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Estimating Total Suspended Sediment Yield With Probability Sampling

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The "Selection At List Time" (SALT) scheme controls sampling of concentration for estimating total suspended sediment yield. The probability of taking a sample is proportional to its estimated contribution to total suspended sediment discharge. This procedure gives unbiased estimates of total suspended sediment yield and the variance of the estimate while automatically emphasizing sampling at higher flows. When applied to real data with known yield, the SALT method underestimated total suspended sediment yield by less than 1%, whereas estimates by the flow duration sediment rating curve method averaged about 51 % underestimation. Implementing the SALT scheme requires obtaining samples with a pumping sampler, stage sensing device, and small battery-powered computer.

INTRODUCTION

Measuring and estimating suspended sediment yields in rivers has long been subject to confusion and uncertainty. Many methods have been developed for collecting data and estimating yields, a fact that suggests the lack of a compelling measurement methodology. The main reason for this situation is the lack of a theoretical framework that defines when discrete samples of suspended sediment should be taken.

The ideal way to estimate the suspended sediment yield of rivers would be to measure suspended sediment discharge continuously. Such data could be integrated over the monitoring period in a way similar to that used to obtain water yield from a hydrograph. There is no technique, however, to monitor suspended sediment discharge directly. A second approach is to measure suspended sediment concentration and water discharge continuously and use the product function as an estimate of suspended sediment discharge.

Obtaining continuous records of concentration, however, is subject to numerous problems. Such measurements are necessarily indirect; turbidity [*Walling,* 1977*a*; *Truhlar*, 1978; *Beschta,* 1980] and water-sediment density [*Skinner and Beverage,* 1982] are two quantities that can be related to suspended sediment concentration. Calibration of these quantities is a continuing problem, the instrumentation is expensive and subject to breakdown, and a 120-volt ac electrical power is usually required.

When cost, remoteness of sites, and technical difficulties preclude collecting continuous concentration data, the usual course is to measure water discharge continuously and to take occasional discrete water samples for gravimetric analysis of suspended sediment concentration. The samples are taken manually, or, more commonly in recent years, with automatic sampling equipment. Regardless of how the samples are collected, there remain the questions of when the measurements of concentration should be made, how they should be used to estimate the total yield, and what the properties of the estimates are.

Walling and Webb [1981] investigated a variety of methods for estimating total suspended yield and compared their properties using Monte Carlo techniques on a "known" sediment record. The tested combinations of estimation technique and data collection method ranged from 70% below to 40% above the true value. Most of the estimates were less than 60% of the correct value. The variance of the estimators tended to increase as the accuracy improved, thus cancelling the benefits, and no approach emerged as the ideal choice for all conditions.

These techniques can be termed nonstatistical because the sampling probabilities are not known. The estimators therefore cannot take the probability structure into account, resulting in bias (i.e., systematic overestimation or underestimation of true values) that depends on unknown and variable factors in the data collection process and on specific site conditions. Bias is particularly prevalent when measuring small flashy streams that drain mountainous terrain.

The other major shortcoming of nonstatistical estimators is that they do not allow a valid estimate of precision of the estimated total yields. This fact prevents making valid comparisons between treatments, setting sample size to obtain desired precision of the estimators, and efficient direction of the sampling process. (In this paper "sample size" refers to the number of population units in a statistical sample rather than to the volume of a sample of water used to determine concentration.)

This paper describes a new sampling strategy using variable probability sampling for monitoring suspended sediment to estimate total suspended sediment yield. The strategy gives estimates with known properties and provides a rational approach to planning and implementing suspended sediment sampling programs. The paper describes the basic sampling philosophy and its application to sampling suspended sediment, the estimators, estimation of sample size, and briefly it discusses how the method would be applied in the field. The method could also be applied to other flow related water quality variables.

VARIABLE PROBABILITY SAMPLING

The method presented here is dependent on the techniques of sample survey theory, that is, statistical methods designed for sampling finite populations. The finite population to be sampled and the units that comprise it will be defined and a sampling method selected to make the best use of the population structure.

The most basic form of probability sampling is called simple

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random sampling (SRS), where each sample of a given size has the same probability of being selected. Probability sampling, however, does not require that selection probabilities be equal, only that they be known. In many cases, using a strategy that restricts the random selection of sampled units according to relevant population characteristics can reduce sample size or improve precision of the estimators.- In this way, other available information can be brought to bear to improve the efficiency of the sampling scheme.

One sampling technique that restricts randomization is termed "sampling with probability proportional to size" (or PPS) [*Raj*, 1968]. "Size" in this context refers to the magnitude of the measured characteristic of the population units. Because these magnitudes are not known until after the sample is collected, and then only for the sampled units, PPS sampling depends on having an easily measured auxiliary variable known to be related to the variable of interest. The auxiliary variable must be easily measurable because it is this variable that defines selection probabilities and therefore must be measured for every unit in the finite population. The auxiliary variable contains outside information that is used to improve the sampling of the primary variable.

