit thinning through a Weibull distribution, which combines the advantages of the exponential and power-law fit of thickness-versus-distance data on semilog plots.

EMPIRICAL DESCRIPTION OF TEPHRA-DEPOSIT THINNING

Volumes of tephra deposits can be derived from the following relationship:

$$V = \int_0^{\infty} T \, dA = \int_0^{\infty} T \, dx^2 = \int_0^{\infty} T(x) x \, dx, \quad (1)$$

where T and x are the thickness and the square root of the isopach area A , respectively. A relationship between T and x can be empirically determined from field observations. Here we propose a new method based on the assumption of a Weibull distribution between thickness and square root of isopach areas, which reconciles the positive features of the exponential and power-law methods. Specifically, the function $xT(x)$ can be described by an even more general distribution (e.g., log-logistic), but the Weibull distribution is chosen because it represents a generalization of the exponential distribution, has the minimum number of parameters (three free parameters), and appears to be able to reproduce a large number of observations.

The Weibull method is based on the assumption that thickness scales with square root of the isopach area according to the following relationship:

$$T = \theta(x/\lambda)^{n-2} \exp\left[-(x/\lambda)^n\right], \quad (2)$$

where λ represents the characteristic decay length scale of deposit thinning (typically expressed in kilometers), θ represents a thickness scale (typically expressed in centimeters; note that $\theta = e \cdot T(\lambda)$ where $e \approx 2.718$ denotes the Euler-Napier constant), and n is a shape parameter (dimensionless). For $n = 1$, the exponential relationship for $xT(x)$ is recovered [for an alternative formulation that recovers exactly the exponential thinning for $T(x)$ see Appendix

TABLE 1. COMPARISON OF VOLUME (in 10⁶ m³) CALCULATED BASED ON THE INTEGRATION OF EXPONENTIAL, POWER-LAW, AND WEIBULL FIT AND ON OTHER METHODS FOR ERUPTIONS WITH VOLCANIC EXPLOSION INDEX (VEI) BETWEEN 1 AND 6

Eruption [no. of isopach lines]	Exponential (± mean error) [no. of exp. segments]	Power law (± mean error) [m]	Weibull (± mean error)	Other methods
Minoan (3.6 ka) [7]	4.4E+4 (21%) [2]	8.7E+4 (71%) [0.6]	4.2E+4 (22%)	2.8E+4
Taupo (A.D. 186) [5]	6.7E+3 (21%) [1]	2.6E+4 (42%) [1.2]	1.2E+4 (14%)	6.4E+3
Askja D (1875) [6]	3.0E+2 (17%) [2]	2.2E+3 (39%) [1.1]	2.1E+3 (26%)	6.0E+2
Fontana Lapilli (>60 ka) [5]	6.0E+2 (22%) [1]	5.0E+3 (37%) [1.5]	1.9E+3 (25%)	2.7E+3 ^a
Hudson (1991) [9]	6.9E+3 (19%) [3]	6.1E+3 (47%) [1.7]	3.8E+3 (36%)	7.6E+3
Mount St. Helens (18 May 1980) [15]	1.1E+3 (15%) [2]	1.2E+3 (99%) [1.3]	1.0E+3 (26%)	1.4E+3 ^b 1.1E+3 ^c
Novarupta A (1912) [7]	5.2E+3 (18%) [2]	6.6E+3 (39%) [1.7]	5.5E+3 (25%)	6.1E+3
Novarupta B (1912) [6]	2.5E+3 (19%) [2]	5.6E+3 (58%) [1.3]	2.5E+3 (19%)	2.7E+3
Novarupta CDE (1912) [8]	2.7E+3 (17%) [3]	3.9E+3 (52%) [1.7]	2.6E+3 (31%)	4.8E+3
Novarupta FGH (1912) [10]	7.5E+3 (22%) [3]	1.2E+4 (53%) [1.2]	7.8E+3 (30%)	3.4E+3
Quizapu (1932) [11]	9.5E+3 (62%) [3]	1.1E+4 (56%) [1.9]	9.0E+3 (59%)	
Santa Maria (1902) [6]	9.2E+3 (21%) [1]	4.8E+4 (64%) [0.8]	8.0E+3 (21%)	2.0E+3
Agnano-Monte Spina B1 (~4100 yr B.P.) [4]	1.9E+2 (17%) [1]	1.5E+3 (45%) [1.1]	1.6E+2 (16%)	3.6E+3 ^d
Agnano-Monte Spina D1 (~4100 yr B.P.) [3]	2.8E+2 (11%) [1]	3.8E+3 (52%) [0.7]	2.2E+2 (11%)	2.8E+3 ^d
Chaiten β (6 May 2008) [12]	2.1E+2 (24%) [3]	1.6E+2 (85%) [1.5]	1.6E+2 (26%)	
Cotopaxi L3 (~820 yr B.P.) [6]	6.0E+2 (18%) [1]	1.8E+3 (42%) [1.8]	5.4E+2 (14%)	2.4E+3 ^a
Cotopaxi L5 (~1180 yr B.P.) [6]	3.0E+2 (17%) [2]	4.5E+2 (32%) [2.1]	2.3E+2 (24%)	4.7E+2 ^a
Fogo (1563) [5]	4.0E+2 (15%) [2]	5.4E+3 (48%) [1.1]	4.8E+2 (13%)	
Hatepe (A.D. 186) [7]	1.0E+3 (15%) [2]	1.5E+3 (39%) [1.4]	5.6E+2 (14%)	6.0E+3
Hekla (1947) [11]	2.0E+2 (18%) [3]	1.5E+2 (55%) [2.2]	2.4E+2 (54%)	2.0E+2
Pululagua (2450 yr B.P.) [9]	3.0E+2 (16%) [1]	1.4E+3 (47%) [1.5]	3.2E+2 (14%)	3.0E+2 ^f
Tarawera (1886) [6]	5.0E+2 (17%) [1]	6.7E+2 (47%) [2.2]	4.8E+2 (14%)	2.4E+4
Cerro Negro (1992) [6]	2.0E+1 (24%) [1]	2.9E+1 (23%) [2.6]	4.5E+1 (12%)	3.0E+1 ^g
Fuego (1974) [6]	3.0E+1 (33%) [1]	4.4E+1 (29%) [2.5]	6.0E+1 (25%)	
Vesuvius AP3-B1 (2.7 ka) [3]	6.2E+0 (21%) [1]	2.0E+1 (34%) [2.0]	1.9E+1 (30%)	
Vesuvius U3 (A.D. 512) [6]	1.1E+1 (14%) [2]	2.2E+1 (23%) [2.0]	5.0E+1 (18%)	8.9E+0 ^h
Etna (1971) [7]	7.0E-1 (28%) [2]	8.0E-1 (18%) [2.6]	1.7E+0 (25%)	6.5E-1 ⁱ
Etna (1998) [10]	2.0E+0 (20%) [3]	2.0E+0 (25%) [2.3]	4.7E+0 (25%)	2.0E+0 ^j
Mount St. Helens (22 July 1980) [7]	6.0E+0 (68%) [2]	4.6E+0 (47%) [2.3]	1.0E+1 (50%)	
Ruapehu (1996) [17]	4.0E+0 (33%) [3]	3.9E+0 (33%) [2.0]	4.4E+0 (11%)	
Montserrat (26 September 1997) [4]	5.0E-1 (23%) [1]	5.1E-1 (25%) [2.7]	5.3E-1 (25%)	
Montserrat (31 March 1997) [7]	2.0E-1 (15%) [1]	5.7E-1 (61%) [1.1]	1.7E-1 (11%)	
Montserrat (21 September 1997) [3]	4.0E-2 (25%) [1]	7.0E-1 (49%) [1.1]	4.7E-1 (11%)	

Note: Decreasing volcanic explosivity index (VEI) is indicated by alternating shading. Estimated uncertainty (mean error) is also indicated in parentheses (see text for details). Weibull fitting was obtained using the software Grace. Number of isopach contours, number of exponential segments, and absolute value of power-law exponent (*m*) are also indicated in square brackets. Distal limits for the power-law integrations are 700–1000 km for VEI 5 and VEI 6; 300–500 km for VEI 4, VEI 3, and VEI 2; 25–50 km for VEI 1; proximal integration limit is calculated as in Equation 7 of Bonadonna and Houghton (2005). Volume derived with other methods (other than exponential, power-law, and Weibull integrations) are described in Bonadonna and Houghton (2005), with the exception of the following:

^aFontana Lapilli: derived solving an inverse problem for mass/area using TEPHRA2.