A complex relationship is required between the primary and auxiliary variables to make PPS efficient. It is not enough for the variables to simply be correlated. The auxiliary variable must be positively .correlated to the square of the primary variable divided by the auxiliary variable [*Raj,* 1968]. That is, if y is a primary variable, to have x be an effective auxiliary variable requires Corr $(x, y^2/x) > 0$. The magnitude of this correlation does not affect the unbiasedness of PPS estimators, but stronger correlations reduce the variance of the estimates.

Suppose an investigator wants to estimate the total volume of channel sediments stored in the tributaries of a watershed. If all tributaries cannot be measured, they can be sampled. If SRS is used, all tributaries would have the same chance of entering the sample. With tributaries of widely differing size, however, this approach can be very inefficient. Large tributaries, which contribute heavily to the total, would not be any more likely to enter the sample than small ones, which contribute little.

The investigator would like to preferentially select the important tributaries while still remaining within a well defined probability context. A reasonable auxiliary variable is tributary length which is likely to be related to the volume of stored sediment and it can easily be measured from maps or photos. The lengths of all tributaries are determined first. The tributaries are listed in any arbitrary order followed by their lengths, and the cumulative sums of the lengths are formed. Suppose n is the desired sample size. Then a set of n uniform random numbers is selected from 0 to the largest cumulative sum. For each. random number the tributary having the next larger cumulative sum is selected for the sample. Because the random numbers are selected uniformly, the probability of selecting any tributary is equal to its length divided by the total of the lengths of all tributaries. The larger tributaries therefore have a greater probability of being chosen for the sample: The estimators of total volume of stored sediment and its variance are weighted to account for the unequal, but known, probabilities of selection, making both statistics unbiased.

While this scheme will work well for many problems, it does require that all values of the auxiliary variable be known before sampling of the .primary variable can begin. This requires two traverses of the population: one to measure the auxiliary variable values of all units and one to measure the primary variable on those units selected for the sample. This procedure will not work therefore when sampling a time-dependent process such as suspended sediment. Any auxiliary variable would have to depend on conditions during the measurement period, and the values would not be known for all units until the end of the period, at which time it would be too late to sample.

An improved PPS sampling technique was developed to avoid this general problem [*Lahiri*,1951] and adapted to sampling forest tree volume [*Grosenbaugh,* 1964]. A futher refinement, also developed for forestry use, is called Selection At List Time (SALT) sampling [*Norick,* 1969]. SALT sampling provides a technique for creating a list of random numbers before the time period being monitored and for using these numbers to determine which units should be sampled as the auxiliary variable values become available during the process. The SALT estimators give unbiased estimates of the total and its variance.

USING SALT TO SAMPLE TOTAL SUSPENDED SEDIMENT YIELD

Basic Concepts

Several ideas must be developed to apply the SALT technique to estimating suspended sediment yields. The first of these is the definition of the finite population and the units that comprise it. The population must be composed of units that are nonoverlapping, exhaustive, and well defined for selection purposes. "Short" periods of time define the population units, and a measure of suspended sediment yield during the period is the characteristic of interest. A common duration (e.g., 5-30 min) must be chosen for the sampling periods for a given time period to be monitored [*Thomas,* 1983]. Let y_i be the measure of suspended sediment yield for the ith time period (i.e., population unit). Then

$$y_i = q_i c_i \Delta t K \tag{1}$$

where q_i *is* the water discharge rate and c_i is the suspended sediment concentration for the ith period, *At is* the time duration chosen for the sampling periods, and K is a constant to convert units. For example, if q_i is in cubic meters per second, c_i is in milligrams per liter, and the sampling period is 1800 s (i.e., 30 min), then $K = 10^{-3}$ (L/kg)/(m^3/mg) gives y_i in killigrams.

If the sampling period duration is short enough we can use the water discharge rate at the midpoint of the period for q_i. In a, similar way, c_i will be a discrete sample of suspended sediment concentration taken at the midpoint, usually with a pumping sampler. A sampling unit therefore is represented by the conditions at the midpoint of the sampling period. This means that the "sampled population" (i.e., the population of y_i) is not identical to the "target population" consisting of the continuous records. By adjusting the sampling period duration, however, these two populations cant be made to match as closely as desired.

The sampled population therefore consists of all y_i for the period being monitored. If resources were adequate, all values of y_i could be measured to determine the total-suspended yield. That is, if there are N sampling periods in the monitoring period, the "true" population total, Y, is given by

$$Y = \sum_{i=1}^{N} y_i \tag{2}$$

We henceforth assume that the sampling period duration has been chosen to satisfy the hydrologist that the target and sampled populations are sufficiently similar for the investigation in question.

An auxiliary variable that can be measured throughout the period being monitored is required to perform PPS (i.e., SALT) sampling on this population. Because water discharge is usually measured continuously, an ideal auxiliary variable is the common sediment rating function that expresses suspend ed sediment concentration as a function of the rate of water discharge. Let f be this empirically determined function and $\hat{c}_i$ denote the estimated concentration. Then

$$\hat{c}_i = f(q_i) \tag{3}$$

is an estimate of the suspended sediment concentration at the midpoint of the ith interval. We can now define x_i as an estimate of the suspended sediment discharge for the ith sampling period. That is,

$$x_i = q_i \, \hat{c}_i \, \Delta t K \tag{4}$$

which is identical to the formula for y_i except that c_i has been replaced by $\hat{c}_i$.