^bMount St. Helens, 18 May 1980: based on measurements of isopach area and thickness inside the 0.5 mm contour (Sarna-Wojcicki et al., 1981).

^cMount St. Helens, 18 May 1980: derived by best-fitting column height, mass/area, and eruption duration using FALL3D with a Buoyancy Plume Theory–based model and assuming a deposit density of 450–550 kg m⁻³ (Folch et al., 2010).

^dAgnano-Monte Spina (AMS) Layer B1 and D1: derived solving an inverse problem for both mass/area and grain size using HAZMAP (Costa et al., 2009).

^eCotopaxi Layer 3 and 5: derived solving an inverse problem for mass/area using TEPHRA2 (Biass and Bonadonna, 2011).

^fPululagua: derived solving an inverse problem for both mass/area and grain size using TEPHRA2 (Volentik et al., 2010).

^gCerro Negro 1992: derived solving an inverse problem for mass/area using TEPHRA2 (Connor and Connor, 2006); erupted mass (3.5 × 10¹⁰ kg) was converted into volume based on an average deposit density of 1170 kg m⁻³ (C.B. Connor, 2011, personal commun.).

^hVesuvius A.D. 512: derived using one exponential segment (Cioni et al., 2011).

ⁱEtna 1971: calculated by Booth and Walker (1973).

^jEtna 1998: derived solving an inverse problem for mass/area using both TEPHRA2 and FALL3D (Bonadonna and Costa, 2012).

All references and Weibull parameters (θ , λ , n) are described in Table DR1 (see text footnote 1).

DR1 in the GSA Data Repository¹). In accordance with Equations 1 and 2, the volume of tephra deposits can be calculated as

$$V = \int_0^{\infty} T dA = 2 \int_0^{\infty} T(x) x dx$$

$$= \frac{2\theta\lambda^2}{n} \int_0^{\infty} \frac{n}{\lambda} \left(\frac{x}{\lambda}\right)^{n-1} e^{-\left(\frac{x}{\lambda}\right)^n} dx$$

$$= \frac{2\theta\lambda^2}{n} \left[1 - e^{-\left(\frac{x}{\lambda}\right)^n} \right]_0^{\infty} = \frac{2\theta\lambda^2}{n}, \quad (3)$$

where θ , λ , and n can be empirically determined from observations

$$\left(\text{note that } V(x) = \frac{2\theta\lambda^2}{n} \left[1 - e^{-\left(x/\lambda\right)^n} \right]\right).$$

RESULTS

The Weibull method was applied to 33 tephra deposits associated with a VEI between 1 and 6, and the new estimates were compared with those of previous methods (Table 1; see also the Data Repository). The error associated with the empirical fit of observed thickness (i.e., relative mean square error, RMSE) was calculated for all techniques (i.e., exponential, power-law, and Weibull). When multiple exponential segments were used, the average error among the different segments was considered. In addition to RMSE, we have also accounted for a 10% error associated with the compilation of isopach maps, as derived from the specific case study of Cotopaxi volcano (Biass and Bonadonna, 2011). For the power-law method, the RMSE and the error related to the compilation of isopachs were added to the relative error due to the choice of different distal integration limits (see Table 1). Nonetheless, such a total mean error does not account for uncertainty due to paucity of data (e.g., lack of proximal or distal data). In fact, scarce data are typically associated with low fitting error, but with high uncertainties. Uncertainties associated with the lack of information are discussed below. In order to find the best fit for the Weibull distribution of tephra-deposit thinning, a range needs to be chosen for each parameter θ , λ , and n . Our data set indicates that the best initial ranges are 2 for θ , λ , and n are 0.1–

¹GSA Data Repository item 2012116, Tables DR1–DR3, Figures DR1–DR4, Appendix DR1 (alternative formulation for recovering exponential thinning for $n = 1$), a description of the Weibull-method spreadsheet, and a Weibull method Excel spreadsheet (template for the calculation of erupted volume and mass based on the integration of the Weibull fit), is available online at www.geosociety.org/pubs/ft2012.htm, or on request from editing@geosociety.org or Documents Secretary, GSA, P.O. Box 9140, Boulder, CO 80301, USA.