The value of x_i will be known for every sampling period in the period to be monitored. We can therefore define the total estimated suspended sediment yield, X, as

$$X = \sum_{i=1}^{N} x_i \tag{5}$$

There is a problem of having to know f before sampling can begin, but having to sample before f can be determined. In many basins, some sediment rating data will exist that can be used to make at least preliminary estimates of the rating function. These estimates can be revised as SALT data accumulate. As a last resort, data from nearby catchments can be used for tentative estimates until data from the monitored stream become available. The quality of f does not affect the unbiasedness of the estimator of suspended yield, but it does affect its variance. That is, the better f predicts c_i, the lower the variance of the estimate of the yield.

Preparing for Sampling

Accomplishing the SALT process in "real time" generally requires additional instrumentation at a gaging station. SALT sampling will usually be used at a station that has a continu ous stage recorder and a pumping sampler. Sampling periods should be short, especially on streams having highly variable suspended sediment concentrations, so determination of x_i will have to be done frequently. This can be accomplished by a small portable battery-powered computer and a float-operated transducer to "sense" stage. R. E. Eads et al. (unpublished manuscript, 1985) have designed an interface circuit that connects a precision potentiometer (attached to the chart recorder shaft) to measure stage, a pumping sampler, and a programmable calculator that controls the SALT sampling process. This system operates the SALT algorithm between station visits, collects information required to make the estimates, and also logs stage data for the period of record.

A set of random numbers must be selected before sampling each period to be monitored. This is done by making a preliminary estimate, Y', of Y, the total suspended sediment yield expected during the period to be monitored. To ensure that the random numbers cover a sufficiently large range to sample the expected yield of suspended sediment, we multiply by a

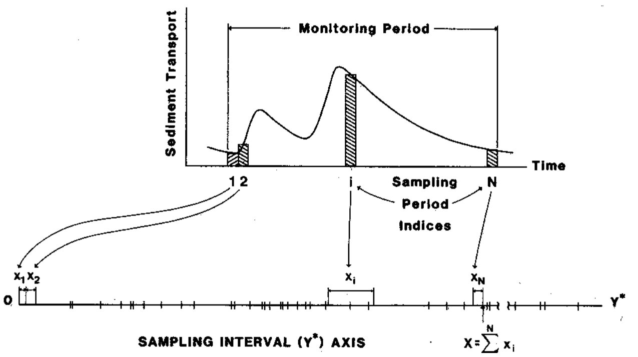

Fig. 1. Suspended sediment transport hydrograph and corresponding sampling interval axis for SALT (selection at list time) sampling. The correspondence is between the equal duration sampling periods on the time axis and the variable length intervals of estimated suspended sediment discharge on the sampling interval axis. Ticks on the sampling interval axis denote random sampling numbers.

factor, W, to obtain

$$Y^* = WY' \tag{6}$$

W is essentially a factor of safety ensuring a near-zero probability that the total estimated suspended sediment yield, X, is greater than Y^*. If X exceeds Y^*, the sampling algorithm will run out of random numbers. The magnitude of W will reflect the quality of existing data and the consequent uncertainty of the estimate Y', but it will usually be in the range from two to 10.

A procedure is described in a following section to establish n^*, the number of random numbers that must be preselected to obtain a specified level of performance for the estimators. By assuming, temporarily, that its value is known, n^* uniform random numbers are selected from the interval $(0, Y^*]$, where the parenthesis indicates exclusion of the boundary point from the interval, and the square bracket indicates inclusion. The actual selection is carried out in the calculator using a pseudorandom number generator. The random numbers are sorted into ascending order (to facilitate their use during sampling) and stored in the computer.

Sampling Algorithm

To understand the SALT sampling process, consider a schematic suspended sediment transport hydrograph and its associated "sampling" (or, Y^*) axis (Figure 1). The n^* random numbers occupy their places along the Y^* axis. N sampling periods occur along the time axis of the hydrograph during the monitored period, some fraction of which will actually be sampled (i.e., a concentration sample will be taken). For each time period, exactly one interval is formed on the Y^* axis using partial sums of the values of x_i. The lower bound of the interval associated with a given sampling period is the cumulative sum of x; through the previous period. The upper bound is that sum plus the x_i for the current period. That is, the interval I_i, formed on the Y^* axis for sampling period i, is given by

$$I_i = \left(\sum^{i-1} x_i , \sum^{i} x_j \right) \tag{7}$$

and has length x_i.