5000 cm, 0.1–1000 km, and 0.2–2, respectively. Best-fit parameters are chosen minimizing the following residual:

$$\sigma^2 = \sum_{i=1}^N w_i [T_i(\text{obs}) - T_i(\text{calc})]^2, \quad (4)$$

where w_i are weighting factors, N is the number of data points, and $T_i(\text{obs})$ and $T_i(\text{calc})$ represent the observed and calculated thickness values, respectively. Weighting factors w_i depend upon the distribution of random errors in the dependent variable. When $w_i = 1$, all values have same weight (i.e., uniform error), and therefore, larger thicknesses have greater effect on the best fit. When $w_i = 1/T_i^2(\text{obs})$, the relative squared errors are minimized (i.e., proportional error). The use of $w_i = 1/T_i(\text{obs})$ is a compromise between minimizing the uniform and proportional errors. In our calculations, we used $w_i = 1/T_i^2(\text{obs})$, except for Askja D, Montserrat (26 September 1997), and Vesuvius AP3-B1, for which we used $w_i = 1/T_i(\text{obs})$. Generally, the best weighting factor is the one that yields a random residual plot with no functional dependencies.

The Weibull function gives an excellent fit for all deposits analyzed. Figures 1A–1C show some selected examples of different VEIs and different scales of square root of isopach area. All the studied cases are reported in Figure DR1 in the Data Repository. The Weibull function is more flexible than a fit with multiple exponen-

tial segments, as it does not require the arbitrary choice of both position and number of straight segments (e.g., Ruapehu 1996 in Fig. 1D) and, similarly to the power-law fit, can reproduce the natural thinning curvature when data are missing (e.g., proximal thinning of Taupo A.D. 186; Fig. 1D). With respect to the power law, the Weibull method shows a less gradual thinning and a better fit of the actual data (Fig. 1D), and does not depend on the integration extremes as it can be integrated between zero and infinity. Volumes estimated based on the integration of the Weibull function are mostly in between the volumes estimated by integrating the exponential fit and the power-law fit (Table 1), reflecting the typical Weibull-type thinning that is more gradual than the exponential but more rapid than the power law (Fig. 1D). The two Weibull empirical parameters λ and θ are strongly related to the eruption characteristics, with λ being a measure of the rapidity of thinning and, therefore, increasing with the erupted volume (Figs. DR2 and DR3). In particular, λ is <20 km for all small and moderate eruptions (i.e., VEI ≤ 3 ; Fig. DR3), while θ tends to increase with a decrease of λ within each VEI class (Figs. DR2 and DR3).

Following the method described by Bonadonna and Houghton (2005), a sensitivity analysis was carried out on two deposits of different magnitude (i.e., Ruapehu 1996 and Novarupta

CDE 1912) to investigate the sensitivity of the Weibull integration when proximal, medial, and distal data are sequentially removed (Fig. DR4; Tables DR2 and DR3). For the Ruapehu 1996 test (VEI 2), when most proximal or most distal data are removed, the Weibull method tends to underestimate by ~50% of the volume calculated based on the complete data set, whereas, when only the medial are missing, and therefore the tails of the thinning are better described, the Weibull function overestimates by only 14% (Table DR2). Bonadonna and Houghton (2005) had already shown that the exponential integration of various segments underestimates the volume by ~40%–70% in all cases investigated, and the power-law integration is relatively stable when medial and distal data are missing, but overestimates up to nearly five times when proximal data are removed. The same test carried out on a larger deposit generated by the Novarupta CDE 1912 eruption (VEI 5) shows that discrepancies are within a similar order of magnitude (Table DR3). In particular, the Weibull integration results in discrepancies of ~58% and 99% when proximal and distal data are removed, respectively. Finally, the exponential integration method depends on two or more parameters (i.e., coefficient and exponent for each of the exponential segments) and the power-law integration method depends on four parameters (i.e., coefficient, exponent, and two arbitrary integration extremes). In contrast, the Weibull integration method depends on three parameters only (i.e., λ , θ , and n), and therefore, the deposit thinning can be described with at least three points distributed over the whole distribution. As an example, differences in volume derived from the Weibull integration in the case when only three points are considered (one in proximal, one in medial, and one in distal area) are 40% and 18% for the Ruapehu 1996 and Novarupta CDE 1912 cases, respectively.