At the midpoint of sampling period i the computer determines the water stage and calculates q_i, $\hat{c}_i$, and x_i. The associated sampling interval I_i is formed on the sampling axis, and the list of random numbers is checked to see if any of them fall within I_i. If no random numbers lie within interval I_i, by far the most frequent case, the computer stores the cumulative sum of x_i and waits until the midpoint of the next sampling period. In cases where one or more random numbers does lie in the interval (as in interval i in Figure 1), the computer activates the pumping sampler to collect one sample. In addition to storing the cumulative sum of x_i, as is done for all sampling periods, the computer stores the specific q_i, x_i, and r_i, the number of random numbers found in the interval. These values are required for the estimators.

Because the random numbers are selected uniformly on the interval (0, Y^*], the probability of selecting sampling period i is proportional to its associated x_i. X is the cumulative sum of x_i at the end of period N. Therefore the probability p_i of the ith sampling period being included in the sample is given by

$$p_i = x_i/X \quad (8)$$

The pumped samples are analyzed in the lab at the end of the monitoring period to determine the values of c_i. The data stored in the computer are then used to calculate the values of y_i and to estimate the total suspended sediment yield and its variance.

Estimating Total Suspended Sediment Yield

The estimator for total suspended sediment yield is an average of individual estimates of the total. These individual estimates are dependent on the probabilities of selection. If the total suspended sediment yield for a sampling period is divided by its probability of selection, the result is an unbiased estimate of the total yield. Therefore the quantity $u_i = y_i/p_i$ is an unbiased estimate of total, or

$$E(u_i) = Y \quad (9)$$

By averaging the ratios u_i over the complete sample we get an unbiased estimate of the total which has lower variance than an SRS estimate provided x_i and y_i have the relationship stated earlier. That is, if $\hat{Y}$ is the SALT estimate of Y, then

$$\hat{Y} = \frac{1}{n}\sum^{N} r_i \frac{y_i}{p_i} = \frac{1}{n}\sum^{N} r_i u_i \quad (10)$$

where r_i is the number of random values contained in the ith interval, and the sample size n is given by

$$n = \sum^{N} r_i \quad (11)$$

The sample size will equal or exceed the number of discrete concentration samples collected. Equality will occur only when there are no intervals containing more than one random point.

The sum in (10) is over all N elements in the population, but there are always exactly n nonzero ratios in the sum. For all periods not sampled, r_i= 0. For each period sampled, r_i equals the number of random value's contained in the associated sampling interval, so that the coefficient r_i effectively repeats the ith ratio that many times in the sum.

The quantity n is a random variable; it depends on the distribution and density of random numbers on the sampling axis and on the pattern of the discharge hydrograph during the monitored period. There are two major effects of this fact: one is to complicate the setting of sample size and the other is to slightly increase the variance of the estimate of the total.

An unbiased estimate $S^2(\hat{Y})$ of the variance of $\hat{Y}$ is given by

$$S^2(\hat{Y}) = \frac{1}{n(n-1)}\sum^{N} r_i (u_i - \hat{Y})^2 \quad (12)$$

where the sum consists of n terms similar to (10). For development of (10) and (12), see *Norick* [1969].

During low discharges, empirically determined rating curves may give estimates, x_i, that differ greatly from the corresponding yi. Therefore under these conditions (10) and (12) perform poorly due to the ratio u; = y;/pi being highly variable. One solution to this problem is to divide the sampling periods into two classes (or strata): one where x_i is "small" and the other where it is "large." The SALT scheme is applied only to the class having large x_i.

If the boundary between the classes is sufficiently low, the class with small x_i can be sampled with a simpler scheme, such as SRS. A rule that has worked well for establishing the class boundary is to require that values of x_i in the SALT stratum be at least 1. The respective estimates of totals and variances for both strata are then added to obtain overall estimates for the monitored period.

COMPARISON OF SALT AND FDSRC

An early but still widely used nonstatistical technique for estimating suspended sediment yield is the flow duration sediment rating curve (FDSRC) method [*Campbell and Bauder,* 1940; *Miller*, 1951; *Vanoni*, 1975; *Walling,* 1977*b*]. The SALT technique and three versions of the FDSRC method will be compared using a simulated "complete" sediment record based on field data collected in the North Fork of Casper Creek near Fort Bragg, California.

The North Fork is part of a long-term watershed study where continuous water discharge and discrete suspended sediment concentration data (both depth integrated and pumped point samples) have been collected since 1962. During hydrologic years (HY) 1978-1980 the pumped concentration data were collected at equal intervals of estimated water discharge and were augmented by fewer depth integrated samples collected largely during storm periods. The suspended sediment rating curve developed from the 541 rating pairs collected during HY 1978-1980 were used with the hydrograph to simulate the sediment concentration for each 30-min period (unit) throughout HY 1980; this rating curve was used only for the simulation. Water discharge was determined at the midpoint of each period, sediment concentration was estimated using the rating curve, and this quantity was then added to a random normal value with zero mean and standard deviation equal to the standard error of estimate of the rating equation. Finally, measured water discharge, simulated sediment concentration, and simulated sediment yield values for all of the 17,568 periods in HY 1980 were stored in a computer file where they were available for sampling.

This data set does not give the actual 30-min suspended sediment concentrations in the North Fork of Casper Creek during HY 1980. It can be expected to follow the major fluctuations, however, and it mimics the variation seen in actual records. This simulation forms a "feasible" record, which we define as the true record for the the purpose of comparing sampling methods. The true sediment yield, Y = 206.0 metric tons, found by summing the simulated 30-min yields, can then be compared to the sample estimates made from the same record.

Fifty SALT samples were collected from the simulated record. The SALT scheme used an SRS stratum composed of units with estimated 30-min yields (i.e., x_i) of 1 kg or less and a SALT stratum having estimated yields greater than 1 kg. Although most of the units are in the SRS stratum, their flows and concentrations are very low; so only 6 values were used to estimate the suspended sediment yield in the SRS stratum. The expected sample size was 39 in the SALT stratum. Because HY 1980 rating curve data would not be available when sampling for 1980 began, the sediment rating curve developed from HY 1979 data was used to estimate the values of $\hat{c}_i$ for the simulated SALT sampling runs. These data were used with the HY 1980 q_i values to calculate x_i required by the SALT algorithm.

Fifty data sets of 45 values each were also chosen from the simulated record to estimate the total yield with the FDSRC technique. The data collection program was designed to simulate regular sampling combined with increased emphasis during storm periods. The regular sampling was simulated by selecting a random starting period in the first 2 weeks of the HY and then sampling every 2 weeks thereafter. This used 26 of the 45 values alloted, leaving 19 to be collected during high-flow periods. Four high-flow classes were established, and the 19 remaining values were randomly selected by requiring 4 to come from the highest class and 5 each from the other 3. A 1980 sediment rating curve was established for each of the 50 FDSRC data sets by using linear regression on the logarithms of the 45 sampled discharge and concentration values.

Fifty FDSRC estimates of total suspended sediment yield were then obtained by applying these rating curves to 154 flow duration classes (1 ft^3/s) classes were used up to the HY 1980 peak flow of 154 ft^3/s formed from the complete set of streamflow data. Log concentrations were estimated for each class using midclass discharges in the rating curves. The log concentrations were detransformed (i.e., exp (log concentration) was formed) at this point to estimate concentration. The concentration was then multiplied by the discharge, the number of periods flowing at that rate, and a unit-adjusting constant to estimate class contribution to suspended sediment yield.

The mean of the 50 SALT estimates underestimated the "actual" yield by less than 1%, whereas the mean of the 50 FDSRC estimates underestimated the yield by nearly 51% (Table 1, Figure 2). This is similar to the magnitude of the majority of underestimates found by *Walling and Webb* [1981].

A source of bias in the FDSRC estimates comes from the method used to estimate concentration. Logarithmic transformations were done on the concentration and discharge data to better satisfy the regression assumptions. The reverse transformations used to estimate concentration, however, produce a negative bias [*Miller,* 1984]. The inverse transformation of regressions performed on logarithmically transformed concentration data estimates the median concentration rather than the mean.

There are several adjustment factors that can be applied to correct for this source of bias. One such factor relies on the assumption that the regression errors on the logarithmic scale are distributed normally. If s^2 is the estimate of variance about the regression line, then an approximate correction factor is given by exp (s^2/2). The results of making this correction on the same 50 FDSRC estimates are also given in Table 1 and Figure 2. These estimates range from 200 to 1968 metric tons with a mean estimate 83.6% above the actual value. The bias is now positive and the mean estimate was moved farther from the true total, implying that the assumption of normality of the regression errors is not valid. Also, the standard deviation of the estimates was increased 31-fold.

TABLE 1. Statistics From Monte Carlo Samplings Using Four Methods for Estimating Total Suspended Sediment Yield

Source of Estimate	Mean of Simulated Yields, metric tons	Standard Deviation of Simulated Yields, metric tons	Mean as Percent of Actual Yield	Estimated Bias, Metric Tons	Estimated Bias, Percent
SALT	204.7	11.0	99.4	-1.3	-0.6
FDSRC(1) (no bias correction)	101.8	10.8	49.4	-104.2	-50.6
FDSRC(2) (lognormal bias correction)	377.9	335.6	183.4	171.9	83.4
FDSRC(3) (smearing bias correction)	241.6	51.0	117.3	35.6	17.3
Actual record	206.0	…	…	…	…

One method used above is SALT, and the other three methods are variations of the FDSRC method. The samplings were taken from a simulated complete record based on data from the North Fork of Caspar Creek for hydrologic year 1980.

A nonparametric bias correction factor is available for situations where the normality assumption is not tenable [*Duan,* 1983]. If $\hat{e}_i$ denotes the *i*th of *n* regression residuals, the bias can be corrected by the factor $1/n\ \Sigma \exp(\hat{e}_i)$. Multiplying the inverse transformation estimate of the concentration by this factor gives the "smearing" estimate of the mean, which was also done for the same 50 FDSRC estimates (Table 1 and Figure 2). The smearing estimates range from 177 to 435 metric tons with a mean of about 242. This method of bias correction also turned the estimation bias positive, but it is only about 17% above the true value. The standard deviation is about 4.7 times that of the uncorrected FDSRC estimates.