DISCUSSION AND CONCLUSIONS

The determination of tephra-deposit volumes is crucial to the characterization of active volcanoes, with obvious implications for environmental and climatic impact, estimation of magma production rate, long-term hazard assessments, and forecasting of future eruptions. Unfortunately, all empirical methods used to derive erupted volume based on integration of deposit thinning strongly depend on the available data. In fact, thinning of distal deposits cannot be easily extrapolated based on the thinning of the proximal deposit because distal and proximal sedimentation are controlled by different regimes (i.e., laminar to turbulent; Bonadonna et al., 1998). For this reason it is very important to assess uncertainties related to each volume estimation method. The errors associated with the exponential and the Weibull fit are comparable

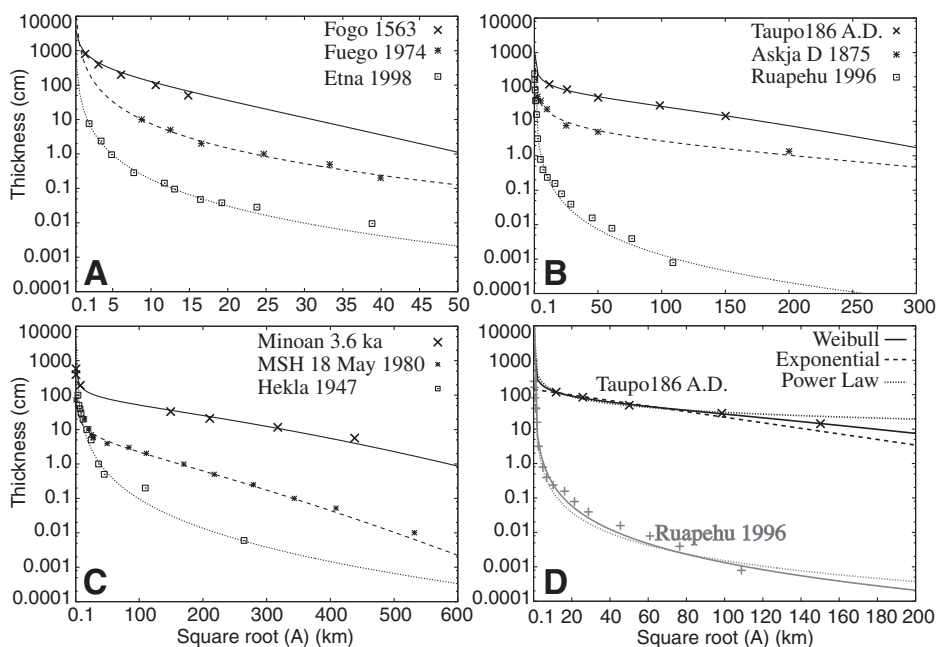


Figure 1. Semilog plots of thickness versus square root of isopach area showing the Weibull best fit for tephra deposits on different length scales. A: 50 km (Etna 1998, VEI 2; Fuego 1974, VEI 3; Fogo 1563, VEI 4). B: 300 km (Ruapehu 1996, VEI 2; Askja D 1875, VEI 5; Taupo A.D. 186, VEI 6). C: 600 km (Hekla 1947, VEI 4; Mount Saint Helens 1980, VEI 5; Minoan 3.6 ka, VEI 6) from the vent. D: Comparison of Weibull, power-law, and exponential fit for Taupo A.D. 186. For Ruapehu 1996, only the comparison between Weibull and power law is shown, as one exponential segment cannot be fit. References are in Table DR1 in the Data Repository (see footnote 1).