The rote application of these two bias correcting techniques is evidently a poor remedy. Because each regression was also performed mechanically, it is possible that other sources of bias derive from outliers or misspecification of the regression models. For a given investigation, these problems could be given adequate attention, but it is not clear what general course to follow for correcting bias to take into Account the idiosyncracies of any given case. Also, even if a bias correction technique was found, it would be a factor of about two for these data, and this would approximately double the standard deviation of the estimates as well as the mean.

The standard deviations were calculated directly from the 50 sample estimates of total suspended sediment yield given by the four methods. Even if a proper bias correction factor was known for the FDSRC method the standard deviation of the estimate cannot be calculated for the individual case. The SALT approach, conversely, provides an estimate of the variance of the total yield with each total estimated. No bias correction factor is required when estimating the probabilities for the SALT method because it would be in both the numerator and the denominator of p_i and therefore would cancel out. Finally, the relationship of the variance to sample size is well defined with SALT sampling (the variance is proportional to $1/n$), a fact that is used to estimate the required sample size. This relationship is obscure with the FDSRC (and any nonstatistical) method and evidently depends on the particular rating curve and the range and distribution of rating discharges.

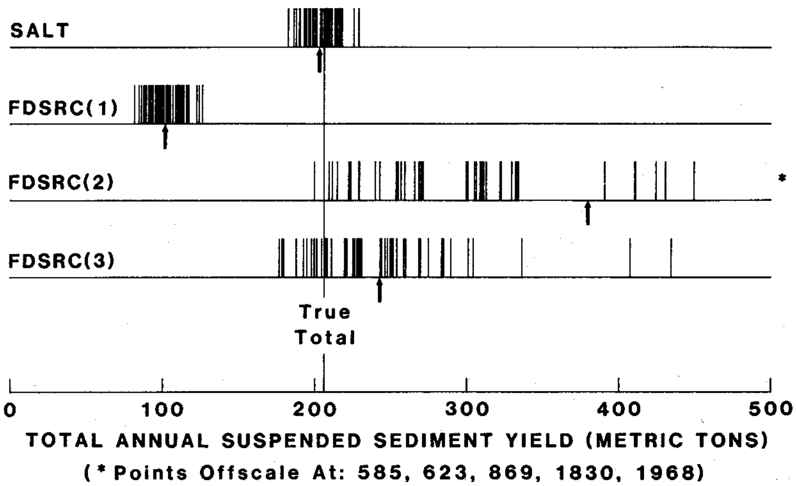

Fig. 2. Distributions of estimates of total annual suspended sediment yield made by four methods. The estimates are based on Monte Carlo sampling of a simulated complete record based on data collected in the North Fork of Caspar Creek, northern California, hydrologic year 1980. The SALT method is a type of variable probability sampling. The FDSRC methods rely on concentration estimates from regressions on log transformations of rating curve data. FDSRC(1) has no bias correction for these estimates, FDSRC(2) corrects for bias using the assumption of normal regression errors, and FDSRC(3) employs a nonparametric bias correction. The true total is the sum of the complete record. The arrows indicate the means of the estimates for the four methods.

Using 154 flow duration classes virtually ensures that the errors came from the rating data rather than from a dearth of classes. The particular values selected to estimate the rating function appear to be critical. Further manipulations of the rating data and specification of the form of the rating function may improve the estimate, but there is no clear recommended procedure that will ensure estimates with known properties for all situations. Also, the rating curve example benefitted from including more high flows than are available in typical sets of rating data. This occurred because data were taken from the complete record and did not have to be selected on a real time basis.

ESTABLISHING SAMPLE SIZE

Establishing sample size is often difficult because it depends on knowing population parameters that the sampling program is undertaken to estimate. To set SALT sample size, the coefficient of variation of the population of ratios u; = y;/p; is needed. Denote the coefficient of variation by

$$C_v = \frac{[Var(u_i)]^{1/2}}{E(u^i)} = \frac{[Var(u_i)]^{1/2}}{Y} \tag{13}$$

Recall that $S^2(\hat{Y})$ is an estimate of the variance of $\hat{Y}$, and $\hat{Y}$ is the mean of the sampled ratios u;, so that $nS^2(\hat{Y})$ is an estimate of the variance of the ratios. Then $\hat{C}_v$ will estimate C_v, where

$$\hat{C}_v = \frac{[nS^2(\hat{Y})]^{1/2}}{\hat{Y}} \tag{14}$$

Because neither the variance or expected value is known, the suspended sediment rating data (i.e., water discharge-suspended sediment concentration pairs) used to calculate $\hat{C}_v$ (and to guide the SALT sampling process) will have to be collected prior to the monitoring period being planned. The data could have come from an earlier sampling program, a pilot study intended solely to tentatively estimate SALT sampling parameters, or from an actual SALT sampling program (e.g., if refining a SALT scheme). As a last resort, data from nearby watersheds with similar hydrological characteristics can be used for tentative estimates of the parameters if no data are available from the stream to be monitored.