(Table 1). However, our sensitivity tests indicate that the Weibull function is less sensitive when proximal, medial, or distal data are missing. In fact, when data are not well distributed or only a few isopach contours are available, uncertainties are typically larger than errors reported in Table 1 (typically up to ~100%; see Tables DR2 and DR3). Moreover, the multiple-exponential-segment method tends to drastically underestimate the error when only few points are used for each segment. Finally, the 10% error we considered for the subjective compilation of isopach maps has to be considered as a minimum estimation, and further studies are needed to better characterize it (e.g., Cioni et al., 2011). Concerning the comparison with other empirical methods, data fitting with one or more exponential segments on a semilog plot is straightforward but requires an arbitrary choice of segments and can often underestimate the volume by a factor of 2.5 when proximal and/or distal data are missing (e.g., Pyle, 1995; Bonadonna and Houghton, 2005; this work). In contrast, the power-law fit can better reproduce the natural thinning of tephra deposits, but cannot be integrated between zero and infinity and can significantly overestimate the volume (up to a factor of 5–6) when proximal or distal data are missing (Bonadonna and Costa, 2012; Bonadonna and Houghton, 2005; this work). In detail, the power-law method is very sensitive to the proximal integration extreme for small deposits (i.e., mostly VEI ≤ 3 ; power-law exponent > 2 ; Table 1) or the distal integration extreme for large deposits (i.e., mostly VEI > 3 ; power-law exponent < 2 ; Table 1) (Bonadonna and Costa, 2012). In brief, the use of the Weibull method is recommended because it shows a better fit with observed thinning and, similarly to the power-law function, it reproduces the natural gradual thinning of tephra deposits even when there are only a few data points available (i.e., when only one exponential segment can be identified), but is associated with smaller errors than the power-law method when large parts of the deposit are missing in either the proximal or distal area (typically within a factor of 2). The method can be implemented by using several free software packages (e.g., Grace, Gnuplot, R), but we have also compiled a Microsoft Excel® template for the easy calculation of both erupted volume and mass in case thickness and mass/area data are used, respectively (see the Excel file in the Data Repository). All bulk vol-

umes can be easily converted into erupted mass, and therefore magnitude, if the deposit density is known. Nonetheless, due to the typical large uncertainties described above, volume and magnitude of explosive eruptions cannot be considered as absolute values regardless of the technique used. It is important that various empirical and analytical methods are applied in order to assess such an uncertainty.

ACKNOWLEDGMENTS

We are grateful to C. Connor, R. Cioni, and two anonymous reviewers for constructive suggestions that improved the manuscript.

REFERENCES CITED

- Biass, S., and Bonadonna, C., 2011, A quantitative uncertainty assessment of eruptive parameters derived from tephra deposits: The example of two large eruptions of Cotopaxi volcano, Ecuador: *Bulletin of Volcanology*, v. 73, p. 73–90, doi:10.1007/s00445-010-0404-5.
- Bonadonna, C., and Costa, A., 2012, Modeling of tephra sedimentation from volcanic plumes, in Fagents, S., et al., eds., *Modeling volcanic processes: The physics and mathematics of volcanism*: Cambridge, UK, Cambridge University Press, ISBN 9780521895439 (in press).
- Bonadonna, C., and Houghton, B.F., 2005, Total grain-size distribution and volume of tephra-fall deposits: *Bulletin of Volcanology*, v. 67, p. 441–456, doi:10.1007/s00445-004-0386-2.
- Bonadonna, C., Ernst, G.G.J., and Sparks, R.S.J., 1998, Thickness variations and volume estimates of tephra fall deposits: The importance of particle Reynolds number: *Journal of Volcanology and Geothermal Research*, v. 81, p. 173–187, doi:10.1016/S0377-0273(98)00007-9.
- Booth, B., and Walker, G.P.L., 1973, Mount Etna and the 1971 eruption: *Philosophical Transactions of the Royal Society of London*, v. 274, p. 147–151.
- Cioni, R., Bertagnini, A., Andronico, D., Cole, P.D., and Mundula, F., 2011, The 512 AD eruption of Vesuvius: Complex dynamics of a small scale subplinian event: *Bulletin of Volcanology*, v. 73, p. 789–810, doi:10.1007/s00445-011-0454-3.
- Connor, L.G., and Connor, C.B., 2006, Inversion is the key to dispersion: Understanding eruption dynamics by inverting tephra fallout, in Mader, H., et al., eds., *Statistics in volcanology*, volume 1: Special Publications of IAVCEI: London, Geological Society, p. 231–242.
- Costa, A., Dell'Erba, F., Di Vito, M.A., Isaia, R., Macedonio, G., Orsi, G., and Pfeiffer, T., 2009, Tephra fallout hazard assessment at the Campi Flegrei caldera (Italy): *Bulletin of Volcanology*, v. 71, p. 259–273.
- Fierstein, J., and Nathenson, M., 1992, Another look at the calculation of fallout tephra volumes: *Bulletin of Volcanology*, v. 54, p. 156–167, doi:10.1007/BF00278005.