From whatever source, suppose there are n' distinct measured values of sediment concentration to estimate C_v The meaning of c_i is now slightly altered to be the measured concentration for the ith of these n' values. Similarly, let $\hat{c}_i$ be the concentration predicted by the rating curve formed from these data at the same water discharge at which c_i was obtained. Now substitute (1), (4), (10), and (12) in (14), note that all r_i are 1, and let n' take the place of n to get

$$\hat{C}_v = \left\{ \frac{n'}{n'-1} \left[\frac{n' \sum^{n'} \left(\frac{c_i}{\hat{c}_i}\right)^2}{\left(\sum^{n'} \frac{c_i}{\hat{c}_i}\right)^2} - 1 \right] \right\}^{1/2} \tag{15}$$

In addition to estimating C_v, the user must specify the performance required of the estimator of the total yield by stating a probability with which an estimate can deviate by a specified amount from the true total. A problem in setting sample size for the SALT method results from n being determined by chance. This problem is solved by generating sufficient random numbers on the sampling axis to give a stated probability of obtaining n large enough for the estimates to per form as desired.

Let m be the minimum sample size required to give specified performance. Suppose that we want the estimate of total suspended sediment yield to be within a proportion, h, above or below the true yield with probability 1 - a. That is, we require

$$P(Y - hY \le \hat{Y} \le Y + hY) = 1 - a \tag{16}$$

Recall that $E(u_i) = Y$, and assume that all u_i are distributed normally with variance Var (u_i). Because $\hat{Y}$ is the mean of, in this case, m values of u_i, $\hat{Y}$ is also distributed normally with the same mean, Y, but with variance Var $(u_i)/m$. Subtract the mean from each expression in the inequality in (16), and divide by the standard deviation of $\hat{Y}$ to obtain an upper cut-off point on the standard normal distribution equal to $hY/[\text{Var}(u_i/m]^{1/2}$. Because C_v is a factor in this expression, its value can be estimated by substituting $\hat{C}_v$ and the result equated to t (the upper 1 - a/2 cut-off point on the t distribution having m - 1 degrees of freedom) to obtain

$$t = \frac{hm^{1/2}}{\hat{C}_v} \tag{17}$$

Finally, solve for m to get

$$m = \left(\frac{t\hat{C}_v}{h}\right)^2 \tag{18}$$

Because m is needed to find t in the tables, an initial value will have to be guessed and the formula evaluated iteratively until the calculated value of m and that used to obtain t are equal.

Lastly, the quantity of random numbers, n^*, is selected to give probability, b, of getting an actual sample size n at least as large as the minimum sample size m. Each selection of a random number from the sampling axis of length Y^* is a Bernoulli trial with probability $p^* = X/Y^*$ that the number will contribute to sample size, that is, that it will lie in the interval (0, X]. Therefore the distribution of n is binomial with n^* trials, each with probability p^*. If we require that

$$P(n \geq m) = b \tag{19}$$

we need to choose m so that

$$B(m - 1; n^*, p^*) = 1 - b \tag{20}$$

where $B(m - 1; n^*, p^*)$ denotes the cumulative binomial probability of getting m - 1 or fewer "successes" out of n^* trials, each having probability p^*. Equation (20) is difficult to solve analytically and tables are not widely available for n^* as large as that needed for sediment sampling. A normal approxi mation will suffice. Suppose $Q\{Z\}$ denotes the probability to the left of the Zth cut-off point on the standard normal distribution. Then an approximate equation for the probability of obtaining j successes out of J Bernoulli trials, each with probability p, is given by

$$B(j; J, p) \cong Q\{[(4j + 3)(1 - p)]^{1/2} - [(4J - 4j - 1)p]^{1/2}\} \tag{21}$$
$$0.05 \leq B(j; J, p) \leq 0.93$$

This form is more accurate than the usual normal approximation to the binomial, especially when p is not close to 1/2 [*Patel and Read*, 1982]. Let Z be the 1 - b cut-off point of the standard normal distribution. Equating Z and the argument of Q in (21), substituting the binomial arguments from (20) into (21), and solving for n^* gives

$$n^* = \frac{1}{4}\left\{\frac{1}{p^*}\left[\left((1-p^*)4m-1\right)^{\frac{1}{2}} - Z\right]^2 + (4m-3)\right\} \tag{22}$$

Not until the end of the monitoring period will X be known so that p^* ($=X/Y^*$) can be calculated. An estimate of p^* is required before sampling, however, to use (22). If the preliminary estimate of the suspended sediment yield Y' is a good estimate of X, $p^* \cong Y'/Y^* = Y'/(WY') = 1/W$, so the reciprocal of the factor of safety can be used as a crude estimate of p^*.

Suppose that rating data are available which give $\hat{C}_v = 0.63$ and that an estimate of total suspended sediment yield for a period to be monitored is required to be within 20% (i.e., h = 0.20) of the true value with a probability of 0.90 (i.e., a = 0.10). If we guess that m = 20, then t = 1.729 for 19 degrees of freedom and a probability of 0.90. By using (18), m *is* estimated to be 30. For 29 degrees of freedom t = 1.699, and another evaluation of (18) gives m = 29. Because further iteration does not change the estimate, a sample of 29 observations is used.