- Fierstein, J., and Nathenson, M., 1993, Reply to comment by W.I. Rose: *Bulletin of Volcanology*, v. 55, p. 375–378, doi:10.1007/BF00301149.
- Folch, A., Costa, A., Durant, A., and Macedonio, G., 2010, A model for wet aggregation of ash particles in volcanic plumes and clouds: 2. Model application: *Journal of Geophysical Research*, v. 115, B09202, doi:10.1029/2009JB007176.
- Gonzalez, A.O., and De la Cruz, S., 2010, A simple semi-empirical approach to model thickness of ash-deposits for different eruption scenarios: *Natural Hazards and Earth System Sciences*, v. 10, p. 2241–2257, doi:10.5194/nhess-10-2241-2010.
- Newhall, C.G., and Self, S., 1982, The Volcanic Explosivity Index (VEI)—An estimate of explosive magnitude for historical volcanism: *Journal of Geophysical Research (Oceans and Atmospheres)*, v. 87, p. 1231–1238, doi:10.1029/JC087iC02p01231.
- Pyle, D.M., 1989, The thickness, volume and grain-size of tephra fall deposits: *Bulletin of Volcanology*, v. 51, p. 1–15, doi:10.1007/BF01086757.
- Pyle, D.M., 1995, Assessment of the minimum volume of tephra fall deposits: *Journal of Volcanology and Geothermal Research*, v. 69, p. 379–382, doi:10.1016/0377-0273(95)00038-0.
- Rose, W.I., 1993, Comment on 'another look at the calculation of fallout tephra volumes' by Judy Fierstein and Manuel Nathenson: *Bulletin of Volcanology*, v. 55, p. 372–374, doi:10.1007/BF00301148.
- Sarna-Wojcicki, A.M., Shipley, S., Waitt, J.R., Dzuri-sin, D., and Wood, S.H., 1981, Areal distribution thickness, mass, volume, and grain-size of air-fall ash from the six major eruptions of 1980, in Lipman, P.W., and Mullineaux, D.R., eds., *The 1980 eruptions of Mount St. Helens*, Washington: U.S. Geological Survey Professional Paper 1250, p. 577–600.
- Sparks, R.S.J., Bursik, M.I., Ablay, G.J., Thomas, R.M.E., and Carey, S.N., 1992, Sedimentation of tephra by volcanic plumes. 2: Controls on thickness and grain-size variations of tephra fall deposits: *Bulletin of Volcanology*, v. 54, p. 685–695, doi:10.1007/BF00430779.
- Volentik, A.C.M., Bonadonna, C., Connor, C.B., Connor, L.J., and Rosi, M., 2010, Modeling tephra dispersal in absence of wind: Insights from the climactic phase of the 2450 B.P. Plinian eruption of Pululagua volcano (Ecuador): *Journal of Volcanology and Geothermal Research*, v. 193, p. 117–136.
- Walker, G.P.L., 1973, Explosive volcanic eruptions—A new classification scheme: *Geologische Rundschau*, v. 62, p. 431–446, doi:10.1007/BF01840108.

Manuscript received 23 August 2011

Revised manuscript received 30 November 2011

Manuscript accepted 6 December 2011

Printed in USA