Suppose the sample size of at least 29 is to be attained with a probability b = 0.95. From tables of the normal distribution, the cut-off point for 1 - b = 0.05 is Z = -1.645. If W is 10, p^* is estimated by 0.1, and evaluation of (22) indicates that n^* = 377 random points are to be selected along the sampling axis. This will give a probability of 0.95 of having a sample size of 29 or greater, which ensures that the estimate will be within 20% of the true value with a probability of 0.90.

SUMMARY

The usual data collected to monitor total suspended sediment load consist of continuous streamflow records and occasional discrete samples of sediment concentrations. Methods used to estimate total suspended yield, such as the flow duration sediment rating curve technique, are generally nonstatistical in that the properties of the estimators are not relat ed to a sampling process based on probability. Higher flows should receive a disproportionately large share of sediment sampling effort, but no clear direction has existed to provide either the size of the sample or its distribution over differing flow conditions. The high cost of collecting and analyzing concentration samples is incentive to use efficient sampling programs to define when to sample and how many samples to collect to obtain estimators with specified properties.

Using pumping samplers, small battery-powered computers, and stage-sensing devices at gaging stations offers an opportunity to employ probability sampling to monitor sediment and estimate suspended sediment yield more efficiently. The selection at list time technique for sampling sediment with probability proportional to the magnitude of estimated sediment transport uses a sediment rating function to calculate an auxiliary variable that directs the sampling process. The auxiliary variable is an estimate of the suspended sediment yield during a short time period (sample unit) and must be calcu lated for every unit in the population. Sampling efficiency is improved because the probability of taking a sample is proportional to the estimated contribution of that unit to the total yield. Sample size is set to obtain estimates with desired performance. This sampling scheme gives unbiased estimates of suspended sediment yield and its variance and requires fewer field measurements than commonly used techniques. It automatically emphasizes concentration sampling at higher flow levels by using presently available technology.

When Monte Carlo sampling methods were applied to real data with known yield, the FDSRC process underestimated the true total by 51%, while the SALT scheme underestimated the true total by less than 1%. Two techniques applied to remove the negative bias from the FDSRC method were not successful. Both techniques changed the negative bias to positive (one overestimated by 83% and the other by 17%) and greatly increased the standard deviation.

The SALT method provides an estimate of variance with each estimate of total suspended sediment yield. The variance estimates of the three FDSRC methods, however, are an artifact of the Monte Carlo process and would not be available for an individual investigation. The SALT method therefore provides an estimate of the error present in estimating total suspended sediment yield and a process for rational selection of sample size; the FDSRC methods provide neither.

NOTATION

- a probability with which Y is to be greater than proportion h above or below Y.

b probability with .which $n \geq m$.
$B(x; n, p)$ binomial probability of having x or fewer successes out of n Bernoulli trials of probability p.
c_i measured suspended sediment concentration for ith time period, mg/L.
$\hat{c}_i$ estimated suspended sediment concentration for ith time period, mg/L.
C_v coefficient of variation for the total estimated in SALT sampling.
$\hat{C}_v$ estimate of coefficient of variation for the total estimated in SALT sampling.
f sediment rating function.
h proportion of Y within which $\hat{Y}$ is to lie with probability a.
I_i interval on the Y^* axis for sampling period i.
K constant to convert product of water discharge, suspended sediment concentration, and sampling period duration to desired units of sediment yield.
m minimum sample size.
n number of units (including repeats) in sample.
n^* number of random points selected on sampling interval (Y^*) axis.
N number of units (time periods) in the population.
p_i probability of selecting ith unit for the sample ($p_i = x_i/X$).
p^* probability of a random point being sampled ($p^* = X/Y^*$).
q_i average water discharge for ith time period, m^3/s.
$Q\{Z\}$ probability of a standard normal random variable being less than the cut-off point Z.
r_i number of random points in the ith sampling interval.
$S^2(\hat{Y})$ estimate of sample variance of estimate of population total in period monitored with SALT sampling.
t $a/2$ cut-off point on the t distribution with m - 1 degrees of freedom.
Δt time duration of sampling intervals.
u_i ratio of measured suspended sediment yield for the ith period to its probability of selection ($u_i = y_i/p_i$).
W factor to inflate Y' to form sampling interval axis.
x_i auxiliary variable (estimated suspended sediment yield) for ith time period, kg.
X auxiliary variable total ($X = \Sigma^N x_i$), kg.
y_i primary variable (measured yield of suspended sediment) for ith time period (known only for those time periods sampled), kg.
Y true population total (suspended sediment yield, kg.
Y' preliminary estimate of population total, kg
Y^* length of sampling axis for SALT sampling, kg.
$\hat{Y}$ estimate of population total (suspended sediment yield), kg.
Z cut-off point on the standard normal distribution.

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